Agricultural Productivity and Structural Change: A Falsifiable Approach to Explain the Structural Break in Relative Price of Manufacturing

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Preliminary

Abstract

In this paper, we examine the role of sectoral productivity in explaining the process of structural change and relative sectoral prices in an economy. We set up a simple two sector general equilibrium model to show that an improvement in agricultural productivity relocates labour away from this sector under certain conditions. Contrary to the conventional wisdom that relative sectoral prices are a mirror image of relative sectoral productivity, we show the possibility of an 'U' shaped relationship between these two variables and also found a structural break in the relationship between them from the US data during the period 1920-1965. We estimate our model parameters using the simulated method of moments after constructing a suitable structural model to the historical data of the US economy for the last two centuries (1820-2013). For most plausible set of parameter values, our model could largely replicate the farm versus non-farm relative price movement and also the agricultural labour share of the US economic history.

***JEL* Classification:** F43, O11, O41. **Key Words:** Agricultural productivity, Structural transformation, Manufacturing productivity, Terms of Trade, Open Economy, Welfare.

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1 Introduction

The role of agricultural productivity on the process of structural transformation of an economy has been one of the major issues of discussion in development economics. The broad consensus is that an improvement in agricultural productivity relocates labour away from agricultural sector and thereby facilitates the process of structural change. With a constant subsistence consumption of food, it requires less labour to produce the same amount of food as agricultural productivity improves. In addition to this, low income elasticity of demand for agricultural good imply that increased income associated with productivity improvement is mostly spent on industrial goods. Therefore, a non-homothetic preference between agricultural and industrial goods makes a strong case for a positive linkage between labor saving technical change in agriculture and the movement of labour out of this sector. These explanations are already prevalent in the literature and particularly relevant in a closed-economy context.[1]

Even though the classical works (e.g., Ragnar [1953], Schultz [1953], Rostow [1960]) argued that an improvement in agricultural productivity is essential for industrialization, the available historical data on industrialization at the country level tells a different story. Economic historians opine that an improvement in agricultural productivity raises the wage rate making labor costly to be hired by industry.[2] This scarcity of cheap labor prohibits local industry to flourish. Historically, Belgium and Switzerland were not much productive in agriculture compared to Netherlands. However the spectacular growth in industrial sector happened first in the former two countries and subsequently occurred in Netherlands (Mokyr [2000]). This negative link between productivity improvement in agriculture and the process of industrialization is cited by the economists as a particular case of the Law of Comparative Advantage (Mokyr [1977]). In this alternative scenario, growth of agricultural productivity makes price of the agricultural goods low which boosts demand for agricultural goods. Therefore, the manufacturing sector does not grow.

In this paper, we asked the following two questions: (a) Is productivity improvement in agriculture essential for the process of industrialization? (b) What is the relationship between agricultural productivity and the relative price (or, terms-of-trade) of manufac-

[1] See for example, Murphy, Shleifer and Vishny [1989] Matsuyama [1992], Kongsamut, Rebelo and Xie [2001], Gollin, Parente and Rogerson [2002].

[2] Mokyr [1977], Field [1978], Wright [1979]

turing? Following the literature, we name it as a 'Push' channel when an improvement in agricultural productivity pushes labour out of this sector. The 'Pull' channel is defined analogously when an improvement in manufacturing productivity pulls labour away from agricultural sector.

In our effort to answer question (a), we show that a productivity improvement in agriculture will push labour out of this sector in countries with a large share of subsistence employment in agriculture (the 'poor' countries). This is a robust result in our formulation. However, for those countries where only a small fraction of population is working in agriculture (the 'rich' countries), a productivity improvement in agriculture may or may not release labour from this sector. The result here depends upon the value of the elasticity of substitution parameter between agriculture and manufacturing goods in the preferences (Table 1). When the elasticity of substitution is less than unity, the employment share in agriculture will unambiguously decrease due to a faster productivity improvement in agriculture relative to the manufacturing sector. This is famously referred to as *'Baumol's cost disease'* in the literature (following, Baumol [1967]) where the stagnant sector will attract labour from progressive sectors of an economy despite rise in production cost and prices of the stagnant sector relative to others. We provide a complete characterization of the effects of relative sectoral productivity changes in our paper.

To answer question (b), we derive a non-monotonic relationship between productivity improvement in agriculture and the relative price of manufacturing. This result goes contrary to the conventional wisdom that relative prices are a mirror image of relative productivity. An improvement in agricultural productivity, as a direct effect, tends to increase the terms of trade for the manufacturing (the "so called" Prebisch-Singer hypothesis).[3] However, as the manufacturing sector grows bigger in its size (in terms of its employment share), this produces an indirect negative effect on its terms of trade. For a country with comparatively lower per capita income, this indirect effect dominates the direct effect. As a consequence, relative price of manufacturing declines with an improvement in agricultural productivity. As the country becomes richer, after a critical level of per-capita income, the direct effect dominates over the indirect one and the relative price of manufacturing rises with any further improvement in agricultural productivity. This

[3] Matsuyama [1992, footnote 8, pp-324] illustrates conditions under which an exogenous growth in agricultural productivity, makes the terms of trade for agriculture deteriorate continuously.

generates a 'U' shaped relationship between agricultural productivity and relative price of manufacturing.

In the empirical part, we construct the two most important variables of our study from the historical US data; namely, the relative price and relative Total Factor Productivity (TFP) between agricultural and manufacturing sector.[4] To our best of our knowledge, the literature, so far, did not model the above two variables separately. For example, Alvarez-Cuadrado and Poschke [2011] considered relative price as the reciprocal of relative TFP with a one-to-one mapping. The characterization of push channel and pull channel as defined in Alvarez-Cuadrado and Poschke [2011] is, as a matter of fact, follows from a non-falsifiable Popper [2005] theory.[5] Moreover, we demonstrated from the US data that a one-to-one correspondence between relative price and relative TFP are not true. Figure 3 illustrates that two different relative prices for the same relative TFP is a common occurrence in the data.

The theoretical results of our model provide an *identification strategy* to calibrate the model parameters in case of historical US data. We quantified our empirical observation with a structural model that explains relative sectoral price and agricultural labour share not only using the relative sectoral productivity but also by the absolute level of productivity in the agricultural sector. We divided the data into two parts to incorporate our observation of structural break: the former part consists of the years, 1820-1919 and the latter part post-1965. The estimates of the structural model parameters are hugely different in latter part of the data compared to their former counterparts.

Contribution to literature

Our results contribute to the literature in multiple ways. **First**, as regards the question, whether agricultural productivity improvement 'pushes' labour out of this sector or 'pulls' labour into this sector, we are able to provide a complete characterisation of the respective conditions. We argue that when agricultural and manufacturing goods are complements in the consumer's preference, a rise in agricultural productivity will always 'push' labour

[4] We will interchangeably use the term 'manufacturing' and 'industrial' sector to mean the non-farm sector in this paper.

[5] We have explained this in Section 6.2.

out of this sector.[6] In contrast, substitutability between these two goods in the preference means that productivity improvement in agriculture may 'pull' labour into this sector when subsistence employment in this sector is negligible. Otherwise, it will 'push' labour out of agricultural sector even with substitutability in the preference. (Table 1)

The result that agricultural productivity improvement may or may not lead to industrialization is consistent with the recent literature. A recent study by Bustos, Caprettini and Ponticelli [2016] showed mixed empirical evidences on the effects of agricultural productivity on industrial development. They studied the recent widespread adoption of new agricultural technologies in Brazil. When agricultural productivity growth came in the form of adoption of genetically engineered soybean seeds (which they posited as a 'labour-augmenting' technological change), it led to an employment growth in the industrial sector. However, in case of adoption of second-harvest maize (which was hypothesised as a 'land-augmenting' technological change), agricultural productivity growth led to a reduction in industrial employment. Their study suggests that the effect of productivity improvement in agriculture depends on the factor-bias of technical change and no uniform view exists in the empirical literature.

In their empirical studies, Foster and Rosenzweig [2004, 2007] argued that in the context of rural Indian economy, agriculturally more productive regions were not accompanied by an expansion of rural industry. In fact, industrial diversity were present in those areas where agriculture was less productive. In their own words:

> *"Our results are striking and, to our minds, unequivocal. Growth in income from the nonfarm sector in rural India over the last 30 years has been substantial, and the primary source of this growth, the expansion of rural industry, is not predicated on expansion of local agricultural productivity. Indeed, as would be anticipated by a model in which rural industry producing tradable goods seeks out low-wage areas, factory growth was largest in those areas that did not benefit from enhancement of local agricultural productivity growth over the study period."*(Foster and Rosenzweig [2004, pp-541])

[6] Our results are consistent with Herrendorf, Rogerson and Valentinyi [2013, section-B, pp-16] who showed that, for lower elasticity of substitution, the prediction of their model fits the US data well based on consumption value added. In fact, a specification close to a Leontief utility function provides a good fit to the data in their work.

Second, our theoretical prediction of a 'U' shaped relationship between agricultural productivity and relative price of manufacturing goods matches well with the available historical data from the developed countries.[7] In the empirical section (Section 5) of our paper, we provided historical data on the relative sectoral price (of manufacturing with respect to agriculture)[8] in the US economy over the last two centuries (1820–2013). A clear pattern of a U-shaped movement of the relative sectoral price emerges over time.

More significantly, a structural break of the US economy is observed in contrasting the 1820-1919 data to post-1965 data: we observe a negative relationship between agricultural labour share and the relative sectoral productivity (Figure 6) during 1820-1919 and a positive relationship between these two variables during post-1965 era. Also, during 1820-1919 the relative sectoral price is more elastic with respect to relative sectoral productivity compared to post-1965 data (Figure 3). We estimated parameters of our model using two different methodologies. Our model generates the structural break observed in the data and also matches the evolution of agricultural labour share over time quite well (Figures 7 and 8).

Third, we extend our model to a small open economy case where the relative price of manufacturing is fixed by the world market condition. In the small open economy set-up, an improvement in agricultural productivity always pushes labour out of this sector thereby facilitating industrialization. This explains a positive link between agricultural productivity and the size of the manufacturing sector in open economy. Our result goes contrary to Matsuyama [1992] where there is a clear negative link between agricultural productivity and the size of the manufacturing sector in the open economy case. That a rise in agricultural productivity can lead to higher manufacturing employment in an open economic context, has been recently validated by Bustos et al. [2016] using the Brazilian economic data.

[7] See Alvarez-Cuadrado and Poschke [2011] who provided historical data for 12 developed countries with present employment share in agriculture at less than ten percent. In most of these cases, relative price of manufacturing evolved in a 'U' shaped pattern over time.

[8] In our empirical exercise, we used relative Total Factor Productivity of manufacturing over agriculture as relative sectoral productivity.

1.1 Brief Literature Survey

Our paper adds to the sizable literature that studies the linkages between agricultural productivity and industrial development. The literature started with classical contribution from Ragnar [1953], took-off with the contribution by Matsuyama [1992] who modelled a two-sector economy with non-homothetic preferences and showed that under closed economy assumption, an improvement in agricultural productivity leads to an increase in the size of the industrial sector. Larger size of the industrial sector then engenders a higher rate of growth for the economy (due to the presence of learning-by-doing kind of technological progress). In our model, however, the size of the industrial sector may or may not grow due to productivity improvements in agriculture.[9] Additionally, under the assumption of a small open economy, we showed that agricultural productivity improvement may facilitate industrialization by drawing labour from the agriculture sector to the manufacturing one, unlike Matsuyama [1992].

The question posed in Matsuyama [1992] paper inspired many follow up papers. We quote Duranton [1998, pp-220] who wrote that: *"Traditional theories of development economics tend strongly to emphasise the positive role of agricultural productivity. Their message is that any shift from agriculture towards industry must be preceded by a significant surge in the productivity of the agricultural sector. This conclusion is widely acknowledged and nearly accepted as an axiom, as noted by Matsuyama [1992] in his review of the literature on this subject."*

Duranton [1998], on the contrary, showed the non-universality of the linkage between agricultural productivity and industrial development. The researcher modelled an economy with agriculture and manufacturing (which can be of traditional or modern type) incorporating transport cost. Under open economy assumption of his model, an improvement in agricultural productivity may reinforce agricultural specialization instead of growth of manufacturing sector. Multiple equilibria can also occur and equilibrium selection becomes a major issue in open economy case of his model. However, under the closed economy, industrialization follows from the progress in the agricultural sector. Like Duranton [1998], we did not allow trade in intermediate inputs in our model; nevertheless, our results in the closed economy case are different from his.

[9] Matsuyama [1992] used a *CES* felicity function in appendix B (pp-332) of his paper. In this paper, we considered a similar felicity function.

Eswaran and Kotwal [2002] used a small open economy to examine the linkage between agricultural productivity and industrialization within the presence of a service sector. The motivating questions in their paper are strikingly similar to ours. Their paper opens with the following question - *"Is high agricultural output (per capita) a help or a hindrance to industrialization?"* They showed that at a high enough level of agricultural productivity, a further increase in agricultural productivity leads to industrialization. Agricultural productivity growth can therefore facilitate industrialization even in a small open economy. Though our model is primarily based on closed economy assumption, we showed similar results hold in an open economy even without any service sector as assumed in their paper.

Most closely related to our paper is Alvarez-Cuadrado and Poschke [2011] where the authors studied labor relocation out of agriculture due to technological improvement in both agricultural and non-agricultural sector. An improvement in agricultural productivity pushes labor out of agriculture and into the industry (they call it 'push factor'). With an improvement in productivity in the non-agriculture sector, labor is attracted toward this sector away from agriculture (they call it 'pull factor'). They provided a simple two sector model under closed economy to study the relative strength of these two effects on structural change. Their major focus is on the movement of the relative price of the manufacturing goods which they relate to its historical trend (as observed in the time-series data). They show that the relative price of the manufacturing good always increases due to an equal proportionate increase in sectoral productivity.

Our result differs from them. We show that, an equal proportionate increase in the productivity of both sectors lead to an unambiguous decline in the relative price of manufacturing. This is exactly opposite to their result and has further implications to the empirical part of their paper. For example, our result indicate that a negative trend in relative manufacturing price (as evident in the historical time series data on relative price of manufacturing in the US during the year 1840-1920) need not be an outcome of a dominant 'labor-pull' effect. Rather, it may very well be the case that the sectoral productivity has grown up at similar rates. In appendix B (pp-154) of their paper, Alvarez-Cuadrado and Poschke [2011] extended the model using a *CES* preferences. They show that their basic results survive under this generalization given that the elasticity of substitution parameter is not too large. In our model, even when elasticity of substitution is higher than unity, structural change can take place. In our model, terms of trade of manufacturing comes out to be an imperfect identification of the relative productivity change. For example, with an improvement in agricultural productivity only, terms of trade may first go down and then go up. Thus, relative price fails to be a mirror image of relative productivity in our model economy. We will run a counterfactual to Alvarez-Cuadrado and Poschke [2011] in Section 6.2.

In another recent paper, Gollin and Rogerson [2014] considered the issue of transport costs and subsistence agriculture in a closed economic system and found out that different channels that can lead to greater allocation of labor to the agricultural sector. One of these channels is the lower agricultural productivity. They showed that improvement in agricultural productivity, though have overall negative impact on the share of labor engaged in agriculture (a result similar to Gollin, Parente and Rogerson [2002]), may actually increase the labour share in agriculture in the nearby-city region. However, welfare impact is large in their calibration exercise following an improvement in agricultural productivity. In our work, part of the workforce in agriculture is engaged in producing subsistence food. Any productivity improvement would unambiguously reduce the size of this subsistence production. Yet overall size of agriculture can be higher due to larger demand effect owing to declining terms-of-trade in agriculture induced by higher productivity in this sector.

Our paper is complementary to the broad literature on structural transformation and the role of agriculture in it.[10] Specifically, the question of whether agricultural productivity can expand the industrial sector has been recently revisited by papers such as Henderson, Mark and Adam [2013], Henderson, Adam and Uwe [2017], Jedwab [2013], and Gollin, Jedwab and Vollrath [2016]. The empirical results in these papers are mostly mixed and depend on the specific set-up of the model. As like them, we also highlight the importance of agriculture on the process of structural transformation.

Rest of our paper is organized as follows. Section (2) lays down our basic model. Comparative static results are provided in section (3). Section (4) introduces trade into the model. In section (5), we look at the quantitative implications of our model to the historical data of US economy on productivity and prices. Finally section (6) concludes

[10] See, for example, Gollin, Parente and Rogerson [2002], Gollin, Parente and Rogerson [2007], Ngai and Pissarides [2007], Kongsamut, Rebelo and Xie [2001], Herrendorf and Valentinyi [2012], Herrendorf, Rogerson and Valentinyi [2013], Caselli [2005], Herrendorf, Rogerson and Ãkos Valentinyi [2014], etc.

the paper.

2 The Economic Environment

2.1 Preliminaries

In the economy, there are a continuum of agents of measure L each endowed with one unit of labor. A unit of labour receives a competitive wage denoted by w. The economy has two sectors - agriculture and manufacturing. Labour is freely mobile across these two sectors equalizing the wage rate between them. The market structure is perfectly competitive in the agriculture sector. Moreover, perfect competition describes the market structure of the final goods production in the manufacturing sector while intermediate goods production in manufacturing sector is characterised by monopolistic competition.

A representative agent's utility maximisation problem is given by

$$\underset{\{c_A,c_i\}}{\text{Max}} \quad U = \left[b(c_A-\gamma)^\theta + c_M^\theta\right]^{\frac{1}{\theta}}; \quad c_A > \gamma > 0, \quad \theta \in (-\infty, 1) \tag{1}$$

$$\text{subject to} \qquad p_A c_A + p_M c_M = w, \tag{2}$$

where c_A and c_M are the consumption levels of the agricultural and manufacturing goods, respectively. The subsistence level of consumption of the agricultural good is denoted by the parameter γ. The relative bias for consumption of the agricultural good as opposed to the manufacturing good is denoted by the parameter b (> 0). Wage income, denoted by w, is the only source of income for the agents in our model. We define $\epsilon \equiv \frac{1}{1-\theta} \in (0, \infty)$ as the elasticity of substitution between the two goods in the agent's preference.

We normalise the price of the agricultural good to unity: $p_A \equiv 1$. With this normalisation, p_M represents the relative price of the manufacturing good, which is alternatively called the manufacturing terms-of-trade. The utility maximisation problem yields the following first order condition:

$$c_A = \gamma + c_M\left(b \cdot p_M\right)^\epsilon.$$

Multiplying both sides of the above condition by L results in the following equation:

$$L \cdot c_A = L \cdot \gamma + L \cdot c_M \big(b \cdot p_M\big)^{\epsilon}.$$

Let us denote the aggregate production of the agricultural good by x_A and that of manufacturing good by x_M. Then, market clearing conditions equating demand to supply (i.e., $L \cdot c_A = x_A$ and $L \cdot c_M = x_M$) are translated into the following equation:

$$x_A = L\gamma + x_M \big(b \cdot p_M\big)^{\epsilon}. \tag{3}$$

The term $L\gamma$ in eq. (3) represents the aggregate subsistence consumption of food in the economy.

2.2 Production

Labour is the only factor of production in the economy. We assume that agricultural goods are produced using the following linear production function,

$$x_A = A \cdot L_A. \tag{4}$$

L_A is the amount of labour required to produce x_A amount of agricultural goods and A is a measure of agricultural productivity. With aggregate subsistence consumption given by $L\gamma$, it requires $\frac{L\gamma}{A}$ workers to be engaged in the subsistence production within the agricultural sector. We assume that the agriculture sector is sufficiently productive so that the following inequality always holds true:

$$L > \frac{L\gamma}{A} \text{ or, } A > \gamma. \tag{5}$$

Note that in an economy of size L, only $\frac{L\gamma}{A}$ workers are engaged in the subsistence sector. Here we define the term $L_S \equiv \frac{L\gamma}{A}$ as the subsistence level of employment in the economy. The fraction of subsistence employment in the economy is free from any scale effect as $\frac{L_S}{L} = \frac{\gamma}{A}$. In an economy where A is very low, a large fraction of the work force are engaged in subsistence production while the opposite is true in an economy with higher productivity in agriculture.[11]

Since the market structure in the agricultural sector is perfect competition, the wage is given by the marginal productivity. Our production technology in eq.(4) along with the normalisation of agricultural price to unity implies that the wage rate is determined by the agricultural productivity parameter, i.e.,

$$w = A. \tag{6}$$

With free mobility of workers across sectors, the same wage rate applies to everywhere and this becomes the per-capita income in this economy.

The production of the manufacturing goods requires n number of differentiated intermediate inputs. These inputs are aggregated using a CES technology to produce the final manufacturing good as given in the the production function below:

$$x_M = M\Big(\sum_{i=1}^{n} z_i^{\delta}\Big)^{\frac{1}{\delta}}; \quad \delta \in (0,1). \tag{7}$$

Here z_i is the amount of i^{th} intermediate input used in the production of the final good. Define the elasticity of substitution between any two intermediate inputs as $\sigma \equiv \frac{1}{1-\delta} > 1$. The parameter M captures the productivity in the manufacturing sector. For higher values of M, the same amount of intermediate inputs produce more of the final goods.

We assume that $\sigma \geq \epsilon$, i.e., the elasticity of substitution among different varieties of intermediate inputs in the production of final good is larger than the elasticity of substitution between agricultural and manufacturing good in the consumption.[12]

The production of the final good is done under perfect competition. Let π_M denote the profit and p_i be the price per unit of i^{th} intermediate input. Then profit maximization in

[11]See Gollin and Rogerson [2014] for empirical evidence on association between high employment in subsistence agriculture and low agricultural productivity in sub-Saharan African economies.

[12]If we had allowed the industrial goods in the utility function to be an aggregate consumption index of n different final good varieties, then the assumption $\sigma \geq \epsilon$ would simply mean that industrial goods are more substitutable among themselves in consumption that they are as a whole with the agricultural good. As an example, it makes sense to assume that two varieties of car are more substitutable to each other than car as a whole with rice or wheat. None of our results would change with such a modification by allowing intermediate inputs to be regarded as consumption varieties in the utility function. We, however, choose to work with intermediate input varieties and assume that $\sigma \geq \epsilon$.

the final manufacturing good sector can be given by

$$\max_{z_i \geq 0} \pi_M = p_M x_M - \sum_{i=1}^{n} p_i z_i; \quad \text{subject to } eq. \text{ (7)}.$$

From solving this profit maximisation, we obtain for the following demand functions for the intermediate inputs,

$$z_i = \frac{p_i^{-\sigma} \left(\sum_{j=1}^{n} z_j p_j \right)}{\sum_{j=1}^{n} p_j^{1-\sigma}}; \quad \forall i \in [1, n]. \tag{8}$$

The above demand function along with the condition that profit must be zero (i.e., $\pi_M = 0$) under perfect competition ensure that the price of the final manufacturing good becomes

$$p_M = \frac{1}{M} \left(\sum_{j=1}^{n} p_j^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{9}$$

Note that an exogenous improvement in manufacturing productivity parameter, M, leads to a decrease in the price of the final manufacturing good. Also an increase in the number of intermediate inputs, denoted by n, leads to an efficiency gain in the manufacturing sector. This gain is purely due to the specialization effect - as the number of inputs grow, each being more specialized leading to an overall efficiency gain in the production process.

2.3 Intermediate inputs

Each variety of the intermediate inputs is being produced by a monopoly producer. The production of variety i needs both fixed cost, denoted by α, as well as marginal cost, denoted by β. The production function of i^{th} intermediate good is given by

$$L_i = \alpha + \beta z_i,$$

where L_i is the amount of labour hired by i^{th} producer. The producer of the i^{th} variety intermediate inputs faces the following profit maximization problem;

$$\max_{p_i} \pi_{z_i} = p_i z_i - (\alpha + \beta z_i) w.$$

subject to the demand function for her product, z_i, given in eq. (8). While maximizing profit, each producer takes p_M as given even though it depends on the choice of p_i (see eq. (9)). The profit maximization problem along with free entry in the intermediate input production sector gives the following solutions of price and quantity.

$$p_i = \frac{A\beta}{\delta}, \quad \text{[using } w = A\text{, by eq.(6)];}$$
$$z_i = \frac{\alpha\delta}{(1-\delta)\beta}; \quad \forall i = 1, 2, ..., n.$$

Aggregate employment in the intermediate goods sector which is equivalent to manufacturing employment is denoted by L_M and is given by

$$L_M = \sum_{i=1}^{n} L_i = \frac{n\alpha}{1-\delta}. \tag{10}$$

With these solutions, aggregate production of the final manufacturing good, in eq. (7), and the price index, in eq. (9), takes the following form,

$$x_M = M\frac{\alpha\delta}{(1-\delta)\beta}n^{\frac{1}{\delta}}; \tag{11}$$
$$p_M = \frac{A\beta}{M\delta}n^{-\left(\frac{1}{\delta}-1\right)};. \tag{12}$$

Note that, labors are not directly employed in the final manufacturing good production (see eq. (7)). However indirectly they are employed through intermediate goods production. To see this, let us re-write eq. (11) using (10) as follows

$$x_M = L_M\frac{M\delta}{\beta}n^{\frac{1}{\delta}-1}.$$

Then the marginal productivity of labors in the final manufacturing goods sector is given by

$$\frac{\partial x_M}{\partial L_M} = \frac{M\delta}{\beta}n^{\frac{1}{\delta}-1}. \tag{13}$$

Multiplying the above expression with p_M (as given in in eq.(12)) we derive the value marginal productivity in the manufacturing sector. This must be equal to the wage rate, A. This verification guarantees that wage rate is equalized across sectors.

Note that the marginal productivity expression in eq.(13) is increasing in n. From eq.(10), see that n rises monotonically with the size of the manufacturing sector. Thus, larger size of the manufacturing sector is associated with higher marginal productivity of its workers.[13] Therefore, given a particular wage, an increase in size of the manufacturing sector lowers the (relative) price of manufacturing.[14] .

Finally the labor market clearing condition can be given by the following.

$$L_A + L_M = L.$$

Using eqs. (4) and (10), above equation can be re-written as

$$x_A = AL - \frac{An\alpha}{1-\delta}. \tag{14}$$

Next, using equations (11), (12), (14) and the definitions of ϵ and σ, we can re-write eq. (3) as follows

$$L - n\alpha\sigma = \frac{L\gamma}{A} + n^{\frac{\sigma-\epsilon}{\sigma-1}} \left(\frac{A}{M}\right)^{\epsilon-1} \alpha\beta^{\epsilon-1} b^{\epsilon} \sigma^{\epsilon} (\sigma-1)^{1-\epsilon}. \tag{15}$$

This equation solves for n uniquely, which was established by the following lemma:

Lemma 1 (Existence and uniqueness of equilibrium). *There exists a unique equilibrium solution of n from equation* (15).

Proof: Define the left hand side of eq.(15) as

$$LHS_{(15)} \equiv L - n\alpha\sigma;$$

and the right hand side as

$$RHS_{(15)} \equiv \frac{L\gamma}{A} + n^{\frac{\sigma-\epsilon}{\sigma-1}} \left(\frac{A}{M}\right)^{\epsilon-1} \alpha\beta^{\epsilon-1} b^{\epsilon} \sigma^{\epsilon} (\sigma-1)^{1-\epsilon}.$$

Clearly, $LHS_{(15)}$ is a monotonically decreasing function of n. The expression $RHS_{(15)}$ is an increasing function of n since $\sigma \geq max\{1, \epsilon\}$. $RHS_{(15)}$ is a concave function of n for

[13] This is in contrast with neo-classical production function where, due to diminishing marginal productivity assumption, larger size of a sector reduces the marginal productivity of its workers.

[14] To see this, note that $w = p_M \times$ (marginal productivity). Since w is fixed at A, an increase in marginal productivity must reduce p_M.

the case $\epsilon > 1$ (see figure 1) and convex function of n for $\epsilon < 1$ (see figure 2). It becomes a linear function of n for the case $\epsilon = 1$. The expression $RHS_{(15)}$ takes the value $\frac{L\gamma}{A}$ at $n = 0$ and it approaches infinity as n approaches infinity. Note that $\frac{L\gamma}{A} < L$ by equation (5). Then, we must have $LHS_{(15)} = RHS_{(15)}$ at some $n = n^*$. This, proves the *existence* of a solution of n. This solution must also be *unique* as the difference $(LHS_{(15)} - RHS_{(15)})$ is a monotonically decreasing function of n. □

3 Comparative statics

To set the stage for comparative statics exercises, let us define the percentage change of any variable, namely x, as $\hat{x} = \frac{dx}{x}$. Using eq.(15), taking logarithm in both sides, performing total derivatives and rearranging expressions, we get the following equation:

$$\hat{n}\left[\frac{n\alpha\sigma}{L - n\alpha\sigma - \frac{L\gamma}{A}} + \frac{\sigma - \epsilon}{\sigma - 1}\right] = -\hat{A}\left[\epsilon - 1 - \frac{\frac{L\gamma}{A}}{L - n\alpha\sigma - \frac{L\gamma}{A}}\right] + (\epsilon - 1)\hat{M}. \tag{16}$$

Similarly, using eq.(12), we get the following expression:

$$\hat{p_M} = \hat{A} - \hat{M} - \left(\frac{1}{\delta} - 1\right)\hat{n}. \tag{17}$$

The exogenous variables are A and M and the endogenous variables are n and p_M. Note that, due to the introduction of endogenous product variety, part of the productivity growth is endogenous to the manufacturing sector. However, by productivity growth we have referred to the growth in exogenous productivity parameter only.

In the literature, there are two competing explanations for structural change to take place: technological and utility-based. In the technological explanation, structural change takes place due to differing rates of sectoral factor productivity growth (e.g., Ngai and Pissarides [2007]). In utility-based approach, different income elasticities can generate structural change even when sectoral productivities grow at equal rates (e.g., Gollin, Parente and Rogerson [2002], Alvarez-Cuadrado and Poschke [2011]). In our model, both these explanations can co-exist. For technological explanations, set $\gamma = 0$ in eq.(16) and see that $\hat{n} \neq 0$ as long as $\hat{A} \neq \hat{M}$. Similarly, for utility-based approach, set $\gamma \neq 0$ and see that $\hat{n} \neq 0$ even when $\hat{A} = \hat{M}$. We will now analyse the effects of changes in productivity on sectoral relocation of labour and relative prices.

3.1 Agricultural productivity changes

With improvement in agricultural productivity only, we put $\hat{A} > 0$ and $\hat{M} = 0$ in equations (16) and (17). This gives us, after some calculations, the following set of equations:

$$\hat{n}\left[\frac{L_M}{L_A - \frac{L\gamma}{A}} + \frac{\sigma - \epsilon}{\sigma - 1}\right] = -\hat{A}\left[\epsilon - 1 - \frac{\frac{L\gamma}{A}}{L_A - \frac{L\gamma}{A}}\right]. \tag{18}$$

and

$$\hat{p_M} = \hat{A}\left[\frac{L\left(1 - \frac{\sigma}{\sigma-1}\frac{\gamma}{A}\right)}{L_M + \frac{\sigma-\epsilon}{\sigma-1}\left(L_A - \frac{L\gamma}{A}\right)}\right] \tag{19}$$

Here we have used the fact that $n\alpha\sigma = L_M$ and $L - L_M = L_A$. As it is evident from eq.(18) that the value of ϵ is crucial in determining the effect of changes of A on n. Similarly, it can be seen from eq.(19) that the values of σ and $\frac{\gamma}{A}$ become crucial in determining the effects of changes of A on P_M.

We first analyse the case where agriculture and manufacturing goods are gross complement in preferences (i.e., $\epsilon \in (0, 1)$). From eq.(18), it is clear that the sign of $\hat{n}$ and $\hat{A}$ are the same when $\epsilon \in (0, 1)$. This implies that an improvement in the agricultural productivity must raise the share of employment of manufacturing. This last result follows as number of intermediate product varieties is directly related to the manufacturing employment share (see equation (10)). To put it differently, an improvement in the agricultural productivity lowers the employment share in agriculture.

To see the effects of change in A on the manufacturing terms of trade we use eq.(19). Here the sign of $\hat{p_M}$ crucially depends on the sign of the term $\left(1 - \frac{\sigma}{\sigma-1}\frac{\gamma}{A}\right)$. It can be seen that for an improvement in agricultural productivity, manufacturing terms of trade will fall if $\frac{\gamma}{A} > 1 - \frac{1}{\sigma}$. We already interpreted the fraction $\frac{\gamma}{A}$ as the share of the subsistence employment in aggregate employment. Thus, when subsistence employment share is relatively high (as it happens in least developed economies), agricultural productivity improvement tends to depress the manufacturing terms of trade. With gradual improvement in A, as soon as the condition $\frac{\gamma}{A} < 1 - \frac{1}{\sigma}$ satisfies, the manufacturing terms of trade starts improving with an increase in A. This gives rise to an inverted 'U' shaped relationship between relative price of manufacturing and agricultural productivity.

To see the effects of A on n in the gross substitute case (i.e., $\epsilon > 1$), we re-write eq.(18) as below:

$$\hat{n} = -\hat{A}(\sigma - 1)\left[\frac{(\epsilon - 1)L_A - \epsilon L_S}{(\sigma - 1)L_M + (\sigma - \epsilon)(L_A - L_S)}\right]. \quad (20)$$

Note that we have previously defined L_S as the subsistence level of employment in the economy. Using the above equation, and for $\hat{A} > 0$, we get

$$\hat{n} < (=) > 0 \quad \text{iff} \quad \frac{L_S}{L_A} < (=) > \frac{\epsilon - 1}{\epsilon}.$$

With gross substitutability in preferences ($\epsilon > 1$), an improvement in agricultural productivity leads to a decrease in manufacturing employment ($\hat{n} < 0$), as long as the share of the subsistence sector in total agricultural employment is less than a critical level given by $\frac{\epsilon-1}{\epsilon}$.

For many underdeveloped countries, the size of subsistence employment within agriculture is very high. In many African countries, almost 50% of the entire work force is engaged into subsistence agricultural employment. In Uganda for example, as many as 43% of all working persons were engaged in subsistence agriculture according to reports of Uganda Bureau of Statistics in the year 2014.[15] The condition $L_S > \frac{\epsilon-1}{\epsilon}L_A$ is likely to be satisfied for the least developed countries. Thus, improvement in agricultural productivity should reduce the size of the agricultural sector in poor countries even accounting for a large degree of substitutability in preferences.

We now summarize these results in the following proposition:

Proposition 1. *Assume that preferences are given as in equation* (1). *An improvement in agricultural productivity leads to*
(i) a decline in the share of employment in the agriculture if there is gross complementarity in the preferences.
(ii) a decline in the share of employment in the agriculture if there is gross substitutability in the preferences and $\frac{L_S}{L_A} > \frac{\epsilon-1}{\epsilon}$.
(iii) a decline in the manufacturing terms of trade for lower values of A *and then raises it.*

[15]See pp-21 of Uganda Bureau of Statistics, http://www.ubos.org/onlinefiles/uploads/ubos/statistical_abstracts/Statistical_Abstract_2014.pdf. Agricultural employment share is 71% in Uganda. Then the ratio $L_S/L_A = 0.61$ implies that for all $\epsilon \leq 2.56$, $\hat{n} > 0$ as long as $\hat{A} > 0$.

This gives rise to an 'U' shaped curve between manufacturing terms of trade and agricultural productivity irrespective of the value of elasticity of substitution in preferences.

3.2 Manufacturing productivity changes

With change in manufacturing productivity only, we use equation (16) to get

$$\hat{n}\left[\frac{n\alpha\sigma}{L_A - L_S} + \frac{\sigma-\epsilon}{\sigma-1}\right] = (\epsilon-1)\hat{M}; \tag{21}$$

and use equation (17) to get

$$\hat{p_M} = -\hat{M}\left[\frac{L-L_S}{L_M + \frac{\sigma-\epsilon}{\sigma-1}(L_A - L_S)}\right] \tag{22}$$

From eq.(22), as manufacturing productivity improves, terms of trade of manufacturing will always decline. This is irrespective of the value of the elasticity of substitutions. However, from eq.(21), the inter-sectoral relocation of labour now crucially depends on the value of the elasticity of substitution parameter, ϵ. Improvement of manufacturing productivity pulls labour toward manufacturing sector if $\epsilon > 1$ and pushes labour out of manufacturing sector if $\epsilon < 1$. For the case where $\epsilon = 1$ (Cobb-Douglas preferences), manufacturing productivity changes do not affect the inter-sectoral labour reallocation.

We summarize these results in the following proposition:

Proposition 2. *Suppose preferences are given as in equation* (1). *An improvement in manufacturing productivity leads to*
(i) a decline (an increase) in the share of employment in the agricultural sector if $\epsilon > 1$ *(*$\epsilon < 1$*). With* $\epsilon = 1$*, inter-sectoral labour allocation becomes independent of any change in manufacturing productivity.*
(ii) a decline in the manufacturing terms of trade.

The result that relative price of the manufacturing goods decline with productivity improvement in manufacturing seems quite intuitive. Note that, an increase in productivity raises the marginal (as well as average) productivity of labour in the manufacturing sector. With competitive market structure, manufacturing workers are paid according to their value marginal product - i.e., $p_M * MP_L^M = A$, where MP_L^M is the marginal product of labour in the manufacturing sector (see equation (13)) and A is the wage rate. As M increases, it raises the MP_L^M and hence, p_M must fall since wage is constant. When, the size of the manufacturing sector grows with M, the relative price falls at a faster rate both due to the direct negative effect of M on p_M as well as through the indirect negative effect of larger n on p_M. However, when n falls with an improvement in M (with $\epsilon < 1$), this indirect effect pacify the fall in p_M but could not change the direction of it's (downward) movement. Thus, relative price falls (albeit, at a slower rate) with an increase in M in this case with $\epsilon < 1$.

The relationship between size of the manufacturing sector and its productivity crucially depends on the elasticity of substitution parameter (as in proposition 2(i)). When $\epsilon < 1$, goods are complement in the preferences. Here, a decrease in the price of manufacturing goods (following an improvement in manufacturing productivity) lead to an increase in the demand for the agricultural goods. This requires more production of the agricultural good. So, labour move away from industry to join the agricultural sector. In fact, expenditure share of the manufacturing sector (in GDP) also declines when $\epsilon < 1$.[16]

Similarly, one can explain the case with $\epsilon > 1$. Here goods are substitute. With a decrease in p_M (induced by manufacturing productivity improvement), demand for agricultural goods go down. This is because, with substitutability, people move toward relatively cheaper (manufacturing) goods. In the process, labours get released from agriculture and finds their way into manufacturing. On net, size of the manufacturing sector goes up along with its expenditure share in GDP as long as $\epsilon > 1$.

[16]Expenditure share of manufacturing in GDP is given by $\frac{p_M x_M}{AL}$. Here, aggregate consumption expenditure of the manufacturing goods are denoted as $p_M x_M$ and aggregate GDP is AL which is the national income in this model. Using equations (10), (11) and (12), we can express this expenditure share as $\frac{p_M x_M}{AL} = \frac{L_M}{L}$. Similarly, expenditure share of the agricultural sector in national income is given by $\frac{x_A}{AL} = \frac{L_A}{L}$. Thus, sectoral employment shares exactly reproduces sectoral GDP shares. This is broadly in line with the empirical facts about relationship between sectoral employment and expenditure shares (see Herrendorf, Rogerson and Ãkos Valentinyi [2014] for detailed evidence on this).

3.3 Relative productivity changes

To analyse the effect of relative productivity change on employment share and relative prices, we re-write equations (16) and (17) as follows:

$$\hat{n} = (\hat{M} - \hat{A})\left(\frac{\epsilon - 1}{k_1}\right) + \hat{A}\left(\frac{k_2}{k_1}\right); \tag{23}$$

$$\hat{p_M} = -(\hat{M} - \hat{A})\left[1 + \frac{1}{\sigma - 1}\frac{\epsilon - 1}{k_1}\right] - \hat{A}\left(\frac{1}{\sigma - 1}\frac{k_2}{k_1}\right). \tag{24}$$

The expressions of k_1 and k_2 take the following form,

$$k_1 = \frac{L_M}{L_A - L_S} + \frac{\sigma - \epsilon}{\sigma - 1} > 0 \quad \text{and} \quad k_2 = \frac{L_S}{L_A - L_S} > 0.$$

From eq.(23), one can find that when $\epsilon < 1$, faster productivity improvement in agriculture relative to manufacturing, i.e., $\hat{A} > \hat{M} > 0$, will always lower the employment share in agriculture. Similarly, when $\epsilon \geq 1$, faster productivity improvement in manufacturing relative to agriculture, i.e., $\hat{M} > \hat{A} > 0$, will always lower the employment share in agriculture.

From eq.(24), we can easily show that the term $\left(1 + \frac{1}{\sigma-1}\frac{\epsilon-1}{k_1}\right)$ is always positive for all values of $\epsilon \in (0, \infty)$. Then a faster productivity improvement in manufacturing relative to agriculture will always lower the manufacturing terms of trade.

One interesting case is an equal proportionate increase in productivity of agriculture and manufacturing, i.e., $\hat{M} = \hat{A} > 0$. In this case, both the employment share in agriculture and manufacturing terms of trade will unambiguously decrease due to productivity improvement.

We summarize these results in the following proposition:

Proposition 3. *Suppose preferences are given as in equation (1) and both agricultural and manufacturing productivity increases. Employment share in agriculture will unambiguously decrease due to - (i) a faster productivity improvement in agriculture relative to manufacturing when goods are complementary in preferences (i.e., $\epsilon < 1$); (ii) a faster productivity improvement in manufacturing relative to agriculture when goods are substitutable in preferences (i.e., $\epsilon > 1$); (iii) an equal proportionate increase in agricultural and manufacturing productivity.*

Relative price of manufacturing will unambiguously decrease due to - (iv) a faster productivity improvement in manufacturing relative to agriculture; (v) an equal proportionate increase in agricultural and manufacturing productivity. For faster productivity improvement in agriculture relative to manufacturing, movement in relative price of manufacturing is ambiguous.

These results can easily be established using eqs.(23) and (24). Note that, result (i) in proposition 3 establishes that employment share moves from the sector with the higher productivity growth (agriculture) to the sector with lower productivity growth (manufacturing) when $\epsilon < 1$. Exactly opposite happens when $\epsilon > 1$ as shown in result in proposition 3(ii). Then the higher productivity growth sector (manufacturing) attracts employment from the lower productivity growth sector.

These results in proposition 3 are similar to those in Ngai and Pissarides [2007] (prop 2, pp-433) but derived in a much simpler setting. Our result in $\epsilon < 1$ case confirms to the facts of structural change as identified by Baumol, Blackman and Wolff [1985]. With price inelasticity of demand, sectors with lower productivity growth rate attracts employment from elsewhere. This may happen despite the rise in their relative price. This is often referred to as 'Baumol's cost disease'.[17]

4 Trade

4.1 Small open economy

In case of small open economy one could reasonably assume that p_M is given by the world market, making $\hat{p_M} = 0$ in eq.(17) (in the home country). Then resource allocation is determined purely by the expression of p_M in eq. (12) in the home economy, i.e., n is determined. So, to explain structural change in the form of declining labor share in agriculture (i.e., to explain $\hat{n} > 0$), it has to be the case that $\hat{A} - \hat{M} > 0$. Thus in an small open economy, faster productivity growth in agriculture relative to non-agriculture is very much consistent with relocation of labor from agriculture to industry. We are stating this result formally in the next proposition.

[17] Baumol [1967] claimed that the stagnant sector will attract labour from progressive sectors of an economy despite rise in production cost and prices of the stagnant sector relative to others. For more discussion on this issue see Ngai and Pissarides [2007] (footnote-1, pp-430).

Proposition 4. *In a small open economy with given terms of trade of manufacturing, faster productivity growth in agriculture facilitates industrialization by attracting labour from agriculture to industry.*

Proof: Set $\hat{p_M} = 0$ in eq.(17) to get

$$\hat{A} - \hat{M} = \left(\frac{1}{\delta} - 1\right)\hat{n}.$$

Then as long as one we assume that $\hat{A} - \hat{M} > 0$, one immediately gets the result that $\hat{n} > 0$. □

The result stated in proposition 4 is important and stands in contrast to the results in the existing literature such as Matsuyama [1992] and Alvarez-Cuadrado and Poschke [2011]. In these models, faster productivity improvement in agricultural leads to migration of labor from industry to agriculture under small open economy assumption. Higher agricultural productivity means relatively higher price of the manufacturing goods in the domestic economy. So, world prices of the manufacturing goods are comparatively low. This makes the industrial sector relatively unattractive to the workers leading to a migration of people from industry to agriculture.

To see the source of the difference in results between our paper and the papers mentioned earlier, let us look at more closely to one of the fundamental assumptions of these class of models which is wage equalization across sectors. In Matsuyama [1992] and Alvarez-Cuadrado and Poschke [2011], this assumption lead to the following condition:[18]

$$AG'(L_A) = p_M\big(MF'(1 - L_A)\big);$$

where agricultural production function is $Y^A = AG(L_A)$ and manufacturing production function is $Y^M = MF(1 - L_A)$ with the standard neoclassical properties. The left hand side is the value marginal productivity of labor in agriculture and right hand is the same in manufacturing. Then, given p_M, faster productivity improvement in agriculture raises the wage rate there which takes away labor from industry. In our model, the same condition

[18]See eq.(4) in both Matsuyama [1992] and Alvarez-Cuadrado and Poschke [2011] paper.

can be given by (using eq.(13))

$$A = p_M \left(\frac{M\delta}{\beta} n^{\frac{1}{\delta}-1} \right).$$

Here, given the value of p_M, an improvement in $\frac{A}{M}$ must increase n and thereby giving the direction of migration from agriculture to industry. Thus increasing returns in the manufacturing sector is the source of this difference. The result that agricultural productivity growth can lead to industrialization in a small open economy is provided in Eswaran and Kotwal [2002]. However, in their model, the service sector plays a crucial role.

4.2 Two country trade

Instead of small open economy, we now assume that there is a foreign country with productivity level A^* and M^* in agriculture and industry respectively ('*' variable denotes foreign). Assume that all other parameters are the same in both home and foreign and that only final goods are tradable. Initially, the condition is such that the following inequality holds.

$$A > A^*; \; M > M^* \text{and} \; \frac{A}{M} = \frac{A*}{M*}.$$

Thus home is (absolutely) more productive in both agriculture and in manufacturing but there is no comparative productivity advantage for home. As an example, say home is twice more productive in both agriculture and manufacturing so that there are no relative productivity differential. Then eqs.(12) and (15) immediately imply that[19]

$$n > n^* \text{ and } p_M < p_M^*.$$

Thus initial size of the industrial sector is larger in home compared to foreign and home would export manufacturing to the foreign. Here trade pattern is purely determined by the absolute productivity advantage in agriculture. In the next proposition we are formalizing this result.

Proposition 5. *Assume a two country world where only the final goods are tradable. Countries are symmetric in every other respect except that home is absolutely more productive in*

[19]To see this, note that the right hand side of eq.(15) will be lower for higher values of A as long as $\frac{A}{M}$ does not change. This would solve for a higher values of n from the left hand side.

both agriculture and manufacturing compared to foreign (i.e., $A > A^*$; $M > M^*$) and there are no comparative advantage (i.e., $\frac{A}{M} = \frac{A*}{M*}$). Then home should export manufacturing goods and import agricultural goods from foreign.

It is also easy to see, using eqs.(12) and (15), that when home has a comparative (but not absolute) advantage in agriculture (i.e., $\frac{A}{M} > \frac{A*}{M*}$ and $A = A^*$ holds), trade pattern will be such that home should export agricultural good and import manufacturing from the foreign (i.e., $p_M > p_M^*$) when $\epsilon \geq 1$. A productivity disadvantage of home in manufacturing goods indicate that these goods are relatively cheaper in foreign. So importing these goods from foreign seems desirable for the home country. For complementarity, i.e., $\epsilon < 1$, this trade pattern will be exactly reversed. In this case, home should import agricultural good and export manufacturing goods from the foreign (i.e., $p_M < p_M^*$)

The fact that agricultural productivity has a negative effect on manufacturing employment for richer countries is not surprising. Gollin, Parente and Rogerson [2002], and Caselli [2005] show that agricultural productivity is the highest in the richest countries. Their manufacturing employment share is on the declining path. These countries also have comparative advantage in production of agricultural goods. Countries such as the U.S., France, Australia, etc. are the largest exporters of agricultural products. They thus export food and processed agricultural goods to poorer countries. This trading pattern between rich and poor world can be explainable in our model when $\epsilon \geq 1$.

5 Structural Model, Structural Break and Estimation Methodology

5.1 Productivity and Price in the United States

We followed Alvarez-Cuadrado and Poschke [2011] in constructing the historical data series. In particular, we constructed the following three historical series from the United States data: (1) the relative price of the manufacturing sector output with respect to the agricultural sector output using the producer price index data, (2) total factor productivity (TFP) for the agricultural sector and (3) total factor productivity (TFP) for the manufacturing sector. Figure 4 illustrates the evolution of both agricultural sector and manufacturing sector TFPs. The evolution of the relative TFP defined as the manufacturing sector TFP over the agricultural sector TFP, is illustrated in Figure 5 along with relative sectoral price. Initially the relative price of manufacturing declined until about 1940 with some ups and down in between. It showed an upward trend since 1940. Overall, this resembles a U-shaped curve. The historical plot for the agricultural labour share (Figure 7) shows a monotonically declining trend over time – from 73.7% in 1800 to 1.5% in 2013.

We plotted the relative price against the relative TFP in the logarithmic scale in Figure 3. The relative price is not uniquely determined by the relative TFP, i.e. a one-to-one relationship between relative price and relative TFP may be ruled out. Therefore, some other exogenous variable besides the relative TFP may determine the relative sectoral price. Our plot of share of agricultural labour against relative TFP (Figure 6) demonstrates that for pre-1920 data points, the agricultural labour share falls with increase in relative TFP. However, the relationship is reversed for the post-1965 data points. As like earlier, this observation is suggestive of some other exogenous variable besides the relative TFP as determinant of the relative price. We concluded of a structural break in the data during 1920-1965 based on in the relationship between relative TFP and relative sectoral price and also between relative TFP and agricultural labour share. We considered 1945 as the year of the structural break in out model.

5.2 Structural Model and Structural Break

We observed that relative sectoral price and agricultural labour share are non-monotonic functions of relative TFP, which we used to create a structural model. In particular, we used eq. (15), derived in section 2. Using eq.(10) and the full employment condition that $L_M + L_A = 1$, we solved for $n = \frac{1-L_A}{\alpha\sigma}$ and replaced this value in place of n in eq. (15). This exercise enabled us to rewrite this equation in terms of agricultural labour share only:

$$L_A \;=\; \frac{\gamma}{A} + (1-L_A)^{\frac{\sigma-\epsilon}{\sigma-1}} \left(\frac{A}{M}\right)^{\epsilon-1} \alpha^{\frac{\epsilon-1}{\sigma-1}} \sigma^{\frac{\sigma(\epsilon-1)}{\sigma-1}} (\sigma-1)^{1-\epsilon} \beta^{\epsilon-1} b^{\epsilon} \tag{25}$$

L_A can also be solved uniquely from eq.(25) in terms of A and $\frac{A}{M}$, as demonstrated in Section 2. In essence, our model predicts that relative price and the agricultural labour share will depend on the agricultural TFP along with the relative TFP. The logarithmic versions of the above equation is:

$$\log L_A = K_1 + K_2 \log A + K_3 \log \frac{A}{M} + \upsilon_2 \tag{26}$$

In this equation, the underlying variables of relative price, agriculture sector TFP, agricultural labour share and relative sectoral price are non-stationary. The regression described by eq. (26) represents a textbook case of spurious correlation indicated by high values of R^2. The consistency and applicability of the estimates are in doubt. To address these concerns, we used the following estimable equations by differencing eq. (26). Let Δ denote the difference operator that represents change over two successive time points in the data. Therefore, the differenced variables in the above regression represent the short run change in the above variables. We found the differenced variables as stationary in the data, and expect the following regression to be free from spuriousness.

$$\Delta \log L_A = k_1 + k_2 \Delta \log A + k_3 \Delta \log \frac{A}{M} + \upsilon_2 \tag{27}$$

We also noted the structural break between agricultural labout share and relative TFP in the data. This structural break happened in during 1920-1965. We posited 1945 as the year of structural break and estimated two regressions for eq. (27) for pre- and post-1945 data. We report the estimation of the structural model (eq. (27)) at the difference level in Table 3. Our estimates for these two periods are different, which supports our conjecture of structural break.

5.3 Structural Break and Estimation: The Simulated Method of Moments

The key question is, what is the source of this structural break. A definite identification strategy is required to identify this structural break as a consequence of values of parameters of our model. One clear possibility for this observed structural break is evolution of the TFP for these two sectors. The evaluation of the merit of this possibility requires estimation of parameters of our model.

We normalised both L and α to unity. This normalisation is done without any loss of generality as L is a measure of size of agents and α is the fixed cost to a firm. Among the other parameters of our model, we calibrated σ using the industry markup. The markup for industry at 50% dictated our choice of σ at 3 going by Hsieh and Klenow [2009, pp-1414].We carried out a sensitivity analysis by considering another values of σ at 5 (equivalent to a 25% markup for the industry).

In order to tackle the problem of mismatch regarding absolute level of relative price and agricultural labour share, we calibrate the parameters β and γ to match the agricultural labour share and the sectoral relative price to their respective empirically observed levels at 1820. The agricultural labour share was 71.12% in 1820 and the relative price between manufacturing and agricultural sector was, then, 1.594. Given a particular set of values for all the other parameters, we computed the values of our two unknowns, estimates of β and γ, by equating the model-predicted agricultural labour share and relative price to these two numbers, respectively. One may note that the expression for relative price in our model is given by eq. (12). The calibration methodology of all the parameters are tabulated in Table 2.

We used the Method of Simulated Moments [Gourieroux and Monfort, 1996] for estimating two remaining key preference parameters: b and ϵ. Under this method, we computed the equilibrium of our model for a given set of values for the parameters. In particular, we calculated the variables like the relative sectoral price and the agricultural labour share from the model for all the periods. We estimated the structural model as outlined by the regression of eq. (27) using these simulated data from the model. We denote the estimates of the structural model from the data as $\widehat{\Theta}^{Data}$ (tabulated in Table **??**) and their counterparts from the simulated data as $\Theta^{Model}(b, \epsilon)$. We minimised the distance between these two sets of estimates of the structural model with respect to the two preference parameters, b and ϵ. We measured the distance using a L^2 norm (the sum of the squared differences). We considered those parameter values for which this distance is minimised, as our model estimates.[20] Mathematically,

$$(\hat{b}, \hat{\epsilon}) = \arg\min_{\{b,\epsilon\}} \left(\widehat{\Theta}^{Data} - \Theta^{Model}(b, \epsilon)\right)' \cdot \left(\widehat{\Theta}^{Data} - \Theta^{Model}(b, \epsilon)\right) \tag{28}$$

We illustrate the above procedure with the numerical example. We included k_1, k_2, and k_3 as parameters of our structural model, estimated separately using the till- and

[20]Numerous applications exist in estimating parameters using the Method of Simulated Moments as we described here. For a recent such application in labour economics, see Blundell et al. [2016].

post-1945 data. These particular estimates from the data are tabulated in Table 3. We minimised[21] the distance between these six numbers estimated from the data and from the simulated data by changing the parameter values.

While considering different values of the parameter, we imposed that all the parameters must be non-negative. Moreover, we have another assumption imposed in the model (discussed in Section 2): $\epsilon \leq \sigma$, which leaves an upper bound for ϵ. The minimum[22] is obtained for the values of $(b, \epsilon) = (0.7231, 1.8739)$. For this set of values for the parameters, we ran regression (27) using the simulated data from our model and reported the estimates in Table 3.

5.4 Long Run Labour Share Match

Papers[23] in this literature often measure the performance of the model by matching the time-series of agricultural labour share. In this case, the focus is on long run evolution of the agricultural labour share rather than any short run change. To compare the performance of our model to the literature, we minimised the distance between agricultural labour share obtained from the simulated data to their data counterparts. More explicitly, we calculated the difference between agricultural labour share from the data and the simulated data for each period. We minimised the sum of squares of differences by changing the parameter values, as given below:

$$(\hat{b}, \hat{\epsilon}) = \arg\min_{\{b,\epsilon\}} \sum_{1820 \leq t \leq 2013} \left(\widehat{\frac{L_{A,t}}{L_t}}^{Data} - \frac{L_{A,t}}{L_t}^{Model}(b, \epsilon) \right)^2 \tag{29}$$

where $\frac{L_{A,t}}{L_t}^{Data}$ represents the agricultural labour share in the t^{th} period from the data and $\frac{L_{A,t}}{L_t}^{Model}(b, \epsilon)$ represents its counterpart in the simulated data.

The frequency of data is not same across our time period of investigation, 1820-2013. For the initial years, the data are available one in a decade or less, whereas in the latter half of the sample, the data are accessed annually. If we match the statistics from the data to the model based on data availability, the latter half of the sample will have unduly

[21] We have used discrete state space algorithm as well as simplex method to compute the optimal values.
[22] This minimum value for the distance is given by
[23] As for instance, Duarte and Restuccia [2010]

more weight. Therefore, we considered the agricultural labour share as a decadal average for the entire time period of our study. Our estimates are reported, again, in Table 3.

6 Concluding Discussion

Our results are noted in Table 4. In particular, when we used structural model (eq. (27)) to estimate our model, we found $(b, \epsilon) = (0.7231, 1.8739)$ which demonstrates that consumption of agricultural food beyond the subsistence level has an weighage of 72% of the consumption of the industrial good in the consumer's utility function. Moreover, the elasticity of substitution between these two sectoral outputs is more than 1 making them substitutes than compliment. Interestingly, the upper bound of this elasticity (that is σ) is not binding for this estimate which is a sign of robustness of our result.

To examine the power of this model in explaining the structural break observed in the data, we compared Figures 3 and 6 to their model counterparts, Figures 7(a) and 7(b). Evidently our model can generate the structural breaks observed in the data. Since our estimation methodology described in is based on the short run change of agricultural labour share, we used two other moments to evaluate the performance of our model: (1) The agricultural labour share over time and (2) The relative price over time. Figure 7(c) contrasts our model to the data and we observed that our model can indeed generate the long run diminishing agricultural labour share observed in the data. However, a certain gap remains between the agricultural labour share of this model to its data counterpart throughout most of the times. As far as relative price is concerned, our model can generate (Figure 7(d)) the initial falling part and somewhat imperfectly the latter rising part.

The other version of our model estimated using the methodology of eq. 29 works wonderfully well in matching the agricultural labour share (Figure 7(c)). *The match is better than any other paper of our knowledge.* This model also can generate structural breaks (Figures 7(a) and 7(b)) though, admittedly, not too successful to generate the U-shaped curve of relative price over time observed in the data (Figure 7(d)).

6.1 Robustness Checks

We also carried out a sensitivity analysis by calibrating σ for which our estimates are noted in Table 4. Qualitatively, it makes no difference. Even quantitatively, most of the estimates remain almost unchanged if we change σ from 3.000 to 5.000. The power of the new estimates is examined in Figures 9 and 10. An important caveat of this exercise is estimation of ϵ in case of agricultural labour share matching described in eq. 29. The estimate of ϵ is quite close to its upper boundary of the value of σ when σ is kept at 3.000. Once we increased the upper boundary by calibrating σ as 5.000, we observed a change in the estimate of ϵ to 3.545. However, the other conclusions remained same or closely similar.

6.2 Comparison with Alvarez-Cuadrado and Poschke [2011]

We discussed in Section 1.1 regarding the fact that Alvarez-Cuadrado and Poschke [2011] treated relative price as a mirror image of relative productivity. We quote from their paper:

"...decreases in the relative price of manufactures are unambiguously associated with faster technological change in the nonagricultural sector, i.e., they indicate that the labor pull effect dominates. If the relative price rises, the situation is less clear. An equal proportionate increase in the productivity of both sectors induces an increase in the relative price of manufactures, resulting from the low income elasticity of demand for food and the high-income elasticity of demand for manufactures. So only a strong increase in the relative price is an unambiguous sign of stronger growth in agricultural productivity, or "labor push.""

Based on this identification strategy, they concluded that labour push channel dominated for the U.S. economy before World War I and the labour push channel being dominant after World War II. In our model, the value of the elasticity parameter, ϵ, determines that push or pull channel. Since this values is much larger than unity in all versions of the model, we can safely say that "pull" is the dominant channel in our model (see Section 3.2). Therefore, our conclusion coincide with those of Alvarez-Cuadrado and Poschke [2011] before World War I. The large change in relative price after World War II, is interpreted as "push" channel by them unlike our identification of the same continuation of the "pull" channel.

Our model is somewhat deficient to quantitatively match the rising relative price observed after World War II. We ran the following counterfactual to consider the impact of relative price on our conclusion. We considered all parameters of our benchmark calculation except ϵ. For every time point, we supplied relative price from data in eq. (12) to derive the variable n which we infused in eq. (15) and solved for ϵ. Our estimates of the time-series of ϵ shows (Figure 11(a)) that both channels worked in the nineteenth century. And, in the twentieth century, till 1980, it is largely the "push" channel and post-1980, it is the pull channel. Therefore, our conclusion are quite different from theirs.

What is the baggage of this estimation? It seems that the dynamics of agricultural labour share is simply unreal (Figure 11(b)). Also this model cannot do justice to explaining structural break in agricultural labour share (11(c)) as observed in the data. This problem was not present in Alvarez-Cuadrado and Poschke [2011] where the model was generating sectoral relative price as a one-to-one function of sectoral TFP. The falsifiability of our model reveals this problem. Our model, on the other hand, matches relative price and agricultural labour share to a large extent (Figure 7(c); a similar model 10(c) offers better match).

6.3 Conclusion

In this paper we take a fresh look at an old issue in development economics literature which is the relationship between agricultural productivity and industrialization. We show that an improvement in agricultural productivity can pose a challenge toward industrialization by attracting labour away from industrial sector even in a closed economy. This possibility takes place in an economy where agriculture is already much productive, or, where subsistence food production sector is relatively small. However, when agriculture is less productive to begin with (or, the subsistence sector in agriculture is relatively larger), further improvement in its productivity would lead to an expansion of the manufacturing sector. Thus, there is an inverted-U shaped relationship between agricultural productivity and size of the industrial sector.

We also show that an improvement in the productivity of the manufacturing sector always draws in more labour toward this sector. However a proportional improvement in both agricultural and manufacturing productivity such that their ratio does not change, leads to a relocation of labor away from agriculture and toward manufacturing. Perhaps the most interesting result that we get is a U-shaped relationship between the terms of trade in manufacturing and agricultural productivity. This is quite new in the literature and explains well the historical trend of the manufacturing prices relative to agricultural goods along the process of structural change in an economy. Welfare is always positively related to productivity improvement across sectors. These results are shown using a broad class of substitutability in preferences between agriculture and manufacturing goods.

Our modelling structure is simple and the approach is standard in the literature. The main innovation here is to bring in monopolistic competition in the manufacturing sector. This is realistic since industry is naturally characterized by product varieties. We focus only on static set up. Bringing in learning-by-doing driven growth into this framework following Matsuyama (1992) would be difficult here since our preference structure is more general. Yet we think the results in this paper are new in the literature and important in understanding the role of agriculture in the process of development.

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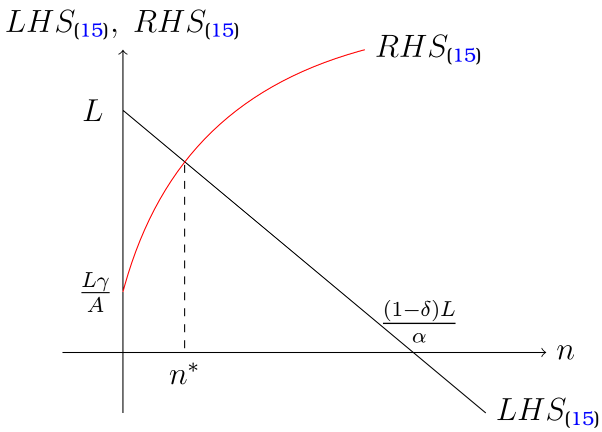


Figure 1: Concave case: $\epsilon > 1$

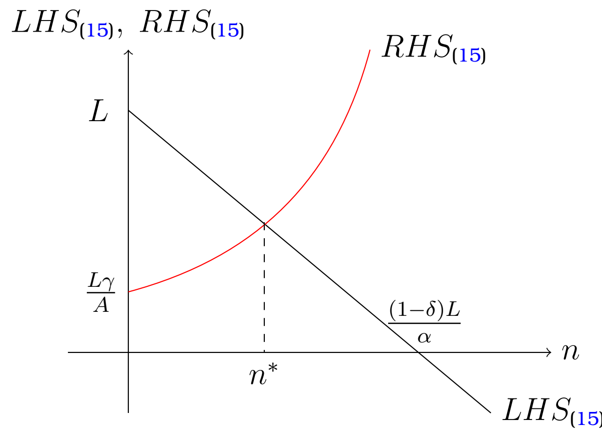


Figure 2: Convex case: $\epsilon < 1$

Figure: Existence and uniqueness of equilibrium.

Preference Parameter	Proportion of agriculture labour employed for subsistence agriculture —>	
	Poor	Rich
Complement	+ve	+ve
Substitute	+ve	-ve

Table 1: $\frac{\partial L_A}{\partial RelativeTFP}$

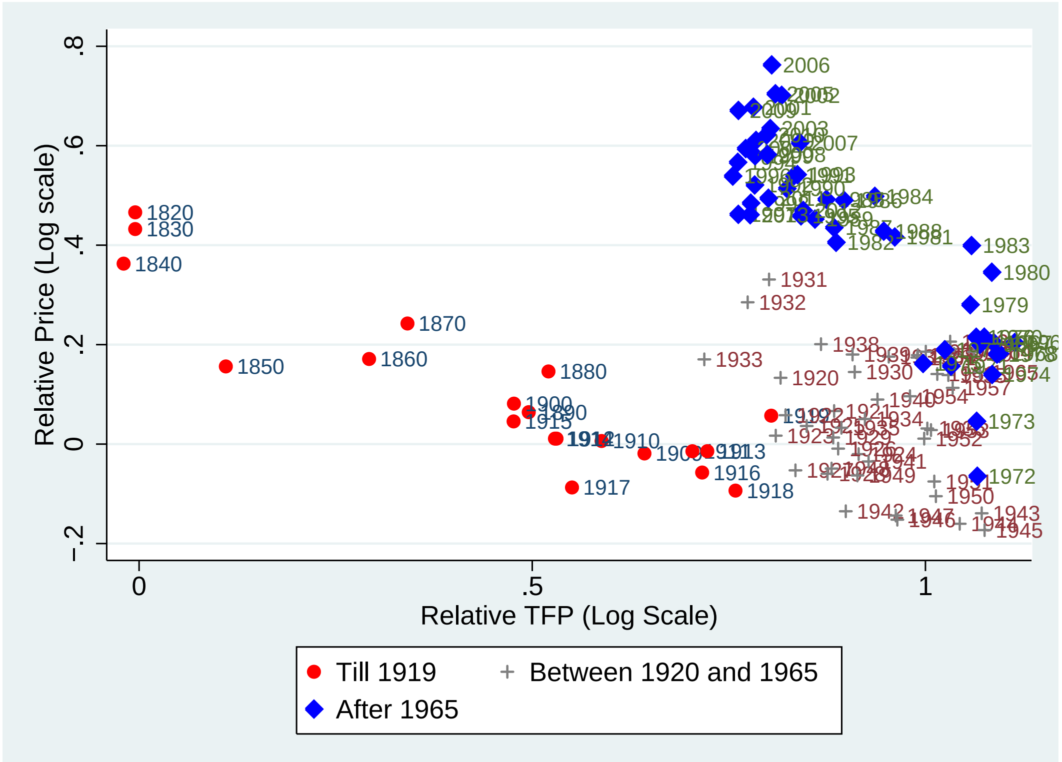


Figure 3: Relative Price plotted against Relative Total Factor Productivity in logarithmic scale. The plot demonstrates that for the same relative TFP multiple relative prices have been observed over time.

match the and the sectoral relative price to their respective empirically observed levels at 1820. The agricultural labour share was 71.12% in 1820 and the between manufacturing and agricultural sector was, then, 1.594.

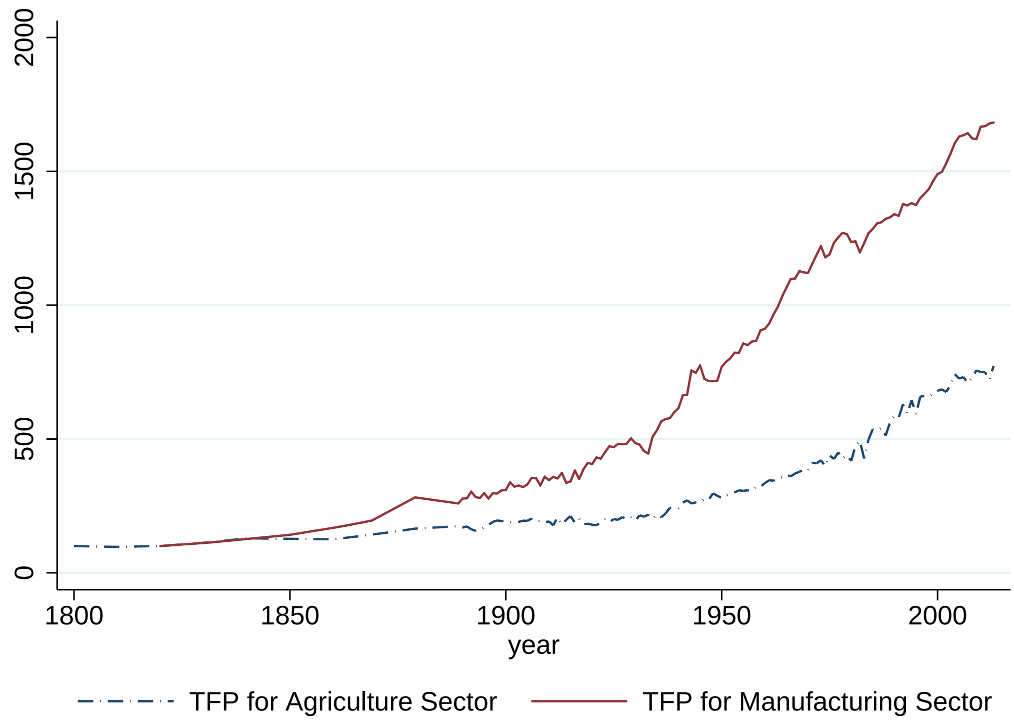


Figure 4: Total Factor Productivity for the agricultural and manufacturing sectors. We have normalised the 1820 TFP of both the sectors to 100.
Source: The farm productivity is gathered various sources: from Gallman [1972] for 1800–1840, from [Craig and Weiss, 2000, Table 3] for 1840–1870, from Kendrick [1961] for 1869–1948, and from the United States Department of Agriculture (USDA) Economic Research Service, Agricultural Productivity Dataset, http://www.ers.usda.gov/Data/AgProductivity/, for 1948–2013. Nonfarm productivity is from Sokoloff [1986] for 1820–1860, from Kendrick [1961] for 1870–1948, and from the Bureau of Labor Statistics (BLS) Multifactor Productivity Trends—Historical SIC Measures 1948–2013, http://www.bls.gov/mfp/historicalsic.htm, for 1948–20013.

Parameter	Variable	Method of Calibration	Value
L	Labour force size	Normalisation	1.000
α	Fixed cost of a firm	Normalisation	1.000
σ	Elasticity of substitution in production	Industry Markup	3.000
β	Variable cost	Agricultural labour share in 1820	Depends
γ	Subsistence agricultural good consumption	Relative sectoral price in 1820	Depends

Table 2: Calibration of Parameters

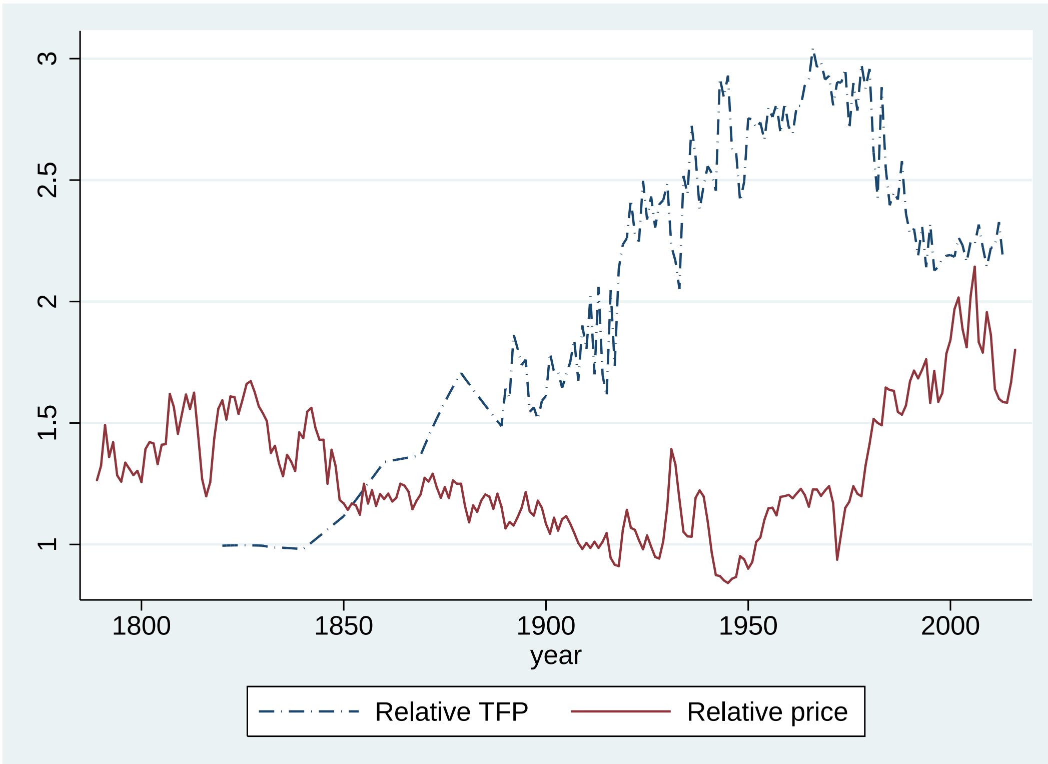


Figure 5: Relative TFP ($\equiv \frac{TFP_M}{TFP_A}$) and Relative Price ($\equiv \frac{p_M}{p_A}$) for agricultural and manufacturing sectors over time: 1820-2013.

Independent Variables	Dependent Variable: $\Delta \log L_A$				
	Data			Model	
	Till 1945	Post 1945	All	Till 1945	Post 1945
Constant	**-0.020**	**-0.030**	-0.027	-0.000	-0.001
	(0.008)	(0.007)	(0.005)	(6.2e-4)	(0.010)
$\Delta \log A$	**-0.309**	**-0.421**	-0.281	-0.452	-1.236
	(0.121)	(0.336)	(0.112)	(0.010)	(0.448)
$\Delta \log \frac{A}{M}$	**-0.277**	**-0.388**	-0.060	-0.305	-0.796
	(0.084)	(0.311)	(0.085)	(0.007)	(0.415)
R^2	0.208	0.024	0.076	0.984	0.143
Number of Observation	45	68	113	45	68

Table 3: Structural Model Estimation at the difference

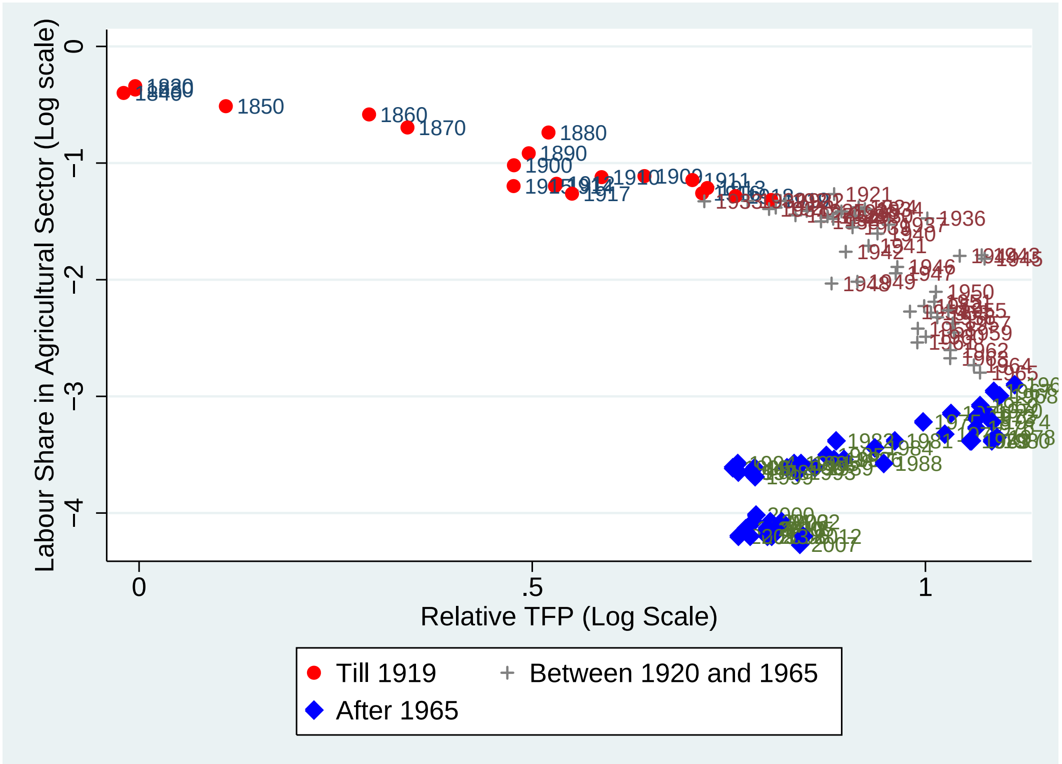
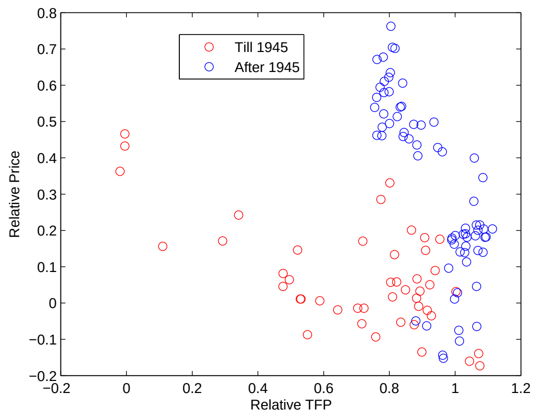
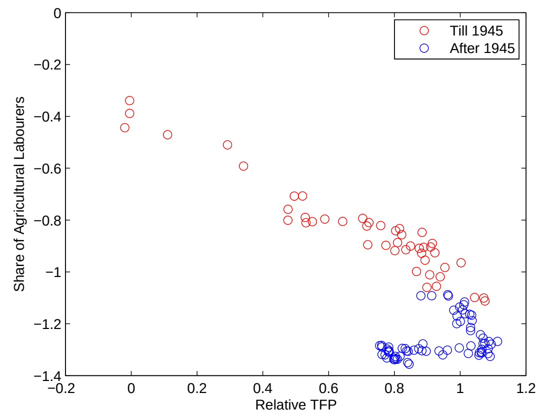


Figure 6: Agricultural Labour Share plotted against Relative Total Factor Productivity in logarithmic scale.The plot demonstrates a negative relationship between the variables during 1820-1919 compared to a positive one during post-1965.

Calibration	Estimation Method	Parameters			
		b	ϵ	β	γ
Benchmark	Structural Model	0.723	1.874	0.328	0.479
Benchmark	Labour Share Match	0.742	2.993	0.328	0.417
$\sigma = 5$	Structural Model	0.672	1.978	0.622	0.507
$\sigma = 5$	Labour Share Match	0.725	3.545	0.622	0.415

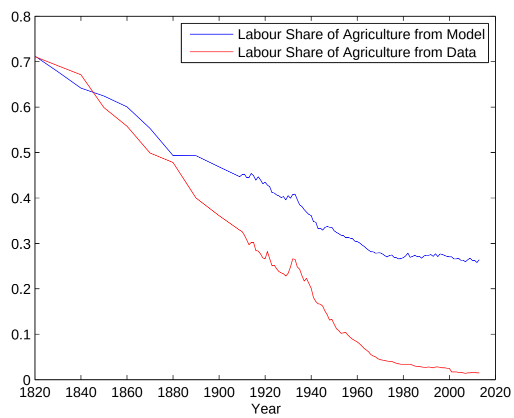
Table 4: Estimated Values for Model Preference Parameters

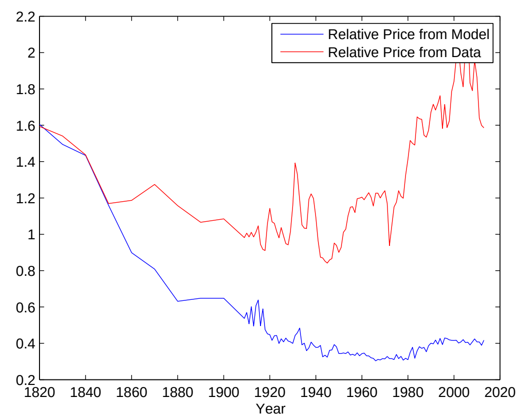




(a) Relative price against Relative TFP (in logarithmic scale): Model contrasts till 1945 to post-1945

(b) Agricultural Labour Share against Relative TFP (in logarithmic scale: Model contrasts till 1945 to post-1945)

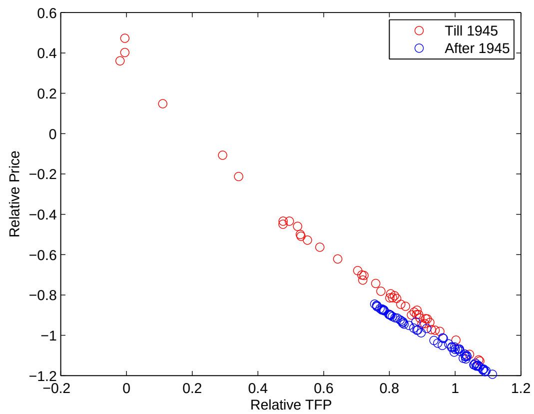




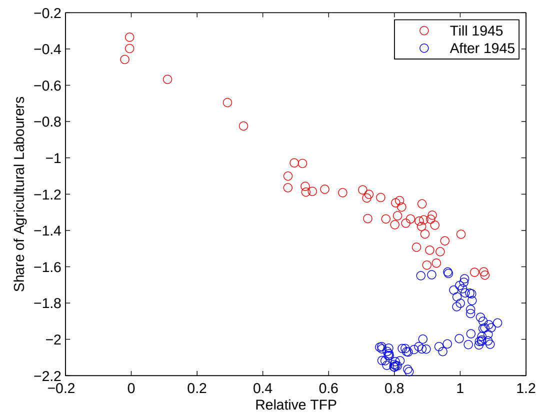
(c) Agricultural Sector Labour Share during 1820–2013: Model versus Data

(d) Relative Price during 1820–2013: Model versus Data

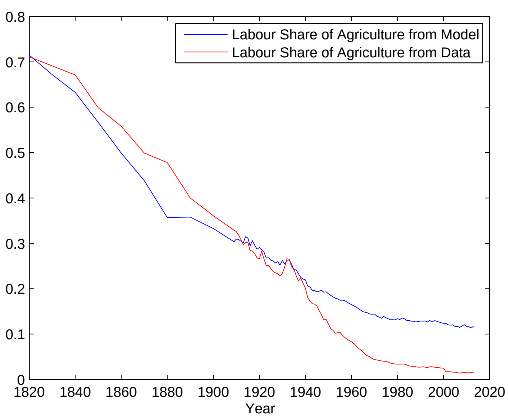
Figure 7: The estimated values of the parameters are: $(b, \epsilon) = (0.723, 1.874)$. These estimates are described in eq. (6). They have been obtained by minimising the distance between the structural model (regression eq. (27)) estimates obtained from the data and their counterparts estimated from the simulated data.



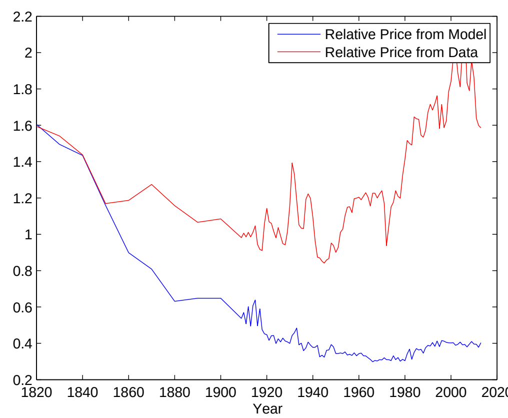
(a) Relative price against Relative TFP (in logarithmic scale): Model contrasts till 1945 to post-1945



(b) Agricultural Labour Share against Relative TFP (in logarithmic scale: Model contrasts till 1945 to post-1945)

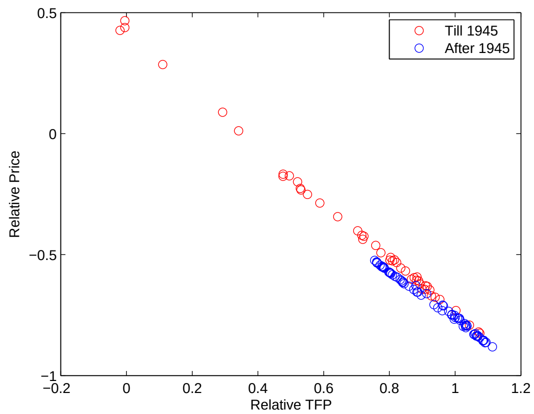


(c) Agricultural Sector Labour Share during 1820–2013: Model versus Data

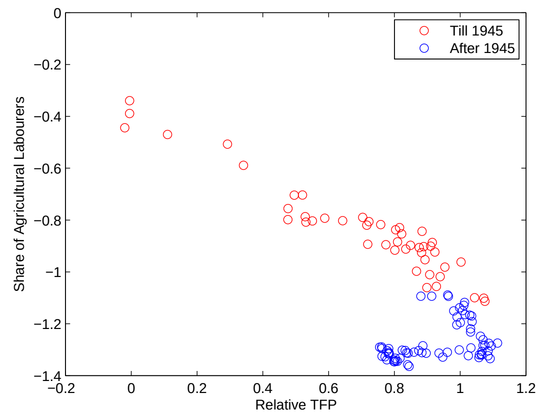


(d) Relative Price during 1820–2013: Model versus Data

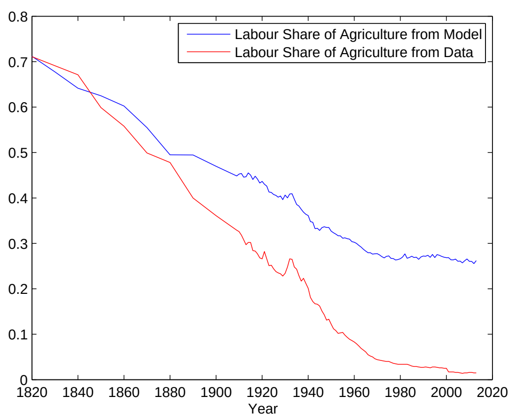
Figure 8: The estimated values of the parameters are: $(b, \epsilon) = (0.742, 2.993)$. These estimates are described in eq. (29). They have been estimated by minimising the distance between period-wise agricultural labour share obtained from the data and their counterparts obtained from the simulated data.



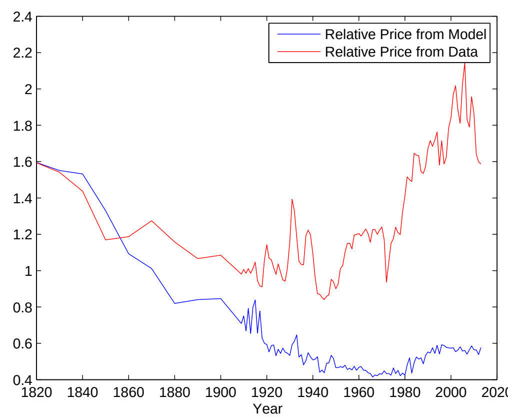
(a) Relative price against Relative TFP (in logarithmic scale): Model contrasts till 1945 to post-1945



(b) Agricultural Labour Share against Relative TFP (in logarithmic scale: Model contrasts till 1945 to post-1945)

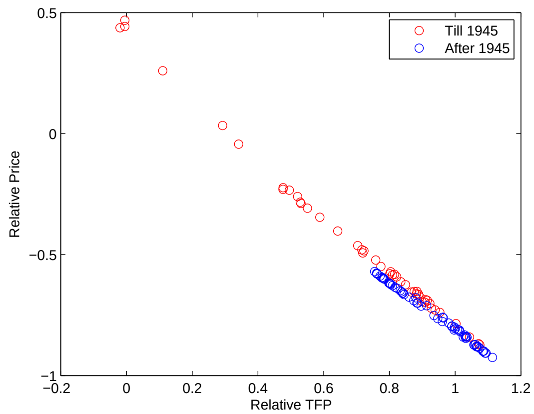


(c) Agricultural Sector Labour Share during 1820–2013: Model versus Data

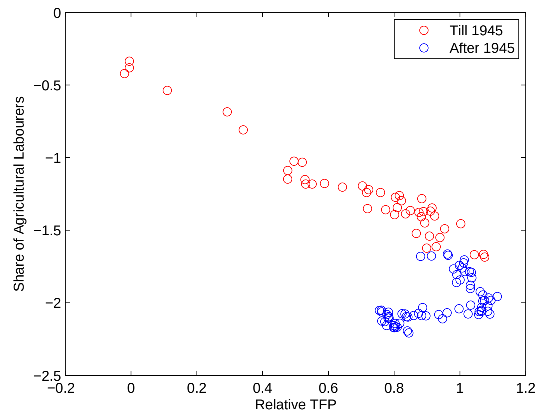


(d) Relative Price during 1820–2013: Model versus Data

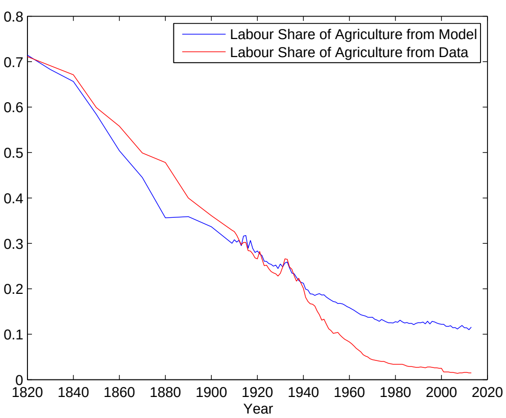
Figure 9: The estimated values of the parameters are: $(b, \epsilon) = (0.672, 1.978)$, given σ is calibrated at 5.000. These estimates are described in eq. (6). They have been obtained by minimising the distance between the structural model (regression eq. (27)) estimates obtained from the data and their counterparts estimated from the simulated data.



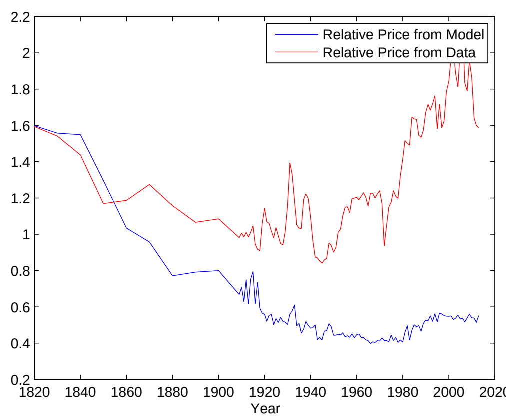
(a) Relative price against Relative TFP (in logarithmic scale): Model contrasts till 1945 to post-1945



(b) Agricultural Labour Share against Relative TFP (in logarithmic scale: Model contrasts till 1945 to post-1945)

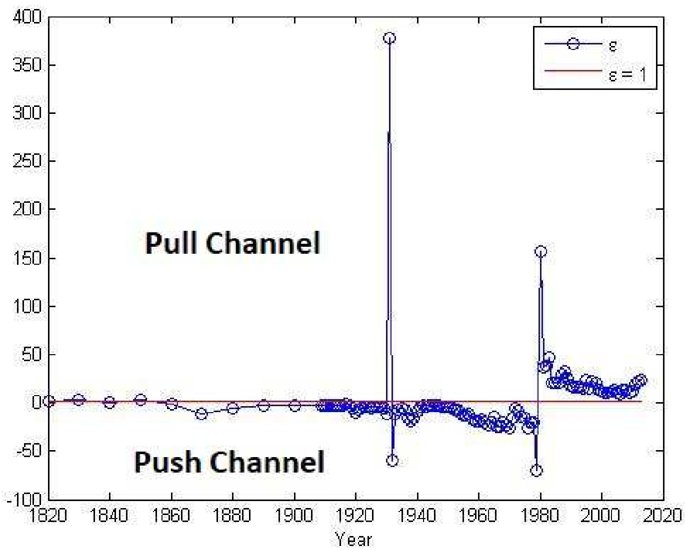


(c) Agricultural Sector Labour Share during 1820–2013: Model versus Data

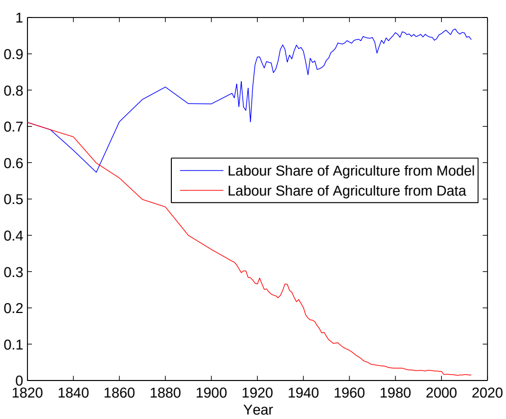


(d) Relative Price during 1820–2013: Model versus Data

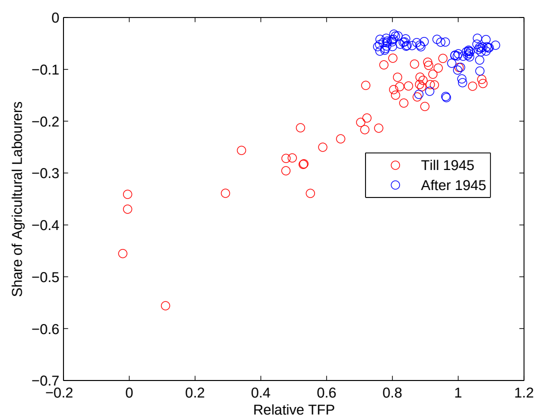
Figure 10: The estimated values of the parameters are: $(b, \epsilon) = (0.725, 3.545)$, given σ is calibrated at 5.000. These estimates are described in eq. (29). They have been estimated by minimising the distance between period-wise agricultural labour share obtained from the data and their counterparts obtained from the simulated data.



(a) Counterfactual: ϵ estimates over during 1820–2013: Push versus Pull channels



(b) Agricultural Sector Labour Share during 1820–2013: Model versus Data



(c) Agricultural Labour Share against Relative TFP (in logarithmic scale: Model contrasts till 1945 to post-1945)

Figure 11: A counterfactual with benchmark values for other parameters (σ = 3, b = 0.723, β = 0.328, γ = 0.479) was run in which the relative price from the data was infused to estimate ϵ at each point in time. $\epsilon = 1$ line is the boundary between these two channels of Push and Pull.