Attention and framing: stochastic choice rules*

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Abstract

We consider a model of individual stochastic choice with framing effects. Alternatives are presented in a display comprising a collection of frames- each one corresponding to an alternative. We characterise *attention-biased stochastic choice rules.* These rules assign attention probabilities to different frames and select an alternative with the joint probability of paying attention to its respective frame and ignoring the frames which contain the more preferred alternatives.

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1 Introduction

The standard model of individual choice assumes that the consumer (or decision maker, DM in short) is perfectly rational and chooses the most preferred alternative(s). Recent works on bounded rationality and behavioural economics show that this is not always the case and that their choices are influenced by the 'frame' or context in which the alternatives are presented. In this paper we consider attention biases of the DM that occur due to framing effects.

Framing affects the consumer's decisions in various ways. Framing effects, like the position of the alternative on the shelf, the quality and attractiveness of packaging, and other types of framing often attract the consumer's attention.

These include the order in which the alternatives are presented (Rubinstein and Salant (2006)), the relative positions of the alternatives on a shelf (Salant and Rubinstein (2008)), packaging over the alternatives, etc.

In this paper, we model framing effects when the decision maker's choice is probabilistic or stochastic. Moreover, our framework is such that the primitives of the DM's choice- the attention parameters and the preference ordering can be identified completely by observing the choice data.

In our model limited attention arises due to the *framing* over the set of alternatives rather than the *alternatives* themselves. And these biases may arise due to differences in the types or quality of frames attached to the alternatives. This is consistent with theory and experimental evidence.[1]

Once the DM has paid attention to a set of frames, intrinsic characteristics of the alternatives play an important part in determining her choice. Our main result captures both these effects and separates the attention biases over frames from the preference over the alternatives based on their intrinsic properties.

For example, an alternative which has been placed too far from the DM's range of sight will be prone to lack of attention irrespective of whether it is ranked high in the DM's preference. Similarly, even if the DM prefers an alternative highly due to its intrinsic qualities, inferior packaging might prevent it from attracting attention. In our model, the DM probabilistically chooses the best alternatives among the set of alternatives the frames to which she pays attention to. We provide an illustration.

We consider a model where each alternative is attached with its respective frame. This is denoted as (x, f_i) where x is the alternative and f_i is the frame attached with x. We call such pairs *products* and collections of these are denoted by G. For each G, the DM has stochastic choice, i.e., a probability distribution over all such pairs (x, f_i) in G.

In our main result, we characterise *attention-biased stochastic choice rules* with four axioms- *difference*, *reflexivity*, *dominance* and *independence*. These rules assign an attention parameter to each frame such that the probability of choosing an alternative is equal to

[1] For a broad survey of this literature see Kahneman and Tversky (2000) and related works.

the joint probability of paying attention to the frames attached to that alternative and the probability of not paying attention to the frames attached to the alternatives which the DM prefers more.

There are various works which deal with attention biases in individual choice.[2] However, the paper that is closely related to ours is Manzini and Mariotti (2014). In their model, the attention bias of the DM arises due to the alternatives themselves. Therefore, they do not directly capture the source of the DM's attention.

Example 1 Suppose the framing of the alternatives is their placement on a 'grid' denoted by G. The dimension of a grid is the number of boxes or positions in that grid. The alternatives appear in the boxes of a grid, where each box accommodates at most one alternative. For example, $a, b \in X$ can be displayed as follows:

$$G = \begin{array}{|c|} \hline b \\ \hline a \\ \hline \end{array}$$

Suppose the preference of the DM is such that a is preferred to b but due to the placement of the alternatives she pays attention to the top box with probability 0.8 and to the lower box with probability 0.2. Her choice probabilities according to *attention-biased revealed stochastic choice rule* are as follows:

(i) Probability of choosing a = Probability of paying attention to the frame attached to $a = 0.2$.

(ii) Probability of choosing b = (Probability of paying attention to the frame attached to b)$\times$ (Probability of not paying attention to the frame attached to a) $= 0.8(1 - 0.2) = 0.64$.

(iii) Probability of choosing nothing $= 1 - 0.2 - 0.64 = 0.16$.

Therefore, even though the alternative a is preferred over b, due to the framing of the alternatives and the fact that the DM hardly ever pays attention to the lower box, b gets chosen with a higher probability.

Contrary to our model, in Manzini and Mariotti (2014) the choice probabilities of the alternatives a and b would remain the same even if their positions were swapped in G. This is due to the fact that the DM pays the same attention to the alternatives irrespective of the framing. This ignores the effect of framing on attention paid by the DM, thereby, affecting choice.

[2]See Masatlioglu et al. (2016) , Manzini and Mariotti (2014)).

There are other papers which study stochastic choice rules. Ahn et al. (2017) characterises a *partially independent* rule, *Luce rule* where the probability of choosing an alternative is the average probability of being chosen among other alternatives from the given menu. However, there are no framing effects and their concern is with the weakening the notion of stochastic independence.[3]

The paper is organised as follows. Section 2 describes the model and lists the axioms. Section 3 presents the benchmark result. Section 4 presents the main result followed by a discussion and conclusion. Appendix contains the proofs and the bibliography is provided in References.

2 Model

Let X be the set of all *alternatives* and F be the set of all *frames*. The pair (x, f_i) is a *product* where alternative x has been displayed in the frame f_i. For simplicity we will write x_i to denote the product (x, f_i).

Let $A : X \times F$ be the set of all products. We allow the decision maker not to choose any product from a given set $G \subseteq A$- we assume the existence of a default product x^*. Let $A^* = A \cup \{x^*\}$.

A *stochastic choice rule* from a set of products is a mapping $P : A^* \times 2^{A^*} \to [0, 1]$ such that $P(x_i, G) \in [0, 1]$ for all $x_i \in G, G \subseteq A^*$; $P(x_i, G) = 0$ for all $x_i \in A \setminus G$; and $\sum_{x_i \in A^*} P(x_i, G) = 1$ for any $G \subseteq A^*$.

Note that when the set of products G is empty, the decision maker chooses the default alternative. Therefore, $P(x^*, \phi) = 1$.

We first characterise the following class of stochastic choice rules.

Definition 1 **(Attention-biased stochastic choice)** A stochastic choice rule from a set of products $P : A^* \times 2^{A^*} \to [0, 1]$ is an *attention-biased stochastic choice rule* if there exists a function $\delta : F \to [0, 1]$ and a complete, asymmetric binary relation $\succ$ over A such that for any $x_i \in G$:

$$P(x_i, G) = \delta_i \Pi_{j: y_j \succ x_i; y_j \in G} (1 - \delta_j)$$

A decision maker with *attention-biased stochastic choice rule* selects a product with the joint probability of paying attention to that frame attached to it and the probability of not paying attention to the frames which contain the products which she prefers more.

We can show that the rule characterised by Manzini and Mariotti (2014) is a special case of the above rule. Simply replace products x_i with alternative a for all such products and therefore removing the frames- then the above given rule is the same as the one characterised by Manzini and Mariotti (2014). We introduce some notation in order to state our axioms.

[3]For other papers on stochastic or random choice rules, see Li and Tang (2017) and Fudenberg et al. (2015), Block et al. (1960) Becker et al. (1963).

Let $G(X) \subseteq X$ and $G(F) \subseteq F$ be the set of elements and the set of frames in G respectively.

Difference. Suppose $G, G' \in 2^A$, $|G|, |G'| > 1$ such that $G(X) = G'(X)$ and $G(F) = G'(F)$. Then $P(x_i, G) \neq P(y_j, G)$ for all $x_i \in G$, $y_j \in G'$.

Difference states that the choice probabilities for two distinct products never coincide irrespective of their framing and for any given non-singleton set of frames G and G' which comprise of the same set of alternatives and frames.

Invariance of singletons For all $x, y \in X$ and $f_i \in F$, $P(x_i, \{x_i\}) = P(y_i, \{y_i\})$.

Invariance of singletons requires that the choice probability of any two products from singleton sets is identical when the two products have the same frames. This axiom emphasizes that stochasticity in choice arises from the frame attached to an alternative in the absence of other alternatives.

Note that there is no conflict with *difference*. The latter compares the probabilities of two products in any two *non-singleton* set of frames while the former compares the same with their respective *singleton* frames.

Dominance. Suppose $G \in 2^A$ such that $x_i \in G$. Then,

$$[P(x_i, \{x_i, y_j\}) > P(y_j, \{x_i, y_j\}) \text{ for all } y_j \in G] \Rightarrow [P(x_i, G) = P(x_i, \{x_i\})].$$

Dominance states that if $x_i \in G$ has a higher choice probability compared to any other product $y_j \in G$ in the binary set of frames $\{x_i, y_j\}$ then the choice probability of x_i in G is equal to the choice probability from the singleton set of frames $\{x_i\}$.

Independence. Suppose $G, G' \in 2^A$ such that $x_i, y_j \in G$ and $y_j \in G'$. Then,

$$[P(x_i, \{x_i, y_j\}) < P(y_j, \{x_i, y_j\})] \Rightarrow \left[\frac{P(x_i, G)}{P(x_i, G \setminus \{y_j\})} = \frac{P(\phi, G')}{P(\phi, G' \setminus \{y_j\})}\right] \text{ for all } G' \in 2^A.$$

Independence states that if $x_i \in G$ has a lower choice probability compared to $y_j \in G$ in the binary set of frames $\{x_i, y_j\}$ then the *effect* of removing y from G (when attached to frame $f_j \in F$) on the choice probability of x_i in G is the same as that on the probability of choosing nothing when y is removed from any other set of frames G'. We now provide the benchmark result.

Theorem 1 A stochastic choice rule P is an *attention-biased stochastic choice rule* if and only if it satisfies *difference*, *dominance*, *reflexivity* and *independence*.

Theorem 1 characterises the benchmark model which is a generalised version of the rule characterised in Manzini and Mariotti (2014). In this rule, the framing effect is adjoined with the alternatives. To obtain their rule the products in our model can be replaced by alternatives as if the framing effects were irrelevant. This points to the fact framing effects have not been fully captured in their model. Our next section will clarify this further as we present our main result.

3 The Main Result

In this section we characterise the following rule:

Definition **2 (Attention-biased stochastic choice (Revealed Preference))** A stochastic choice rule $P : A^* \times 2^{A^*} \to [0,1]$[4] is *attention-biased stochastic choice (RP) rule* if there exists a function $\delta : F \to [0,1]$[5] and a complete, asymmetric binary relation $\succ$ over X such that for any $x_i \in G$:

$$P(x_i, G) = \delta_i \Pi_{j:y \succ x; y_j \in G}(1 - \delta_j)$$

The above rule states that the choice probability of a product x_i depends on the attention paid to the frame attached to the alternative x and on the attention paid to the frames attached to the alternatives that dominate x according to the binary relation $\succ$ over X.

In contrast to the previous rule this rule separates the framing effect on attention biases from the underlying preferences which are based solely inherent properties of the alternatives.

We characterize the *attention-biased stochastic choice (RP) rule* with the following axioms. We introduce some notation in order to state our axioms. Let $G(X) \subseteq X$ and $G(F) \subseteq F$ be the set of elements and the set of frames in G, respectively.

The axiom *invariance of singletons* is defined in the same way as before. However, we introduce new versions of *difference*, *independence* and *dominance* axioms.

Difference. For any $x, y \in X$ and $f_i, f_j \in F$, $P(x_i, \{x_i, y_j\}) \neq P(y_i, \{x_j, y_i\})$.

Difference requires that for any two alternatives and frames, the choice probability of a product from a binary set depends on the alternative that constitutes the product and also on the presence of the other product. This axiom ensures that the choice probabilities are not independent of the alternatives in binary sets. When x appears in frame f_i, while y appears in f_j, the probability with which the decision maker chooses x differs from the probability with which y is chosen when the frames are reversed.

Dominance. Suppose $G \subseteq A^*$ such that $x_i \in G$. Then,

[4] $P(x^*, \phi) = 1$

[5] δ_i is interpreted as the attention drawn by the frame f_i. Our results remain unchanged if $\delta \in (0,1)$.

(i) $P(x_i, \{x_i\}) > P(x^*, \{x_i\})$
(ii) $[P(x_i, \{x_i, y_j\}) > P(y_j, \{y_i, x_j\})$ for all $y_j \in G] \Rightarrow [P(x_i, G) = P(x_i, \{x_i\})]$.
(iii) $[P(x_i, \{x_i, y_j\}) > P(y_i, \{y_i, x_j\})$ for $f_i, f_j \in F] \Rightarrow [P(x_k, \{x_k, y_l\}) > P(y_k, \{y_k, x_l\})$ for any $f_k, f_l \in F]$.

Dominance (i) requires that the choice probability of a product from a singleton set is higher than the probability with which the decision maker refrains from making a choice, i.e., chooses the default alternative x^*.

Dominance (ii) is similar as *dominance* in the previous section but with the alternatives x and y swapped in their respective frames f_i and f_j. For any two alternatives x and y, $P(x_i, \{x_i, y_j\})$ and $P(y_i, \{y_i, x_j\})$ are the choice probabilities of x and y from binary sets, when they occur in the same frame f_i, while the other alternative appears in the frame f_j. When x_i is chosen with a higher probability from $\{x_i, y_j\}$ than y_i from $\{x_j, y_i\}$, we say that x "dominates" y.

Dominance (iii) requires that if an alternative x in frame f_i has a higher choice probability when another alternative y (which was previously in some frame $f_j \in F$) takes its place in frame $f_i \in F$ then its 'dominance' continues to hold for any other set of frames f_k and f_l corresponding to x and y respectively.

For any alternative $y \in X$, we construct a set $H(y)$ that contains those alternatives that are dominated by y. Formally, $H(y) = \{x \in X^* | P(x_i, \{x_i, y_j\}) < P(y_i, \{y_i, x_j\}) \forall f_i, f_j \in F\}$.

Independence For any $x, z \in H(y)$; $G, G' \subseteq A^*$ such that $x_j, y_i \in G$ and $y_i, z_j \in G'$:

$$\frac{P(x_j,G)}{P(x_j,G \setminus \{y_i\})} = \frac{P(z_j,G')}{P(z_j,G' \setminus \{y_i\})}$$

Independence requires that the influence of y_i on the choice probabilities of any two products that are dominated by y is the same. The influence of y_i on a product x_j is the ratio $\frac{P(x_j,G)}{P(x_j,G \setminus \{y_i\})}$. This ratio is the same for any $x, z \in H(y)$, and is also independent of the sets from which the choice is made.

THEOREM **2** A stochastic choice rule P is an *attention-biased stochastic choice (RP) rule* if and only if it satisfies *difference*, *invariance of singletons*, *independence* and *dominance.*

Theorem 2 characterises *attention-biased stochastic choice rule (RP).* The binary relation represents the preferences of the DM over *the set of alternatives* and not products. Therefore, we separate the effect of framing on the attention from the pure preference effect.

The attention-biased stochastic choice rule (RP) recognizes that the frame attached to an alternative may influence the probability with which it is chosen. A binary relation over alternatives may differ from a binary relation over products. An alternative x may dominate y, but in the presence of different frames, the product (y, f_j) may dominate (x, f_i). The following example illustrates this.

Example 2 Let $\succ_f$ be a complete and asymmetric binary relation over the set of products A, and $\succ$ be a complete and asymmetric binary relation over the set of alternatives X. For $x, y \in X$ and $x_i, y_j \in A$:

$$x_i \succ_f y_j \iff [P(x_i, \{x_i, y_j\}) > P(y_j, \{x_i, y_j\})]$$

$$x \succ y \iff [P(x_i, \{x_i, y_j\}) > P(y_i, x_j, y_i\})]$$

Let $P(x_i, \{x_i\} = \delta_i = 0.4$, $P(x_j, \{x_j\}) = \delta_j = 0.8$ and $x \succ y$. According to the attention biased stochastic choice rule (RP): $P(x_i, \{x_i, y_j\}) = 0.4$, $P(y_i, \{x_j, y_i\}) = 0.4(1-0.8) = 0.08$, $P(y_j, \{x_i, y_j\}) = 0.8(1-0.4) = 0.48$. Clearly, $y_j \succ_f x_i$ even though $x \succ y$.

3.1 Applications

The above rules are a special case of the following types of choice rules,

(i) Consideration sets. A decision maker only pays attention to subset of frames called the consideration set. She chooses the most preferred alternative from this set of frames. In such a case the $\delta = 1$ for all frames in the consideration set and $\delta = 0$ for all frames that are outside it. Therefore, the most preferred alternative is picked with probability 1.

(ii) Rational choice rule: If the consideration set if the full set of frames then the DM picks the most preferred alternative.

(iii) Ordered attention-biased RP rules: Suppose frames have a quality factor and the attention parameters are ordered i.e. a higher quality frame has a higher attention parameter. Alternatives in higher quality frames will have a higher probability of being chosen depending on the preferences of the DM.

3.2 Suitable frames

A frame f_i may influence the choice probability of two alternatives x and y in different ways. This implies that $P(x_i, \{x_i\}) \neq P(y_i, \{y_i\})$. The attention biased stochastic choice rule characterized in the previous section does not allow this possibility- the choice probabilities of two products with the same frame is the same when they appear in singleton sets.

The role of a frame may not be limited to attracting the decision maker's attention. It is possible that certain alternatives are chosen more frequently when they occur in some specific frames. For example, a decision-maker may prefer *cotton candy* to *cheesecake* while at a picnic in a park, but may prefer *cheesecake* to *cotton candy* while dining at a restaurant. Here, the frame is the environment in which the dessert is consumed. Another example of a frame that is suitable for some alternative, but not for every alternative are combination meals offers: several restaurants offer a fixed combination such as *burgers* with a portion of *fries and coke*. An offer of *pasta* with *fries and coke* is unlikely to be equally attractive as the combination with the *burger*. In this example, *burger* and *pasta* are the alternatives, while the offer of *fries and coke* creates the frame. The frame is suitable for *burgers*, but is not as suitable for *pasta*. In this section we develop a general characterization of the broad class of attention biased stochastic choice rules, which contains the choice rules characterized in the previous section, and also allows for the stochastic parameter to depend on the frame, and also on the alternative that is placed in the frame.

We introduce the notion of *suitability* of a frame for an alternative. Consider two frames f_i,$f_j \in F$, and two products $x_i, y_j \in A^*$. We term a frame f_i as *suitable* for the alternative x relative to y as compared to frame f_j, if the following inequality is satisfied:

$$\frac{P(x_i,\{x_i,y_j\})}{P(x_j,\{x_j,y_i\})} \geq \frac{P(y_i,\{x_j,y_i\})}{P(y_j,\{x_i,y_j\})}$$

We abbreviate the ratios $\frac{P(x_i,\{x_i,y_j\})}{P(x_j,\{x_j,y_i\})}$ and $\frac{P(y_i,\{x_j,y_i\})}{P(y_j,\{x_i,y_j\})}$ as $I(xy_{ij})$ and $I(yx_{ij})$. Notice that:

$$I(xy_{ij}) \geq I(yx_{ij}) \iff I(yx_{ji}) \geq I(xy_{ji})$$

i.e., for any two products $x_i, y_j \in A^*$, if f_i is suitable to x relative to y, then f_j is suitable to y relative to x. Thus, for any two frames and two alternatives, both frames cannot be suitable for the same alternative. We also use the terminology *unsuitable* in the following manner: if $I(xy_{ij}) < I(yx_{ij})$, then x_i and y_j are products with *unsuitable* frames.

Further, we interpret $I(xy_{ij}) = I(yx_{ij})$ as a situation where neither frame f_i nor f_j is suitable to x relative to y or vice versa.

We now introduce the following stochastic choice rule:

Definition **3** *(Frame biased stochastic choice rule) A stochastic choice rule from frames* $P : A^* \times 2^{A^*} \to [0,1]$ *is a frame biased stochastic choice rule if there exists a function* $\delta : A \to [0,1]$ *and a complete binary relation* $\succeq$ *over* $F \times A(2)$*, where* $A(2)$ *is the set of all ordered pairs of alternatives in* X*, such that for any* $x_i \in G$*:*

$$P(x_i, G) = \delta(x_i).\Pi_{j:f_j \succeq_{(y,x)} f_i; y_j \in G}[1 - \delta(y_j)]$$

The parameter δ represents the attention paid to a product. For any product x_i, $\delta(x_i)$ depends on the frame f_i as well as the alternative x, which together constitute the product. The same frame applied to two different alternatives may lead to different attention parameters for each of them. Similarly, when the same alternative is placed in different frames, the attention paid to each would differ.

The *frame biased stochastic choice rule* states that the probability with which a decision-maker chooses a product x_i from the set G is the probability that he pays attention to x when it is placed in frame f_i and he does not pay attention to all those products that contain alternatives in frames that are suitable to them, relative to x.

This captures the idea that a suitable frame enhances the attractiveness of an alternative relative to another alternative. Thus, a decision-maker's choice of a product x_i depends on the probability of paying attention to the product, given the frame, and also on the suitability of the frames attached to the alternatives in the other products relative to x_i. A product y_j may affect the choice probability of x_i only if it's frame f_j is suitable to y relative to x. If a product's frame is unsuitable, it is unlikely to draw the decision-maker's attention in a way that affects another product.

For each product $x_i \in A$, we define the set $H(x_i)$ containing all those products that appear in suitable frames, relative to x when it appears in the frame f_i. Formally, $H(x_i) = \{y_j \in X^* | I(yx_{ji}) \geq I(xy_{ji})\}$. Notice that if $y_j \in H(x_i)$ then $x_i \in H(y_j)$.
We introduce the following axioms:

Frame invariance. For any $x \in X$ and for all $f_i, f_j \in F$,

$$P(x_i, \{x_i\}).P(x^*, \{x_i\}) = P(x_j, \{x_j\}).P(x^*, \{x_j\})$$

Frame invariance requires that the joint probability with which a decision maker chooses and does not choose an alternative from a singleton set is the same for all frames. This axiom enables us to interpret suitability of frames for an alternative when the decision maker is choosing from a singleton set. Notice that according to frame invariance $P(x_i, \{x_i\}) \geq P(x_j, \{x_j\}) \Rightarrow P(x^*, \{x_i\}) \leq P(x^*, \{x_j\})$, i.e. if the decision maker chooses x when it appears in frame f_i with weakly higher probability than when it appears in frame f_j from a singleton set, then the default alternative must be chosen with weakly lower probability from $\{x_i\}$ than from $\{x_j\}$. In comparison with the default alternative frame f_i is suitable to x relative to the frame f_j.

Independence of suitable frames. For any $y_j, z_k \in H(x_i)$; $G, G' \subseteq A^*$ such that $x_i, y_j \in G$ and $y_j \in G'$:

$$\frac{P(x_i,G)}{P(x_i,G\setminus\{y_j\})} = \frac{P(z_k,G')}{P(z_k,G'\setminus\{y_j\})}$$

Independence of suitable frames requires that the influence of a product y_j on the choice probability of x_i when both x and y appear in suitable frames is the same as the influence of y_j on any other alternative that appears in a suitable frame as compared to x_i.

Independence* Suppose $G, G' \subseteq A^*$ such that $x_i, y_j \in G$ and $y_j \in G'$:

$$[I(xy_{ij}) \geq I(yx_{ij})] \Rightarrow [\frac{P(x_i,G)}{P(x_i,G\setminus\{y_j\})} = \frac{P(x^*,G')}{P(x^*,G'\setminus\{y_j\})}]$$

*Independence** implies the following weaker version:
Weak Independence* Suppose $G \in 2^A$. For any $x_i, y_j \in G$:

$$[I(xy_{ij}) \geq I(yx_{ij})] \Rightarrow [\frac{P(x_i,G)}{P(x_i,G\setminus\{y_j\})} = \frac{P(x^*,\{y_j\})}{P(x^*,\phi)}]$$

*Independence** states that if two alternatives x and y are present in G within frames f_i, f_j that are suitable to them respectively, then the effect of removing a product y_j on another product x_i is the same as the effect of y_j on choosing the default alternative when y_j is removed from any other set of frames G'.

Frame dominance. Suppose $G \subseteq A^*$ such that $x_i, y_j \in G$

$$[I(xy_{ji}) > I(yx_{ji}) \forall y_j \in G] \Rightarrow [P(x_i, G) = P(x_i, \{x_i\})]$$

Frame dominance states that if the frames attached to the alternatives are unsuitable for all the products in the set G relative to some $x_i \in G$, then the probability of choosing x_i from the set G is the same as the probability of choosing x_i from a set that contains no other products. Thus, the probability with which the decision-maker chooses x_i is not adversely influenced by the frames of other alternatives, as none of them can enhance the alternatives relative to x.

THEOREM **3** A stochastic choice rule P is a frame biased stochastic choice rule if and only if it satisfies frame invariance, independence of suitable frames and frame dominance.

4 Conclusion

We characterise a class of stochastic choice rules which separate the effect of framing on attention biases from the preferences based on their inherent qualities.

5 Appendix

Proof of Theorem 1. We show necessity. By Axiom 1, exactly one of the following holds, (i) $P(x_i, \{x_i, y_j\}) > P(y_j, \{x_i, y_j\})$ or (ii) $P(x_i, \{x_i, y_j\}) < P(y_j, \{x_i, y_j\}$.
Define a complete and asymmetric binary relation $\succ$ over $X \times F$ as follows:
$x_i \succ y_j$ iff $P(x_i, \{x_i, y_j\}) > P(y_j, \{x_i, y_j\})$.
Consider $x_i \in G$. We partition G, the set of alternatives as follows: $G = G_1 \cup G_2 \cup \{x_i\}$ where $G_1 = \{y_j \in G : P(x_i, \{x_i, y_j\}) < P(y_j, \{x_i, y_j\})\}$ and $G_2 = \{w_r \in G : P(x_i, \{x_i, w_r\}) > P(w_i, \{x_i, w_r\})$. Moreover, $G_1 \cap G_2 = \phi$ and $x_i \notin G_1 \cup G_2$.

Notice that using the definition of the binary relation $\succ$ we have the following:

$$y_j \succ x_i \quad \forall y_j \in G_1 \subset G \text{ and } x_i \succ w_r \forall W_r \in G_2 \subset G.$$

Pick an arbitrary $y_j \in G_1 \subset G$. By axiom 4:

$$\frac{P(x_i, G)}{P(x_i, G \setminus \{y_j\})} = \frac{P(\phi, \{y_j\})}{P(\phi, \{\phi\})} \tag{1}$$

As $\Sigma_{x_i \in G} P(x_i, G) = 1$, we know that $P(\phi, \{y_j\}) = 1 - P(y_j, \{y_j\})$ and $P(\phi, \{\phi\}) = 1$. Therefore, we have,

$$\frac{P(x_i, G)}{P(x_i, G \setminus \{y_j\})} = 1 - P(y_j, \{y_j\}).$$

This in turn implies that,

$$P(x_i, G) = P(x_i, G \setminus \{y_j\})[1 - P(y_j, \{y_j\})]. \tag{2}$$

We pick another arbitrary alternative $q_l \in G_1 \setminus \{y_j\}$.

By axiom 4,

$$\frac{P(x_i, G \setminus \{y_j\})}{P(x_i, G \setminus \{y_j, q_l\})} = [1 - P(q_l, \{q_l\})].$$

which implies,

$$P(x_i, G \setminus \{y_j\}) = [1 - P(q_l, \{q_l\})] P(x_i, G \setminus \{y_j, q_l\}). \tag{3}$$

Using equations 5 and 6 we get:

$$P(x_i, G) = [1 - P(y_j, \{y_j\})][1 - P(q_l, \{q_l\})] P(x_i, G \setminus \{y_j, q_l\}).$$

By the repeated application of axiom 4 for every $y_j \in G_1$,

$$P(x_i, G) = P(x_i, G \setminus G_1) \Pi_{y_j \in G_1} [1 - P(y_j, \{y_j\})].$$

which implies:

$$P(x_i, G) = P(x_i, G_2 \cup \{x_i\}) \Pi_{y_j \in G_1} [1 - P(y_j, \{y_j\})] \qquad (5)$$

Now consider $G_2 \cup \{x_i\}$. By construction of G_2, for all $w_r \in G_2 \cup \{x_i\}$ such that $w_r \neq x_i$, $P(x_i, G_2 \cup \{x_i\}) > P(w_i, G_2'' \cup \{x_r\})$, where $w_i, x_r \in G_2''$ and $G \setminus \{x_i, w_r\} = G'' \setminus \{w_i, x_r\}$. Therefore, by axiom 3:

$$P(x_i, G_2 \cup \{x_i\}) = P(x_i, \{x_i\}) \tag{6}$$

From (5) and (6) we get:

$$P(x_i, G) = P(x_i, \{x_i\}).\Pi_{y_j \in G_1} [1 - P(y_j, \{y_j\})]$$

Using the definition of the binary relation $\succ$ and by the construction of G_1, the above expression can be written as:

$P(x_i, G) = P(x_i, \{x_i\}).\Pi_{y_j \succ x_i} [1 - P(y_j, \{y_j\})]$

Define:

$P(x_i, \{x_i\}) = P(y_i, \{y_i\}) = \delta_i$

$P(x_j, \{x_j\}) = P(y_j, \{y_j\}) = \delta_j$ Therefore, $P(x_i, \{x_i\}) = \delta_i$ and $P(y_j, \{y_j\}) = \delta_j$. Therefore, $P(x_i, G) = \delta_i.\Pi_{y_j \succ x_i} [1 - \delta_j]$

□

Proof of Theorem 2. We show necessity. By *difference*, exactly one of the following holds, (i) $P(x_i, \{x_i, y_j\}) > P(y_i, \{x_j, y_i\})$ or (ii) $P(x_i, \{x_i, y_j\}) < P(y_i, \{x_j, y_i\}$. Define a binary relation $\succ$ over X as follows: for any $x, y \in X$ and $f_i, f_j \in F$, $x \succ y \iff P(x_i, \{x_i, y_j\}) > P(y_i, \{y_i, x_j\})$. By (i) or (ii) above and *dominance (iii)*, $\succ$ is complete and asymmetric.
Consider $x_i \in G$. We partition G as follows: $G = G_1 \cup G_2 \cup \{x_i\}$ where $G_1 = \{y_j \in G : P(x_i, \{x_i, y_j\}) < P(y_i, \{x_j, y_i\})\}$ and $G_2 = \{w_r \in G : P(x_i, \{x_i, w_r\}) > P(w_i, \{x_r, w_i\})$. Moreover, $G_1 \cap G_2 = \phi$ and $x_i \notin G_1 \cup G_2$.
Notice that using the definition of the binary relation $\succ$ we have the following:

$$y \succ x, \forall y_j \in G_1 \text{ and } x \succ w, \forall w_r \in G_2.$$

Pick an arbitrary $y_j \in G_1$. Let $H(y) = \{x \in X^* | P(y_j, \{x_i, y_j\}) > P(x_j, \{y_j, x_i\})\}$. By *dominance (i)*, $P(y_j, \{y_j\}) > P(x^*, \{y_j\})$. Therefore $x^* \in H(y)$. By *independence*:

$$\frac{P(x_i, G)}{P(x_i, G \setminus \{y_j\})} = \frac{P(x^*, \{y_j\})}{P(x^*, \phi)} \tag{4}$$

As $\Sigma_{x_i \in G} P(x_i, G) = 1$, we know that $P(x^*, \{y_j\}) = 1 - P(y_j, \{y_j\})$ and $P(x^*, \phi) = 1$. Therefore, we have

$$\frac{P(x_i, G)}{P(x_i, G \setminus \{y_j\})} = 1 - P(y_j, \{y_j\}).$$

This in turn implies that

$$P(x_i, G) = P(x_i, G \setminus \{y_j\})[1 - P(y_j, \{y_j\})]. \tag{5}$$

Notice that $\frac{P(x^*, \{y_j\})}{P(x^*, \phi)} = 1 - P(y_j, \{y_j\})$.

We pick another arbitrary alternative $q_l \in G_1 \setminus \{y_j\}$.
By *independence* and $\frac{P(x^*, \{y_j\})}{P(x^*, \phi)} = 1 - P(y_j, \{y_j\})$,

$$\frac{P(x_i, G \setminus \{y_j\})}{P(x_i, G \setminus \{y_j, q_l\})} = [1 - P(q_l, \{q_l\})].$$

which implies,

$$P(x_i, G \setminus \{y_j\}) = [1 - P(q_l, \{q_l\})] P(x_i, G \setminus \{y_j, q_l\}) \tag{6}$$

Using equations 5 and 6 we get:

$$P(x_i, G) = [1 - P(y_j, \{y_j\})][1 - P(q_l, \{q_l\})] P(x_i, G \setminus \{y_j, q_l\}).$$

By the repeated application of *independence* for every $y_j \in G_1$,

$$P(x_i, G) = P(x_i, G \setminus G_1) \Pi_{y_j \in G_1} [1 - P(y_j, \{y_j\})].$$

which implies:

$$P(x_i, G) = P(x_i, G_2 \cup \{x_i\})\Pi_{y_j \in G_1}[1 - P(y_j, \{y_j\})] \tag{7}$$

Now consider $G_2 \cup \{x_i\}$. By construction of G_2, for all $w_r \in G_2 \cup \{x_i\}$ such that $w_r \neq x_i$, $P(x_i, G_2 \cup \{x_i\}) > P(w_i, G_2'' \cup \{x_r\})$, where $w_i, x_r \in G_2''$ and $G \setminus \{x_i, w_r\} = G'' \setminus \{w_i, x_r\}$. Therefore, by dominance (ii), $P(x_i, G_2 \cup \{x_i\}) = P(x_i, \{x_i\})$ (8)

From (7) and (8) we get: $P(x_i, G) = P(x_i, \{x_i\}).\Pi_{y_j \in G_1}[1 - P(y_j, \{y_j\})]$. Using the definition of the binary relation $\succ$ and by the construction of G_1, $P(x_i, G) = P(x_i, \{x_i\}).\Pi_{y_j : y \succ x}[1 - P(y_j, \{y_j\})]$. Define $P(x_i, \{x_i\}) = P(y_i, \{y_i\}) = \delta_i$ and $P(x_j, \{x_j\}) = P(y_j, \{y_j\}) = \delta_j$. Using the definition of δ in $P(x_i, G)$, we get $P(x_i, G) = \delta_i.\Pi_{j:y \succ x; y_j \in G}[1 - \delta_j]$.

$\square$

Proof of Theorem 3. We show necessity. Define a binary relation $\succeq$ over $F \times A(2)$ where $A(2)$ is the set of all ordered pairs of alternatives in X, as follows:

$$f_i \succeq_{(x,y)} f_j \iff I(xy_{ij}) \geq I(yx_{ij})$$

Consider $x_i \in G$. We partition G, the set of alternatives as follows: $G = G_1 \cup G_2 \cup \{x_i\}$ where $G_1 = \{y_j \in G : I(xy_{ij}) \geq I(yx_{ij})\}$ and $G_2 = \{w_r \in G : I(xw_{ir}) < I(wx_{ir})$. Moreover, $G_1 \cap G_2 = \phi$ and $x_i \notin G_1 \cup G_2$.

Note that using the definition of the binary relation $\succeq$ we have the following:

$$f_i \succeq_{(x,y)} f_j \text{ for all } y_j \in G_1 \tag{1}$$

By frame invariance, for some $f_z \in F$, $\frac{P(x_i, \{x_i\})}{P(x_z, \{x_z\})} = \frac{P(x^*, \{x_z\})}{P(x^*, \{x_i\})}$. Therefore, $x^* \in H(x_i)$. (2)
Pick an arbitrary $y_j \in G_1$. By independence of suitable frames and (2):

$$\frac{P(x_i, G)}{P(x_i, G \setminus \{y_j\})} = \frac{P(x^*, \{y_j\})}{P(x^*, \phi)}$$

As $P(x^*, \{y_j\}) = 1 - P(y_j, \{y_j\})$ and $P(x^*, \phi) = 1$, the above expression can be written as $P(x_i, G) = P(x_i, G \setminus \{y_j\}).[1 - P(y_j, \{y_j\})]$.
By the repeated application of independence of suitable frames for all $y_j \in G_1$, we get:

$$P(x_i, G) = P(x_i, G_2 \cup \{x_i\}).\Sigma_{y_j \in G_1}[1 - P(y_j, \{y_j\})]$$

By frame dominance, $P(x_i, G_2 \cup \{x_i\}) = P(x_i, \{x_i\})$. Therefore,

$$P(x_i, G) = P(x_i, \{x_i\}).\Sigma_{y_j \in G_1}[1 - P(y_j, \{y_j\})]$$

Using (1) in the above expression, we get:
$P(x_i, G) = P(x_i, \{x_i\}).\Sigma_{y_j \in G: f_i \succeq_{(x,y)} f_j}[1 - P(y_j, \{y_j\})]$ (3)

Define $\delta(x_i) = P(x_i, \{x_i\})$; and $\delta(y_j) = P(y_j, \{y_j\})$. Using $\delta(x_i)$ and $\delta(y_j)$ in (3):
$P(x_i, G) = \delta(x_i).\Sigma_{y_j \in G: f_i \succeq_{(x,y)} f_j}[1 - \delta(y_j)]$
We know that $I(xy_{ij}) \geq I(yx_{ij}) \iff I(yx_{ji}) \geq I(xy_{ji})$. This implies $f_i \succeq_{(x,y)} f_j \iff f_j \succeq_{(y,x)}$.
Therefore, $P(x_i, G) = \delta(x_i).\Sigma_{j: f_j \succeq_{(y,x)} f_i; y_j \in G}[1 - \delta(y_j)]$ □

We show that the axioms Axiom 1, difference, revealed dominance and weak independence are independent of each other:

- Let $G(2)$ be a set contaning all $G \subseteq X$ such that $|G| \leq 2$. Define a function $g : F \to I_{++}$, where I_{++} is the set of all strictly positive integers. g assigns each frame in F an integer $k_i \in I_{++}$. $\succ$ is a complete, asymmetric binary relation over X such that for any $x_i \in G; G \in G(2)$

$$P(x_i, G) = \frac{1}{(k_i+1)^{|N(x)|+1}}$$

  where $|N(x)| = |\{y_j : y_j \succ x_i; y_i \in G\}|$
  This choice rule satisfies axiom 1, difference, revealed dominance (i) and (ii), but does not satisfy independence.

- Define $\delta_i : F \to (0, 1)$. $\succ$ is a complete, asymmetric binary relation over X. For all $x_i \in G$, consider the following modification of the luce rule:

$$P(x_i, G) = \frac{\alpha_x.\delta_i}{\Sigma_{y_j \in G}\delta_j}$$

  where $\alpha_x = \begin{cases} 1, \text{ if } \nexists y_j \in G \text{ such that } y \succ x;\ y \neq x \\ 0, otherwise \end{cases}$
  The above rule satisfies axiom 1, difference, weak independence and revealed dominance (ii), but does not satisfy revealed dominance (i).

- Define $\alpha_x : X \to [0, 1]$. $\succ$ is a complete, assymetric binary relation over X such that for all $x_i \in G$

$$P(x_i, G) = \alpha_x \Pi_{y_j : y \succ x}(1 - \alpha_y)$$

  The above rule is similar to the stochastic choice rule characterized in Manzini and Mariotti (2014). It satisfies difference, revealed dominance (i) and (ii) and weak independence but does not satisfy axiom 1.

- Let $\delta_i : F \to (0, 1)$. For all $x \in X$, consider the following interpretation of the Luce rule: we attach weights to frames.

$$P(x_i, G) = \frac{\delta_i}{\Sigma_{y_j \in G} \delta_j}$$

The above choice rule satisfies axiom 1. It vacuously satisfies revealed dominance (i) and (ii), and independence, but does not satisfy difference.

- Let $G(2)$ be a set containing all $G \subset X$ such that $|G| \leq 2$; for any $x_i, y_j \in G, x \neq y$. $\succ$ is a complete, asymmetric binary relation over X. For any $x_i \in G$

$$P(x_i, G) = \begin{cases} 1 \text{ if } [x \succ y \forall y_j \in G \text{ and } i \neq i^*] \text{ or } [y \succ x \text{ for some } y_j \in G \text{ and } i = i^*] \\ 0, otherwise \end{cases}$$

for some $f_{i^*} \in F$.

The above choice rule states that for some frame f_{i^*}, the alternative which is dominated by the other alternative according to $\succ$ is chosen when it occurs in this frame. This rule satisfies axiom 1, difference, revealed dominance (i) and weak independence. It does not satisfy revealed dominance (ii).

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