Okun's Law, Business Cycles and Unemployment Insurance*

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WORK IN PROGRESS

Abstract

Okun's Law and the positive correlation of labor productivity with unemployment are two important facts documented in the data. A necessary condition to match these facts simultaneously is to model labor force participation. We develop a business cycle model where the individual's labor force participation choice is indivisible; to overcome indivisibilities, individuals have access to lotteries over labor force participation. The labor market is characterised by thick market search externalities that satisfy Okun's Law. The economy features two steady states: low and high unemployment. At the low unemployment steady state, a novel mechanism of self-fulfilling fluctuations emerges. Expectations of individuals about labor market conditions - through a search-based labor market wedge - feed back into Okun's Law, in turn validating those expectations. The labor wedge allow us to capture features of the business cycle that existing theories fail to match. The high unemployment steady state is characterised by a rat race for jobs channel: high participation rates is associated with high real wages, low demand for labor and high unemployment rates. Finally, on the policy side, if the economy is at the low steady state, unemployment insurance (UI) - replacing a fraction of market wages - is a powerful automatic stabiliser since it can counterbalances (belief-

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driven) fluctuations in the labor market wedge. Moreover, UI polices are desirable in terms of welfare since they move the equilibrium allocations closer to the constrained optimum. However, if the economy is at the high unemployment steady state, UI policies exacerbate the rat race for jobs and move the equilibrium allocation away from the constrained optimum.

Keywords: Okun's law, search externalities, indeterminacy, automatic stabilizers.
JEL Classification: E32, E62, J20.

1 Introduction

To understand how unemployment evolves over the business cycle, Okun's law (Okun, 1962) has long been the first port of call, with its prediction of a positive (and greater than unity) semi-elasticity of the output gap to changes in the unemployment rate. Recent estimates confirm the robustness of this law, and find that each percentage point reduction in the unemployment rate is associated with roughly 2% more output (Ball et al., 2013). For this reason, the real business cycle (RBC) framework requires large total factor productivity (TFP) shocks to match the comovement of employment and output, implying a strongly procyclical real wage. Unfortunately, the empirical evidence in support of procyclical TFP shocks as the main driver of unemployment fluctuations is very weak at best (see, for example: Galí, 1999; Gordon, 2010; Hall, 2017). On the other hand, in the frictionless RBC model, other types of disturbances that are believed to be important to explain the business cycle phenomena, such as shocks to the relative price of investment, counterfactually generate a negative conditional correlation between consumption and investment (a point famously made by Barro and King, 1984). Thus, the frictionless RBC model fails to identify a plausible source of business cycles, consistent with the correlation between unemployment and output.

In this paper we develop a business cycle model featuring a "three-states" labor market, including employment, unemployment and non-participation. We show that, given the behavior of average labor productivity over the business cycle, including an endogenous participation choices is essential for any theory of the business cycle to be consistent with Okun's law. To see this, consider the following representation of output

$$\begin{aligned} Y_t &= (\text{APL})_t \, N_t, \\ &= (\text{APL})_t \, \Pi_t \left(1 - u_t\right), \end{aligned} \tag{1}$$

where APL $= Y/N$ denotes average labor productivity (defined as the output/employment ratio), $N = \Pi\,(1-u)$ is employment, Π is the number of participants in the labor force and u is the rate of unemployment. If we ignore participation choices and assume that Π is constant, from (1) the semi-elasticity of labor productivity to changes in the unemployment rate is given by

$$\frac{d\ln ALP}{du} \simeq 1 + \underbrace{\frac{d\ln Y}{du}}_{\text{Okun's coefficient} \simeq -2} = -1,$$

where we made the approximation $\ln\left(1-u_t\right) \simeq -u_t$. Thus, given an Okun's coefficient of -2 (the estimate in Ball et al., 2013), this implies a semi-elasticity of labor productivity to changes in the unemployment rate of -1 and, hence, a negative correlation between labor productivity and unemployment over the business cycle. But, although average labor productivity was procyclical until the 1980's, it has become markedly countercyclical after that (a fact well documented in e.g.: Galí and Van Rens, 2014; Hall, 2017), and the correlation between unemployment and average labor productivity became positive (as illustrated in Figure 1). Thus, based on equation (1), it follows that endogenous changes in labor force participation over the business cycle are required to match simultaneously the Okun's relationship and the negative correlation between unemployment and average labor productivity and, in particular, that labor force participation must be procyclical. Notice that this finding is entirely data driven and does not rely on any theoretical restrictions.

To model endogenous labor force participation, we abstract from the large family assumption (Merz, 1995), as is commonly used in the literature. Instead, our approach is closer, but with notable differences, to the RBC models of Hansen (1985) and Rogerson (1988) where the labor supply decision is indivisible and individuals play lotteries over employment outcomes. The primary choice of individuals in the labor market is to decide whether to participate or not. Conditional on participation, individuals search for a job, in a frictional labor market, by incurring a disutility cost; a job-finding probability describes the likelihood of

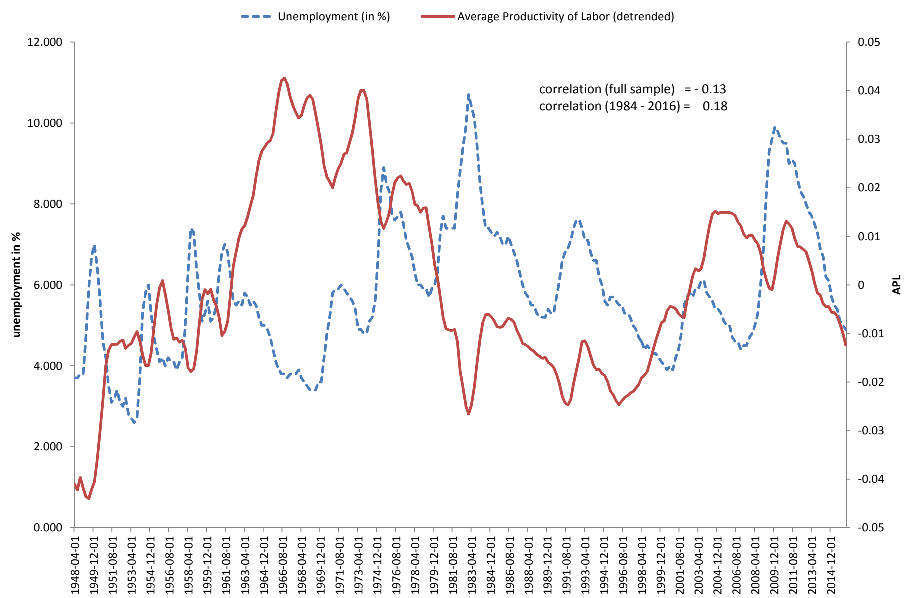


Figure 1: labor productivity and unemployment

finding employment. Individuals that choose not to participate, do not search - and, hence, do not incur a disutility cost - but are allowed to accumulate assets. We convexity the choice of individuals by allowing them to play lotteries over labor force participation; also, individuals have access to private insurance against the realisation of the lottery. Lotteries over labor force participation allow us to construct a search-based labor market wedge: the MRS between consumption and participation is equal to the real wage times the job-finding probability.

The labor market is characterised by thick market search externalities akin to that in Diamond's (1982) "coconuts' model": the higher the number of individuals with a coconut in the market, the easier it is to meet someone with a coconut and trade. Here, the probability of a match between a firm and a worker is an increasing function of aggregate output and, as a result, it is easier for job seekers to meet job opportunities when output is high. As we will demonstrate, the more participants in the market, the easier it is for job seekers to meet job opportunities.

An important assumption throughout the paper is that matches last only one period[1]. Also,

[1] In a companion paper, Kokonas and Santos Monteiro (2017), we extend the current set up by allowing long-lasting matches and jobs of different quality. However, departing from the big family assumption, as

since search for a job is not directed but random, equilibrium requires that the fraction of participants with a job (employed individuals) is equal to the fraction of firms (recruiters) that were able to form a match. This implies that the probability to form a match equal the employment rate which, in turn, is consistent with Okun's Law.

The economy features two steady state equilibria: low and high unemployment steady states. The low unemployment steady state might be indeterminate with many paths converging to it. The intuition for the possible indeterminacy is as follows. If individuals expect unemployment to be high, labor force participation and output are low, which in turn validates the high unemployment expectations because of Okun's law. We provide necessary conditions for local indeterminacy, offering an intuitive explanation for the emergence of self-fulfilling fluctuations. The crucial insight is that, for a given real wage, the existence of unemployment introduces a search-based wedge between the marginal utility of consumption and the marginal utility of leisure. A necessary condition for indeterminacy requires sufficiently higher elasticity of the wedge with respect to participation relative to the elasticity of real wage with respect to participation. Thus, increasing returns to scale are not required for indeterminacy and, contrary to other endogenous business cycle models, our model does not imply a negative correlation between consumption and employment conditional on sunspot shocks (overcoming the critique of Schmitt-Grohé, 2000).

On the other hand, the high unemployment steady state is determinate. The labor market is characterise by a rat race for jobs. In particular, more participants in the labor market raise unemployment and wages. As we will explain, firms face downward-sloping labor demands. Hence, demand for labor decreases and since the number of participants in the market increase, labor market clearing requires higher unemployment. In effect, the labor market is "overcrowded" by market participants who search for a job, firms reduce hiring because the real wage is too high and as a consequence, unemployment increases. Comparing our mechanisms with the rat race model of Michaillat (2012), we do not assume exogenously fixed real wages that are "too high" which imply rationing in the labor market; the rat race for jobs channel arises endogenously due to multiplicity of steady states.

By assuming that the job finding rate is determined by the level of output, we hardwire Okun's law into our model, effectively treating the Okun's relationship as a structural equa-

we do, it is far from trivial how to model job destruction and matches that last more than one period.

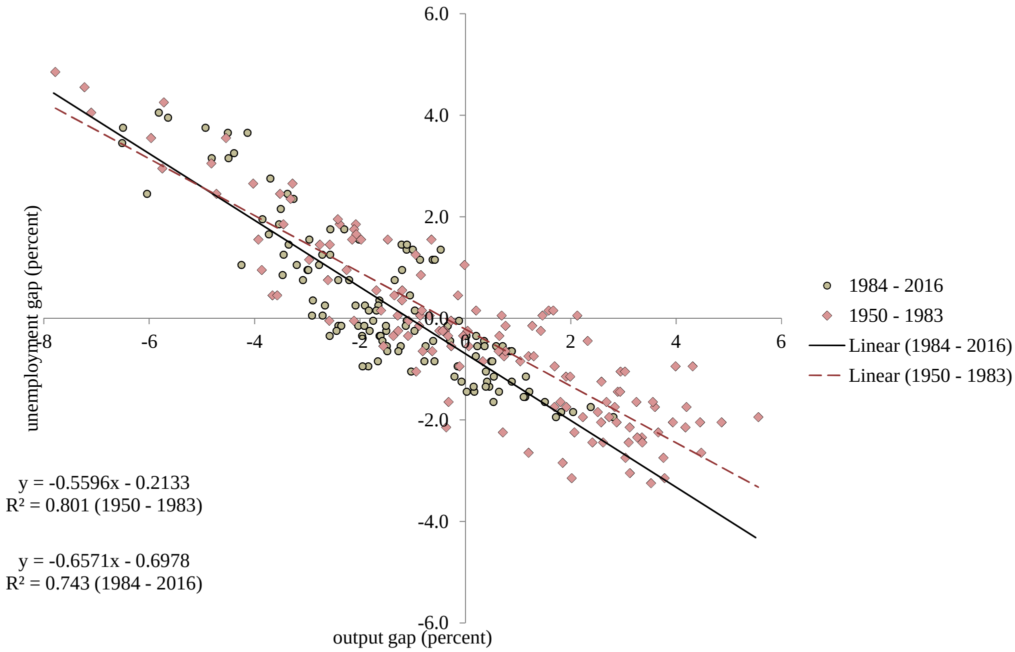


Figure 2: Okun's law before and after 1984

tion. We justify this approach with the impressive stability of the relationship between unemployment and the output gap in the post-war US economy, despite the substantial changes in the properties of the business cycle.[2] In particular, it is remarkable that average labor productivity has shifted from being procyclical before the 1980's to countercyclical after that, but Okun's relationship remained stable, with the estimated semi-elasticity of output to changes in the unemployment rate still between -1.5 and -2, as illustrated in Figure 2. The upshot of assuming that Okun's law is structural, is that our model will match successfully the relationship between unemployment and output, something which we consider an essential feature of the business cycle, but that previous models featuring endogenous labor force participation have struggled to achieve (see Veracierto, 2008, for an important early attempt to embed a "three-states" labor market in an RBC framework).

The search based labor wedge allows us to capture two important features of business cycles that previous endogenous business cycle models and the canonical RBC model failed to capture. The first feature relates to the positive correlation between consumption and

[2]The stability of Okun's relationship over time is carefully documented by Ball et al. (2013).

employment conditional on sunspot shocks and the second to the Barro-King criticism. The labor wedge and, in particular, the higher elasticity of the wedge relative to the elasticity of the real wage allows us to obtain a positive correlation between consumption and employment conditional on sunspot shocks and overcome the Barro-King criticism.

We contribute to an important literature that includes unemployment in equilibrium business cycle models. A prominent recent example is the paper by Christiano et al. (2016). Their paper makes the important point that the classic RBC framework and the subsequent work including search frictions à la Mortensen and Pissarides (1994) in RBC models, as in Andolfatto (1996), have problems matching the volatility of unemployment over the business cycle because the real wage is too procyclical in these models. Because the existence of involuntary unemployment in our model introduces a cyclical wedge between the marginal utility of consumption and the marginal utility of leisure, we are able to overcome this problem and at the same time obtain a mildly countercyclical real wage.

Our paper also contributes to the important literature on automatic stabilizers, understood broadly as features of the tax and transfer system that respond automatically to current conditions in the economy, thereby lowering business cycle volatility. The stabilizing effect of automatic stabilizers, in particular unemployment insurance, is traditionally thought to be most effective in environments featuring incomplete opportunities for private insurance (McKay and Reis, 2016). For example, an unemployment insurance may dampen fluctuations in disposable income and thus stabilize the business cycle in environments with nominal rigidities and market incompleteness (Brown, 1955). Similarly, unemployment insurance may redistribute income across individuals that have different marginal propensity to spend and, thus, contribute to aggregate demand stabilization when markets are incomplete (Blinder, 1975). Instead, we use the model developed in this paper to study the role of unemployment insurance in a setting with perfect private insurance markets and, consequently, no motivation for redistribution. Unemployment insurance is shown to make local indeterminacy less likely and, therefore, is a powerful automatic stabilizer. This result resonates well with the empirical finding that raising the replacement ratio of unemployment insurance lowers business cycle volatility (see, for example, Di Maggio and Kermani, 2016).

UI policies can be effective in our set-up if the economy is at the low unemployment steady state since they can counteract (belief-driven) volatility in the labor wedge. For any given arbitrary beliefs, UI acts as a subsidy that pushes in the opposite direction of individual's beliefs. Moreover, UI policies are desirable from an optimality standpoint since they bring the equilibrium allocation closer to the constrained optimum.

However, UI policies exacerbate the rat race for jobs if the economy is at the high unemployment steady state. In particular, UI increase participation in the labor market which in turn, is associated with higher real wages, lower demand for labor and higher unemployment rates. This result is in sharp contrast to the paper of Landais et al. (2010), where they demonstrate that UI policies alleviate the rat race channel.

The rest of the paper is organized as follows. Section 2 introduces the search externality which is central to our model. Section 3 embeds this externality in a complete general equilibrium model. Section 4 looks at the properties of the model in steady state. Section 5 looks at the low unemployment steady state dynamics and, in particular, derives necessary conditions for multiplicity of equilibrium to arise, Section 6 analyses the rat race channel for jobs and Section 7 studies the role of unemployment insurance.

2 Search Externalities and Okun's Law

The purpose of this section is to establish a simple relationship between output and unemployment, which is consistent with Okun (1962) formulation, and has sound theoretical foundations. The theory that we propose is based on the search model of Diamond (1982), which emphasizes the importance of search externalities. We define $\widetilde{Y}_t$ to be aggregate output (in deviation from a trend component which is defined in the following section). At the start of date t, there is a continuum of individuals of mass $\pi_t \in (0,1)$ searching for work, corresponding to the size of the labor force, and a continuum of recruiters in the unit interval. Each recruiter posts a single vacancy (at zero cost), and each worker is matched with a recruiter with probability

$$\mathscr{P}\left(\widetilde{Y}_t\right) = \left(1 + \mu \widetilde{Y}_t^{-\eta}\right)^{-1}. \tag{2}$$

with $\mu > 0$ and $\eta > 0$.

Thus as in Diamond (1982), there is a search externality, as workers are only able to sell their output (labor) with a given probability, $\mathscr{P} \in (0,1)$, which is an increasing function of aggregate output. The choice of functional form for $\mathscr{P}$ is made for its tractability and is not essential for our results, and the parameter η controls the output elasticity of the matching probability. If a match is formed, an employed individual produces one unit of intermediate output (labor services), which is sold by the recruiters to the the final good producers at price w_t. In the Appendix C we show that because recruiters are allowed to create vacancies at zero cost and individuals have access to perfect insurance markets, employed workers earn $w_t = \varpi_t h_0$ (where ϖ_t denotes the "hourly" wage rate).

Since an individual whose search fails remains unemployed, we obtain the following relationship between unemployment and output

$$1 - u_t = \left(1 + \mu \widetilde{Y}_t^{-\eta}\right)^{-1}, \tag{3}$$

where u_t corresponds to the unemployment rate. Although, as we show in Section 4, the natural rate of unemployment is generically indeterminate, we define $u^\star$ to be it. Then, after taking the log-linear approximation of equation (3), we obtain the gap formulation of Okun's Law, given by

$$u_t - u^\star = -\theta \ln\left(\widetilde{Y}_t / \underline{Y}\right), \tag{4}$$

with $\underline{Y}$ that denotes the steady state level of $\widetilde{Y}_t$, and where

$$\theta = \eta \left(1 - u^\star\right) \left(1 + \mu \underline{Y}^{-\eta}\right)^{-1} \mu \underline{Y}^{-\eta}, \tag{5}$$

is the gradient of the Okun's relationship which, as is illustrated in Figure 1, has been remarkably stable in post-war US data. Thus, we henceforth consider equation (4) as a structural relationship, with fluctuations in involuntary unemployment driven by search frictions in a way that is consistent with the empirical evidence on the comovement of unemployment and the output gap.

3 Equilibrium Model

We consider an indivisible labor economy in which labor market adjustment occurs entirely along the extensive margin and there are three possible labor market states: employment, unemployment and non-participation. The formulation of the problem assumes that an individual who is part of the labor force (either employed or unemployed) uses her endowment of time instead of enjoying leisure. In particular, we consider the Hansen (1985) and Rogerson (1988) economy, but with individuals playing lotteries over labor market participation. Thus, the opportunity cost of employment is the same as that of unemployment, with the upshot that any equilibrium with unemployment in this economy is not Pareto efficient. The problem solved by the stand-in agent in the household sector is given by

$$\max \mathbf{V} = E_0 \sum_{t=0}^{\infty} \beta^t \Big[\ln (c_t) + \psi \pi_t \ln (1 - h_0) \Big], \tag{6}$$

subject to the constraints

$$c_t + i_t = \pi_t (1 - u_t) w_t + r_t k_t, \tag{7}$$

$$k_{t+1} = V_t i_t + (1 - \delta) k_t, \tag{8}$$

with $\psi > 0$ and $h_0 \in (0, 1)$, and where V_t is the level of investment-specific technology (following the formulation in Fisher, 2006). In particular, the individual's problem is as in Hansen (1985) involving lotteries, but with lotteries played over the labor force participation/non-participation outcomes, instead of over the employment/unemployment outcomes.[3] Hence, $\pi_t \in [0, 1]$ denotes the probability of the individual being part of the labor force.

Since consumption and leisure are separable in the utility function and their are complete insurance markets, individuals participating in the labor force (either employed or unemployed) enjoy the same level of consumption as the individuals who do not participate, denoted $c_t > 0$. Individuals in the labor force are either employed or unemployed and the unemployment rate is denoted $u_t \in (0, 1)$. The rental rate of capital is r_t and is set competitively, while w_t is the wage rate and corresponds to the surplus generated by each job match, as explained above. Finally, x_t denotes investment and k_{t+1} the end of period capital stock holdings of the stand-in household.

[3]See Appendix A for details.

The first-order conditions solving the stand-in household's problem are given by

$$1 = \frac{(1-u_t)\, w_t}{\phi c_t}, \tag{9}$$

$$\frac{1/V_t}{c_t} = \beta \left[\frac{(1-\delta)\,(1/V_{t+1}) + r_{t+1}}{c_{t+1}} \right], \tag{10}$$

with $\phi = -\psi \ln(1-h_0) > 0$. These conditions are standard, except for the presence of the unemployment rate in (9).

Indeed, the distinct feature of this economy is the existence of involuntary unemployment. In particular, in the neighborhood of the steady state, the rate of unemployment satisfies Okun's Law given by equation (4) and which we repeat here for convenience

$$u_t - u^\star = -\theta \ln\left(\widetilde{Y}_t/\underline{Y}\right). \tag{4$'$}$$

Equation (4′) is a structural feature of the economy and is, therefore, taken as given by agents. Taken together with (6), it leads to coordination problems in trade similar to those in Diamond (1982). In his framework, agents are randomly presented with production opportunities and, once they produce a fixed quantity of output, they must search for a buyer and cannot undertake production if they have unsold output. Our framework offers similar opportunities and constraints. In particular, individuals receive production opportunities (in our case, the ability to search for work) with a given probability π_t and, conditional on participation, they must find a buyer for their fixed supply of labor h_0, subject to the search frictions described in Section 2.[4]

Final output is produced by competitive firms combining capital K_t and intermediate output (labor services) N_t , through the following Cobb-Douglas technology

$$Y_t = Z_t K_t^{\alpha} \left(A_t N_t\right)^{1-\alpha}, \tag{11}$$

with $\alpha \in (0,1)$, and where Z_t is the transitory component of TFP and its logarithm follows

[4] In Diamond (1982), the arrival rate of production opportunities is exogenous and individuals must choose if they pursue production if given the opportunity. Instead, in our model all individuals who participate (receive a production opportunity) also search for work, but the probability of participation π_t is chosen endogenously. The opportunity cost of participation is h_0 units of leisure. This small difference in the protocol does not change the fundamental coordination problem that emerges when there are search externalities.

a stationary autoregressive process; in turn, A_t is the permanent component of technology and its logarithm follows a random walk process with drift.

The equilibrium factor prices are given by

$$w_t = (1-\alpha)\left(Y_t/N_t\right), \tag{12}$$

$$r_t = \alpha\left(Y_t/K_t\right). \tag{13}$$

We let Π_t denote the labor force participation rate and use capital letters to denote aggregate variables. Then, the market clearing conditions are given by $\widetilde{c}_t = \widetilde{C}_t$, $\widetilde{i}_t = \widetilde{I}_t$, $\widetilde{k}_t = \widetilde{K}_t$, $\pi_t = \Pi_t$, $N_t = \Pi_t\left(1-u_t\right)$, where the notation $\widetilde{X}_t$ denotes the stationary version of X_t, given by $\left(X_t/\Omega_t\right)$ with $\Omega_t = A_t V_t^{\alpha/(1-\alpha)}$, except for $\widetilde{K}_t$ which is defined as $K_t/\left(V_{t-1}\Omega_{t-1}\right)$. Combining the market clearing conditions with the efficiency conditions (9) and (10), production function (11), the factor prices (12) and (13), and Okun's equation (4′), yields the following equilibrium conditions

$$\phi\widetilde{C}_t = (1-\alpha)\left(\widetilde{Y}_t/\Pi_t\right), \tag{14}$$

$$\frac{1}{\widetilde{C}_t} = \beta E_t\left[\frac{(1-\delta)\,\mathcal{X}_{t+1} + \alpha\widetilde{Y}_{t+1}/\widetilde{K}_{t+1}}{\widetilde{C}_{t+1}}\right], \tag{15}$$

$$\widetilde{Y}_t = Z_t\left(\mathcal{X}_t\widetilde{K}_t\right)^{\alpha}\left[\Pi_t\left(1-u_t\right)\right]^{1-\alpha}, \tag{16}$$

$$\widetilde{C}_t + \widetilde{I}_t = \widetilde{Y}_t, \tag{17}$$

$$\widetilde{K}_{t+1} = \widetilde{I}_t + (1-\delta)\,\mathcal{X}_t\widetilde{K}_t, \tag{18}$$

$$\mathcal{X}_t = \left(V_{t-1}/V_t\right)\left(\Omega_{t-1}/\Omega_t\right), \tag{19}$$

$$u_t - u^{\star} = -\theta\ln\left(\widetilde{Y}_t/\underline{Y}\right), \tag{20}$$

with $\ln\left(\mathcal{X}_t\right) = \left(g^v + \epsilon_t^v\right)/\left(\alpha-1\right) - \left(g^a + \epsilon_t^a\right)$, and $\ln\left(Z_t\right) - \rho\ln\left(Z_{t-1}\right) = \epsilon_t^z$, and where ϵ_t^z, ϵ_t^a and ϵ_t^v are, respectively, the transitory TFP shock, the permanent neutral technology shock and the permanent investment-specific shock, and g^a and g^v are the growth rates of A and V along the deterministic balanced growth path (BGP) equilibrium. Finally, the net growth rate of output, consumption and investment along the deterministic BGP is $g = g^a + g^v\alpha/\left(1-\alpha\right)$.

4 Steady State

In what follows, we look at the properties of the steady state and next characterize the equilibrium dynamics of the model in the neighborhood of its deterministic steady-state. One interesting feature of this model economy is that it is only possible to define an unique steady state up to a choice for the steady state unemployment rate (henceforth, the natural rate). In turn, the natural rate $u^\star$ may not be uniquely defined, depending on the form of the function $\mathscr{P}$. Thus, if we do not specify a natural rate the model may exhibits many steady state equilibria. For a given choice of the natural rate, the deterministic steady state of this economy corresponds exactly to the steady state of the neoclassical growth model, as shown in Appendix D.

A second curious feature of the steady state, is that the participation rate $\underline{\Pi}$, which is given by equation (D.5), is independent of the natural unemployment rate. This result follows from the fact that the preferences exhibit unit elasticity of substitution between consumption and leisure: an increase in the natural unemployment rate lowers the opportunity cost of non-participation (substitution effect), but this effect is exactly offset by the negative income effect implied by the lower expected labor income.

Proposition 1 *The economy may exhibit several steady state equilibria, indexed by the natural rate $u^\star \in (0,1)$, depending on the shape of the function $\mathscr{P}$. However, the steady state participation rate $\underline{\Pi}$ is independent of the natural rate. Finally, for a given natural rate, the steady state of this economy corresponds exactly to that of the neoclassical growth model.*

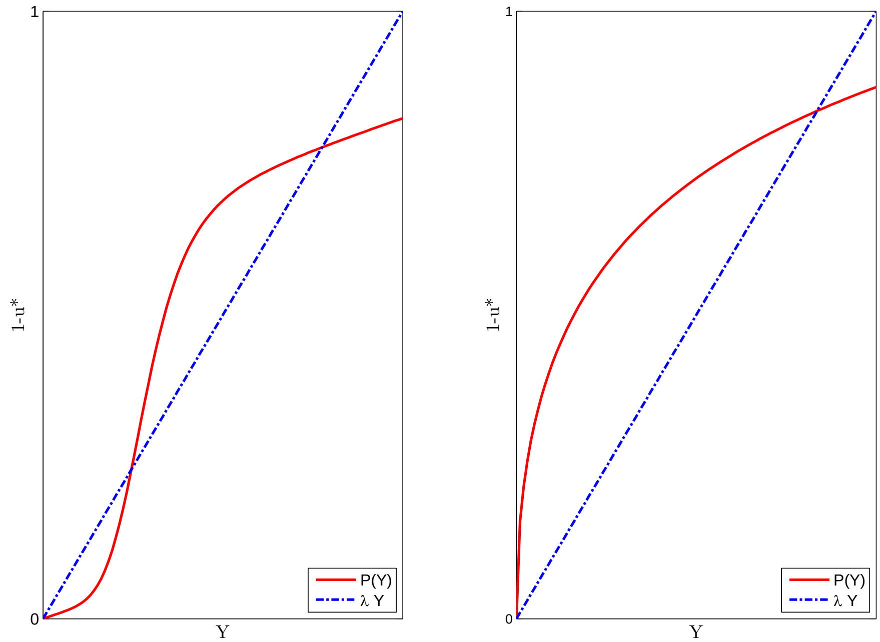
To show that the steady state equilibria are indexed by the natural unemployment rate, is suffices to note that in steady state the capital-output ratio is independent of $u^\star$, given by

$$(\underline{K}/\underline{Y}) = \left[\frac{\alpha}{\mathcal{G}/\beta - (1-\delta)}\right]. \tag{21}$$

In turn, as shown in Appendix D, the steady state capital stock is proportional to $(1-u^\star)$, so that $\underline{K} = \Xi\,(1-u^\star)$. The upshot, is that the steady state aggregate output is also proportional to $(1-u^\star)$. Thus, we may write

$$(1-u^\star) = \lambda\underline{Y}, \tag{22}$$

Figure 3: The natural rate of unemployment



with $\lambda = \Xi \left[\frac{\mathcal{G}/\beta-(1-\delta)}{\alpha} \right]$ a positive constant. All other variables of interest are proportional to the capital-output ratio as in the standard neoclassical growth model. Thus, the steady state is fully determined, conditional on the level of $u^{\star}$, the natural rate of unemployment. However, the economy may exhibit a large number of natural unemployment rates and, hence, steady state equilibria, depending on the form of the function $\mathscr{P}$. These equilibria are Pareto ranked and, as in Diamond (1982), each steady state equilibrium is locally inefficient.

It turns out that with the functional form proposed for $\mathscr{P}$ in Section 2 there may be at most two interior solutions for the natural rate $u^{\star}$. To see this, notice that equation (3) implies the following in steady state

$$\begin{aligned} (1-u^{\star}) &= \left(1+\mu \underline{Y}^{-\eta}\right)^{-1}, \\ u^{\star} &= \mu Y^{-\eta}\left(1+\mu \underline{Y}^{-\eta}\right)^{-1}, \end{aligned} \tag{23}$$

and from (23) it follows that the gradient of the Okun's law is given by

$$-\theta = -\eta\left(1-u^{\star}\right) u^{\star}. \tag{24}$$

Finally, combining (22) and (23), $u^\star$ and $\underline{Y}$ are found. That there can be at most two natural rate follows from the fact that $\left(1+\mu\underline{Y}^{-\eta}\right)^{-1}$ is an increasing function, bounded between $(0,1)$, and it is strictly concave if $\eta \in (0,1)$, while it is first convex and then concave if $\eta > 1$. Thus, setting $\eta \in (0,1)$ guarantees the existence of a unique steady state equilibrium and, if $\eta > 1$ there will be two steady states. The latter possibility is illustrated in the left-hand side panel of Figure 3, and the former in the right-hand side panel.

5 Search Externalities and Local Indeterminacy

In what follows, we show that because of the existence of search externalities, there will be parameter regions for which the low unemployment steady state is locally indeterminate. In doing this analysis, we follow the method of Wen (2001) who obtains necessary and sufficient conditions for local indeterminacy in RBC models. The local indeterminacy of the perfect-foresight equilibrium implies the existence of stationary sunspot equilibria. In particular, our focus in this section is to provide necessary conditions for local indeterminacy that offer an intuitive explanation for the possible emergence of self-fulfilling fluctuations.

Consider the following proposition:

Proposition 2 *A necessary condition for indeterminacy is*

$$\xi\left(\underline{R},\underline{\Pi}\right) < \xi\left(1-u^\star,\underline{\Pi}\right) + \xi\left(\underline{w},\underline{\Pi}\right), \tag{25}$$

where $\xi(R,\Pi)$ denotes the elasticity of the gross rate of capital return to participation, $\xi(1-u^\star,\Pi)$ denotes the elasticity of the employment rate to participation, and $\xi(w,\Pi)$ the elasticity of wages to participation. Condition (25) imposes restrictions on η, that is,

$$\eta u^\star \in \left(\frac{1-\beta(1-\alpha)(1-\delta)\underline{\mathcal{X}}}{1-\alpha}, \frac{1}{1-\alpha}\right). \tag{26}$$

Proof. See Appendix F ■

It is possible to give concrete economic intuitions for the necessary condition (26). In particular, in Appendix H, we show that in the neighborhood of the steady state the total elasticity of output to changes in participation is given by

$$\xi\left(\bar{Y},\bar{\Pi}\right)=\frac{dY}{d\Pi}\frac{\Pi}{Y}=\left[\frac{(1-\alpha)}{1-\eta\,(1-\alpha)\,u^{\star}}\right]. \tag{27}$$

This elasticity is positive as long as $\eta u^{\star}<1/\left(1-\alpha\right)$, which corresponds to the upper bound in condition (26). Thus, a necessary condition for indeterminacy is that the elasticity of output to changes in participation is positive, $\xi\left(\bar{Y},\bar{\Pi}\right)>0$. If this condition is satisfied, an increase in participation leads to an increase in output, which in turn may generate a multiplier effect if search externalities are sufficiently strong. This is the case if the "effective wage", defined as $\left(1-u_t\right)w_t$, increases sufficiently following a rise in participation. For this to be the case, the participation elasticity of the "effective wage", given by $\xi\left(1-u^{\star},\underline{\Pi}\right)+\xi\left(\underline{w},\underline{\Pi}\right)$, must exceed the participation elasticity of the return to capital, $\xi\left(\underline{R},\underline{\Pi}\right)$. The "effective wage", as we just defined, is the relevant measure of the return to labor in an economy with involuntary unemployment because, conditional on labor market participation, only the fraction $\left(1-u_t\right)$ of labor market participants earns a wage.

The relevant elasticities are given by

$$\xi\left(\underline{R},\underline{\Pi}\right)=\left[\frac{1-\alpha-\beta\,(1-\alpha)\,(1-\delta)\,\underline{\mathcal{X}}}{1-\eta(1-\alpha)u^{\star}}\right], \tag{28}$$

$$\xi\left(1-u^{\star},\underline{\Pi}\right)=\left[\frac{\eta\,(1-\alpha)\,u^{\star}}{1-\eta\,(1-\alpha)\,u^{\star}}\right], \tag{29}$$

$$\xi\left(\underline{w},\underline{\Pi}\right)=-\left[\frac{\alpha}{1-\eta\,(1-\alpha)\,u^{\star}}\right], \tag{30}$$

Thus, notice that when $\eta u^{\star}<1/\left(1-\alpha\right)$, the participation elasticity of the wage rate, $\xi\left(\underline{w},\underline{\Pi}\right)$, is negative. Indeed, as there are diminishing returns to labor, the slope of the equilibrium wage-employment loci is negative. Moreover, labor supply in our model is standard and, in particular, preferences feature a unit elasticity of substitution between consumption and leisure. However, there may still be local indeterminacies if the search externalities are sufficiently strong, so that the employment rate $\left(1-u^{\star}\right)$ increases sufficiently following a rise in participation.[5]

[5] This is in contrast to the Benhabib and Farmer (1994) seminal model, that requires the slope of the labor demand curve to be sufficiently steeper than that of the labor supply curve, to obtain a sufficiently positive labor elasticity of wages.

To see how self-fulfilling equilibria may emerge in our economy if search externalities are sufficiently strong, consider the following argument. Suppose the economy is on a given equilibrium path, and there is a shock to agents' beliefs about the shadow price of capital. For example, suppose agents believe that the shadow price of capital has declined and, thus, increase consumption. From condition (14), the increase in consumption has to be associated with an increase in the "effective wage rate", $(1 - u_t)\, w_t$. If there are no search externalities and, thus, the unemployment rate stays at its natural level, the participation rate must fall to raise wages. This, in turn, implies a decline in output and in the capital stock, raising the return to capital in the next period. This implies further declines in the shadow price of capital to support the initial change in beliefs. But these dynamics would violate boundary conditions and, thus, are not an equilibrium.

If, instead, search externalities are sufficiently strong, an increase in the "effective wage rate" is possible without a decline in the participation rate, if the unemployment rate falls sufficiently. Thus, consumption and participation may both increase. In turn, the increase in participation raises output and further lower unemployment, allowing the return to capital and investment to increase initially. However, the increase in the capital stock eventually leads to a decline in the return to capital, which leads to an appreciation of the price of capital, and declines in consumption and participation, as the economy returns to its balanced growth path.[6] Because it is the "effective wage" which matters in our economy, employment and consumption may both increase, conditional on an extrinsic shock. Instead, in endogenous business cycles models based on increasing return to scale à la Benhabib and Farmer (1994), an extrinsic shock that raises consumption is associated with an increase in wages and, hence, a decline in employment. This counterfactual prediction about the conditional correlation of consumption and employment is singled out by Schmitt-Grohé (2000) as an important shortcoming of the canonical endogenous business cycle model. Search externalities in the labor market, as proposed in this paper, overcomes this problem.

[6] We have shown that if search externalities are sufficiently large, for any initial equilibrium path it is possible to construct an alternative equilibrium path supported by a change in beliefs (sunspots). Of course, if at least two equilibria can be obtained, then the set of equilibria under indeterminacy is a continuum.

5.1 Diagrammatic exposition

The preceding analysis provided necessary conditions offering an intuition behind the mechanism that generates self-fulfilling fluctuations. Next, we show that equilibrium can be presented in a two-dimensional graph as intersection of two schedules: Okun's Law and equilibrium in the final producers sector. This graph can be used to analyse the effects of both fundamental shocks and non-fundamental (sunspot) shocks.

From section 2, the log-linear form of Okun's relation is given by

$$\hat{y}_t = -\left[\frac{1}{\eta u^\star (1-u^\star)}\right](u_t - u^\star).$$

This relation describes the matching process between recruiters and labor market participants; thus, it is as an equilibrium condition in the recruiters market and we denote it as recruiters equilibrium and represent it with the RE locus in Figure 4.

In turn, recruiters sell labor to final good producers. The latter combine aggregate labor services and capital services to produce output. The final producers equilibrium (FPE) condition is given by

$$\hat{y}_t = -\frac{1-\alpha}{\alpha(1-u^\star)}(u_t - u^\star) + \hat{k}_t - \frac{1-\alpha}{\alpha}\widehat{c}_t + \frac{z_t}{\alpha},$$

which is obtained from combining the production function, the demand equations for labor and capital services, and the intra-temporal first order condition of households - we have shut down the effect of investment shocks to focus on transitory technology shocks.

The FPE and RE schedules are drawn in the $(\hat{y}, u - u^\star)$-space in Figure 4, holding consumption and the productivity shock fixed (as the capital stock is predetermined). The following Lemma compares the slopes of FPE and RE schedules:

Lemma 1 *If the necessary condition for indeterminacy is satisfied, then the slope of the FPE schedule is steeper than the slope of the RE schedule.*

Proof. The proof is straightforward. The slope of FPE is steeper than the slope of RE if and only if

$$\frac{1-\alpha}{\alpha} > \frac{1}{\eta u^*}.$$

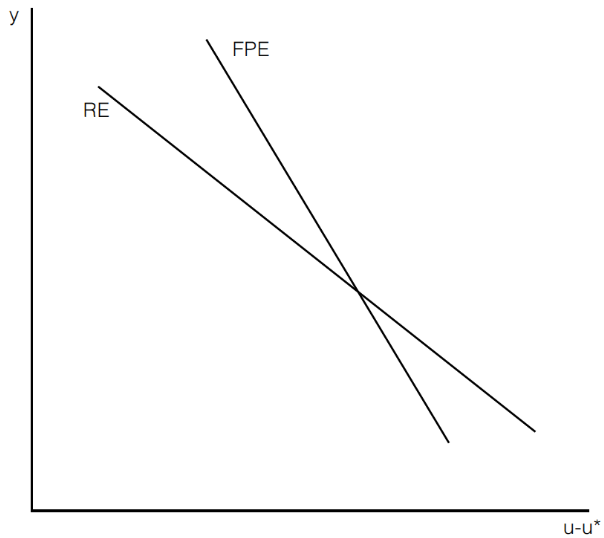


Figure 4: FPE and RE schedules

The latter is always satisfied if ηu^* is restricted as in Proposition 2 $\blacksquare$

Figure 4 is useful to understand the effects of sunspot shocks and, also, fundamental shocks. Let us start with sunspot shocks. Following the argument of the previous section, suppose households hold beliefs that justify higher consumption than the current equilibrium path (suppose initially the economy is at the steady state, that is, FPE and RE intersect at zero). The FPE schedule moves to the left since $\widehat{c} > 0$. At the new intersection, output increases and unemployment falls. Lower unemployment implies that the participation rate increases. Moreover, households increase investment since the return on capital is higher. As more capital is accumulated, the marginal product falls and dynamics are reversed back to the steady state. Since consumption falls, the FPE schedule moves back to the right until it crosses RE at zero.

The upshot of the previous analysis is that sunspot shocks, by changing consumption, change the position of the FPE schedule relative to the RE schedule and affect the real allocation of resources. The next experiment considers the effects of a transitory productivity shock. Starting from the steady state, we uncover the following surprising result. A transitory expansionary shocks to technology implies, on impact, a contraction of output, investment and consumption, an increase of unemployment, and a decrease of participation. This result is in sharp contrast with the benchmark RBC model (without search externalities), in which expansionary technology shocks generate a boom. However, it is consistent with the findings in Galí (1999) and Basu et al. (2006), who found that aggregate technology shocks in the U.S. economy lower employment, investment, and the real interest rate in the short run.

The intuition can be explained with the help of Figure 4. Consider a positive shock to technology, so that $\epsilon^z > 0$. Holding consumption fixed at the steady state, $\widehat{c} = 0$, the FPE schedule moves to the right. At the new point of intersection, and since the FPE is steeper than the RE schedule, output falls and unemployment increases. Higher unemployment rates lower participation by households since search in the labor market becomes less attractive. In turn, the lower employment lowers consumption - through the intratemporal condition - and also, lowers the marginal product of capital and investment.

5.2 Labor wedge and business cycles

Consider first the positive correlation between employment and consumption, conditional on sunspot shocks, that most endogenous fluctuations models fail to match. From the intratemporal and the necessary conditions we obtain that increases in participation imply higher employment which, in turn, imply higher consumption. Thus, consumption and employment are positively correlated.

Combining the intratemporal and the necessary conditions, we can overcome the Barro-King criticism as well. Suppose an investment shocks hits the economy that makes investment in capital an attractive option. On impact, the real wage falls and the return on capital increases. If we were to shut down the search externalities, the intratemporal condition would imply that consumption has to fall. Thus, consumption falls and capital investment increases in the short-run. However, if search externalities are active, lower real wages imply higher labor demand which increases the probability of matching which, in turn, and through Okun's Law, increases the employment rate. Taking into account the intratemporal and necessary conditions, consumption increases. Thus, investment and consumption are positively correlated conditional on an investment shock.

6 Rat race for jobs

The left panel of figure 3 features the possibility of two steady state equilibria. The previous section analysed the local dynamics around the low unemployment stated state - here, we are assuming that η is greater than one and there exist two steady states. Although the low steady state might be indeterminate, the high unemployment one is determinate. Consider the following argument. Suppose participation increases, then the low steady state moves up and to the right of figure 3 whereas the high steady state moves down and to the left - this is an artefact of the $S-$shaped function. Higher participation rates increase unemployment at the high steady state, which in turn imply that the elasticity of employment with respect to participation, expression (29), is negative. This elasticity is negative if and only if $\eta u^*(1-\alpha) > 1$. The latter implies that the necessary condition is violated from above and hence, the stated state is determinate.

Since $\eta u^*(1-\alpha) > 1$, the elasticity of employment with respect to participation is negative and the elasticity of real wages with respect to participation, expression (30), is positive. The rat race channel is as follows. High participation rates impose an upward pressure on wages. Firms lower the demand for labor since they face downward sloping demands. Low labor demand and more participants in the market looking for jobs imply that equilibrium in the labor market is achieved only with higher rates of unemployment which, in turn, puts a downward pressure on the job-finding probability. The labor market is overcrowd with many participants looking for jobs, but high real wages, effectively, create a rationing mechanism which results in high unemployment.

7 Policy

The point of this section is twofold. First, in section 6.1 we demonstrate how unemployment insurance (UI) policies can stabilise the economy by lowering or, even, eliminating the likelihood of indeterminacy. Subsequently, we consider the issue of optimal policy.

7.1 Fluctuations and automatic stabilisers

Suppose the government provides unemployment insurance (UI). In particular, it replaces a fraction γ of the wage income of households that participate in the labor market but are unemployed. The government finances this policy by taxing all households with lump-sum taxes T_t. The period t budget constraint of the stand-in agent modifies as[7]

$$c_t + i_t = \pi_t(1 - u_t)w_t h_0 + \gamma \pi_t u_t w_t h_0 + r_t k_t - T_t, \tag{31}$$

where $\gamma \in [0, 1]$ is the replacement ratio. A policy of $\gamma = 1$ is defined as full replacement policy. Moreover, the government follows a balance-budget policy, requiring

$$T_t = \gamma u_t w_t h_0 \Pi_t. \tag{32}$$

The equilibrium conditions for the economy with UI are the same as those for the baseline economy without UI, except for the intra-temporal equilibrium condition (14) which is replaced by

$$\phi \Pi_t \widetilde{C}_t = \left[\frac{1 - (1 - \gamma)\, u_t}{1 - u_t} \right] (1 - \alpha)\, \widetilde{Y}_t. \tag{14$'$}$$

Of course, the economy with $\gamma = 0$ is identical to the baseline economy, since then (14) and (14′) are the same. However, with a positive UI replacement ratio the mechanism which could generate equilibrium indeterminacy in the baseline economy are weaker. In the baseline economy individuals would participate less in the labor market when high unemployment was expected and this could lead to self-fulfilling high unemployment. However, in an economy with UI, expectations of high unemployment have a less detrimental effect on participation as as unemployed workers still receives the government transfer. Thus, the self-fulfilling multiplier effect is less salient.

More formally, we establish the following proposition:

[7] Although the economy already offers perfect private insurance opportunities, the introduction of the government sponsored UI program affects the budget constraint of the stand-in agent, as it does not have to be purchased and, thus, crowds-out some of the private insurance. See Appendix B for details.

Proposition 3 *A necessary condition for indeterminacy is*

$$\xi\left(\underline{R},\underline{\Pi}\right) < (1-\frac{\gamma}{1-u^{\star}(1-\gamma)})\xi\left(1-u^{\star},\underline{\Pi}\right)+\xi\left(\underline{w},\underline{\Pi}\right). \tag{33}$$

Condition (30) imposes restrictions on η, that is,

$$\eta u^{\star} \in \left(\frac{1-\beta\left(1-\alpha\right)\left(1-\delta\right)\underline{\mathcal{X}}}{\left(1-\alpha\right)\left(1-\frac{\gamma}{1-u^{\star}(1-\gamma)}\right)}, \frac{1}{1-\alpha}\right). \tag{34}$$

Proof. See Appendix G ■

There exist a unique replacement ratio[8] such that (31) collapses to a single point and, as a result, the steady state is determinate. In particular,

$$\gamma^{*} = \frac{(1-u^{\star})\beta(1-\alpha)(1-\delta)\mathcal{X}}{1-u^{\star}\beta(1-\alpha)(1-\delta)\mathcal{X}}$$

and γ^{*} is less than one since $\beta(1-\alpha)(1-\delta)\mathcal{X} < 1$ - the latter is a requirement for a well-defined steady state. UI policies can eliminate belief-driven fluctuations in the labor wedge.

It is important to contrast the previous policy with a policy of progressive income taxes. Suppose the government taxes labor and capital income and pays lump-sum subsidies:

$$\tau_t Y_t = T_t.$$

Following Guo and Lansing (1998), consider the following tax function

$$\tau = 1-\zeta\left(\frac{\bar{Y}}{Y}\right)^{\psi},$$

where $\zeta \in (0,1]$, $\psi \in ((\alpha-1)/\alpha, 1)$ and $\bar{Y}$ is SS level of output.

The following proposition applies.

[8]We can verify that steady state unemployment is a continuous function of γ and it is locally unique.

Proposition 4 *A necessary condition for indeterminacy is*

$$\eta u^* \in \left(\frac{1-(1-\alpha)\beta(1-\delta)(1-\psi)}{1-\alpha}, \frac{1}{1-\alpha} \right). \tag{35}$$

Proof. See Appendix H. ■

7.2 Optimality

The constrained optimal allocation is attained when a planner chooses feasible allocations and internalises Okun's Law to maximise discounted intertemporal utility. To understand optimality, we demonstrate whether UI and progressive income taxation policies can "push" the economy closer to the constrained best allocation.

Constrained optimality requires

$$\begin{aligned}
&\max_{\{C,K,\Pi\}} \sum_t^\infty \beta^t \left(\log(C_t) - \phi\Pi_t\right), \\
&s.t \\
&C_t + K_{t+1} = F(K_t, \Pi_t(1-u_t)) + (1-\delta)K_t, \\
&1 - u_t = \mathscr{P}\left(F(K_t, \Pi_t(1-u_t))\right),
\end{aligned}$$

where F is Cobb-Douglas but here we abstract from technological progress and $\mathscr{P}$ is given by (2).

The first order conditions of the planner's problem are

$$\phi C_t = (1-u_t)\left[1+\xi(1-u_t,\Pi_t)\right](1-\alpha)\frac{Y_t}{(1-u_t)\Pi_t}, \tag{36}$$

$$\frac{1}{C_t} = \beta\frac{1}{C_{t+1}}\left(1-\delta+\alpha\frac{Y_{t+1}}{K_{t+1}}\left[1+\xi(1-u_{t+1},\Pi_{t+1})\right]\right), \tag{37}$$

where (33) and (34) are the intratemporal and Euler equation respectively. The first order conditions of the planner's problem include a wedge which is measured by the elasticity of employment with respect to participation, that is,

$$\xi(1-u_t, \Pi_t) = \frac{\eta(1-\alpha)u_t}{1-\eta(1-\alpha)u_t}. \tag{38}$$

To get a better understanding of the properties of the constrained optimum solution, we substitute the competitive equilibrium allocations of both the low and high unemployment steady states into (33), (34).

Let us start with the competitive equilibria at the low unemployment steady state. We are interested in cases where the necessary condition for indeterminacy is satisfied and hence, there might exist multiple paths to converge back the steady state. If the necessary condition is satisfied - expression (23) -, then it follows that the elasticity of employment with respect to participation, ξ, is positive. In turn, this implies that the competitive equilibrium solution is characterised by lower participation and lower investment relative to the constrained optimum.

The intratemporal condition at the competitive solution implies that the marginal cost of putting an additional individual in the labor force is lower than the marginal benefit of the additional wage income. This implies under-participation at competitive equilibrium. Similarly, the Euler equation implies that the marginal cost of sacrificing one unit of consumption today is lower than the marginal benefit of an additional unit of consumption tomorrow, which in turn, implies under-investment at competitive equilibrium. To decentralise the constrained optimum solution, a government must subsidise labor and capital income - and use lump-sum taxes to finance subsidies - in order to induce more participation and higher investment.

From the previous argument, it should be clear that if the economy is at the low steady state equilibrium, then a stabilisation policy of progressive income taxes is not desirable from an optimality perspective. In that case, government policy moves the economy in the wrong direction relative to the constrained optimum.

However, UI policies are desirable not only in terms of stabilisation but also in terms of optimality. Intratemporal optimality under UI becomes

$$\phi C_t = (1 - u_t) \left[1 + \gamma \frac{u_t}{1 - u_t} \right] (1 - \alpha) \frac{Y_t}{\Pi_t (1 - u_t)}. \tag{39}$$

Comparing (33) with (36), it is evident that UI, effectively, subsidises labor income. This policy moves the equilibrium allocations in the direction of the constrained optimum - however, it does not attain the constrained best. In particular, UI policies incentivise more participation and higher investment, and through the Okun's Law, the unemployment rate falls. In effect, aggregate demand management policies, like UI, are the optimal policies if the economy is stuck at the low unemployment steady state. This is consistent with Diamond (1982).

Consider the high unemployment steady state. We have shown that in that case the steady state is determinate since the unemployment rate is above the upper bound of the necessary condition (23). It follows that the elasticity of employment with respect to participation is negative. Substituting the competitive solution into (33), (34), we conclude that the high unemployment steady state is characterised by over-participation and over-investment relative to the constrained optimum.

At the high unemployment steady state, UI policies exacerbate the "rat race" phenomenon that we described before. In particular, UI polices increase participation and since $\xi(1-u, \Pi)$ is negative, the unemployment rate increases; since ξ, in absolute value, is greater than one, higher participation and higher unemployment lower the total employment and create an upward pressure on wages. Equivalently, when wages increase, firms lower the demand for labor and with more participants in the labor market, unemployment has to increase to restore equilibrium.

Appendix

A Decentralized Equilibrium and the Lottery Mechanism

This appendix explains in greater detail the institutional and market arrangements used to support the competitive equilibrium. The set-up follows the that of Hansen (1985) and also Andolfatto (1996), but extended to allow for three states $s \in \{1, 2, 3\}$ in the labor market: corresponding to employment, unemployment and non-participation, respectively. As in Andolfatto (1996), although individuals experience different employment histories, the existence of perfect insurance markets guarantee that labor income (net of insurance premia) will be independent of the employment history. Therefore, we may describe the problem of the stand-in agent. This appendix shows this formally.

Individuals choose a probability of participation each period, and conditional on participation face a probability of unemployment u, taken as given. As in Andolfatto (1996), individuals who participate in the labor market are exogenously shuffled around the available jobs regardless of employment history and, thus, u also corresponds to the unemployment rate. Those who participate in the labor market (either in employment or unemployed) give up $h_0 \in (0, 1)$ units of time. Finally, individuals have access to competitive insurance markets which implement complete markets allocations. In particular, at the start of each period, individuals may purchase the quantity of insurance y_2 and y_3, at price $p_2(\pi, u)$ and $p_3(\pi, u)$, where y_2 and y_3 are the units of consumption to be received in case of, respectively, unemployment and non-participation.

The problem solved by the stand-in agent can be represented as follows

$$\begin{aligned}\max_{\pi,c,y,k'} \mathbf{V}(k; u, K) = \pi(1-u)\left[\ln(c_1) + \psi \ln(1-h_0) + \beta \mathbf{V}(k'_1; u', K')\right] \\ + \pi u \left[\ln(c_2) + \psi \ln(1-h_0) + \beta \mathbf{V}(k'_2; u', K')\right] \\ + (1-\pi)\left[\ln(c_3) + \beta \mathbf{V}(k'_3; u', K')\right],\end{aligned} \tag{A.1}$$

subject to

$$c_1 + i_1 = w\,(u, K) + r\,(u, K)\,k - p_2\,(\pi, u)\,y_2 - p_3\,(\pi, u)\,y_3, \tag{A.2}$$

$$c_2 + i_2 = y_2 + r\,(u, K)\,k - p_2\,(\pi, u)\,y_2 - p_3\,(\pi, u)\,y_3, \tag{A.3}$$

$$c_3 + i_3 = y_3 + r\,(u, K)\,k - p_2\,(\pi, u)\,y_2 - p_3\,(\pi, u)\,y_3, \tag{A.4}$$

$$k'_s = (1-\delta)k + i_s, \quad \text{for} \quad s = 1, 2, 3, \tag{A.5}$$

where for simplicity, but without loss of generality, we consider a stationary economy with $\mathcal{G} = 1$ and without investment shocks, and where c_s, i_s and y_s are the allocations chosen contingent on the realization of state $s \in \{1, 2, 3\}$, that are employment, unemployment and non-participation, respectively.

The insurance company maximizes expected profits, given by

$$\wp = p_2\,(\pi, u)\,y_2 + p_3\,(\pi, u)\,y_3 - \pi u y_2 - (1-\pi)y_3, \tag{A.6}$$

and, with competitive insurance markets, we have that $p_2\,(\pi, u) = \pi u$, and $p_3\,(\pi, u) = (1-\pi)$.

Turning again to the stand-in agent's problem, we denote λ_1, λ_2 and λ_3 the multipliers of (A.2), (A.3) and (A.4). The first order condition with respect to y_2 and y_3 are

$$\lambda_1 = \lambda_2 \left(\frac{1-\pi u}{\pi u} \right) - \lambda_3, \tag{A.7}$$

$$\lambda_1 = \lambda_3 \left(\frac{\pi}{1-\pi} \right) - \lambda_2. \tag{A.8}$$

Combining these two necessary conditions obtains

$$\frac{\lambda_1}{\lambda_2} = \frac{1-u}{u}, \quad \frac{\lambda_2}{\lambda_3} = \left(\frac{\pi}{1-\pi} \right) u. \tag{A.9}$$

The first order conditions with respect to c_s, for $s \in \{1, 2, 3\}$, are

$$\pi(1-u) = \lambda_1 c_1, \quad \pi u = \lambda_2 c_2, \quad (1-\pi) = \lambda_3 c_3. \tag{A.10}$$

Combining (A.9) and (A.10) yields $c_1 = c_2 = c_3 = c$. From the envelope condition, we have that $1/c = \beta d\mathbf{V}(k_s; u, K)/dk_s = \mu$. Thus, this implies $k'_s = k'$ for all $s \in \{1, 2, 3\}$ and, given (A.5), $i_s = i$ for all $s \in \{1, 2, 3\}$. Finally, it follows from the budget constraints (A.2), (A.3) and (A.4) that $y_2 = y_3 = w$. Thus, the existence of perfect insurance markets guarantee that labor income (net of insurance premia) will be independent of the employment history.

It follows that the stand-in agent's problem can be equivalently written as follows

$$\max_{c_t, \pi_t} \mathbf{V}(k; u, K) = \max_{c, \pi, k'} \ln(c) + \psi \pi \ln(1 - h_0) + \beta \mathbf{V}(k'; u', K'), \tag{A.11}$$

with $c + k' - (1 - \delta) k = \pi (1 - u) w + rk$.

B Unemployment Insurance and the Lottery Mechanism

In addition to private insurance markets, unemployed households also benefit from government sponsored unemployment insurance. Specifically, the government replaces a fraction γ of wage income of the individuals who are unemployed. The government finances this policy by taxing everyone lump-sum, that is, subject to the following balanced-budget rule

$$T = \pi u \gamma w. \tag{B.1}$$

Compared to Appendix A, the stationary individual problem modifies as

$$\begin{aligned} \max_{\pi, c, y, k'} V(k; u, K) = {} & \pi(1 - u) \left[\ln(c_1) + \psi \ln(1 - h_0) + \beta V(k'_1; u', K')\right] \\ & + \pi u \left[\ln(c_2) + \psi \ln(1 - h_0) + \beta V(k'_2; u', K')\right] \\ & + (1 - \pi) \left[\ln(c_3) + \beta V(k'_3; u', K')\right], \end{aligned} \tag{B.2}$$

subject to

$$c_1 + i_1 = w(u, K) + r(u, K) k - p_2(\pi, u) y_2 - p_3(\pi, u) y_3 - T(\pi, u, K), \tag{B.3}$$

$$c_2 + i_2 = y_2 + \gamma w(u, K) + r(u, K) k - p_2(\pi, u) y_2 - p_3(\pi, u) y_3 - T(\pi, u, K), \tag{B.4}$$

$$c_3 + i_3 = y_3 + r(u, K) k - p_2(\pi, u) y_2 - p_3(\pi, u) y_3 - T(\pi, u, K), \tag{B.5}$$

$$k_s' = (1-\delta)k + i_s, \quad \text{for} \quad s = 1, 2, 3. \tag{B.6}$$

As in Appendix A, $c_1 = c_2 = c_3 = c$, $k_1' = k_2' = k_3' = k'$, and $i_1 = i_2 = i_3 = i$. Making use of the above budget constraints, the upshot is $y_2 = w(1-\gamma)$ and $y_3 = w$. Thus, the previous problem can be written as

$$\max_{c_t, \pi_t} \mathbf{V}(k; u, K) = \max_{c, \pi, k'} \ln(c) + \psi\pi \ln(1-h_0) + \beta \mathbf{V}(k'; u', K'), \tag{B.7}$$

with $c + k' - (1-\delta)k + T = \pi(1-u)w + \pi u \gamma w + rk$.

C Wage Determination

In this appendix we describe in greater detail the bargaining solution used to determine the wage received by workers that are successfully matched with a recruiter.

Each recruiter is matched to a single employee who supplies h_0 units of labor. In turn, each match produces one unit of the intermediate good (labor services), which is sold to final good producers at price w. Let ϖ denote the "hourly" wage received by the worker. We assume the Nash bargaining protocol, implying the following bargaining problem solved by each employer-employee pair

$$\max_{\varpi} \mathcal{H}(u, K)^{\chi} \mathcal{J}(u, K)^{1-\chi} \tag{C.1}$$

where $\mathcal{H}$ and $\mathcal{J}$ are the match surplus of, respectively, the employee and the employer, and with χ the employee's bargaining power.

The match surplus earned by the worker is

$$\mathcal{H}(u, K) = \mu(\varpi h_0 - (c_1 - c_2) - y_2) + \ln(c_1) - \ln(c_2), \tag{C.2}$$

where, following the notation in Appendix A, c_1 and c_2 are the consumption levels contingent on employment and unemployment, y_2 is the payment received by the private insurance in case of unemployment and μ is the marginal utility of wealth for the stand-in agent.

Next, the match surplus earned by the recruiter is

$$\mathcal{J}(u,K) = \mu(w - \varpi h_0) + \rho\beta\mu'\mathcal{J}(u',K'), \tag{C.3}$$

where ρ is the probability that the match is pursued the following period and x' denotes the continuation value of variable x.

The solution to the Nash bargaining problem must satisfy the necessary condition

$$\chi\mathcal{J}(u,K)\left(\frac{\partial\mathcal{H}}{\partial\varpi}\right) + (1-\chi)\mathcal{H}(u,K)\left(\frac{\partial\mathcal{J}}{\partial\varpi}\right) = 0. \tag{C.4}$$

We conjecture that an equilibrium solution to the bargaining problem of the is given by $\varpi h_0 = w$, no matter the bargaining power distribution, and proceed to verify this conjecture. First, notice that, as shown in Appendix A, in equilibrium $y_2 = w$ and $c_1 = c_2$. Thus, if $\varpi h_0 = w$, we have that $\mathcal{H}(u,K) = 0$, and condition (C.4) may be written as

$$\chi\mathcal{J}(u,K)\left(\frac{\partial\mathcal{H}}{\partial\varpi}\right) = 0. \tag{C.5}$$

In turn, if $\varpi h_0 = w$ the functional equation (C.3) becomes

$$\mathcal{J}(u,K) = \rho\beta\mu'\mathcal{J}(u',K'), \tag{C.6}$$

which has solution $\mathcal{J}(u,K) = 0$ for all (u,K). This solution is an admissible equilibrium solution, as it satisfies the free entry condition. In particular, since there are no costs of creating a vacancy, firms must have zero capital value. Finally, notice that with $\mathcal{J}(u,K) = 0$ condition (C.5) is satisfied and the conjectured solution is an equilibrium solution. Thus, although we allow for a bargaining protocol, the equilibrium outcome coincides with the competitive equilibrium in which workers are paid their marginal product. This is the upshot of two features of the economy: the zero cost of creating vacancies which forces the capital value of a job to zero, and the perfect insurance markets.

D Steady State

Let $\underline{X}$ denote the steady state of $\widetilde{X}$. For a given natural rate of unemployment $u^{\star} \in (0,1)$, the following equations uniquely characterize the steady state of the model

$$\underline{\mathcal{X}} = \exp\left(\frac{g^{v}}{\alpha - 1} - g^{a}\right), \tag{D.1}$$

$$(\underline{K}/\underline{Y}) = \left[\frac{\alpha}{1/\beta - (1-\delta)\,\underline{\mathcal{X}}}\right], \tag{D.2}$$

$$(\underline{C}/\underline{K}) = \left[\frac{1/\beta - \alpha - (1-\alpha)\,(1-\delta)\,\underline{\mathcal{X}}}{\alpha}\right], \tag{D.3}$$

$$(\underline{I}/\underline{K}) = 1 - (1-\delta)\,\underline{\mathcal{X}}, \tag{D.4}$$

$$\underline{\Pi} = \left[\frac{(1-\alpha)\,(\underline{Y}/\underline{C})}{\phi}\right], \tag{D.5}$$

$$\underline{K} = \underline{\Pi}\,(1-u^{\star})\,(\underline{K}/\underline{Y})^{1/(1-\alpha)}\,\underline{\mathcal{X}}^{\alpha/(1-\alpha)}. \tag{D.6}$$

Thus, if we do not specify a natural rate the model exhibits a multiplicity of steady state equilibria, for each possible $u^{\star}$. In turn, from (3) the natural rate of unemployment $u^{\star}$ is given by

$$u^{\star} = \frac{\mu \underline{Y}^{-\eta}}{1 + \mu \underline{Y}^{-\eta}}, \tag{D.7}$$

This equation has a unique interior solution for $\eta \in (0,1)$, and has at most two solutions.

E Log-linear Equilibrium Conditions

Let $\hat{x} \equiv \ln\left(\widetilde{X}/\underline{X}\right)$ denote the variable $\widetilde{X}$ in log-deviation from steady state. The log-linearized equilibrium conditions (around the deterministic steady state) are given by

$$\hat{\pi}_t + \hat{c}_t = \hat{y}_t, \tag{E.1}$$

$$E_t\left(\hat{c}_{t+1} - \hat{c}_t\right) = \alpha\beta\,(\underline{Y}/\underline{K})\,E_t\left(\hat{y}_{t+1} - \hat{k}_{t+1}\right), \tag{E.2}$$

$$\hat{y}_t = \alpha\left(\hat{k}_t - \epsilon_t^{a} - \frac{\epsilon_t^{v}}{1-\alpha}\right) + (1-\alpha)\,\hat{n}_t + z_t, \tag{E.3}$$

$$(1-u^{\star})\,\hat{n}_t = (1-u^{\star})\,\hat{\pi}_t - (u_t - u^{\star}), \tag{E.4}$$

$$(\underline{C}/\underline{K})\,\hat{c}_t + (\underline{I}/\underline{K})\,\hat{i}_t = (\underline{Y}/\underline{K})\,\hat{y}_t, \tag{E.5}$$

$$\hat{k}_{t+1} = (\underline{I}/\underline{K})\,\hat{i}_t + (1-\delta)\,\underline{\boldsymbol{\mathcal{X}}}\left(\hat{k}_t - \epsilon_t^a - \frac{\epsilon_t^v}{1-\alpha}\right), \tag{E.6}$$

$$u_t - u^\star = -\theta \hat{y}_t, \tag{E.7}$$

with the unemployment gap, $(u - u^\star)$, included in levels instead of logs.

Thus, the deterministic version of the model in log-linear form can be written as

$$\begin{bmatrix} \hat{k}_{t+1} \\ \hat{c}_{t+1} \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} = \Gamma \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} \tag{E.8}$$

with

$$\Gamma_{11} = \left[\frac{\alpha\,(\underline{Y}/\underline{K})}{\alpha - (1-\alpha)\,\sigma} + (1-\delta)\,\underline{\boldsymbol{\mathcal{X}}}\right],$$

$$\Gamma_{12} = -\left[\frac{(1-\alpha)\,(\underline{Y}/\underline{K})}{\alpha - (1-\alpha)\,\sigma} + (\underline{C}/\underline{K})\right],$$

$$\Gamma_{21} = \left[\frac{\alpha\,(1-\alpha)\,\beta\,(\underline{Y}/\underline{K})\,\sigma\Gamma_{11}}{\alpha - (1-\alpha)\,\sigma + \alpha\,(1-\alpha)\,\beta\,(\underline{Y}/\underline{K})}\right],$$

$$\Gamma_{22} = \left[\frac{\alpha - (1-\alpha)\,\sigma + \alpha\,(1-\alpha)\,\beta\,(\underline{Y}/\underline{K})\,\sigma\Gamma_{12}}{\alpha - (1-\alpha)\,\sigma + \alpha\,(1-\alpha)\,\beta\,(\underline{Y}/\underline{K})}\right].$$

and where $\sigma = \eta u^\star$. The eigenvalues of the matrix Γ are

$$\begin{bmatrix} \kappa_1 \\ \kappa_2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \Gamma_{11} + \Gamma_{22} - \sqrt{(\Gamma_{11}+\Gamma_{22})^2 - 4\,(\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21})} \\ \Gamma_{11} + \Gamma_{22} + \sqrt{(\Gamma_{11}+\Gamma_{22})^2 - 4\,(\Gamma_{11}\Gamma_{22} - \Gamma_{12}\Gamma_{21})} \end{bmatrix}. \tag{E.9}$$

Indeterminacy arises if the real part of both eigenvalues are less than one in absolute value.

F Proof of Proposition 2

Following the approach of Wen (2001), consider the consumption of the stand-in household at date t

$$c_t = (1-\beta)\left[R_t k_t + w_t\left(1-u_t\right)\pi_t + \sum_{j=1}^{\infty}\left(\prod_{i=1}^{j} R_{t+i}^{-1}\right) w_{t+j}\left(1-u_{t+j}\right)\pi_{t+j}\right], \tag{F.1}$$

where $R_t = r_t + (1-\delta)$ is the gross rate of return on capital. Making use of the intratemporal condition (9) and the market clearing conditions, we obtain

$$\widetilde{C}_t = (1-\beta)\left[\left(R_t/\mathcal{G}\right)\widetilde{K}_t + \phi\widetilde{C}_t\Pi_t + \phi\sum_{j=1}^{\infty}\left(\prod_{i=1}^{j} R_{t+i}^{-1}\right)\mathcal{G}^j\widetilde{C}_{t+j}\Pi_{t+j}\right]. \tag{F.2}$$

In turn, solving forward the aggregate Euler equation yields

$$\left(\prod_{i=1}^{j} R_{t+j}^{-1}\right)\mathcal{G}^j\widetilde{C}_{t+j} = \beta^j\widetilde{C}_t. \tag{F.3}$$

Substituting the latter into the previous relationship, we obtain

$$\widetilde{C}_t = \left(R_t/\mathcal{G}\right)\widetilde{K}_t\left[\sum_{j=0}^{\infty}\beta^j\left(1-\phi\Pi_{t+j}\right)\right]^{-1}. \tag{F.4}$$

Finally, substituting for consumption using the intra-temporal first order condition and market clearing conditions, yields

$$\mathcal{G}\sum_{j=0}^{\infty}\beta^j\left(1-\phi\Pi_{t+j}\right) = \frac{\phi R\left(u_t,\Pi_t\right)\widetilde{K}_t}{(1-u_t)\widetilde{w}\left(u_t,\Pi_t\right)}, \tag{F.5}$$

with $R\left(u_t,\Pi_t\right) = (1-\delta) + \alpha\left[\frac{(1-u_t)\mathcal{G}\Pi_t}{\underline{K}}\right]^{1-\alpha}$, and $\widetilde{w}\left(u_t,\Pi_t\right) = (1-\alpha)\left[\frac{\underline{K}}{\mathcal{G}(1-u_t)\Pi_t}\right]^{\alpha}$.

We want to obtain necessary conditions for local indeterminacy. Thus, the next step is to log-linearise the difference equation (F.5) around the steady state given the initial condition

$\widetilde{K}_t = \underline{K}$. The log-linear version of (F.5) is

$$\frac{-(1-\beta)\,\phi\underline{\Pi}}{1-\phi\underline{\Pi}}\sum_{j=0}^{\infty}\beta^j\hat{\pi}_{t+j} = \left(\xi\left(\underline{R},\underline{\Pi}\right) - \xi\left(1-u^\star,\underline{\Pi}\right) - \xi\left(\underline{w},\underline{\Pi}\right)\right)\hat{\pi}_t, \tag{F.6}$$

where $\xi\left(x,y\right) = \frac{dx}{dy}\left(y/x\right)$ denotes the total elasticity.

The relevant elasticities are given by

$$\xi\left(\underline{R},\underline{\Pi}\right) = \left[\frac{1-\alpha-\beta\left(1-\alpha\right)\left(1-\delta\right)\underline{\mathcal{X}}}{1-\eta(1-\alpha)u^\star}\right], \tag{F.7}$$

$$\xi\left(1-u^\star,\underline{\Pi}\right) = \left[\frac{\eta\left(1-\alpha\right)u^\star}{1-\eta(1-\alpha)u^\star}\right], \tag{F.8}$$

$$\xi\left(\underline{w},\underline{\Pi}\right) = -\left[\frac{\alpha}{1-\eta(1-\alpha)u^\star}\right], \tag{F.9}$$

where (F.8), in particular, is obtained from the total differentiation of (3).

Starting from an equilibrium path, a small deviation of Π_t from this path will not violate (F.6) if and only if the change in both sides of this equation caused by the change in Π_t are exactly the same. Since the elasticity of the left-hand side of equation (F.6) is negative, a necessary condition for local indeterminacy is that

$$\xi\left(\underline{R},\underline{\Pi}\right) - \xi\left(1-u^\star,\underline{\Pi}\right) - \xi\left(\underline{w},\underline{\Pi}\right) \leq 0, \tag{F.10}$$

and condition (F.10) is equivalent to

$$\left[\frac{(1-\alpha)(1-\delta)\left(\beta/\mathcal{G}\right)}{1-\eta(1-\alpha)u^\star}\right] \geq 1, \tag{F.11}$$

This condition is satisfied if

$$\eta u^\star \in \left[\frac{1-\beta\left(1-\alpha\right)\left(1-\delta\right)\underline{\mathcal{X}}}{1-\alpha}, \frac{1}{1-\alpha}\right]. \tag{F.12}$$

G Proof of Proposition 3

To derive a necessary condition, we follow the logic of proposition 2. The consumption of the stand-in household at date t

$$c_t = (1-\beta)\Bigg[R_t k_t + w_t\left(1-u_t\right)\pi_t + \gamma w_t \pi_t u_t - T_t +$$
$$\sum_{j=1}^{\infty}\left(\prod_{i=1}^{j} R_{t+i}^{-1}\right)\left(w_{t+j}\left(1-u_{t+j}\right)\pi_{t+j} + \gamma w_{t+j}\pi_{t+j}u_{t+j} - T_{t+j}\right)\Bigg], \tag{G.1}$$

Making use of the intratemporal condition and the market clearing conditions, we obtain

$$C_t = (1-\beta)\left[R_t K_t + \phi C_t \Pi_t - T_t + \sum_{j=1}^{\infty}\left(\prod_{i=1}^{j} R_{t+i}^{-1}\right)\left(\phi C_{t+j}\Pi_{t+j} - T_{t+j}\right)\right], \tag{G.2}$$

Solving the Euler forward and substituting into (G.8), we obtain

$$C_t = (1-\beta)\left[R_t K_t + \phi C_t \Pi_t - T_t + \phi C_t \sum_{j=1}^{\infty}\beta^j \Pi_{t+j} - C_t\sum_{j=1}^{\infty}\beta^j \frac{1}{C_{t+j}}T_{t+j}\right], \tag{G.3}$$

which simplifies to

$$\sum_{j=0}^{\infty}\beta^j\left(1-\phi\Pi_{t+j}\right) + \sum_{j=0}^{\infty}\beta^j\frac{T_{t+j}}{C_{t+j}} = \frac{R_t K_t}{C_t} \tag{G.4}$$

Next, substituting the government's budget and the intratemporal condition, we obtain

$$\sum_{j=0}^{\infty}\beta^j\left(1-\phi\Pi_{t+j}\right) + \sum_{j=0}^{\infty}\beta^j\phi\Pi_{t+j}\frac{\gamma\frac{u_{t+j}}{1-u_{t+j}}}{1+\gamma\frac{u_{t+j}}{1-u_{t+j}}} = \frac{\phi R_t K_t}{(1-u_t)w_t + \gamma w_t u_t}. \tag{G.5}$$

Log-linearising (G.11), we obtain that the left hand side has a negative elasticity - we verify this at the end. As a result, we need to determine the sign of the right hand side. Log-linearising the right hand side, we obtain

$$\left(\xi\left(\underline{R},\underline{\Pi}\right)-\left(1-\frac{\gamma}{1-u^{\star}(1-\gamma)}\right)\xi\left(1-u^{\star},\underline{\Pi}\right)-\xi\left(\underline{w},\underline{\Pi}\right)\right)\hat{\pi}_t. \tag{G.6}$$

A necessary condition for local indeterminacy is

$$\left(\xi\left(\underline{R},\underline{\Pi}\right)-\left(1-\frac{\gamma}{1-u^{\star}(1-\gamma)}\right)\xi\left(1-u^{\star},\underline{\Pi}\right)-\xi\left(\underline{w},\underline{\Pi}\right)<0 \tag{G.7}$$

or, equivalently,

$$\eta u^{\star}\in\left[\frac{1-\beta\left(1-\alpha\right)\left(1-\delta\right)\underline{\mathcal{X}}}{\left(1-\alpha\right)\left(1-\frac{\gamma}{1-u^{\star}(1-\gamma)}\right)},\frac{1}{1-\alpha}\right]. \tag{G.8}$$

Finally, to complete the proof we need the following restrictions:

$$\frac{\partial K_{t+j}}{\partial \Pi_{t+j-1}}>0,\ \ \frac{\partial K_{t+j}}{\partial K_{t+j-1}}>0, \frac{\partial u_t}{\partial \Pi_t}<0,\ \ \frac{\partial u_t}{\partial K_t}<0. \tag{G.9}$$

From the Okun's Law, it is straightforward to see that the last two restrictions of (G.9) are satisfied. For the first two, combine the resource constraint and intratemporal condition, to get

$$K_{t+j}=Y_{t+j-1}\left(1-\frac{1-\alpha}{\phi\Pi_{t+j-1}}\left(1+\gamma\left(\frac{1}{1-u_{t+j-1}}-1\right)\right)\right)+(1-\delta)K_{t+j-1}. \tag{G.10}$$

From (G.10), it is straightforward to see that the first two restrictions of (G.9) hold.

Log-linearising the left hand side of (G.5), we obtain

$$\frac{1-\beta}{1-\phi\underline{\Pi}+\phi\underline{\Pi}\frac{\gamma\frac{u^{\star}}{1-u^{\star}}}{1+\gamma\frac{u^{\star}}{1-u^{\star}}}}\left[-\phi\underline{\Pi}\left(1-\frac{\gamma\frac{u^{\star}}{1-u^{\star}}}{1+\frac{\gamma u^{\star}}{1-u^{\star}}}\right)\sum_{j=0}^{\infty}\beta^j\widehat{\pi}_{t+j}+\right.$$
$$\frac{\gamma\underline{\Pi}\widehat{\pi}_t}{(1-u^{\star})^2+\gamma^2(u^{\star})^2+2\gamma u^{\star}(1-u^{\star})}\left(\frac{\partial u_t}{\partial \pi_t}+\beta\frac{\partial u_{t+1}}{\partial K_{t+1}}\frac{\partial K_{t+1}}{\partial \pi_t}+\beta^2\frac{\partial u_{t+2}}{\partial K_{t+2}}\frac{\partial K_{t+2}}{\partial K_{t+1}}\frac{\partial K_{t+1}}{\partial \pi_t}+\cdots\right)+$$

$$\frac{\beta\gamma\underline{\Pi}\widehat{\pi}_{t+1}}{(1-u^{\star})^2+\gamma^2(u^{\star})^2+2\gamma u^{\star}(1-u^{\star})}\left(\frac{\partial u_{t+1}}{\partial \pi_{t+1}}+\beta\frac{\partial u_{t+2}}{\partial K_{t+2}}\frac{\partial K_{t+2}}{\partial \pi_{t+1}}+\beta^2\frac{\partial u_{t+3}}{\partial K_{t+3}}\frac{\partial K_{t+3}}{\partial K_{t+2}}\frac{\partial K_{t+2}}{\partial \pi_{t+1}}+\cdots\right)+ \tag{G.11}$$

$$\cdots+\cdots\Bigg] \tag{G.12}$$

H Proof of Proposition 4

Additional Appendix (not for publication)

Log-linear Equilibrium Conditions (deterministic model)

Let $\hat{x} \equiv \ln\left(\widetilde{X}/\underline{X}\right)$ denote the variable $\widetilde{X}$ in log-deviation from steady state. The log-linearized equilibrium conditions are given by

$$\begin{aligned}
\hat{\pi}_t + \hat{c}_t &= \hat{y}_t, \\
E_t\left(\hat{c}_{t+1} - \hat{c}_t\right) &= \alpha\beta\left(\underline{Y}/\underline{K}\right) E_t\left(\hat{y}_{t+1} - \hat{k}_{t+1}\right), \\
\hat{y}_t &= \alpha\hat{k}_t + (1-\alpha)\,\hat{n}_t, \\
(1-u^\star)\,\hat{n}_t &= (1-u^\star)\,\hat{\pi}_t - (u_t - u^\star), \\
\left(\underline{C}/\underline{K}\right)\hat{c}_t + \left(\underline{I}/\underline{K}\right)\hat{i}_t &= \left(\underline{Y}/\underline{K}\right)\hat{y}_t, \\
\hat{k}_{t+1} &= \left(\underline{I}/\underline{K}\right)\hat{i}_t + (1-\delta)\,\underline{\mathcal{X}}\hat{k}_t, \\
u_t - u^\star &= -\theta\hat{y}_t,
\end{aligned}$$

with the unemployment gap, $(u - u^\star)$, included in levels instead of logs. From the intratemporal f.o.c., the production function and the labor market clearing condition, we obtain

$$\alpha\,(1-u^\star)\,\hat{y}_t = \alpha\,(1-u^\star)\,\hat{k}_t - (1-\alpha)\,(1-u^\star)\,\hat{c}_t - (1-\alpha)\,(u_t - u^\star),$$

and plugging the Okun relationship obtains

$$\alpha\hat{y}_t = \alpha\hat{k}_t - (1-\alpha)\,\hat{c}_t + (1-\alpha)\,\sigma\hat{y}_t,$$

with $\sigma = \eta u^\star$. Solving for $\hat{y}_t$ yields

$$\hat{y}_t = \left[\frac{\alpha}{\alpha - (1-\alpha)\,\sigma}\right]\hat{k}_t - \left[\frac{1-\alpha}{\alpha - (1-\alpha)\,\sigma}\right]\hat{c}_t.$$

Next, we make use of the above equation to substitute for $\hat{y}_t$ and $\hat{y}_{t+1}$ in, respectively, the resources constraint and the Euler equation, as follows

$$
\begin{aligned}
\hat{k}_{t+1} &= (\underline{Y}/\underline{K})\,\hat{y}_t - (\underline{C}/\underline{K})\,\hat{c}_t + (1-\delta)\,\underline{\mathcal{X}}\hat{k}_t, \\
&= \left[\frac{\alpha\,(\underline{Y}/\underline{K})}{\alpha-(1-\alpha)\,\sigma} + (1-\delta)\,\underline{\mathcal{X}}\right]\hat{k}_t - \left[\frac{(1-\alpha)\,(\underline{Y}/\underline{K})}{\alpha-(1-\alpha)\,\sigma} + (\underline{C}/\underline{K})\right]\hat{c}_t \\
&= \Gamma_{11}\hat{k}_t + \Gamma_{12}\hat{c}_t.
\end{aligned}
$$

and

$$
\begin{aligned}
E_t\left(\hat{c}_{t+1}-\hat{c}_t\right) &= \alpha\beta\,(\underline{Y}/\underline{K})\,E_t\left(\hat{y}_{t+1}-\hat{k}_{t+1}\right), \\
&= \left[\frac{\alpha\,(1-\alpha)\,\beta\,(\underline{Y}/\underline{K})}{\alpha-(1-\alpha)\,\sigma}\right]\left(\sigma\hat{k}_{t+1}-\hat{c}_{t+1}\right).
\end{aligned}
$$

Finally, combining the above equations we obtain

$$
\begin{aligned}
E_t\left(\hat{c}_{t+1}-\hat{c}_t\right) &= \left[\frac{\alpha\,(1-\alpha)\,\beta\,(\underline{Y}/\underline{K})}{\alpha-(1-\alpha)\,\sigma}\right]\left(\sigma\hat{k}_{t+1}-E_t\hat{c}_{t+1}\right), \\
&= \left[\frac{\alpha\,(1-\alpha)\,\beta\,(\underline{Y}/\underline{K})}{\alpha-(1-\alpha)\,\sigma}\right]\left(\sigma\Gamma_{11}\hat{k}_t+\sigma\Gamma_{12}\hat{c}_t-E_t\hat{c}_{t+1}\right),
\end{aligned}
$$

$$
\begin{aligned}
E_t\left(\hat{c}_{t+1}\right) &= \left[\frac{\alpha\beta\,(\underline{Y}/\underline{K})\,\sigma\Gamma_{11}}{\alpha/\,(1-\alpha)-\sigma+\alpha\beta\,(\underline{Y}/\underline{K})}\right]\hat{k}_t + \left[\frac{\alpha/\,(1-\alpha)-\sigma+\alpha\beta\,(\underline{Y}/\underline{K})\,\sigma\Gamma_{12}}{\alpha/\,(1-\alpha)-\sigma+\alpha\beta\,(\underline{Y}/\underline{K})}\right]\hat{c}_t, \\
&= \Gamma_{21}\hat{k}_t + \Gamma_{22}\hat{c}_t.
\end{aligned}
$$

Additional derivations and results

Consider the following total derivative

$$
\begin{aligned}
\frac{dY}{d\Pi} &= (1-\alpha)\,K^{\alpha}\,(1-u)^{-\alpha}\,\Pi^{-\alpha}\left[1-u+\frac{d\,(1-u)}{d\Pi}\Pi\right], \\
&= (1-\alpha)\,Y\left[\frac{1}{\Pi}+\frac{\theta}{Y}\left(\frac{dY}{d\Pi}\right)\frac{1}{1-u}\right].
\end{aligned}
$$

Next, from (3), we obtain

$$\begin{aligned}
\frac{d\,(1-u)}{d\Pi} &= \frac{\eta Y^{-\eta-1}}{\left(1+Y^{-\eta}\right)^2}\left(\frac{dY}{d\Pi}\right), \\
&= \frac{\eta}{1+Y^{-\eta}}\frac{Y^{-\eta-1}}{1+Y^{-\eta}}\left(\frac{dY}{d\Pi}\right), \\
&= \theta\left(\frac{dY}{d\Pi}\right)Y^{-1} = \theta\left(\frac{dY}{d\Pi}\frac{\Pi}{Y}\right)\Pi^{-1}.
\end{aligned}$$

Combining equations (H) and (H) yields

$$\frac{dY}{d\Pi}\frac{\Pi}{Y} = \left[\frac{(1-\alpha)\,(1-u)}{1-u-\theta\,(1-\alpha)}\right],$$

the total elasticity of output to changes in participation.

Making use of the above result, we obtain

$$\begin{aligned}
\frac{dR}{d\Pi} &= \alpha\left(\frac{dY}{d\Pi}\right)K^{-1}, \\
&= \alpha\left[\frac{(1-\alpha)\,(1-u)\,(Y/K)}{1-u-\theta\,(1-\alpha)}\right]\Pi^{-1},
\end{aligned}$$

and, using the fact that $R = 1-\delta+\alpha\,(Y/K)$, yields

$$\frac{dR}{d\Pi}\frac{\Pi}{R} = \left[\frac{(1-\alpha)\,(1-u)}{1-u-\theta\,(1-\alpha)}\right]\left[\frac{\alpha\,(Y/K)}{1-\delta+\alpha\,(Y/K)}\right].$$

Similarly, making use of the fact that $w = (1-\alpha)\,(Y/\Pi)\,/\,(1-u)$, we obtain

$$\begin{aligned}
\frac{dw}{d\Pi} &= \frac{(1-\alpha)}{(1-u)\,\Pi}\left(\frac{dY}{d\Pi}\right) - \frac{(1-\alpha)\,Y}{(1-u)^2\,\Pi^2}\left[1-u+\frac{d\,(1-u)}{d\Pi}\Pi\right], \\
&= \frac{w}{(1-u)\,\Pi}\left[\frac{dY}{d\Pi}\frac{\Pi}{Y}\,(1-u-\theta)-(1-u)\right],
\end{aligned}$$

which yields the total elasticity

$$
\begin{aligned}
\frac{dw}{d\Pi}\frac{\Pi}{w} &= \frac{dY}{d\Pi}\frac{\Pi}{Y}\frac{1-u-\theta}{1-u} - 1, \\
&= \left[\frac{(1-\alpha)(1-u-\theta)}{1-u-\theta(1-\alpha)}\right] - 1, \\
&= -\left[\frac{\alpha(1-u)}{1-u-\theta(1-\alpha)}\right].
\end{aligned}
$$

Finally, notice that

$$
\begin{aligned}
\frac{du}{d\Pi}\frac{\Pi}{u} &= -\frac{d(1-u)}{d\Pi}\frac{\Pi}{1-u}, \\
&= -\theta\left(\frac{dY}{d\Pi}\frac{\Pi}{Y}\right)\frac{1}{1-u}, \\
&= -\left[\frac{\theta(1-\alpha)}{1-u-\theta(1-\alpha)}\right].
\end{aligned}
$$

Return to Capital (deterministic BGP)

As long as $g^v > 0$, the rental cost of capital $r_t = \alpha(Y_t/K_t)$ and, hence, also the capital to output ratio decline over time. However, the cost of investment in terms of the consumption good also declines at the same rate. Thus, the net rate of return on capital ρ remains constant over time, and satisfies the condition

$$
1+\rho = \left[(1-\delta)\frac{V_t}{V_{t+1}} + \alpha\left(\underline{Y}/\underline{K}\right)\frac{\Omega_{t+1}}{\Omega_t}\right],
$$

with

$$
\left(\underline{Y}/\underline{K}\right) = \left[\frac{1/\beta - (1-\delta)\underline{\mathcal{X}}}{\alpha}\right].
$$

In the calibration we set $(1+\rho)^4 = 1.0516$, to match an annual rate of return on capital of 5.16% reported by Gomme et al. (2011), and solve for the implied discount factor β.

Proposition 5 *Consider a baseline economy without UI, and suppose the baseline economy is parameterized such that there are multiple equilibrium paths converging to the same BGP (local indeterminacy). It is always possible to transform the baseline economy by introducing an UI policy in order to guarantee a unique equilibrium path around the BGP. In particular,* *the full replacement UI policy always guarantees uniqueness. Moreover, there exists a range of replacement ratios* $\gamma \in \left(\underline{\gamma}, 1\right]$ *for which there is a unique saddle path stable equilibrium.*

Proof. First, we compute the new steady state with UI and then we verify the claim in Proposition **??**. The new steady state modifies as

$$(\underline{K}/\underline{Y}) = \left[\frac{\alpha}{1/\beta - (1-\delta)\,\underline{\mathcal{X}}}\right], \tag{H.1}$$

$$\underline{\Pi} = \left[\frac{(1-\alpha)\,(\underline{Y}/\underline{C})}{\phi}\right]\frac{1-u^{\star}(1-\gamma)}{1-u^{\star}}, \tag{H.2}$$

$$\underline{K} = \underline{\Pi}\,(1-u^{\star})\,(\underline{K}/\underline{Y})^{1/(1-\alpha)}\,\underline{\mathcal{X}}^{\alpha/(1-\alpha)}. \tag{H.3}$$

As in the model without UI, steady state unemployment is given by

$$u^{\star} = \frac{\mu \underline{Y}^{-\eta}}{1+\mu \underline{Y}^{-\eta}}, \tag{H.4}$$

The log-linear equilibrium conditions are identical to those for the economy without UI, except for the intratemporal condition governing consumption and leisure choices, which is now given by

$$\widehat{c}_t + \widehat{\pi}_t = \widehat{y}_t + \left[\frac{\gamma}{(1-u^{\star})\,(1-u^{\star}\,(1-\gamma))}\right](u_t - u^{\star})\,. \tag{H.5}$$

Thus, the system of equilibrium conditions is still given by (E.8), with the only difference that the coefficient σ is now given by

$$\sigma = \left[\frac{\theta\,(1-\gamma)}{1-u^{\star}\,(1-\gamma)}\right].$$

We now turn to Proposition **??**. Consider a baseline economy without UI, which is obtained when $\gamma = 0$, and let the parametrization of this economy be such that both eigenvalues κ_1 and κ_2 (defined in Appendix E) are inside the unit circle. Thus, in the baseline economy there is local indeterminacy of equilibrium around the BGP. Next, consider an otherwise identical economy, but for which there is UI with full replacement ratio, so that $\gamma = 1$. For

$\gamma = 1$, we obtain $\sigma = 0$ and the eigenvalues κ_1 and κ_2 simplify as

$$\begin{aligned} \kappa_1 &= \Gamma_{11} > 1, \\ \kappa_2 &= \Gamma_{22} \in (0,1), \end{aligned} \tag{H.6}$$

given $(\beta/\mathcal{G}) \in (0,1)$. Therefore, the economy with full UI replacement ratio ($\gamma = 1$) must be locally determinate (saddle path stable).

Finally, since the eigenvalues are continuous functions of γ, it follows that, in the neighborhood of the economy with full UI replacement ratio, there exists a range of $\gamma \in \left(\underline{\gamma}, 1\right]$ for which the equilibrium is saddle path stable. ■

Proposition 6 *Suppose the steady state tax rate satisfies the following relation:*

$$\frac{\underline{\tau}\frac{\alpha}{1-\alpha}\frac{1}{u^\star}}{1+\frac{\alpha}{1-\alpha}\underline{\tau}} + \frac{1-\beta(1-\delta)\underline{\mathcal{X}}}{1-\underline{\tau}}\underline{\tau}\frac{1}{u^\star} < (1-\alpha)\beta(1-\delta)\underline{\mathcal{X}} < 1. \tag{H.7}$$

Then, a necessary condition for indeterminacy is

$$\xi\left(\underline{R}, \underline{\Pi}\right) - \left(1 - \frac{\gamma}{1-u^\star(1-\gamma)}\right)\xi\left(1-u^\star, \underline{\Pi}\right) - \xi\left(\underline{w}, \underline{\Pi}\right) < 0. \tag{H.8}$$

Condition (31) imposes restrictions on η, that is,

$$\eta u^\star \in \left(\frac{1}{1-\alpha}\frac{1-(1-\alpha)\beta(1-\delta)\underline{\mathcal{X}}}{1-\frac{\underline{\tau}\frac{\alpha}{1-\alpha}\frac{1}{u^\star}}{1+\frac{\alpha}{1-\alpha}\underline{\tau}} - \frac{1-\beta(1-\delta)\underline{\mathcal{X}}}{1-\underline{\tau}}\underline{\tau}\frac{1}{u^\star}}, \frac{1}{1-\alpha}\right); \tag{H.9}$$

with

$$\underline{\tau} = \gamma\frac{1-\alpha}{\alpha}\frac{u^\star}{1-u^\star}. \tag{H.10}$$

Proof. Following a similar argument as with the proof of proposition 2, we obtain

$$\mathcal{G}\sum_{j=0}^{\infty}\beta^j\left(1-\phi\Pi_{t+j}\right) = \frac{\phi R\left(u_t, \Pi_t\right)\widetilde{K}_t}{(1-u_t)(1+\gamma(\frac{1}{1-u_t}-1))\widetilde{w}\left(u_t, \Pi_t\right)}, \tag{H.11}$$

with $R\left(u_t, \Pi_t\right) = (1-\delta)\,\mathcal{X}_t + (1-\tau_t)\alpha\left[\frac{(1-u_t)\mathcal{G}\Pi_t}{\underline{K}}\right]^{1-\alpha}$, $\widetilde{w}\left(u_t, \Pi_t\right) = (1-\alpha)\left[\frac{\underline{K}}{\mathcal{G}(1-u_t)\Pi_t}\right]^{\alpha}$ and $\tau_t = \gamma\frac{1-\alpha}{\alpha}\frac{u_t}{1-u_t} = \gamma\frac{1-\alpha}{\alpha}\left(\frac{1}{1-u_t}-1\right)$.

We want to obtain necessary conditions for local indeterminacy. Thus, the next step is to log-linearise the difference equation (I.1) around the steady state given the initial condition $\widetilde{K}_t = \underline{K}$. The log-linear version of (I.1) is

$$\frac{-\left(1-\beta\right)\phi\underline{\Pi}}{1-\phi\underline{\Pi}}\sum_{j=0}^{\infty}\beta^j\hat{\pi}_{t+j} = \left(\xi\left(\underline{R},\underline{\Pi}\right) - \left(1-\frac{\gamma}{1-u^\star(1-\gamma)}\right)\xi\left(1-u^\star,\underline{\Pi}\right) - \xi\left(\underline{w},\underline{\Pi}\right)\right)\hat{\pi}_t. \tag{H.12}$$

The relevant elasticities are given by

$$\begin{aligned}\xi\left(1-u^\star,\underline{\Pi}\right) &= \left[\frac{\eta\left(1-\alpha\right)u^\star}{1-\eta(1-\alpha)u^\star}\right],\\ \xi\left(\underline{w},\underline{\Pi}\right) &= -\left[\frac{\alpha}{1-\eta(1-\alpha)u^\star}\right],\end{aligned}$$

and $\xi\left(\underline{R},\underline{\Pi}\right)$ is given by

$$\begin{aligned}\xi\left(\underline{R},\underline{\Pi}\right) = &\frac{1}{\underline{R}}\left[-\alpha\frac{\underline{Y}}{\underline{K}}\frac{\partial\tau_t}{\partial(1-u_t)}\frac{\partial(1-u_t)}{\partial\pi_t}\underline{\Pi} + (1-\underline{\tau})(1-\alpha)\alpha\frac{\underline{Y}}{\underline{K}} + \alpha\frac{\underline{Y}}{\underline{K}}(1-\underline{\tau})(1-\alpha)\frac{\partial(1-u_t)}{\partial\pi_t}\frac{\underline{\Pi}}{1-u^\star}\right]\\ =&\alpha\beta\frac{\underline{Y}}{\underline{K}}\left[-\underline{\tau}\xi(\underline{\tau},1-u^\star)\xi(1-u^\star,\underline{\Pi}) + (1-\underline{\tau})(1-\alpha)(1+\xi(1-u^\star,\underline{\Pi}))\right]\\ =&\alpha\beta\frac{\underline{Y}}{\underline{K}}\left[-\underline{\tau}(-\frac{1}{u^\star})\frac{\eta\left(1-\alpha\right)u^\star}{1-\eta(1-\alpha)u^\star} + (1-\underline{\tau})(1-\alpha)\frac{1}{1-\eta(1-\alpha)u^\star}\right]\\ =&\frac{1-\beta(1-\delta)\underline{\mathcal{X}}}{1-\underline{\tau}}\left[\underline{\tau}\frac{1}{u^\star}\frac{\eta\left(1-\alpha\right)u^\star}{1-\eta(1-\alpha)u^\star} + (1-\underline{\tau})(1-\alpha)\frac{1}{1-\eta(1-\alpha)u^\star}\right].\end{aligned}$$

Following the same logic as in proposition 2, a necessary condition for indeterminacy is given by

$$\xi\left(\underline{R},\underline{\Pi}\right) - \left(1-\frac{\gamma}{1-u^\star(1-\gamma)}\right)\xi\left(1-u^\star,\underline{\Pi}\right) - \xi\left(\underline{w},\underline{\Pi}\right) < 0. \tag{H.13}$$

Substituting into (I.3), we obtain

$$\frac{1-\beta(1-\delta)\underline{\mathcal{X}}}{1-\underline{\tau}}\left[\underline{\tau}\frac{1}{u^\star}\frac{\eta\,(1-\alpha)\,u^\star}{1-\eta(1-\alpha)u^\star}+(1-\underline{\tau})(1-\alpha)\frac{1}{1-\eta(1-\alpha)u^\star}\right]-$$
$$\left(1-\frac{\underline{\tau}\frac{\alpha}{1-\alpha}\frac{1}{u^\star}}{1+\frac{\alpha}{1-\alpha}\underline{\tau}}\right)\frac{\eta\,(1-\alpha)\,u^\star}{1-\eta(1-\alpha)u^\star}+\frac{\alpha}{1-\eta(1-\alpha)u^\star}<0, \tag{H.14}$$

or, equivalently,

$$\frac{1}{1-\eta(1-\alpha)u^\star}\left[1-(1-\alpha)\beta(1-\delta)\underline{\mathcal{X}}-\eta(1-\alpha)u^\star\left(1-\frac{\underline{\tau}\frac{\alpha}{1-\alpha}\frac{1}{u^\star}}{1+\frac{\alpha}{1-\alpha}\underline{\tau}}-\frac{1-\beta(1-\delta)\underline{\mathcal{X}}}{1-\underline{\tau}}\underline{\tau}\frac{1}{u^\star}\right)\right]<0. \tag{H.15}$$

If the steady state tax rate is bounded above, that is,

$$\frac{\underline{\tau}\frac{\alpha}{1-\alpha}\frac{1}{u^\star}}{1+\frac{\alpha}{1-\alpha}\underline{\tau}}+\frac{1-\beta(1-\delta)\underline{\mathcal{X}}}{1-\underline{\tau}}\underline{\tau}\frac{1}{u^\star}<(1-\alpha)\beta(1-\delta)\underline{\mathcal{X}}<1, \tag{H.16}$$

then (I.6) is satisfied if and only if

$$\eta u^\star\in\left[\frac{1}{1-\alpha}\frac{1-(1-\alpha)\beta(1-\delta)\underline{\mathcal{X}}}{1-\frac{\underline{\tau}\frac{\alpha}{1-\alpha}\frac{1}{u^\star}}{1+\frac{\alpha}{1-\alpha}\underline{\tau}}-\frac{1-\beta(1-\delta)\underline{\mathcal{X}}}{1-\underline{\tau}}\underline{\tau}\frac{1}{u^\star}},\frac{1}{1-\alpha}\right]; \tag{H.17}$$

(I.7) guarantees that (I.8) is a well-defined interval. ■

Constant income taxes

Suppose the government collects income taxes and distribute subsidies:

$$\tau Y_t = T_t.$$

To derive a necessary condition, we follow the logic of proposition 2. The consumption of the stand-in household at date t

$$c_t = (1-\beta)\left[R_t k_t + w_t\left(1-u_t\right)\pi_t + T_t + \sum_{j=1}^{\infty}\left(\prod_{i=1}^{j} R_{t+i}^{-1}\right)\left(w_{t+j}\left(1-u_{t+j}\right)\pi_{t+j} + T_{t+j}\right)\right], \tag{H.18}$$

Making use of the intratemporal condition and the market clearing conditions, we obtain

$$C_t = (1-\beta)\left[R_t K_t + \phi C_t \Pi_t + T_t + \sum_{j=1}^{\infty}\left(\prod_{i=1}^{j} R_{t+i}^{-1}\right)\left(\phi C_{t+j}\Pi_{t+j} + T_{t+j}\right)\right], \tag{H.19}$$

Solving the Euler forward and substituting into (G.8), we obtain

$$C_t = (1-\beta)\left[R_t K_t + \phi C_t \Pi_t + T_t + \phi C_t \sum_{j=1}^{\infty}\beta^j \Pi_{t+j} + C_t \sum_{j=1}^{\infty}\beta^j \frac{1}{C_{t+j}} T_{t+j}\right], \tag{H.20}$$

which simplifies to

$$\sum_{j=0}^{\infty}\beta^j\left(1-\phi\Pi_{t+j}\right) - \sum_{j=0}^{\infty}\beta^j \frac{T_{t+j}}{C_{t+j}} = \frac{R_t K_t}{C_t} \tag{H.21}$$

Next, substituting the government's budget and the intratemporal condition, we obtain

$$\sum_{j=0}^{\infty}\beta^j\left(1-\phi\Pi_{t+j}\right) - \sum_{j=0}^{\infty}\beta^j \phi\Pi_{t+j}\frac{\tau}{(1-\tau)(1-\alpha)} = \frac{\phi R_t K_t}{(1-u_t)(1-\tau)w_t}. \tag{H.22}$$

Log-linearising H.22 we can show that the necessary condition coincides with the case where there is no policy.

Constrained Optimum

Suppose a social planner can internalise the externalities arising from the Okun's relation. In particular, the planner solves the following problem:

$$\max_{C,K,\Pi}\sum_{t=0}^{\infty}\beta^t\left(\log(C_t) - \phi\Pi_t\right),$$

$$
\begin{aligned}
&s.t \\
C_t + K_{t+1} &\leq K_t^{\alpha}\Pi_t^{1-\alpha}(1-u_t)^{1-\alpha} + (1-\delta)K_t, \\
1 - u_t &= (1+\mu Y^{-\eta})^{-1}.
\end{aligned}
$$

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