Should Victory Bets in Contests be Banned?*

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Abstract

Should players in contests be allowed to publicly back themselves by placing bets on own victories? Victory bets serve as commitment to increase efforts and thus have the potential to make the contest more intense and close. It is shown that, rather than making the contest more exciting, such provisions typically will spread the efforts apart: the favorite places a positive bet and the underdog abstains, with the favorite increasing his effort to depress rival's effort. Also it is possible that an underdog turns into a favorite by betting, shifting the balance of the contest. Whether allowing player participation in betting depresses or spurs competition and changes player hierarchy in the contest will depend on (i) the contestants' risk preferences, (ii) any asymmetry in the technology of contest, and (iii) whether one considers local or large shifts in the betting player's reaction function.

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1 Introduction

Sporting events are attracting larger crowds. In the past few decades this development has fed exponential growth in the sports betting industry; the volume of the global online gaming market, 35.5 billion U.S. dollars in 2013, is expected to rise to 56.05 billion in 2018.[1] Yet, sports regulations may not always permit players to place bets involving their own games.[2] Strict prohibitions are found in team sports,[3] whereas regulations in two-player (or individualistic) contests seem more permissive. The positions of various sporting authorities towards player participation in betting are not uniform either. A detailed discussion of these positions and the related laws appear in Standen (2006), who cites specific two-player contests such as golf or motor racing where the contestants can and often do wager money before playing the game.

Why should players not be allowed to bet on the outcome of their own games, at least their own wins? Given that public proclamations or banter boasting in individual superiority is not uncommon in some sports such as wrestling and boxing, it is not clear what could go wrong by allowing the same player to publicly back himself by putting "money where his mouth is." The standard criticism against player betting in team sports is that allowing bets on specific aspects of a contest, rather than just the own team's win, would encourage personal gains by undermining team cause. Another more substantial argument is that allowing players to place bets on sporting events would facilitate their involvement in match-fixing networks, often through illegal bookmakers, with potentially harmful consequences for the sports industry.[4] Finally, one may be concerned about the signaling effect of players' betting choices, revealing private information about the odds of the game. These signals may raise many questions

[1]See https://www.statista.com/statistics/270728/market-volume-of-online-gaming-worldwide/.

[2]In horse races in the UK, rules concerning betting by jockeys state that (http://www.valuehorsetips.co.uk/racehorse-ownership-gambling-within-the-rules/), "Whilst, there is nothing illegal about betting on a horse you own, betting against it is another matter. Rule E 92.2 of the BHA's horse racing rules states an owner must not lay any horse in their ownership to lose a race, instruct another person to do so on their behalf, or receive the whole or any part of any proceeds of such a lay."

An older version of Rule 707(1) in New Zealand horse-racing (https://www.nzracing.co.nz/OnHorseFiles/Downloads/10-09-14%20NZTR%20Announces%20New-%20Jockey%20Betting%20Rule%20Effective%2011%20September%202014.pdf) stipulated that, "A Rider may only bet on a race and/or sports event (including but not limited to a Race) in New Zealand or in any other jurisdiction provided that where he is betting on a race and he is riding a horse in that race he may only bet on the horse he is riding..." Rule 707(1) has now been changed prohibiting a Rider from having any interest in a bet in relation to any Race in which the Rider rides.

[3]Footballers in England are prohibited from betting on football matches. See the Daily Mail (2014) report at http://www.dailymail.co.uk/sport/football/article-2635217/Footballers-playing-England-not-allowed-bet-ANY-matches-worldwide.html .

[4]With intense monitoring and heavy sanctions – strict zero-tolerance policies of sports federations – it seems doubtful whether allowing players to bet on their own wins through legal bookmakers would substantially enhance the likelihood of their involvement in match-fixing activities.

and reduce the game's value for the viewing public.[5] Overall, the dominant public opinion seems to favor the ban on players' betting.

We pitch an economic argument to scrutinize any rationale for banning over allowing player betting, based on contest efforts. The higher the players' total efforts and the smaller the difference between their efforts, the more exciting should be the contest. To abstract from potential signaling effects of betting, we consider a complete information model with no uncertainty about player types. The organizers focus on contest excitement, reflected in total as well as relative efforts.

Formally, we combine a sporting contest between two players, A and B, with a market for betting on the binary contest outcome—either player A wins, or player B wins. The probability of a particular player's victory depends on relative efforts and the winner receives a prize. We assume Bertrand competition by bookmakers leading to odds on the two outcomes at actuarially fair values. All punters have rational expectations about the behavior of the two players in the betting-and-efforts game. Our focus will be primarily on the players' behavior: whether and how much they bet on their victories and what efforts they exert in the eventual contest. Based on the findings we answer our main question—the desirability of allowing player betting.

We borrow and apply ideas from two different literatures, contests and commitment. In the extensive literature on contests the main concern is how to induce players to exert more efforts; see, for example, Moldovanu and Sela (2001), and Olszewski and Siegel (2017). Whereas this may be appropriate for certain contests such as promotion and R&D tournaments or status games (Lazear and Rosen, 1981; Che and Gale, 2003; Moldovanu, Sela and Shi, 2007), for sporting contests the organizers' primary concern is whether the contest turns out to be close and exciting. Surprisingly, *closeness of contest* as part of the designer's objective is an overlooked aspect. To the best of our knowledge, there is no theoretical analysis of how a contest organizer should design incentives for such an objective. The idea of bringing ex-ante different ability contestants closer *prior to* the actual contest has earlier appeared in Myerson (1981) and Che and Gale (1998), among others. But these are either with a primary motivation of making contestants compete more vigorously as in Myerson's optimal auction, or limiting campaign spending in Che and Gale's political lobbying game. And for the *excitement* objective, while aggregate efforts could be a useful proxy, there could be other aspects that are at least equally as important. For example, *suspense and surprise* as in Ely, Frankel and Kamenica (2015) should be high on sports organizers' agenda. Our analysis will show how allowing player betting may generate surprises.

[5] See for instance the reply by Chuck Klosterman (Nov 14, 2014) to a reader's question: "An athlete who bets against his team—or himself—clearly has a conflict of interest in the outcome of the game. It's not obvious to me, however, why an athlete betting in favor of his team (or himself) would be doing anything unethical whatsoever. What am I missing?" in New York Times Magazine, at https://www.nytimes.com/2014/11/16/magazine/what-if-an-athlete-wants-to-bet-on-himself.html?.

Commitment games (Bagwell, 1995; Kreps and Scheinkman, 1983) involve sequential moves, as in Stackelberg games with observable actions, where early mover has the advantage to influence late mover's action via pre-commitment. Dixit (1987) analyzed such a game of contest with sequential efforts.[6] None of the players in our model have an exogenous built-in first-mover advantage. Simultaneously placed bets on own victory set up a game with equal pre-commitment opportunity to tilt the effort contest to one's favor. The idea of betting before the contest can be likened to Horner and Sahuguet's (2007) two-stage auction game where the first-stage bid sets a threshold for the second-stage bid, thus acting as commitment. High initial bids may spur the contestants into more frenzied bidding or contrastingly may even diffuse subsequent bidding. Besides the difference in the contest forms (winner-pay auction vs. all-pay contest), Horner-Sahuguet model is about signaling of bidder types whereas ours is a complete information model.

An important component of our model is the players' risk preference with regard to eventual wealth. Though a larger part of our analysis is carried out under the assumption of *risk aversion*, we also consider the case of *risk loving* players. We begin with a standard one-shot contest without player betting and show that the players exert strictly positive efforts. Against this, we study a two-stage game of bet-and-compete: players can first place bets on their own wins followed by a real contest where they exert efforts and a winner is determined stochastically. The bets can be *private* (unobservable bets) or *public* (observable bets).

When bets are private and the players are risk averse, in the unique equilibrium players will place zero bets and thus engage in the same amount of efforts as in the reference no-betting case. A bet on own victory increases the wealth gap between win and loss outcomes, which is costly for the player. Thus, judging by the impact on contest efforts, with risk-averse players it wouldn't make a difference to ban betting or allow private betting (Proposition 2).

When bets are public, one might expect the contest to be more competitive and exciting. On the contrary, we show that the betting-and-efforts game has no (pure strategy) equilibrium in which both players bet; either no player places a bet on own victory, or at most one player places a positive bet. Compared to the no-betting case, the player who places a positive bet puts in more effort while the player who does not bet may or may not lower his effort. This does not say whether the post-betting game is more or less competitive. The answer depends on which player places the bet and whether the impact on the effort reaction function of the betting player, i.e. the shift, is *local* or *large*.

The player who is the favorite in the no-betting equilibrium (i.e., the player who

[6]See also Baik and Shogren (1992) for a follow-up analysis.

exerts higher effort and hence is more likely to win) will have an incentive to place a positive bet while the underdog will abstain from betting, so long as one considers (i) only "local" incentives, i.e., small bets that shift the effort reaction function not significantly, and (ii) the favorite is risk neutral and the underdog either risk neutral or risk averse. In such a scenario (e.g., in the general logit contest success function),[7] after betting the favorite increases his contest effort and the underdog lowers his effort, thus making the contest more uneven (Proposition 3, Corollary 1). But it is also possible that the underdog places a positive bet shifting his reaction function "large" enough to overturn his position from being an underdog to favorite. This possibility could be due to a combination of two factors: (i) the initial (pre-bet) disadvantage in the contest is not significant (success probability less than but not too far from $1/2$), (ii) the underdog's cost of betting due to risk aversion is much less than that of the favorite. Overall, for local shifts (in the reaction function) the contest may become more uneven whereas for large shifts the contest may turn more or less uneven, with even the possibility that both players' efforts increase (Proposition 3).

The above results mirror the 'widening gap' (or ϵ-preemption) and 'aggressive bids with toehold' results in r&d race and takeover battles, and the 'leapfrogging' result in r&d race. In an r&d race a competitor increasing his r&d may act as discouragement to rivals' r&d (Harris and Vickers, 1985), or there can be leapfrogging (Fudenberg et al., 1983) where a laggard may get ahead in the race.[8] In takeover battles (Bulow, Huang and Klemperer, 1999), a bidder having even a small toehold in the target's shares makes the bidder bid aggressively, causing rival bidders to be conservative (winner's curse) that in turn makes the toehold bidder bid even more aggressively, thus creating a feedback loop. We should add that betting in our analysis does not directly influence the contest outcome, whereas in a dynamic r&d race early round investments will increase cumulative investments and thus influence the outcome of the race. Our leapfrogging result obtains through pre-commitment via positive bet, whereas in Fudenberg et al. leapfrogging happens either by chance when a less experienced firm gets lucky with success in the early stage of the race or due to the leader's lack of information about the rival's progress.[9] Differential effort inducements of asymmetric

[7]This contest success function has been used in many applications, in r&d race, sports, and conflict. See, for example, Hirshleifer (1991).

[8]Less directly related is the observation of escalation in arms race models (Baliga and Sjostrom, 2004; 2008), where adversaries end up in mutually harmful arms accumulation due to the fear of the rival's increased strength. At the heart of this prisoner's dilemma situation is the coordination failure or ambiguity of cheap talk messages such as refusal to arms inspection. Pre-contest betting in our formulation is the opposite of cheap talk and ambiguity but even then effort escalation in the contest is a distinct possibility.

[9]Player betting in own games bears similarities with two different financial practices, insider trading in stocks and stock options for executives. Just like players' own victory bets in a market with competitive odds, CEOs of firms in rivalry over innovation patenting can buy own stocks at competitive prices, which incorporate in a forward looking manner all implications of insider trading.

bets is in fact much closer in spirit to the link between toehold and bidding aggression.

In the risk-loving case whether bets are private or public, at least one player (both players in the private bets case) will stake his entire wealth in betting (Proposition 4). Such aggressive strategy translates into higher effort by one of the players, if not both, in the eventual contest. However, there is a real chance that the contest can become more uneven, although strictly higher efforts by both players is also possible (Proposition 5). This last possibility bears a closer analog of arms race escalation noted in footnote 8, with higher efforts abetted by the contestants' love for risky bets.

Given the impacts noted above on contest efforts, we argue that if the players are risk neutral or risk averse then there might be a strong case to ban victory bets. No such definite conclusion is possible if players are risk loving.

In the next section we present a basic version of the betting-effort game. The paper's main findings appear in Sections 3 and 4. Section 5 concludes. Proofs not contained in the text are relegated to the Appendix.

2 Betting-Effort Game

Two players, A and B, will engage in a sporting contest with two possible outcomes, A wins and B wins. The winner gets a prize $V > 0$. Each player has an initial liquid wealth of w_i for gambling. We assume (w_A, w_B) to be common knowledge. Modest variations in (w_A, w_B) and incomplete information associated with the gambling budget should not alter the qualitative nature of our analysis.

We develop our main results under the assumption that each player's utility (in wealth) function $u_i(\cdot)$ is strictly concave. To the best of our knowledge, there are no empirical studies on risk preferences of sports players, with regard to economic prospects, to guide us in modelling. We also consider the case of risk-loving players (convex wealth utility functions), in Section 4.[10,11]

Although popular debates on insider trading revolve around unfair use of private information, our complete information analysis, appropriately interpreted, can highlight a different aspect to it: insider own-stock purchases may serve as commitments to work harder or invest more into the race. Contrary to sporting events where both players' efforts contribute to contest excitement, increased disparity between rival firms' research efforts and investments may be socially more desirable because the loser's effort will be wasted to a large extent. In a duopolistic context, if the CEOs are risk-averse only one of them (typically, the favorite) would buy own stocks and due to the cross-investment effect of insider trading, investments would spread further apart. Under complete information and risk aversion, then, our analysis favors own-stock acquisitions. The use of stock options for executives is slightly different in that the costly incentivizing of commitment (to compete) are done by firms rather than directly chosen by the executives (Hall and Murphy, 2003, pp. 58-59; Rappaport, 1999).

[10] It is well understood that players will adopt different strategies, risk-taking or safe, in how they play the game depending on the match situation – whether one is ahead or behind. This does not tell us anything about the players' general risk preferences with regard to financial stakes, however.

[11] A study by Stanton et al. (2011) correlates sex hormones and economic risk preferences in a sample of 298 men and women, associating high and low testosterone levels to risk neutrality, intermediate testosterone levels to risk aversion. Taken and applied to sports, such observations are

We assume that competing bookmakers collect bets from the public by setting fair odds, leading to zero expected profits. We revisit this assumption at the end of Section 3.

Regulations permitting, the two players may be allowed to place bets but only on their own victories. Player i's bet is denoted by $b_i \geq 0$. We keep the bets non-negative implying the players cannot short bets.

Below we outline the sequence of betting-and-contest events.

Stage 1. Risk-neutral bookmakers set betting odds on each player's win competitively, determining the reward of r_i dollars for a dollar bet on player i's win, if i indeed goes onto winning the contest.[12]

Stage 2. Each player $i = A, B$ simultaneously places a bet on his own win, $0 \leq b_i \leq w_i$.

Stage 3. Players simultaneously exert efforts e_A, e_B. Player A wins with probability $q(e_A, e_B)$ and loses (player B wins) with probability $1 - q(e_A, e_B)$. ||

Remarks. Bets can be private (unobservable bets case) or public at least among the players (observable bets case), depending on the rules governing player betting. The last two stages will be lumped together in a single stage for unobservable bets.

For bets to be public, either sports law must mandate disclosure or the players voluntarily make it public, before the game begins. When law allows players to place bets, it would be in the players' interest to make their bets public. ||

We impose the following assumption on the contest technology:

Assumption 1 (Contest technology) (i) $q(e_A, e_B)$ *is continuously differentiable over a compact range* $[0, \bar{e}]^2$, $\bar{e}$ *appropriately large.*

(ii) $0 < q(0, e_B) < 1$ *for all* $e_B \in [0, \bar{e}]$ *with* $q_1 > 0,\ q_2 < 0,\ q_{11} < 0,\ q_{22} > 0.$

The first two signs in part (ii) are easy to understand; $q_{11} < 0$ and $q_{22} > 0$ state diminishing marginal winning probabilities in own effort. Note that Assumption 1 does not impose a sign restriction on q_{12} $(= q_{21})$, i.e., how the marginal impact of a player's own effort on his winning probability changes as the other player increases his effort. The sign of q_{12} (likewise, q_{21}) is likely to depend on the relative magnitudes of the two players' efforts. Also, there is no presumption that if both players exert the same effort, including zero effort, they will win with probability $1/2$. There could be

not very useful even if we assume that majority of sports players have testosterone levels in the intermediate to high range.

[12] We deliberately leave the number of bookmakers unspecified as it could be two or more. What is important is that the odds are set to drive expected profit down to zero.

asymmetry in player skills directly through the $q(.,.)$ technology, though we are not ruling out symmetric $q(.,.)$, i.e., $q(e,\tilde{e}) = 1 - q(\tilde{e}, e)$.[13]

Assumption 1 will be satisfied by the exponential contest success function $q(e_A, e_B) = \frac{\exp(e_A)}{\exp(e_A)+\exp(e_B)}$ (also known as logit function), but not by the Tullock contest success function $q = \frac{e_A}{e_A+e_B}$.

Assumption 2 (Cost function) *Let $\psi_i(\cdot)$ be player i's ($i = A, B$) effort cost function, continuously differentiable, satisfying $\psi_i'(e_i) > 0$ and $\psi_i''(e_i) > 0$ for all $e_i > 0$. Moreover, $\psi_i(0) = \psi_i'(0) = 0$.*

Players may differ in their costs, just like the contest technology can be asymmetric.

Let W_i^x denote player i's wealth from his own outcome in the contest, $x \in \{win, lose\}$. Thus,

$$\begin{aligned} W_A^{win} &= V + w_A + (r_A - 1)b_A, \qquad & W_A^{lose} &= w_A - b_A, \\ W_B^{win} &= V + w_B + (r_B - 1)b_B, \qquad & W_B^{lose} &= w_B - b_B. \end{aligned} \tag{1}$$

Since $V + r_i b_i > 0$, we have

$$W_i^{win} > W_i^{lose}. \tag{2}$$

Player A's expected utility is

$$Eu_A(e_A, e_B, b_A, b_B) = q(e_A, e_B)u_A(W_A^{win}) + (1 - q(e_A, e_B))u_A(W_A^{lose}) - \psi_A(e_A). \tag{3}$$

Similarly, player B's expected utility is

$$Eu_B(e_A, e_B, b_A, b_B) = (1 - q(e_A, e_B))u_B(W_B^{win}) + q(e_A, e_B)u_B(W_B^{lose}) - \psi_B(e_B). \tag{4}$$

A strategy for player i in this game, denoted by $\{b_i, e_i\}$, consists of a bet $b_i \in [0, w_i]$ and an effort level $e_i \in R_+$ if bets are not observable, $e_i : R_+^2 \to R_+$ if bets are observable, given his and player j's bets. We focus on strategies forming a subgame perfect or Nash equilibrium, depending on whether bets are observable or not.

The following result, Theorem 1 in Hellwig and Leininger (1987) adapted to our context, guarantees existence of equilibrium:[14]

Proposition 0 (Hellwig and Leininger, 1987) *In the two-stage betting-and-efforts contest game, with compact set of feasible player bets in Stage 1, compact set of feasible*

[13] The assumption that a player wins with a positive probability despite exerting zero effort for any positive effort by the other player helps avoid discontinuity of $q(.,.)$ at $(e_A, e_B) = (0, 0)$. See Proposition 0 for its relevance.

[14] See second paragraph of p. 60 of Hellwig and Leininger's article, where they assert the existence of SPE (in pure strategies) in finite player, infinite action games with simultaneous moves within stages (their footnote 1 being relevant for this last point).

efforts in Stage 2, and continuously differentiable contest technology $q(e_A, e_B)$ *and cost functions* $\psi_i(\cdot)$ *(Assumptions 1 and 2), the following will hold:*[15]

(i) *The players' payoff functions (i.e., expected utility functions) are continuously differentiable.*

(ii) *There exists a subgame-perfect equilibrium in pure strategies.*

We will therefore assume in the rest of our paper *existence of SPE in pure strategies* for observable betting-and-efforts contest games.[16]

3 Victory Bets: Pro- or Anti-Competitive?

We develop the analysis keeping *observable bets* as our primary focus. Below we consider effort decisions first, followed by betting. The treatment of private bets will follow easily by lumping together Stages 2 and 3 in a single stage, with players choosing their bets and efforts simultaneously.

■ **Determination of equilibrium efforts.** Consider effort decisions in Stage 3 for any (r_A, r_B) chosen by competitive bookmakers in Stage 1, and a pair of bets (b_A, b_B) in Stage 2. Player A chooses e_A to maximize (3) and player B chooses e_B to maximize (4). The Nash equilibrium efforts e_i are positive and characterized by the first-order conditions below, given $W_i^{win} > W_i^{lose}$ in (2):

$$u_A(W_A^{win}) - u_A(W_A^{lose}) = \frac{\psi_A'(e_A)}{q_1(e_A, e_B)}, \tag{5}$$

$$u_B(W_B^{win}) - u_B(W_B^{lose}) = \frac{\psi_B'(e_B)}{-q_2(e_A, e_B)}. \tag{6}$$

Eqs. (5) and (6) determine, implicitly, the effort reaction functions $\hat{e}_A(e_B, b_A)$ and $\hat{e}_B(e_A, b_B)$. For any given pair of bets (b_A, b_B), solving the reaction functions yield Nash equilibrium efforts which we denote by $(e_A^*(b_A, b_B), e_B^*(b_A, b_B))$ or simply (e_A^*, e_B^*).

Differentiating both sides of (5) and (6), we obtain the slopes of the two reaction functions as follows:

$$\left.\frac{de_A}{de_B}\right|_{\hat{e}_A} = \frac{\psi_A'(e_A)q_{12}}{\psi_A''(e_A)q_1 - \psi_A'(e_A)q_{11}}, \qquad \left.\frac{de_B}{de_A}\right|_{\hat{e}_B} = \frac{\psi_B'(e_B)q_{21}}{\psi_B''(e_B)q_2 - \psi_B'(e_B)q_{22}}. \tag{7}$$

We can now make the following observation:

[15] While Tullock contest success function is discontinuous at zero efforts, it is easy to show that zero effort by either player is never an equilibrium.

[16] For pure contest games the issue of existence of Nash equilibrium has been addressed by Cornes and Hartley (2012).

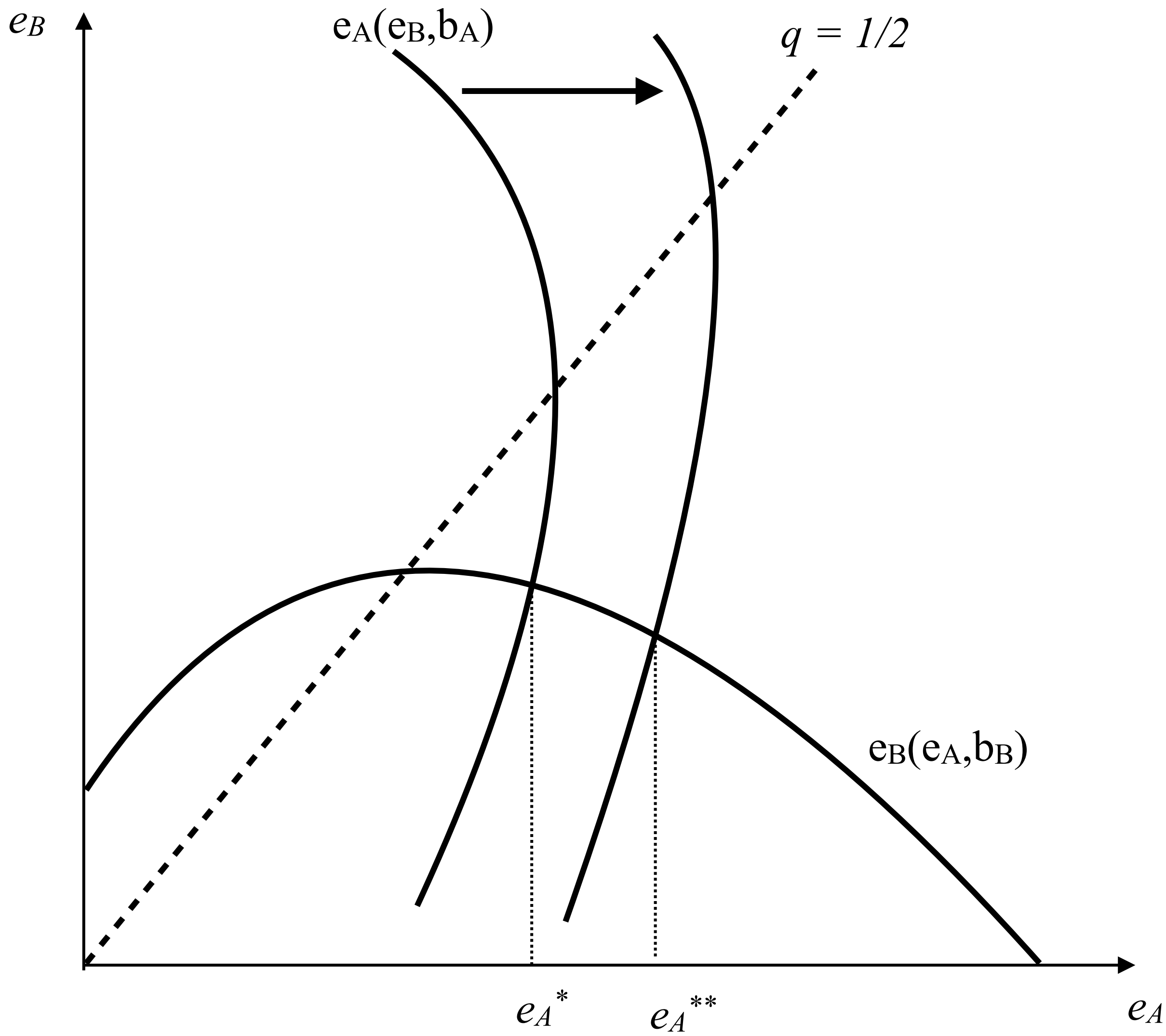

Fig. 1: Impact of an increase in A's victory bet b_A on equilibrium efforts

Lemma 1 *At any Nash equilibrium* (e_A^*, e_B^*) *the slopes of the players' reaction functions* $\hat{e}_A(e_B, b_A)$ *and* $\hat{e}_B(e_A, b_B)$ *will be of opposite signs. If at equilibrium efforts* $q_{12} > 0$*, player* A*'s reaction function will be upward-sloping whereas player* B*'s reaction function will be downward-sloping, as in* (7) *and plotted in Fig. 1. The slopes will reverse in signs if* $q_{12} < 0$*.*

First note that the slopes may reverse in signs as effort levels, (e_A, e_B), vary.

Second, when contest efforts are *strategic substitutes* for one player, for the other player the efforts are *strategic complements*. Assume that an increase in e_B increases the marginal impact of e_A on the probability that A wins (the case $q_{12} > 0$). Then, A responds to an increase in e_B by raising e_A. In this same case $(q_{21}\ (= q_{12}) > 0)$, an increase in e_A increases the marginal impact of e_B on the probability that A wins (i.e., lowers the negative impact of incremental e_B on A's win probability). So, B responds to an increase in e_A by lowering e_B. In the opposite case, q_{12} $(= q_{21}) < 0$, an increase in e_B reduces the marginal impact of e_A on the probability that A wins and, equivalently, an increase in e_A reduces the marginal impact of e_B on the probability that A wins. Then, $de_A/de_B < 0$ and $de_B/de_A > 0$. It follows that in any interior effort Nash equilibrium, at the intersection point the slopes of the reaction functions must have opposite signs.

The asymmetry in effort responses is illustrated by Fig. 1 where we assume contest success function such that $q_{12}(= q_{21}) > 0 \Leftrightarrow q > \frac{1}{2}$. For such technologies, player A is called the *favorite* and player B the *underdog*, using Dixit's (1987) terminologies. Given a pair of efforts such that $q(e_A, e_B) > \frac{1}{2}$, A responds by raising whereas B responds by reducing own effort, if the opponent increases his effort. When B is more likely to win, these effort responses are reversed.

Given the ambiguous sign of q_{12} and especially the fact that the slopes' signs may change as effort levels vary, uniqueness of Nash equilibrium efforts is usually not guaranteed. In the rest of the paper we restrict attention to contests in which players' effort reaction functions are "sufficiently well-behaved":

Assumption 3 (Single-peakedness) *Each player's reaction function $\hat{e}_i(e_j)$ is continuous and single-peaked, i.e., admits a unique maximum in e_j.*

Admittedly, the assumption is not on the primitives, so it is not entirely satisfactory. However, it is satisfied by many contest functions including the general logit function of the form $q(e_A, e_B) = \frac{f(e_A)}{f(e_A)+g(e_B)}$ where $f(.)$ and $g(.)$ are increasing functions. This is verified in Lemma 4 in the Appendix.

Lemma 2 (Uniqueness of efforts equilibrium) *Given Assumption 3, the Nash equilibrium efforts in the contest stage, with or without observable bets, will be unique.*

As a reference case, in the proposition below (proof follows from simple first-order conditions) we present the equilibrium in the contest where players are not allowed to place bets, $(b_A, b_B) = \mathbf{0}$ (bold-faced $\mathbf{0}$ indicates the null-vector, $(0, 0)$).

Proposition 1 (Contest without betting) *If betting is forbidden for the players, the unique Nash equilibrium efforts e_i^0 are both positive-valued, $e_A^0 = e_A^*(\mathbf{0}) > 0$, $e_B^0 = e_B^*(\mathbf{0}) > 0$, satisfying (5) and (6), and player A wins with probability $q(e_A^0, e_B^0)$. The competitive betting market sets the return on player A's win at $r_A^0 = 1/q(e_A^0, e_B^0)$, B's win at $r_B^0 = 1/[1 - q(e_A^0, e_B^0)]$.*

Since $W_i^{win} > W_i^{lose}$ and $\psi_i'(0) = 0$, (5) and (6) imply $e_i^0 > 0$ for $i = A, B$. The two equilibrium efforts need not be identical because the players may have different utility

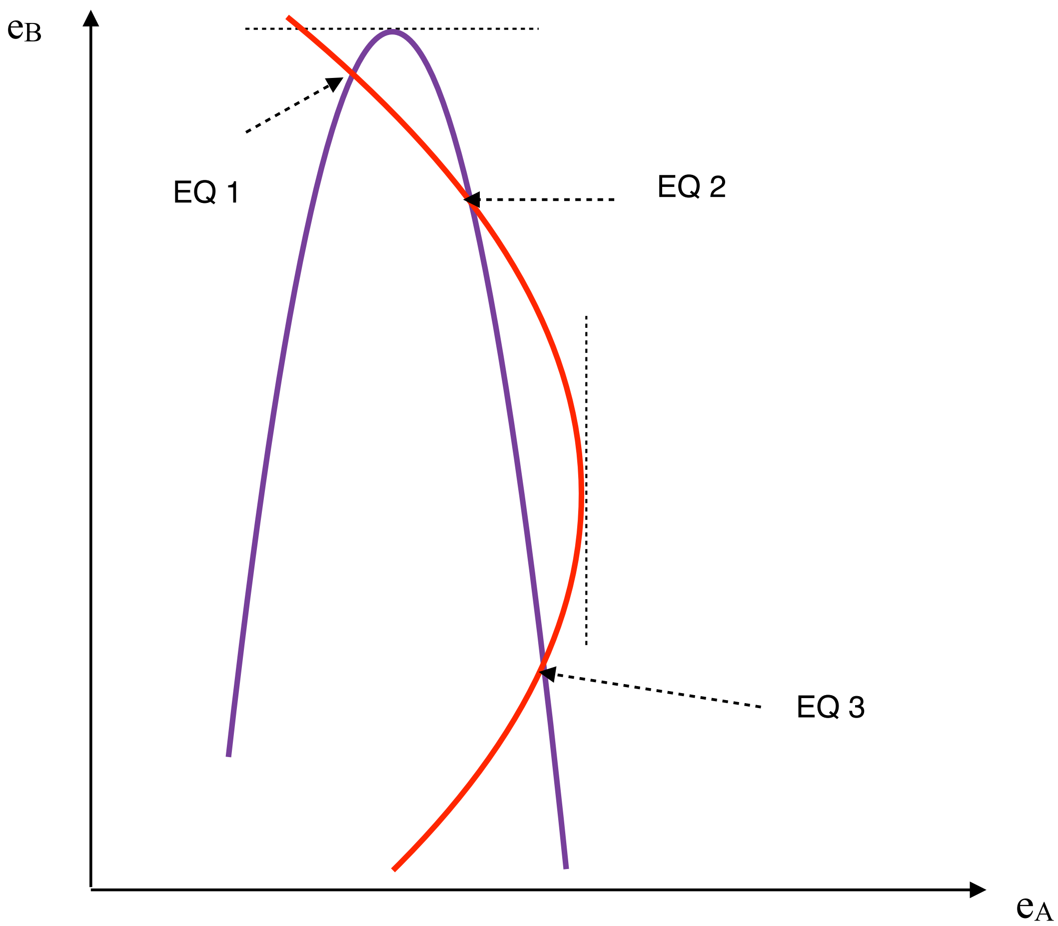

Fig. 2: Uniqueness of equilibrium under single-peaked reaction functions. Multiple Nash equilibria must involve reversal of signs of the slopes of two reaction functions with an intermediate equilibrium exhibiting the same sign slopes -- a contradiction to Lemma 1.

and effort cost functions, or the probability of win function may be ***asymmetric***, i.e., $q(e, \tilde{e}) \neq 1 - q(\tilde{e}, e)$ for $\tilde{e} \neq e$.

■ **Determination of equilibrium bets.** Let us now turn to betting decisions. To see the impact of betting on equilibrium effort strategies, differentiate the first-order conditions (5) and (6). First holding e_B fixed at any arbitrary level, we have

$$\frac{\partial \hat{e}_A}{\partial b_A} = \frac{[u_A'(W_A^{win})(r_A - 1) + u_A'(W_A^{lose})]q_1}{\psi_A''(e_A) - \{u_A(W_A^{win}) - u_A(W_A^{lose})\}q_{11}} > 0,$$

for $b_A \geq 0$. Similarly, holding e_A fixed at an arbitrary level, the effects on player B's reaction function are given by:

$$\frac{\partial \hat{e}_B}{\partial b_B} = \frac{[u'_B(W_B^{win})(r_B - 1) + u'_B(W_B^{lose})]q_2}{-[\psi''_B(e_B) + \{u_B(W_B^{win}) - u_B(W_B^{lose})\}q_{22}]} > 0,$$

for $b_B \geq 0$. Observe that placing a larger bet on one's victory makes him more aggressive in the effort game. As illustrated in Fig. 1, an increase in b_i by player i would shift (only) player i's effort reaction function upwards and to the right if it is downward-sloping, and downwards and to the right if it is upward-sloping.

3.1 Private bets

Putting betting and effort decisions within a single frame for an overall equilibrium yields the following result:

Proposition 2 (Bet abstention) *Suppose the bets are private. In any Nash equilibrium the players will completely abstain from betting:* $b_i^* = 0$, $i = A, B$.

With bets unobservable and hence deprived of its commitment value, the only way a player's bet can alter contest efforts is through its influence on one's own effort. However, because players are risk averse, they stand to lose by placing a victory bet at actuarially fair odds and thereby increasing the spread between wealth in win and loss states. So they do not bet and there is no real difference between completely banning betting by players or allowing them to bet on their own victory.

3.2 Observable bets

When bets are observable, the game is modified in one important respect: every pair of bets generates a proper subgame in efforts, therefore players will consider the impact of their betting choices on the equilibrium of the effort game. The extensive form game is thus the three-stage game presented in Section 2. The appropriate solution concept for this game is subgame perfect equilibrium (SPE).

Given (r_A, r_B), a pure strategy for player i consists of, first choosing a bet $b_i \leq w_i$, and then given the bets by both players, an effort $e_i : [0, w_i] \times [0, w_j] \to R^+$. A subgame-perfect equilibrium (SPE) in pure strategies is defined in the usual way: the effort pair $(e_A^*(b_A, b_B), e_B^*(b_A, b_B))$ must constitute a Nash equilibrium in the effort contest subgame given the bets (b_A, b_B), and the strategies $\left(b_A^*, e_A^*(b_A^*, b_B^*), b_B^*, e_B^*(b_A^*, b_B^*)\right)$ must constitute a Nash equilibrium of the overall betting-effort game. In addition, the bets are priced actuarially fair under rational expectations by competitive bookmakers, i.e., $qr_A = (1 - q)r_B = 1$ where q is consistent with the SPE bet and effort choices.

The new aspect in players' betting strategies is that each player takes into account the impact of his betting strategy on the rival's effort response.

The following lemma describes own- and cross-effort effects of betting along an SPE path of play, with the continuation Nash equilibrium (e_A^*, e_B^*).

Lemma 3 (Effort effects of bets) *Suppose bets are observable. Fix any pair of bets* (b_A, b_B) *in Stage 2. The following hold* <u>*irrespective*</u> *of whether the players are risk averse or risk loving:*

(i) $\frac{\partial e_i^*}{\partial b_i} > 0$.

(ii) *If* $q_{21}(e_A^*, e_B^*) > 0$, *then* $\frac{\partial e_B^*}{\partial b_A} < 0$ *and* $\frac{\partial e_A^*}{\partial b_B} > 0$;

if $q_{21}(e_A^*, e_B^*) = 0$, *then* $\frac{\partial e_B^*}{\partial b_A} = \frac{\partial e_A^*}{\partial b_B} = 0$.

If $q_{12}(e_A^*, e_B^*) < 0$, *then* $\frac{\partial e_B^*}{\partial b_A} > 0$ *and* $\frac{\partial e_A^*}{\partial b_B} < 0$;

if $q_{12}(e_A^*, e_B^*) = 0$, *then* $\frac{\partial e_B^*}{\partial b_A} = \frac{\partial e_A^*}{\partial b_B} = 0$.

The own-effort effect in Part (i) is fairly intuitive as already discussed in the private bets case. Part (ii) describes the cross-effort effect, absent in the private bets case, on the impact of a player's betting on the other player's effort that operates through the change in his own effort. Here cross-effect is determined by the sign of q_{12} ($\equiv q_{21}$) in the continuation equilibrium. For instance, when $q_{21} > 0$ so that an increase in B's effort improves the marginal impact of A's effort on the probability that A wins, a higher bet on own victory by B (which raises B's effort by part (i)) leads A to raise his effort as well. In this same case, an increase in effort by A decreases the marginal impact of B's effort on the probability that B wins, so a higher bet by A on own victory (which improves A's effort) leads B to lower his effort.

We now focus on the players' betting incentives in Stage 1.

For any given pair of bets (b_A, b_B) and the continuation equilibrium efforts (e_A^*, e_B^*), denote the expected utility of player A, stated in (3), by Eu_A^*. Differentiating (3) w.r.t. b_A yields:

$$\begin{aligned}
\frac{\partial Eu_A^*}{\partial b_A} &= qu_A'(W_A^{win}) \cdot (r_A - 1) - (1-q)u_A'(W_A^{lose}) \\
&\quad + (q_1 \frac{\partial e_A^*}{\partial b_A} + q_2 \frac{\partial e_B^*}{\partial b_A})[u_A(W_A^{win}) - u_A(W_A^{lose})] \\
&\quad - \psi_A'(e_A^*) \cdot \frac{\partial e_A^*}{\partial b_A} \\
&= (1-q)[u_A'(W_A^{win}) - u_A'(W_A^{lose})] + q_2 \frac{\partial e_B^*}{\partial b_A}[u_A(W_A^{win}) - u_A(W_A^{lose})]. \quad (8)
\end{aligned}$$

The first term in this expression follows from the fact that actuarially fair (betting) odds imply $qr_A = 1$. The second term is the residual after using (5). Evaluating $\frac{\partial Eu_A^*}{\partial b_A}$

at $b_A = 0$ gives

$$\frac{\partial Eu_A^*}{\partial b_A} = (1-q) \cdot \underbrace{[u_A'(V+w_A) - u_A'(w_A)]}_{<0} + \underbrace{q_2}_{<0} \cdot \frac{\partial e_B^*}{\partial b_A} \cdot [u_A(V+w_A) - u_A(w_A)]. \quad (9)$$

Similarly, differentiating player B's expected utility (4) w.r.t. b_B we obtain:

$$\frac{\partial Eu_B^*}{\partial b_B} = q \cdot [u_B'(W_B^{win}) - u_B'(W_B^{lose})] - q_1 \cdot \frac{\partial e_A^*}{\partial b_B} \cdot [u_B(W_B^{win}) - u_B(W_B^{lose})]. \quad (10)$$

Evaluating this marginal utility at $b_B = 0$ yields:

$$\frac{\partial Eu_B^*}{\partial b_B} = q \cdot \underbrace{[u_B'(V+w_B) - u_B'(w_B)]}_{<0} - \underbrace{q_1}_{>0} \cdot \frac{\partial e_A^*}{\partial b_B} \cdot [u_B(V+w_B) - u_B(w_B)]. \quad (11)$$

From expressions (9) and (11) it is quite conceivable that $\frac{\partial Eu_i^*}{\partial b_i}\Big|_{b_i=0} > 0$ for some i, which means that a *risk-averse player would place a positive bet on his own victory.* This, however, depends on the sign of $\frac{\partial e_j^*}{\partial b_i}$, the other player's cross-effort response to an increase in b_i.

Let $q^* = q(e_A^*, e_B^*)$ and $q^0 = q(e_A^0, e_B^0)$. Similarly, $q_{12}^* = q_{12}(e_A^*, e_B^*)$ and $q_{12}^0 = q_{12}(e_A^0, e_B^0)$. We have the following result.

Proposition 3 (Widening contest/leapfrogging) **(i)** *In any SPE of the game with observable betting, either no player bets, or one player only places a bet.*

(ii) *Suppose $q_{12}^0 > 0$; under general logit contest functions, $q^0 > 1/2$, hence, player A is the favorite in the no-betting equilibrium. An SPE with positive betting can take one of the following two forms, which may coexist.*

(a) A *bets,* B *does not.* A*'s effort increases and* B*'s effort falls. In this SPE,* $q_{12}^* > 0$.

(b) B *bets,* A *does not.* B*'s effort increases whereas* A*'s effort may rise or fall. In this SPE, $q_{12}^* < 0$, hence under general logit contest functions, $q^* < 1/2$: the underdog B becomes the favorite via betting.*

(iii) *The description of SPE betting strategies in the case $q_{12}^0 < 0$ is fully symmetric to part (ii).*

Some general features of the possible equilibria are worth highlighting. An important aspect is the absence of positive bets by *both* players. For risk-averse players, betting is valuable only as an instrument of commitment to exert higher efforts. But contest is a zero-sum game and escalation of efforts in order not to concede substantial

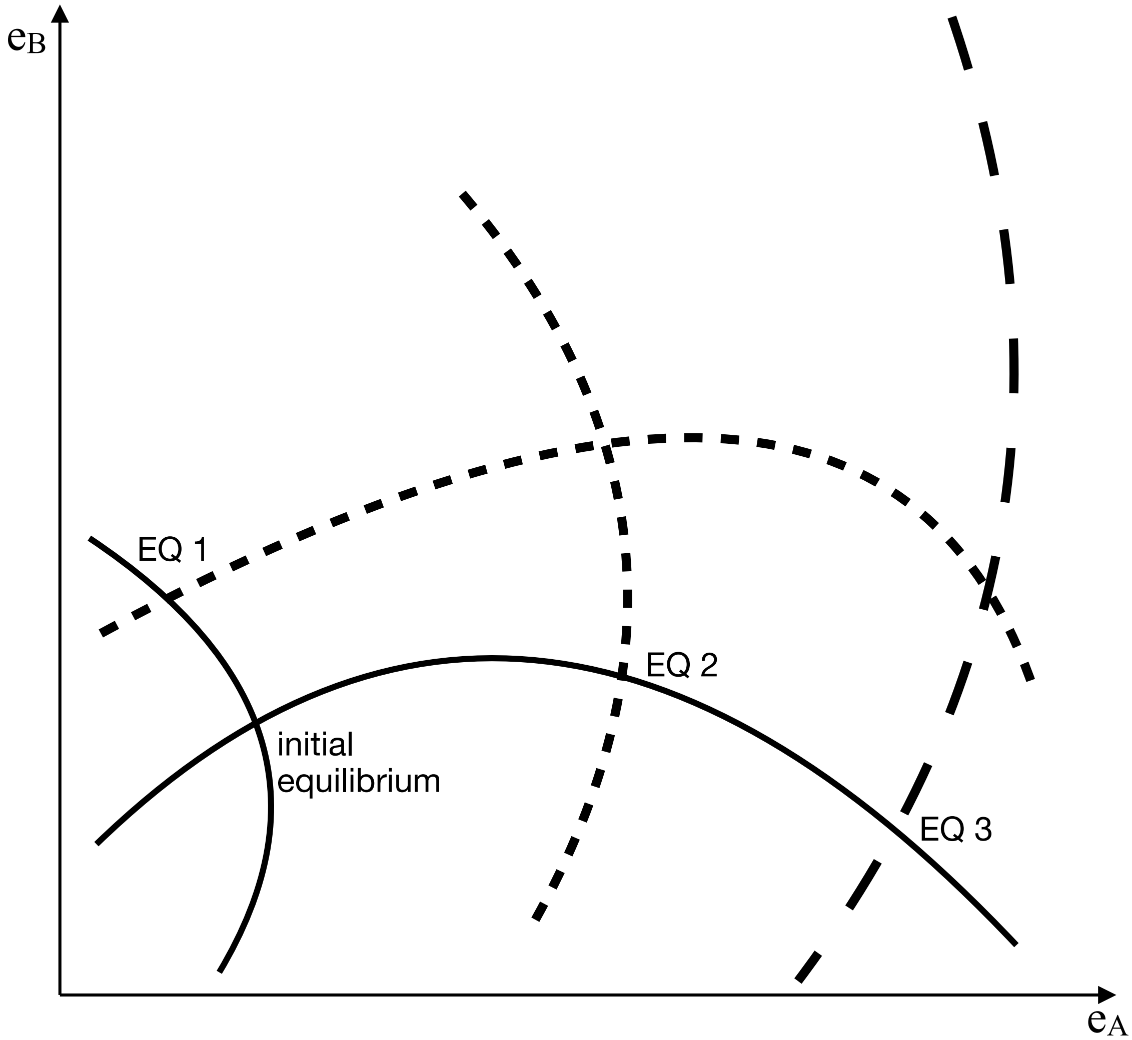

Fig. 3: e_B may increase/decrease

ground in the contest, through commitment, cannot yield both players positive gains from bets – a feature we call *non-escalation in betting.*[17]

Second, although betting escalation does not happen, we cannot rule out *effort escalation* as indicated in the case of the underdog becoming the favorite. The favorite-turned-underdog, player A in part (ii.b), while not betting, may increase his effort in response to the newly turned favorite player B's increased effort.[18] (In Fig. 3, the move from the initial equilibrium to EQ_2 involves A being the underdog-turned-favorite; here the initial-favorite B becomes the mild underdog and his effort escalates, relative to the initial equilibrium, to protect his losing turf in the contest.) The underdog becoming

[17] We restrict to pure strategies in betting as well as efforts contest. It is conceivable that the players choose mixed strategies in the betting stage and thus both end up with positive bets.

[18] In r&d race (Harris and Vickers, 1985; Fudenberg et al., 1983), arms race (Baliga and Sjostrom, 2004; 2008), and auction contest (Horner and Sahuguet, 2007), escalation of contest initiatives by adversaries is a prominent feature. But most of these models involve some uncertainty about player types or actions, the signaling of which drives the escalation.

the favorite is parallel with the idea of leapfrogging in patent races, as in Fudenberg et al. (1983). The difference is that leapfrogging in Fudenberg et al. is a chance event, whereas in ours it occurs through the rational choice of bets. In sports, one well-known instance of the underdog betting and turning the tables on is the famous boxing match on September 29, 2001 in New York city, *Bernard Hopkins vs. Félix Trinidad* billed as *And Then There Was One*: "For the first time in many years, Hopkins was an underdog in the betting, which led the confident Hopkins to place a $\$100,000$ bet on himself to win the bout." Hopkins went onto winning the contest in a distinct display of superiority.[19]

Third, according to part (ii), both types of SPE with positive betting, one in which only A bets and the other in which only B bets, can coexist. Nor can we rule out the possibility that one of the players never bets, in any SPE. These issues are context specific and depend on the curvatures of wealth utilities, effort cost functions and the contest probability function. Coexistence of SPE of types (a) and (b) is best illustrated in the fully symmetric betting-effort game, with identical players. In the no-betting equilibrium, effort reaction functions intersect at their peaks, so, $q_{12}^0 = 1/2$. Although none has an incentive to marginally raise his bet above zero, one player, say, A, can beneficially deviate to a large bet $b_A^* = \bar{b}$ as best response to $b_B = 0$. However, if these bets are part of an SPE, then so are the bets $b_A^* = 0$ and $b_B^* = \bar{b}$, by full symmetry. Clearly, introducing slight asymmetry and making one player mildly favorite is not going to change the possibility that SPEs of types (a) and (b) coexist. We discuss these equilibria in the sequel.

Fourth, an equally important observation, is the possibility of 'widening gap' in the contest due to bet allowance, somewhat in contrast to effort escalation. We postpone the discussion of this point, temporarily.

■ **Tradeoff: Risk-aversion cost against technological advantage.** Let us probe a bit more into individual expected utility functions to understand the players' behavior, the impact of betting on continuation equilibrium efforts and the factors that would make them more likely to place a bet on own victory despite risk aversion, under fair betting odds. It will be useful to treat separately asymmetries in risk preferences and asymmetries in the contest. The latter can be rooted in the contest probability function $q(.,.)$ and differences in individual effort cost functions $\psi_i(.)$. Differences in individual wealth levels can also generate different contest efforts by affecting a player's "utility prize", $u_i(W_i^{win}) - u_i(W_i^{lose})$. If these utility prizes are not too different, the player with higher effort cost and/or a contest disadvantage (embedded in an asymmetric q function) will enter the contest stage as the *underdog* when betting is

[19] See https://en.wikipedia.org/wiki/Bernard_Hopkins_vs._F%C3%A9lix_Trinidad and https://en.wikipedia.org/wiki/Bernard_Hopkins.

not allowed: In the Nash equilibrium to come, he is expected to land on the declining portion of his effort reaction function, where his marginal effort raises the productivity of the opponent's (the *favorite*'s) effort.

The players' risk preferences, that is, the curvature of $u_i(.)$ functions, have no impact on the contest outcome when betting is not allowed—only the utility prizes and contest-related attributes count. But the zero-betting outcome may persist even after betting is allowed because risk-averse players incur a loss from betting at fair odds. The only motivation for placing a positive *observable* bet on own victory is its *commitment effect* which, by raising own utility prize, strengthens own effort incentives and (more likely than not) weakens the opponent's effort response. Whether this commitment effect is sufficiently strong to offset the cost of placing an observable bet depends on the player's degree of risk aversion. In a match between a slightly risk-averse player B and an extremely risk-averse player A, player B is quite likely to place a bet whereas A would never do so. This behavioral difference may persist even if we introduce a contest advantage for the extremely risk-averse player A.

■ **Further interpretation of Proposition 3.** Proposition 3 gains a more intuitive interpretation under general logit contest functions where $q_{12} > 0 \Leftrightarrow q > 1/2$: the "favorite" is the player "more likely to win." Then, of the two forms of SPE involving unilateral betting stated in (a) and (b) of Part (ii), there is a sense in which the favorite's unilateral betting outcome in (a) is more likely than the underdog's unilateral betting outcome in (b). In (a), all the favorite player A has to do if he is not too risk averse is to take his status quo advantage in the contest (the fact that $q_{12}^0 > 0$) and slip down along the underdog B's effort reaction function, by betting on own victory. In terms of Fig. 1, A's reaction function $\hat{e}_A(e_B, b_A, 0)$, upward-sloping in the no-betting Nash equilibrium, shifts to the right along the downward sloping portion of B's reaction function $\hat{e}_B(e_A, b_A, 0)$. Given the direct marginal cost from increased spread in wealth outcomes, $u_B'(V + w_B) - u_B'(w_B)$, betting on own victory to become more aggressive in the effort subgame will not pay off for B because it also raises the marginal productivity of A's effort. Thus, B's reaction function does not change and as a result, A's equilibrium effort increases from $e_A^*(0, 0)$ to $e_A^*(b_A, 0)$ and B's equilibrium effort decreases from $e_B^*(0, 0)$ to $e_B^*(b_A, 0)$.[20] That is, if the favorite player A places a positive bet (i.e., $b_A > 0$) when allowed to bet on own victory and hence by Proposition 3 player B places zero bet (i.e., $b_B = 0$), the contest can only become more effort-uneven, with effort levels ranked as[21]

$$e_A^* > e_A^0 > e_B^0 > e_B^*. \tag{12}$$

[20]Symmetrically, B as the favorite may place a positive bet when the $\hat{e}_A(e_B, 0, b_B)$ is downward-sloping and the $\hat{e}_B(e_A, 0, b_B)$ is upward-sloping (which corresponds to the case $q_{12}^0 < 0$).

[21]In the initial no-betting equilibrium, the favorite A also exerts more effort than the underdog B.

Both players never choose positive bets, so, if it is the effort-aggressive player who bets, he further pacifies his opponent in the contest, runs away with the match and robs the fans off a good spectacle. The possibility of *widening gap* in the contest due to betting lends some justification for why player betting might be banned. The widening gap possibility is also reminiscent of a result by Bulow, Huang and Klemperer (1999), who have argued in the context of corporate takeover battles with incomplete information about a target's value that a bidder owning a toehold would be incentivized to bid more aggressively for the target. In our case, placing a bet on one's own win is like gaining a toehold.

The above possibility is an intuitive/natural prediction if, say A, the favorite in the contest without betting, is almost risk neutral so that his marginal expected utility in (9) is strictly positive (thus, B's marginal expected utility in (11) is strictly negative). One can then construct a sequence of bets $S = \{\Delta, 2\Delta, \cdots, n\Delta, \cdots\}$ by player A, where $\Delta > 0$ is "small", such that $\frac{\partial Eu_A^*}{\partial b_A} > 0$ and $\frac{\partial Eu_B^*}{\partial b_B} < 0$ for all $b_A \in S$ and given fixed $b_B = 0$, by continuity of the marginal expected utility functions.[22] Note that B's best response to all $b_A \in S$ remains $b_B^* = 0$ because $q_{12} > 0$, as we gradually move "down" along B's effort reaction function. Although we do not characterize the solution b_A^* explicitly, the implied effect in the effort contest subgame is indicated by the shift in equilibrium efforts, $e_A^* \to e_A^{**}$ in Fig. 1. This observation we summarize in the following corollary:

Corollary 1 (Local analysis) *Suppose in the initial no-betting equilibrium,* $q_{12}^0 > 0$ *with* $e_A^0 > e_B^0$. *If player* A *is (almost) risk neutral and player* B *(weakly) risk averse, then limiting to only "local" shifts under bet allowance,* A *would place a strictly positive bet and* B *would place zero bet. In the follow-up contest, the initially favorite player* A *becomes stronger and the underdog* B *becomes even more underdog, with contest efforts widening as in* (12).

The alternative possible SPE (case (b)) where only the underdog bets is at least as interesting: Allowing the players to bet on own victory now turns contest expectations upside down. If the underdog B ever bets in equilibrium, he bets a sufficiently large amount that makes him the new favorite, switching the sign of q_{12}, from $q_{12}^0 > 0$ to $q_{12}^* < 0$. He must thus generate for himself a powerful incentive to raise his contest effort up to the region where his effort reduces the marginal productivity of the ex-favorite. That the bet be sufficiently large is a hurdle that the underdog must benefit from surmounting, which depends on his degree of risk aversion and how much underdog he is – how far below $1 - q^0$ is from $1/2$ – at the equilibrium of the game without betting.

This SPE is not a remote possibility, however. To see this, suppose that B has only

[22] Utility functions are assumed to be second-order differentiable.

a slight disadvantage in the contest without betting, so that the effort equilibrium is near the peak but at the downward sloping portion of A's (the favorite) reaction function. If B has an extremely small cost from betting on own victory, that is, if he is almost risk neutral whereas A is very risk averse, given also that he has only a small contest disadvantage B can bet a large amount to shift his own reaction function up along A's reaction function. In the resulting new Nash equilibrium the effort pair will lie in the region where A's reaction function is negatively sloped and (by Lemma 1) B's reaction function is positively sloped. At a small betting cost the ex-underdog pacifies the ex-favorite. The SPE effort response of the ex-favorite, A, depends on the amount that B bets and the curvatures of the two reaction functions; A's contest effort may rise or fall, as illustrated in Fig. 3.

■ **Implications of commitment: Comparison with sequential contest.** Dixit (1987), and Baik and Shogren (1992) studied in a sequential efforts contest game how effort commitment by the first mover can encourage or depress efforts relative to a simultaneous move contest. Dixit observed that a favorite would overcommit whereas an underdog undercommit efforts. Endogenizing the order of moves Baik and Shogren argue that the underdog should like to move first while the favorite preferred moving last, resulting in lower efforts by both players. Importantly, both papers considered only "local" analysis limiting attention to "small" changes in the first mover's effort along the second mover's reaction function. While our game form is very different with players placing bets first and then exerting efforts simultaneously, betting creates an opportunity for "large shift" in the betting player's reaction function. Our observation that the initial favorite may become the underdog with the rival player increasing his effort after placing a large bet contrasts with Dixit's result, while our result that when one player places a positive bet his effort will surely rise and the rival player's effort may rise or fall contrasts with the uniformly negative consequence of commitment as predicted by Baik and Shogren. With large shifts possible due to betting, predictions of both Dixit, and Baik and Shogren can thus come to be completely overturned.

Changing of player hierarchy also introduces an element of surprise to most followers of a sports contest and is in the same spirit as in Ely, Frankel and Kamenica (2015). Allowing player betting can thus serve a positive role, assuming spectators may derive greater entertainment value from an upset. At the minimum, an occasional upset generates greater coverage in the media that helps lift the tournament's profile.

■ **Examples: Widening efforts, changing player hierarchy.** In the no-betting regime $e_A^0 > e_B^0$ if and only if $q_{12} > 0$. Whenever A's effort is larger than B's effort so that A is more likely to win ($q > 1/2$), an increase in B's effort raises the marginal impact of A's effort on the probability that A wins ($q_{12} > 0$). It follows that if A is more likely to win in the no-betting regime, A has an advantage in betting on own victory, regulations permitting. Player A will do so if in addition $\frac{\partial Eu_A^*}{\partial b_A} > 0$ at $b_A = 0$. In the new effort equilibrium, the first-order conditions reduce to:

$$u_A(V + w_A + (r_A - 1)b_A) - u_A(w_A - b_A) = \frac{\psi_A'(e_A^*)}{q_1(e_A^*, e_B^*)}, \tag{13}$$

$$u_B(V + w_B) - u_B(w_B) = \frac{\psi_B'(e_B^*)}{-q_2(e_A^*, e_B^*)}. \tag{14}$$

By Lemma 3, A's effort will be higher whereas B's effort will be lower than their respective efforts in the no-betting case, hence the equilibrium efforts will rank as in (12).

We next present two numerical examples to illustrate Proposition 3(ii-a,b) with the following specifications:

Utility functions : $u_A(m) = m^\alpha,\ u_B(m) = m^\beta,\ \ \alpha = \beta = 0.8$

Cost functions: $\psi_A(e) = (k/2)e^2,\ \ \psi_B(e) = (1/2)e^2,\ \ k = 0.65$

Wealth: $w_A = w_B = 120$

Contest success function: $q = \dfrac{\exp(e_A)}{\exp(e_A) + \exp(e_B)}$.

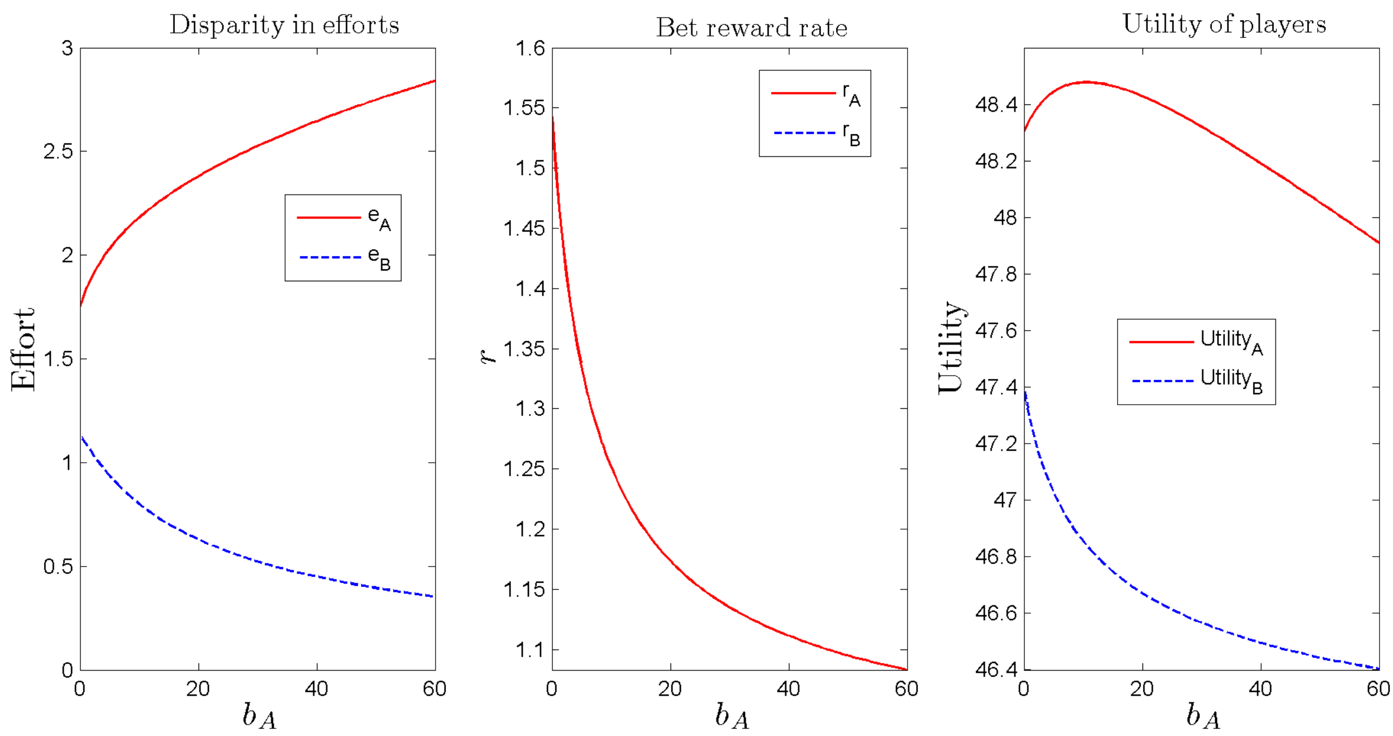

Fig. 4: Favorite gets stronger; r_B cannot be seen as it is outside the displayed range

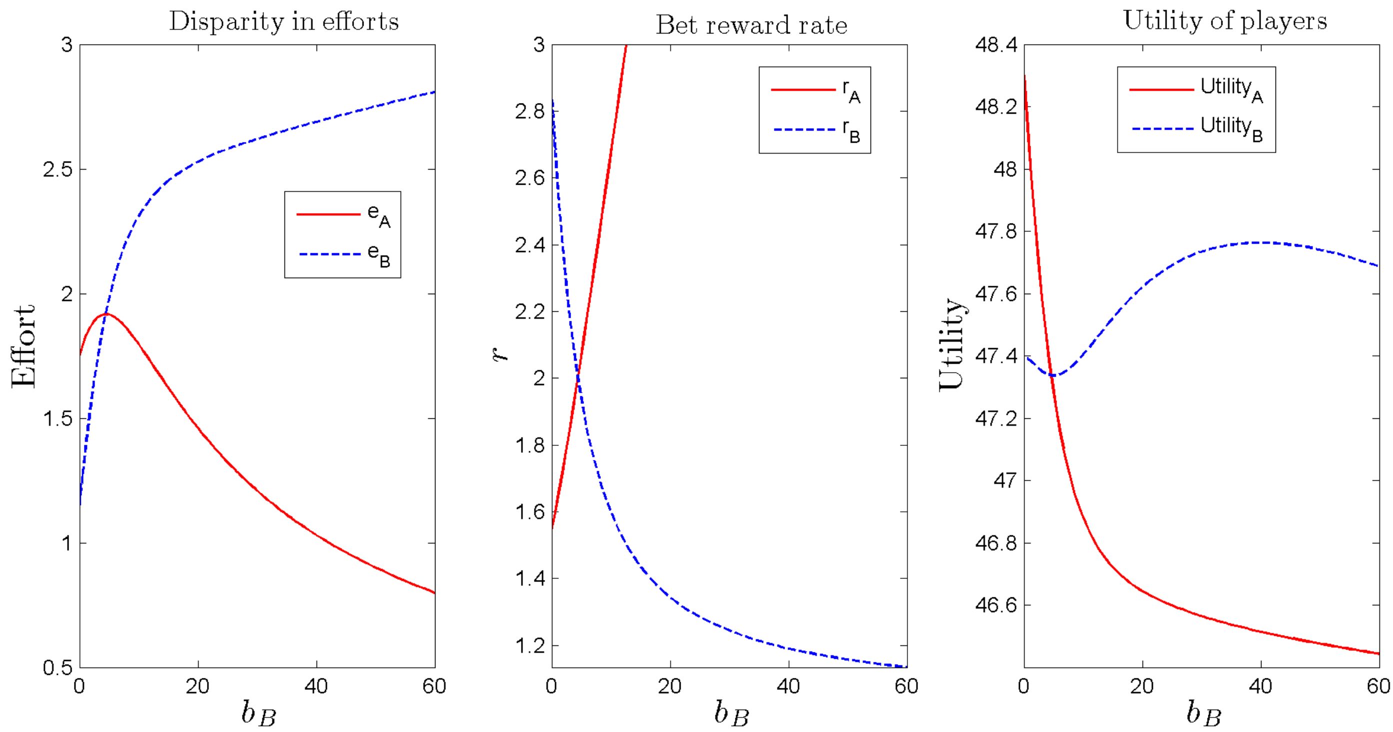

Fig. 5: Underdog turns favorite

Widening-efforts-gap possibility. Note that

$$\frac{\partial q}{\partial e_A} = \frac{\exp(e_A + e_B)}{\big(\exp(e_A) + \exp(e_B)\big)^2}, \quad \frac{\partial q}{\partial e_B} = \frac{-\exp(e_A + e_B)}{\big(\exp(e_A) + \exp(e_B)\big)^2},$$
$$\frac{\partial^2 q}{\partial e_B \partial e_A} = \frac{\exp(e_A + e_B)[\exp(e_A) - \exp(e_B)]}{\big(\exp(e_A) + \exp(e_B)\big)^3} > 0 \text{ if } e_A > e_B.$$

In the no-betting regime, first-order conditions (5) and (6) can now be written as

$$(\exp(2.8) + 120)^{0.8} - (120)^{0.8} = \frac{e_A \cdot 0.65}{\frac{\exp(e_A + e_B)}{\left(\exp(e_A) + \exp(e_B)\right)^2}} = \frac{0.65 e_A \big(\exp(e_A) + \exp(e_B)\big)^2}{\exp(e_A + e_B)},$$
$$(\exp(2.8) + 120)^{0.8} - (120)^{0.8} = \frac{e_B}{-\frac{-\exp(e_A + e_B)}{\left(\exp(e_A) + \exp(e_B)\right)^2}} = \frac{e_B \big(\exp(e_A) + \exp(e_B)\big)^2}{\exp(e_A + e_B)}.$$

The equilibrium efforts can be solved (using MATLAB) as follows:

$$\hat{e}_A = 1.748 \quad \text{and} \quad \hat{e}_B = 1.136, \tag{15}$$

leading to $\hat{r}_A = 1.542$.

Consider now the bet-allowance regime, where first-order conditions (13) and (14) are:

$$(\exp(2.8) + 120 + (r_A - 1)b_A)^{0.8} - (120 - b_A)^{0.8} = \frac{0.65e_A\big(\exp(e_A) + \exp(e_B)\big)^2}{\exp(e_A + e_B)}, \quad (16)$$

$$(\exp(2.8) + 120)^{0.8} - (120)^{0.8} = \frac{e_B\big(\exp(e_A) + \exp(e_B)\big)^2}{\exp(e_A + e_B)}. \quad (17)$$

By setting $b_A = 12$ exogenously,[23] we solve for $(e_A^*, e_B^*) = (2.228, 0.757)$ satisfying (16) and (17), where $r_A^* = 1.23$. Thus,

$$e_A^* = 2.228 > 1.748 = \hat{e}_A \quad \text{and} \quad e_B^* = 0.757 < 1.136 = \hat{e}_B. \quad (18)$$

Thus allowing betting leads player A to bet and increase his effort while prompting B to lower his effort. Note from Fig. 4 that through betting player A has improved his utility relative to the no-betting regime. This corresponds to the case depicted in Fig. 1 where A's reaction function shifts along B's reaction function and accordingly the effort gap widens as shown in Fig. 4. Although by becoming even more favorite player A decreases the market return on his own bet, he would raise his bet up to the point where his expected utility is maximum as shown in the rightmost chart in Fig. 4. ||

Reversal of player hierarchy. We now illustrate the effort reversal possibility when we allow the pre-bet underdog, player B, to bet. Consider the bet-allowance regime, where first-order conditions (13) and (14) are:

$$(\exp(2.8) + 120)^{0.8} - (120)^{0.8} = \frac{0.65e_A\big(\exp(e_A) + \exp(e_B)\big)^2}{\exp(e_A + e_B)}, \quad (19)$$

$$(\exp(2.8) + 120 + (r_B - 1)b_B)^{0.8} - (120 - b_B)^{0.8} = \frac{e_B\big(\exp(e_A) + \exp(e_B)\big)^2}{\exp(e_A + e_B)}. \quad (20)$$

By setting $b_B = 40$ exogenously and $b_A = 0$, we solve for $(e_A^*, e_B^*) = (1.030, 2.689)$ satisfying (19) and (20), where $r_B^* = 1.190$.[24] Thus,

$$e_B^* = 2.689 > 1.136 = \hat{e}_B \quad \text{and} \quad e_A^* = 1.030 < 1.748 = \hat{e}_A. \quad (21)$$

While player B exerts a lower effort when his bet is zero, with higher bets he gradually increases his effort prompting A to lower efforts. For sufficiently high b_B, the effort by B overtakes A's effort, so the pre-bet underdog becomes the favorite under bet

[23]The choice of b_A roughly corresponds to the highest point of the utility plot for player A.
[24]Again, the choice of b_B roughly corresponds to the highest point of the utility plot for player B.

allowance. Fig. 5 confirms that player B has improved his utility relative to the no-betting regime.

Observe in the rightmost chart in Fig. 5 that the underdog B's expected utility falls as b_B is raised above zero. Because $q_{12} > 0$, the favorite increases his effort in response to the underdog's higher effort. Eventually, as b_B reaches a critical level, B's reaction function shifts beyond the peak of A's reaction function, reversing the sign of q_{12}. Further increases in b_B pacify A and raise B's expected utility which reaches a maximum at B's SPE bet. $\|$

■ **Player types and betting market assumptions.** Our analysis has made no mention of the relevance of player types, i.e., attributes that would shift the winning odds in favor of one player or the other. In a symmetric contest, players' types are implicitly captured by the success probability function $q(e_A, e_B)$ and the disutility-of-effort function $\psi_i(e_i)$. We say that in a symmetric contest player A is the "strong" player if $q(e, e) \geq \frac{1}{2}$ and $\psi_A(e) \leq \psi_B(e)$, with at least one inequality holding strictly, for all $e > 0$. That is, a strong player's effort may be relatively effective and/or his cost of effort may be smaller.

The numerical example above has a symmetric success probability function but $\psi_A(e) < \psi_B(e)$, hence, player A is the strong player. As illustrated in this example with asymmetric player power, if players are risk averse and bets are observable, the "strong" player may bet; if he does so, he will put a more aggressive effort and further reduce the weak player's already relatively small chance of winning. Thus, in as much as public enjoyment of the contest is a function of the players' relative efforts, allowing players with asymmetric power to bet on their own victory amplifies the asymmetry and makes the contest outcome even more predictable, the contest, less exciting.

One can also introduce a pure victory component into the players' preferences to account for post-victory effects from audience praise and public attention. Part of the motivation to win may come from this emotional satisfaction of being just the winner, besides the monetary stakes attached to contest outcomes. Wealth utility functions would then be outcome-dependent, $u_i^{win}(w_i)$ and $u_i^{lose}(w_i)$, both strictly concave in the case of risk-averse players, such that $u_i^{win}(w_i) > u_i^{lose}(w_i)$ for all $w_i > 0$. Qualitatively, our effort contest equilibria would not be affected because the only modification in the equilibrium conditions (5) and (6) would be to replace the left-hand side utility prize by the new expression $u_i^{win}(W_i^{win}) - u_i^{lose}(W_i^{lose})$. Nor would betting behavior be any different from the present analysis, provided the marginal utility of wealth in the victory outcome is not higher than the loss outcome (i.e., $u_i^{lose}(w_i)$ is a concave transformation of $u_i^{win}(w_i)$).[25]

[25] A simple example is $u_i^{win}(w_i) - u_i^{lose}(w_i) = d$, a positive constant, where victory-wealth utility is a parallel upward shift of the loss-wealth utility. On the other hand, note that a higher marginal

We assumed actuarially fair betting odds, for analytical convenience. Though the empirical literature on pricing in betting markets for the case of individualistic sports is very small, the realism of our assumption might be questionable. In horse races, for example, the odds on the favorite understate the true probability of winning whereas the odds on the underdogs often exaggerate the corresponding horses' true probability of winning.[26] Departure from the fair odds assumption to the range of expected returns below the betting cost will obviously decrease risk-averse players' incentives to place bets. The case for betting on own victory as commitment to exert higher effort will be weaker, though it will not disappear. Nor will our conclusion about the desirability of allowing players to bet on their own games change.

4 Risk-Loving Players

Betting on own victory generated commitment to aggressive play. The assumption of risk aversion, adding to the cost of betting, ensured that not every player would bet. Difference in players' risk attitudes thus could lead to different costs of commitment and separate the players naturally into aggressive and conceding plays in the contest effort stage.[27] This was partly responsible for both the widening efforts result and the changing player hierarchy result.

Here we adapt the analysis for risk-loving players. Betting now becomes attractive for its own sake that in turn would lead to more aggressive play in the contest stage. We want to see the direction of effort rivalry due to the allowance of betting.

Let us assume strictly convex wealth utility functions $u_i(\cdot)$, maintaining Assumptions 1 and 2 on, respectively, the contest success function and the effort cost functions. The returns (r_A, r_B) are competitive and the structure of the overall game is the same, so the only modification is in the curvature of the utility functions.

Proposition 4 (Private victory bets) *Suppose the players are allowed to place private bets on own victories.*

(i) *In any Nash equilibrium a risk-loving player i will bet his entire wealth, $b_i^* = w_i$.*

(ii) *Suppose that both players are risk loving. Compared to the no-betting regime, at least one player will exert strictly higher effort (A, if $q_{12} \geq 0$; B, if $q_{12} \leq 0$).*

utility of wealth in the victory outcome incentivizes betting by risk-averse players, making it possible that both players gain by placing a small bet on own victory.

[26] This stylized bias, known as the favorite-longshot bias, is probably due to *information asymmetry* about the true odds among various market participants (Ali, 1977). On the other hand, for American baseball Woodland and Woodland (1994) show that the odds are of the opposite nature—underdogs tend to be underbet. The authors cite favorite-longshot bias of racetrack betting as one where the favorite is underbet and underdogs, overbet.

[27] Of course the contest technology did also matter, so the direction of effort responses were not entirely due to risk preferences.

That at least one player will exert higher effort when allowed to bet on own victory follows directly from the fact that by betting entire wealth on own victory each player is shifting "up" his reaction function. As the new Nash equilibrium lies at the intersection of these new reaction functions, at least one player's effort must rise relative to his Nash equilibrium effort in the no-betting case.

Remark. Proposition 4 is based on the assumption that the sign of q_{12} does not change between the initial no-betting equilibrium and the equilibrium under bet allowance, so the slopes of the respective reaction functions are of the same signs. (This would be true if the equilibrium effort pairs under the two bet allowance rules do not differ by too much.) However, the uniformity of the sign of q_{12} is not strictly required; see footnote 30 in the proof. $\|$

If bets are unobservable, a risk-loving player will bet maximally on his own victory regardless of the risk preference of the other player. As for the impact of jointly aggressive betting on effort levels, intuition suggests that each player's incentives to exert effort should be stronger with their entire wealth staked on own victories. However, with each engaging in aggressive betting they also partly tend to stifle each other's effort incentives. So while intuition fails to give a clear guidance about the overall effects of betting on efforts, we are able to say a bit more – which player will certainly increase his effort depending on the sign of q_{12}. Overall, due to (unobservable) betting the contest may become more uneven (similar to that in Proposition 3 and Corollary 1) or both players may end up exerting higher efforts.

When bets are observable, the new element in the equilibrium analysis is the cross-effort effects of individual bets:

Proposition 5 (Observable victory bets) *Suppose that the players are risk loving and allowed to place victory bets but with the requirement of full disclosure before the actual contest. In any SPE at least one player will bet all his wealth (A, if $q_{12} > 0$; B, if $q_{12} < 0$).*

Under observable bets, even under risk-loving preferences not necessarily both players stake all their wealth in betting or even place positive bets. One of the players' marginal utility from betting remains ambiguous. This can be easily seen by re-evaluating (11) (for the risk-loving case). Here we cannot rule out the possibility that, say, player B bets less than his full wealth on own victory when under the SPE pair of efforts, $q_{12} > 0$, hence $\frac{\partial e_A^*}{\partial b_B} > 0$, because marginally raising b_B leads player A to increase his effort, reducing B's chances of victory. If this is the case and the effect is strong enough, i.e., if $\frac{\partial e_A^*}{\partial b_B}$ is positive and sufficiently large, in the SPE player B may choose to bet less aggressively.

Consider Fig. 6 to see how bet and effort decisions interact as suggested above. The

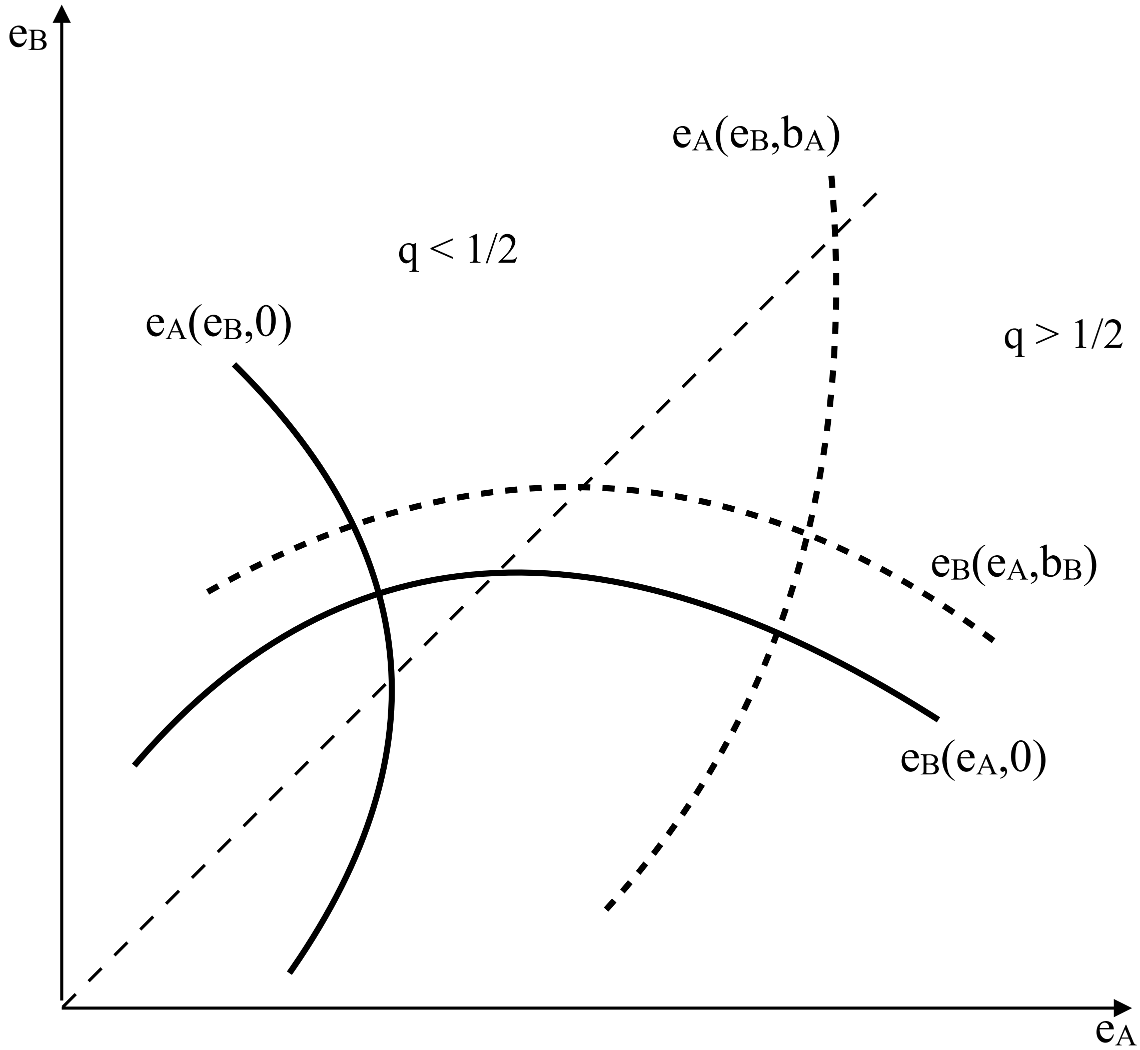

Fig. 6: Compare with Fig. 1. Unbroken reaction functions under no-betting case. Broken reaction functions under bet allowance. A's effort increases and B's effort may or may not increase. Without betting B is the favorite; with betting, B becomes the underdog. B's reaction function shifts up by less than the rightward shift in A's reaction function. Both could bet their entire wealth on own victory. The resulting contest can be more exciting or less.

unbroken reaction functions represent the no-betting case and the broken functions represent the bet-allowance case. Due to bet allowance both equilibrium efforts have actually gone up in the drawn figure, but if the reaction functions shift with different intensities then one of the efforts could come down.

The possibility that (say) B may even set $b_B^* = 0$ with $e_B^* < e_B^0$ while $b_A^* = w_A$ and $e_A^* > e_A^0$ can be seen by inspecting the marginal expected utility of B in (10). The first term, marginal utility difference in victory and loss outcomes, vanishes if player B is only slightly risk loving (take, for example, $u_B(W) = W^{1+\epsilon}$ where ϵ is a small positive real). As for the second term in (10), it is negative if in the no-betting equilibrium $q_{12} > 0$, which can only stay negative in the equilibrium with betting where player A bets on own victory (see (8) which, evaluated at $b_A = 0$, is unambiguously positive), shifting his reaction function along B's reaction function further away from its peak. The first term in (10), positive but very small, would be dominated by the negative second term, so (10) would be negative, implying $b_B^* = 0$. Basically, player B's behavior in this SPE is that of a risk-averse player B in Proposition 3 equilibrium, which should not be surprising because he is only slightly risk loving—almost risk neutral. Allowing players to bet on own victory may thus split equilibrium efforts further apart even if both players are risk loving, the equilibrium shifting from slightly above to much below the the 45^o-line, as in Fig. 6.

Betting on own victory, when fairly priced, produces two effects: (i) it raises the expected utility of a risk-loving player, and (ii) affects the contest efforts and expected outcome. While the first is beneficial for both players if they are risk loving, the second effect benefits only one player at a time. If in the no-betting effort equilibrium $q_{12} > 0$, the player who benefits from affecting contest efforts via betting is player A. Raising his effort along B's reaction function increases his expected utility, which betting achieves for him in concordance with his risk preference. Player B, on the other hand, would prefer to commit to an effort below his effort in the no-betting equilibrium, so betting on own victory can only reduce his contest-stage utility. Despite this fact he may bet some of his wealth on own victory if the first effect is strong enough (if he has a strong preference for wealth risk).

Proposition 5, just like Proposition 4, indicates that when contestants are risk loving, allowing them to place bets makes at least one player to uphold the standard intuitions on the value of commitment and aggressive play. But still it is impossible to establish with certainty the positive value of bets for contest excitement. Proposition 5 leaves open any of the possibilities such as *deterrence effect* (strong player's aggressive bet and effort lowering weak player's effort) or *escalation effect* (both players go at each other through aggressive bets and efforts in the contest) (Harris and Vickers, 1985; Horner and Sahuguet, 2007). The commitment and competition and the insights of divergent impacts on contest efforts in this paper have some parallels, albeit superficial, with the insights of Horner and Sahuguet (2007),[28] and Bulow, Huang and Klemperer (1999).

5 Conclusion

Observable own victory bets have asymmetric cross-effort effects and, for risk-averse players, are costly commitment instruments to raise own effort in the contest. In any

[28]However, as mentioned earlier, the difference between our complete information setup and Horner and Sahuguet's incomplete information signaling model is a significant one, besides the difference in the nature of applications.

pure strategy equilibrium only one player, if any, would bet. Typically this player is the favorite, who then becomes a stronger favorite due to the negative cross-effort effect, which makes the contest less exciting for the ordinary viewing public. Thus, it is not a good idea to let players bet on own victory especially in first elimination rounds of sporting tournaments where the big favorites are matched with the underdogs. In final rounds or in competitive matches where the odds are near equal, bet allowance may not substantially destroy the excitement.

In the case of risk-loving players can one possibly find a strong justification for allowing players to bet on their own victories. If pre-contest public proclamations of superiority can be associated with similar extravagant attitudes towards risks, then letting the players make credible commitment through observable victory bets might be an equally effective way of raising the contest excitement. Risk-loving players would bet maximally on their own victory, which as a by-product would make them more aggressive to win in the contest stage. Although the impact on relative efforts is ambiguous, total efforts would increase.

Our analysis of bet-augmented contests offers new insights on endogenous commitment. Besides the potential to make the strong player stronger and the weak player weaker, an equal access to commitment opportunity can turn contest expectations upside down. It is possible that the ex-favorite with a mild advantage finds himself the underdog after the bets are closed. Some of these results stand in stark contrast with the literature's earlier findings on how commitment affects rivalry in sequential contests (Dixit, 1984; Baik and Shogren, 1992).

We did not analyze the most permissive bet allowance case where a player can take a punt on the rival's victory. It is not difficult to imagine that such allowance might lead to a race for the bottom, especially if the players are risk averse. By placing a bet on rival's win a player partly insures himself from losing, which clearly has adverse implications for effort initiatives. Sports regulations would always legislate against such extreme bet allowance. Finally, we did not consider the case of incomplete information where players' bets can signal private information about motivation and effort costs. How the single instrument of betting on own victory can serve the dual role of signaling and commitment in contests should be an interesting question.

A Appendix

Proof of Lemma 2. We provide a geometric argument based on Fig. 2. Existence of a pure strategy Nash equilibrium is already guaranteed by Proposition 0, so the two reaction functions, red and violet, must cross at least once. If there are at least two Nash equilibria, say EQ_3 and EQ_1, the reaction functions' slopes must reverse in signs. Further, because of the continuity of the reaction functions (Assumption 3), by Brouwer's fixed-point theorem the reaction functions must intersect at some effort levels in between the two equilibria, which we indicate by EQ_2. At this additional equilibrium EQ_2, both reaction functions will be negatively sloped, contradicting Lemma 1 which is an equilibrium property. **Q.E.D.**

Proof of Proposition 2. By Proposition 0, there exists a pure strategy Nash equilibrium in the simultaneous-move contest with private bets. In the text we observed that the equilibrium efforts (e_A^*, e_B^*) will be strictly positive satisfying (5) and (6), as shown in Fig. 1.

Taking partial derivative of Eu_A with respect to b_A and using strict concavity of $u_A(.)$ obtain:

$$\frac{\partial Eu_A}{\partial b_A} = (1-q)[u_A'(W_A^{win}) - u_A'(W_A^{lose})] < 0.$$

This implies $b_A^* = 0$. Similarly,

$$\frac{\partial Eu_B}{\partial b_B} = q[u_B'(W_B^{win}) - u_B'(W_B^{lose})] < 0,$$

so that $b_B^* = 0$. **Q.E.D.**

Proof of Lemma 3. Recall the Nash equilibrium efforts $(e_A^*(b_A, b_B), e_B^*(b_A, b_B))$ given by (5) and (6). Differentiating (5) and (6) w.r.t. b_A and arranging in a matrix form, we obtain

$$\underbrace{\begin{bmatrix} \frac{q_1\psi_A''(e_A) - q_{11}\psi_A'(e_A)}{(q_1)^2} & -\psi_A'(e_A)\cdot\frac{q_{12}}{(q_1)^2} \\ -\psi_B'(e_B)\cdot\frac{q_{21}}{(q_2)^2} & \frac{q_2\psi_B''(e_B) - q_{22}\psi_B'(e_B)}{(q_2)^2} \end{bmatrix}}_{=D} \times \begin{bmatrix} \frac{\partial e_A^*}{\partial b_A} \\ \frac{\partial e_B^*}{\partial b_A} \end{bmatrix} = \begin{bmatrix} u_A'(W_A^{win})(r_A - 1) + u_A'(W_A^{lose}) \\ 0 \end{bmatrix},$$

solving which yields:

$$\begin{bmatrix} \frac{\partial e_A^*}{\partial b_A} \\ \frac{\partial e_B^*}{\partial b_A} \end{bmatrix} = \frac{1}{|D|}\cdot\begin{bmatrix} \frac{q_2\psi_B''(e_B) - q_{22}\psi_B'(e_B)}{(q_2)^2}\cdot\{u_A'(W_A^{win})(r_A-1) + u_A'(W_A^{Lose})\} \\ \psi_B'(e_B)\cdot\frac{q_{21}}{(q_2)^2}\cdot\{u_A'(W_A^{win})(r_A-1) + u_A'(W_A^{lose})\} \end{bmatrix},$$

where

$$|D| = \underbrace{\frac{q_1\psi_A''(e_A) - q_{11}\psi_A'(e_A)}{(q_1)^2}}_{>0}\cdot\underbrace{\frac{q_2\psi_B''(e_B) - q_{22}\psi_B'(e_B)}{(q_2)^2}}_{<0} - \underbrace{\{\psi_A'(e_A)\cdot\frac{q_{12}}{(q_1)^2}\cdot\psi_B'(e_B)\cdot\frac{q_{21}}{(q_2)^2}\}}_{>0} < 0.$$

It now follows that

$$\begin{aligned}
\frac{\partial e_A^*}{\partial b_A} &> 0; \\
\frac{\partial e_B^*}{\partial b_A} &> 0 \quad \text{if } q_{21}(e_A^*, e_B^*) < 0, \\
&= 0 \quad \text{if } q_{21}(e_A^*, e_B^*) = 0, \\
&< 0 \quad \text{if } q_{21}(e_A^*, e_B^*) > 0.
\end{aligned}$$

The proof of the symmetric case: $\frac{\partial e_B^*}{\partial b_B} > 0$, and $\frac{\partial e_A^*}{\partial b_B} < (=, \text{or} >) 0$ if $q_{12}(e_A^*, e_B^*) <$ (respectively $=$, or $>$) 0, follows the same arguments. **Q.E.D.**

Proof of Proposition 3. **(i)** That in any SPE at most one player will place a positive bet follows from the fact that the marginal (expected) utility of bet for the players in (8) and (10) cannot both be positive; by Lemma 3, $\frac{\partial e_A^*}{\partial b_B}$ and $\frac{\partial e_B^*}{\partial b_A}$ are of opposite signs or both are equal to zero, so at least one of (8) and (10) must be negative-valued.

(ii) Assume $q_{12}(e_A^0, e_B^0) > 0$. In each of the cases (a) and (b) we verify the sign of q_{12}^* and the possibility of an SPE of the corresponding form, and compare the players' efforts with their efforts in the no-betting equilibrium.

(a) To see the possibility for $b_A^* > 0$, set $b_B = 0$ and fully differentiate A's expected utility in (3) along B's effort reaction function. This yields:

$$\frac{dEu_A}{db_A} = (1-q)[u_A'(W_A^{win}) - u_A'(W_A^{lose})] + [(q_1 + q_2 \frac{d\hat{e}_B}{de_A})(u_A(W_A^{win}) - u_A(W_A^{lose})) - \psi'(e_A)] \frac{de_A}{db_A}. \tag{22}$$

Since $q_{12}(e_A^0, e_B^0) > 0$, in the no-betting Nash equilibrium, $\frac{d\hat{e}_B}{de_A} < 0$ by Lemma 1. We also know that $\frac{de_A}{db_A} > 0$ and, by assumption, $q_2 < 0$. Therefore, using also (5), the second term in (22) is positive. If the first term, though negative, is smaller in absolute value than the second term, then $\frac{dEu_A}{db_A} > 0$ at $(b_A, b_B) = (0, 0)$ and continuation equilibrium efforts (e_A^0, e_B^0): Player A can increase his expected utility by raising b_A above zero. By continuity, the expected utility function must have a strictly positive maximand b_A^* in the compact set $[0, w_A]$. This $b_A^* > 0$ is therefore a best response to $b_B^* = 0$ and by part (i) $b_B^* = 0$ is B's best bet response.

In this SPE, $q_{12}(e_A^*, e_B^*) > 0$, for otherwise the second term in (22) cannot be positive; more precisely, by Lemma 1, $q_{12} < 0$ implies $\frac{d\hat{e}_B}{de_A} > 0$ along B's reaction function and hence it would be optimal for A to reduce b_A^*, which contradicts the fact that b_A^* maximizes Eu_A given $b_B^* = 0$. Using in the effort-optimality conditions (5) and (6) the fact that under these SPE bets player A's win-lose wealth difference increases while B's win-lose wealth difference is constant implies $e_A^* > e_A^0$ and $e_B^* < e_B^0$. The equilibrium betting-effort possibility (a) can be seen in Fig. 1.

(b) Fix $b_A = 0$ and fully differentiate B's expected utility in (4). Using (6) and arranging terms yields

$$\frac{dEu_B}{db_B} = q \cdot [u'_B(W_B^{win}) - u'_B(W_B^{lose})] - q_1 \frac{d\hat{e}_A}{de_B}(u_B(W_B^{win}) - u_B(W_B^{lose})) \cdot \frac{de_B}{db_B}. \quad (23)$$

The expression in (23) is B's marginal expected utility of betting, incorporating the impact on efforts along A's reaction function (which remains at its original position because A does not bet). We also know that $q_{12}(e_A^0, e_B^0) > 0$ implies $\frac{d\hat{e}_A}{de_B} = \frac{\psi'_A(e_A)q_{12}}{\psi''_A(e_A)q_1 - \psi'_A(e_A)q_{11}} \geq 0$, by (7). Hence the expression in (23) is negative at the bets $(b_A, b_B) = (0, 0)$, and remains negative at all $b_B > 0$ such that $q_{12}(e_A, e_B) > 0$ in the continuation effort equilibrium. Assuming player B's wealth w_B is not too small, there exists a bet $b_B^P \in (0, w_B)$ such that the continuation equilibrium effort pair $(\hat{e}_A(e_B^P), e_B^P)$ lies at the peak of A's reaction function, where $q_{12}(\hat{e}_A(e_B^P), e_B^P) = 0$. Because reaction functions are single-peaked, the sign of $q_{12}(e_A, e_B)$ is negative for all $e_B > e_B^P$. Thus, if $b_B > b_B^P$, $e_B > e_B^P$ and the second term in (23) becomes positive, which may offset the negative first term and revert the sign of $\frac{dEu_B}{db_B}$ from negative to positive. The expected utility of B may not be monotonic in b_B, first declining and then possibly increasing, once continuation equilibrium efforts move beyond the peak of A's effort reaction function. It follows that Eu_B can have a maximum at $b_B^* \in (b_B^P, w_B]$, given $b_A = 0$.

Suppose such a b_B^* exists and denote the continuation effort equilibrium by (e_A^*, e_B^*). Then, b_B^* and $b_A^* = 0$ must constitute a pair of SPE betting strategies (by the same arguments used in part (a), the symmetric case to the present one). The claim $q_{12}(e_A^*, e_B^*) < 0$ follows from the fact that at (e_A^*, e_B^*) we must have $\frac{d\hat{e}_A}{de_B} < 0$ because (e_A^0, e_B^0) and (e_A^*, e_B^*) are located at opposite sides of the peak of A's reaction function. Under general logit functions, $q_{12} < 0 \Leftrightarrow q < 1/2$, therefore $q^0 > 1/2$ and $q^* < 1/2$: the underdog becomes the favorite in this SPE, when betting is allowed. Finally, that B's SPE effort e_B^* is larger than e_B^0 follows from the fact that $\frac{de_B^*}{db_B} > 0$. As for the change in A's effort, a definite prediction is impossible because whereas the left-hand side of (5) is constant, the direction of change in the right-hand side depends on the change in q_1, which is ambiguous. This completes the proof of part (b).

(iii) The arguments for the case of $q_{12}(e_A^0, e_B^0) < 0$ will be symmetric to that of part (ii). **Q.E.D.**

Proof of Proposition 4. **(i)** Consider player A's expected utility in (3). Given strict convexity of the utility function and $W_A^{win} > W_A^{lose}$ (by (2)), in any equilibrium we obtain:

$$\frac{\partial Eu_A^*}{\partial b_A} = (1-q)[u'_A(W_A^{win}) - u'_A(W_A^{lose})] > 0.$$

Therefore, $b_A^* = w_A$: player A bets his entire wealth on his victory. The proof of $b_B^* = w_B$ is identical.

(ii) It is easy to verify that $W_i^{win} - W_i^{lose}$ is larger if $b_i^* = w_i$ than in the case of $b_i^0 = 0$. Given this fact and the result in part (i), combining the effort optimality conditions under private victory-bets and no-betting regimes (see eqs. (5) and (6)) we can write:

$$\frac{\psi_A'(e_A^*)}{q_1(e_A^*, e_B^*)} > \frac{\psi_A'(e_A^0)}{q_1(e_A^0, e_B^0)}, \qquad \frac{\psi_B'(e_B^*)}{-q_2(e_A^*, e_B^*)} > \frac{\psi_B'(e_B^0)}{-q_2(e_A^0, e_B^0)}. \tag{24}$$

We first rule out a situation of a completely null betting effect with $(e_A^*, e_B^*) = (e_A^0, e_B^0)$: none of the two first-order conditions (5) and (6) can be satisfied given the different wealth gaps between win and loss states under betting and no-betting regimes.

Give this last assertion we know that for at least one player the effort under bet allowance must be different from his effort under no-betting regime. Below we verify two stronger claims about the players' equilibrium efforts under bet allowance: (a) if $q_{12} \geq 0$ then player A's effort will increase, and (b) if $q_{12} \leq 0$ then player B's effort will increase.[29]

Suppose claim (a) is false. First consider the possibility that $e_A^* < e_A^0$ and $e_B^* \geq e_B^0$. Then $\psi_A'(e_A^*) < \psi_A'(e_A^0)$, which to guarantee the first inequality in (24) must imply:

$$q_1(e_A^*, e_B^*) < q_1(e_A^0, e_B^0). \tag{25}$$

However, we know that

$$q_1(e_A^*, e_B^*) > q_1(e_A^0, e_B^*) \geq q_1(e_A^0, e_B^0),$$

since $q_{11} < 0$ and $q_{12} \geq 0$.[30] This contradicts (25).

Consider the other possibility that $e_A^* < e_A^0$ and $e_B^* < e_B^0$. Then $\psi_B'(e_B^*) < \psi_B'(e_B^0)$ that implies by the second inequality in (24),

$$-q_2(e_A^*, e_B^*) < -q_2(e_A^0, e_B^0). \tag{26}$$

But we also know that

$$q_2(e_A^*, e_B^*) < q_2(e_A^*, e_B^0) \leq q_2(e_A^0, e_B^0),$$

since $q_{22} > 0$ and $q_{21} \geq 0$. This contradicts (26).

[29] We cannot rule out a situation of $q_{12} = 0$ in equilibrium both before and after admitting bet allowance so that one of the reaction functions has vertical slope while the other one has horizontal slope (at the point of intersection), in which case both players' efforts would increase.

[30] In fact, we do not strictly require $q_{12} \geq 0$. The last part of the above inequality may be true even with q_{12} changing sign. Similarly, the sign of q_{12} may change and still we might be able to arrive at the required contradictions of (26)–(28), below.

Next suppose claim (b) is false. So first consider $e_B^* < e_B^0$ and $e_A^* \geq e_A^0$. Then, $\psi_B'(e_B^*) < \psi_B'(e_B^0)$, which to guarantee the second inequality in (24) must imply

$$-q_2(e_A^*, e_B^*) < -q_2(e_A^0, e_B^0). \tag{27}$$

However, we know that

$$q_2(e_A^*, e_B^*) < q_2(e_A^*, e_B^0) \leq q_2(e_A^0, e_B^0),$$

since $q_{22} > 0$ and $q_{21} \leq 0$. This contradicts (27).

Consider the remaining possibility which is $e_B^* < e_B^0$ and $e_A^* < e_A^0$. Then $\psi_A'(e_A^*) < \psi_A'(e_A^0)$ which implies by the first inequality in (24),

$$q_1(e_A^*, e_B^*) < q_1(e_A^0, e_B^0). \tag{28}$$

But we also know that

$$q_1(e_A^*, e_B^*) > q_1(e_A^0, e_B^*) \geq q_1(e_A^0, e_B^0),$$

since $q_{11} < 0$ and $q_{12} \leq 0$. This contradicts (28). **Q.E.D.**

Proof of Proposition 5. Fix an SPE pair of strategies and consider the marginal expected utilities from betting, which are as stated in (8) and (10). Since $W_i^{win} > W_i^{lose}$ (by (2)), in any equilibrium the marginal utility difference $u_i'(W_i^{win}) - u_i'(W_i^{lose})$ is positive for a risk-loving player. Given that the results in Lemma 3 on the signs of own- and cross-effort responses to marginal changes in bets are independent of risk preferences, if under the SPE pair of efforts $q_{12} > 0$, then by Lemma 3 the following inequalities must hold:

$$\frac{\partial e_B^*}{\partial b_A} < 0, \quad \frac{\partial e_A^*}{\partial b_B} > 0.$$

Using $\frac{\partial e_B^*}{\partial b_A} < 0$ in (8) and recalling $q_2 < 0$ yield $\frac{\partial E u_A^*}{\partial b_A} > 0$. Hence, $b_A^* = w_A$.

Symmetric arguments can be used to establish, given $q_1 > 0$, that $\frac{\partial E u_B^*}{\partial b_B} > 0$ when $q_{12} < 0$. Player B will then set $b_B^* = w_B$. **Q.E.D.**

■ Logit contest function

Lemma 4 *Under logit contest functions of the form* $q = \frac{f(e_A)}{f(e_A)+g(e_B)}$ *where* $f(.)$ *and* $g(.)$ *are increasing functions, each player's reaction function is single-peaked.*

Proof: The proof argument will follow Fig. 7 closely. The following facts are useful:

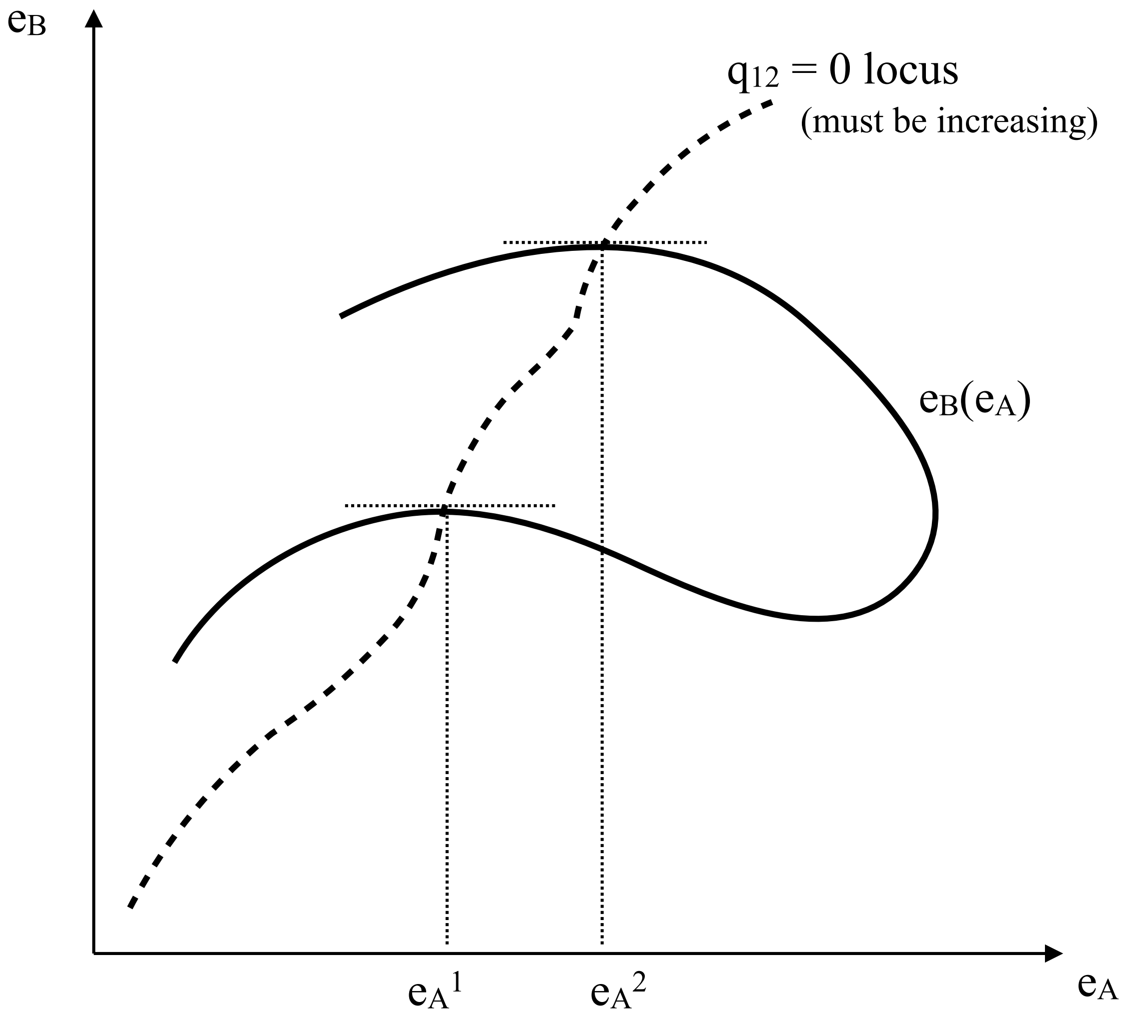

Fig. 7: The solid curve is B's effort reaction function $e_B(e_A)$ under the assumption that it has two peaks (for j=A) at $e_A{}^1$ and $e_A{}^2$. Note that B's reaction function cannot be drawn to cross the dotted curve because whenever it does, there must be a peak and its slope must be zero.

Fact 1. $\left.\frac{de_i}{de_j}\right|_{\hat{e}_i} = 0 \quad \Leftrightarrow \quad q_{12} = 0$; *moreover, given any positive utility difference from win and loss outcomes,* $u_i(W_i^{win}) - u_i(W_i^{lose})$, *in stage 3 of the game player* i*'s best response to any* $e_j \geq 0$ *is a singleton.*

Fact 2. Under logit contest functions of the form above, the locus of effort combinations (e_A, e_B) *such that* $q_{12} = 0$ *is an increasing curve in the* R_+^2 *plane.*

The first part of Fact 1 follows from (7), the second part follows from the first-order conditions (5) and (6), Assumption 1 ($q_{11} < 0$, $q_{22} > 0$), and Assumption 2. Fact 2 can be verified by taking partial derivatives of the contest function, which yields: $q_{12} = 0$ if and only if $f(e_A) = g(e_B)$. Therefore the locus $q_{12} = 0$ is defined by $e_B = g^{-1}(f(e_A))$, where $g^{-1}(f(.))$ is an increasing function because $f(.)$ and $g(.)$ are both increasing functions.

Combining the two facts above, we conclude that player i's reaction function must have zero slope (a "peak"), $\frac{de_i}{de_j}\Big|_{\hat{e}_i} = 0$, when it intersects the $q_{12} = 0$ locus, which for contest functions of the form $q = \frac{f(e_A)}{f(e_A)+g(e_B)}$ is defined by $e_B = g^{-1}(f(e_A))$.

We claim that any such "peak" must be unique. Suppose not, and consider any two consecutive peaks of player B's reaction function, one at $\hat{e}_B(e'_A)$ (lower peak) and the other at $\hat{e}_B(e''_A)$ (higher peak). Because both peaks must be on the *increasing* $q_{12} = 0$ locus, $e'_A < e''_A$ and $\hat{e}_B(e'_A) < \hat{e}_B(e''_A)$. Note that the slope of $\hat{e}_B(e_A)$ must be negative in the right neighborhood of e''_A, the peak. On the other hand, $\hat{e}_B(e_A)$ must also be positively sloped in some segment of the interval $[e'_A, e''_A]$ to reach its higher peak at e''_A. Then, by continuity of $\hat{e}_B(e_A)$, there must exist e_A in the right neighborhood of e''_A where $\hat{e}_B(e_A)$ takes two different values. This contradicts Fact 1, that at each e_j player i's best-response effort $\hat{e}_i(e_j)$ is a singleton. **Q.E.D.**

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