Quantifying the Fiscal Cushion for the US:1960-2013 *

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Abstract

The debt valuation equation reveals that the real value of government debt equals the expected discounted present value of primary surplus. A shock to the primary surplus can get absorbed by surprise change in the price level or as a change in the bond prices given the level of nominal debt. Therefore, nominal debt can act as a Fiscal Cushion. In this paper I consider two channels of debt revaluation: surprise changes in in ation and bond prices. Using quarterly US data on debt and other macro variables between 1960-2013, I find that unanticipated return on the US government debt due to surprise inflation has been a negative 0.02% and the capital gain/loss due to changes in the bond price on one-year securities, medium and long term securities as a percent of nominal GDP, has been 0.62% , 0.8% and 0.15% respectively. Monetary and fiscal policy interactions affect these revaluation components and they take different values depending on whether the regime is active fiscal and passive monetary (AFPM) or vice versa. Prior Predictive analysis in a simple endowment economy model shows that in the AFPM regime a unit fiscal policy shock results in a change in surprise inflation ranging between 0 and -0.025% in the first four quarters of the shock and a change in the bond price ranging between 0 and 0.025%. Using sign restriction to identify the fiscal shock for the US between 1960-2013, I find that on impact due to a fiscal shock inflation falls to a little below -0.4% and after four quarters it is about -0.3%. The bond price rises by 0.5% on impact and by a year the change is about 0.2%.

Key Words: Public Debt, Fiscal Policy, Vector Autoregression
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1 Introduction

Debt held by economic agents is subject to capital gains/losses due to changes in inflation and nominal interest rate. Due to inflation an agent holding a government bond with fixed coupon payment will lose in real terms both in terms of coupon payment and principal. Bond price and interest rate offered by a government security are inversely related. If the nominal interest rate rises then the price of that security falls and the agent loses if the security is sold in the market. By surprise inflation and changes in the interest rate the level of nominal debt can be affected. The literature on Fiscal Theory of the Price Level (Leeper, 1991), (Sims, 1994), (Woodford, 1995), (Cochrane, 1998) point out that fiscal policy changes can lead to changes in inflation whereby fiscal shocks working its way through the price level can help revalue debt. Hence nominal debt can act as a "Fiscal Cushion" for fiscal shocks.

Fiscal and monetary policy interactions play a role in affecting the components that can revalue debt. The debt valuation equation implies that real debt equals the expected discounted present value of primary surplus. As a result shocks to primary surplus can be absorbed by changes in the general price level and/or bond prices if nominal debt remains same. Also, monetary policy plays a role in debt valuation by affecting the nominal interest rate that directly affects inflation. Changes in the nominal interest rate by the monetary authority also affects debt by affecting the bond prices. Accordingly, I focus on two major channels through which debt can get revalued: surprise inflation and changes in bond prices.

Marketable debt at any time period t, undergoes change due to surprise inflation. So that following (Sims, 2001, 2013) these "unanticipated returns"(X_t) can be calculated as follows:

$$X_t = B_t(1 - \tilde{\pi}_t) + S_t - (1 + r_{t-1})B_{t-1} \tag{1}$$

where, $\tilde{\pi}$ is the quarterly forecast error of inflation calculated from a VAR, B_t is debt, S_t is the primary surplus, r_{t-1} is the nominal interest rate at time $(t-1)$. Specifically, at the end of period t, the level of debt B_t undergoes change due to unanticipated changes in inflation captured by the term $B_t(1 - \tilde{\pi}_t)$. The unanticipated changes in inflation occur due to the difference between actual inflation π_t and the expected inflation $E_{t-1}\pi_t$. If $\tilde{\pi}_t > 0$ (or < 0) then actual inflation is higher (or lower) than the expected inflation so that $B_t(1 - \tilde{\pi}_t)$ falls (or rises) and ceteris paribus, X_t rises (or falls). The government's primary surplus position and repayment of past debt (principal and interest rate) needs to be accounted for when considering how the debt position changes. Accordingly, the expression for X_t incorporates the surplus keeping in mind that the government retires part of the debt using primary surplus S_t during period t and repays past debt given by $(1 + r_{t-1})B_{t-1}$. [1]

Figure (1) shows the series x_t (X_t as a fraction of GDP). The average annual return during the whole period 1961-2013 turns out to be a negative 0.02% (as shown in Table 1) and that during 1961-2006 is a negative 0.017% with a lower standard deviation at 0.08% compared to the whole period at about 0.09% so that the returns get more volatile post

[1] The debt series is marketable treasury debt (in dollar billions) series. The primary surplus series (in dollar billions) is calculated from National Income and Products Accounts and the interest rate series is the three month treasury bill rate. Although it makes sense to use the one year treasury constant maturity rate for the calculations, it was dropped in favor of the three month treasury bill rate as the former series is available only from 1963 onwards. The inflation forecast error is calculated from a three variable quarterly VAR at four lags consisting of inflation, unemployment and interest rate.

2007. The maximum value of the returns are recorded for the year 2006 when it is about 0.16% and this is due to the fact that inflation forecast error for this period was about -0.72 and that when multiplied by a high level of nominal debt led to such a positive return. The minimum value for the return series is recorded for the year 2010 when the return is about -0.35%. This is partly due to the fact that the inflation forecast error was positive during the year rendering the first term in equation (1) to be positive. Also, during the period the debt level to be repaid was higher compared to the other periods so that the repayment term, $(1+r_{t-1})B_{t-1}$, in equation (1) was high thereby making the X_t term a big negative.

Figure 1: The Annual Unanticipated Return Series

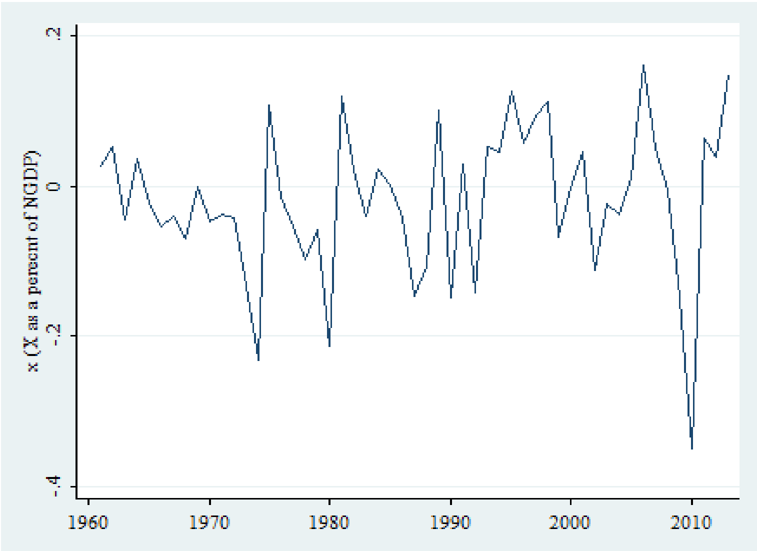


Table 1: The Annual Unanticipated Return Series

Variable	Obs	Mean	Std. Dev.	Min	Max
Unanticipated Return	53	-0.02	0.10	-0.35	0.16

Note: Unanticipated return as a share of nominal GDP

At the beginning of time period t government has to repay the interest along with the principle i.e., $(1 + r_{t-1})B_{t-1}$, that it owes from the previous period. However, due to bond price changes the interest amount undergoes change. If for a j period security at time t the bond price is denoted by $a_{j,t}$ and the amount of nominal debt outstanding is given by $s_{j,t}$, then the market value of the debt is given by $a_{j,t}s_{j,t}$. Since at any time period t there are securities of different maturities so that $j = 1, 2, ..., n$ years then the total market value of debt is given by $\sum_{j=1}^{n} a_{j,t}s_{j,t}$. The market value of the past debt that comes due at time t is given by, $\sum_{j=1}^{n} a_{j-1,t}s_{j,t-1}$[2]. Accordingly, the total debt due at time t can be expressed in terms of bond price and amount outstanding as follows:

$$(1 + r_{t-1})B_{t-1} = \sum_{j=1}^{n} a_{j-1,t}s_{j,t-1} = \sum_{j=1}^{n}(a_{j-1,t} - a_{j,t-1})s_{j,t-1} + \sum_{j=1}^{n} a_{j,t-1}s_{j,t-1} \qquad (2)$$

[2]See Appendix for an explanation.

The above equation can be manipulated to capture the capital gain/loss on securities of different maturity due to changes in the bond prices (Hall and Sargent, 1997). I consider here securities of maturities one-year, 2-10 years (Treasury Notes) and 11-30 years (Treasury Bonds)[3]. Specifically, re-writing, the first term of the last equality from the above equation leads to the following:

$$\begin{aligned}\sum_{j=1}^{30}(a_{j-1,t}-a_{j,t-1})s_{j,t-1} = \; & \underbrace{(a_{0,t}-a_{1,t-1})s_{1,t-1}}_{\text{The capital gain/loss on one-period securities}} \\ + \; & \underbrace{\sum_{j=2}^{10}(a_{j-1,t}-a_{j,t-1})s_{j,t-1}}_{\text{The capital gain/loss on medium term securities}} \\ + \; & \underbrace{\sum_{j=11}^{30}(a_{j-1,t}-a_{j,t-1})s_{j,t-1}}_{\text{The capital gain/loss on long term securities}} \qquad (3)\end{aligned}$$

Figure 2 shows the capital gain/loss as a percent of GDP between 1961-2013.[4] On an average, the capital gain/loss on one-year securities as a percent of nominal GDP, has been 0.62% and those for the medium and long term has been 0.8% and 0.15% as in Table 2. The numbers suggest that the capital gain has been highest on the medium term securities compared to that of the short-term and long-term securities. The reason for this being the fact that between 1961-2013 as per the sample the US government issued more securities that are medium term compared to that of the short-term (one period) securities and the long term bonds.

Table 2: Capital Gain/Loss as Percent of GDP Across Maturities

Capital gain/loss	**Obs**	**Mean**	**Std. Dev.**	**Min**	**Max**
Short-term	53	0.62	0.39	0.04	1.46
Medium-term	53	0.80	0.80	-0.58	3.14
Long-term	53	0.15	0.35	-0.56	1.16

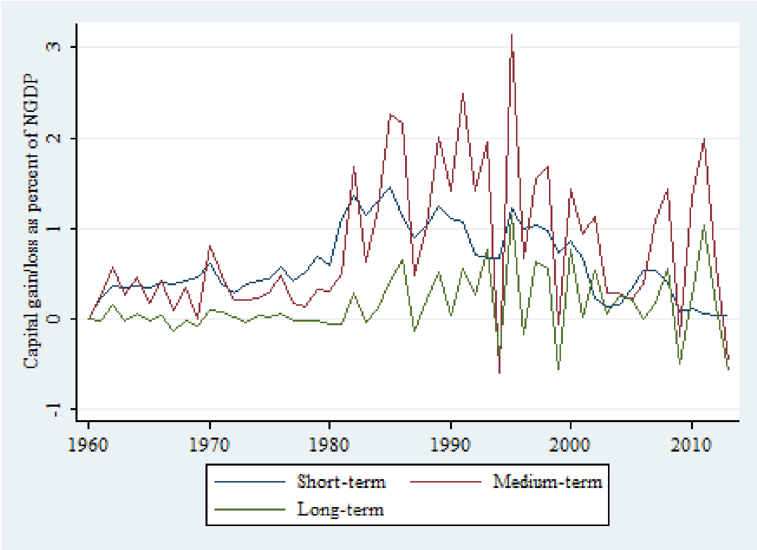
Note: All capital gains/losses are calculated as a share of nominal GDP

However, during 1961-2013 the bond prices of the long bonds have fluctuated most compared to those of the medium term securities and the short-term ones as observed in figure 3. The standard deviation of the bond price change per dollar for the long term securities is highest at \$0.05, followed by the medium term at \$0.04 and then the short

[3]I consider securities of maturity only until 30 years due to the lack of availability of reliable yield to maturity data beyond maturity of 30 years. Besides, post WWII the average maturity of the US government securities have come down. (Hall and Sargent, 2011) find that during the 1940s long-term debt was a sizeable portion of total US government debt. The maturity dropped during the 50s and 60s so that by the 70s the US government had very few securities that were more than 15 years in maturity.

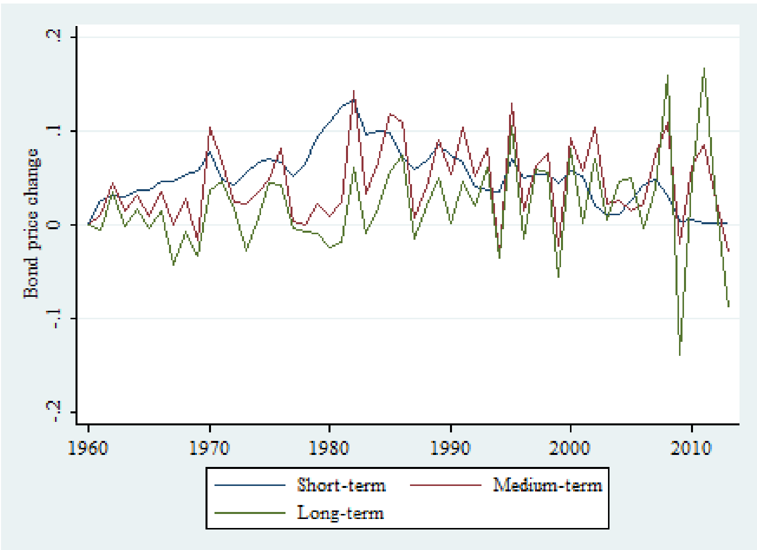
[4]To calculate these series I follow (Hall and Sargent, 1997) and look at the amount outstanding in December of every year and their corresponding bond prices, so that these capital gains/losses are at annual frequency.

Figure 2: Capital Gain/Loss as Percent of GDP Across Maturities



term at \$0.03 as shown in Table 3. Since the bond price and interest rate are related the fluctuations in the bond price implies fluctuations in the interest rate. As the bond prices of the longest maturity bonds record the highest fluctuations it implies higher fluctuations for the interest rate of long term bonds as well. This result is in sync with the findings of (Hall and Sargent, 2011) who find that the interest rate fluctuations have been highest on the longest term bonds in the US between 1940-2010.

Figure 3: Bond Price Change Across Maturities



In the light of the above findings then it is evident that there has been fluctuations in the marketable value of US debt from the sixties until 2013 and that it is pronounced in the medium term debt that comprises a significant portion of the US government debt. Besides, there is evidence of the bond price fluctuations with the highest bond price volatility recorded in the longest term securities. Therefore, to understand the underlying factors affecting such fluctuations in the market value of debt and bond prices I undertake a vector autoregression analysis to see how innovation in inflation and bond

Table 3: Bond Price Across Maturities in the Sample

Bond Price	Obs	Mean	Std. Dev.	Min	Max
Short-term	53	0.05	0.03	0.00	0.13
Medium-term	53	0.05	0.04	-0.03	0.14
Long-term	53	0.02	0.05	-0.14	0.17

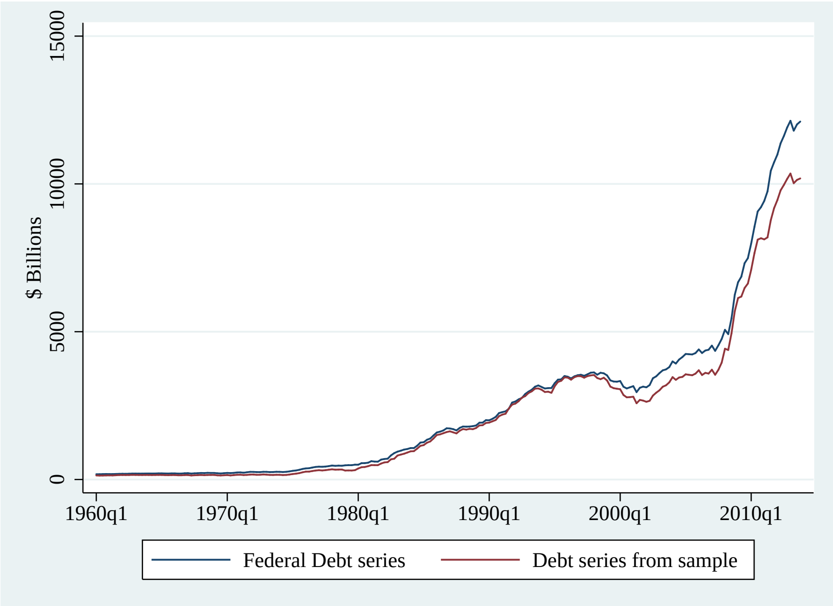
prices affect the market value of nominal debt. For the analysis I use quarterly data on marketable portion of the debt held by the public and calculate bond prices from yield data for the US between 1960:Q1 to 2013:Q4. In my analysis the revaluation components are surprise inflation and changes in bond prices. A portion of the lagged debt is subject to contemporaneous shocks as it is composed of bond prices and past debt. I subsequently use lagged debt as a proxy for bond pries and undertake the VAR analysis. I begin by considering a simple model of policy interaction to understand the responses of the variables namely inflation and debt to fiscal and monetary policy shocks and following (Leeper, 1991) I focus on the active fiscal and passive monetary region. In order to identify the fiscal shock in the active fiscal passive monetary regime I use sign restrictions on the impulse responses of the relevant variables. As a method sign restriction requires an priori condition to be imposed on the sign of the relevant impulse responses. In my analysis, I use a prior predictive analysis on the model parameters to arrive at the required a priori condition for the sign restriction.

This paper is close in spirit to several earlier studies that have analyzed the various channels through which fiscal financing has been achieved. For instance (Hall and Sargent, 2011) consider how the US debt to GDP level between 1940-2009 has been affected by nominal interest rate, growth rate of GDP, inflation and primary deficit. However, they do not analyze how surprise changes in inflation or interest rate stand to affect the debt to GDP level. The current analysis is also close to (Sims, 2001, 2013) however Sims only consider the effect of surprise change in inflation. It is also proximate to another growing body of work, for instance (Giannitsarou and Scott, 2006) and (Hasanov and Cherif, 2012) who consider in a typical vector autoregression setting how sovereign debt is affected by aggregate macroeconomic variables and economic policy in general. The current study deviates from this body of work by focussing on the effect of surprise inflation as an instrument that revalues debt. Also, the current work differs from the macro-finance literature where the focus is on delineating the different factors including aggregate macroeconomic ones in affecting the yield to maturity of securities. Some of the papers in that tradition are (Diebold et al., 2006),(Ang and Piazzesi, 2003), (Evans and Marshall, 2007) where they analyze various factors that affect the yield curve that affects the corresponding bond prices of the treasury securities. In the current work I abstain from such analyses and instead consider how innovations in the bond prices affect the overall debt level and thereby consider the impact of bond prices on the debt level.

2 Data

I create marketable nominal net public debt series for securities excluding the Treasury Inflation Protected Securities (TIPS). Net public debt is the debt held by the public excluding the debt held by the government agencies and comprises a major portion of the gross public debt. Since nominal debt comprises about 80% of the total marketable net public debt I focus on nominal US debt excluding TIPS. Figure 4 shows how the marketable treasury net public debt series from the Dallas Federal Reserve compares with the series constructed from the sample.

Figure 4: Marketable Treasury Debt Series Comparison:1960-2013



I begin by creating a quarterly data series for the securities by looking at securities issued by the US government every month since 1960 until 2013 from the CRSP monthly treasury series. The data for the bills are collected from two sources. The monthly bills data from 1960-1994 are taken from the FRASER database. For the monthly data beyond 1994 I obtain the data from Treasury Bulletin available online. Specifically, for 1960-1982 the monthly data is taken from "Treasury Survey of Ownership", Table 2: Summary of Interest-Bearing Public Marketable Securities and I pick up the columns "Total Amount Outstanding" (column 1) and "US govt. investment account and Federal Reserve Banks" (column 2) corresponding to the row "Treasury bills". The required bills data for 1960-1981 is then given by subtracting column 2 from column 1. The data for October 1982-September 1982 are taken from the section "Federal Debt" (Table FD-7: Maturity Distribution and Average Length of Marketable Interest Bearing Public Debt Held by Private Investors) issues of the Treasury Bulletin. The rest of the monthly data were taken from the March issues of each year that had the Treasury Bills data for all of the months of the previous year. From the monthly series for the securities I select those pertaining to the months of March, June, September and December to make the dataset quarterly.

Accordingly, I consider US marketable nominal security issued to the public between 1960 and 2013. Following (Hall and Sargent, 1997, 2011) each US government coupon bond is viewed as a bundle of zero coupon bonds. Each constituent zero coupon bond is then valued individually and then added together to arrive at the value of the whole bundle. In other words, the theory involves stripping the coupons from a bond, and pricing the bond as though it is a weighted sum of pure discount bonds of maturities $1, 2, ..., j$. More specifically, let s_{jt} denote the number of dollars at time $t + j$ that the government has promised to deliver as of time t and is computed from historical data by adding all principal and coupon payments that the government has promised to deliver at date $t+j$ as of date t. To calculate the bond prices, let, a_{jt} be the number of time t dollars that it takes to buy a dollar at time $t+j$. The prices a_{jt} are then calculated from the yield to maturity as:

$$a_{jt} = \frac{1}{(1+\rho_{jt})^j}$$

where, ρ_{jt} is the yield to maturity (ytm) on a j-period pure discount bond. This expression shows how to convert the ytm,ρ_{jt} on a j-period nominal pure discount bond, into the price of a promise (sold at time t, to one dollar at time $t+j$).

The monthly yield data to derive the bond prices are obtained from two sources. Until October 1985, the data are taken from (Hall and Sargent, 2011) [5] and from November 1985 until December 2013 it is taken from Federal Reserve [6] that are updated using (Gurkaynak et al., 2007). The Federal Reserve series (SVENY) has complete data for all maturities (1 year to 30 years) only starting from November 1985. I consider the highest maturity to be 30 years though the highest maturity is about 40 years for the bonds issued between 1963-2013. This is due to the fact that reliable yield to maturity data for earlier years in the sample are only available until maturity 30 years. Besides, restricting maturity to 30 years does not cause major difference in the marketable debt series I calculate and those available from the Dallas Fed.

The data for primary surplus is obtained from Bureau of Economic Analysis [7]. By definition, primary surplus is the difference between government receipts less expenses excluding the interest payments. Specifically, the quarterly primary surplus data in current dollar billions is obtained from line 45 Net Lending or Borrowing from Table 3.2 Federal Government Current Receipts and Expenditures. The primary surplus data so obtained matches the primary surplus/deficit series from FRED[8] and Office of Management and Budget [9].

The other macroeconomic data series-GDP, GDP deflator, inflation, short-term and long-term interest rates are obtained from FRED. The quarterly nominal GDP is in billions of dollars, and seasonally adjusted, real GDP is in billions of chained 2009 dollars and seasonally adjusted. The GDP deflator series is the chain-type price index, with the index 2009=100 and seasonally adjusted and the quarterly inflation series is created from the GDP deflator series[10]. The short-term interest rate used in the analysis is the three month treasury bill (secondary market rate and not seasonally adjusted). The long-term interest rate is the ten year treasury constant maturity rate and not seasonally adjusted. Figure 5 shows the time series plot of the variables used in the analysis.

3 Methodology

In this section I present the vector autoregression (VAR) model that I use for the analysis.

[5](Hall and Sargent, 2011) obtain the data from (Waggoner, 1997) till December 1999

[6]http://www.federalreserve/pubs/feds/2006

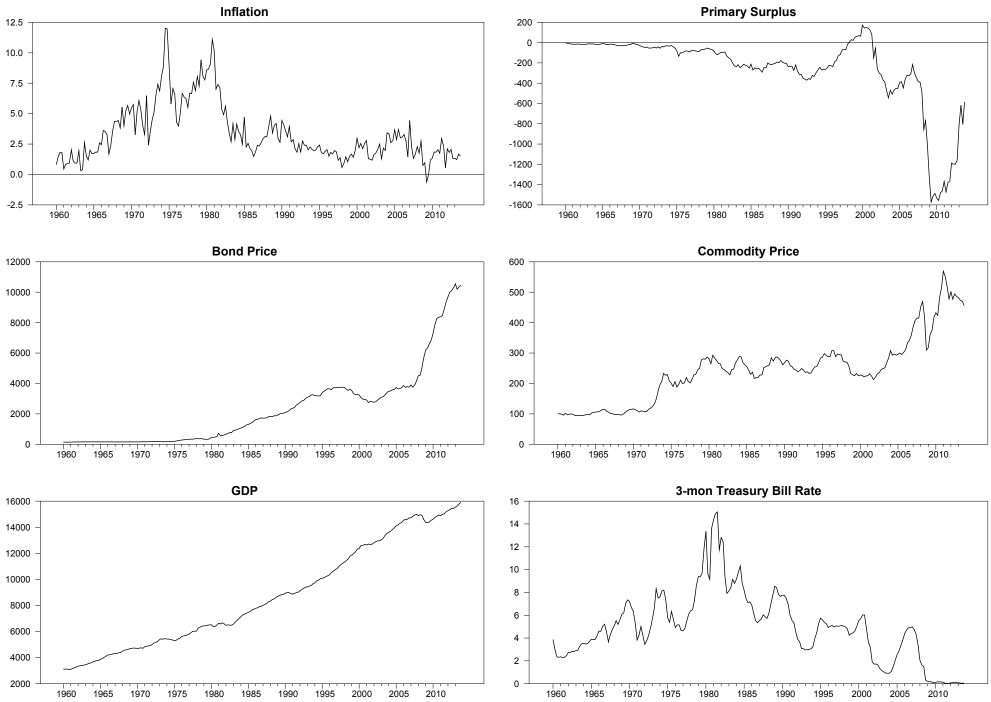
[7]http://www.bea.gov/

[8]Compared the primary surplus/deficit series with FYFSD in http://research.stlouisfed.org/fred2/series/FYFSD/

[9]My primary surplus/deficit series matches with those in Table 1.3 available at http://www.whitehouse.gov/omb/budget/Historicals/

[10]Inflation series is transformed following (Stock and Watson, 2001) as $400*(ln(\text{gdpctpi}_t/\text{gdpctpi}_{t-1}))$

Figure 5: Time Series plot of variables: 1960-2013



Following (Lutkepohl, 2007) A $p-$th order vector autoregression or VAR(p), with exogenous variables x can be written as:

$$y_t = \nu + A_1 y_{t-1} + ... + A_p y_{t-p} + B_0 x_t + B_1 x_{t-1} + ... + B_s x_{t-s} + u_t \text{ for } t \in \{-\infty, \infty\} \quad (4)$$

where, $y_t = (y_{1t}, ..., y_{Kt})$ is a $K \times 1$ random vector, A_1 through A_p are $K \times K$ matrices of parameters, x_t is an $M \times 1$ vector of exogenous variables, B_0 through B_s are $K \times M$ matrices of coefficients, ν is a $K \times 1$ vector of constant parameters and u_t is a $K \times 1$ vector of reduced-form disturbances with $E[u_t] = 0$ and $E[u_t u_t'] = \Sigma$ and $E[u_t u_s] = 0$ for $s \neq t$.

The joint distribution of y_t is determined by the distributions of x_t, ν, B_i, A_i and estimating the parameters requires that the variables in y_t and x_t be covariance stationary [11]. If u_t is a mean zero i.i.d process and x_t and y_t are covariance stationary and are uncorrelated with u_t then consistent and efficient estimates of B_i, A_i and ν can be obtained by the Seemingly Unrelated Regression (SUR) that yield estimates that are asymptotically normally distributed. When the equations for the variables in the y_t vector have the same set of regressors then the equation-by-equation OLS estimation is possible.

The structural form of the above reduced form equation can be written as :

$$W_0 y_t = a + W_1 y_{t-1} + ... + W_p y_{t-p} + \tilde{W}_1 x_t + ... \tilde{W}_s x_{t-s} + e_t \quad (5)$$

where a is a $K \times 1$ vector of constant parameters, each W_i, $i = 0, ..., p$ is a $K \times K$ matrix of parameters and e_t is $K \times 1$ vector of disturbance vector. Assuming that W_0 is non-singular equation 4 can be written as:

$$y_t = W_0^{-1} a + W_0^{-1} W_1 y_{t-1} + ... + W_0^{-1} W_p y_{t-p} +$$

[11] Covariance stationarity implies that the first two moments exist and are time invariant.

$$W_0^{-1}\tilde{W}_1 x_t + ... + W_0^{-1}\tilde{W}_s x_{t-s} + W_0^{-1}e_t \tag{6}$$

such that, $\nu = W_0^{-1}a$, $A_i = W_0^{-1}W_i$, $B_i = W_0^{-1}\tilde{W}_i$, $u_t = W_0^{-1}e_t$.

If the VAR in equation 4 is stable then y_t can be re-written in the moving average form as:

$$y_t = \mu + \sum_{i=0}^{\infty} d_i x_{t-i} + \sum_{i=0}^{\infty} \Phi_i u_{t-i} \tag{7}$$

where μ is the $K \times 1$ time-invariant mean of the process and d_i and Φ_i are $K \times M$ and $K \times K$ matrices of parameters, respectively. The equation 7 also known as the "Vector Moving Average" representation of equation 4 shows how the process y_t fluctuates around the mean μ and that it is completely determined by the parameters in d_i and Φ and the infinite past history of the exogenous variables x_t and the identically distributed and independent(i.i.d)shocks $u_{t-1}, u_{t-2}, ...$. The coefficients d_i are the dynamic multiplier or transfer functions and the coefficients Φ are also known as the moving-average coefficients or the impulse response functions (IRF) at horizon i.

In the absence of exogenous variables, the disturbance variance-covariance matrix Σ contains all relevant information about the contemporaneous correlation among the variables in y_t. The reduced form VARs do not account for this contemporaneous correlation but a recursive VAR does, where the K variables are assumed to form a recursive dynamic structural model such that each variable only depends upon those above it in the vector y_t. An IRF provides the effect over time of a one-time unit increase to one of the shocks, holding all else constant. Therefore, IRFs from a reduced form does not lend itself to any causal inference because it does not account for any contemporaneous correlation among variables. To the extent that the shocks are contemporaneously correlated, the other shocks cannot be held constant. One solution to this problem is to orthogonalize the shocks.

Orthogonalization involves taking the $E(u_t u_t') = \Sigma$, the covariance matrix of the shocks and finding a matrix P such that $PP' = \Sigma$ and $P^{-1}\Sigma P^{-1'} = I_K$ so that the vector of shocks are then orthogonalized by P^{-1}. Thus, for a VAR with no exogenous variables, the process y_t can be written as:

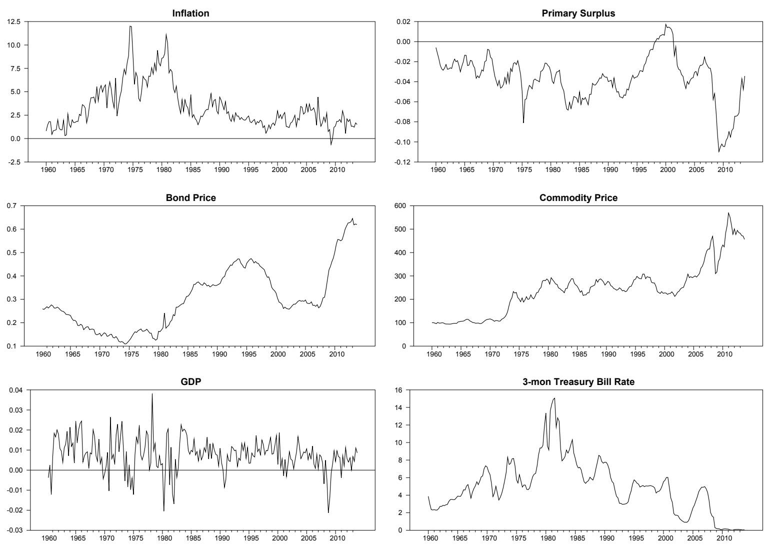
$$\begin{aligned} y_t &= \mu + \sum_{i=0}^{\infty} \Phi_i u_{t-i} \\ &= \mu + \sum_{i=0}^{\infty} \Phi_i P P^{-1} u_{t-i} \\ &= \mu + \sum_{i=0}^{\infty} \Theta_i w_{t-i} \end{aligned} \tag{8}$$

where, $\Theta_i = \Phi_i P$ and $w_{t-i} = P^{-1}u_{t-i}$. Obtaining such a P matrix renders the new transformed errors w_t orthogonal and the new transformed coefficients can then be used for causal inferences. Due to (Sims, 1980) P can be written as a Cholesky Decomposition of Σ^{-1}. As a VAR can be considered to be the reduced form of a dynamic structural equation (DSE) model, choosing P is equivalent to imposing a recursive structure on the corresponding DSE model. The ordering of the recursive structure is that imposed in the Cholesky decomposition, which in turn is the order in which the endogenous variables appear in the VAR estimation.

4 Results from VAR

In this section I present the results from a simple VAR analysis to analyze the interaction of the revaluation components with the policy and other macro variables. I begin by describing the variables used in the analysis whose time series plot is shown in figure 6 followed by the results from the impulse responses and the variance decomposition.

Figure 6: Time Series plot of VAR variables (transformed): 1960-2013



4.1 Bond Price

At any period t the debt that comes due is given by, say, $D_t \equiv (1 + r_{t-1})B_{t-1}$, where, r_{t-1} is the nominal interest on the debt and B_{t-1} is the market value of past debt that comes due in the current period t. Using the data described in the last section the total marketable past debt due at the current period is given by:

$$D_t \equiv (1 + r_{t-1})B_{t-1} = \sum_{i=1}^{n} a_{j-1,t} s_{j,t-1}$$

where, $a_{j-1,t}$ is the bond price and $s_{j,t-1}$ is the total debt outstanding as of time $t-1$ corresponding to a security of maturity j. Given the information set at the beginning of time t, say, Ω_{t-1}, $s_{j,t-1}$ is known, however, $a_{j-1,t}$ is unknown at the beginning of time period t and is subject to change due to shocks in time period t. Thus, any change in D_t at time t would be brought about due to changes in $a_{j-1,t}$ or in other words due to innovations in the bond prices. If Q_t represents the bond price component that affects debt D_t then:

$$Q_t \equiv \tilde{a}_{j-1,t} = a_{j-1,t} - E_{t-1}a_{j-1,t} \approx \tilde{D}_t = D_t - E_{t-1}D_t$$

Accordingly, I consider a VAR with the following variables (also entered in the following order): Inflation (π_t), Bond price (Q_t), Growth rate, Primary Surplus, Commodity Price, 3 months Treasury Bill rate. The variables Q_t and Primary Surplus are entered as share of nominal GDP [12]. Rest of the variables are entered without any transformation. The data being quarterly I use four lags for the VAR analysis.

4.2 Impulse Responses and FEVD

The impulse response function of the six variables VAR is shown in Figure 7 and the FEVD results are in provided in Table 5. Due to a positive shock to primary surplus inflation is observed to fall and so does the bond price. From the debt valuation equation an increase in the primary surplus could lead to a fall in prices because the government securities are now backed by the primary surplus. This induces the economic agents to shift from goods and services towards bonds leading to a fall in the price level. Now bond prices fall as a result of an increase in the primary surplus. Since bond prices and interest rates are inversely related an increase in the primary surplus leads to an increase in the interest rate. This again holds good considering the fact that agents now demand more of the government securities and that leads to an increase in the interest rate. A positive shock to the three month treasury bill leads to a fall in inflation and a rise in the bond price. One plausible reason could be that lower inflation serves as an indicator that securities will not fall in value and therefore the bond prices rise. [13]

From the FEVD Table 5 it is observed that primary surplus plays a relatively more important role for the bond prices accounting for 15-30% between four and sixteen quarters. Also, for inflation primary surplus seems to gradually play a role going from about 1% to 6.5% over a period of one to four years. Compared to the primary surplus the federal funds rate do not seem to affect much of the variance of inflation or bond prices.

Therefore, gauging from the impulse responses and the variance decompositions it appears that both fiscal and monetary policy affect the revaluation components. However, comparatively the impact of fiscal policy is more prominent than that of monetary policy.

Identification of the relevant shocks using the Cholesky Decomposition has its limitations. For one, different ordering of the variables can lead to different results. Second, there can be numerous ordering and evaluating the model each time with a new ordering can be tedious and also inefficient if there are many variables in the model. I therefore attempt to identify the relevant shocks by resorting to a theoretical model described in the next section.

5 Simple Endowment Economy

In this section I consider a simple economy consisting of representative households, a fiscal authority that imposes lump-sum tax and issues bonds and a monetary authority that sets the nominal interest rate and responds to inflation. The choice of the model is guided by the fact that it lends itself to the analysis of policy interactions.

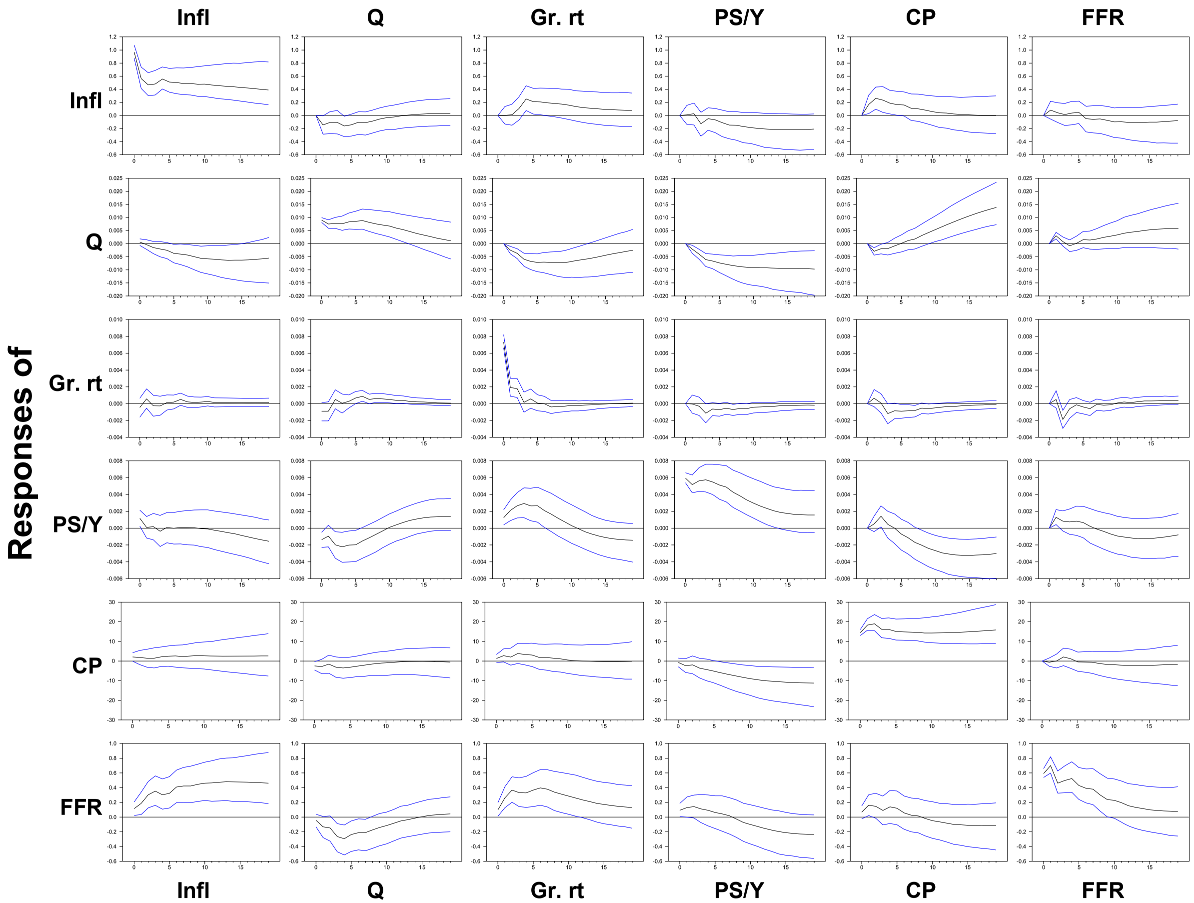
5.1 Household

A representative household receives a constant endowment of output Y every period. For simplicity I also assume that the government spending is zero every period i.e., $G_t = 0$.

[12] The variable Q_t is entered as a share of one period lagged nominal GDP so that innovations in the series reflect solely that coming from bond prices.

[13] I include the commodity price in the VAR to account for the “price puzzle” (Balke and Emery, 1994), (Hanson, 2004).

Figure 7: Impulse Response from the Single VAR



Notes: Shaded areas show 95% confidence interval. 95% Monte Carlo bands.

Consequently, the representative household optimizes inter-temporally by maximizing the following:

$$\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t U(C_t)$$

Subject to,

$$C_t + Q_t \frac{B_t}{P_t} + T_t = Y + (1 + \rho Q_t) \frac{B_{t-1}}{P_t}$$

Every period the household consume, C_t, hold real government bonds $\frac{B_t}{P_t}$ of various maturity represented by the parameter ρ with price, Q_t and pay lump-sum taxes, T_t, out of the endowment Y and the return from the holding of government bonds. Following (Woodford, 2001) it is assumed that coupon payments from bond are perpetual and decay exponentially. A bond issued in period t pays ρ^j dollars $j+1$ periods later for each $j \geq 0$ with decay factor $0 \leq \rho < \beta^{-1}$. In such a setup $\rho = 0$ would represent only one-period bonds and $\rho = 1$ would represent a consol and $0 \leq \rho < \beta^{-1}$ would mean a bond of medium-term maturity. The optimization leads to the following equations:

$$E_t \Big(\frac{\beta}{\pi_{t+1}} \Big) \Big(\frac{1 + \rho Q_{t+1}}{Q_t} \Big) = 1 \tag{9}$$

The Fisher equation linking the nominal interest rate i_t on one-period debt is given by:

$$E_t \frac{\beta}{\pi_{t+1}} = \frac{1}{(1 + i_t)} \tag{10}$$

From equations (9) and (10) the following condition emerges that links the interest rate on one-period debt to the bond prices.

$$E_t \frac{(1 + \rho Q_{t+1})}{(1 + i_t)} = Q_t \tag{11}$$

Letting $R_t = (1 + i_t)$, and iterating on the above equation leads to the following relationship between the bond price and the long term interest rates, showing the relationship between short term and long term interest rates.[14]:

$$Q_t = E_t \sum_{j=0}^{\infty} \rho^j \prod_{i=0}^{j} \frac{1}{R_{t+i}}$$

5.2 Government

The government collects taxes and issues bonds as per the following budget constraint:

$$T_t + Q_t \frac{B_t}{P_t} = (1 + \rho Q_t) \frac{B_{t-1}}{P_t} \tag{12}$$

The tax rule evolves as per the following:

$$T_t = e^{\gamma_0} (\frac{B_{t-1}}{P_{t-1}})^{\gamma} \psi_t = e^{\gamma_0} b_t^{\gamma} \psi_t \tag{13}$$

where, ψ_t is an AR(1) process that evolves as follows:

$$\psi_t = \rho_\psi \psi_{t-1} + \epsilon_t^{\psi} \text{ and } 0 < \rho_\psi < 1 \tag{14}$$

[14] If there is only one-period debt then $\rho = 0$ so that from the FOC for bonds and the Fisher equation, $Q_t = \frac{1}{R_t}$

5.3 Monetary Authority

The monetary authority sets the nominal interest rate and responds to inflation as per the following equation:

$$R_t = e^{\alpha_0}\pi_t^{\alpha}\theta_t \tag{15}$$

where, θ_t is an AR(1) process that evolves as follows:

$$\theta_t = \rho_\theta\theta_{t-1} + \epsilon_t^\theta \text{ and } 0 < \rho_\theta < 1 \tag{16}$$

6 Equilibrium

Log-linearizing the relevant equations around the steady state leads to the following system of equations:

$$\hat{\pi}_{t+1} = \alpha\hat{\pi}_t + (1-\beta)\hat{\theta}_t + \eta_{t+1}^{\pi} \tag{17}$$

$$\beta^{-1}\hat{\pi}_{t+1} + \hat{b}_{t+1} + (1-\rho)\hat{Q}_{t+1} + \beta^{-1}(1-\beta)\hat{\psi}_{t+1} = \beta^{-1}(1-\gamma(1-\beta))\hat{b}_t \tag{18}$$

$$\rho\beta\hat{Q}_{t+1} = \hat{Q}_t + \alpha\hat{\pi}_t + \hat{Q}_t + \rho\beta\eta_{t+1}^{Q} \tag{19}$$

$$\hat{\psi}_{t+1} = \rho_\psi\hat{\psi}_t + \epsilon_{t+1}^{\psi} \tag{20}$$

$$\hat{\theta}_{t+1} = \rho_\theta\hat{\theta}_t + \epsilon_{t+1}^{\theta} \tag{21}$$

where, it is understood that, $\hat{x}_t = ln(x_t) - ln(x)$ and the variable without a time subscript implies the steady state value and, $\eta_{t+1}^{x} = x_{t+1} - E_t x_{t+1}$ is the one step ahead forecast error.

In matrix notation the system can be represented as per the following:

$$X_{t+1} = AX_t + B\Pi_{t+1} + C\Psi_{t+1} \tag{22}$$

where, $X_t = \{\hat{\pi}_t, \hat{b}_t, \hat{Q}_t, \epsilon_t^{\psi}, \epsilon_t^{\theta}\}$, Π_t and Ψ_t are matrices containing the shocks and forecast errors, respectively. In order for the system in equation (22) to deliver a unique solution it is required that two eigenvalues of matrix A be greater than unity. The eigenvalues are given by: $\rho_\psi, \rho_\theta, \beta^{-1}(1-\gamma(1-\beta)), \alpha, \frac{1}{\rho\beta}$. Now ρ_ψ, ρ_θ are both less than one and $\frac{1}{\rho\beta}$ is greater than one (since $0 < \beta < 1$, $0 < \rho < 1$). Therefore, a unique equilibrium is delivered when either of the following occur:

$$\text{Case I: } \beta^{-1}(1-\gamma(1-\beta)) > 1 \text{ and } \alpha < 1 \tag{23}$$

$$\text{Case II: } \beta^{-1}(1-\gamma(1-\beta)) < 1 \text{ and } \alpha > 1 \tag{24}$$

Following (Leeper, 1991) I define an "Active" authority to be one who satisfies the budget constraint independently, whereas, the "Passive" authority takes into consideration the optimization exercise of the household and the other authority and adjusts its budget accordingly. Therefore, Case I (or Case II) above represents a scenario when Fiscal Policy(or Monetary Policy) is "Active" and Monetary Policy (or Fiscal Policy) is "Passive" and represents the AFPM (or PFAM) region.

In the current analysis I focus on the AFPM region only and analyze the behavior of three variables $\hat{\pi}_t, \hat{b}_t, \hat{Q}_t$ following a shock to fiscal and monetary policy. Solving for the parameter restrictions from Case I for AFPM leads to $\gamma < 1$ and $\alpha < 1$. The impulse responses to a positive shock to fiscal policy and monetary policy in the AFPM region is shown in Figures 8 and 9, respectively, under the parameter values: $\beta = 0.99, \rho =$

Figure 8: Responses for Fiscal Policy Shock in AFPM ($\gamma = 0.9, \alpha = 0.9$)

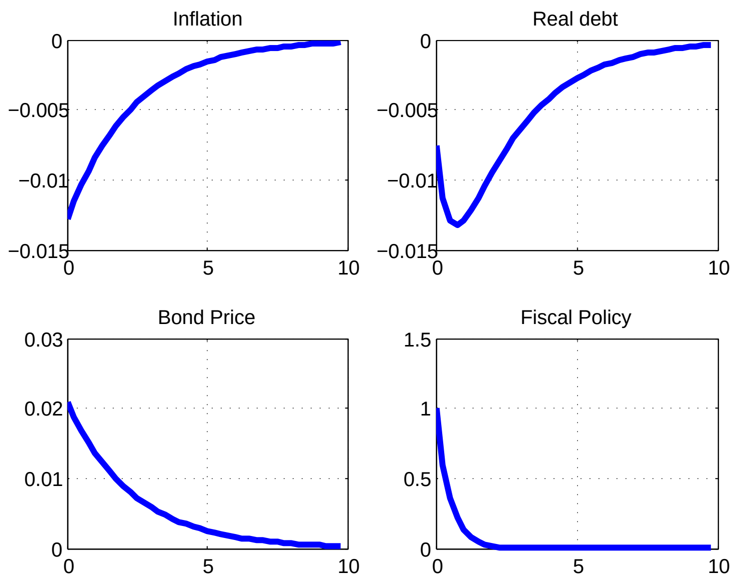
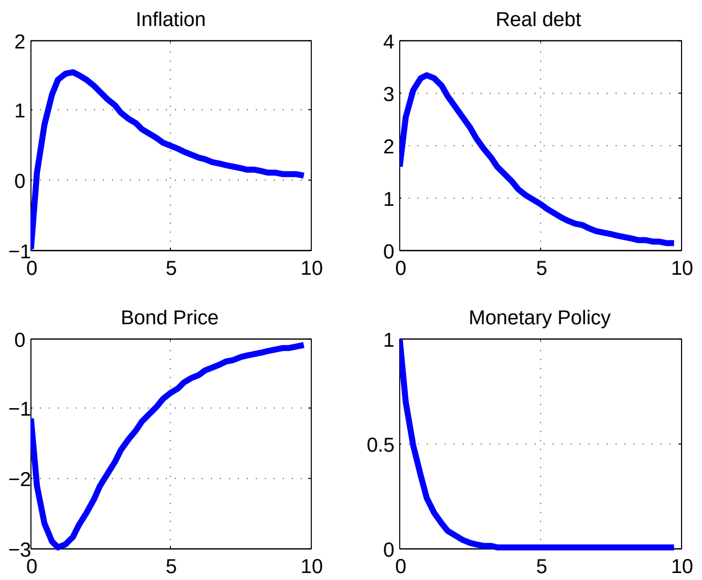


Figure 9: Responses for Monetary Policy Shock in AFPM ($\gamma = 0.9, \alpha = 0.9$)



$0.5, \rho_\psi = 0.6, \rho_\theta = 0.7, \alpha = 0.9, \gamma = 0.9$. The value of $\beta = 0.99$ delivers a steady state value of the nominal interest rate to be 1% and the value chosen for $\rho = 0.5$ implies that the government issues only medium-term bonds.

Due to a positive shock to tax given that it is an endowment economy, aggregate demand falls and hence the price level. Due to a fall in the price level real bonds gain in value and agents move from goods and services to buying bonds. As a result of an increase in the demand for bonds the bond price rise as shown in figure 8. Following a monetary policy shock nominal interest rate rises whereby bond price falls (since nominal interest rate and bond price are inversely related) and real bond rises. Due to rising interest rate agents feel wealthier which in turn leads to rise in price level given that output does not change.

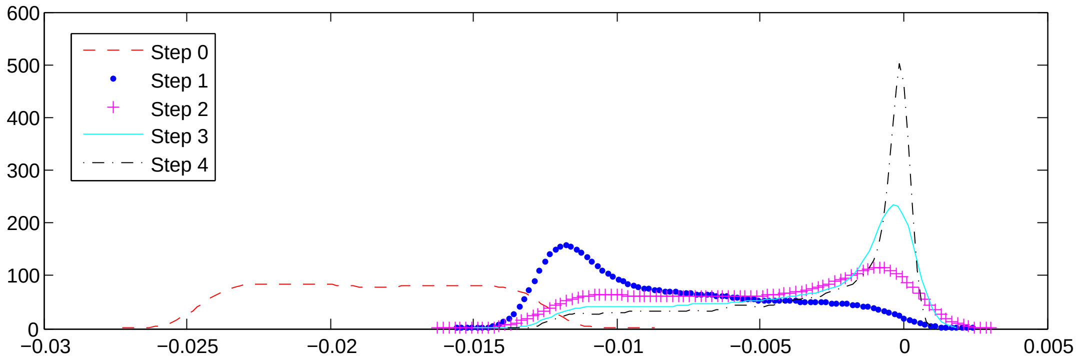
7 Prior Predictive Analysis

The impulse response functions in the AFPM region depend on the parameter values and the parameters can take on a range of values. Subsequently there can be a wide range of models that can be used to generate the impulse responses. Since the goal here is to understand the effect of a fiscal shock from the data a prior predictive analysis can be helpful. For a complete model A, prior predictive analysis involves the prior density $P(\theta_{A,T}|A)$, conditional density $P(y_T|\theta_{A,T}, A)$, and the vector of interest density $P(\omega|\theta_{A,T}, A)$. The posterior distribution of the parameter θ given by $P(\theta_{A,T}|y_T) = \frac{P(\theta_{A,T})P(y_T|\theta_{A,T})}{P(y_T)}$ which is then used to find $p(\omega|y_T, A_T) = \int p(\omega_T|y_T, \theta_{A,T}, A_T)p(\theta_{A,T}|y_T, A_T)d\theta_{A,T}$. The vector of interest in this case is to find the range of values that the impulse responses can take.

For the prior predictive analysis I fix the parameters, $\beta = 0.99, \rho = 0.5, \rho_\psi = 0.6, \rho_\theta = 0.7$. Since the region of interest in the current analysis is AFPM following (Leeper et al., 2011) I consider the prior for the monetary policy parameter to be uniformly distributed ($\alpha \sim U[0, 1]$). The fiscal policy parameter is taken to be normally distributed ($\gamma \sim N(0, 0.03)$). Consequently, I take 5000 draws from the parameter vector, solve the model for each draw of the parameter and collect impulse responses of all variables to a fiscal shock.

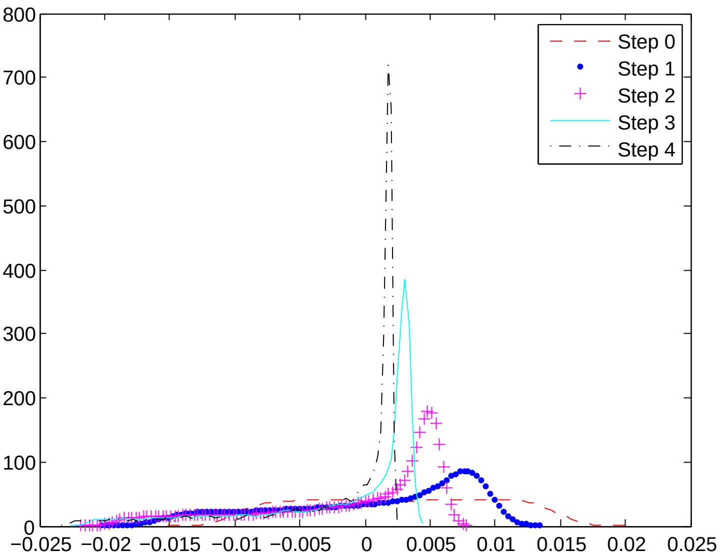
The kernel density of the distribution of impulse responses of inflation to a fiscal shock is shown in figure 10. The figure depicts the kernel densities from period zero until five. During the zero to five time period the bulk of the distribution is observed to be shifting from negative to zero suggesting a high probability of the sign of the impulse response to be negative for inflation. This fact is confirmed by the probability figures for the impulse responses of inflation in Table 4 showing a probability of one for the negative sign of impulses for inflation. Figure 11 shows the kernel density across periods zero to five for the impulse responses of real debt to a fiscal shock. For real debt density impulses with negative signs do bear some probability however the bulk of the probability is centered around positive values for impulses. Accordingly, the probability of a positive sign of response for real debt is 0.6 as is seen from table 4. By similar reasoning, the impulse response of bond price can be taken to be positive reading from figure 12 and the probability table.

Figure 10: Kernel Density of Responses of Inflation to Fiscal Shock



Note:Vertical axis represents the number of draws. Total draws equal 5000.

Figure 11: Kernel Density of Responses of Real Debt to Fiscal Shock



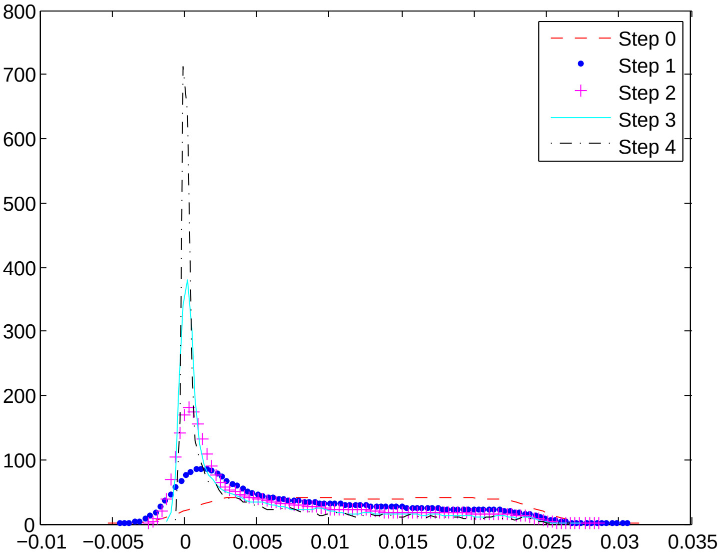
Note:Vertical axis represents the number of draws. Total draws equal 5000.

Table 4: Probability of sign of Impulse Responses

	Probability of Sign of Impulse Responses		
Horizon	**Prob(Inflation < 0)**	**Prob(Real debt > 0)**	**Prob(Bond price > 0)**
Impact	1	0.6	1
1-qtr	1	0.6	1
2-qtr	1	0.6	1
3-qtr	1	0.6	1
4-qtr	1	0.6	1

Note:Probability is fraction calculated by taking the number of favorable draws out of total draws(5000).

Figure 12: Kernel Density of Responses of Bond Price to Fiscal Shock



Note:Vertical axis represents the number of draws. Total draws equal 5000.

8 Identification: Sign Restriction

In this section I present the results from the identification using sign restriction. Popularized by (Uhlig, 2005) the premise of the sign restriction is one where the method entails a search in the space of impulse responses that match an a priori condition. The impulse responses that match the criterion are then attributed to the shock that led to the creation of such responses. In the current analysis I seek to identify the fiscal shock using quarterly data on inflation (π), real debt(b), bond prices (Q), primary surplus(PS) and federal funds rate (FFR) for the US between 1960-2013.

The a priori condition for identifying the fiscal shock is obtained from the prior predictive analysis. Specifically, the task at hand requires the identification of the fiscal shock in the AFPM region. From the prior predictive analysis it is observed that in the AFPM region a positive fiscal shock leads to a fall in inflation, rise in real bond and a rise in the bond price. Therefore, if the impulse responses match the sign (on the impulse responses) spelled out from the prior predictive analysis then the shock can be identified as a fiscal shock (or primary surplus shock).

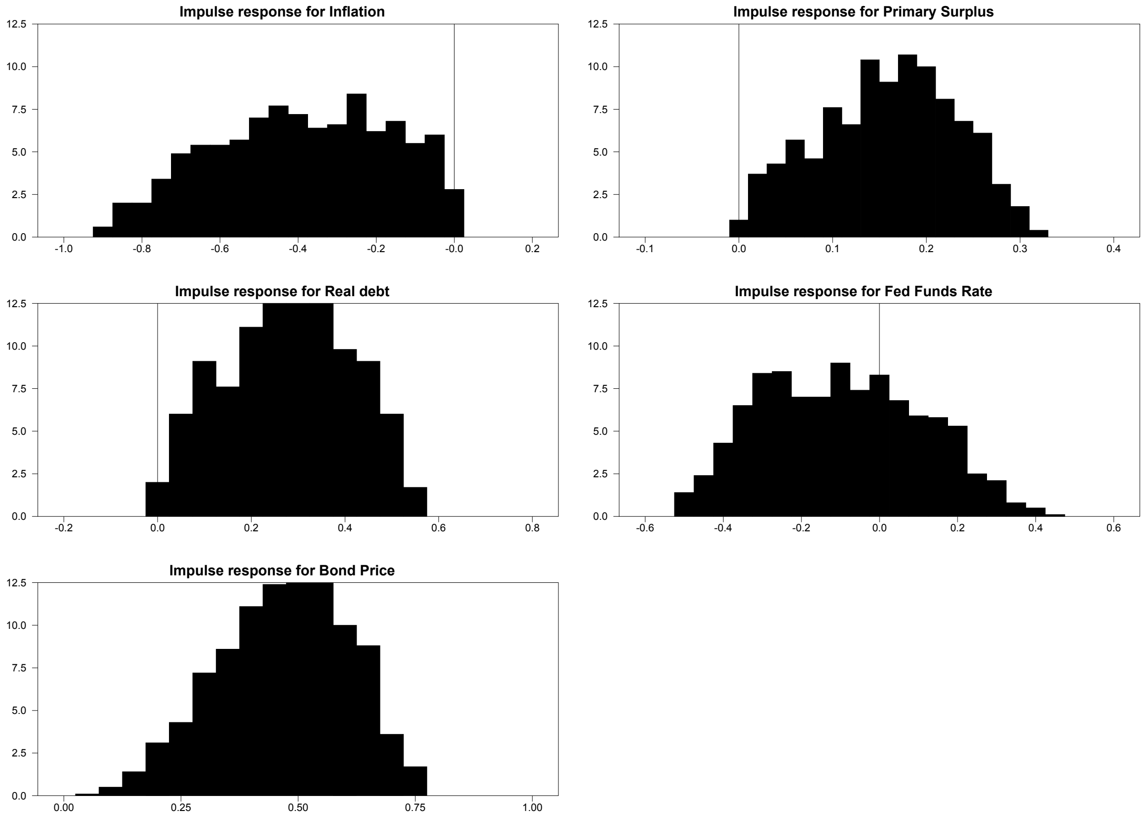
Hence, *a fiscal policy impulse vector would be one so that the impulse responses to that vector of real debt and bond price are not negative and inflation is not positive and the impulse responses for primary surplus is not negative, at horizons* $s = 0, 1, ..., K$. Accordingly, I begin by estimating the reduced form model, followed by finding out the Cholesky Decomposition of the reduced form variance-covariance matrix[15]. Next, I take draws from the random orthonormal matrix and calculate the corresponding impulse responses retaining only the ones satisfying the above mentioned sign restrictions[16].

Figure 13 shows the distribution of the accepted responses for all the variables on impact. The bulk of the distribution of the inflation responses are concentrated around negative values, with that of the real debt and bond prices around positive values thereby satisfying the criterion at least on impact. The prior predictive analysis shows that response of inflation to a fiscal shock is negative and that of real debt and bond price are both positive until about time period five. Therefore, restricting attention to about time period five in figure 14 reveals that the range of the impulse responses of the above three variables over time period zero to five satisfy the sign criterion. Finally, figure 15 shows the impulse responses obtained using sign-restriction with the average draw for all variables along with error bands at 16th and 84th percentile. It is observed that due to a positive shock to primary surplus, inflation falls but eventually rises, real debt rises and then gradually declines with bond price rising on impact and eventually declining.

[15]See Appendix for description of the sign restriction method of identification

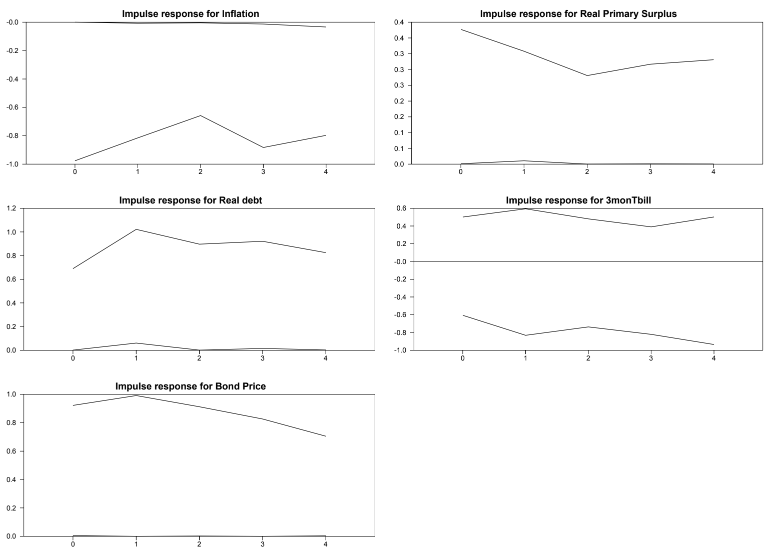
[16]I consider 50000 draws from the orthonormal matrix and retain 1000 draws for calculating the distribution of the impulse responses

Figure 13: Impact Impulse Responses from Sign Restriction due to Fiscal Shock



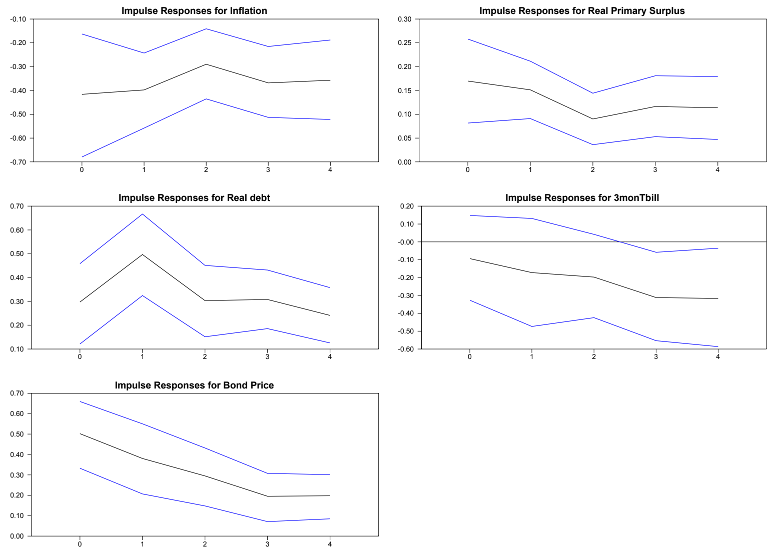
Note: Distribution of Impact Impulse Response

Figure 14: Range of Impulse Responses from Sign Restriction due to Fiscal Shock with K=5



Note: Range of Impulse Responses with K=5

Figure 15: Impulse Responses from Sign Restriction due to Fiscal Shock



Note: Impulse Responses with Pure-Sign Approach

9 Conclusion

At a time when most of the advanced nations are facing a debt to GDP ratio close to those after WWII the issue gains relevance as to what instruments could help tackle the debt burden. Different countries have used different instruments at different times to tackle the debt overhang so that there exist no fixed prescription to a way out of such a situation. Typically the instruments that adjust are: inflation, nominal interest rate, economic growth rate and primary surplus/deficit. Post WWII economic growth in the US has been a major factor in affecting the level of debt. However, given state of demographics in the advanced nations economic growth and primary surplus do not seem to be a viable option and hence the remaining two factors emerge as the important ones.

Debt held by agents are affected by surprise jumps in inflation. Both the coupon payment and principal in real terms are affected if there are surprise jumps in inflation. Also, when interest rate on fixed income securities like government debt changes then market value of the debt gets affected. Thus, surprise inflation affects real debt and nominal interest rate (or bond price) affects market-value of debt. Though not substantial, both surprise inflation and bond price changes have affected the market value of government debt in the US between 1960-2013.

Fiscal and monetary policy both can affect inflation and bond prices. Specifically, the attempt in this paper has been to analyze if government debt can act as a cushion for fiscal policy shocks. From the VAR analysis it appears that comparatively fiscal policy plays a more important role by affecting both inflation and bond prices for the US between 1960-2013. Also, from the sign restriction analysis appears to support the view that fiscal policy is the relevant shock and therefore government debt can act as a "fiscal cushion" to such shocks. However, it would be incorrect to arrive at a final conclusion regarding the identification given that the current analysis has been carried out in a simple model. As a future course of action one can carry out a similar analysis in a more sophisticated model where household behavior and firms' optimization are also taken into consideration in addition to the policy interactions.

Table 5: FEVD:Single VAR model

Horizon	Inflation	Q	Gr. Rt.	rim. Surplı	CP	FFR
(Quarter)						
Variance Decomposition of Inflation						
1	100.00	0.00	0.00	0.00	0.00	0.00
4	87.80	2.23	0.64	0.92	7.97	0.44
8	80.79	3.30	6.06	1.73	7.58	0.54
12	79.28	2.69	6.89	3.99	6.07	1.08
16	78.16	2.20	6.42	6.52	4.96	1.74
Variance Decomposition of Q						
1	0.51	99.50	0.00	0.00	0.00	0.00
4	1.69	63.20	13.91	14.51	4.05	2.64
8	5.08	45.50	21.33	24.69	1.85	1.55
12	8.54	33.58	20.43	28.84	5.78	2.85
16	10.33	24.24	16.52	29.51	14.46	4.93
Variance Decompositin of Gr. Rt						
1	0.41	1.50	98.09	0.00	0.00	0.00
4	0.98	2.68	85.75	1.97	2.65	5.98
8	1.38	4.47	77.96	3.93	6.17	6.09
12	1.50	5.69	74.75	4.99	7.15	5.92
16	1.56	5.98	73.57	5.31	7.39	6.19
Variance Decompositin of Prim. Surplus						
1	3.35	4.67	3.75	88.23	0.00	0.00
4	0.91	6.86	12.90	76.11	1.45	1.78
8	0.50	7.54	14.12	74.07	2.40	1.37
12	0.42	6.29	11.92	70.65	8.92	1.80
16	0.73	6.25	10.68	62.79	16.71	2.85
Variance Decomposition of CP						
1	1.94	2.61	0.81	0.21	94.43	0.00
4	0.91	1.94	2.22	1.77	92.80	0.36
8	1.42	2.28	2.26	6.28	87.47	0.28
12	1.72	1.63	1.60	12.49	82.03	0.53
16	1.78	1.18	1.17	17.86	77.23	0.77
Variance Decomposition of FFR						
1	3.11	0.49	2.54	2.26	1.08	90.52
4	11.98	5.27	15.27	2.80	2.84	61.85
8	18.49	8.18	20.43	1.75	2.29	48.86
12	27.51	7.29	21.39	1.85	1.82	40.14
16	35.33	5.98	19.70	3.48	2.05	33.45

10 Appendix

Recall the reduced form vector autoregression be given by:

$$Y_t = \mu + A(L)Y_{t-1} + u_t \text{ , with } E(u_t u_t') = \Sigma \tag{25}$$

The corresponding structural model is obtained once one is able to pin down the matrix W_0 such that:

$$u_t = W_0^{-1} e_t$$

It is known that:

$$\Sigma_u = E(u_t u_t') = E[W_0^{-1} e_t (W_0^{-1} e_t)'] = W_0^{-1} E(e_t e_t') W_0'^{-1} = W_0^{-1} W_0'^{-1}$$

since $E(e_t e_t') = I_K$

Let the Cholesky Decomposition of Σ_u be given by:

$$\Sigma_u = P'P$$

where, P' is the Cholesky factor and is a lower triangular matrix. Since $\Sigma_u = W_0^{-1} W_0'^{-1}$ and $\Sigma_u = PP'$, it implies, $W_0^{-1} = P'$. Hence for a random orthonormal matrix S, such that $S'S = I$, one can have:

$$\Sigma_u = W_0^{-1} W_0'^{-1} = P'S'SP = \tilde{P}'\tilde{P} \text{ where } \tilde{P} = SP$$

Therefore, $W_0^{-1} = \tilde{P}'$ is a valid solution to the identification problem. Since S' is a random matrix identification is achieved when the impulse responses implied by a particular choice of the S' matrix satisfies the a priori sign restriction.

10.1 Capital Gain/Loss

$s_{j,t}$: Number of dollar at time $t+j$ that the government has promised to deliver as of time t. It is computed by adding all the principal and coupon payment that the government has promised to pay.
$a_{j,t}$: Number of time t goods that it takes to buy a dollar at time $t+j$.
$\rho_{j,t}$: Yield to maturity of a security of maturity j at time t.
Now, $a_{j,t} = \frac{1}{(1+\rho_{j,t})^j}$

Suppose two securities, a period security and another two-period security, $s_{1,t-1}$ and $s_{2,t-1}$ that comes due at the beginning of period t. By definition, $s_{1,t-1}$ is in current dollars but $s_{2,t-1}$ is not in current dollars. Hence, the security of maturity one year i.e, $s_{1,t-1}$ requires no discounting. But for a security that was issued two periods (or years) back i.e., $s_{2,t-1}$ at time t will require to be discounted appropriately in order that it can be added to the security that comes due at the beginning of time period t. Therefore, $s_{2,t-1}$ is to be discounted by $(1 + \rho_{1,t})$. Hence if there are only two securities, $s_{1,t-1}$ and $s_{2,t-1}$ then the total amount that comes due at the beginning of period t is given by $s_{1,t-1} + \frac{1}{(1+\rho_{1,t})} s_{2,t-1} = s_{1,t-1} + a_{1,t} s_{2,t-1}$. Thus, if there are securities of maturity $j = 1, 2, ...n$, then at the beginning of time period t the total amount that comes due is given by :
$\sum_{j=1}^{n} a_{j-1,t} s_{j,t-1} = a_{0,t} s_{1,t-1} + a_{1,t} s_{1,t-1} + ... + a_{n-1,t} s_{n,t-1}$
$= s_{1,t-1} + a_{1,t} s_{1,t-1} + ... + a_{n-1,t} s_{n,t-1}$ (since $a_{0,t} = 1$).

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