

Inefficiencies due to Skill Choice

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September 13, 2017

Abstract

Some college degrees, such as engineering, provide multiple job options, which incentivize individuals to invest in them while making skill choices. However, this may lead to over investment in this skill type, which can eventually lead to higher unemployment among engineers, owing to the search frictions that are present in the economy. In this paper, I use a two-sector two-skill search theoretic model to analyze this problem and I find that this may lead to inefficient outcomes. A government intervention which penalizes individuals who invest in these skills and rewards workers who invest in skills which has less job options can help to reduce this inefficiency.¹

JEL classification: D61, E24, J24, J64

1 Introduction

Engineering education in India comprises of a large percentage of the total enrollment in colleges. According to the Planning Commission of India, as of 2012, approximately 5.46 million students enrolled in the engineering degree, which contributes to approximately 25% of the total enrollment in colleges (Figure 1). Engineering is widely regarded as a popular choice among students owing to various factors: societal mindset, multiple carrier options, financial stability. Engineering is often perceived as a ‘base’ degree with a flexible career path. Students with an engineering degree can work in engineering firms, which

¹I would like to thank Julia Thomas, Aubhik Khan and Sanjay Chugh for their helpful comments

are well suited for the education that they have obtained, or alternatively can work in other sectors such as software, business administration, finance, consulting among others. Figure 2 (Source: Banerjee and Muley, 2008) shows that over time real GDP per capita has increased and the number of engineers per million population has increased as well. However Banerjee and Muley point out, the increase in engineers is higher than the rise of real GDP per capita; greater than the actual demand for engineers. According to the All India Council for Technical Education the present scenario is such that every year, out of the total engineers graduating from technical institutions, around 60% stay unemployed.

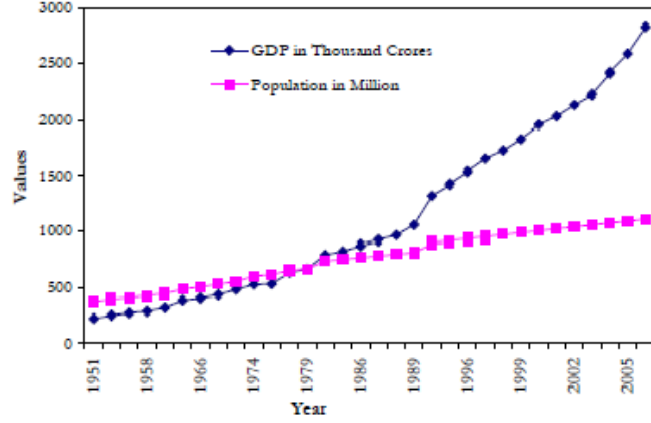
Table 1: Growth of Enrollment by Field of Study (in millions)				
FACULTY	2006-2007		2011-2012	
	TOTAL	PERCENT	TOTAL	PERCENT
Engineering/Technology	1.80	13.00	5.46	25.00
Education	0.62	4.50	1.30	6.00
Medicine, Nursing and Pharmacy	0.59	4.30	1.20	5.50
Commerce & Management	2.28	16.50	3.43	15.80
Agriculture and Veterinary Science	0.09	0.70	0.12	0.60
Science	2.54	18.40	3.05	14.00
Arts	5.48	39.60	6.57	30.2
Law	0.30	2.20	0.34	1.60
Others	0.11	0.80	0.27	1.30
Total	13.85	100.00	21.78	100.00

Source: Planning Commission Government of India. 2013. *Twelfth Five Year Plan (2012-2017), Social Sector*, Vol. III. New Delhi
Note: Does not include distance education

World Education Services, 2015
Mobility Patterns and Pathways of Indian Engineers to the U.S., wes.org/RAS 

Figure 1: Growth of enrollment by field of study

People opt for engineering degrees even though they do not eventually work in engineering firms because they would have more employment choices with engineering, which provides more insurance against future unemployment; however, this may lead to over supply of engineers and owing to job search frictions present in the economy, may lead



Population and GDP Growth for India during 1947-2006

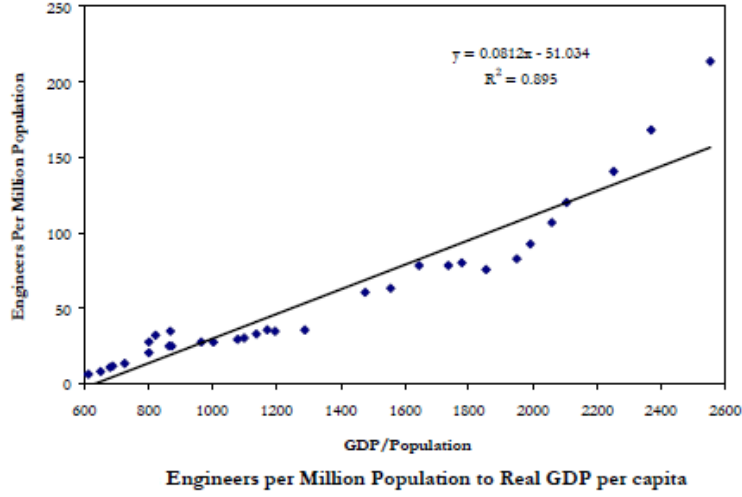


Figure 2: Rise in engineering relative to real GDP

to unemployment among engineers. If engineering graduates end up working in sectors which do not need them to invest in this skill, there could be a wastage of resources in terms of the human capital acquired as well as financial resources, which are required in an engineering degree. The purpose of my paper is to analyze whether this over-investment in a skill such as engineering leads to market inefficiency, and if there is a way to reduce this inefficiency.

To analyze this problem, I use a Diamond-Mortensen-Pissarides type search theoretic model with 2 sectors and 2 types of worker skills (high and low). The setup of my paper is similar to other papers in the literature such as Gautier (2002), Albrecht-Vroman (2002). I assume that a high-skilled worker can work in both the sectors whereas a low-

skilled worker can only work in one of the sectors. In this paper, I allow workers to endogenously decide the type of skill that they would like to invest in depending on their lifetime utilities. I use this setup to look at the efficiency of the market outcome. I find that in the decentralized economy, there are more number of high-skilled workers than is efficient. Further, there are more number of high-skilled workers searching for a job in the high-skilled market than is efficient. Both these factors contribute substantially to inefficiency in other market outcomes, in particular, they substantially lead to higher unemployment among high-skilled workers.

The paper is organized as follows: Section 2 describes the model in detail. Section 3 discusses the parameter choices and the solution approach to this model. Section 4 illustrates the results that I obtain in this paper. Section 5 concludes.

2 Model

2.1 Overview

I consider an economy with a unit measure of risk-neutral workers. Workers can be of two types: high-skilled(H) or low-skilled(L) and there are two types of jobs(markets) available: 1 and 2. At the start of every period, ν fraction of workers are born. Each of these workers need to decide which skill type to invest in. Once they acquire a skill, they along with other unemployed workers in the economy, search for a job. H-type workers can search for a job in both the markets whereas L-type can only search in market 1. Let the number of unemployed in each market i be denoted as u_i , $i \in \{1, 2\}$.

There is a continuum of risk-neutral profit maximizing firms. Each firm decides (a) whether to post a vacancy and (b) which market to post a vacancy in. I assume that each firm can post only 1 vacancy. The number of vacancies in each market i is given by v_i , $i \in \{1, 2\}$. The market tightness in each market (θ) is the ratio of the number of vacancies to the number of unemployed in the market, that is, $\theta_i = \frac{v_i}{u_i}$, $i \in \{1, 2\}$.

A CRS matching technology $M(u, v)$ governs the meetings between vacancies and unemployed workers in each market. I assume that $M(u, v)$ is concave and is increasing

in both its arguments, that is, higher the number of vacancies or higher the number of unemployed workers, the number of matches increase. A firm's probability of being matched to a worker is given by $q(\theta_i) = \frac{M(u_i, v_i)}{v_i}$, $i \in \{1, 2\}$. A worker's probability of being matched to a firm is given by $\frac{M(u_i, v_i)}{u_i} = \frac{M(u_i, v_i)}{v_i} \frac{v_i}{u_i} = \theta_i q(\theta_i)$, $i \in \{1, 2\}$. Once the matches are established, wages to be paid by the firm to the worker are negotiated upon. At the end of each period, ν fraction of each type of worker and ν fraction of each type of firm which posted a vacancy dies. Output production and wage payment takes place between the surviving matches at the beginning of next period. Home production by unemployed workers in each period is given by b . The sequence of events in each period is given by Figure 3.

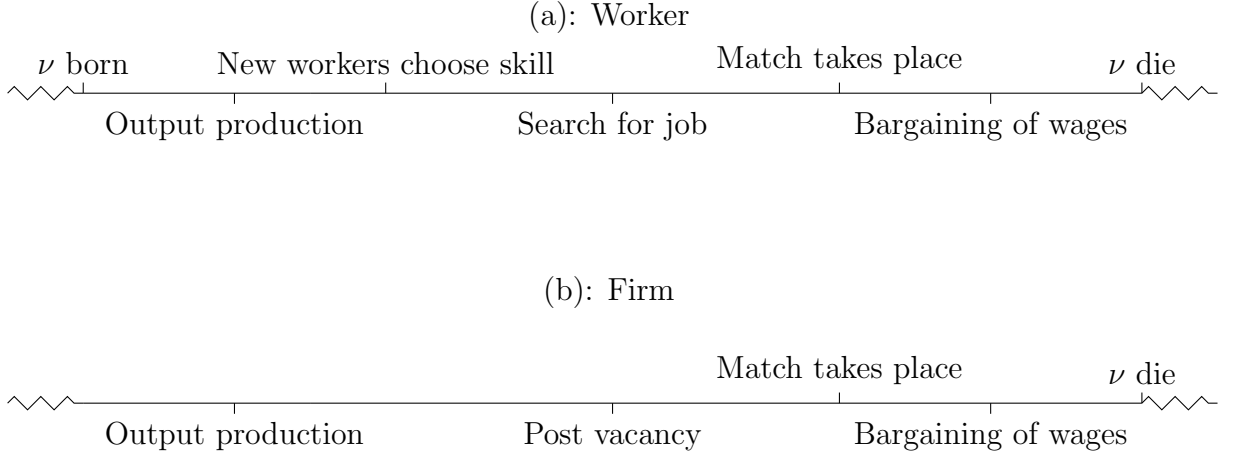


Figure 3: Sequence of events

2.2 Workers

Investing in high-skill requires a worker to pay an idiosyncratic cost ζ ($\zeta > 0$) which is drawn from a distribution $G(\zeta)$. This could be interpreted in terms of the financial cost that is required in order to pay for the high-skill, which can be partially correlated with the ability of the individual. Cost of investing in the low-skill is normalized to 0. The decision problem for a newly-born unskilled worker is defined below:

$$V(\zeta) = \max \{ -\zeta + EV_U^H, V_U^L \}$$

where $V(\zeta)$ represents the lifetime utility of a worker who has drawn a cost of ζ . The lifetime utility of an unemployed worker of type- S is given by V_U^S , where $S \in \{H, L\}$. The threshold value of the cost which makes a worker indifferent between investing in either of the skills is given by $\zeta^T = EV_U^H - V_U^L$. The fraction of low-skilled workers in the economy is given by λ , where

$$\lambda = \nu[1 - G(\zeta^T)] + \lambda^{-1} - \nu\lambda^{-1} \quad (1)$$

where λ^{-1} represents the fraction of low-skilled workers in the previous period. The number of low-skilled workers in the economy is the sum of low-skilled workers who survived from the previous period and the fraction of newly-born workers who chose to be low-skilled.

An unemployed low-skilled worker produces b units of home production while searching for a job in market 1. With probability $\theta_1 q(\theta_1)$, the worker gets employed, and with probability $1 - \theta_1 q(\theta_1)$, the worker remains unemployed. V_U^L represents his lifetime utility when unemployed and V_E^L is his lifetime utility when employed. His future discount rate is given by β . If he is employed, he receives a wage of w_{1L} .

$$V_U^L = b + \beta(1 - \nu) \{ \theta_1 q(\theta_1) V_E^L + [1 - \theta_1 q(\theta_1)] V_U^L \} \quad (2)$$

$$V_E^L = w_{1L} + \beta(1 - \nu)[\delta V_U^L + (1 - \delta) V_E^L]$$

where δ is the exogenous probability that an existing match breaks.

An unemployed high-skilled worker produces b units of home production and decides which market to search in. Each worker has to pay an idiosyncratic cost of ξ , drawn from a distribution $H(\xi)$, if he decides to search in market 2². This cost could be interpreted as the cost of preparing resumes, applications for the job; it could also reflect the relative quality of the high-skilled worker in the market, such that if the worker is of high quality,

²In principle, ξ can take any non-zero value. Hence if $\xi < 0$, it implies that there is a relative cost of ξ units if the worker decides to search in market 1

his cost would be lower. The lifetime utility for a worker who has drawn a cost of ξ units this period is given by:

$$\begin{aligned}
V_U^H(\xi) &= u(b) + \beta(1 - \nu) \max \{ \Delta_1, \Delta_2 - \xi \} \\
\Delta_1 &= \theta_1 q(\theta_1) V_E^{1H} + [1 - \theta_1 q(\theta_1)] EV_U^H \\
\Delta_2 &= \theta_2 q(\theta_2) V_E^{2H} + [1 - \theta_2 q(\theta_2)] EV_U^H \\
V_E^{1H} &= w_{1H} + \beta(1 - \nu) [\delta EV_U^H + (1 - \delta) V_E^{1H}] \\
V_E^{2H} &= w_{2H} + \beta(1 - \nu) [\delta EV_U^H + (1 - \delta) V_E^{2H}].
\end{aligned}$$

where EV_U^H and V_E^{iH} represent the expected lifetime utility of an unemployed and employed high-skilled worker in market $i, i \in \{1, 2\}$ respectively; Δ_i is the lifetime utility of searching for a job in market $i, i \in \{1, 2\}$; w_{1H} and w_{2H} are the wages earned by the high-skilled worker in markets 1 and 2 respectively.

$$\begin{aligned}
EV_U^H &= \int_0^{\xi^{max}} V_U^H(\xi) d\xi \\
&= u(b) + \beta(1 - \nu) [\rho \Delta_1 + (1 - \rho) \Delta_2 - E(\xi | \xi \leq \xi^T)]
\end{aligned} \tag{3}$$

where fraction of high-skilled workers who choose to search in market 1 is given by ρ ,

$$\rho = 1 - H(\xi^T) = 1 - H(\Delta_2 - \Delta_1) \tag{4}$$

The fraction of workers of type- S employed in market i are given by $E_{Si}, i \in \{1, 2\}, S \in \{H, L\}$. The law of motion for the employed workers is given by

$$E'_{L1} = (1 - \nu) \{ E_{L1}(1 - \delta) + \theta_1 q(\theta_1) U_L \} \tag{5}$$

$$E'_{H1} = (1 - \nu) \{ E_{H1}(1 - \delta) + \theta_1 q(\theta_1) \rho U_H \} \tag{6}$$

$$E'_{H2} = (1 - \nu) \{ E_{H2}(1 - \delta) + \theta_2 q(\theta_2) (1 - \rho) U_H \} \tag{7}$$

where U^H and U^L are the fractions of high-skilled and low-skilled workers respectively

who are unemployed and are given by

$$U_L = \lambda - E_{L1} \quad (8)$$

$$U_H = 1 - \lambda - E_{H1} - E_{H2} \quad (9)$$

2.3 Firms

Each firm decides whether to post a vacancy and in which market to post in . I assume that firms and workers discount the future at the same rate β . Posting a vacancy requires a firm to incur a cost of p units. With probability $q(\theta)$, the unmatched firm gets matched to a worker in that market. If it gets matched to a worker and they survive the death shock at the end of the period, the firm pays the worker his wage in return of the output produced by him. An S -type worker's productivity in market i is given by z_{iS} , where $i \in \{1, 2\}$, $S \in \{H, L\}$. I assume that the high-skilled workers are more productive than low-skilled workers on average.

Each firm in market 1 can be matched to either type of worker. The expected profit of an unmatched firm in market 1 is given by J_U^1 as defined by

$$J_U^1 = -p + \beta(1 - \nu) \{q(\theta_1)[\pi_L J_E^{1L} + (1 - \pi_L) J_E^{1H}] + [1 - q(\theta_1)] J_U^1\} \quad (10)$$

where π_L is the probability that the matched worker is of type-L, that is, it is the ratio of unemployed low-skilled workers to the total number of unemployed workers searching in market 1,

$$\pi_L = \frac{U_L}{U_L + \rho U_H} \quad (11)$$

and J_E^{1S} is the expected profit of a firm in market 1 who has been matched to a worker of type-S, $S \in \{H, L\}$ as defined by

$$\begin{aligned} J_E^{1L} &= (z_{1L} - w_{1L}) + \beta(1 - \nu) \{ \delta J_U^1 + (1 - \delta) J_E^{1L} \} \\ J_E^{1H} &= (z_{1H} - w_{1H}) + \beta(1 - \nu) \{ \delta J_U^1 + (1 - \delta) J_E^{1H} \} \end{aligned}$$

Each firm in market 2 can be matched only to an H -type worker. The expected profit of an unmatched firm in market 2 is given by J_U^2 as defined by

$$J_U^2 = -p + \beta(1 - \nu) \{q(\theta_2)J_E^{2H} + [1 - q(\theta_2)]J_U^2\} \quad (12)$$

where, J_E^{2H} is the expected profit of a matched firm as defined by

$$J_E^{2H} = (z_{2H} - w_{2H}) + \beta(1 - \nu)[\delta J_U^2 + (1 - \delta)J_E^{2H}]$$

I assume free entry of firms, which implies that

$$J_U^1 = 0 \quad (13)$$

$$J_U^2 = 0 \quad (14)$$

Firms are indifferent between posting vacancies in each market.

2.4 Wage determination

Wages are determined between workers and firms when the expected gains from trade are split according to a Nash Bargaining solution. Firms can observe the skill of the worker before the bargaining starts. The worker can threaten to remain unemployed and the firm can threaten to remain unmatched. The surplus beyond these threats is then split where the worker gets to keep a fraction of the surplus, which constitutes his bargaining power. The bargaining power for a worker of skill type- S in a match with firm i is given by ϕ_{iS} . Renegotiation of the wages is not allowed for in a match. The Nash Bargaining problems are stated as follows.

For a match between a low skilled worker in market 1:

$$\max_{w_{1L}} (V_E^L - V_U^L)^{\phi_{1L}} (J_E^{1L} - J_U^1)^{1-\phi_{1L}} \quad (15)$$

$$\text{subject to } V_E^L - V_U^L \geq 0 \text{ and } J_E^{1L} - J_U^1 \geq 0 \quad (16)$$

For a match between a high skilled worker in market 1:

$$\max_{w_{1H}} (V_E^{1H} - EV_U^H)^{\phi_{1H}} (J_E^{1H} - J_U^1)^{1-\phi_{1H}} \quad (17)$$

$$\text{subject to } V_E^{1H} - EV_U^H \geq 0 \text{ and } J_E^{1H} - J_U^1 \geq 0 \quad (18)$$

For a match between a high skilled worker in market 2:

$$\max_{w_{2H}} (V_E^{2H} - EV_U^H)^{\phi_{2H}} (J_E^2 - J_U^2)^{1-\phi_{2H}} \quad (19)$$

$$\text{subject to } V_E^{2H} - EV_U^H \geq 0 \text{ and } J_E^2 - J_U^2 \geq 0 \quad (20)$$

2.5 Equilibrium

A steady state equilibrium is a list $\{V_U^L, EV_U^H, J_U^1, J_U^2, \lambda, \rho, \theta_1, \theta_2, w_{1L}, w_{1H}, w_{2H}, E_L, E_{H1}, E_{H2}, \pi_L\}$ that satisfies:

- The First Order Conditions from the Nash Bargaining problems as given by (15), (17), (19).
- The definitions given by equations (2), (3), (4), (12), (10), (13), (14).
- The steady state conditions for (5), (6), (7), (1), (11).

2.6 Socially efficient outcome

This section characterizes the solution when the planner chooses allocations of in order to maximize the total utility of the society subject to the feasibility constraints. The efficient allocation is given by the solution to the planner's problem as defined by the following:

$$\begin{aligned} W(E_{L1}, E_{H1}, E_{H2}, \lambda^{-1}) = & \max_{\{\lambda, \rho, \theta_1, \theta_2, E'_{L1}, E'_{H1}, E'_{H2}\}} c + \beta(1 - \nu)W(E'_{L1}, E'_{H1}, E'_{H2}, \lambda) \\ & - \nu \int_0^{\zeta^T} \zeta dG(\zeta) - (1 - \lambda - E_{H1} - E_{H2}) \int_0^{\xi^T} \xi dH(\xi) \end{aligned} \quad (21)$$

subject to

$$c \leq z_{1L}E_{L1} + z_{1H}E_{H1} + z_{2H}E_{H2} + b(1 - E_{L1} - E_{H1} - E_{H2}) - p[\theta_1 u_1 + \theta_2 u_2] \quad (22)$$

$$E'_{L1} \leq (1 - \nu) \{E_{L1}(1 - \delta) + \theta_1 q(\theta_1)(\lambda - E_{L1})\} \quad (5)$$

$$E'_{H1} \leq (1 - \nu) \{E_{H1}(1 - \delta) + \theta_1 q(\theta_1)\rho(1 - \lambda - E_{H1} - E_{H2})\} \quad (6)$$

$$E'_{H2} \leq (1 - \nu) \{E_{H2}(1 - \delta) + \theta_2 q(\theta_2)(1 - \rho)(1 - \lambda - E_{H1} - E_{H2})\} \quad (7)$$

$$u_1 = \lambda - E_{L1} + \rho(1 - \lambda - E_{H1} - E_{H2}) \quad (23)$$

$$u_2 = (1 - \rho)(1 - \lambda - E_{H1} - E_{H2}) \quad (24)$$

$$\lambda = \nu(1 - G(\zeta^T)) + \lambda^{-1} - \nu\lambda^{-1} \quad (1)$$

$$\xi^T = H^{-1}(1 - \rho) \quad (4)$$

The total utility of the economy per period is given by $\int_0^1 u(c_i)di = \int_0^1 c_i di = c$. The total utility costs paid by the economy to invest in high-skill is given by the fraction of newly born individuals who draw a cost of less than ζ^T , which is the cost at which individuals are indifferent between investing in either skill. The total utility costs paid by the economy in order to search in market 2 over market 1 is incurred only by high-skilled workers and is paid only by those whose random cost draws are less than ξ^T , the cost at which the high-skill individuals are indifferent between searching for a job in either of the markets, as defined by Equation (4). Equation (22) illustrates the resource constraint of the economy. The total consumption of the economy should be less than or equal to the sum of resources available for the economy which is equal to the sum of the total output produced by the employed individuals in the market and the unemployed workers at home minus the total cost of posting vacancies given by $p(v_1 + v_2) = p(\theta_1 u_1 + \theta_2 u_2)$. The total number of unemployed workers in market 1 as stated in equation (23) is equal to the sum of all low skilled unemployed workers and the fraction of high-skilled workers

searching for a job in market 1 whereas the number of unemployed workers in market 2 as stated in equation (24) comprises only of the fraction of high-skilled workers who choose to search in market 2. Equations (5)-(7) describe the law of motion of employed workers and (1) describes the law of motion of the fraction of low-skilled workers in the economy. I use the first order conditions for the choice variables and the Benveniste-Scheinkman conditions for the state variables and impose steady state conditions in order to obtain a set of equations which can be solved to obtain the efficient allocations.

3 Model Solution and Parameterization

I use numerical methods to quantitatively solve the model. I make specific assumptions about the functional forms for the matching function as well as the probability distribution functions for the two idiosyncratic costs. I assume, as is common in the literature, that the matching function is Cobb-Douglas. In particular, $M(u, v) = Au^\alpha v^{1-\alpha}$, where α represents the elasticity of a match with respect to an unemployed worker and A is the matching efficiency parameter. I assume that A and α are the same in both the markets. Further, I assume that the idiosyncratic costs are drawn from Uniform distributions, that is, $\zeta \sim U[0, \zeta^{max}]$ and $\xi \sim U[0, \xi^{max}]$. For the decentralized economy, I use a non-linear function solver³ to simultaneously solve the set of fifteen variables from the fifteen equations as has been described in Section 2.5. For the social planner's problem, I use the method of iterations to obtain the efficient allocations. I start with an initial guess of values and iterate till the variables converge. This gives the efficient allocation.

Table 1 lists the parameter choices I make in this paper. My model is not calibrated. I borrow some of the standard parameter choices from Shimer (2005). Similar to Shimer, I set the value of leisure to $b = 0.4$. The mean income in my baseline model is 1.0355 which corresponds to a 39% replacement rate which is comparable to Shimer's 40%. I normalize a time period to be 1 year, and therefore set the annual discount rate, $\beta = 0.954$ which corresponds to an annual real rate of interest of 5%. Shimer suggests that the quarterly job separation rate is 0.1 which corresponds to an annual rate of $\delta = 0.34$. I assume that

³To avoid a case of local convergence, I use 10 different vectors of initial guesses

once workers join the labor force, the average work life expectancy is 45 years, which corresponds to the death probability, $\nu = 1/45$. I use the elasticity parameter for the matching function $\alpha = 0.72$, similar to Shimer. In the standard Mortensen-Pissarides one firm one worker search framework, the Hosios condition states that when the bargaining weight for a worker in a Nash Bargain is the same as the elasticity of a match with respect to the unemployed, the market outcome is efficient. Hence, in the baseline case, I start with the assumption that the bargaining weights for a worker in any of the matches is equal to the elasticity parameter. The efficiency parameter in the matching function, A is chosen in a way to ensure that the probabilities $\theta q(\theta)$ and $q(\theta)$ are less than 1 at all times. p is chosen such that the cost of investing in vacancies lies around 5% of the total output produced. The upper bound for the two costs, ζ^{max} and ξ^{max} are chosen so as to ensure that λ and ρ have interior solutions at all times.

Without loss of generality, I normalize the productivity of a low-skilled worker in market 1 to be equal to 1. I assume that high-skilled workers are more productive than low-skilled workers in both the markets; high-skilled workers are 40% more productive in market 2, whereas they are only marginally, that is 5% more productive in market 1.

Parameter	Definition	Value
β	Discount factor	0.954
ν	Death rate	1/45
δ	Job separation rate	0.34
α	Elasticity of a match w.r.t unemployed	0.72
ϕ_{1L}	Bargaining wt of 'u' in a match between firm 1 and worker L	0.72
ϕ_{1H}	Bargaining wt of 'u' in a match between firm 1 and worker H	0.72
ϕ_{2H}	Bargaining weight of 'u' in a match between firm 2 and worker H	0.72
A	Efficiency parameter in matching function	0.25
p	Fixed cost of posting a vacancy	0.005
z_{1L}	Productivity of L-type in market 1	1
z_{1H}	Productivity of H-type in market 1	1.05
z_{2H}	Productivity of H-type in market 2	1.4
b	Home production	0.4
ζ^{max}	Upper bound of cost of investing in H-skill	5
ξ^{max}	Upper bound of cost of search in market 2	1

Table 1: Parameter choices

4 Results

In Table 2, I present the comparison between the allocations obtained under the social planner's problem (Column 1) and in the decentralized economy (Column 2) under the baseline set of parameter choices.

	Planner (1)	Baseline (2)
λ	0.82438	0.63534
ρ	0.78972	0.66841
v_1	7.9646	1.5913
v_2	0.86859	5.7407
w_{1L}	-	0.90146
w_{1H}	-	0.96766
w_{2H}	-	1.1394
E_L	0.52554	0.32677
E_{H1}	0.08274	0.096635
E_{H2}	0.029561	0.12512
θ_1	22.831	3.938
θ_2	65.241	121.15
U_L	0.2989	0.3086
U_H	0.0633	0.1429

Table 2: Comparison under baseline set of parameters

The results illustrate that as compared to the efficient allocation, the decentralized economy has: (a) more number of high-skilled workers in the economy (a lower λ), (b) a higher fraction of high-skilled workers searching for a job in market 2 (a lower ρ). These two inefficiencies affect the other allocations in the economy. In this economy, a high-skilled worker working in market 2 has to incur two kinds of costs: cost of skill investment and the cost of searching in market 2. The benefit of having an unemployed high-skilled worker searching for a job in market 2 is that the productivity of this worker when employed is higher than any other worker. However, due to the presence of search frictions, a high-skilled worker who chooses to search in market 2 does not always get matched but has to pay the cost of search nonetheless. When a high-skilled worker decides which market to search in, the wages that would be negotiated as well as the market tightness in each market is accounted for. When z_{1H} and z_{2H} are substantially

different, w_{2H} is substantively higher than w_{1H} , which induces more workers to search for a job in market 2. However, they do not account for the fact that when each worker decides to search in market 2, there is a congestion externality effect that it has for all other workers in the same market, due to which it becomes more difficult to find a job in that market. This is why the efficient allocation has lower fraction of high-skilled workers searching in market 2. Further, workers in market 1 do not significantly differ in their productivities, but high-skilled workers need to incur a cost in order to obtain their skill. Thus, the efficient outcome would have a higher fraction of low-skilled workers in the market. Together these two effects result in a higher number of vacancies in market 1 and lower number of vacancies in market 2 in the efficient allocation as compared to the market. Further, this results in a significantly higher unemployment in the market among high-skilled workers than is efficient.

In order to isolate the effect of each of the fractions (ρ and λ) on the other allocations in the economy, that is, to discuss how much of these contribute to the inefficient allocations, I conduct three exercises. In the first exercise, I fix the supply of skills in the decentralized economy to the efficient allocation (that is fix $\lambda_{Decentralized} = \lambda_{Planner}$), and look at the extent of inefficiency that still remains. In the second exercise, I fix the fraction of high-skilled workers searching in each market to be equal to the allocation obtained from the planner's problem (that is fix $\rho_{Decentralized} = \rho_{Planner}$) and do the same. In the third exercise I fix both λ and ρ and look at the outcomes. Table 3 reports these results.

Over-investment in high-skill has a significant impact on the other market outcomes, as seen in Column (3). when I control for the excess supply of high-skilled workers, as compared to the baseline economy, vacancies in each market are significantly closer to the efficient outcomes. The fraction of unemployed high-skilled workers are significantly reduced, whereas the fraction of employed low-skilled workers is significantly increased. This illustrates that an over-supply of high-skilled workers in the market contributes significantly to the inefficiency, especially leading to an increase in the unemployment of high-skilled workers.

Controlling for the inefficiency caused due to an over-supply of high skilled workers

	Planner (1)	Baseline (2)	w/ λ -fixed (3)	w/ ρ -fixed (4)	w/ λ, ρ -fixed (5)
λ	0.82438	0.63534	0.82438	0.78206	0.82438
ρ	0.7897	0.66841	0.5575	0.7897	0.7897
v_1	7.9646	1.5913	7.1965	7.2117	7.0662
v_2	0.86859	5.7407	1.7347	1.1525	0.9361
w_{1L}	-	0.90146	0.91728	0.91678	0.91639
w_{1H}	-	0.96766	0.98306	0.97467	0.97437
w_{2H}	-	1.1394	1.2347	1.2263	1.2259
E_L	0.52554	0.32677	0.52192	0.49285	0.5175
E_{H1}	0.08274	0.096635	0.055147	0.10077	0.0807
E_{H2}	0.029561	0.12512	0.059593	0.037687	0.0305
θ_1	22.831	3.938	21.393	20.489	19.75
θ_2	65.241	121.15	64.398	68.962	69.203
U_L	0.29885	0.30856	0.30246	0.28922	0.3069
U_H	0.063313	0.14291	0.060875	0.079481	0.06434

Table 3: Isolating components of inefficiency

Column (1) lists the efficient allocation; Column (2) lists the baseline market outcomes; Column (3) lists the market outcomes under exercise 1 and Column (4) lists the market outcomes under exercise 2; Column (5) lists the outcomes under exercise 3.

searching in market 2 also results in market outcomes which closely resemble the efficient outcomes, as illustrated in Column (4). As compared to the baseline economy, similar to the previous case, vacancies in each market are significantly closer to that in the planner's problem. Fraction of employed high-skilled workers in market 2 reduces but in market 1 increases. Similar to the previous case, the fraction of unemployed high-skilled workers are significantly reduced, whereas the fraction of employed low-skilled workers is significantly increased. This illustrates that controlling for ρ also contributes significantly to the inefficiency.

Next, when I fix both ρ and λ in the decentralized economy to their efficient allocations, as is clear from the results in Column (5), most of the inefficiency is accounted for. Number of vacancies in market 2 decreases substantially. Unemployment among high-skilled workers reduces significantly, whereas employment among low-skilled workers rises. This suggests that both these factors contribute substantially to the inefficiency that is generated in the model and hence both ρ and λ need to be controlled for in order to get the market outcome to resemble the efficient outcome.

In an attempt to get both ρ and λ in the decentralized economy to closely resemble

the efficient allocations, I change the bargaining weights associated with each type of match. I had started with the assumption that all unemployed workers have the same bargaining weight ($\phi_{1L} = \phi_{1H} = \phi_{2H} = 0.72$) irrespective of the type of match. Now, I let $\phi_{1L} = 0.72$ and change the bargaining weights of unemployed workers in the other two matches so as to reduce the inefficiency. From the previous exercises it is clear that if ρ and λ in the decentralized could be affected in a way such that they are close to the efficient allocations, a large part of the inefficiency can be taken care of. Now, the baseline results show that both λ and ρ are lower than the efficient outcomes. In order to have more low-skilled workers in the economy, I reduce the bargaining weight of the high-skilled workers in both the matches relative to the bargaining weight of the low-skilled worker. Further, in order to have more high-skilled workers searching for a job in market 1, I reduce the bargaining weight of the high-skilled worker in market 2 by a much larger fraction as compared to it in market 1. An example of this is illustrated in Table 4. In particular, I use $\phi_{1H} = 0.94\phi_{1L}$ and $\phi_{2H} = 0.65\phi_{1L}$. The resultant outcomes are given in Column (3)⁴. As the results indicate, changing the bargaining weights helps to reduce

	Planner (1)	Baseline (2)	Changing wts (3)
λ	0.82438	0.63534	0.83084
ρ	0.78972	0.66841	0.80246
v_1	7.9646	1.5913	6.7322
v_2	0.86859	5.7407	2.0939
w_{1L}	-	0.90146	0.91577
w_{1H}	-	0.96766	0.95481
w_{2H}	-	1.1394	1.0566
E_L	0.52554	0.32677	0.51838
E_{H1}	0.08274	0.096635	0.074882
E_{H2}	0.029561	0.12512	0.034636
θ_1	22.831	3.938	18.684
θ_2	65.241	121.15	177.71
U_L	0.29885	0.30856	0.31246
U_H	0.063313	0.14291	0.059646

Table 4: Changing bargaining weights

⁴I concede that it is possible that there are other combinations of bargaining weights which also reduce the inefficiency. I merely use this as an indication towards one kind of intervention which can work towards reducing the inefficiency

the inefficiency in the market outcomes. Employment among low-skilled workers rises and among high-skilled workers falls, but unemployment among high-skilled workers falls significantly. There is an overall shift towards output production by low-skilled workers from high-skilled workers. In terms of government intervention, these results hint that if the high-skilled workers are taxed, in particular, those who look to search in the high-skilled market, and low-skilled workers are subsidized, the inefficiency can be reduced.

So far I had assumed that high-skilled workers are marginally more productive than low-skilled workers in market 1. In the final exercise, I relax this assumption. I assume that high-skilled workers are less productive than low-skilled workers in market 1, and compare the decentralized market and social planner's outcomes. In particular, I assume that $z_{1H} = 0.95z_{1L}$. The results are given in Table (5)

	Planner (1)	Market (2)
λ	0.94833	0.6354
ρ	0.73696	0.55419
v_1	8.177	1.9082
v_2	0.33607	5.9831
w_{1L}	-	0.90282
w_{1H}	-	0.89885
w_{2H}	-	1.1728
E_L	0.60474	0.33898
E_{H1}	0.022401	0.080996
E_{H2}	0.010916	0.14958
θ_1	22.897	5.1476
θ_2	69.615	100.13
U_L	0.34359	0.29642
U_H	0.018354	0.13403

Table 5: Comparison when $z_{1H} < z_{1L}$

I find that the results do not qualitatively change. In the decentralized economy, λ and ρ are lower than the efficient outcomes; v_1 is lower than efficient and v_2 is higher. Hence, in this scenario, the inefficiency increases. In fact, the gap between the market outcomes and the efficient outcomes, seem to have increased. This might be owing to the fact that now there is inefficiency due to productivity loss in market 1, when high-skilled workers get employed, as they produce less than the low-skilled workers, but still have to

pay a cost of investing in this skill.

5 Conclusion

In this paper, I analyze the problem of having a large number of engineering enrollment as well as a large number of unemployed engineers in India. To do this, I use a search-theoretic environment where workers decide which skill type to invest in, based on future job options. I find that there is an over-investment in skill types which provide workers with a multiple number of job options. This leads to inefficient levels of job vacancies and as a result of search frictions present in the market, higher than efficient levels of unemployment in the economy, particularly for the high-skilled workers (or engineers). I find that in order to reduce the inefficiency, high-skilled workers should be penalized, in particular, those searching for a job in the high-skilled market (engineering sector) and low-skilled workers should be subsidized. This would disincentivize the high-skill workers from investing in the high-skill such that the fraction of each type of worker in the economy is closer to the efficient level. This would, in the process lead to a reduction in the other forms of inefficiency.

References

- [1] Acemoglu, Daron. "Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence." *American Economic Review* 89.5 (1999): 1259-1278.
- [2] Albrecht, James, and Susan Vroman. "A matching model with endogenous skill requirements." *International Economic Review* 43.1 (2002): 283-305.
- [3] Banerjee, Rangan, and Vinayak P. Muley. "Engineering education in India." Sponsored by Observer Research Foundation, Department of Energy Science and Engineering, Indian Institute of Technology, Bombay 16 (2008).
- [4] Gautier, Pieter A. "Unemployment and search externalities in a model with heterogeneous jobs and workers." *Economica* 69.273 (2002): 21-40.

- [5] Hosios, Arthur J. "On the efficiency of matching and related models of search and unemployment." *The Review of Economic Studies* 57.2 (1990): 279-298.
- [6] Mortensen, Dale T., and Christopher A. Pissarides. "Job creation and job destruction in the theory of unemployment." *The review of economic studies* 61.3 (1994): 397-415.
- [7] Pissarides, Christopher A. *Equilibrium unemployment theory*. MIT press, 2000.
- [8] Shimer, Robert. "The cyclical behavior of equilibrium unemployment and vacancies." *The American Economic Review* 95.1 (2005): 25-49.