Strategy-Proof Resource Allocation in a Network

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Abstract

A divisible resource needs to be divided amongst the agents in a society who are connected in a network. Agents care about the amount of money they receive as well as the amounts that their neighbors receive. The existence (or non-existence) of a link between a pair of agents is private information of these two agents. We introduce and characterize the class of strategy-proof mechanisms for different shapes in the utility function.

Keywords: Strategyproof, resource allocation, network externality

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1 Introduction

The study of externalities in social networks has found prolific applications in many disciplines ranging across Economics, Sociology and Computer Sciences. An entire field of network economics is devoted to the study of externalities in networks. In the presence of externalities the structure of the network becomes very crucial for a planner who wants to allocate some resources to the agents and has a goal of, say, maximizing the total sum of the welfare of the society. We study the case where each agent cares not only about the resources he is allocated, but also about the resources allocated to his immediate neighbors.[1] We can think of the example of a charitable organization distributing books, computers, or other resources to poor students. If two students are connected (friends, families, neighbors, etc.), they may share the resources. Similarly, one can think of the allocation of research funds to various universities or laboratories when scientists and researchers have collaboration across laboratories and universities. Other examples include the allocation of money to different groups where there are spillover effects or the allocation of houses where people care about neighbors. The positive network externalities in groups have been successfully utilized in microfinance where one of the main mechanisms for the delivery of financial services requires enterpreneurs to come together to apply for loans and other services as a group. There may also be negative externalities where people care about their relative allocation in the neighborhood, e.g., the allocation of salaries in a department where people care about the salary of their peers.

When the network is known, the planner may have objectives for the allocation of resources, such as welfare efficiency or a fairness criteria like no-envy, equal treatments of equals, maximizing the welfare of least happy agent, etc. Sharing the cost of the network goods (see, e.g., Hougaard and Moulin[17]) and injection points for resources or information for distribution on a network (see, e.g., Banerjee et al.[4]) are other avenues for the planner when the network is known. However, the problem for the planner becomes more difficult if the structure of the network is not known to the planner and is known only to the agents. In such a scenario the planner might want to extract the information about the network from the agents. The goal of this project is to study the allocation of resources when the planner lacks information about the connections of the agents.

Extracting information about the network might come at a cost for the planner's objectives. In particular, given an allocation rule, it may not be in the best interest of the agents to reveal their true connections to the planner. To see this, suppose that the planner objective is to allocate more resources to agents with the highest degree. If the agents report their connections to the planner, then agents have the incentive to over-report the number of connection in order to increase their share of the resource.[2] Therefore, in order for the planner to

[1] This can be extended to neighbors of neighbors and longer chains but we stick to immediate neighbors for simplicity.

[2] This is true assuming that the marginal utility of receiving a good is higher than giving

achieve his objective more effectively, he needs to find an incentive compatible mechanism. In particular, incentive compatibility of the mechanism is interpreted as strategy-proofness, which requires truthful reporting of each agents connections to be a dominant strategy.

1.1 Overview of the results

1.1.1 Strategyproof mechanisms

We study the problem when the planner has a fixed amount of a divisible resource that will be fully distributed to the agents. Agents have preferences for the distribution of resources to them and their neighbors. These preferences can be represented by a utility function, which depends on both the agents connections as well as their intensities. Thus, for instance, two agents might be connected to the same group of agents but might care differently about them.[3] The planner does not know the agents, and thus lacks information about the agent's connections and their intensities.

We study the case where the planner has the ability to elicit the connections of the agents but not their intensities. That is, every agent will report to the planner who they are connected to in a network but may not necessarily report the degree of their externalities (intensity).[4] The goal of the planner is to characterize the entire class of strategy-proof mechanisms for a wide range of utility functions.[5]

The first difficulty of the problem comes from a simple observation: A mechanism is strategyproof for any type of utility function if and only if it is a fix sharing mechanisms, where the allocation of the agents is independent on their reports (Lemma 1). Thus, in the absence of information about the preference of the agents, the planner has no choice but to use a highly restrictive fix sharing mechanism.[6]

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it to your neighbors, which is a standard assumption in the paper. One can also find similar issues of misrepresentation in everyday life, for instance, when filing income taxes in the USA. The government allocation policy may require to charge different amount of taxes if a couple is filing their taxes jointly (i.e., they are connected) or independently (not connected). These two cases mostly end up with different amount of taxes and the couple may choose the one where they end up paying less tax in total.

[3] We focus in the most simplistic model, where the intensities are symmetric among their connections, although the non-symmetric intensities is also an interesting example that is studied in our companion paper.

[4] Our work also different with the more traditional approach in the literature to require agents to report their entire utility profile. Simple reporting of the preferences is desirable, and even in such a model, our results already show the robustness of the characterized mechanisms. The case when agents report their intensities is address in our companion paper, mainly characterizing impossibilities.

[5] Every strategy-proof mechanism that we characterize is also Pareto efficient. As we try to be as general as possible, we do not use more simplistic measure of efficiency, like utilitarian, which will greatly limit our mechanisms. Depending on the specific application, future studies should focus on selecting the appropriate mechanism among the class of mechanisms that we introduce.

[6] Fix sharing mechanisms are highly restrictive, for instance in terms of efficiency.

Therefore, in order for the planner to find non-trivial mechanisms, it is crucial for him to have more information about the agents' preferences. Our work assumes that the planner has some minimal information about the *shape of the externality* that agents receive about their friends, but not about the intensity. For instance, the planner knows whether agents care about the aggregate resources send to their friends (additive externality), the least-well off (minimum), or a more general quasi-concave utility function.

The classes of mechanisms that are strategyproof will vary widely depending on the shape of the externality factor. To overcome this difficulty, we introduce a key lemma that will greatly simplify the analysis and proofs for a general function. This lemma will provide the classes of mechanisms that are strategy-proof in the absence of externalities, that is, when agents care only about their own allocation. All the mechanisms that will be studied for different externality factors will be a subset of the ones provided by this lemma. This lemma states that in order to construct a mechanism that is strategy-proof in the absence of externalties, we can just assign budgets to every coalition of agents, and let them send money to people outside of this coalition in an arbitrary way. Since every coalitions send the money outside the coalition, then they cannot influence their own share, thus, making the mechanism strategyproof. We call this result the *inverse revelation Lemma*.

In the case of externalities, only a subset of the mechanisms provided by the inverse revelation Lemma will be strategy-proof for different functions. Our two main results, characterize the class of strategy-proof mechanisms depending on the curvature of the externality of the agents.

When at least one agent have an an externality factor that is quasi-concave (e.g., when agents care about the least well off individual among their friends), the class of strategy-proof and symmetric mechanisms resemble a fix-sharing mechanism, except when agents receive connections from everyone else.

This contrast with the case where all agents have additive externality, for instance, when agents care about the average, or the sum of the allocation of their friends. In such as case, a large class of strategy-proof mechanisms, where every coalition of the same size is assigned a budget to re-distribute to others outside the coalition. The redistribution depend on whether the intersection of the reports of the agents is non-empty. When non-empty, all the agents in the intersection are assigned an equal share. On the other hand, if the intersection is empty, then the distribution is equal among the agents outside this coalition.[7] The second main result of the paper shows that the class of strategy-proof mechanisms when all the agents have an externality factor that is additive only contains these mechanisms. (Theorem 2)

[7] This can be interpreted as the case where a coalition agrees on the distribution or not. When there is an agreement, these agents get an equal share. However, when there is no agreement, everyone outside the coalition get an equal share.

1.2 Related literature

The strategyproof mechanisms for resource allocation has been widely studied in literature. The main fucus of this literature has been on the allocation of indivisible goods because of the existence of mechanisms that implement desirable Social Choice Functions (SCF), see e.g., Bogomolnaia and Moulin [6], Bogomolnaia et al. [7], Dutta et al. [12], etc. Although there are specific domains with divisible goods where desirable SCFs can be implemented even by stronger notions than strategyproofness (see, e.g., Kumar [23]), not much study has been done for allocation of divisible goods because of its dependence on the specific form of utility functions. Studies of allocation mechanisms in divisible goods framework include de Clippel et al. [11], Tideman and Plassmann [26], Juarez and Kumar [18], Juarez, Nitta and Vargas [19], Han and Juarez [15], Hougaard and Moulin [17] among others.
There is also a large literature looking at varios network effects. Bourlès and Bramoullé [8] studying the rule of altruism in networks. Bramoullé and Kranton [9] study public goods under network effects. Bramoullé and Kranton [10] looking at strategic interactions in networks. The model of local externalities introduced by Farrell and Saloner [13], and Katz and Shapiro [21] has been used for identifying influential agents in a network and pricing by the firms in various market set up. The local externality has been used by Jullien [20] and Banerji and Dutta [3] to analyze competition between two price setting firms. Sundarajan [25] studies monololy pricing on a network in a model where consumers make a deterministic choice between adopting a new product or not. Saaskilahti [24] studies uniform monopoly pricing on social networks whereas Ghinglino and Goyal [14] focus on a model of conspicuous consumption with negative externalies from neighbors consumption. Hartline et al. [16] and Arthur et al. [1] compute optimal pricing strategies by identifying influential agents in a network. Our work is the first to study the allocation of a divisible resource to agents under network effects when the planner has imperfect information about the network.

2 Preliminaries

One unit of a divisible resource (e.g. money) should be divided among the agents $N = \{1, 2, ..., n\}$. Let $\Delta(N) = \{x \in \mathbb{R}^N_+ | \sum x_i = 1\}$ be the simplex for the agents in N. An allocation $x \in \Delta(N)$ is a vector where x_k is the amount received by agent k.
Agent might or might not be connected to other agents in the network. Let V the network of connections. Let $N_i(V)$ (or simply N_i) the set of agents connected to agent i in the network V.
Given the network V, let

$$g_{ij} = \begin{cases} 1, \text{if there is a link between agent } i \text{ and agent } j \\ 0, \ \text{otherwise.} \end{cases}$$

Let $x_{N_i} = [x_j g_{ij}]_{j \in N \setminus \{i\}}$ the projection of the allocation x over the space of neighbors of agent i.
The utility of agent i is denoted by

$$U_i^f(x) = u_i(x_i, f_i(x_{N_i}))$$

where the function $f_i : \mathbb{R}^{n-1} \to \mathbb{R}$ is an increasing function in every coordinate, the function $u_i : \mathbb{R}^2 \to \mathbb{R}_+$ is also an increasing function on each coordinate, differentiable and such that $\frac{\partial u_i(x,y)}{\partial x} > \frac{\partial u_i(x,y)}{\partial y}$.
Thus, the utility of agent i is a separable function that depends in his own allocation of money and the allocation of money of his neighbors.
Agents may vary on the externality factor f_i as well as the intensity in which they care for others (u_i). We assume that agents care more about their own allocation instead of the allocation of others, thus $\frac{\partial u_i(x,y)}{\partial x} > \frac{\partial u_i(x,y)}{\partial y}$. We focus first in the case of positive externalities, thus the function u_i is increasing in both coordinates.

Example 1 *The following are examples of utility functions incorporated by our model:*

- *The **sum** of the money received by the neighbors*

$$U_i^{sum}(x) = u_i(x_i, \sum_{j \neq i} x_j g_{ij})$$

  The sum utility functions appear when there is substitutability on the allocation of the resources allocated to the neighbors.

- *The **average** of the amount received by the neighbors*

$$U_i^{avg}(x) = u_i(x_i, (\sum_{j \neq i} x_j g_{ij}) / \sum_{j \neq i} g_{ij}))$$

  The average utility depicts the substitutability of resources, but also takes into account the number of neighbors.

- *The **minimum** of the amount received by the neighbors*

$$U_i^{\min}(x) = u_i(x_i, \min_{j \neq i, g_{ij}=1} \{x_j\})$$

  The minimum utility function includes situations of extreme complementaries among individuals, where agents care about the worst-off individual in your social network.

- *The **Cobb-Douglas** of the amount received by the neighbors*

$$U_i^{c-d}(x) = u_i(x_i, \prod_{j \in N_i} x_j^{\alpha})$$

  The Cobb-Douglas utility is a concave utility function that shows some extent of complementarity and substitutability.

3 Mechanisms and desirable properties

We study mechanisms where agents report their connections to other agents. Agents do not report the intensity of their connection.

Definition 1 *A mechanism is a function* $F : \Pi_i 2^{N \setminus i} \rightarrow \Delta(N)$.

To implement an allocation satisfying desirable properties e.g., the one maximizing the sum of utilities (welfarist) or the one maximizing the minimum utility (egalitarian), it is important to know the exact network. However, the knowledge of links being private information, the allocation rule must induce the agents to reveal their links. Strategyproofness of the allocation rule requires that it be in the interest of the agents to report their connections wihout worrying about the report of other agents.

Definition 2 *A mechanism is strategy-proof if for any utility functions* $(U_1, \ldots, U_n)$, *for any network* V *and any agent* i*:*

$$U_i(F(N_i(V), M_{-i})) \geq U_i(F(\tilde{S}, M_{-i}))$$

for any $\tilde{S} \subset N \setminus \{i\}$ *and any report of the other agents* M_{-i}.

A mechanism is strategyproof if every agent i has no incentive to report any subset $\tilde{S}$ that is different than their neighbors $N_i((V)$.
A necessary condition for strategyproofness is that agent i has no way to influence his own payment. This condition is necessary but not sufficient, since agents might also benefit in the way in which the resource is allocated to other agents. The goal of this paper is to find the class of strategyproof mechanisms.

Definition 3 (Fix-sharing mechanism) *Let* $x \in \Delta(N)$. *The fix sharing mechanism* FIX *allocates* x *regardless of the reports of the other agents. That is,* $FIX(N) = x$ *for any* N.

The fix sharing mechanism is clearly strategyproof for any utility of the agents, since the report of the agents will not influence the allocation of money of them or anyone else.
As seen in footnote above, the class of fix sharing mechanisms are the only mechanisms that belong to the intersection of all the strategyproof mechanisms for arbitrary utility functions.

Definition 4 (S-sharing mechanism) *Given* $S \subsetneq N$, *consider a function* $f^S : \Pi_{j \in S} 2^{N \setminus j} \rightarrow \Delta(N \setminus S)$. *The* S*-sharing mechanism allocates:*

$$F^S(S_1, \ldots, S_n) = f^S([S_j]_{j \in S}).$$

An S-sharing mechanism, the reports of the agents in S do not influence their payoff. Therefore, it meets a necessary condition for strategyproofness. If $S = \emptyset$, the $\emptyset$-sharing mechanism is defined as the FIX mechanism.

Definition 5 (Separable mechanism) *Given a collection of S-sharing mechanisms $\{F^S\}_{S\in 2^N\setminus N}$ and a vector of allocation of the resource to every coalition $y \in \Delta(2^N \setminus \{N\})$, a separable mechanisms SEP is defined as:*

$$SEP(T) = \sum_{S\in 2^N\setminus N} y_S f^S(T_S)$$

A separable mechanism allocates to every coalition S a share of the resource y_S that will be split to the agents in $N \setminus S$ according to the reports of the agents in S. Thus, a separable mechanism is a convex combination of S-sharing mechanisms with vector of weights y.

Example 2

- *If $x_\emptyset = 1$, then the full resource will be split to the agents in N in a fix way and we get the fix-sharing mechanism.*
- *If $x_i = \frac{1}{n}$, then every agent will have a share of $\frac{1}{n}$ to split to the other agents.*

In general the separable mechanisms are not strategyproof for all utility functions. However, as we will see below, they will be the core of our characterizations for additive utility functions such as sum and average.

4 Main Results

4.1 Strategyproofness under minimal information

The following negative result shows that in the absence of information about the utility function of the agents, the planner is forced to implement a fix sharing mechanism.

Lemma 1 *A mechanism F is strategyproof for any common externality and any utility function if and only if it is a fix-sharing mechanism.*

In light of this result, the planner is forced to acquire more information about the utility function of the agents in order for him to get a larger class of strategyproof mechanisms.
The following sections deals with this problem, when the planner is informed about the statistic of the agents, but not about thei utility function.

4.2 Inverse Revelation Lemma

The classes of strategyproof mechanisms will be very different depending on the externality factor, however, the following Lemma provides a subset of mechanisms that contain all the mechanisms discussed for particular utility functions.

Lemma 2 *Suppose that every agent has utility a function without externality, $U_i(x) = x_i$ for all i. A mechanism is strategyproof if and only if it is separable.*

The proof of this result is a consequence of Farkas' lemma.

4.3 Strictly quasi-concave statistic

We study the case where there exists at least one agent with a strictly quasi-concave statistic. For instance, an agent might care about the less worse-off individual (min) or might evaluate the allocation of his friends using a Cobb-Douglass statistic. The rest of the agents may have quasi-concave statistics, some of which might not be strict, like Sum or Average statistics.
Given a profile $T = (T_1, \ldots, T_n)$, let $C^i(T) = |\{k | i \in T_k, T_k \in T\}|$ be the number of agents who point to agent i in the profile T. Let $\mathcal{S} = \{(x, y) \in \mathbb{N}^2 | x \leq n-2, y \leq n-2\}$ represent the possible cardinality of the reports that $n-2$ agents have over the remaining two agents. Thus, at the pair (x, y), the number x represents the number of agents who are connected to the first agent and y represents the number of agents who are connected to the second.
Let $\mathcal{T} = \{(x, 0), (0, x) \in \mathbb{N}^2 | 2 \leq x \leq n-2\}$ be a subset from $\mathcal{S}$ where one of the agents receive no connections and another receive at least two.

Definition 6 *Given a function $g : \mathcal{S} \to \Delta^2_+$ such that $g_1(x, y) = g_2(y, x)$ for any $(y, x) \in \mathcal{S}$, the scoring mechanism φ^g allocates to agent i:*

$$\varphi^g_i(S_1, \ldots, S_n) = \frac{2}{n(n-1)} \sum_{j \neq i} g(C^i(S_{-ij}), C^j(S_{-ij}))$$

A scoring mechanism is a particular case of a separable mechanism, where every coalition of size $n-2$ is given the budget $\frac{n(n-1)}{2}$. The payment of agent i in the scoring mechanism takes into account his number of connections in relation to every other agent. This division is given by the function g. Note that the scoring mechanism is symmetric since the function g is symmetric.
When, $g(x, y)$ is a constant function equal to $(\frac{1}{2}, \frac{1}{2})$, the scoring mechanism determines the fix sharing mechanism. The flexibility on the function g, for instance by dividing based on proportion, provides a rich class of mechanisms. Most of these mechanism are omitted in the following class of mechanisms.

Definition 7 *The almost-fixed mechanism is a scoring mechanism generated by the function g such that:*

i. $g(x, y) = (\frac{1}{2}, \frac{1}{2})$ *for* $(x, y) \notin \mathcal{T}$,

ii. $g_1(x, 0) \geq g_1(x-1, 0)$ *for* $n-1 \geq x \geq 1$

Under the almost-fixed mechanism generated the function g, the relative payments for a pair of agents is constant when both agents are pointed by at least one agent (condition *i*). Thus, an almost-fixed mechanism is determined by exactly $n-3$ variables,

$$g(2, 0), g(3, 0), \ldots, g(n-2, 0).$$

Furthermore, the relative share of an agent in relation with an agent who receives no connection does not decrease as more agents point to him (condition *ii*).

The contrast on the allocations of almost-fixed mechanisms can be seen in the star networks shown in figures 1 and 2. In the first figure, there is no distinction in the allocation between the center and periphery agents. In the second picture, the center agent is pointed by every other agent, thus beating every periphery agent in pairwise comparisons. Anything in between these two extremes can be feasible.
While we do not consider any efficiency measures, we want to highlight than an allocation rewarding an agent with the larger number of connections may be more desirable than say an equal allocation, especially for utilitarian measures of efficiency.

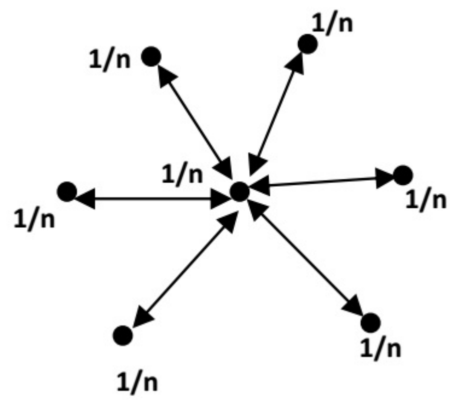

Figure 1: The fix-sharing mechanism generated by $g(x, 0) = (1/2, 1/2)$ for $x \geq 2$.

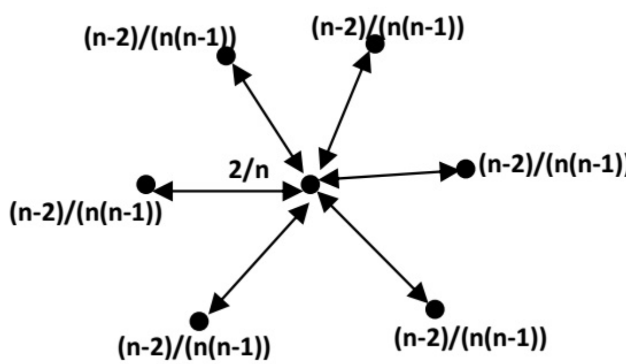

Figure 2: An almost-fixed mechanism generated by $g(x, 0) = (1, 0)$, $x \geq 2$.

Theorem 1 *Suppose that at least one agent has a strictly quasi-concave statistic, while the rest have quasi-concave statistic. A mechanism is strategy-proof and symmetric if and only if such mechanism is almost-fixed.*

4.4 Additive statistic

In this section, we consider the case of a utility function with externality factor that is additive. That is, $f(x_T) = \gamma_T \sum_{i\in N} x_i$. For instance, the sum or average utility functions discussed above.

Definition 8 (S-sharing monotonic mechanism) *Given $S \subsetneq N$, consider a function $f^S : \Pi_{j\in S} 2^{N\setminus j} \to \Delta(N \setminus S)$ such that, for any reports T^i and $\tilde{T}^i$ of the agent i, and reports T^{-i} of the agents in $S \setminus i$:*

$$\sum_{j\in T^i} f_j^S(T^i, T^{-i}) \geq \sum_{j\in T^i} f_j^S(\tilde{T}^i, T^{-i}).$$

The S-sharing monotonic mechanism allocates

$$F(S_1, \ldots, S_n) = f^S([S_j]_{j\in S}).$$

An S-sharing monotonic mechanism is an S-sharing mechanism such that when an agent report to be connected to T, this coalition received the largest overall transfer relative to any other reported coalition.
If the externality factor is additive, then the S-sharing monotonic mechanisms are strategyproof. This is because the payment of the agents in T

Example 3

- *The following is an i-sharing monotonic mechanism: Assign one unit of the good to i and divide it equally between his neighbors.*
- *The following is an S-sharing monotonic mechanism: Assign one unit of the good to the agents in S and divide it equally to the agents from $N \setminus S$ who are in the interesection of the reports of the agents in S (or equally to $N \setminus S$ if the intersection is empty).*

Definition 9 (Cross-monotonic mechanism) *Given a collection of S-sharing monotonic mechanisms $\{F^S\}_{S\in 2^N\setminus N}$ and a vector of allocation of the resource to every coalition $y \in \Delta(2^N \setminus \{N\})$, a cross-monotonic mechanisms SEP is defined as:*

$$SEP(T) = \sum_{S\in 2^N\setminus N} y_S f^S(T_S)$$

Note that, for additive utility functions, strategyproofness is preserved under convex combinations of strategyproof mechanisms. Since cross-monotonic mechanisms are convex combinations of S-sharing mechanisms, then they are strategyproof.

Theorem 2 *Suppose that all of the agents have an additive statistic. A mechanism is strategyproof for any utility function if and only if it is cross-monotonic.*

Example 4 (Symmetric cross-monotonic mechanisms) *Let an S-sharing symmetric mechanism divides one unit of the good among the agents outside S*

who are in the intersection of the reports of the agents in S. If the intersection in empty then we divide equally among the agents in $N \setminus S$.
For instance, the i-sharing symmetric mechanism will divide one unit of the good amoung the neighbors of i. Clearly, an S-sharing symmetric mechanism is an S-monotonic mechanism.
The symmetric cross-mononotic mechanisms can be found by taking convex combination of S-sharing symmetric mechanisms if adding over all coalitions of the same size, and coalitions of the same size get the same weight.

Corollary 3 *Consider a fix externality factor f that is additive. A mechanism is strategyproof and symmetric for any utility function with externality factor f if and only if it is a convex combination of S-sharing symmetric mechanisms with the same weight over coalitions of the same size.*

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