Between-group contests over group-specific public goods with within-group fragmentation

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Abstract

We model a contest between two groups of equal population size over the division of a group-specific public good. Each group is fragmented into sub-groups. Each sub-group allocates effort between production and contestation. There is perfect coordination within sub-groups, but sub-groups cannot coordinate with one another. All sub-groups choose effort allocations simultaneously. We find that aggregate rent-seeking rises, social welfare falls, and *both* communities are worse off when the dominant sub-groups within both communities increase their population shares relative to the respective average sub-group population. Any unilateral increase in fragmentation within a group reduces conflict andmakes its opponent better off. The fragmenting community itself may however be better off as well, even though its share of the public good falls. Thus, a reduced share of public good provisioning cannot be used to infer a negative welfare implication for the losing community.

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1. Introduction

When two communities contest one another for a politically determined division of some public good, how does coordination-inhibiting internal fragmentation (intuitively, 'factionalization') within each community affect the outcome? Can greater fragmentation generate aggregate welfare gains? Does greater asymmetry in internal fragmentation between the communities, i.e. one community getting fragmented into more factions even as its opponent consolidates into fewer, expand or reduce conflict, measured as total social wastage due to rent-seeking rather than production? How does it affect aggregate welfare? Does a community benefit when its opponent fragments? Perhaps most interestingly, can greater internal fragmentation be to the benefit of the fragmenting community itself? This paper addresses these issues.

The questions we pose acquire particular importance in the context of the revival of ethnic, in the general sense of non-class (especially religious) identities in recent years, and the consequent increasing salience of mass political conflict, both among rival ethnic identities and between religious and secular identities, over *extra-economic* aspects of life. These inter-group conflicts often occur over items of within-group non-rival and non-excludable intrinsic benefit ('culture/religion') rather than income, consumption of which imposescollective costs on members of another group. As Dasgupta and Kanbur (2005a, 2005b, 2007) have argued,identity groups can be visualized as held together by the common consumption of certain forms of group-specific public goods, which do not yield monetary benefits, but are deemed intrinsically valuable by all group members. These very same group-specific public goods may however turn out to bepublic 'bads' for another identity group. Esteban and Ray (2011) and Dasgupta (2017) have accordingly modeled such collective consumption as generating conflict between groups. To illustrate with a concrete example, one group may pressurize the state to impose a common secular legal code regarding marriage and sexual behavior over the entire country, while another wishes to impose religious (e.g. Sharia) law. The outcome is a composite legal code exhibiting both secular and religious features, with their shares (proportions) determined by the political efforts deployed by the contending groups. Particular ethnic, especially religious, communitiestypically espouse a set of core values and norms with regard to the private behavior of individuals. This is especially so in matters of marriage, sexual behavior, divorce, abortion, inheritance, dietary habits, religious practices and dress codes for women. Greater enforcement of such values and norms among the population at large then generates greater non-rival and non-excludable psychic benefits for those espousing them. Different communitieslobby authorities to act in their favor, for and against the status quo, or engage in direct action. Examples include mass protests for and against cow slaughter (in India), public statues (in Bangladesh), perceived blasphemy (in Pakistan and Europe) or banning of polygyny and juvenile marriage (in many Muslim countries). Direct action may also involve the mobilization of activists to physically destroy places of worship or monuments belonging to other communities, terrorize other communities to force them to desist from observing certain practices (e.g. consumption of beef or alcohol) or rituals (e.g. the routine bombing of Shia processions and Sufi shrines by Salafists in Iraq and Pakistan), and countervailing efforts to defend or expand. For analytical purposes, all such inter-group conflicts may be thought of as occurring over the division of a public good between communities, the benefits of which are mutually exclusive between communities, but both non-rivalrous and non-excludable within a community. Of course, inter-community conflicts may also occur over the sharing of state investment in public goods of localized benefit more traditionally interpreted, such as schools, roads, hospitals, security, public art, local anti-pollution measuresetc., when the communities exhibit locational segregation. This second interpretation, in terms of politicalconflict over jurisdiction-specific local public goods, has been the one originally deployed in the literature (e.g. Katz *et al*. 1990, Ursprung 1990 and Gradstein 1993), and equally compatible with our analysis.

Typically, in large diverse societies, two groups contesting one another also exhibit non-class internal cleavages.Religious Hindus demanding greater restrictions on cow slaughter in India are fragmented along castelines, while the opposition to them includes secularists, Muslims, Christians and Buddhists. Local Pashtuns, Pakistani Pashtuns, non-Pashtun Pakistanis and Arab volunteers are all well-represented among Taliban fighters in Afghanistan, while local Pashtuns, Tajiks, Hazaras and Uzbeks have all constituted large components of government forces since the overthrow of the Taliban in 2001.Both Christians and Muslims in Nigeria are internally fragmented along ethno-linguistic triballines. This common phenomenon of internal divisions (despite common interests) within both contending groups, which can be expected to impede internal coordination in conflict with the opposing group, motivates us.

A large literature has developed in recent years on how ethnic fragmentation (measured by the ethnic fractionalization index) and ethnic polarization affect social conflict.[1]However, internal cleavages within contending groups have not received attention in this literature. Nor has the question been addressed in the literature on collective action stemming from the seminal contribution by Olson (1965) and synthesized by Esteban and Ray (2001), which investigates the consequences of size asymmetry between contending groups. Likewise, the literature on rent-seeking specifically over group-specific public goods (e.g. Katz *et al.* 1990,Ursprung 1990, Gradstein 1993, Riaz*et al*. 1995, Baik 2008, Epstein and Mealem 2009, Lee 2012, Kolmar and Rommeswinkel 2013 and Chowdhury *et al.* 2013)appears not

[1] See Montalvo and Reynal-Querol (2012) for a recent survey.

to have addressed the issue.[2] A parallel literature developing from the seminal contributions by Alesina*et al.* (1999) and Miguel and Gugerty (2005) emphasizes the (typically) negative impact of ethnic heterogeneity on local public good provision, but conflict among groups over sharing of such goods does not figure in this literature. Our paper relates to all these literatures, while belonging most closely to that on rent-seeking over group-specific public goods in its formal structure.

We consider a situation where two communities of equal population size contest the division of a public good in standard Tullock (1980) fashion. Each community is fragmented into a finite number (two or more) of sub-groups. The number of constituent sub-groups may vary across communities, but the population share of the largest sub-group within a community is some constant proportion of the average sub-group size within that community. Thus, the sub-groups may, but need not, be of equal size within a community. Each individual is endowed with one unit of effort which she allocates between production for private consumption and contestation over the public good. All individuals have identical preferences overprivate and public consumption given by an additively separable utility function. The utility function has a linear component denoting benefit from private consumption. It also has a non-linear component denoting disutility from the opposing community's share of the public good, given specifically by an increasing, convex and iso-elastic disutility function. This feature of endogenous marginal valuation of the public good distinguishes our model from most of the literature. The linear utility function commonly used in the literature (e.g.Katz *et al.* 1990, Baik 2008, Cheikbossian 2008, Lee 2012, and Kolmar and Rommeswinkel 2013)is one limiting case in our model, and our utility specification is in turn a sub-class of the general quasi-linear utility function used by Gradstein (1993).[3]There is perfect coordination within each sub-group, so that each sub-group can be modeled as an individual endowed with effort equal to its population share, maximizing a utility function that is the simple aggregate of the utilities of all its

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[2]Katz *et al.* (1990) investigate the consequences of asymmetry in size and wealth between groups with and without risk aversion.Ursprung (1990) concentrates on rent-dissipation. Gradstein (1993) focuses on the comparison between politically determined public provision and private provision of jurisdiction-specific local public goods. Riaz*et al.* (1995) consider a general expected utility set-up with von Neumann Morgenstern utility functions and highlight the consequences of changes in relative group size. Baik (2008) examines free-riding with preference differences among group members with a linear utility function. Epstein and Mealem (2009) also focus on free-riding. Lee (2012) offers a 'weakest-link' contest model over a group-specific public good, while Kolmar and Rommeswinkel (2013) develop the implications of a contest success function where individual group members' contest efforts aggregate to group conflict effort in a constant elasticity of substitution fashion. Chowdhury *et al.* (2013) examine free riding in 'best-shot' group contests over public goods.A broadly related contribution is by Cheikbossian (2008), who develops a linear utility model with preference difference and size asymmetry between groups, and examines how these factors affect politically determined public good provision. The public good however is not group-specific in his model.

[3]Esteban and Ray (2001) also deploy a general quasi-linear utility function. However, the benefit from the public good is the linear component in their utility function, whereas it is the benefit from the private good that is the linear component in ours. The benefit from the public good is assumed constant by Chowdhury *et al.* (2013) as well.

members. However, sub-groups within a community cannot coordinate with one another, intuitively reflecting the consequences of linguistic, sectarian, ethnic or caste cleavages within the community. All sub-groups choose their effort allocations simultaneously. Thus, our model bears a family resemblance with those advanced in the literature on simultaneous internal versus external rent-seeking (e.g. Hausken 2005, Munster 2007, Dasgupta 2009 and Choi *et al*. 2016), but differs fundamentally from them in two major ways. First, conflicts occur in these models only over sharing of private goods, whereas the sharing of a public good constitutes our site of conflict. Second, unlike these contributions, there is no explicit conflict among constituent sub-groups within a community in our model. Instead, internal cleavages affect external conflict solely through their impact on within-group coordination.

Examining the Nash equilibrium, we find the following. The group that is more fragmented internally receives the lower share of the public good. Given the extent of inter-community asymmetry in internal fragmentation, measured by the relative number of sub-groups, greater overall fragmentation (i.e. an increase in the total number of sub-groups in society) reduces conflict and improves the aggregate welfare of both communities. Conversely, given overall fragmentation, greater inter-community asymmetry in internal fragmentation increases conflict and reduces social welfare. It improves the welfare of the consolidating community and reduces that of the fragmenting community when the elasticity of the disutility function is sufficiently low. The opposite however holds when it is sufficiently high. When the disutility function is ofsufficientlylow elasticity, greater overall fragmentation implies lower conflict. Aggregate rent-seeking rises, social welfare falls, and *both* communities are worse off when the dominant sub-groups within both communities increase their population shares relative to the respective average sub-group population. Any unilateral increase in fragmentation within a community reduces total conflict and makes its opponent better off. Aggregate social welfare rises at both high and low elasticities of the disutility function. Strikingly, the fragmenting community itself is better off as well when the disutility function is sufficiently elastic, though it is worse off when the latter has a low elasticity. The higher the relative dominance of the dominant sub-groups, the more likely it is that a unilateral increase in fragmentation within a community will make both communities better off.Thus, a reduced share of public good provisioning cannot be used to infer a negative welfare implication for the losing community: an aggregate welfare improvement for that community is consistent with such a reduction.

The intuition behind these findings is the following. Greater unilateral fragmentation within a community, by reducing internalization of community-wide benefits from the public good, reduces its political effort allocation. This increases its output (and thus private consumption), which has a positive effect on the community's aggregate welfare. It also reduces the fragmenting community's share of the public good. The positive effect prevails when the disutility function is sufficiently elastic. For the opponent of the fragmenting community,the positive effect of receiving a higher share always dominates.

Section 2 sets up the model. Our comparative static results are presented in Section 3. Section 4 discusses some possible variants and generalizations of the model. Section 5 concludes. Detailed proofs of our formal results are provided in an appendix.

2. The model

Consider a society with a continuum of population consisting of two communities M and N, with equal population shares. Total population in the society is of measure 2, so that the size of each community is 1. Each community $c \in \{M, N\}$ is *fragmented*, i.e. partitioned into, a finite number of subgroups $g_c \in \{2,3,\ldots\}$, though g_M need not be equal to g_N.We denote the total number of sub-groups in society by $G \equiv g_M + g_N$; to make the analysis non-trivial, we assume $5 \leq G.G$ measures overall fragmentation in society. The population size of a sub-group $j \in \{1,2,\ldots,g_c\}$ in community c is p_{jc}, so that $\sum_{j=1}^{g_c} p_{jc} = 1$. We shall assume that the population size of the largest sub-group within a community, i.e. $Max\{p_{1c},\ldots,p_{g_c c}\}$, is $\frac{\gamma}{g_c}$, where $\gamma \in [1,2)$. The special case$\gamma = 1$ obtains when each community is equally divided among its constituent subgroups. The higher the value of γ, the greater the population share of the largest sub-group (of which there may be more than one) relative to the average, i.e. the greater its relative dominance within the community. Notice that we put no restrictions on the size of any sub-group except the largest.

Each individual in society is endowed with one unit of effort, which she can allocate between production and rent-seeking activities to influence the cross-community division of one unit of a public good. Each sub-group within a community can perfectly coordinate its internal effort allocation decisions, so that it can be modeled as an individual endowed with effort p_{jc} maximizing the total utility of that sub-group. However, sub-groups can neither coordinate effort allocation decisions with, nor internalize benefits accruing to, other sub-groups. Thus, there is complete centralization within each sub-group, but complete decentralization across sub-groups. Marginal productivity of effort (in output generating activities) is $k > 0$. Total amount of effort allocated to political (i.e. rent-seeking) efforts by a sub-group j within community c is denoted by $x_{jc} \in [0, p_{jc}]$, so that the community as a whole allocates total political effort $x_c \equiv \sum_{j=1}^{g_c} x_{jc}$. Outputs produced by the sub-group and the community are therefore, respectively, $k(p_{jc} - x_{jc})$ and $k(1 - x_c)$. Total political effort in society is given by $X \equiv x_M + x_N$, which also provides the measure of social resource wastage due to diversion of effort to rent-seeking activities instead of production. Given any community $c \in \{M, N\}$, we shall refer to the other community as $-c$. The share of the public good received by community $c$is given by the standard Tullock (1980) contest success function:

$$\lambda_c = \frac{x_c}{X} \text{ if } X > 0, \text{ and } \lambda_c = \frac{1}{2} \text{ otherwise.} \tag{1}$$

All members of a community have identical utility functions. Pay-off to a sub-group j in community c is the aggregate of its members' utilities, and is given by the sub-group utility function:

$$\pi_{jc} = k(p_{jc} - x_{jc}) - (1 - \lambda_c)^\alpha p_{jc}, \tag{2}$$

where $\alpha > 1$. Hence, total pay-off to a community c is given by:

$$\pi_c \equiv \sum_{j=1}^{g_c} \pi_{jc} = k(1 - x_c) - (1 - \lambda_c)^\alpha. \tag{3}$$

The parameter α measures the elasticity of the disutility function $(1 - \lambda_c)^\alpha$ with respect to the share of the public good lost. The higher the value ofα, the lower the monetary, i.e. private consumption, equivalent of the utility loss due to the other community receiving a given (partial) share of the public good. In this sense, a higher αintuitively implies lower importance of losing part of the public good. The utility function converges to a linear form in the limiting case of unit elasticity ($\alpha = 1$). It converges to the case of the public good losing its group-specific character in the other limiting case of infinite elasticity ($\alpha \to \infty$).

All sub-groups choose their political effort allocation x_{jc} simultaneously, so as to maximize the sub-group utility function given by (2), subject to the contest success function (1) and the sub-group effort constraint $x_{jc} \in [0, p_{jc}]$. The first order conditions yield:

$$\frac{kg_c}{\alpha\gamma} = (1 - \lambda_c)^{\alpha - 1} \left(\frac{x_{-c}}{X^2}\right). \tag{4}$$

From (4), using (1), we get:

$$x_{-c} = \left(\frac{k}{\alpha\gamma}\right)^{\frac{1}{\alpha}} g_c^{\frac{1}{\alpha}} X^{\frac{1+\alpha}{\alpha}}, \tag{5}$$

$$X = \frac{\left(\frac{\alpha\gamma}{k}\right)}{[(g_c)^{\frac{1}{\alpha}} + (g_{-c})^{\frac{1}{\alpha}}]^\alpha}; \tag{6}$$

$$x_{-c} = \frac{\left(\frac{\alpha\gamma}{k}\right) g_c^{\frac{1}{\alpha}}}{[(g_c)^{\frac{1}{\alpha}} + (g_{-c})^{\frac{1}{\alpha}}]^{\alpha+1}}; \tag{7}$$

$$\lambda_c = \frac{1}{1+\left(\frac{g_c}{g_{-c}}\right)^{\frac{1}{\alpha}}}. \tag{8}$$

Remark 1. From (8), $\lambda_c > \frac{1}{2}$ iff $g_{-c} > g_c$, and λ_c is decreasing in$\frac{g_c}{g_{-c}}$. Thus, the community that is internally more fragmented gets the lower share of the public good. The greater its internal fragmentation relative to that of its opponent, the less successful it is in the rent-seeking contest.Hence, the community with the smaller-sized dominant (largest) sub-group receives the lower share, but whether a community has one or more dominant sub-groups makes no difference to its equilibrium share. Since $\frac{\lambda_c}{1-\lambda_c} = \left(\frac{g_{-c}}{g_c}\right)^{\frac{1}{\alpha}}$, given relative fragmentation, the inter-community ownership division of the public good is more equal, the more elastic the disutility function.Hence, greater disutility elasticity makes shares less sensitive to relative fragmentation. Only the largest sub-group(s) of a community allocate effort to rent-seeking; all smaller ones free-ride and allocate their effort entirely to production.[4]In cases where there are multiple largest sub-groups within a community, of which all sub-groups being of equal size is one, there are multiple equilibria, so that political effort allocation of individual largest sub-groups is indeterminate. However, the total political effort allocation by a community is always uniquely determinate. Total rent-seeking effort declines as productivity of effort in output-generating activities rises.

3. Intra-community fragmentation, rent-seeking and social welfare

How do aggregate social wastage due to rent-seeking, aggregate social welfare, and its distribution between communities depend on intra-community fragmentation? We now proceed to answer these questions. For convenience, we recall that, by construction,$[g_M, g_N \geq 2]$, $\left[\frac{2}{G-2} \leq \frac{g_M}{g_N} \leq \frac{G-2}{2}\right]$, $[0 \leq |g_M - g_N| \leq G - 4\}]$, and$[5 \leq G]$. All our formal statements below (Propositions 1 and 2 and Corollary 1) are to be read as implicitly referring only to variable values and changes thereinthat satisfy the restrictions specified above.

[4]This is the counterpart in our model of the result derived by Baik (2008) for his model of a group-contest for a group-specific public good that,in each group, only thehighest-valuation players expend positive effort and the rest expend zero effort. That contribution uses a linear utility function and ignores within-group fragmentation.

Proposition 1. (i)X decreases with any increase in g_c. X decreases with any increase in Ggiven either g_M/g_N or $|g_M - g_N|$; it increases with any increase in $|g_M - g_N|$, given G.

(ii) Given any pair G_1, G_2, $G_1 > G_2$, there exists $\varepsilon(G_1, G_2) > 0$ such that, for all $\alpha \in (1, 1 + \varepsilon(G_1, G_2))$, X is lower under G_1 relative to G_2.

(iii) Given any $\frac{g_c}{g_{-c}} \in (0,1]$, there exists $\bar{\bar{\alpha}}\left(\frac{g_c}{g_{-c}}\right) \in (1, \infty)$ such that $\frac{\partial X}{\partial \alpha} < 0$ if $\alpha > \bar{\bar{\alpha}}\left(\frac{g_c}{g_{-c}}\right)$.$\bar{\bar{\alpha}}\left(\frac{g_c}{g_{-c}}\right)$ falls as $\frac{g_c}{g_{-c}}$ rises, and when $\frac{g_c}{g_{-c}} = 1, \frac{\partial X}{\partial \alpha} > 0$ for all $\alpha \in (1, \bar{\bar{\alpha}}(1))$.

Proof. See the Appendix.

By Proposition 1(i),a unilateral increase in fragmentation within either community reduces conflict. Given inter-community asymmetry in internal fragmentation, measured either as the absolute differencein the number of sub-groups, or in relative terms, greater overall fragmentation reduces rent-seeking. Conversely, given overall fragmentation, greater inter-community asymmetry in internal fragmentation (i.e. a rise in the absolute difference in the number of sub-groups) increases rent-seeking.By (8), such a rise also leads to greater inequality in the division of the public good. Proposition 1(ii) implies that, when the elasticity of the disutility function is sufficiently low,greater overall fragmentation implies lower aggregate rent-seeking, regardless of how that fragmentation is distributed between the two communities. By Proposition 1(iii), given any level of relative fragmentation, there exists a certain threshold level of disutility elasticity, above which more elastic disutility monotonically implies lower conflict. When both communities are equally fragmented, it is also the case that more elastic disutility will monotonically imply higher conflict below this threshold level.

The next question we turn to is the impact of intra-community fragmentation on social welfare. Using (3), aggregate social welfare, i.e. the sum of the two communities' pay-offs, is given by:

$$\pi \quad = k(2 - X) - [(1 - \lambda_M)^\alpha + {\lambda_M}^\alpha]. \tag{9}$$

The following conclusions can then be drawn in light of Proposition 1.

Corollary 1.(i) Given g_M/g_N, any increase in G increases social welfare; furthermore, given G, any increase in $|g_M - g_N|$ reduces social welfare.

(ii) Given any pair G_1, G_2, $G_1 > G_2$, there exists $\varepsilon(G_1, G_2) > 0$ such that, for all $\alpha \in (1, 1 + \varepsilon(G_1, G_2))$, social welfare is higher under G_1 relative to G_2.

Proof. See the Appendix.

Corollary 1(i) implies that,given the extent of inter-community asymmetry in internal fragmentation, measured in ratio terms, greater overall fragmentation improves social welfare by reducing rent-seeking. Given overall fragmentation, greater inter-community asymmetry in internal fragmentation reduces social welfare both by increasing rent-seeking and generating a more unequal distribution of the public good. Corollary 1(ii) implies that, when the disutility function exhibits sufficiently low elasticity, greater overall fragmentation in society implies higher aggregate social welfare due to reduced rent-seeking.

Since, by Proposition 1, aggregate conflict falls with unilateral fragmentation, i.e. a rise in g_c, given g_{-c}, and since such a rise generates a more equal division of the public good when $g_c < g_{-c}$ (Remark 1), it is clear that greater unilateral fragmentation within the less fragmented community must increase total social welfare. By Corollary 1(ii), greater unilateral fragmentation within the more fragmented community will also increase aggregate social welfare when the elasticity of the disutility function is sufficiently low. As we shall show below, greater unilateral fragmentation within the more fragmented community must increase social welfare when the elasticity of the disutility function is sufficiently high as well. However, whether it is possible to have a reduction in social welfare from greater unilateral fragmentation within the more fragmented community for an intermediate range of elasticity values appears an open question.[5]

Remark 2. By Corollary 1(i), the social welfare maximizing combination of intra-communal fragmentation levels is given by $g_M = g_N$ when G is even, and by $g_M = \frac{G+1}{2}$ otherwise. It can be checked from (6), (8), (11) and (14) below that aggregate rent-seeking rises, social welfare falls, and *both*communities are worse off as γ increases, i.e., as the dominant sub-groups within both communities increase their population shares relative to the average sub-group population ($\frac{1}{g_c}$). Thus, given any level of fragmentation (any $\langle g_M, g_N \rangle$), the aggregate welfare-maximizing population distribution across sub-

[5]A general proof to the contrary has proved elusive so far, but so has an example of such a reduction.

groups is that of equality within each community; such a population distribution also maximizes the welfare of both communities individually, given $\langle g_M, g_N \rangle$.

Our next set of results characterizes how the distribution of welfare between communities is affected by intra-community fragmentation. Recall that aggregate welfare of a community is measured simply as the sum of the pay-offs of its constituent sub-groups, as noted in (3) above.

Proposition 2.

(i) Given $\frac{g_M}{g_N}$, any increase in *G* increases both π_M and π_N; given *G*, there exist$\breve{\alpha} > 1$ and $\tilde{\alpha} > \breve{\alpha}$ such that, for all $c \in \{M, N\}$, any increase in $\frac{g_{-c}}{g_c}$increases (resp. decreases)π_c if $\alpha \in (1, \breve{\alpha})$ (resp. $\alpha > \tilde{\alpha}$).

(ii) Given g_c, any increase in g_{-c}increases π_c.

(iii) Given anyg_{-c}, (a) any increase in g_c decreases π_c if $\left[\alpha < \frac{4}{\gamma} - 1\right]$; and (b) for every $\overline{g} > g_{-c}$, any increase in $g_c$over $[2, \overline{g}]$ increasesπ_c if $\left[\alpha > \frac{2\overline{g}}{\gamma} - 1\right]$.

Proof. See the Appendix.

By Proposition 2(i), an equi-proportionate increase in fragmentation within both communities implies an aggregate welfare improvement for both. However, given total fragmentation, greater asymmetry in fragmentation across communities improves the welfare of the consolidating community and reduces that of the fragmenting community when the disutility elasticity is sufficiently low. The opposite holds when it is sufficiently high. By Proposition 2(ii) any unilateral increase in fragmentation within a community makes its opponent better off. However, by Proposition 2(iii), the fragmenting community itself is better off as well when the disutility function is sufficiently elastic. Thus, in this case, a unilateral fragmentation within one community leads to an aggregate welfare improvement for both communities. A unilateral increase in fragmentation makes the fragmenting community worse off ifthe elasticity of the disutility function is sufficiently low. In the first case, the relevant threshold elasticity is lower, the higher the population share of the dominant sub-group within a community relative to the average sub-group population size therein. In the second case, the relevant threshold elasticity ishigher, the lower the relative population share of the dominant sub-group within a community. Thus, the higher the relative dominance of the dominant sub-groups, the more likely it is that a unilateral increase in fragmentation within a community will make both communities better off. Between the two threshold elasticities specified in Proposition 2(i), the welfare distribution can either increase or decrease, depending on the exact parameter values, initial situation and the magnitude of the change, so that a general conclusion cannot be drawn. The same holds for the interval between the threshold elasticities specified in Proposition 2(iii).

Remark 3. Since $\gamma \in [1,2)$, Proposition 2(iii) implies greater unilateral fragmentation must make the fragmenting community worse off when preferences are linear, as is commonly assumed in the literature (e.g. Katz *et al.* 1990, Baik 2008, Cheikbossian 2008, Lee 2012 and Kolmar and Rommeswinkel 2013).

The intuition behind these findings is the following. Greater unilateral fragmentation within a community, by reducing internalization of community-wide benefits from the public good, reduces its political effort allocation. Thisincreases its output, which has a positive effect on that community's aggregate welfare. The larger the relative population share of the dominant sub-group within a community,the larger this positive effect (recall (7)). Of course, this also has a negative consequence: it reduces the fragmenting community's share of the public good.The negative effect dominates at low values of α(and/or γ), while the positive effect dominates at high values of one or both of these variables. The same mechanism drives the findings when one community fragments and the other consolidates in a compensating fashion, so as to keep overall fragmentation constant. For the opponent of the fragmenting community, the positive effect of receiving a higher share always dominates (notice (14) and (15) below).

4. Variants

We now discuss some variants of our model that can be easily incorporated in our analysis.

4.1. Discrete population

We have assumed a continuum of population in our model. This is purely for convenience of exposition. One can have a discrete population version of the model with total population given by some even number $P \geq 4$, with each community having $\frac{P}{2}$ members. Under the assumption of equal population shares for all sub-groups within a community, this version is especially useful for comparing the two polar cases of complete centralization ($g_c = 1$) and complete decentralization ($g_c = \frac{P}{2}$) within a community. Complete centralization within both communities has sometimes been studied in contraposition to complete decentralization within both communities in the literature (e.g. Cheikbossian 2008). It can be shown, in a manner exactly analogous to that developed earlier, that aggregate conflict is lower, aggregate social welfare higher, and both communities better off, if both communities are completely decentralized, relative to the case where both are completely centralized. As in our benchmark model, unilateral fragmentation by a community makes its opponent better off. The fragmenting community itself is better off if disutility is sufficiently elastic. Specifically, it can be shown that, given any $g_{-c} \in [1, \frac{P}{2}]$, π_c is increasing in g_c over $[1, \frac{P}{2}]$ if $\alpha > P - 1$. All other substantive findings continue to hold as well.

4.2. Preference differences across sub-groups

With equal population size $(\frac{1}{g_c})$ across sub-groups, the model can be reinterpreted to include preference differences across subgroups within a community. We can amend the sub-group pay-off function in (2) to:

$$\pi_{jc} = k(p_{jc} - x_{jc}) - (1 - \lambda_c)^{\alpha} \left(\frac{\gamma_{jc}}{g_c}\right).$$

The sub-group specific parameters $\gamma_{jc} \in (0,2)$ then capture possible preference differences in terms of valuation of the public good across sub-groups within a community. Defining γ as the maximum value of the sub-group preference parameter, we may assume that this maximum remains constant across communities and regardless of the level of fragmentation (i.e., for all $c \in \{M, F\}$ and all $g_c \geq 2$, $\gamma = Max\{\gamma_{1c}, \dots, \gamma_{g_c c}\}$). Our entire analysis remains unchanged under this alternative interpretation, which also implies a more permissive parameter restriction $\gamma \in (0,2)$.

4.3. General utility function and contest success function

Our quasi-linear specification of the utility function in (2) can be generalized to the form:

$$\pi_{jc} = k(p_{jc} - x_{jc})^{\beta} - (1 - \lambda_c)^{\alpha} p_{jc};$$

where $\beta \in (0,1]$. Likewise, the contest success function in (1) can be generalized to the form:

$$\lambda_c = \frac{x_c{}^{\varepsilon}}{x_c{}^{\varepsilon} + x_{-c}{}^{\varepsilon}};$$

where $\varepsilon \in (0,1]$. It is intuitively evident that, while greatly increasing the notational burden, these generalizations do not yield any additional substantive insights.

5. Concluding remarks

In this paper, we have examined the consequences of coordination-inhibiting within-community cleavages on conflict between communities over sharing of a public good. We have found that an increase in such divisions may be socially beneficial, in that it may reduce inter-community conflict and increase social welfare. Greater unilateral fragmentation within its opponent makes a community better off. Greater unilateral fragmentation may make the fragmenting community itself better off in the aggregate as well, even though it makes that community receive a lower share of the public good. Thus, the fact of losing out in the public goods contest cannot be used to infer welfare implications: the losing/fragmenting community may be better off nonetheless. Sub-communal identity politics, such as caste exclusivism among Hindus in India and ethno-linguistic assertion among Muslims and Christians in large parts of Africa,seek to highlight and emphasize ethnic, linguistic, regional or caste divisions and distinctions within a broader religious community. Our analysis suggests the intriguing possibility that such internally 'divisive' politics may actually work to the overall benefit of the broader community when it is engaged in conflicts with another community, even if such politics tilt the outcome of the inter-community conflict against the former. Furthermore, we have found that inter-group conflict rises as the dominant sub-groups within both communities increase their population shares relative to the average sub-group population. This is a hypothesis that can be usefully confronted with empirical evidence.

The literature on simultaneous between and within group contests (e.g. Hausken 2005, Munster 2007, Dasgupta 2009 and Choi *et al*. 2016) typically modelsconflicts solely over private goods. One may however visualize a scenario where two communities contest the division of a public good even asall constituent sub-groups individually contest the distribution of private consumption alongside their engagement in production.One may examine the impact of within-group fragmentation in such a context.Second, one may use alternatives to our perfect substitutes summative specification for each community's aggregate group conflict effort, such as a constant elasticity of substitution aggregation (Kolmar and Rommeswinkel 2013), the 'best-shot' specification (Chowdhury*et al.* 2013) or the weakest-link formulation (Lee 2012). The consequences of within-group fragmentation in such contests over group-specific public goods constitute another promising avenue of future enquiry. Lastly, by focusing on shares rather than success probabilities, we have abstracted from risk-related issues. Explicit incorporation of risk aversion, and of wealth effects on risk aversion (along the lines, for example, of Katz *et al.* 1990), may yield useful insights. We look forward to these and other extensions in future work.

Appendix

Proof of Proposition 1. That X decreases with any increase in g_c, and that it decreases with any increase in G, given g_M/g_N, follow from (6). Let $\Delta \equiv g_c - \frac{G}{2} \geq 0$ for some $c \in \{M, N\}$. Consider the term $[(g_M)^{\frac{1}{\alpha}} + (g_N)^{\frac{1}{\alpha}}]$, which can be rewritten as: $\left[\left(\Delta + \frac{G}{2}\right)^{\frac{1}{\alpha}} + \left(\frac{G}{2} - \Delta\right)^{\frac{1}{\alpha}}\right]$. Since $\alpha > 1$, $\left[\left(\Delta + \frac{G}{2}\right)^{\frac{1}{\alpha}} + \left(\frac{G}{2} - \Delta\right)^{\frac{1}{\alpha}}\right]$ is falling in Δ , and rising in G. Proposition 1(i) follows in light of (6). Furthermore, $\lim_{\alpha \to 1}\left[\left(\Delta + \frac{G}{2}\right)^{\frac{1}{\alpha}} + \left(\frac{G}{2} - \Delta\right)^{\frac{1}{\alpha}}\right] = G$, so that $\lim_{\alpha \to 1} X = \left(\frac{\gamma}{Gk}\right)$. Proposition 1(ii)follows by continuity and (6).

Now note that $\frac{\partial\left((g_c)^{\frac{1}{\alpha}}\right)}{\partial \alpha} = \frac{-(g_c)^{\frac{1}{\alpha}} \ln g_c}{\alpha^2}$. Let $Z \equiv [(g_c)^{\frac{1}{\alpha}} + (g_{-c})^{\frac{1}{\alpha}}]^{-\alpha}$. Then:

$$\frac{1}{Z}\frac{\partial Z}{\partial \alpha} = -\ln[(g_c)^{\frac{1}{\alpha}} + (g_{-c})^{\frac{1}{\alpha}}] + \frac{(g_c)^{\frac{1}{\alpha}} \ln g_c + (g_{-c})^{\frac{1}{\alpha}} \ln g_{-c}}{\alpha[(g_c)^{\frac{1}{\alpha}} + (g_{-c})^{\frac{1}{\alpha}}]}.$$

Assume, without loss of generality, that $g_{-c} \geq g_c$. Hence,

$$-\ln[(g_c)^{\frac{1}{\alpha}} + (g_{-c})^{\frac{1}{\alpha}}] + \ln(g_c)^{\frac{1}{\alpha}} \leq \frac{1}{Z}\frac{\partial Z}{\partial \alpha} \leq -\ln[(g_c)^{\frac{1}{\alpha}} + (g_{-c})^{\frac{1}{\alpha}}] + \ln(g_{-c})^{\frac{1}{\alpha}} = \ln\left(\frac{(g_{-c})^{\frac{1}{\alpha}}}{(g_c)^{\frac{1}{\alpha}} + (g_{-c})^{\frac{1}{\alpha}}}\right) < 0.$$

Recalling that $Z \equiv [(g_c)^{\frac{1}{\alpha}} + (g_{-c})^{\frac{1}{\alpha}}]^{-\alpha}$, the inequality above can be rewritten to yield:

$$\left[Z \ln(Z g_c)^{\frac{1}{\alpha}} \leq \frac{\partial Z}{\partial \alpha} \leq Z \ln(Z g_{-c})^{\frac{1}{\alpha}} < 0\right].$$

Since, by (6), $\left[\left(\frac{k}{\gamma}\right) X = \alpha Z\right]$, so that $\left[\left(\frac{k}{\gamma}\right)\frac{\partial X}{\partial \alpha} = Z + \alpha \frac{\partial Z}{\partial \alpha}\right]$, we thus get:

$$Z(1 + \ln(Z g_c)) \leq \frac{k}{\gamma}\frac{\partial X}{\partial \alpha} \leq Z(1 + \ln(Z g_{-c})).$$

Hence, $\frac{\partial X}{\partial \alpha} < 0$ if $\ln(Zg_{-c}) < -1$, i.e. if $Zg_{-c} < \frac{1}{e} < \frac{1}{2}$. Notice now that $Zg_{-c} = \left(\frac{1}{\left(\frac{g_c}{g_{-c}}\right)^{\frac{1}{\alpha}}+1}\right)^{\alpha}$. Since $\frac{g_c}{g_{-c}} \leq 1$, $\lim_{\alpha\to 1} Zg_{-c} \geq \frac{1}{2}$, and, given any $\frac{g_c}{g_{-c}} \in (0,1]$, $\lim_{\alpha\to\infty} Zg_{-c} = 0$. Furthermore, $\frac{\partial Z}{\partial \alpha} < 0$. Hence, there must exist $\bar{\bar{\alpha}}\left(\frac{g_c}{g_{-c}}\right) \in [1, \infty)$ such that$\ln(Zg_{-c}) = -1$, and $\frac{\partial X}{\partial \alpha} < 0$ if $\alpha > \bar{\bar{\alpha}}\left(\frac{g_c}{g_{-c}}\right)$.Since Zg_{-c} falls as $\frac{g_c}{g_{-c}}$rises, and $\frac{\partial Z}{\partial \alpha} < 0$, $\bar{\bar{\alpha}}\left(\frac{g_c}{g_{-c}}\right)$ falls as $\frac{g_c}{g_{-c}}$ rises. Now, $\frac{\partial X}{\partial \alpha} \leq 0$ only if $\ln(Zg_c) \leq -1$, i.e., only if $Zg_c \leq \frac{1}{e} < \frac{1}{2}$. Notice that $\lim_{\alpha\to 1} Zg_c = \frac{g_c}{G} \leq \frac{1}{2}$, with the equality holding when $g_c = g_{-c}$. Hence, $\lim_{\alpha\to 1} \frac{\partial X}{\partial \alpha} > 0$ when $\left(\frac{g_c}{g_{-c}}\right) = 1$. Part (iii) of Proposition 1 follows by continuity. •

Proof of Corollary 1. The first claim in part (i) follows directly from Proposition 1(i), (8) and (9). Now recall that, using (4), the FOCs yield:

$$\frac{kGX}{\alpha\gamma} = (1-\lambda_M)^{\alpha} + (\lambda_M)^{\alpha}. \tag{10}$$

Together, (9) and (10) yield:

$$\frac{\pi}{k} = 2 - X[1 + \frac{G}{\alpha\gamma}]. \tag{11}$$

In light of (11), the second claim in part (i) of Corollary 1 follows from Proposition 1(i). Lastly, from (11), recalling that $\lim_{\alpha\to 1} X = \left(\frac{\gamma}{Gk}\right)$, we have:

$$\lim_{\alpha\to 1} \pi = 2k - \left[k + \frac{kG}{\gamma}\right](\lim_{\alpha\to 1} X) = (2k-1) - \left(\frac{\gamma}{G}\right). \tag{12}$$

Part (ii) of Corollary 1 follows from (12) by continuity. •

Proof of Proposition 2. Using (7) and (8), we get:

$$x_c = \frac{\left(\frac{\alpha\gamma}{k}\right)(1-\lambda_c)^{\alpha}\lambda_c}{g_c}; \tag{13}$$

which, in light of (3), implies:

$$\pi_c = \left[k - (1-\lambda_c)^{\alpha}\left(\frac{\alpha\gamma\lambda_c}{g_c} + 1\right)\right]. \tag{14}$$

It follows from (8) and (14) that π_c increases with an increase in *G*, given $\frac{g_M}{g_N}$. Now note that:

$\frac{\partial \pi_c}{\partial \lambda_c} = -\left(\frac{\alpha}{g_c}\right)(1-\lambda_c)^{\alpha-1}[(1-\lambda_c)\gamma - \alpha\gamma\lambda_c - g_c] > 0;$ (15)

since $g_c > \gamma$. Now let $g_{-c} \equiv \rho G$. Rewriting (8) as: $\lambda_c = \frac{(\rho)^{\frac{1}{\alpha}}}{\left[(\rho)^{\frac{1}{\alpha}}+(1-\rho)^{\frac{1}{\alpha}}\right]}$, we have: $\frac{d\lambda_c}{d\rho} = \frac{\lambda_c(1-\lambda_c)}{\alpha\rho(1-\rho)}$. From (14), $\frac{\partial \pi_c}{\partial \rho} = -\left[(1-\lambda_c)^{\alpha}\left(\frac{\alpha\gamma\lambda_c}{(1-\rho)g_c}\right)\right]$. Thus, using (14)-(15), we have:

$$\frac{d\pi_c}{d\rho} = \frac{\partial \pi_c}{\partial \lambda_c}\left(\frac{d\lambda_c}{d\rho}\right) + \frac{\partial \pi_c}{\partial \rho}$$

$$= -\left(\frac{1}{(1-\rho)^2\rho G}\right)(1-\lambda_c)^{\alpha}\lambda_c\left[(\gamma - g_c) + \rho\gamma[\alpha(1-\frac{\lambda_c}{\rho}) - \left(\frac{\lambda_c}{\rho}\right)]\right]. \quad (16)$$

The term $[\gamma - g_c]$ is negative since $g_c > \gamma$. By (8), $\frac{\lambda_c}{\rho}$ declines as α increases. Hence $Z \equiv [\alpha(1-\frac{\lambda_c}{\rho}) - \left(\frac{\lambda_c}{\rho}\right)]$ rises as α increases, with $\lim_{\alpha\to 1} Z = -1$ and $\lim_{\alpha\to\infty} Z = \infty$. The second claim in Proposition 2(i) follows.

Recalling (8), $\frac{\partial \lambda_c}{\partial g_{-c}} > 0$. Part (ii) of Proposition 2 follows from (14)-(15). Now, from (8),

$$\frac{d\lambda_c}{dg_c} = \frac{-\left(\frac{g_c}{g_{-c}}\right)^{\frac{1}{\alpha}}}{\alpha g_c\left(1+\left(\frac{g_c}{g_{-c}}\right)^{\frac{1}{\alpha}}\right)^2} = \frac{-(1-\lambda_c)\lambda_c}{\alpha g_c}. \quad (17)$$

Using (14), (15) and (17), we get:

$$\frac{d\pi_c}{dg_c} = \left(\frac{1}{{g_c}^2}\right)(1-\lambda_c)^{\alpha}\lambda_c[(1-\lambda_c)(1+\alpha)\gamma - g_c]. \quad (18)$$

Using (8),

$$[(1-\lambda_c)(1+\alpha)\gamma - g_c] = g_c\left[\frac{(1+\alpha)\gamma}{(g_c)^{\frac{\alpha-1}{\alpha}}\left[(g_c)^{\frac{1}{\alpha}}+(g_{-c})^{\frac{1}{\alpha}}\right]} - 1\right].$$

Since $\alpha > 1$, $\left[\frac{(1+\alpha)\gamma}{(g_c)^{\frac{\alpha-1}{\alpha}}\left[(g_c)^{\frac{1}{\alpha}}+(g_{-c})^{\frac{1}{\alpha}}\right]} - 1\right] > \left[\frac{(1+\alpha)\gamma}{2(\overline{g})} - 1\right]$ for $g_c \leq \overline{g}, g_{-c} < \overline{g}$. Its maximum value is $\left[\frac{(1+\alpha)\gamma}{4} - 1\right]$. Part (iii) of Proposition 2 follows in light of (18). •

References

Alesina, A., Baqir, R., and Easterly, W., 1999. "Public goods and ethnic divisions", *Quarterly Journal of Economics* 114(4): 1243–1284.

Baik, K.H., 2008. "Contests with group-specific public-good prizes", *Social Choice and Welfare* 30 (1): 103-117.

Cheikbossian, G., 2008. "Heterogeneous groups and rent-seeking for public goods", *European Journal of Political Economy* 24: 133–150.

Choi, J.P.,Chowdhury, S.M.,and Kim, J., 2016. "Group contests with internal conflict and power asymmetry",*Scandinavian Journal of Economics*118 (4):816–840.

Chowdhury, S.M., Lee, D., and Sheremeta, R., 2013. "Top guns may not fire: Best-shot group contests with group-specific public good prizes", *Journal of Economic Behavior and Organization* 92: 94-103.

Dasgupta, I., 2017. "Linguistic assimilation and ethno-religious conflict", in W. Buchholz and D. Rübbelke (eds.)*The Theory of Externalities and Public Goods*:*Essays in Memory of Richard C. Cornes*; 219- 242. Berlin: Springer.

Dasgupta, I., 2009. "'Living' wage, class conflict and ethnic strife", *Journal of EconomicBehavior and Organization* 72 (2): 750-765.

Dasgupta, I., and Kanbur, R., 2007. "Community and class antagonism", *Journal of Public Economics* 91 (9): 1816-1842.

Dasgupta, I., and Kanbur, R., 2005a. "Community and anti-poverty targeting", *Journal of Economic Inequality* 3 (3): 281-302.

Dasgupta, I., and Kanbur, R., 2005b. "Bridging communal divides: separation, patronage, integration", in C. Barrett (ed.) *The Social Economics of Poverty: On Identities, Groups, Communities and Networks*;146-170. London: Routledge.

Epstein, G.S., andMealem, Y., 2009. "Group specific public goods, orchestration of interest groups with free riding",*Public Choice* 139 (3): 357–369.

Esteban, J., and Ray, D., 2011. "A model of ethnic conflict", *Journal of the European Economic Association* 9(3):496–521.

Esteban, J., and Ray, D., 2001. “Collective action and the group size paradox”,*American Political Science Review* 95: 663–672.

Gradstein, M., 1993. “Rent-seeking and the provision of public goods”, *Economic Journal* 103: 1236–1243.

Hausken, K., 2005. “Production and conflict models versus rent-seeking models.” *Public Choice* 123: 59-93.

Katz, E., Nitzan, J., and Rosenberg, S., 1990. “Rent-seeking for pure public goods”, *Public Choice* 65: 49– 60.

Kolmar, M., and Rommeswinkel, H., 2013. “Contests with group-specific public goods and complementarities in efforts”, *Journal of Economic Behavior and Organization* 89: 9-22.

Lee, D., 2012. “Weakest-link contests with group-specific public good prizes”, *European Journal of Political Economy* 28 (2): 238-248.

Miguel, E., and Gugerty, M.K., 2005. “Ethnic diversity, social sanctions, and public goods in Kenya”, *Journal of Public Economics* 89:2325–2368.

Montalvo, J.G., and Reynal-Querol, M., 2012. “Inequality, polarization, and conflict”, in M. Garfinkel and S. Skaperdas (eds.) *The Oxford Handbook of the Economics of Peace and Conflict*; 152-178. Oxford: Oxford University Press.

Münster, J. 2007. "Simultaneous inter- and intra-group conflicts",*Economic Theory* 32: 333–352.

Olson, M., 1965. *The Logic of Collective Action: Public Goods and the Theory of Groups*; Harvard:Harvard University Press.

Riaz, K., Shogren, J.F., Johnson, S.R., 1995. “A general model of rent-seeking for pure public goods”, *Public Choice* 82: 243–259.

Tullock, G., 1980. "Efficient rent seeking", in J.M. Buchanan, R.D. Tollison, and G. Tullock(eds.) *Toward a Theory of the Rent-seeking Society*; 97-112. College Station: Texas A and M University Press.

Ursprung, H.W., 1990. “Public goods, rent dissipation, and candidate competition”,*Economics and Politics* 2: 115-132.