Growth in a Graying Economy

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Abstract

Population aging – falling fertility coupled with rising longevity – is expected to slow economic progress. *Directed* technological change may (partly) redress the problem. In an R&D-based general equilibrium life cycle model, technology responds to factor scarcity brought about by a higher dependency ratio. Falling fertility shrinks the workforce, causing wages to rise. Rising longevity, in contrast, expands the supply of capital, lowering its cost. This change in relative prices causes technology to become more capital-biased. That effect is somewhat attenuated if rising longevity also encourages human capital accumulation. Quantitative results show that the net effect depends on credit market distortions, the importance of intergenerational transfers and how much of population aging is due to rising longevity versus falling fertility.

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1 Introduction

Population aging due to rising longevity and falling fertility has been identified as a major impediment to future economic prosperity in industrialized nations (United Nations, 2007). The most discussed of its effects is a rising old-age dependency ratio. It is feared that fewer and fewer workers will have to provide for more and more retirees, substantially lowering aggregate welfare. Another concern relates to changes in savings behavior as, given everything else, population aging implies the ratio of dissavers (retired people) to savers (working people) will be higher, lowering the aggregate saving rate, capital accumulation and slowing the pace of economic growth (e.g., Futagami and Nakajima, 2001).[1]

This paper takes a less alarmist view. It argues that, in the long run, technological progress will respond to the adverse effects of population aging and redress part of the economic costs. This would occur as a response to relative price changes. In a life-cycle model, an exogenous improvement in old-age longevity encourages physical capital accumulation in order to provide for future consumption. If credit markets are imperfect, it can also promote greater investment in skills due to arbitrage opportunities. This change in the supply of factor inputs would move their relative prices and create profit opportunities for technological change directed towards the cheaper, more abundant factor (Acemoglu, 2003). In addition, the human capital response improves the productivity of R&D. Quantitative results shows that when technological change is not directed, higher life expectancy costs 0.48 percentage points of growth per year while lower fertility costs 0.63 percentage points. Under directed technological change, higher life expectancy costs little, but the effect of lower fertility is more severe, 0.72 percentage points of growth.

These result have several policy implications. The amortization effect of increased longevity on education can be made more potent by investing in the quality of public education. Public policy can also facilitate growth through incentives and investment in the research sector. Since the consequences of fertility decline are particularly severe, pro-natal and pro-immigration policies would be especially effective in aging economies.

The macroeconomic effects of population aging have been extensively analyzed in the recent literature (see Bloom *et al.* 2008, 2010, 2011). Among others, Gertler (1999) and Gruescu (2007) highlight the fiscal implications of rising number of retirees supported by an average worker. Bloom *et al.* (2007) find that increasing longevity encourages saving rate under a fixed retirement-age regime. This raises a possibility of higher supply of financial assets reducing their prices. However, Poterba (2005) finds little evidence of demographic effects on asset returns in the US, Canada, and the UK. Another set of studies focuses on the effect of interest rates. Given fixed length of work-life, an increase in life expectancy (fall in mortality rate), leads to an increase in the effective rate of

[1]Traditional neoclassical growth models suggest that aging will have transitory effects on the growth rate of output per worker and per capita. The latter will be permanently lower, however.

return to savings, encouraging savings. This effect depends on lack of access to annuity markets, and the substitution effect on savings of higher effective interest rates outweighing the income effect. However, we rule out this mechanism by assuming perfect annuities market in our model.

Though not the primary focus, the long-run effects of declining work-force can be deduced from the large body of literature on endogenous growth models with purposeful R&D investments. For example, in Romer (1990) and Aghion and Howitt (1992), a larger population is favorable for long-run growth because of the scale effect: a larger population not only makes more scientists available for R&D, but the innovative firms also benefit from more profit opportunities made available by a larger market-size. Among more recent works, Prettner (2013) specifically looks at the effect of rising longevity when technology responds to changing population age structure. Using endogenous and semi-endogenous growth frameworks developed by Romer (1990) and Jones (1995) he shows not only that an increase in longevity raises growth of per capita income through a net positive effect on R&D intensity, the effect can be strong enough to ameliorate the negative effect of falling fertility rate.

Yet another body of work related to this study concerns how increase in life expectancy induces private investment in human capital. The seminal work of Ben-Porath (1967) and subsequent theoretical literature concluded that increasing life expectancy induces individuals to invest more in education by allowing a longer horizon over which the returns from education accrue. The empirical validity of this mechanism is much debated. For example, Hazan (2009) shows that a period of increasing life expectancy coincided with declining expected lifetime working hours, because individuals decreased their lifetime labor supply. This evidence does not, however, imply an absence of a causal effect of life expectancy on education, only that the impact does not work through expected lifetime working hours. In our model, increased longevity raises private incentive for education for credit-constrained households as their return on financial investment falls.

That technology can respond to factor scarcity is not a novel premise. In a series of works Acemoglu has shown how directed technological change can explain the rise of the skill premium in the US since the eighties. Indeed our work here borrows from Acemoglu (2003). Two recent works offer interesting applications of directed technological change in different contexts. Hanlon (2012) looks at the disruption of American cotton supplies to the British cotton textile industry due to the U.S. Civil War. He finds that the supply shock initially lowered the relative price of Indian cotton, inducing innovations directed towards making use of that cheaper substitute. He also finds evidence for strong induced-bias, that is, an eventual increase in the relative price of Indian cotton.

Allen (2009) builds on Hicks' insight to reinterpret the causes of the British industrial revolution.[2] He argues that British economic progress in the nineteenth century was made possible not

[2] "The real reason for the predominance of labour saving inventions is surely that . . . a change in the relative prices of the factors of production is itself a spur to innovation and to inventions of a particular kind – directed at economizing the use of a factor which has become relatively expensive" (*The Theory of Wages*, 1932), quoted in Allen (2009), p. 141.

due to cultural values or the spread of "Industrial Enlightenment" but by labor-saving innovations. Higher real wages for British workers relative to continental Europe and cheaper energy prices, the result of a geographical accident, made innovations capital-biased. Cheaper energy had a direct effect through energy-saving innovations, and an indirect effect on the cost of capital (metals and bricks). Even when a technological breakthrough followed from the application of a scientific discovery that was disconnected from profit-seeking, for example the steam engine, its subsequent perfection owed much to energy costs that made similar improvements unprofitable elsewhere.

The rest of this paper is organized as follows. We begin by specifying household behavior in the next section, followed by the production sector in section 3. Section 4 studies the general equilibrium properties of the economy and characterizes how its long-run behavior depends on population aging. In section 5 we attempt to quantify the macroeconomic effects of population aging.

2 Household Behavior

The economy is populated by an infinite sequence of four period-lived overlapping generations of families. The four stages of life are childhood, youth, middle age and old age. At each date $t = 1, 2, .., \infty$, a young agent gives birth to $1 + n \geq 1$ children who remain passive until they attain maturity at $t + 1$.[3]

Each active individual works in youth and middle-age and retires thereafter. His labor time endowment vector is $(1, \varpi, 0)$ where $\varpi \leq 1$. If he chooses to, he can invest time during youth in human capital accumulation through college education or vocational training. Prior to doing so he realizes an idiosyncratic "ability draw" v that determines the productivity of human capital investment. A middle aged individual does not invest in human capital since he retires in old age. Survival in old age is uncertain, determined by the probability $\phi \in [0, 1]$. Accordingly we will refer to ϕ as old-age longevity. An increase in ϕ constitutes one aspect of population aging, a decrease in n the other one.

It will be helpful to tag household variables in youth, middle-age and old-age by 1, 2 and 3 respectively. Each young adult at time t starts his active life with an endowment vector of human and financial capital, (h_t, a_{1t}). The financial capital is received as inheritance from a middle-aged parent. It can be thought of as parental contribution towards college education (Ionescu, 2009). Additionally, the child acquires his parent's human capital via parent-to-child transmission.[4]

[3] Fertility has fallen below replacement in many industrialized nations. Negative population growth is inconsistent with balanced growth paths which is why we restrict n to be non-negative. A world in which a country's population is imperiled would look quite different from the one we have. To understand that would require a model of endogenous fertility, one where fertility can respond to a population implosion. Possibly, relative price changes would eventually make fertility desirable enough to avoid an implosion in the long run.

[4] While we do not explicitly model this transmission, it can be rationalized as the outcome of childhood schooling

Households of a particular generation are *ex ante* heterogeneous in their human capital and financial assets, apart from *ex post* realizations of an ability shock (that affects them in youth) and survival (that affects them in old age). A young household with human capital h_t and assets a_{1t} at time t maximizes expected lifetime utility

$$\ln c_{1t} + \beta \left[\ln c_{2t+1} + \gamma_1 \ln\{(1+n)(\gamma_{2t} + b_{t+1})\}\right] + \phi\beta^2 \ln c_{3t+2}$$

subject to

$$\begin{aligned}
c_{1t} &\leq w_t h_t (1 - s_t) + R_t a_{1t} - a_{2t+1} \\
c_{2t+1} &\leq \varpi w_{t+1} h_{t+1} + R_{t+1} a_{2t+1} - (1+n) b_{t+1} - a_{3t+2} \\
c_{3t+2} &\leq \hat{R}_{t+2} a_{3t+2} \\
h_{t+1} &= (1 - \delta_h) h_t + \nu (h_t s_t)^\theta, \theta \in (0,1), \nu > 0 \\
a_{2t+1} &\geq -\mu \left[R_t a_{1t} + (1 - s_t) w_t h_t\right]
\end{aligned}$$

given $a_{1t} > 0$, $h_t > 0$ and utility from death normalized to a large negative number. Here a_{1t} denotes the inheritance (without interest) received from the parent while b_t denotes bequests made to each child. The last inequality is a borrowing constraint that limits a young individual's ability to borrow to a multiple $\mu \geq 0$ of his wealth. As mentioned earlier, the productivity of human capital investment ν is individual-specific, drawn before the start of its planning horizon at t. We denote the interest factor in any period as $R_t = 1 + r_t$. $\hat{R}_{t+2}$ is the annuity return adjusted for mortality risk, that is, $\hat{R}_{t+2} = R_{t+2}/\phi$. The subjective discount rate is $\beta \in (0,1)$. $\gamma_1 > 0$ is the intensity of warm-glow bequest motive towards children, and $\gamma_{2t} > 0$ is a time-varying parameter. Assuming later that γ_{2t} grows at the same rate as aggregate consumption will ensure that bequest remains a luxury good in the long run, consistent with the empirical evidence.

The production function for human capital follows Ben-Porath (1967), $\delta_h \in [0,1]$ denoting the depreciation rate of human capital. The one difference is we do not include resource investment, only time: the cost of acquiring human capital is purely the opportunity cost of foregone labor income. A young household's human capital endowment positively affects its return from schooling, subject to diminishing returns.

2.1 Optimization

Non-satiated preferences imply the three budget constraints will be satisfied with equality in equilibrium. The only inequality constraint we need to explicitly consider is that for a_{2t+1}. De-

where the production function of a child's human capital is CRS in schooling investment and parental human capital. The child's human capital would be a linear function of the parent's under the logarithmic preferences adopted below.

noting by λ_t the Lagrange multiplier associated with this constraint, the necessary and sufficient Kuhn-Tucker conditions are:

$$\begin{aligned}
\frac{1}{c_{1t}} w_t h_t &= \frac{\beta}{c_{2t+1}} \varpi w_{t+1} \theta \nu h_t^{\theta} s_t^{\theta-1} - \lambda_t \mu w_t h_t && (1)\\
\frac{1}{c_{1t}} &= \frac{\beta}{c_{2t+1}} R_{t+1} + \lambda_t && (2)\\
\frac{1+n}{c_{2t+1}} &= \frac{\gamma_1}{\gamma_{2t} + b_{t+1}} && (3)\\
\frac{1}{c_{2t+1}} &= \frac{\beta\phi}{c_{3t+2}} \hat{R}_{t+2} && (4)
\end{aligned}$$

for s_t, a_{2t+1}, b_{t+1} and a_{3t+1} respectively, and the complementary slackness condition

$$\begin{aligned}
a_{2t+1} + \mu\{R_t a_{1t} + (1-s_t) w_t h_t\} &\geq 0, \quad \lambda_t \geq 0, \\
\lambda_t \left[a_{2t+1} + \mu\{R_t a_{1t} + (1-s_t) w_t h_t\}\right] &= 0. && (5)
\end{aligned}$$

Unconstrained Equilibrium

Take the case of a household for which the borrowing constraint is non-binding, that is, $a_{2t+1} > -\mu[R_t a_{1t} + (1-s_t) w_t h_t]$. The household's schooling choice simply maximizes its lifetime budget constraint to give

$$s_t = \frac{1}{h_t}\left[\theta\nu\varpi\left(\frac{w_{t+1}}{w_t}\right)\frac{1}{R_{t+1}}\right]^{1/(1-\theta)}. \tag{6}$$

At least in partial equilibrium, this investment does not depend on old-age longevity ϕ or population growth. Human capital investment and endowment are inversely related since the opportunity cost rises linearly in human capital while returns are subject to diminishing returns.

Define the constant $\chi \equiv \beta(1+\gamma_1+\beta\phi)$. It follows from the first order conditions and budget constraints above that

$$c_{1t} = \frac{W_{2t+1} + (1+n)\gamma_{2t}}{\chi R_{t+1}} \tag{7}$$

$$a_{2t+1} = \frac{1}{R_{t+1}}\left[W_{2t+1} - \varpi w_{t+1} h_{t+1} - \frac{(1+n)\gamma_{2t}}{1+\chi}\right] \tag{8}$$

$$c_{2t+1} = \frac{\beta}{\chi}\left[W_{2t+1} + \frac{\chi(1+n)\gamma_{2t}}{1+\chi}\right] \tag{9}$$

$$a_{3t+2} = \frac{\phi\beta^2}{\chi}\left[W_{2t+1} + \frac{\chi(1+n)\gamma_{2t}}{1+\chi}\right] \tag{10}$$

$$b_{t+1} = \beta\gamma_1\left[\frac{W_{2t+1}}{\chi(1+n)} + \frac{\gamma_{2t}}{1+\chi}\right] - \gamma_{2t} \tag{11}$$

$$c_{3t+2} = \frac{\beta^2}{\chi} R_{t+2}\left[W_{2t+1} + \frac{\chi(1+n)\gamma_{2t}}{1+\chi}\right] \tag{12}$$

where financial and non-financial wealth in youth is

$$W_{1t} = (1-s_t) w_t h_t + R_t a_{1t} \tag{13}$$

and in middle age

$$W_{2t+1} = \frac{\chi}{1+\chi}\left[R_{t+1} W_{1t} + \varpi w_{t+1} h_{t+1}\right] \tag{14}$$

for the human capital investment choice given by (6). For the borrowing constraint to not bind in this equilibrium, the household's initial endowment needs to be large enough. That is,

$$a_{1t} \geq \frac{1}{R_t}\left[\frac{\varpi}{\chi + \mu(1+\chi)}\left(\frac{w_{t+1} h_{t+1} + (1+n)\gamma_{2t}}{R_{t+1}}\right) - (1-s_t) w_t h_t\right] \tag{15}$$

The right hand side this inequality is increasing in schooling s_t which, among other factors, is increasing in the productivity of the investment ν and the growth rate of wages w_{t+1}/w_t that will be pinned down by the rate of innovation in a stationary equilibrium.

Constrained Equilibrium

When (15) is not satisfied and households borrow up to their maximum ability, the human capital investment decision depends on consumption smoothing. From (1) and (2)

$$\frac{c_{2t+1}}{c_{1t}} w_t h_t = \left(\frac{\beta}{1+\mu}\right)\left[\mu R_{t+1} w_t h_t + \theta\nu\varpi w_{t+1} h_t^{\theta} {s_t}^{\theta-1}\right] \tag{16}$$

Optimal decisions are now

$$c_{1t} = (1+\mu)W_{1t} \tag{17}$$

$$c_{2t+1} = \frac{\beta}{\chi}[W_{2t+1} + (1+n)\gamma_{2t}] \tag{18}$$

$$a_{2t+1} = -\mu\,[R_t a_{1t} + (1-s_t)\,w_t h_t] \tag{19}$$

while a_{3t+2}, b_{t+1}, and c_{3t+2} are given by the same expressions as equations (10)-(12) above but for middle-age wealth level of

$$W_{2t+1} = \varpi w_{t+1} h_{t+1} - \mu R_{t+1} W_{1t} \tag{20}$$

and W_{1t} defined by (13) as before.

Using these optimal choices, the optimality condition for human capital investment can be simplified to

$$\Gamma(s_t) \equiv \frac{\varpi w_{t+1} h_{t+1} + (1+n)\gamma_{2t}}{w_t h_t (1-s_t) + R_t a_{1t}} - \mu(1+\chi) R_{t+1} = \theta \chi \nu \varpi \frac{w_{t+1}}{w_t} (h_t s_t)^{\theta-1}$$

Eliminating h_{t+1} using the accumulation equation, this equation implicitly solves for optimal schooling as a function of current and future prices and the household's human and financial assets

$$s_t = \mathrm{H}\,(w_t, w_{t+1}, R_t, R_{t+1}, h_t, a_{1t}, n) \tag{21}$$

The left hand side of this equation is monotonically increasing in s_t, the right hand side monotonically decreasing. Hence there is typically a unique equilibrium schooling choice for each constrained household. Note that human capital investment (college education) is now increasing in ϕ.

There are two ways human capital can respond to longevity. First it does so if the expected return on human capital investment directly depends on it. A change in the survival probability in middle-age, for example, would have such an effect since survival uncertainty affects the amortization period. Since aging is driven in industrialized countries by post-retirement health improvements (need citation), such a channel is absent in the model. This is similar to Hazan (2009). Instead, the effect of ϕ on s is driven by rate of return arbitrage. Recall that mortality risk against financial investment is fully insured through annuities, return on human capital investment is not. When human capital is very productive, the individual wants to invest only in it, not in financial capital. That makes his return sensitive to ϕ. In an interior equilibrium, instead, return to human capital is pinned down by the (exogenous) return on saving, which we know is independent of ϕ.

2.2 Aggregate Labor Supply

Individuals differ in three dimensions: human capital endowment, financial asset and learning ability. We assume that learning ability is drawn (i.i.d.) at the beginning of his working life from the distribution $\mathscr{F}(v)$ with positive support. The cumulative distribution function $F_t(a,h,v)$ gives the fraction of period-t young individuals with financial asset at or below a, human capital endowment at or below h and learning ability at or below v. Optimal choices (policy functions) of the form $a' = q^a(a,h,v)$ and $h' = q^h(a,h,v)$ studied above induce a law of motion for the distribution. Let f_t be the density function associated with F_t.[5] Then, based on the policy functions noted above and a fresh draw of ability in the new cohort, the cumulative distribution evolves as per

$$F_{t+1}(a,h,v) = \frac{N_t}{N_{t+1}} \int_a\int_h\int_v (1+n)\mathscr{I}(a_{1t+1} \leq a, h_{t+1} \leq h, v_{t+1} \leq v) f_t(a,h,v)\mathrm{d}a\,\mathrm{d}h\,\mathrm{d}v$$

where $\mathscr{I}$ is an indicator function and N_t denotes the size of the cohort that is young at t. As before we have $N_{t+1} = (1+n)N_t = (1+n)^t N_0$ and the expressions for P_t and d_t from above.

Labor supply at t comes from young individuals who divide their time between working and education and from middle-aged individuals who work full time. A young individual i with endowments (a^i_{1t}, h^i_t) supplies $(1-s^i_t)h^i_t$ units of effective labor and a middle-aged individual supplies $(1-\delta_h)h^i_{t-1} + v^i(h^i_{t-1}s^i_{t-1})^\theta$ times his labor time endowment of ϖ. Hence, aggregate labor supply is

$$L_t = N_t \int_a\int_h\int_v (1-s_t)hf_t(a,h,v)\mathrm{d}a\,\mathrm{d}h\,\mathrm{d}v + N_{t-1}\int_a\int_h\int_v \left[(1-\delta_h)h + v(hs_{t-1})^\theta\right]\varpi f_{t-1}(a,h,v)\mathrm{d}a\,\mathrm{d}h\,\mathrm{d}v. \tag{22}$$

For homogeneous households, that is, identical ability (mean ability of unity) and endowments this simplifies to

$$L_t = \left[1 - s_t + \frac{\varpi}{1+n}\right] N_t h_t = \left[(1-s_t)(1+n) + \varpi\right] N_{t-1} h_t$$

increasing in the population growth rate. In general equilibrium, there will be additional effects through time-allocation towards human capital accumulation.

Innovation and production of specialized intermediate goods require labor. Individuals can work either in goods production or in innovation (R&D) both of which pay the same wage per effective unit of labor supply.

2.3 Aggregate Consumption and Savings

Aggregate (net) savings S_t consists of the (positive) savings by middle-aged workers, (possibly negative) saving by young workers and (positive) bequests that are invested in the capital market

[5] The distribution will not be usually invariant except in the balanced growth path, which we allow for by a time subscript on F and f. Obviously $f_t(a,h,v) = \wr(v) f_t(a,h|v)$, the latter conditional distribution being independent of v.

for use in $t+1$. That is,

$$S_t = (1+n)N_t \int_a\int_h\int_v b_t(a,h) f_t(a,h,v)\mathrm{d}a\mathrm{d}h\mathrm{d}v + N_t \int_a\int_h\int_v a_{2t+1}(a,h) f_t(a,h,v)\mathrm{d}a\mathrm{d}h\mathrm{d}v \\ + N_{t-1}\int_a\int_h\int_v a_{3t+1}(a,h) f_{t-1}(a,h,v)\mathrm{d}a\mathrm{d}h\mathrm{d}v. \tag{23}$$

Similarly aggregate consumption is given by

$$C_t = N_t \int_a\int_h\int_v c_{1t}(a,h) f_t(a,h,v)\mathrm{d}a\mathrm{d}h\mathrm{d}v + N_{t-1} \int_a\int_h\int_v c_{2t}(a,h) f_{t-1}(a,h,v)\mathrm{d}a\mathrm{d}h\mathrm{d}v \\ + N_{t-3}\int_a\int_h\int_v c_{3t}(a,h) f_{t-2}(a,h,v)\mathrm{d}a\mathrm{d}h\mathrm{d}v. \tag{24}$$

3 Production (Acemoglu, 2003)

The production side consists of three sectors: a final goods sector, an intermediate goods sector, and a research sector that invents new blueprints.[6]

The economy has two productive resources – labor and capital. Production of the final good uses two types of goods – those that are produced using labor and those produced using capital. Both require specific types of intermediate goods. The research sector hires researchers and scientists to develop blueprints for new varieties of capital- and labor-complementing intermediate goods. We use a basic model of R&D that relies on horizontal innovations, i.e. development of new product varieties. Following Acemoglu (2003), it nests the Romer (1990) framework with strong positive spillovers and Jones' (1995) insight about scale effects. The Romer-Jones specification has been modified to incorporate heterogeneous population cohorts and labor quality. We assume perfect competition in the final goods and research sectors and monopolistic competition in the intermediate goods sector.

Aggregate output Y_t is produced using a capital-intensive good Y_{Kt} and labor intensive good Y_{Lt}

$$Y_t = \left[\kappa Y_{Kt}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\kappa) Y_{Lt}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{25}$$

where $\varepsilon \in [0,\infty)$ is the elasticity of substitution between the two.

The capital-intensive good is produced from specialized goods and capital. Technical change takes the form of an expanding variety of these specialized goods which makes capital more productive

$$Y_{Kt} = \left[\int_0^{A_t} y_{kt}(i)^\alpha \mathrm{d}i\right]^{\frac{1}{\alpha}}, \alpha \in (0,1) \tag{26}$$

[6] This closely follows Acemoglu (2003) where steady-state growth results from expanding product variety of goods that improve labor productivity. The main differences here are endogenous supply of skilled labor and non-scale effects in ideas production.

Likewise, for the labor-intensive good,

$$Y_{Lt} = \left[\int_0^{B_t} y_{lt}(i)^{\alpha} \mathrm{d}i\right]^{\frac{1}{\alpha}} \tag{27}$$

Evidently an expansion of A corresponds to capital-augmenting technical change, an increase in B to labor-augmenting technical change.

Intermediate goods are supplied by technology monopolists who hold the relevant patents, and are produced linearly from their respective factors:

$$\begin{aligned} y_k(i) &= k(i), \\ y_l(i) &= l(i) \end{aligned} \tag{28}$$

where $k(i)$ and $l(i)$ are capital and labor used in the production of good i. Denoting the labor force employed in the y_l sector as L_t^y, market clearing for capital and labor requires that

$$\begin{aligned} \int_0^{A_t} k_t(i)\mathrm{d}i &= K_t \\ \int_0^{B_t} l_t(i)\mathrm{d}i &= L_t^y \end{aligned} \tag{29}$$

where the left-hand side denotes demand and the right-hand-side supply of the factor input.

3.1 Final Goods Production

Perfectly competitive final goods producers maximize their profit flow $P_{Yt}Y_t - P_{Kt}Y_{Kt} - P_{Lt}Y_{Lt}$ subject to (25), taking as given the prices $\{P_{Yt}, P_{Kt}, P_{Lt}\}$. Optimality conditions for capital and labor-intensive inputs are

$$\begin{aligned} \kappa P_{Yt}(Y_t/Y_{Kt})^{1/\varepsilon} &= P_{Kt}, \\ (1-\kappa)P_{Yt}(Y_t/Y_{Lt})^{1/\varepsilon} &= P_{Lt}. \end{aligned} \tag{30}$$

Since zero equilibrium profits entail that the price of final goods equal the average cost of production, that is, $P_{Yt} = P_{Kt}\,(Y_{Kt}/Y_t) + P_{Lt}\,(Y_{Lt}/Y_t)$, we have

$$P_{Yt} = \left[\kappa^{\varepsilon} P_{Kt}^{1-\varepsilon} + (1-\kappa)^{\varepsilon} P_{Lt}^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{31}$$

from (30). Then normalizing $P_{Yt} = 1\ \forall\, t$ we get

$$P_{Kt} = \left[\kappa^{\varepsilon} + (1-\kappa)^{\varepsilon} p_t^{\varepsilon-1}\right]^{1/(\varepsilon-1)} \tag{32}$$

$$P_{Lt} = \left[\kappa^{\varepsilon} p_t^{1-\varepsilon} + (1-\kappa)^{\varepsilon}\right]^{1/(\varepsilon-1)} \tag{33}$$

where $p_t \equiv P_{Kt}/P_{Lt}$ is the relative price of the capital-intensive good.

3.2 Capital- and Labor-intensive Goods Production

The sectors producing Y_K and Y_L are also perfectly competitive. Denote by η_K and η_L the prices of K- and L-complementing intermediate inputs y_k and y_l respectively.[7] Capital intensive goods producers maximize profits

$$P_{Kt}Y_{Kt}(i) - \int_0^{A_t} \eta_{Kt} y_k(i)\mathrm{d}i$$

subject to the production function (26). From this the demand for each specialized input $y_k(i)$ is obtained as

$$y_{kt}(i) = y_{kt} = \left(\frac{\eta_{Kt}}{P_{Kt}}\right)^{-\sigma} Y_{Kt} \quad \forall i \in [0, A_t]. \tag{34}$$

Similarly, for producers of the labor-intensive intermediate good $y_l(i)$

$$y_{lt}(i) = y_{lt} = \left(\frac{\eta_{Lt}}{P_{Lt}}\right)^{-\sigma} Y_{Lt} \quad \forall i \in [0, B_t]. \tag{35}$$

Given these isoelastic demands, profit maximization by the monopolists implies that prices will be set as a constant markup over the marginal costs:

$$\eta_{Kt}(i) = \left(1 - \frac{1}{\sigma}\right)^{-1} \rho_t = \frac{\rho_t}{\alpha} \tag{36}$$

$$\eta_{Lt}(i) = \left(1 - \frac{1}{\sigma}\right)^{-1} w_t = \frac{w_t}{\alpha}. \tag{37}$$

Finally from (28) and (29)

$$\begin{aligned} y_{kt} = k_t = \frac{K_t}{A_t} \\ y_{lt} = l_t = \frac{L_t^y}{B_t} \end{aligned} \tag{38}$$

Substituting these into (26) and (27) yields two "reduced-form" aggregate productions for capital and labor-intensive goods

$$Y_{Kt} = \left[\int_0^{A_t} \left(\frac{K_t}{A_t}\right)^{\alpha} \mathrm{d}i\right]^{\frac{1}{\alpha}} = A_t^{\frac{1-\alpha}{\alpha}} K_t \tag{39}$$

$$Y_{Lt} = \left[\int_0^{B_t} \left(\frac{L_t^y}{B_t}\right)^{\alpha} \mathrm{d}i\right]^{\frac{1}{\alpha}} = B_t^{\frac{1-\alpha}{\alpha}} L_t^y \tag{40}$$

[7] We assume physical capital fully depreciates upon use, not unrealistic since a model period is 25 years. Since financial savings is converted into physical capital, as in the standard Neoclassical model, arbitrage implies that $r_t = \rho_t - 1$ in equilibrium.

that are linear in capital and labor. Manipulating equations (34) – (40) gives the wage rate and rental rate of capital as:

$$\rho_t = \alpha A_t^{\frac{1-\alpha}{\alpha}} P_{Kt} \tag{41}$$

$$w_t = \alpha B_t^{\frac{1-\alpha}{\alpha}} P_{Lt}. \tag{42}$$

3.3 Intermediate Goods Production

Patent holding producers of the capital- and labor-complementing intermediate goods are perpetual monopolists as long as those goods do not become obsolete (see below). The profit of a capital intensive intermediate goods producer is $\pi_{Kt} = \eta_{Kt} y_{kt} - \rho_t k_t$. From (36) and (38), the maximal profit of a monopoly producer of capital intensive intermediate goods

$$\pi^*_{Kt} = \left(\frac{1-\alpha}{\alpha}\right)\frac{\rho_t K_t}{A_t}. \tag{43}$$

Similarly, the maximal profit of a monopoly producer of labor intensive intermediate goods

$$\pi^*_{Lt} = \left(\frac{1-\alpha}{\alpha}\right)\frac{w_t L^y_t}{B_t}. \tag{44}$$

3.4 Innovations Possibilities Frontier

The innovations possibilities frontier is the locus of technological possibilities for transforming resources into blueprints of new varieties of capital and labor intensive intermediate goods. The R&D sector innovates new types of capital- and labor-intensive intermediate goods.[8] It does so by hiring workers. Denote the effective labor force of the sector producing capital-intensive innovations as L^A_t and that producing labor-intensive innovations as L^B_t. Clearly, in a full employment equilibrium, $L^A_t + L^B_t = L^I_t$.

The production of blueprints for new types of capital- and labor-complementing intermediate goods are:

$$A_{t+1} - A_t = \left[\xi_A \left(\frac{L^A_t}{L_t}\right)^{1-\varsigma} - \delta_A\right] A_t, \tag{45}$$

$$B_{t+1} - B_t = \left[\xi_B \left(\frac{L^B_t}{L_t}\right)^{1-\varsigma} - \delta_B\right] B_t. \tag{46}$$

These imply that, for a given level of human capital, more workers in the R&D sectors would suc-

[8]Section 5.1 in Acemoglu (2003) shows that the Kiley-style lab-equipment model of innovations is inconsistent with a balanced growth path. Hence we closely follow Acemoglu's formulation.

cessfully develop more types of intermediate goods and that, an increase in the human capital of each of those workers would increase the innovation even more. Moreover, there is a knowledge externality similar to Romer (1990) in that a larger body of knowledge makes it easier to invent new goods. The ξ's denote productivity of researchers in the two types of innovation, whereas $\varsigma \in (0,1)$ ensures that innovative activities are subject to decreasing returns in the number of scientists. Note that we deviate from Acemoglu (2003) in that we eliminate scale effects by positing it is the *share* of scientists in the labor force, not their absolute number, that drives the respective R&D. Each intermediate becomes obsolete at the rate δ_j for $j = A, B$, so that when there is no research effort devoted to a particular type of intermediate, its stock declines exponentially.[9] Finally, while the patent for a new blueprint can be held forever, it becomes worthless should the technology become obsolete.

4 General Equilibrium

We begin by noting that in, general equilibrium, the usual financial asset market clearing condition whereby one unit of savings is costlessly transformed into one unit of future physical capital holds

$$K_{t+1} = S_t$$

given the initial capital stock K_0, initial distribution F_0 and S_t determined by (23) above.

4.1 Prices

From (30) we have $p_t \equiv P_{Kt}/P_{Lt} = [\kappa/(1-\kappa)]\,(Y_{Kt}/Y_{Lt})^{-1/\varepsilon}$, or

$$p_t = \left(\frac{\kappa}{1-\kappa}\right)\left[\left(\frac{A_t}{B_t}\right)^{\frac{1-\alpha}{\alpha}} \frac{K_t}{L_t^y}\right]^{-1/\varepsilon} \tag{47}$$

using (39) and (40).

In the balanced growth path (BGP) the relative price of the intermediate goods will be constant. We anticipate that such a BGP exists only when technical progress is labor augmenting (Acemoglu, 2003), which means along this growth path A_t will remain constant and B_t will grow at some rate g. We also expect human capital investment decisions $\{s^*\}_i$ and, hence, dynastic human capital stocks to be constant on the BGP. That implies the growth rate of $L_t - L_t^I$ is pinned down by population growth rate in this BGP. Thus, from (47) it follows that the growth rate of aggregate capital is

[9] Setting these obsolescence rate to zero opens up the possibility of multiple balanced growth paths all with the same growth rate. See Proposition 4 in Acemoglu (2003).

given by

$$1+\hat{g}=(1+n)(1+g)^{\frac{1-\alpha}{\alpha}} \tag{48}$$

in the BGP.

Now consider the supply of intermediate inputs in the BGP where the interest rate remains constant at r. Suppose that invented blueprints can be immediately put to production. As mentioned above, technologies can get obsolete over time once they go into production, the rates of obsolescence being $\delta_A>0$ and $\delta_B>0$ for capital- and labor-augmenting intermediate goods respectively. Using the maximal profit flows (43) and (44), the PDV of expected monopoly rents in the labor-complementing intermediate goods sector is

$$\begin{aligned}
V_t^L &= \sum_{\tau=0}^{\infty}(1-\delta_B)^{\tau}\frac{\pi^*_{Lt+\tau}}{(1+r)^{\tau}} \\
&= \left(\frac{1-\alpha}{\alpha}\right)\left[\sum_{\tau=0}^{\infty}\left(\frac{1-\delta_B}{1+r}\right)^{\tau}\frac{w_{t+\tau}L^y_{t+\tau}}{B_{t+\tau}}\right] && (49)\\
&= \left(\frac{1-\alpha}{\alpha}\right)\left[\sum_{\tau=0}^{\infty}\left(\frac{1-\delta_B}{1+r}\right)^{\tau}\left\{(1+n)(1+g)^{\frac{1-2\alpha}{\alpha}}\right\}^{\tau}\right]\frac{w_tL^y_t}{B_t} \\
&= \left(\frac{1-\alpha}{\alpha}\right)\left[\frac{1+r}{(1+r)-(1-\delta_B)(1+n)(1+g)^{\frac{1-2\alpha}{\alpha}}}\right]\frac{w_tL^y_t}{B_t}\equiv\Gamma_L(r,g,n)\frac{w_tL^y_t}{B_t} && (50)
\end{aligned}$$

where the third line anticipates that the wage rate per effective unit of labor, w, grows at the gross rate $(1+g)^{(1-\alpha)/\alpha}$ via (42), dynastic schooling choices and human capital remain constant in the BGP as does the relative allocation of labor in the various sectors which implies sector-specific employment grows at the population growth rate of n. Similarly

$$\begin{aligned}
V_t^K &= \sum_{\tau=0}^{\infty}(1-\delta_A)^{\tau}\frac{\pi^*_{Kt+\tau}}{(1+r)^{\tau}} \\
&= \left(\frac{1-\alpha}{\alpha}\right)\left[\frac{1+r}{(1+r)-(1-\delta_A)(1+n)(1+g)^{\frac{1-\alpha}{\alpha}}}\right]\frac{\rho_tK_t}{A_t}\equiv\Gamma_K(r,g,n)\frac{\rho_tK_t}{A_t} && (51)
\end{aligned}$$

using the expression for $1+\hat{g}$ from above. Note that we need

$$1+n<(1+r)\min\left\{\frac{1}{(1-\delta_A)(1+g)^{\frac{1-\alpha}{\alpha}}},\frac{1}{(1-\delta_B)(1+g)^{\frac{1-2\alpha}{\alpha}}}\right\}$$

for V_t^K and V_t^L to be well defined.

Newly invented blueprints are sold at competitive prices p_{At} and p_{Bt}. Perfectly competitive bidding for the blueprints lead to

$$p_{At}=V_t^K,\; p_{Bt}=V_t^L \tag{52}$$

so that the net return from intermediate goods production is zero.

4.2 Labor Allocation

Now consider the allocation of labor between production of intermediate goods versus production of ideas. The value of marginal product of researchers in the two sectors are $\xi_A(1-\varsigma)A_t V_t^K (L_t^A)^{-\varsigma} L_t^{\varsigma-1}$ and $\xi_B(1-\varsigma)B_t V_t^L (L_t^B)^{-\varsigma} L_t^{\varsigma-1}$. Hence, in equilibrium,

$$w_t = \frac{(1-\varsigma)}{L_t} \max\left\{ \xi_A A_t V_t^K \left(\frac{L_t^A}{L_t}\right)^{-\varsigma}, \xi_B B_t V_t^L \left(\frac{L_t^B}{L_t}\right)^{-\varsigma} \right\}. \tag{53}$$

In the BGP, A remains constant. So some research has to be devoted to inventing new capital-intensive intermediates to balance obsolescence. This means we must have

$$w_t = \frac{(1-\varsigma)}{L_t}\xi_A A_t V_t^K (L_t^A/L_t)^{-\varsigma} = \frac{(1-\varsigma)}{L_t}\xi_B B_t V_t^L (L_t^B/L_t)^{-\varsigma} \tag{54}$$

Next note that in the BGP, $A_{t+1} - A_t \equiv \Delta A_t = 0$ while $\Delta B_t = g B_t$. This, along with the labor market clearing condition, means

$$L_t^A = (\delta_A/\xi_A)^{\frac{1}{1-\varsigma}} L_t, \tag{55}$$

$$L_t^B = [(g+\delta_B)/\xi_B]^{\frac{1}{1-\varsigma}} L_t \tag{56}$$

Using the labor market constraint, we then get,

$$L_t^y = L_t - \left(L_t^A + L_t^B\right) = L_t \left[1 - \left(\frac{\delta_A}{\xi_A}\right)^{1/(1-\varsigma)} - \left(\frac{g+\delta_B}{\xi_B}\right)^{1/(1-\varsigma)} \right] \tag{57}$$

Given (r, g, n) and L_t, equations (55) – (57) fully determine the equilibrium allocations in the R&D and labor-intensive goods sectors. They also validate our prior assumption that L_t^A/L_t, L_t^B/L_t and L_t^y/L_t will be constant on the BGP.

Finally, using (50) in (54) we arrive at the equilibrium labor allocation for the labor intensive intermediate good, y_l:

$$L_t^y = \left[\Gamma_L(r,g,n)\xi_B(1-\varsigma)\right]^{-1} (L_t^B/L_t)^{\varsigma} L_t = \left[\Gamma_L(r,g,n)\xi_B(1-\varsigma)\right]^{-1} [(g+\delta_B)/\xi_B]^{\frac{\varsigma}{1-\varsigma}} L_t \tag{58}$$

Now substituting (58) back in (57) gives two equations that define an equilibrium condition in two unknowns, g, and r, given L_t:

$$\left[\Gamma_L(r,g,n)\xi_B(1-\varsigma)\right]^{-1} [(g+\delta_B)/\xi_B]^{\frac{\varsigma}{1-\varsigma}} = 1 - (\delta_A/\xi_A)^{\frac{1}{1-\varsigma}} - [(g+\delta_B)/\xi_B]^{\frac{1}{1-\varsigma}} \tag{59}$$

4.3 The Balanced Growth Path

It is well known that an economy with non-homothetic preferences does not have a balanced growth equilibrium with constant rates of growth. In this model, this means we need $\gamma_{2t} = \gamma_2 \; \forall t$. That is, asymptotically bequests are no longer a luxury good.

Let us define a normalized capital stock

$$\tilde{k}_t \equiv \frac{A_t^{\frac{1-\alpha}{\alpha}} K_t}{B_t^{\frac{1-\alpha}{\alpha}} L_t^y} \equiv \frac{M_t K_t}{Z_t L_t^y}.$$

Along a BGP $\tilde{k}_t$ is constant. From (47) we see that this means $p_t \equiv P_{Kt}/P_{Lt}$ is constant. Since the price of aggregate output has been normalized to one, from (32) and (33), both P_{Kt} and P_{Lt} will be constant in the BGP. We conjectured earlier that the interest rate is constant along a BGP, which requires the rental rate to be constant too. From (41) $\rho_t = \alpha A_t^{\frac{1-\alpha}{\alpha}} P_{Kt}$,which is constant since A_t is constant and if P_{Kt} is constant. Finally, from (41) and (42)

$$\frac{w_t}{\rho_t} = \left(\frac{P_{Lt}}{P_{Kt}}\right)\left(\frac{B_t}{A_t}\right)^{(1-\alpha)/\alpha} \tag{60}$$

Since prices of capital- and labor-intensive goods, the rental rate and A_t are constant in the BGP, the wage rate increases at $(1+g)^{(1-\alpha)/\alpha}$. The household decision rules, when normalized by Z_t are stationary.[10] More formally,

Definition 1 *A balanced growth path (BGP) of this economy is a long-run equilibrium path ($t \to \infty$) where*

- *The number of labor-augmenting intermediate inputs (B_t) grows at a constant rate $g > 0$ while the number of capital-augmenting intermediate inputs (A_t) remains constant,*
- *Aggregate labor (L_t) grows at the exogenous rate of population growth $n \geq 0$ while aggregate capital (K_t), output (Y_t) and consumption (C_t) all grow at the rate $\hat{g} = (1+n)(1+g)^{\frac{1-\alpha}{\alpha}} - 1$,*
- *The effective capital-labor ratio ($M_t K_t / Z_t L_t^y$) and labor allocation shares in the different sectors (L_t^A / L_t and L_t^B / L_t) are constant, and*
- *The rental rate of capital (ρ_t), the prices of capital- and labor-intensive goods (P_{Kt}, P_{Lt}) and the relative price of these intermediate goods (p_t) are constant, while the wage rate per effective unit of labor (w_t) grows at the gross rate $(1+g)^{(1-\alpha)/\alpha}$.*

[10] Note that for bequest to remain a luxury good in the long run, γ_{2t} has to grow at the rate of consumption. Hence we specify $\gamma_{2t} = \gamma_2 Z_t$.

4.4 The Effects of Population Aging on Long-Run Growth

Several partial and general equilibrium effects are embedded in the model by which an aging population affects the macroeconomy. To see the equilibrium effects more clearly, take the equilibrium factor price ratio ρ_t / w_t from equation (60) above and substitute for the relative price of capital- and labor-intensive goods from (47) to get

$$\frac{\rho_t}{w_t} = \left(\frac{\kappa}{1-\kappa}\right)\left[\frac{A_t}{B_t}\right]^{\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\varepsilon-1}{\varepsilon}\right)}\left[\frac{K_t}{L_t^y}\right]^{-\frac{1}{\varepsilon}}$$

This is the inverse relative factor demand curve, decreasing in the factor ratio K/L^y and in the relative productivity A/B (for $\varepsilon < 1$).

First of all, households tend to save more as a result of extension of longevity, increasing supply of capital. The higher need for old-age saving detracts from bequest and human capital formation. Hence effective labor supply goes down at a given population growth rate. Second, if increased longevity is also accompanied by lower fertility, effective labor supply shrinks even more relative to capital execerbating the initial effect of longevity extension. These changes tend to drive innovation towards capital-intensive goods. Secondly there is the effect of higher longevity on the supply of capital. This tends to reinforce the first effect by making capital more abundant and cheaper relative to labor. The third effect, the human capital channel, depends on whether households are credit constrained. Higher old-age longevity, in this case, raises the effective supply of the workforce which, *ceteris paribus*, would shift innovation towards labor-intensive goods.

We have established above that in the steady state, capital intensive innovations have only a level effect on output per worker, while labor-augmenting innovations can affect the steady-state growth rate. This means, the first two effects identified above would tend to raise the level of output per worker but, by themselves, may not help growth in the long run. For that one has to rely on the human capital channel, whose ability to make an appreciable difference depends on what proportion of the household sector is credit-constrained and how severely.

5 Quantitative Results (incomplete)

We start by calibrating the benchmark economy. Our quantitative assessment of the macroeconomic effects of population aging will focus on the long run, that is, the balanced growth path. We begin by calibrating the benchmark economy, the U.S. in 1970. Table 1 reports the assigned parameter values. Subsequently we "shock" the economy by extending old-age longevity and lower fertility to replacement levels and examine the outcomes. Model predictions with and without directed technological change are compared to ascertain to what extent the technology response can compensate for the conventional costs of population aging.

5.1 Parameter Values

We assume that childhood lasts for 20 years in our model and each of the other periods for 25 years. That sets maximum lifetime at 95 years, well within current projections into the next century.[11] Taking 65 to be the mandatory retirement age, we then set $\varpi = 0.8$. Between 1965–1970, the US total fertility rate was 2.58, or 1.29 in terms of our model agents. Hence we set the birth rate to be 29%. US life expectancy at birth was 70.3 years in 1970 (UN 2012) which implies $\phi = 0.012$.

Household	**Production**
$\beta = 0.34$	$\varepsilon = 0.95$
$\gamma_1 = 0.80$	$\kappa = 0.90$
$\gamma_2 = 0$	$\alpha = 0.35$
$\phi = 0.012$	$\xi_A = 0.33$
$\varpi = 0.8$	$\xi_B = 0.72$
$\mu = 0.54$	$\delta_A = 0.29$
$\theta = 0.80$	$\delta_B = 0.03$
$\delta_h = 0.5$	$\zeta = 0.50$
$\bar{v} = 1.00$	
$\sigma_v = 0.49$	
$n = 0.30$	

Table 1: Parameter Values

For the subjective discount rate, we follow the business cycle literature in picking a value of 0.99 for quarterly data or 0.99^{100} for each model period. The parameter γ_1 determines the intensity of warm-glow altruism. We set it equal to 0.8.

The distribution of abilities $\mathscr{F}(v)$ is assumed to be Normal with mean $\bar{v} = 1$.[12] We pick the standard deviation $\sigma_v = 0.49$.Since we focus on the balanced growth path, not transitional dynamics, it is not necessary to parameterize the initial distribution of endowments $G_1(a_{11}, h_1)$. Instead, we start with a guess of the distribution of agents over the steady-state endowment vectors and iterate using optimal policy rules until we converge to the stationary distribution. For μ, we follow two early contributions on credit constraints in the US economy. Using PSID data, Hall and Mishkin (1982) estimate that 20% of US families were liquidity constrained in the 1970s. This is similar to Jappelli's (1990) estimate that 19% of US households were credit constrained in the 1983 Survey of Consumer Finances. In our model this percentage depends on the severity of the borrowing constraint μ and the underlying distribution of household wealth. We set $\mu = 0.54$ so that about 19.5% of households are credit constrained in the steady state.

[11] Japan, for example, is projected to have $e_0 = 93.3$ by 2100-2105 (UN 2004, Table 11).

[12] Households with productivity draws below zero are all imputed the lowest value of v, which is positive, on the grid.

The parameter δ_h is the depreciation rate of human capital for workers with college education. We use $\delta_h = 0.5$ over 25 years, which is close to the estimate of 2.17% depreciation per year in Ionescu (2009). Browning *et al* (1999) report a range of estimates for θ, the elasticity of human capital with respect to college time investment, between 0.5 and 0.9. We use an estimate of 0.8.

Turn next to the production parameters. Most multi-sector models of technological change, particularly applications of Acemoglu's model of directed technological change, feature goods that differ in their skill intensity alone. Quantitative applications of Acemoglu (2003) are sparse, if non-existent. We follow closely the skill-biased technology literature for parameter choices, unless the choice is obviously flawed.

The elasticity of substitution between capital- and labor-intensive outputs in the aggregate production function is given by ε. Capital and labor are gross substitutes if $\varepsilon > 1$, and complements if $\varepsilon < 1$. Estimates for this elasticity vary in the literature. We present baseline results for $\varepsilon = 0.95$ and robustness checks (coming soon) for $\varepsilon = 1.09$ and 2.25.

The value of κ determines the share of capital-intensive goods in aggregate output. Based on OECD data, Jin (2012) calibrates the share of capital-intensive goods in total value-added as 0.6 and the share of labor-intensive goods as 0.4. Accordingly, we choose $\kappa = 0.6$. Jin also calibrates the capital share to be 0.52 and 0.11 in the capital- and labor-intensive sectors respectively. Our production technologies do not match the ones used in his study. In the absence of suitable alternatives we pick $\alpha = 0.35$, the weighted average of Jin's estimates and not that far from the conventional capital share estimate of one-third.

Finally, consider the innovation frontier. Estimates of the obsolescence rate of knowledge capital are typically significantly higher than depreciation rates for physical capital. Pakes and Schankerman (1984), for instance, estimate the annual rate to be 25% on average, while Nadiri and Prucha (1996) estimate it to be about 12%. These compute to 96-100% depreciation rates in our model. In contrast, depreciation rates of physical capital are about 10% on an annual basis (Nadiri and Prucha, 1996) or 92% over the quarter century that defines each model period. Since we have assumed full depreciation of physical capital in the model, to be consistent with the evidence we also assume full depreciation of knowledge capital each model period.

Research productivities ξ_A and ξ_B are scaling parameters. For the ratio of researcher productivities ξ_B/ξ_A, we pick a value of 2.2 from Schaefer (2010) and set their individual values to to get a steady-state annual growth rate of 2% for GDP per worker,[13] or 64.06% over 25 years.

5.2 Aging Shocks

Now consider an aging shock to the benchmark economy where there is an exogenous change in the old-age survival and fertility rates to $\phi' = 0.80$ and $n' = 0$. Since the depreciation rates of

[13] This value is based on the US average annual growth rate for 1900-1970.

knowledge and physical capital are hundred percent and the effect of ϕ is felt immediately (recall Figure 1), the economy would adjust immediately to a new steady-state if the only shock were to old-age survival. A change in the fertility rate takes three generations to be fully felt in the model, so it would be associated with transitional dynamics. We focus solely on the properties of the steady state.

The growth rate of labor-augmenting technological progress, hence output per capita, falls from 0.64 to 0.56. On an annual basis that's a drop in the growth rate from 2% to 1.79%. That implies output per capita doubles every 39 years instead of 35. Over an individual's lifetime that is not an enormous welfare loss but, of course, with time the cumulative loss will be substantial. The drop in the relative price of capital-intensive goods indicates that the higher supply of physical capital dominates price changes, in particular, changes emanating from fewer workers but higher human capital. Finally note that in both economies the share of *effective* labor is similar.

Under a life expectancy shock, aggregate saving and hence, aggregate capital rises. Under credit constraints, it also raises human capital investment in youth which raises the supply of labor input. The first effect dominates however: Y_K/Y_L rises and lowers the relative price P_K/P_L. Since the relative return to capital-intensive intermediate goods falls, it drags down the profitability of producing A-type blueprints. In other words, the "price effect" dominates the market size effect as should be when $\epsilon < 1$. The effect of the fertility shock is similar with one difference: capital accumulation rises because parents allocate more towards their future consumption than to their (fewer) children's. Schooling and overall human capital fall, lowering labor input, L_A/N and L_B/N. In other words, in this case the final goods-sector attracts more workers, not the R&D sector. As a result growth falls. The effect of fertility decline turns out to be the more dominant channel by which growth falls. Higher life expectancy by itself lower growth by only 0.03 percentage points annually. Lower fertility, on the other hand, costs the economy 0.72 percentage points of annual growth.(INCOMPLETE)

6 Conclusion

The paper presents an R&D-led endogenous growth model to analyze the long-run growth effects of population aging. The key mechanism in the model lies in the responsiveness of technological innovation to changing demographics caused by extension of longevity and falling fertility. We find that despite increasing dependency ratio, brought about by exogenous decline in old-age mortality, the long-run growth prospects are not compromised. The reason is that with fixed fertility rate and retirement age, improvements in old-age survival rates unambiguously increase private saving and supply of capital. This makes capital cheaper and production of capital-using machines more profitable. Technological innovation in the model is therefore capital-biased. This process can be further aided by greater accumulation of skill in response to longer life-time as well as higher wage-rental ratio. Our quantitative exercise shows that with capital-skill complementarity in production, long-run growth rate may falter, but much less than commonly believed under the assumption of exogenous technological progress.

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