Social Reform as Path to Political Leadership: A Dynamic Model*

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Abstract

Leader that wishes to overthrow an unpopular ruling government chooses the optimal strategy of its opposition. Every period the leader chooses the nature of its opposition. Opposition can either be in the form of a political or a non political protest. The non-political protest does not threaten the existence of the present regime, whereas a political protest can. A leader is characterised by her intentions- which can be political or non political and her ability. The success of a protest depends upon the unknown ability of the leader and the strength of mass participation that the leader can garner. We find that for intermediate ranges of the ability of the leader, the leader with politic to follow a strategy of gradualism in which it undertakes non political protest initially to favorably update the belief about his ability and mobilize a higher participation for the political protest. For very low and high values of the ability of the leader, it is optimal to do the political protest in the first period.

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1 Introduction

> *"Effective leadership is putting first things first. Effective management is discipline, carrying it out."*- Stephen Covey

A leader is an architect of change. Leaders or heads of organizations, be it political parties, corporates or any institution play an important role in choosing optimal actions and coordinate with the followers to bring about the desired change. We observe substantial variation in the outcomes of organizations depending upon the ability of the leader. Some leaders be it in business or politics are better able to manage resources and direct the followers effectively and hence achieve the desired change while others fail. Apart from an individual's leadership ability, one cannot be a leader without followers. The most important aspect of successful leadership in any organization or setting is to have a sufficient pool of dedicated followers. However the question is then how does a leader able to draw a set of dedicated followers to bring about a successful change.

It is widely agreed that the "Salt March" by Mahatma Gandhi in 1930 was the first shot that eventually brought down the British Empire in India. However, Gandhi's effectiveness in transforming a novel protest into a broad movement for change was also driven his ability to draw on a cadre of followers that he had attracted by this time (Dalton, 1993). The question is how was he able to draw this pool of followers. Looking back at history, Gandhi's first great experiment in *Satyagraha* came in 1917, in Champaharan in Bihar, followed by Kheda satyagraha (1918) and then the Ahmedabad Mill workers strike (1918) and none of these events were a direct revolt against the British regime and hence a threat to their existence. However Gandhi emerged as one of the most popular and acceptable figure in Indian politics by his technique of mass mobilization through smaller protests that he initially undertook after coming back to India in 1915. Turning to modern India, Arvind Kejriwal formed a new political party named the Aam Admi Party (AAP) and is now the chief minister of Delhi where his party swiped the assembly elections winning 67 seats out of 70 in 2015. However Kejriwal started his career as a leader with formation of a movement named "Parivartan" in December 1999 which addressed citizens' grievances related to Public Distribution System (PDS), led many other smaller protests by filing public interest litigation (PIL) demanding transparency in public dealings of the Income Tax department and then in 2011 joined several other activists to form the India Against Corruption (IAC) group. By this time he was successful in gathering enough momentum to have a dedicated pool of followers which he leveraged to contest the assembly elections in which his party won with a massive mandate. On the other hand, the Lok Satta party started by Jayaprakash Narayan in 2006 which wanted to project itself as an alternative in Indian politics has hardly been successful.

In the examples above on Gandhi and Arvind Kejriwal, the leaders took a strategy of gradualism through which they were successful in mobilizing the mass before attacking the regime directly. On the other hand leaders can also choose to attack the regime directly rather than following a process of gradualism. In this paper, we show that under what conditions it might be optimal to take a gradual path and then announce a revolution against the regime versus announcing a revolution against the regime immediately.

In this paper there are two types of leaders - a "political" leader and a "non-political" leader. The political leader, P, aims at overthrowing the present regime while the non-political leader, NP, is one who aims at protesting against social injustice and tries to bring about social reforms. We call the protest to overthrow the regime as a "revolution" and a protest against social injustice and reforms as "social protest". A leader with a political objective, despite her aim being to overthrow the present regime might still undertake social protest initially to favorably update the belief about his ability and mobilize a higher participation for the revolution. The underlying assumption is that revolution directly threatens the existence of the regime while any social protests do not directly threaten the existence of the regime.

In this model, there are three types of agents - the present regime or the Government, a Leader and a unit mass of citizens. We assume that there are two types of leaders who have different objectives or motives - a leader with a social objective (NP) who never intends to overthrow the regime. However, a leader with a political motive (P) can choose to do so. A leader can also be of two different abilities, high and low. Given the same resources a high ability leader is able to manage more efficiently and hence has a higher probability of success in a small protest or revolution as compared to a low ability leader. The probability of success in a small protest or revolution depends upon the unknown ability of the political leader and mass participation. In this paper the leader is assumed to be inexperienced and does not know his own ability. However the objective is known to the leader privately. All players in the society have initial priors about the objective as well as about the ability of the leader. The political leader might still do a social protest because upon success in the social protest, the beliefs about his ability is revised upwards and hence helps her to mobilize more masses in future which ultimately helps in overthrowing the present regime by announcing a revolution. The mass is assumed to be myopic and enjoys some benefit from a successful small protest and revolution but also bears a cost of participation in either of the movements. We assume that the objective of the leader P is aligned with the broader populace and wants to overthrow the present regime. Hence the mass enjoys a higher payoff from a successful revolution as compared to a successful social protest.[1]

The Government can exert force to suppress a revolution and also a social protest but

[1]In the background it is assumed that the leader has enforcement as well as persuasive powers.

is costly to do so. The problem that the present regime faces is that if there is a social protest, then it does not know with certainty whether it is by a leader, NP or it is by a leader with a political objective, P and is being used as a device to mobilize mass. If the case is the former and the government knows with certainty, then it does not need to exert any force while it would probably like to suppress the movement if it is by a leader with a political objective. In this paper we solve for a two period model and we characterize the equilibrium.

2 Model

There are three types of agents - government (G), leader of a movement (L) and citizens, (C). The leader does not belong to the government but can overthrow the government by garnering sufficient support from the citizens. The leader has two characteristics- efficiency in executing a movement, θ and a motive for conducting a movement, ζ. The leader's efficiency can either be high, θ_H or low, θ_L, i.e. $\theta \in \{\theta_H, \theta_L\}$. The actual efficiency of the leader is not known to either the government or the masses. To begin with we assume that the leader is inexperienced, i.e. he does not know his own efficiency. [2] The common initial prior that the political leader is of high type is α_1 i.e., $Pr(\theta = \theta_H) = \alpha_1$. The leader's motive of conducting a movement can either be political, $\zeta = P$ or non-political, $\zeta = NP$. A leader with a non-political objective, $\zeta = NP$, never intends to overthrow the government. However, a leader with a political motive, $\zeta = P$ can choose to do so. ζ is known to the leader but unknown to others. Let β_1 be the common initial prior that the leader is non-political, i.e. $Pr(\zeta = NP) = \beta_1$. We denote the type of the leader by $\tau = \theta \times \zeta \in \mathbb{T}$, where $\mathbb{T} = \{\theta_H, \theta_L\} \times \{P, NP\}$.

We consider a two-period model. At the begining of each period, $t \in \{1, 2\}$ the leader of type, τ, chooses the nature of movement it conducts, a_t. The movement can either be a revolution, r or a social protest, s. A successful revolution overthrows the government. A successful social protest never does so. Upon hearing L's announcement in period t, the government and citizen's update their belief about the leader's objective, $\hat{\beta}_t$.[3] Next, the government announces the extent or level of force with which it combats the leader's announced movement, $g_t \in \{0, W\}$. We assume that the government can either put no effort, $g_t = 0$ or maximum effort, $g_t = W$. After observing the nature of the movement, a_t and government's force, g_t, each citizen decides either to partcipate, p or not participate, np in the announced movement in that period. Each citizen bears a private cost of

[2] In Section we solve the game when leader knows his own efficiency.

[3] The nature of movement announced by the leader does not reveal anything about the efficiency of the leader. The prior about the efficiency of the leader changes only upon the success or failure of the movement, as described below.

participating in the movement, $e_i \sim U[-e_L, e_H]$. We allow the private cost of participation to be negative, implying a positive utility to the citizen from participation in the protests, irrespective of the outcome of the movement. Citizens also bear a common cost equal to the force implemented by the government, g_t. Thus, the total cost of participating in a movement for a citizen is $c_i = e_i + g_t$. Let the number of citizens who choose to participate in the movement at period t be m_t. Once the participation in the movement has been decided, nature determines the success or failure of the movement, i.e. $\gamma_t \in \{S, F\}$. The success of the movement announced at t, depends upon the quality of the leader, θ and the mass of citizens that participate in the movement, m_t, i.e. $Pr(\gamma_t = S) = \theta m_t$. The success or failure of the movement is common knowledge at the end of each period. Upon revelation of γ_t, the common prior about the quality of the political leader is updated at the end of every period, i.e. $\hat{\alpha}_t$. The leader that announces a revolution in period 1, does not get a chance to conduct any movement in the subsequent period. i.e. If a revolution is announced in period 1, $a_1 = r$, the game ends after the success or failure of the movement is revealed. However, a leader that announces a social protest in period 1, $a_1 = s$ can announce a movement in the second period. Let the prior about the ability and intention of the leader at the beginning of the second period be $\alpha_2 = \hat{\alpha}_1$, and $\beta_2 = \hat{\beta}_1$, respectively.

Let $h_t = (a_t, g_t, m_t, \gamma_t)$ be the public history at the beginning of time period t, with $h_0 = \phi$ and $\mathbb{H}_t$ be the set of all possible histories at the beginning of time period t, where, a_t is the nature of the movement chosen by the leader of type τ, g_t, is government's force, m_t, fraction of mass that participated, and γ_t, the success or failure of the movement in the period. The time line of the game is as follows. At the beginning of the game, the type of the leader is determined, $\tau = \theta \times \zeta$. After observing a_t, at the beginning of every period, the leader chooses the nature of the movement, a_t. Upon seeing the nature of movement, prior on the intention of the leader is updated, $\hat{\beta}_t$. Next, the government chooses force to combat the movement, g_t. Citizens observe, a_t and g_t and decide whether to participate in the movement or not. Depending upon the ability of the leader, θ and citizen participation, success of the movement is determined, γ. All agents observe, γ and update their prior about leader's ability, $\hat{\alpha}_t$.

Payoffs and Strategies

Ex-ante per period utility of a leader depends upon its type $\tau = (\theta, \zeta)$, nature of movement announced, a_t, and the success or failure of the movement, γ_t. The ex-ante per period utility of a leader with political intentions, $\zeta = P$ is given as follows:

$$U_t^L(\tau = (\theta, P), a_t, \gamma_t) = \begin{array}{llll} 0 & if \quad a_t = s & \& & \gamma_t = S/F, \quad \forall\theta \\ 0 & if \quad a_t = r & \& & \gamma_t = F, \quad \forall\theta \\ W & if \quad a_t = r & \& & \gamma_t = S, \quad \forall\theta \end{array}$$

The ex-ante per period utility of a leader with non-political intentions, $\zeta = NP$ is given as follows:

$$U_t^L(\tau = (\theta, NP), a_t, \gamma_t) = \begin{array}{llll} W & if \quad a_t = s & \& & \gamma_t = S, \quad \forall\theta \\ 0 & if \quad a_t = s & \& & \gamma_t = F, \quad \forall\theta \\ 0 & if \quad a_t = r & \& & \gamma_t = S/F, \quad \forall\theta \end{array}$$

A leader that has political objectives, $\zeta = P$ derives a positive payoff only from a successful revolution and gains nothing from a social protest. However, a leader that has non political objectives, $\zeta = NP$ derives positive payoff only from a successful social protest.[4] It gains nothing from conducting a revolution that overthrows the government. The utility derived by the leader is independent of its efficiency. The cost of implementing a movement is assumed to be zero irrespective of the type of the movement and the quality of the leader. A pure strategy of the leader of type $\tau \in \mathbb{T}$ at time period $t \in \{1, 2\}$ is a function $\sigma_t^\tau : \mathbb{H} \rightarrow [0, 1]$ that maps for every history, $h_{t-1} \in \mathbb{H}$ to a probability that the leader would take a social movement, $a_t = s$ at time period t.

Ex-ante utility of the government depends upon the the nature of the movement announced, a_t, the extent of force announced by the government in that period, g_t, and the success or failure of the movement, γ_t. The ex-ante per period utility of the government, that exerts a force, g_t is given as follows:

$$U_t^G(a_t(\tau), g_t, \gamma_t) = \begin{array}{llll} W - cg_t & if\, a_t(\tau) = s & \& & \gamma_t = S/F \\ W - cg_t & if\, a_t(\tau) = r & \& & \gamma_t = F \\ 0 - cg_t & if\, a_t(\tau) = r & \& & \gamma_t = S \end{array}$$

We assume that the government can be thrown out of power only if the movement is a successful revolution. We assume that the benefit from being in power is the same for the government and political leader. The government incurs a cost, cg_t for implementing

[4]Intituition for this

force g_t, where $c \in [0,1]$. A pure strategy of the government at time period t is a function $G_t : \mathbb{H} \times \{r,s\} \to [0,1]$ that maps for every history, $h_{t-1} \in \mathbb{H}$ and announcement of the political leader, $a_t \in \{r,s\}$, to a probability that the government will use force of level, $g_t = W$ at time period t. The leader and the government, discount the future with the same discount factor, $\delta \in [0,1]$.

Ex-ante utility of the citizen depends upon the the nature of movement announced, a_t, and the success or failure of the movement, γ_t and is given as follows.

$$U_t^C(a_t, \gamma_t) \quad = \quad W \quad if \quad a_t = r, s \qquad \& \qquad \gamma_t = S$$

We assume that the benefit to the citizen derives positive utility only from any successful revolution is W. The benefit to the citizen from a regime change is equal to the benefit from being in power to the leader and government. The failure of any movement gives the citizens a benefit of 0. We assume that citizens are myopic. The pure strategy of a citizen of type $e_i \in [-e_L, e_H]$ at time period t depends upon nature of movement in time period t, $a_t \in \{r,s\}$ and government effort in time period t, $g_t \in \{0, W\}$. Thus, the pure strategy of a citizen is a function $\Omega_t : \mathbb{H} \times \{r,s\} \times \{0,W\} \times [-e_L, e_H] \to \{p, np\}$ that maps, a_t, g_t to an action, $\{p, np\}$ of the citizen of type e_i. Citizens decide to participate in a movement at time period t if their current period payoff is greater than the cost of doing so in that period. We assume $e_L > W$ and $e_H > \theta_H W$. [5]

Updating

Leader's Objective, β

Nature of movement, a_t announced by the leader at time period t, reveals private information about his/her intentions or objective. It does not provide any further information about the efficiency or quality of the leader in executing a movement. After hearing the nature of movement, a_t, the updated belief about the intention of the leader at time period t, is defined as

$$\begin{aligned}
\hat{\beta}_t = \hat{\beta}_t(h_{t-1}, a_t) = Pr(\zeta = NP | h_{t-1}, a_t) \quad &= \quad \frac{\sigma_t^{NP}\beta_t}{\sigma_t^{NP}\beta_t + \sigma_t^{P}(1-\beta_t)} \quad if \quad a_t = s \\
&= \quad \frac{(1-\sigma_t^{NP})\beta_t}{(1-\sigma_t^{NP})\beta_t + (1-\sigma_t^{P})(1-\beta_t)} \quad if \quad a_t = R
\end{aligned}$$

Given the payoffs and the fact that there is no cost of a revolution to a leader, a leader with non-political objective, i.e. $\zeta = NP$ will always call for a non-political protest in

[5]This assumption ensures that for any type of movement at every period there is a non degenerate fraction of mass participation.

both periods. i.e. $\sigma_t^{NP} = 1$ and a leader with a political objective, i.e. $\zeta = P$ will always announce a revolution in the second period, i.e. $\sigma_2^P = 0$. Thus,

$$\hat{\beta}_2(h_1, a_2 = s) = 1$$

and

$$\hat{\beta}_t(h_{t-1}, a_t = R) = 0 \quad \forall t \in \{1, 2\}$$

Let, $\beta_t = \hat{\beta}_{t-1}$.

Leader's Efficiency, α

At the end of every period, common prior about the efficiency of the political leader is updated after observing the nature of the movement, a_t and its success or failure γ_t, which inturn depends upon the observed, force of the government, g_t and the mass participation, m_t.[6]

$$\hat{\alpha}_t = Pr(\theta = \theta_H|h_t) \quad = \quad \frac{Pr(\gamma_t|\theta = \theta_H, a_t, m_t, g_t)Pr(\theta = \theta_H)}{Pr(\gamma_t|\theta = \theta_H, a_t, m_t, g_t)Pr(\theta = \theta_H) + Pr(\gamma_t|\theta = \theta_L, a_t, m_t, g_t)Pr(\theta = \theta_L)}$$

If the first period movement is a success, i.e. $\gamma_1 = S$, the updated belief about the quality of the leader at the end of the period is given by

$$\hat{\alpha_1} = \alpha^S(\alpha_1) = Pr(\theta = \theta_H|h_1 = (a_1, g_1, m_1, \gamma_1 = S)) = \frac{\theta_H \alpha_1}{\theta_H \alpha_1 + \theta_L(1 - \alpha_1)}$$

If the first period movement is a success, i.e. $\gamma_1 = F$, the updated belief about the quality of the leader in the second period is given by

$$\hat{\alpha_1} = \alpha^F(\alpha_1, g_1) = Pr(\theta = \theta_H|h_1 = (a_1, g_1, m_1, \gamma_1 = F)) = \frac{\alpha_1[1 - \theta_H m_1(g_1)]}{\alpha_1[1 - \theta_H m_1(g_1)] + (1 - \alpha_1)[1 - \theta_L m_1(g_1)]}$$

It is interesting to note that $\alpha^S(\alpha_1)$ is independent of the level of mass participation and government effort. However, $\alpha^F(\alpha_1, g_1)$ depends on the level of mass participation which in turn depends on the level of governement's force in period 1. We solve for pure strategy Perfect Bayesian Equilibrium (PBE) for this game.

3 Analysis

We first consider the decision of a citizen to partcipate in a movement, a_t annouced by the leader at time period t. Expected payoff of each participant of type, e_i from participating

[6]The non political leader is assumed to be of High type. Hence, the success or failure of the movement is not informative about the non political leader's efficiency.

in a movement, a_t at time period t is given by

$$Pr[\gamma_t = S \mid a_t, g_t, \alpha_t, \hat{\beta_t}]W - c_i$$

where where $c_i = e_i + g_t$ is the cost of participation in a movement. The probability of success of a movement is given by

$$\begin{aligned} Pr[\gamma_t = S \mid a_t, g_t, \alpha_t, \hat{\beta_t}] &= \sum_{\theta \in \{\theta_H, \theta_L\}} \sum_{\zeta \in \{P, NP\}} [Pr(\zeta \mid a_t, g_t) Pr(\theta \mid a_t, g_t, \zeta) Pr(\gamma_t = S \mid \theta; a_t, g_t)] \\ &= [(1 - \hat{\beta_t})(1 - \alpha_t)\theta_L + [(1 - \hat{\beta_t})\alpha_t + \hat{\beta_t}]\theta_H] m_t(a_t, g_t, \alpha_t, \hat{\beta_t}) \end{aligned}$$

A citizen i will partcipate only if

$$Pr[\gamma_t = S \mid a_t, g_t, \hat{\alpha}_t, \hat{\beta_t}]W - c_i \geq 0$$

Hence, the equilibrium level of participation in a movement of type a_t, given that the government announces force g_t, at any period t is given by

$$m_t^*(g_t, a_t, \alpha_t, \hat{\beta_t}) = \frac{e_L - g_t}{(e_H + e_L) - [(1 - \hat{\beta_t})(1 - \alpha_t)\theta_L + [(1 - \hat{\beta_t})\alpha_t + \hat{\beta_t}]\theta_H] V_{at}}$$

The equilibrium level of mass participation in period t decreases as government increases its effort level, i.e.

$$m_t^*(g_t = 0, a_t, \alpha_t, \hat{\beta_t}) > m_t^*(g_t = W, a_t, \alpha_t, \hat{\beta_t})$$

intuition Also, the equilibrium level of mass participation increases as the quality of the leader increases i.e., m_t^* increases with α_t for any given $\hat{\beta}_t$.*intuition*

3.1 Second Period

In this section we solve the last period of the game. Given the payoffs and the fact that there is no cost of a revolution to a leader, a leader with non-political objective, i.e. $\zeta = NP$ will always call for a non-political protest in both periods, $\sigma_t(NP) = 1, \forall t$. Similarly, a leader with a political objective, i.e. $\zeta = P$ will always announce a revolution in the second period, $\sigma_2(P) = 0$.

Now, consider the problem of the government in the second period. The government observes the nature of movement announced by the leader in period 2, a_2 and updates its belief about the objective of the leader, $\hat{\beta}_2$. Since, the government is not overthrown by a non-political movement and its payoffs are same irrespective of the success of a non-political movement, the government exerts no force when a non political movement is announced in the second period. i.e.

$$g_2(a_2 = s) = 0$$

However, if a revolution is announced in the second period i.e., $a_2 = R$ then the government updates its belief about the motive of the leader as political, $i.e.\hat{\beta}_2(a_2 = R) = 0$. The choice of government's force in the second period, g_2^* maximizes the following expected payoff

$$\begin{aligned} EU^G(g_2|a_2 = R; \alpha_2, \hat{\beta}_2 = 0) &= Max_{g_2} Pr[\gamma_2 = F \mid a_2 = R; \alpha_2, \hat{\beta}_2 = 0] U^G(a_2 = R, g_2, \gamma_2 = F) \\ & \quad -cg_2 \\ &= [(1-\alpha_2)(1-\theta_L m_2^*(a_2, g_2, \alpha_2, \hat{\beta}_2 = 0)) \\ &+ \alpha_2(1-\theta_H m_2^*(a_2, g_2, \alpha_2, \hat{\beta}_2 = 0))]W - cg_2 \end{aligned}$$

We can write the difference in expected utility of the government from exerting no effort, $g_2 = 0$ and maximum, $g_2 = W$ effort is as follows:

$$EU^G(g_2 = 0) - EU^G(g_2 = W) = \frac{-[(1-\alpha_2)\theta_L + \alpha_2\theta_H]W^2}{[(e_H + e_L) - [(1-\alpha_2)\theta_L + \alpha_2\theta_H]W} + cW$$

The difference in expected utility of the government from exerting no effort, $g_2 = 0$ and maximum, $g_2 = W$ is a continuous and decreasing function in α_2. Hence, there exists a threshold value of $\alpha_2 = \bar{\alpha}$, such that for all $\alpha_2 < \bar{\alpha}$, the government exerts $g_2 = 0$ while it exerts an effort $g_2 = W$ for all $\alpha_2 > \bar{\alpha}$. The value of $\bar{\alpha}$ is given by

$$\bar{\alpha} = \frac{1}{(\theta_H - \theta_L)}[\frac{c(e_H + e_L)}{W + cW} - \theta_L]$$

Given the assumptions on the parameters above, $0 < \bar{\alpha} < 1$.

Lemma 1. *In the second period, the leader with political objective i.e.,* $\zeta = P$ *announces a revolution*$(a_2 = R)$*while a leader with a non-political objective i.e.,* $\zeta = NP$ *annouces a non-political protest* $(a_2 = s)$*. Upon hearing,* $a_2 = s$*, the governement puts effort* $g_2 = 0$*. However, if* $a_2 = R$*, then the governement's effort is* $g_2 = W$ $\forall \alpha_2 > \bar{\alpha}$ *while* $g_2 = 0$*,* $\forall \alpha_2 \leq \bar{\alpha}$*.1*

3.2 First Period

In this section, we solve the first period problem. Since citizens are myopic, their problem remains the same as that in the second period. Hence, mass particpation in the first period, $m_1^*(a_1,\, g_1,\, \alpha_1, \hat{\beta_1})$ in a movement, a_1 when government puts force g_1, is determined in the same way as that in the second period.

We consider an equilibrium where the political leader follows a threshold policy. The threshold policy of a political leader is defined by endogenously determined thresholds $\alpha_L(\beta_1)$ and $\alpha_H(\beta_1)$ such that

$$\begin{aligned} \sigma_1(P) &= 0 \quad \forall \alpha_1 < \alpha_L(\beta_1) \\ &= 1 \quad \forall \alpha \in [\alpha_L(\beta_1), \alpha_H(\beta_1)) \\ &= 0 \quad \forall \alpha_1 \geq \alpha_H(\beta_1) \end{aligned}$$

Next, we determine government's optimal action in the first period, g_1. Government's optimal action, g_1, depends upon the nature of movement announced in the first period, a_1. If the leader announces a revolution in the first period $i.e. a_1 = R$, the government's optimal action is the same as that in the second period. Thus, government's optimal strategy in the first period $\forall \alpha_1 < \alpha_L(\beta_1)$ and $\forall \alpha_1 \geq \alpha_H(\beta_1)$ is as given in Lemma 1.

To determine the government's strategy in the first period when a non political movement is announced in the first period, $a_1 = s$, i.e. $\forall \alpha_1 \in \alpha_L(\beta_1), \alpha_H(\beta_1))$, we consider three thresholds of the initial prior, α_1: α_1^S, $\alpha_1^F(g_1 = W)$ and $\alpha_1^F(g_1 = 0)$. From the second period analysis, we know that the government puts force in the second period if and only if the updated belief at the beginning of the second period,$\hat{\alpha_2} > \bar{\alpha}$. Updated belief at the beginning of the second period depends upon initial prior α_1 and the history $h_1 = (a_1 = s, g_1, \gamma_1)$. Let α_1^S be the initial belief about the political leader's ability such that upon the success of a non political movement in the first period movement, the updated belief is equal to $\bar{\alpha}$. Similarly, $\alpha_1^F(g_1 = W)$ (and $\alpha_1^F(g_1 = 0)$) be the initial belief about the political leader's ability such that upon failure of a non political movement in the first period and government effort, $g_1 = W$ (and $g_1 = 0$), the updated belief is equal to $\bar{\alpha}$.

$$\alpha_2^S(\alpha_1^S) = \bar{\alpha}$$

$$\alpha_2^F(\alpha_1^F(g_1 = W)) = \bar{\alpha}$$

$$\alpha_2^F(\alpha_1^F(g_1 = 0)) = \bar{\alpha}$$

The following lemma describes the relation between these three thresholds.

Lemma 2. $\alpha_1^S < \bar{\alpha} < \alpha_1^F(g_1 = W) < \alpha_1^F(g_1 = 0)$

Proof. We first show that $\alpha_2^S - \alpha_2^F(g_1)$ is always positive for any given α_1.

$$\alpha_2^S - \alpha_2^F(g_1) = \frac{\alpha_1(1-\alpha_1)(\theta_H - \theta_L)}{[\theta_H\alpha_1 + \theta_L(1-\alpha_1)][(1-\theta_H m_1)\alpha_1 + (1-\alpha_1)(1-\theta_L m_1)]}$$

The denominator is always positive. Given that $\theta_H > \theta_L$, $\alpha_2^S > \alpha_2^F(g_1)$ Since $\frac{\partial \alpha_2^F}{\partial m_1} < 0$ and $\frac{\partial m_1}{\partial g_1} < 0$, then $\frac{\partial \alpha_2^F}{\partial g_1} = \frac{\partial \alpha_2^F}{\partial m_1}\frac{\partial m_1}{\partial g_1} > 0$. Hence,

$$\alpha_2^S > \alpha_2^F(g_1 = w) > \alpha_2^F(g_1 = 0)$$

Next, $\frac{\partial \alpha_2^S}{\partial \alpha_1} > 0$ Since these functions are increasing and $\alpha_2^S > \alpha_2^F(g_1 = w) > \alpha_2^F(g_1 = 0)$, then $\alpha_1^S < \alpha_1^F(g_1 = w) < \alpha_1^F(g_1 = 0)$. Since, $\theta_H > \theta_L$, it can be shown that $\alpha_1^S < \bar{\alpha} < \alpha_1^F(g_1 = w) < \alpha_1^F(g_1 = 0)$. □

The expected payoff of the government that chooses effort g_1, when the leader announces a non political movement in the first period, $a_1 = s$ is given as:

$$\begin{aligned}
EU^G(g_1, a_1 = s, \hat{\alpha_2}, \hat{\beta}_2) &= W - cg_1 + \delta[Pr(\zeta = P)[Pr(\gamma_1 = S \mid \zeta = P, a_1 = s) \\
&* \ [Pr(\gamma_2 = S \mid a_2 = R, \zeta = P, \gamma_1 = S, g_2, \hat{\beta}_2 = 0)(-cg_2)] \\
&+ \ [Pr(\gamma_2 = F \mid a_2 = R, \zeta = P, \gamma_1 = S, g_2, \hat{\beta}_2 = 0)(W - cg_2)]] \\
&+ \ [Pr(\gamma_1 = F \mid \zeta = P, a_1 = s) \\
&* \ [Pr(\gamma_2 = S \mid a_2 = R, \zeta = P, \gamma_1 = F, g_2, \hat{\beta}_2 = 0)(-cg_2)] \\
&+ \ [Pr(\gamma_2 = F \mid a_2 = R, \zeta = P, \gamma_1 = F, g_2, \hat{\beta}_2 = 0)(W - cg_2)]] \\
&+ \ Pr(\zeta = NP)W]
\end{aligned}$$

The difference in the expected payoffs of the government from exerting $g_1 = 0$ and $g_1 = W$, i.e., $EU^G(g_1 = 0) - EU^G(g_1 = W)$ depends upon the intial priors α_1 and β_1.

Range I: $\alpha_1 \in [\alpha_L, \alpha_1^S]$
In this range, α_1 is such that even if the non-political protest is succesful, the updated belief, i.e. $\alpha_2 S$ is less than $\bar{\alpha}$. Thus, the government will put zero effort in the second

period.

$$
\begin{aligned}
EU^G(g_1=0)-EU^G(g_1=W) = \;& cW+\delta W(1-\beta_1)[(\alpha_1\theta_H+(1-\alpha_1)\theta_L)* \\
& [1-(\alpha_2^S\theta_H+(1-\alpha_2^S\theta_L)m_2(g_2=0,\hat{\beta}_2=0,\alpha_2^S)]* \\
& [m_1(g_1=0)-m_1(g_1=W)] \\
& +[1-(\alpha_1\theta_H+(1-\alpha_1)\theta_L)m_1(g_1=0)]* \\
& [1-(\alpha_2^F(0)\theta_H+(1-\alpha_2^F(0))\theta_L)m_2(g_2=0,\hat{\beta}_2=0,\alpha_2^F(0)] \\
& -[1-(\alpha_1\theta_H+(1-\alpha_1)\theta_L)m_1(g_1=W)]* \\
& [1-(\alpha_2^F(W)\theta_H+(1-\alpha_2^F(W))\theta_L)m_2(g_2=0,\hat{\beta}_2=0,\alpha_2^F(W)]
\end{aligned}
$$

Range II: $\alpha_1 \in [\alpha_S, \alpha_1^F(g_1=W)]$
In this range, initial prior about political leader's efficiency is such that if the non-political protest is successful then the updated belief, i.e., α_2^S is greater than $\bar{\alpha}$. Thus, the government will exert effort $g_2=W$ upon a successful first period movement. However, if the non-political movement is unsuccessful then the updated belief is less than $\bar{\alpha}$ and government's effort in the second period would be $g_2=0$.

$$
\begin{aligned}
EU^G(g_1=0)-EU^G(g_1=W) = \;& cW+\delta W(1-\beta_1)[(\alpha_1\theta_H+(1-\alpha_1)\theta_L)* \\
& [[1-(\alpha_2^S\theta_H+(1-\alpha_2^S\theta_L)m_2(g_2=0,\hat{\beta}_2=0,\alpha_2^S)]-c]* \\
& [m_1(g_1=0)-m_1(g_1=W)] \\
& +[1-(\alpha_1\theta_H+(1-\alpha_1)\theta_L)m_1(g_1=0)]* \\
& [1-(\alpha_2^F(0)\theta_H+(1-\alpha_2^F(0))\theta_L)m_2(g_2=0,\hat{\beta}_2=0,\alpha_2^F(0)] \\
& -[1-(\alpha_1\theta_H+(1-\alpha_1)\theta_L)m_1(g_1=W)]* \\
& [1-(\alpha_2^F(W)\theta_H+(1-\alpha_2^F(W))\theta_L)m_2(g_2=0,\hat{\beta}_2=0,\alpha_2^F(W)]
\end{aligned}
$$

Range III: $\alpha_1 \in [\alpha_1^F(g_1=W), \alpha_1^F(g_1=0)]$
In this range, initial prior about political leader's efficiency is such that if the non-political protest in the first period is successful then the updated belief, i.e., α_2^S is greater than $\bar{\alpha}$. Thus, the government will exert effort $g_2=W$ upon a successful first period movement. However, if the non-political movement is unsuccessful then the updated belief crosses the threshold $\bar{\alpha}$ depending upon government's action in first period. If government puts no effort in period 1, i.e.$g_1=0$ the updated belief after an unsuccessful movement is

higher than $\bar{\alpha}$ but remains below $\bar{\alpha}$ if $g_1 = W$. Thus,

$$\begin{aligned} EU^G(g_1=0) - EU^G(g_1=W) = \; & cW + \delta W(1-\beta_1)[(\alpha_1\theta_H + (1-\alpha_1)\theta_L)* \\ & [[1-(\alpha_2^S\theta_H + (1-\alpha_2^S\theta_L)m_2(g_2=0, \hat{\beta}_2=0, \alpha_2^S)] - c]* \\ & [m_1(g_1=0) - m_1(g_1=W)] \\ & +[1-(\alpha_1\theta_H + (1-\alpha_1)\theta_L)m_1(g_1=0)]* \\ & [1-(\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)m_2(g_2=0, \hat{\beta}_2=0, \alpha_2^F(0)] \\ & -[1-(\alpha_1\theta_H + (1-\alpha_1)\theta_L)m_1(g_1=W)]* \\ & [1-c-(\alpha_2^F(W)\theta_H + (1-\alpha_2^F(W))\theta_L)m_2(g_2=W, \hat{\beta}_2=0, \alpha_2^F(W)] \end{aligned}$$

Range IV: $\alpha_1 \in [\alpha_1^F(g_1=0), \alpha_H]$
In this range the initial prior about the efficiency of the leader is sufficiently high such that irrespective of the success or failure of the first period non political movement, the updated belief is always greater than $\bar{\alpha}$. Thus, the government's effort in the second period is $g_2 = W$.

$$\begin{aligned} EU^G(g_1=0) - EU^G(g_1=W) = \; & cW + \delta W(1-\beta_1)[(\alpha_1\theta_H + (1-\alpha_1)\theta_L)* \\ & [[1-(\alpha_2^S\theta_H + (1-\alpha_2^S\theta_L)m_2(g_2=0, \hat{\beta}_2=0, \alpha_2^S)] - c]* \\ & [m_1(g_1=0) - m_1(g_1=W)] \\ & +[1-(\alpha_1\theta_H + (1-\alpha_1)\theta_L)m_1(g_1=0)]* \\ & [1-c-(\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)m_2(g_2=W, \hat{\beta}_2=0, \alpha_2^F(0)] \\ & -[1-(\alpha_1\theta_H + (1-\alpha_1)\theta_L)m_1(g_1=W)]* \\ & [1-c-(\alpha_2^F(W)\theta_H + (1-\alpha_2^F(W))\theta_L)m_2(g_2=W, \hat{\beta}_2=0, \alpha_2^F(W)] \end{aligned}$$

The following lemma describes the strategy of the government in the first period upon observing a non-political protest. The strategy of the government is crucially dependent on the marginal cost of exerting effort for the government, c.

Lemma 3. *Government's first period strategy upon observing a non political movement in the first period, $a_1 = s$, depends upon the marginal cost of exerting effort for the government, c.*

- *If $c > \bar{c}$, the government exerts no effort in the first period $g_1 = 0$ for all the ranges of initial belief of α_1.*
- *If $c < \bar{\bar{c}}$, then government exerts maximum effort, i.e. $g_1 = W$, in the range of initial belief, $\forall \alpha_1 \in [\alpha_1^S, \alpha_1^F(g_1 = 0)]$. For all other ranges of initial beliefs, government exerts no effort, i.e.$g_1 = 0$.*

The lemma suggests that if it is sufficiently costly for the government to exert effort then the government doesn't put any effort upon observing a non-political protest. On the other hand when the marginal costs of exerting effort are sufficiently low, then the government exerts effort for intermediate ranges of initial prior about the efficiency of the political leader, α_1.

Proof. First we refer to region I. We redefine equation3.2 as

$$EU^G(g_1 = 0) - EU^G(g_1 = W) = cW + \delta W(1 - \beta_1)[A(\alpha_1) + B(\alpha_1) - C(\alpha_1)]$$

Where

$$A(\alpha_1) = (\alpha_1\theta_H + (1-\alpha_1)\theta_L)[1-(\alpha_2^S\theta_H + (1-\alpha_2^S\theta_L)m_2(g_2 = 0, \hat{\beta}_2 = 0, \alpha_2^S)][m_1(g_1 = 0) - m_1(g_1 = W)]$$

$$B(\alpha_1) = [1-(\alpha_1\theta_H + (1-\alpha_1)\theta_L)m_1(g_1 = 0)][1-(\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)m_2(g_2 = 0, \hat{\beta}_2 = 0, \alpha_2^F(0))]$$

$$C(\alpha_1) = [1-(\alpha_1\theta_H + (1-\alpha_1)\theta_L)m_1(g_1 = W)][1-(\alpha_2^F(W)\theta_H + (1-\alpha_2^F(W))\theta_L)m_2(g_2 = 0, \hat{\beta}_2 = 0, \alpha_2^F(W)]$$

We can verify that $A(\alpha_1) + B(\alpha_1) - C(\alpha_1)$ is always positive for all values of α_1. Hence, $EU^G(g_1 = 0) - EU^G(g_1 = W) > 0$ in range I. Thus, the optimal strategy of the government in the first period is $g_1 = 0$.

Consider range II. As before we can redefine equation3.2 as

$$EU^G(g_1 = 0) - EU^G(g_1 = W) = cW + \delta W(1 - \beta_1)[\bar{A}(\alpha_1) + B(\alpha_1) - C(\alpha_1)]$$

We can verify that the expression $\bar{A}(\alpha_1) + B(\alpha_1) - C(\alpha_1)$ is monotone in α_1. The expression $EU^G(g_1 = 0) - EU^G(g_1 = W)$ evaluated at $\alpha_1 = 0$ is an increasing function in c.

$$\begin{aligned} EU^G(g_1 = 0)(\alpha_1 = 0, c) - EU^G(g_1 = W)(\alpha_1 = 0, c) \quad = \quad & cW + \delta W(1 - \beta_1) \\ & (\frac{W\theta_L}{e_H + e_L - (\beta_1\theta_H + (1 - \beta_1)\theta_L)V_s}) \\ & [\frac{\theta_L W}{e_H + e_L - \theta_L W} - c] \end{aligned}$$

We define c^1 such that $EU^G(g_1 = 0)(\alpha_1 = 0, c^1) - EU^G(g_1 = W)(\alpha_1 = 0, c^1) = 0$. Thus, $\forall c > c^1$, $EU^G(g_1 = 0)(\alpha_1 = 0, c) - EU^G(g_1 = W)(\alpha_1 = 0, c) > 0$. Similarly $\forall c < c^1$, $EU^G(g_1 = 0)(\alpha_1 = 0, c) - EU^G(g_1 = W)(\alpha_1 = 0, c) < 0$.
Evaluating the expression $EU^G(g_1 = 0) - EU^G(g_1 = W)$ at $\alpha_1 = 1$ gives an equation

increasing in c.

$$EU^G(g_1 = 0)(\alpha_1 = 1, c) - EU^G(g_1 = W)(\alpha_1 = 1, c) = cW + \delta W(1 - \beta_1) \frac{W\theta_H}{e_H + e_L - (\beta_1\theta_H + (1 - \beta_1)\theta_L)V_s} [\frac{\theta_H W}{e_H + e_L - \theta_H W} - c]$$

We define c^2 such that $EU^G(g_1 = 0)(\alpha_1 = 1, c^2) - EU^G(g_1 = W)(\alpha_1 = 1, c^2) = 0$. Thus, $\forall c > c^2$, $EU^G(g_1 = 0)(\alpha_1 = 1, c) - EU^G(g_1 = W)(\alpha_1 = 1, c) > 0$. Similarly $\forall c < c^2$, $EU^G(g_1 = 0)(\alpha_1 = 1, c) - EU^G(g_1 = W)(\alpha_1 = 1, c) < 0$.

Hence $\forall c > max\{c^1, c^2\}$, $EU^G(g_1 = 0)(\alpha_1, c) - EU^G(g_1 = W)(\alpha_1, c) > 0$ evaluated at $\alpha_1 = 0$ and $\alpha_1 = 1$. Given the expression $\bar{A}(\alpha_1) + B(\alpha_1) - C(\alpha_1)$ is monotone in α_1, $EU^G(g_1 = 0)(\alpha_1, c) - EU^G(g_1 = W)(\alpha_1, c) > 0$, $\forall \alpha_1 \in [0, 1]$ and $\forall c > max\{c^1, c^2\}$. Thus, government's optimal strategy is to exert no effort in the first period, i.e. $g_1 = 0$. By similar reasoning $\forall c < min\{c^1, c^2\}$, $EU^G(g_1 = 0)(\alpha_1, c) - EU^G(g_1 = W)(\alpha_1, c) < 0$, $\forall \alpha_1 \in [0, 1]$. Thus, government's optimal strategy is to exert maximum effort in the first period, i.e. $g_1 = W$.

Consider range III. As before we can redefine equation3.2 as

$$EU^G(g_1 = 0) - EU^G(g_1 = W) = cW + \delta W(1 - \beta_1)[\bar{A}(\alpha_1) + B(\alpha_1) - \bar{C}(\alpha_1)]$$

We can verify that the expression $\bar{A}(\alpha_1) + B(\alpha_1) - \bar{C}(\alpha_1)$ is monotone in α_1. The expression $EU^G(g_1 = 0) - EU^G(g_1 = W)$ evaluated at $\alpha_1 = 0$ is an increasing function in c. We define c^3 such that $EU^G(g_1 = 0)(\alpha_1 = 0, c^3) - EU^G(g_1 = W)(\alpha_1 = 0, c^3) = 0$. Hence $\forall c > c^3$, $EU^G(g_1 = 0)(\alpha_1 = 0, c) - EU^G(g_1 = W)(\alpha_1 = 0, c) > 0$. Similarly $\forall c < c^3$, $EU^G(g_1 = 0)(\alpha_1 = 0, c) - EU^G(g_1 = W)(\alpha_1 = 0, c) < 0$.

Thr expression $EU^G(g_1 = 0) - EU^G(g_1 = W)$ evlautaed at $\alpha_1 = 1$ is an equation increasing in c.

We define $c = c^4$such that $EU^G(g_1 = 0)(\alpha_1 = 1, c^4) - EU^G(g_1 = W)(\alpha_1 = 1, c^4) = 0$. Thus, $\forall c > c^4$, $EU^G(g_1 = 0)(\alpha_1 = 1, c) - EU^G(g_1 = W)(\alpha_1 = 1, c) > 0$. Similarly $\forall c < c^4$, $EU^G(g_1 = 0)(\alpha_1 = 1, c) - EU^G(g_1 = W)(\alpha_1 = 1, c) < 0$.

Hence $\forall c > max\{c^3, c^4\}$, $EU^G(g_1 = 0)(\alpha_1, c) - EU^G(g_1 = W)(\alpha_1, c) > 0$ evaluated at $\alpha_1 = 0$ and $\alpha_1 = 1$. Given the expression $\bar{A}(\alpha_1) + B(\alpha_1) - \bar{C}(\alpha_1)$ is monotone in α_1, $EU^G(g_1 = 0)(\alpha_1, c) - EU^G(g_1 = W)(\alpha_1, c) > 0$, $\forall \alpha_1 \in [0, 1]$. Thus, government's optimal strategy is to exert no effort in the first period, i.e. $g_1 = 0$. By similar reasoning $\forall c < min\{c^3, c^4\}$, $EU^G(g_1 = 0)(\alpha_1, c) - EU^G(g_1 = W)(\alpha_1, c) < 0$, $\forall \alpha_1 \in [0, 1]$. Thus, government's optimal strategy is to exert maximum effort in the first period, i.e. $g_1 = W$.

Define, $\bar{c} = max\{c^1, c^2, c^3, c^4\}$. $\forall c > \bar{c}$, $g_1 = 0$ for initial beliefs in the range II and III. Similarly, define $\bar{\bar{c}} = min\{c^1, c^2, c^3, c^4\}$. $\forall c < \bar{\bar{c}}$, $g_1 = W$.

Consider range IV and redefine equation3.2 as

$$EU^G(g_1 = 0) - EU^G(g_1 = W) = cW + \delta W(1 - \beta_1)[\bar{A}(\alpha_1) + \bar{B}(\alpha_1) - \bar{C}(\alpha_1)]$$

We can verify that the expression $\bar{A}(\alpha_1) + \bar{B}(\alpha_1) - \bar{C}(\alpha_1)$ is always positive for all values of α_1. Hence, $EU^G(g_1 = 0) - EU^G(g_1 = W) > 0$ in this range. Thus, the optimal strategy of the government is $g_1 = 0$.

□

Proposition 1. *If $\delta > \bar{\delta}$ and $c > \bar{c}$, leader with a political objective i.e. $\zeta = P$ follows a threshold policy in the first period such that*

$$\begin{aligned} \sigma_1(P) &= 0 \quad \forall \alpha_1 < \bar{\alpha} \\ &= 1 \quad \forall \alpha \in [\bar{\alpha}_1, \alpha_1^F(g_1 = 0)) \\ &= 0 \quad \forall \alpha_1 \geq \alpha_1^F(g_1 = 0) \end{aligned}$$

In the second period, the political leader announces a revolution, i.e. $\sigma_2(P) = 0$. Leader with a non-political objective i.e.,$\zeta = NP$ always announces non-political protest in both the periods, i.e.$\sigma_t(NP) = s, \forall t \in \{1, 2\}$ irrespective of α_1. The government, upon observing a non-political protest exerts no force in the first period, i.e. $g_1 = 0$ and follows a strategy according to Lemma 1 upon hearing a revolution in either period.

Proof. As stated in Lemma3, if $\delta > \bar{\delta}$ and $c > \bar{c}$, the government will exert an effort $g_1 = 0$. The expected payoff of a political leader that announces a revolution in the first period depends upon the initial common prior about the leader's efficiency. If $\alpha_1 < \bar{\alpha}_1$, the expected payoff of a political leader, when it announces a revolution is

$$\begin{aligned} H_0(\alpha_1, \beta_1 = 0) &= EU^R(\alpha_1, g_1 = 0) \\ &= \frac{[\alpha_1\theta_H + (1 - \alpha_1)\theta_L]e_L W}{[e_H + e_L - [\alpha_1\theta_H + (1 - \alpha_1)\theta_L]W]} \end{aligned}$$

If $\alpha_1 > \bar{\alpha}_1$, the expected payoff of a political leader, when it announces a revolution is

$$\begin{aligned} \bar{H}_0(\alpha_1, \beta_1 = 0) &= EU^R(\alpha_1, g_1 = W) \\ &= \frac{[\alpha_1\theta_H + (1 - \alpha_1)\theta_L](e_L - W)W}{[e_H + e_L - [\alpha_1\theta_H + (1 - \alpha_1)\theta_L]W]} \end{aligned}$$

The expected payoff of a political leader that announces a non political movement in the first period depends upon the initial common prior about the leader's efficiency. If

$\alpha_1 < \alpha_1^S$, the expected payoff of a political leader, when it announces a non political movement is

$$\begin{aligned} H_1(\alpha_1, \beta_1) &= EU^s(\alpha_1, g_1 = 0, \hat{\beta}_1 = \beta_1, g_2 = 0) \\ &= \delta W K(\alpha_1, \beta_1) \frac{(\alpha_2^s \theta_H + (1-\alpha_2^s)\theta_L) e_L}{e_H + e_L - (\alpha_2^s \theta_H + (1-\alpha_2^s)\theta_L) W} \\ &+ \delta W [1 - K(\alpha_1, \beta_1)] \frac{(\alpha_2^F(0) \theta_H + (1-\alpha_2^F(0))\theta_L) e_L}{e_H + e_L - (\alpha_2^F(0) \theta_H + (1-\alpha_2^F(0))\theta_L) W} \end{aligned}$$

where $K(\alpha_1, \beta_1) = \frac{[\alpha_1 \theta_H + (1-\alpha_1)\theta_L] e_L}{e_H + e_L - [\beta_1 \theta_H + (1-\beta_1)(\theta_H \alpha_1 + (1-\alpha_1)\theta_L)] W}$

The expected payoff when the leader announces a non political movement, $a_1 = s$ and $\alpha_1^S \leq \alpha_1 < \alpha_1^F(0)$ is as follows. In this range, upon success of the non-political protest in the first period, the updated α at the start of the second period is above $\bar{\alpha}$ while on failure it is below $\bar{\alpha}$.

$$\begin{aligned} \bar{H}_1(\alpha_1, \beta_1) &= EU^s(\alpha_1, g_1 = 0, \hat{\beta}_1 = \beta_1, g_2) \\ &= \delta W K(\alpha_1, \beta_1) \frac{(\alpha_2^s \theta_H + (1-\alpha_2^s)\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^s \theta_H + (1-\alpha_2^s)\theta_L) W} \\ &+ \delta W [1 - K(\alpha_1, \beta_1)] \frac{(\alpha_2^F(0) \theta_H + (1-\alpha_2^F(0))\theta_L) e_L}{e_H + e_L - (\alpha_2^F(0) \theta_H + (1-\alpha_2^F(0))\theta_L) W} \end{aligned}$$

The expected payoff when the leader announces a non political movement, $a_1 = s$ and $\alpha_1 \geq \alpha_1^F(0)$ is as follows. In this range, irrespective of success or failure of the non-political protest in the first period, the updated α is always greater than $\bar{\alpha}$ which means the government will put effort in the second period.

$$\begin{aligned} \hat{H}_1(\alpha_1, \beta_1) &= EU^s(\alpha_1, g_1 = 0, \hat{\beta}_1 = \beta_1, g_2 = W) \\ &= \delta W K(\alpha_1, \beta_1) \frac{(\alpha_2^s \theta_H + (1-\alpha_2^s)\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^s \theta_H + (1-\alpha_2^s)\theta_L) W} \\ &+ \delta W [1 - K(\alpha_1, \beta_1)] \frac{(\alpha_2^F(0) \theta_H + (1-\alpha_2^F(0))\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^F(0) \theta_H + (1-\alpha_2^F(0))\theta_L) W} \end{aligned}$$

$H_0(\alpha_1, \beta_1 = 0)$, $\bar{H}_0(\alpha_1, \beta_1 = 0)$, $H_1(\alpha_1, \beta_1)$, $\bar{H}_1(\alpha_1, \beta_1)$ and $\hat{H}_1(\alpha_1, \beta_1)$ are all increasing in α_1. We show through elimination that $\alpha_L = \bar{\alpha}$ and $\alpha_H = \alpha_1^F(0)$.

Let us assume that $\alpha_L < \alpha_1^S$. For this to hold, $H_0(\alpha_1, \beta_1 = 0) < H_1(\alpha_1, \beta_1)$, $\forall \alpha_1 \in [\alpha_L, \alpha_1^S)$. However, $H_0(\alpha_1 = 0, \beta_1 = 0) > H_1(\alpha_1 = 0, \beta_1)$ and also $H_0(\alpha_1 = 1, \beta_1 = 0) > H_1(\alpha_1 = 1, \beta_1)$. Since, $H_0(\alpha_1, \beta_1 = 0)$ and $H_1(\alpha_1, \beta_1)$ are increasing in α_1, this implies that $H_0(\alpha_1, \beta_1 = 0) > H_1(\alpha_1, \beta_1), \forall \alpha_1$. Thus, $H_0(\alpha_1, \beta_1 = 0) < H_1(\alpha_1, \beta_1)$,

$\forall\alpha_1 \in [\alpha_L, \alpha_1^S)$ does not hold and hence $\alpha_L \not< \alpha_1^S$.

Let us assume that $\alpha_L = \alpha_1^S$. For this to hold, $H_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)$, $\forall\alpha_1 \in [\alpha_1^S, \bar{\alpha})$. However, $H_1(\alpha_1, \beta_1) > \bar{H}_1(\alpha_1, \beta_1)$, $\forall\alpha_1$. Since, $H_0(\alpha_1, \beta_1 = 0) > H_1(\alpha_1, \beta_1), \forall\alpha_1$, therefore, $H_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1)$,$\forall\alpha \in [0, 1]$. Hence, the necessary condition $H_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)$, $\forall\alpha_1 \in [\alpha_1^S, \bar{\alpha})$ does not hold and therefore $\alpha_L \neq \alpha_1^S$.

Let us assume that $\alpha_L \in (\alpha_1^S, \bar{\alpha})$. For this to hold $H_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)$, $\forall\alpha_1 \in [\alpha_L, \bar{\alpha})$. However, $H_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1)$,$\forall\alpha \in [0, 1]$ and hence $\alpha_L \notin (\alpha_1^S, \bar{\alpha})$.

Let us assume that $\alpha_H > \alpha_1^F(0)$. For this to hold, $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \hat{H}_1(\alpha_1, \beta_1)$, must hold $\forall\alpha_1 \in [\alpha_1^F(0), \alpha_H)$. However, $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) > \hat{H}_1(\alpha_1 = 0, \beta_1)$ and also $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) > \hat{H}_1(\alpha_1 = 1, \beta_1)$. Since, $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\hat{H}_1(\alpha_1, \beta_1)$ are increasing in α_1, this implies that $\bar{H}_0(\alpha_1, \beta_1 = 0) > \hat{H}_1(\alpha_1, \beta_1), \forall\alpha_1$. Thus, the condition, $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \hat{H}_1(\alpha_1, \beta_1)$ $\forall\alpha_1 \in [\alpha_1^F(0), \alpha_H)$, does not hold and hence $\alpha_H \not> \alpha_1^F(0)$.

Now the only possibility is that $\alpha_L, \alpha_H \in [\bar{\alpha}, \alpha_1^F(0)]$. We consider the case where $\alpha_L = \bar{\alpha}$ and $\alpha_H = \alpha_1^F(0)$. For this to hold the following conditions should true

1. $\forall\alpha_1 < \alpha_1^S : H_0(\alpha_1, \beta_1 = 0) > H_1(\alpha_1, \beta_1 = 1)$
2. $\forall\alpha_1 \in [\alpha_1^S, \bar{\alpha}) : H_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1 = 1)$
3. $\forall\alpha_1 \geq \alpha_1^F(0) : \bar{H}_0(\alpha_1, \beta_1 = 0) > \hat{H}_1(\alpha_1, \beta_1 = 1)$
4. $\forall\alpha_1 \in [\bar{\alpha}, \alpha_1^F(0)) : \bar{H}_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)$

As proved previously conditions 1, 2 and 3 holds true. Now consider the last condition. Let $\delta_1 = \frac{e_L - W}{e_L[1 - \frac{\theta_L W}{e_H + e_L - \theta_H W}]}$, such that $\forall\delta > \delta_1, \bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 0, \beta_1), \forall\beta_1$. Similarly, $\delta_2 = \frac{e_L - W}{e_L[1 - \frac{\theta_L W}{e_H + e_L - \theta_L W}]}$, such that $\forall\delta < \delta_2, \bar{H}_0(\alpha_1 = 0, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 0, \beta_1), \forall\beta_1$.
Let $\delta_3 = \frac{1}{[1 - \frac{\theta_L W}{e_H + e_L - \theta_L W}]}$ be such that if $\delta > \delta_3$, then $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 1, \beta_1)$ and $\forall\delta < \delta_3$, $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 1, \beta_1)$.
Define, $\bar{\delta} = max\{\delta_1, \delta_3\}$ such that if $\delta > \bar{\delta}$ then $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 0, \beta_1)$ and $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 1, \beta_1)$. Since, $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\bar{H}_1(\alpha_1, \beta_1)$ are increasing in α_1, then $\bar{H}_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)\forall\alpha_1, \beta_1$. Thus, condition 4 follows.

Now we can have three other cases, i.e. *Case 1*: $\alpha_L > \bar{\alpha}, \alpha_H < \alpha_1^F(0)$, *Case 2*: $\alpha_L = \bar{\alpha}, \alpha_H < \alpha_1^F(0)$ and *Case 3*: $\alpha_L > \bar{\alpha}, \alpha_H = \alpha_1^F(0)$. In all these cases we need condition $\bar{H}_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1 = 1)$ to be satisfied for some range of α_1. However, given the range of $\delta > \bar{\delta}$ the above condition can never hold. Hence, $\alpha_L = \bar{\alpha}$ and $\alpha_H = \alpha_1^F(0)$.

The non-political leader always have a positive expected payoff by announcing $a_1 = s$ and hence calls for a non-political protest. □

Proposition 2. *If $\delta < \hat{\bar{\delta}}$ and $c > \bar{c}$, leader with a political objective i.e. $\zeta = P$ always announces a revolution ($a_1 = R$), $\forall \alpha_1 \in [0, 1]$ and the game ends in the first period. Leader with a non-political objective i.e.,$\zeta = NP$ always announces non-political protest in both the periods, i.e.$\sigma_t(NP) = s, \forall t \in \{1, 2\}$ irrespective of α_1. The government, follows a strategy according to Lemma 1 upon hearing a revolution in first period and exerts no force in the first period, i.e. $g_1 = 0$ upon observing a non-political protest.*

Proof. This follows from the proof of previous proposition. Define $\hat{\bar{\delta}} = min\{\delta_2, \delta_3\}$. $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 0, \beta_1), \forall \beta_1$ and $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 1, \beta_1), \forall \beta_1$. Since, $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\bar{H}_1(\alpha_1, \beta_1)$ are increasing in α_1, then $\bar{H}_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1), \forall \alpha_1, \beta_1$. Thus, a non-political protest cannot be sustained by a political leader in equilibrium and her expected payoff is always higher from conducting a revolution $a_1 = R$ for all values of α_1. Given that the leader announces $a_1 = R$, and the government's action is the same as in lemma1. □

Proposition 3. *If $\delta > \bar{\bar{\delta}}$ and $c < \bar{\bar{c}}$, leader with a political objective i.e. $\zeta = P$ follows a threshold policy in the first period such that*

$$\begin{aligned}\sigma_1(P) &= 0 \quad \forall \alpha_1 < \bar{\alpha}_1 \\ &= 1 \quad \forall \alpha \in [\bar{\alpha}_1, \alpha_1^F(g_1 = W)) \\ &= 0 \quad \forall \alpha_1 \geq \alpha_1^F(g_1 = 0)\end{aligned}$$

In the second period, the political leader announces a revolution, i.e. $\sigma_2(P) = 0$. Leader with a non-political objective i.e.,$\zeta = NP$ always announces non-political protest in both the periods, i.e.$\sigma_t(NP) = s, \forall t \in \{1, 2\}$ irrespective of α_1. The government, upon observing a non-political protest exerts no force in the first period, i.e. $g_1 = 0$ and follows a strategy according to Lemma 1 upon hearing a revolution in either period.

Proof. As stated in lemma 3, given the value of c, the governnment will exert an effort $g_1 = W$, $\forall \alpha_1 \in [\alpha_1^S, \alpha_1^F(g_1 = 0)]$ and $g_1 = 0$ for all other ranges of α_1. Now we write the expected payoff of the leader, $\zeta = P$ for different actions it takes in period 1 and the value of α_1 and α_2. The expected payoff when the leader announces a revolution and $\alpha_1 < \bar{\alpha}_1$ is given by

$$\begin{aligned}H_0(\alpha_1, \beta_1 = 0) &= EU^R(\alpha_1, g_1 = 0) \\ &= \frac{[\alpha_1\theta_H + (1 - \alpha_1)\theta_L]e_L W}{[e_H + e_L - [\alpha_1\theta_H + (1 - \alpha_1)\theta_L]W]}\end{aligned}$$

The expected payoff when the leader announces a revolution and $\alpha_1 > \bar{\alpha}_1$ is given by

$$\begin{aligned}\bar{H}_0(\alpha_1, \beta_1 = 0) &= EU^R(\alpha_1, g_1 = W) \\ &= \frac{[\alpha_1\theta_H + (1-\alpha_1)\theta_L](e_L - W)W}{[e_H + e_L - [\alpha_1\theta_H + (1-\alpha_1)\theta_L]W]}\end{aligned}$$

The expected payoff when the leader announces $a_1 = s$ and $\alpha_1 < \alpha_1^S$ is given by

$$\begin{aligned}H_1(\alpha_1, \beta_1) &= EU^s(\alpha_1, g_1 = 0, \hat{\beta}_1 = \beta_1, g_2 = 0) \\ &= \delta W K(\alpha_1, \beta_1)\frac{(\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)e_L}{e_H + e_L - (\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)W} \\ &+ \delta W[1 - K(\alpha_1, \beta_1)]\frac{(\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)e_L}{e_H + e_L - (\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)W}\end{aligned}$$

where $K(\alpha_1, \beta_1) = \frac{[\alpha_1\theta_H + (1-\alpha_1)\theta_L]e_L}{e_H + e_L - [\beta_1\theta_H + (1-\beta_1)(\theta_H\alpha_1 + (1-\alpha_1)\theta_L)]W}$

Now we calculate the expected payoff when the leader announces $a_1 = s$ and $\alpha_1^S \leq \alpha_1 < \alpha_1^F(W)$. In this range, upon success of the non-political protest with government effort $g_1 = W$ in the first period, the updated α at the start of the second period is above $\bar{\alpha}$ while on failure it is below $\bar{\alpha}$.

$$\begin{aligned}\bar{H}_1(\alpha_1, \beta_1) &= EU^s(\alpha_1, g_1 = W, \hat{\beta}_1 = \beta_1, g_2) \\ &= \delta W \bar{K}(\alpha_1, \beta_1)\frac{(\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)W} \\ &+ \delta W[1 - \bar{K}(\alpha_1, \beta_1)]\frac{(\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)e_L}{e_H + e_L - (\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)W}\end{aligned}$$

where $\bar{K}(\alpha_1, \beta_1) = \frac{[\alpha_1\theta_H + (1-\alpha_1)\theta_L](e_L - W)}{e_H + e_L - [\beta_1\theta_H + (1-\beta_1)(\theta_H\alpha_1 + (1-\alpha_1)\theta_L)]W}$

Now we calculate the expected payoff when the leader announces $a_1 = s$ and $\alpha_1 \leq \alpha_1^F(W) < \alpha_1^F(0)$. In this range, irrespective of success or failure of the non-political protest in the first period and the government exerting $g_1 = W$, the updated α is always greater than $\bar{\alpha}$ which means the government will put effort in the second period in case of $a_2 = R$.

$$\begin{aligned}\hat{H}_1(\alpha_1, \beta_1) &= EU^s(\alpha_1, g_1 = W, \hat{\beta}_1 = \beta_1, g_2 = W) \\ &= \delta W \bar{K}(\alpha_1, \beta_1)\frac{(\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^s\theta_H + (1-\alpha_2^s)\theta_L)W} \\ &+ \delta W[1 - \bar{K}(\alpha_1, \beta_1)]\frac{(\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)W}\end{aligned}$$

At last we need to calculate the expected payoff when the leader announces $a_1 = s$ and $\alpha_1 \geq \alpha_1^F(0)$. In this range, the government doesn't exert force in the first period, i.e., $g_1 = 0$ and irrespective of success or failure of the non-political protest in the first period, he updated α is always greater than $\bar{\alpha}$ which means the government will put effort in the second period in case of $a_2 = R$.

$$\begin{aligned}\tilde{H}_1(\alpha_1, \beta_1) &= EU^s(\alpha_1, g_1 = 0, \hat{\beta}_1 = \beta_1, g_2 = 0) \\ &= \delta W K(\alpha_1, \beta_1) \frac{(\alpha_2^s \theta_H + (1-\alpha_2^s)\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^s \theta_H + (1-\alpha_2^s)\theta_L)W} \\ &+ \delta W[1 - K(\alpha_1, \beta_1)] \frac{(\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)(e_L - W)}{e_H + e_L - (\alpha_2^F(0)\theta_H + (1-\alpha_2^F(0))\theta_L)W}\end{aligned}$$

where $K(\alpha_1, \beta_1) = \frac{[\alpha_1\theta_H + (1-\alpha_1)\theta_L]e_L}{e_H + e_L - [\beta_1\theta_H + (1-\beta_1)(\theta_H\alpha_1 + (1-\alpha_1)\theta_L)]W}$
It is easy to verify that $H_0(\alpha_1, \beta_1 = 0)$, $\bar{H}_0(\alpha_1, \beta_1 = 0)$, $H_1(\alpha_1, \beta_1)$, $\bar{H}_1(\alpha_1, \beta_1)$, $\hat{H}_1(\alpha_1, \beta_1)$ and $\tilde{H}_1(\alpha_1, \beta_1)$are all increasing in α_1. We are considering the equilibrium where $\forall \alpha_1 \in [0, \alpha_L)$ and $\forall \alpha_1 \in [\alpha_H, 1]$, the leader does revolution in the first period, $a_1 = R$ while $\forall \alpha_1 \in [\alpha_L, \alpha_H)$, the leader does non-political protest, $a_1 = s$. Now we will show that $\alpha_L = \bar{\alpha}$ and $\alpha_H = \alpha_1^F(W)$. We show by the method of eliminating different cases.

Let us assume that $\alpha_L < \alpha_1^S$. For this to be true we need the condition that $\forall \alpha_1 \in [\alpha_L, \alpha_1^S)$, the following holds, $H_0(\alpha_1, \beta_1 = 0) < H_1(\alpha_1, \beta_1)$. However we can show that $H_0(\alpha_1 = 0, \beta_1 = 0) > H_1(\alpha_1 = 0, \beta_1)$ and also $H_0(\alpha_1 = 1, \beta_1 = 0) > H_1(\alpha_1 = 1, \beta_1)$. Since $H_0(\alpha_1, \beta_1 = 0)$ and $H_1(\alpha_1, \beta_1)$ are increasing in α_1, this implies that $H_0(\alpha_1, \beta_1 = 0) > H_1(\alpha_1, \beta_1) \forall \alpha_1$. Thus the condition does not hold and hence $\alpha_L \not< \alpha_1^S$.

Let us assume that $\alpha_L = \alpha_1^S$. For this to hold, we need the condition that at $\alpha_1 = \alpha_1^S$, the following holds, $H_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)$. Now one can easily show that $H_1(\alpha_1, \beta_1) > \bar{H}_1(\alpha_1, \beta_1)$ $\forall \alpha_1$ and since we have proved that $H_0(\alpha_1, \beta_1 = 0) > H_1(\alpha_1, \beta_1) \forall \alpha_1$, therefore we have $\forall \alpha \in [0, 1]$, $H_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1)$. Hence the necessary condition does not hold and therefore $\alpha_L \neq \alpha_1^S$.

Now lets assume that $\alpha_L \in (\alpha_1^S, \bar{\alpha})$. For this to be true we need the condition that $\forall \alpha_1 \in [\alpha_L, \bar{\alpha})$, the following holds, $H_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)$. However we have proved that this condition cannot hold and hence $\alpha_L \notin (\alpha_1^S, \bar{\alpha})$.

Next we show that $\alpha_H \not> \alpha_1^F(0)$. Let us assume that $\alpha_H > \alpha_1^F(0)$. For this to hold, we need that $\forall \alpha_1 \in [\alpha_1^F(0), \alpha_H)$, the following condition holds, $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \tilde{H}_1(\alpha_1, \beta_1)$. However we can show that $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) > \tilde{H}_1(\alpha_1 = 0, \beta_1)$ and also $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) > \tilde{H}_1(\alpha_1 = 1, \beta_1)$. Since $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\tilde{H}_1(\alpha_1, \beta_1)$ are increasing in α_1, this implies that $\bar{H}_0(\alpha_1, \beta_1 = 0) > \tilde{H}_1(\alpha_1, \beta_1) \forall \alpha_1$. Thus the condition does not hold and hence $\alpha_H \not> \alpha_1^F(0)$.

Let us now assume that $\alpha_H \in (\alpha_1^F(W), \alpha_1^F(0))$. For this to be true we need the condition that $\forall \alpha_1 \in [\alpha_1^F(W), \alpha_H)$, the following condition holds, $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \hat{H}_1(\alpha_1, \beta_1)$. However we can show that $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) > \hat{H}_1(\alpha_1 = 0, \beta_1)$ and also $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) > \hat{H}_1(\alpha_1 = 1, \beta_1)$. Since $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\hat{H}_1(\alpha_1, \beta_1)$ are increasing in α_1, this implies that $\bar{H}_0(\alpha_1, \beta_1 = 0) > \hat{H}_1(\alpha_1, \beta_1)\forall\alpha_1$.

Now let us assume that $\alpha_H = \alpha_1^F(0)$. Now for this to be true, we need the condition that for $\forall \alpha_1 \in [\alpha_1^F(W), \alpha_H)$, the following condition holds, $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \hat{H}_1(\alpha_1, \beta_1)$. However we have already shown that $\bar{H}_0(\alpha_1, \beta_1 = 0) > \hat{H}_1(\alpha_1, \beta_1)\forall\alpha_1$ and hence $\alpha_H \neq \alpha_1^F(0)$.

Now the only possibility therefore we have is that $\alpha_L, \alpha_H \in [\bar{\alpha}, \alpha_1^F(W)]$. We consider the case where $\alpha_L = \bar{\alpha}$ and $\alpha_H = \alpha_1^F(W)$. For this to hold we need the following conditions to be true

1. $\forall \alpha_1 < \alpha_1^S : H_0(\alpha_1, \beta_1 = 0) > H_1(\alpha_1, \beta_1 = 1)$
2. $\forall \alpha_1 \in [\alpha_1^S, \bar{\alpha}) : H_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1 = 1)$
3. $\forall \alpha_1 \in [\alpha_1^F(W), \alpha_1^F(0)) : \bar{H}_0(\alpha_1, \beta_1 = 0) > \hat{H}_1(\alpha_1, \beta_1 = 1)$
4. $\forall \alpha_1 \geq \alpha_1^F(0)) : \bar{H}_0(\alpha_1, \beta_1 = 0) > \tilde{H}_1(\alpha_1, \beta_1)$
5. $\forall \alpha_1 \in [\bar{\alpha}, \alpha_1^F(W)) : \bar{H}_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)$

As proved previously conditions 1, 2, 3 and 4 holds true. Now we consider the last condition and check whether it holds or not. We evaluate $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\bar{H}_1(\alpha_1, \beta_1)$ at $\alpha_1 = 0$ and $\alpha_1 = 1$. We can find that if $\delta > \delta_1 = \frac{1}{[\frac{e_L}{e_L - W} - \frac{\theta_L W}{e_H + e_L - \theta_H W}]}$, then $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 0, \beta_1)\forall\beta_1$. We can also find another $\delta = \delta_2$ such that if $\delta < \delta_2 = \frac{1}{[\frac{e_L}{e_L - W} - \frac{\theta_L W}{e_H + e_L - \theta_L W}]}$, then $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 0, \beta_1)\forall\beta_1$.

Now comparing $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0)$ and $\bar{H}_1(\alpha_1 = 1, \beta_1)$, we can show that there exists a $\delta = \delta_3 = \frac{1}{[\frac{e_L}{e_L - W} - \frac{\theta_H W}{e_H + e_L - \theta_H W}]}$ such that if $\delta > \delta_3$, then $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 1, \beta_1)$ and if $\delta < \delta_3$, then $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 1, \beta_1)$.
We can then define $\bar{\bar{\delta}} = max\{\delta_1, \delta_3\}$ such that if $\delta > \bar{\bar{\delta}}$ then $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 0, \beta_1)$ and $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) < \bar{H}_1(\alpha_1 = 1, \beta_1)$. Since $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\bar{H}_1(\alpha_1, \beta_1)$ are increasing in α_1, then $\bar{H}_0(\alpha_1, \beta_1 = 0) < \bar{H}_1(\alpha_1, \beta_1)\forall\alpha_1, \beta_1$ and therefore condition 4 follows.

The values of δ_1, δ_2 and δ_3 lies between 0 and 1 as long as $\theta_H < 1$ and $\theta_L < 1$ which is the case in our model.Now we can have three other cases, i.e. *Case 1*: $\alpha_L > \bar{\alpha}, \alpha_H < \alpha_1^F(W)$, *Case 2*: $\alpha_L = \bar{\alpha}, \alpha_H < \alpha_1^F(W)$ and *Case 3*: $\alpha_L > \bar{\alpha}, \alpha_H = \alpha_1^F(W)$. In all these cases we need the condition $\bar{H}_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1 = 1)$ to be satisfied for some range of α_1 which varies according to the case considered. However given the range of δ we consider the above condition can never hold and hence we prove that $\alpha_L = \bar{\alpha}$ and $\alpha_H = \alpha_1^F(W)$. The non-political leader always have a positive expected payoff by announcing $a_1 = s$ and hence calls for a non-political protest. □

Proposition 4. *If $\delta < \tilde{\delta}$ and $c < \bar{\bar{c}}$, leader with a political objective i.e. $\zeta = P$ always announces a revolution ($a_1 = R$), $\forall \alpha_1 \in [0, 1]$ and the game ends in the first period. Leader with a non-political objective i.e.,$\zeta = NP$ always announces non-political protest in both the periods, i.e.$\sigma_t(NP) = s, \forall t \in \{1, 2\}$ irrespective of α_1. The government, follows a strategy according to Lemma 1 upon hearing a revolution in first period and exerts no force in the first period, i.e. $g_1 = 0$ upon observing a non-political protest.*

Proof. This follows from the proof of previous proposition. Now suppose we define $\tilde{\delta} = min\{\delta_2, \delta_3\}$. Then we can claim that $\bar{H}_0(\alpha_1 = 0, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 0, \beta_1) \forall \beta_1$ and $\bar{H}_0(\alpha_1 = 1, \beta_1 = 0) > \bar{H}_1(\alpha_1 = 1, \beta_1) \forall \beta_1$. Since we know that $\bar{H}_0(\alpha_1, \beta_1 = 0)$ and $\bar{H}_1(\alpha_1, \beta_1)$ are increasing in α_1, then $\bar{H}_0(\alpha_1, \beta_1 = 0) > \bar{H}_1(\alpha_1, \beta_1) \forall \alpha_1, \beta_1$. Under this circumstances, we cannot sustain a non-political protest by the political leader in equilibrium and her expected payoff is always higher from $a_1 = R$ for all values of α_1. Given that the leader announces $a_1 = R$, then the government's action is the same as in lemma1. □