Group Size and Political Representation Under Alternate Electoral Systems*

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October 12, 2017

Abstract

We examine the effect of group size on access to state power for minorities under majoritarian (MR) and proportional (PR) electoral systems. We establish a robust empirical pattern using an ethnicity-country level panel data comprising 438 ethno-country minority groups across 102 democracies spanning the period 1946-2013. We show that an ethnic group's population share has no relation with its absolute access to power in the national executive under PR but has an inverted U-shaped relation in countries that employ the MR system. The pattern is stable over time and holds up under various alternate specifications. The developmental outcomes for a group proxied using stable nightlight emissions in a group's settlement area follow the same pattern. We reproduce the main results by two separate identification strategies, namely (i) instrumenting colony's voting system by that of the primary colonial ruler and, (ii) comparing the same ethnicity across countries within a continent. We provide a theoretical framework that takes into account the spatial distribution of groups in a two party probabilistic voting model and justifies these patterns as equilibrium behavior. Our work, therefore, has important implications for how electoral systems can affect group *inequality* - an issue largely ignored in the literature.

JEL Classification: D72

Keywords: Electoral systems; minorities; political inclusion

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1 Introduction

Broad categories of electoral systems, namely the majoritarian rule (MR) and proportional representation (PR), and their relationship with public policy have been of interest to the researchers for a long time.[1] There is a large literature that discusses various aspects of this relationship both theoretically as well as in empirical contexts.[2] In the present paper, we look at electoral systems across countries and investigate how differential population shares of minority groups in a country affect their representations in the national executive under the two electoral systems.[3] Trebbi, Aghion and Alesina (2008) answer a similar question by examining the choice of electoral systems by the incumbent municipal councils in U.S. cities following the Voting Rights Act of 1965. Guided by their context they model the representation of two groups, namely the white majority and the black minority under the two systems for various group size configurations. In our context, however, countries may have multiple minority groups. Our work shows how allowing for multiple minorities may modify their conclusions.[4]

Using a dataset covering 102 democracies and spanning the post World War II period (1946-2013), we empirically demonstrate that the effect of group share on its access to political power is starkly different across the two systems. We show that under PR, group size of minorities has no effect on its representation in the national executive, while it has an inverted U-shaped relationship under MR. The existing theoretical framework, as provided by Trebbi, Aghion and Alesina (2008), is inadequate in explaining these

[1]In majoritarian rule legislators are usually elected through elections in single member constituencies (examples include India, Ethiopia, Australia etc). In proportional representation the seats are allocated to parties based on their vote shares across the country (examples include Argentina, Belgium, South Africa etc). For details about the institutions and their trends over time see the Institutional Setting section.

[2]For discussions and a review of literature on the theoretical aspects of this issue see Myerson (1999) and Persson and Tabellini (2002). In empirical examinations some of the key outcome variables studied are corruption (Kunicova and Ackerman, 2005), public attitude towards democracy (Banducci, Donovan and Karp, 1999), voter turnout (see Herrera, Morelli and Palfrey, 2014 and Kartal, 2014), and incentive to engage in conflict (Fjelde and Hoglund, 2014).

[3]We define a group to be a minority if it is not the largest group in its country. We consider this definition to include countries where the largest group does not have absolute majority in the population. Our results, both theoretical and empirical, do not change if we only focus on minority groups in countries that have a group with absolute majority.

[4]In contrast to Trebbi, Aghion and Alesina (2008), answering our question requires us to take the electoral system of a country as given. However, the electoral system of a country is not necessarily exogenous to the power structure of the various groups within the country, as argued by many including Persson and Tabellini (2003), Colomer (2004) and Trebbi, Aghion and Alesina (2008). We address this potential endogeneity problem in our empirical analysis, which we discuss later in the Introduction.

patterns in the data. In their model, for example, access to power never falls with group size under MR, and increases eventually with group size under PR. We develop a model of representation of (three or more) groups in a two party probabilistic voting set up, where we explicitly model the spatial distribution of groups as a function of group sizes. We show that the equilibrium outcome under reasonable parameter restrictions is consistent with the observed patterns. We then provide empirical evidence in favor of the parameter restriction required for the results and further empirically verify additional comparative static results of the model.

For our empirical analysis we use an ethnicity-country level panel dataset comprising 438 ethno-country minority groups. We restrict attention to country-year observations where the country is democratic and further, is run by a parliamentary system. The dataset contains, among other things, a power status variable that codes, for every year, a group's access to the national executive.[5] We define a group to be "included" in the national executive if its power status is not coded as being powerless or being discriminated by the state.[6] In our sample, only about one-third of the group-year observations are "included," and therefore, we consider "inclusion" in the national executive as an important marker of power for minorities.[7] This motivates our focus on the minorities and the "inclusion" dummy constitutes our primary outcome variable.

We empirically establish the effect of a minority group's size on the group's likelihood of being "included" in three steps. We first show the pattern using a linear probability model. The result is true even when we compare minority groups within a country-year observation. In the second step we carry out a number of alternate specifications to test the stability of the observed patterns. We show that these patterns are true in both halves of the time period separately, indicating that the relationship has not significantly changed over time. The result is also robust to using the original (ordered) power rank variable[8] as the dependent variable and using relative population share as the main explanatory variable instead of the actual population share.[9] Importantly, the developmental outcomes

[5]There are six primary power statuses for any group, as coded by the data, indicating the degree of power enjoyed by the group in the national executive. These are, in the descending order of power, *monopoly*, *dominant*, *senior partner*, *junior partner*, *powerless* and *discriminated*. This information comes from the Ethnic Power Relations (EPR) core dataset 2014. We describe the dataset in detail in the Data Descriptions section.

[6]Stated otherwise, a group is "included" if its power status is one of the following: monopoly, dominant, senior partner, or junior partner.

[7]The largest groups, on the other hand, are "included" in our sample in 94% of the cases.

[8]Power rank is coded as a numbers from 1 to 6 where 1 corresponds to being being discriminated, 2 to being powerless and so on.

[9]The relative population share for a minority group is the ratio of its population share and the

for a group proxied using stable nightlight emissions in a group's settlement area follow the same pattern.

In the final step, we verify whether the observed relationships in the data are indeed causal. To that end we employ two separate identification strategies. In the first strategy we address the potential endogeneity issues concerning the electoral system. Though changes in electoral systems of a country are infrequent, their occurrences can depend on the existing power structure of the groups. In fact, the initial choice of the system during a country's transition to democracy can itself be endogenous as noted by Colomer (2004) and Persson and Tabellini (2003). This motivates us to focus on the set of countries which were once colonized by other countries. Reynolds, Reilly and Ellis (2008) point out that these countries often adopted the electoral systems of their former colonial rulers. However, the electoral system of the colonial ruler is unlikely to have a direct effect on the group politics in the colonies post-independence. We therefore use the electoral system of the primary colonial ruler as an instrument of the electoral system of a colony. We show that the first stage holds in our data and the second stage results confirm the results from the OLS specification.

In the second identification strategy we address the concern that there could be unobservable characteristics of groups which could affect both their population size and their access to power in the national executive. To address this concern we compare the same group that is present in more than one country within a continent and exploit the plausibly exogenous variation in its group size across those countries to identify our coefficients of interest. The variation in group sizes in this specification comes primarily due to a group falling unequally on the two sides of a national boundary. An identification strategy similar to this has previously been used in Dimico (2016). This exercise drastically, evidently, reduces our sample. However, even in the restricted sample the data exhibit the same pattern as before and our coefficients of interest retain their statistical significance.

We explain the empirical results by extending the two party probabilistic voting model (*à la* Persson and Tabellini, 2002) with multiple minorities. The political parties announce as platforms proportions of executive positions in the government allocated to each group. Group members care about a group's representation in the government since it determines the par capita transfer of state resources to that group. This framework straight away predicts that group size wouldn't affect the share of executive positions

population share of the largest group in the country-year observation.

offered to any minority under the PR system. This is driven by the fact that the gain in number of votes for a party from an additional allocation to a group is same across all minority groups of various sizes. Since in a PR system both parties are effectively maximizing vote shares, this results in equal allocation of executive positions across minorities.

In order to examine the case with MR, we need to consider how the groups are spatially located across constituencies. We postulate a settlement pattern of minorities which relates their population shares with the area that they occupy. Specifically, we claim that the relationship is *inelastic*. Given that, we show that there is an inverted U-shaped relationship between population share of a minority and its share of executive positions when there are enough constituencies where multiple minorities reside. The presence of multiple minorities in our model is key to get this result. In presence of multiple minorities, increasing the size of one minority would necessitate decreasing another's (holding the size of the largest group fixed). This changes the aggregate population share of minorities within the constituencies non-monotonically. This in turn determines their equilibrium representation.

Our work is related to the literature that discusses the role of electoral systems in providing representation to minorities. The initial set of studies considered PR to be a more inclusive system than MR (see for example, Lijphart, 2004). A number of country-specific studies, however, demonstrate that minority representation under MR depends on the settlement patterns of these groups. Moser (2008), for example, shows in the Russian context that small, geographically concentrated and less assimilated minorities are better represented under single member district MR compared to the PR system. Wagner (2014) shows that a transition to PR system in Macedonia did not increase representation for Albanians.[10] He attributes this to concentration of Albanians in north-west Macedonia, where they form a local majority.

We note that most of the studies that look at group based outcomes across the two systems compare differences in their levels. We on the other hand, focus on difference in the *slope* of the relationship between group size and political power. Our work therefore sheds light on the differential access to power received by groups of differing sizes *within* a system. In the PR system, size inequality of minorities has no bearing on their power inequality, whereas under the MR system it does. Specifically, under the MR system the power inequality between minority groups may either be greater or smaller than their size inequality depending on the size distribution of groups. This work, therefore, comments

[10] Albanians comprise around 26% of politically relevant population in Macedonia and are the largest minority.

on how electoral systems affect group inequality in access to power and consequently, in their overall welfare - an issue that is discussed rarely in the literature.

2 Institutional Setting

The decline of colonialism and autocratic rule, and a transition towards democracy has characterized the world in the post World War II period. An interesting aspect of this wave of democratization is the choice of electoral system made by the newly emerging democracies. On one hand there is MR in which elections are typically contested over single member districts. The candidate or party with a plurality or an absolute majority in these districts wins. The advantage of this system is formation of a stable government. In contrast, in a PR system, seats are allocated to parties in proportion to their vote share in multimember districts. This reduces the disparity in vote share at the national level and the seat share of a party in the parliament.[11] In the period from 1950s to the 1970s, a larger fraction of countries had adopted the MR system. However, the past few decades have seen a trend towards the adoption of PR (Figure 1). This is driven by adoption of the PR system by the new democracies in Latin America, Africa, and Mediterranean, Central and Eastern Europe in the 1970s and 1980s.[12] Apart from country specific factors, colonialism and the influence of neighboring countries have also been important in this choice of electoral system (Reynolds, Reilly and Ellis, 2008). This has resulted in a regional clustering of the systems (Figure 2).

The transition to PR has not been accompanied by a substantive political inclusion of minorities overall (Figure 3). There has been a gradual decline in the state administered discrimination over the years accompanied by an increase in share of groups in the powerless category. The proportion of groups in power sharing arrangements with other groups (i.e., junior and senior partner) and of those who rule virtually alone (dominant and monopoly groups) has remained stable.

3 Data Description

In this section, we describe the various data sources that we have put together for this project and discuss the main variables that concern us.

[11] Some countries also use mixed systems which are a combination of both MR and PR. However, we do not include them in our empirical analysis.

[12] The possible reasons for adoption of PR system by these countries are discussed in Farrell (2011).

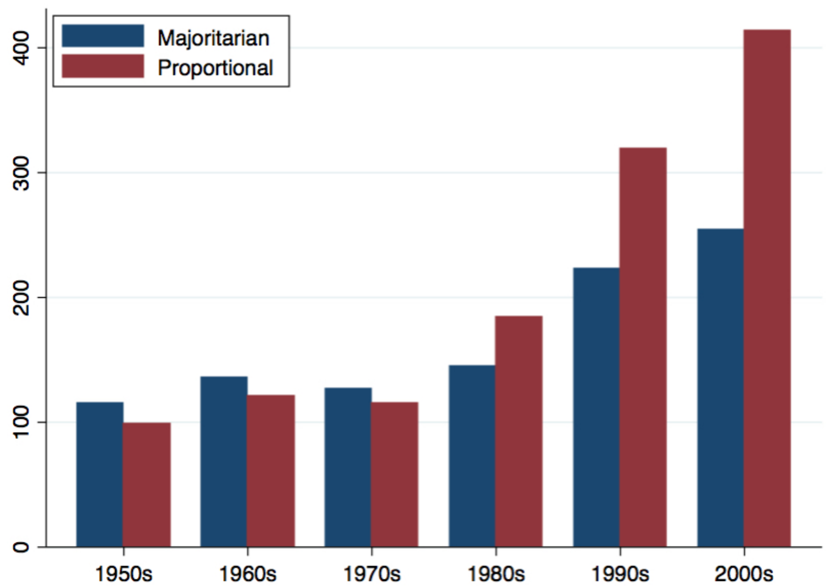

Figure 1: Electoral systems by decade

3.1 Data Sources

EPR Dataset Our primary source of data is the Ethnic Power Relations (EPR) core dataset 2014 (Vogt et al., 2015). The dataset contains various characteristics of well-identified groups ("ethnicities") within countries for about 155 countries across the world at an annual level for the period 1946-2013. The dataset defines a group "as any subjectively experienced sense of commonality based on the belief in common ancestry and shared culture."[13] (Cederman, Wimmer and Min, 2010)

The dataset reports various group level characteristics for all politically relevant groups residing in sovereign states with a total population of at least 500,000 in 1990. A group is politically relevant if at least one political organization has at least once claimed to represent it at the national level or the group has been explicitly discriminated against by the state during any time in the period 1946-2013. It provides annual group-country

[13] Cederman, Wimmer and Min (2010) further point out that in different countries different "markers may be used to indicate such shared ancestry and culture: common language, similar phenotypical features, adherence to the same faith, and so on." Further, in some societies there may be multiple dimensions of identity along which such "sense of commonality" may be experienced. The dataset, however, is concerned with groups that are *politically relevant* (more on this later) and that aligns with our interest as well. As long as there is some marker of identity which is salient in the society and is also politically meaningful, our analysis should be applicable.

level data on population shares, settlement patterns, trans-border ethnic kinship, as well as religious and linguistic affiliations for the period 1946-2013. However, most importantly for us, it also codes a group's access to national executive. A group's access to absolute power in the national executive is coded based on whether the group rules alone (power status = *monopoly*, *dominant*), shares power with other groups (power status = *senior partner*, *junior partner*) or is excluded from executive power (power status = *powerless*, *discriminated by the state*). We rank these six categories in a separate variable called "power rank"; they range from 6 to 1 in decreasing order of power (i.e., from *monopoly* to *discriminated*).[14] The EPR dataset also provides information about the settlement patterns of the groups. Specifically, it categorizes the groups as being *dispersed*, i.e., those who do not inhabit any particular region and, *concentrated*, i.e., settled in a particular region of the country which is easily distinguishable on a map. For concentrated groups, it further gives information about the fraction of the country's land area that they occupy.[15]

The EPR dataset was created by scholars who work on group based conflict. The first version of the dataset was created as part of a research project between scholars at Zurich ETH and University of California, Los Angeles (UCLA), which was then updated and released by Vogt et al. (2015). The information about the attributes about the groups, including their power statuses were coded by taking inputs from about one hundred country experts. This consultation period lasted for about two years followed by a workshop where the final coding of attributes was decided after taking into account the inputs provided by the experts and accumulated knowledge available for the countries.

This dataset has certain advantages for our paper over other existing datasets about political outcomes of groups. Some of the prominent datasets used by scholars of conflict are the Minorities At Risk (MAR) dataset, the All Minorities at Risk (A-MAR) dataset and the dataset used by Fearon (2003). Though most of these datasets give information about group sizes, none of the datasets provide any detail about the settlement patterns of the groups. This is critical for us since we demonstrate that the pattern observed in our data is driven by groups which are geographically concentrated. Also, the EPR dataset provides information about the power status of all groups; this is in contrast to the MAR dataset which systematically excludes the groups who are in the government from the sample.

[14]There is an additional categorization in the data, known as *self-exclusion*. This applies to groups which have declared independence from the central state. They constitute only 0.7% of our sample and we do not consider them for our analysis.

[15]The GIS shape file of their area of settlement is also provided on the EPR website.

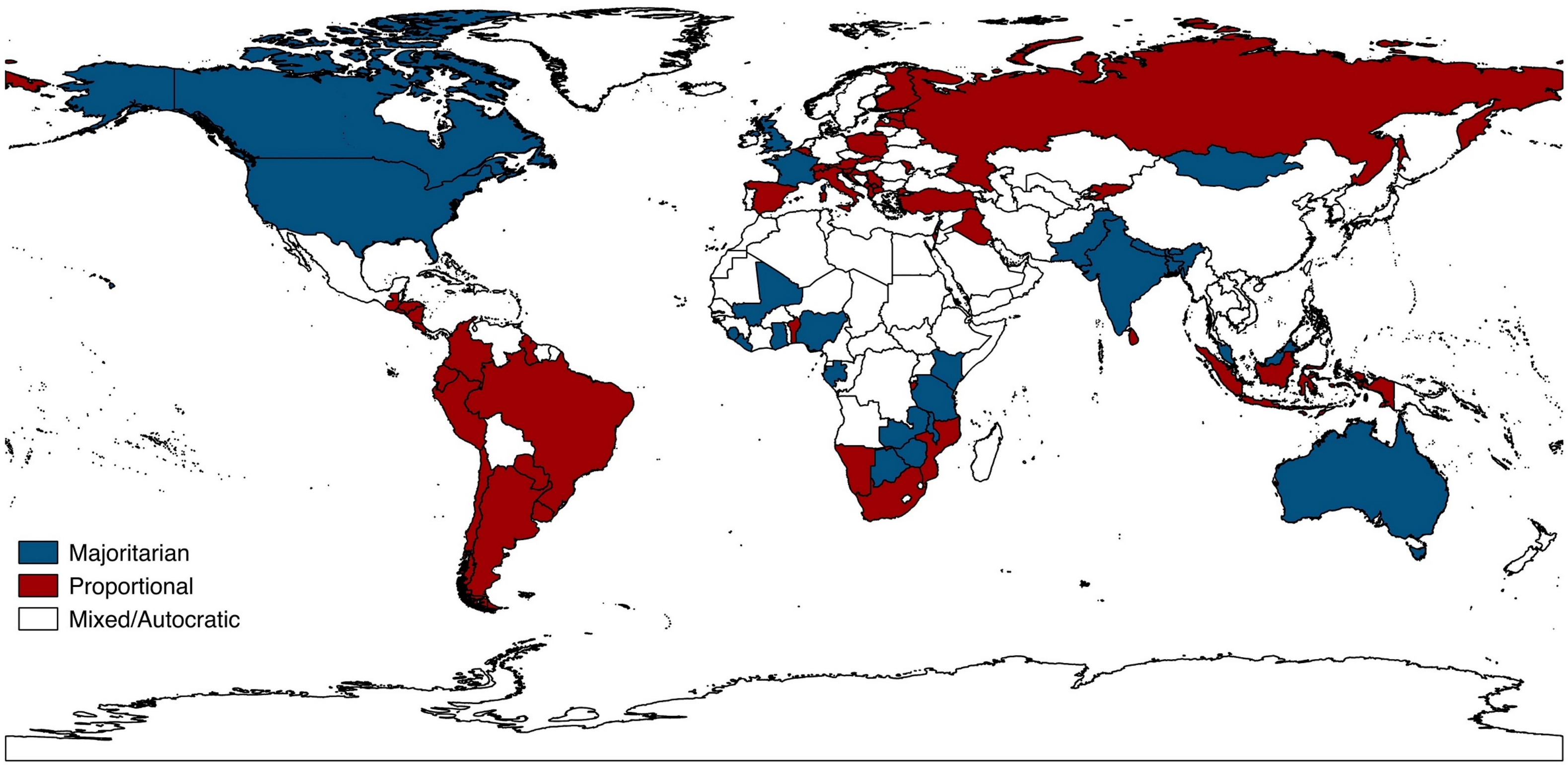

Figure 2: Electoral system distribution in 2013

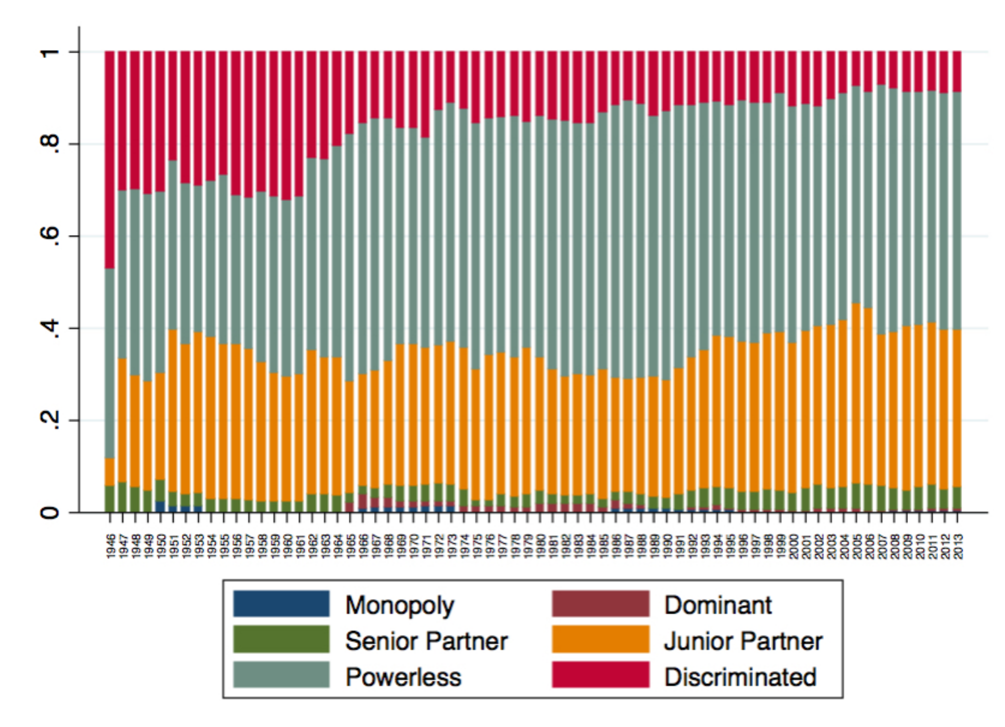

Figure 3: Minority power status over time

Electoral systems The data for electoral rules used for national legislative elections come from merging two datasets. The first of these is the Democratic Electoral Systems (DES) data compiled by Bormann and Golder (2013). It contains details about electoral systems used for about 1200 national elections for the period 1946-2011. We compliment this with a second source of data - the IDEA Electoral System Design Database, which gives us information about the electoral systems for some additional countries. The classification into broad electoral systems is based on the DES dataset. For any given year, the electoral system in a country is the electoral system used in the most recently held election. We restrict our main analysis to Majoritarian and Proportional systems.

Polity characteristics and colonial history Polity IV Project is used to identify periods of autocratic rule in a country. We include only those country-year pairs where the position of the chief executive is chosen through competitive elections. The ICOW Colonial History Dataset 1.0 compiled by Hensel (2014) recognizes the primary colonial ruler and the year of independence for each country that was colonized. To obtain the electoral systems of the colonial rulers we use the data on electoral systems provided in

The Handbook of Electoral System Choice (HESC) (Colomer, 2004). The HESC provides information about electoral systems of democracies since 1800. We use this to find the electoral rule followed by the primary colonial ruler in the colony's year of independence. We use this information for our instrumental variable analysis which we describe later.

3.2 Summary statistics

Table 1 reports summary statistics for both the ethnicity level (Panel A) and the country level (Panel B) variables. In our final data, 43.87 percent of country-year observations have MR system, whereas 56.13 percent have PR system. The countries with the MR system are more fractionalized, have greater number of relevant groups, but allow lesser political competition and place fewer constraints on decision making powers of the chief executive compared to the PR system. These differences, however, are not statistically significant at 10% level. On an average, the largest group comprises 73.5 percent of the politically relevant population and in 84.9 percent of country-year observations, the largest group has an absolute majority in the country (i.e. population share over 50 per cent of politically relevant population). Overall 36.7 percent of minorities are politically included and 75.1 percent are geographically concentrated. The ethnicity level characteristics are not significantly different between countries with MR and PR systems.

4 Empirics

4.1 Empirical strategy

Baseline We use the linear probability model to estimate the effect of group size on political inclusion under MR and PR. Following is the baseline specification:

$$Y_{ict} = \delta_{ct} + \beta_1 n_{ict} + \beta_2 n_{ict}^2 + \beta_3 P_{ct} * n_{ict} + \beta_4 P_{ct} * n_{ict}^2 + \gamma X_{ict} + \epsilon_c \tag{1}$$

Where Y_{ict} can be a dummy for whether the group i is politically included in country c in year t or the nightlight intensity in the group's settlement area; δ_{ct} denotes country-year fixed effects; n_{ict} is the population of group i in the country relative to the total politically relevant population of country c in year t; P_{ct} is a dummy for whether the proportional electoral system is used for national legislative elections or not; X_{ict} is a vector of ethnicity level controls (i.e. years of peace, settlement patterns, transethnic kin inclusion/exclusion and fraction of the group associated with the largest language and religion for the group). The error term ϵ_c is clustered at the country level.

Identification For identification, we first restrict our sample to ethnic groups living in more than one country in the same continent and estimate the following model:

$$I_{ict} = \delta_{irt} + \theta P_{ct} + \beta_1 n_{ict} + \beta_2 n_{ict}^2 + \beta_3 P_{ct} * n_{ict} + \beta_4 P_{ct} * n_{ict}^2 + \gamma X_{ict} + \epsilon_{ic}$$

Where δ_{irt} denotes ethnicity-region-year effects; error term ϵ_{ic} is double clustered at the level of group and country to adjust standard errors against potential autocorrelation within ethnicity and country.

Secondly, the electoral system of a country may be endogenous to other country specific factors which might differ between the two electoral systems. To correct for this potential source of bias, we use a dummy that indicates whether the electoral system of the primary colonial ruler in the colony's year of independence is proportional as an instrument for the electoral system in our baseline specification.

4.2 Results

Baseline Column (6) in table 2 shows the results from our baseline specification. The coefficient of n_{ict} (population share) is positive and coefficient of n_{ict}^2 (population share-squared) is negative. The two coefficients are statistically significant at 1% and 5% levels respectively. This demonstrates an inverted U-shaped relation between population share of an ethnic group and its probability of political inclusion under the MR electoral system. Probability of political inclusion for a group is maximized when the population share is 25.7 percent. F-tests for statistical significance of sum of linear and quadratic coefficients give p-values of .091 and .648 respectively. This indicates that there is no statistically significant relation between population share and political inclusion under the PR system.

The coefficients of ethnicity level controls are reported in table A1 in the appendix. An additional decade without any conflict incidence experienced by an ethnicity is associated with a 5.11 percent more likelihood of its political inclusion. The coefficient of trans-ethnic kin exclusion dummy is suggestive of politically excluded ethnic groups immigrating to countries where they might get political representation. Political exclusion of the trans-ethnic kin of a group is associated with 10.90 percent higher likelihood of political inclusion for a group. An indicator of an ethnic group's cohesiveness is the

Table 1: Descriptive statistics

	All data	Majoritarian system	Proportional system	Difference
	(1)	(2)	(3)	(4)
Panel A: Ethnicity level				
Political inclusion	0.367	0.444	0.275	0.170
	(0.482)	(0.496)	(0.446)	(0.114)
Power rank	2.295	2.391	2.180	0.212
	(0.793)	(0.770)	(0.805)	(0.189)
Population share	0.074	0.070	0.079	-0.009
	(0.099)	(0.090)	(0.108)	(0.024)
Years peace	31.434	29.227	34.058	-4.831
	(20.287)	(19.170)	(21.246)	(4.168)
Statewide settlement	0.032	0.026	0.040	-0.014
	(0.176)	(0.158)	(0.195)	(0.045)
Concentrated settlement	0.751	0.741	0.763	-0.022
	(0.433)	(0.438)	(0.426)	(0.010)
Urban settlement	0.087	0.103	0.067	0.036
	(0.282)	(0.305)	(0.251)	(0.061)
Dispersed settlement	0.109	0.118	0.098	0.020
	(0.312)	(0.323)	(0.298)	(0.074)
Migrant settlement	0.020	0.011	0.031	-0.020
	(0.140)	(0.103)	(0.173)	(0.028)
Transethnic-kin inclusion	0.417	0.403	0.435	-0.032
	(0.493)	(0.491)	(0.496)	(0.103)
Transethnic-kin exclusion	0.521	0.459	0.595	-0.135
	(0.500)	(0.498)	(0.491)	(0.105)
Fraction largest religion	0.719	0.750	0.682	0.069
	(0.209)	(0.222)	(0.186)	(0.052)
Fraction largest language	0.878	0.889	0.866	0.022
	(0.223)	(0.215)	(0.233)	(0.045)
Observations	9,304	5,054	4,250	9,304
Panel B: Country level				
Ethnic fractionalization	2.433	2.885	2.079	0.806
	(1.989)	(2.201)	(1.723)	(0.494)
Number of relevant groups	4.596	5.470	3.913	1.557
	(3.772)	(4.221)	(3.221)	(0.944)
Largest group size	0.735	0.687	0.772	-0.086
	(0.219)	(0.238)	(0.195)	(0.054)
Absolute majority	0.849	0.753	0.923	-0.170*
	(0.359)	(0.432)	(0.266)	(0.086)
Competitiveness of participation	3.989	3.873	4.079	-0.207
	(1.056)	(1.252)	(0.962)	(0.232)
Constraints chief executive	6.121	5.978	6.233	-0.256
	(1.291)	(1.370)	(1.497)	(0.270)
Observations	2,601	1,141	1,460	2,601

Notes: The data is at the ethnicity-country-year level for 102 countries and 68 years. Standard deviation in parenthesis. Standard errors clustered at the country level in parenthesis in the last column. ***p<0.01, **p<0.05, *p<0.1.

Table 2: Baseline specification

	Political inclusion					
	(1)	(2)	(3)	(4)	(5)	(6)
Population share	5.201***	5.027***	4.753***	4.475***	5.212***	5.613***
	(0.889)	(1.069)	(1.123)	(1.012)	(1.462)	(1.454)
Population share - squared	-8.511***	-8.433**	-7.648**	-6.340**	-9.535**	-10.91**
	(2.684)	(3.382)	(3.280)	(3.001)	(4.500)	(4.620)
Proportional*Population share	-0.715	-0.995	-1.356	-1.768	-3.217*	-3.832**
	(1.440)	(1.480)	(1.375)	(1.331)	(1.755)	(1.784)
Proportional*Population share - squared	0.506	1.319	2.013	2.524	7.395	9.468*
	(4.227)	(4.179)	(3.792)	(3.759)	(5.459)	(5.638)
Proportional	-0.127	-0.131	-0.0332	0.0624	0.260*	
	(0.0980)	(0.0902)	(0.0664)	(0.0652)	(0.149)	
$H_0 : \beta_1 + \beta_3 = 0$ (p-value)	.000	.000	.001	.011	.060	.091
$H_0 : \beta_2 + \beta_4 = 0$ (p-value)	.016	.017	.046	.172	.508	.648
Mean inclusion	0.367	0.367	0.367	0.367	0.367	0.367
Observations	9,304	9,304	9,304	9,304	9,304	8,712
R-squared	0.240	0.282	0.412	0.450	0.640	0.675
Ethnicity-year Controls	NO	YES	YES	YES	YES	YES
Country-year Controls	NO	NO	YES	YES	YES	NO
Year FE	NO	NO	YES	YES	YES	NO
Region FE	NO	NO	NO	YES	NO	NO
Country FE	NO	NO	NO	NO	YES	NO
Country-year FE	NO	NO	NO	NO	NO	YES

Notes: The data is at ethnicity-country-year level for 102 countries and 438 ethno-country groups. There are 87 countries and 422 ethno-country groups in column (6). The time period is 1946-2013. Standard errors are clustered at the country level. *** p<0.01, ** p<0.05, * p<0.1.

fraction of its members associated with the largest language for the group. Groups that are linguistically more cohesive find it easier to organize themselves and put forth their demands and therefore are more likely to be politically included. This is supported by the result that a 10 percent increase in fraction of group members associated with the largest language for the group is related with a 2.89 percent increase in likelihood of political inclusion for the group.

Robustness and effect on development Table 3 tests the robustness of our results. The broad pattern depicted in our baseline specification of inverted U-shaped relation for MR and weakening of that relation under PR continue to hold over time. This can be seen in columns (2) and (3) which show results over time periods 1946 - 1979 and 1980-2013,

Table 3: Robustness and development

	Political inclusion			Power rank	Nightlight intensity	Political inclusion	Power rank	Nightlight intensity
	1946-2013	1946-1979	1980-2013					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Population share	5.613***	7.182**	5.175***	6.322***	1.485**			
	(1.454)	(2.679)	(1.089)	(1.880)	(0.641)			
Population share - squared	-10.91**	-16.17*	-9.550***	-9.478	-3.417**			
	(4.620)	(9.272)	(3.137)	(6.221)	(1.638)			
Proportional*Population share	-3.832**	-6.386**	-2.938*	-5.290*	-1.225*			
	(1.784)	(2.401)	(1.538)	(2.825)	(0.624)			
Proportional*Population share - squared	9.468*	16.38*	6.754	12.46	2.599			
	(5.638)	(8.478)	(4.589)	(9.451)	(1.654)			
Relative population share						2.699***	2.713***	0.715***
						(0.475)	(0.747)	(0.239)
Relative population share-squared						-2.372***	-1.725	-0.736***
						(0.527)	(1.063)	(0.238)
Proportional*relative population share						-1.675***	-2.054*	-0.527**
						(0.620)	(1.061)	(0.203)
Proportional*relative population share-squared						1.864**	2.398	0.452**
						(0.712)	(1.542)	(0.208)
$H_0 : \beta_1 + \beta_3 = 0$ (p-value)	.091	.498	.048	.635	.185	.023	.394	.066
$H_0 : \beta_2 + \beta_4 = 0$ (p-value)	.648	.942	.408	.684	.281	.344	.559	.108
Mean dependent	0.367	0.332	.379	2.277	0.026	0.367	2.227	0.026
Observations	8,712	2,296	6,416	8,712	3,469	8,712	8,712	3,469
R-squared	0.675	0.660	0.691	0.667	0.710	0.682	0.671	0.722
Ethnicity-year Controls	YES	YES	YES	YES	YES	YES	YES	YES
Country-year FE	YES	YES	YES	YES	YES	YES	YES	YES

Notes: The data is at ethnicity-country-year level. 70 countries and 278 ethno-country groups in column (4) and (8). Standard errors are clustered at the country level and reported in parenthesis. *** p<0.01, ** p<0.05, * p<0.1.

Table 4: Comparing ethnic group in countries in the same continent

	Political inclusion	Power rank	Nightlight intensity
	(1)	(2)	(3)
Population share	12.62***	10.69**	0.746***
	(2.545)	(4.254)	(0.118)
Population share - squared	-30.17***	-20.75*	-2.001***
	(6.620)	(10.11)	(0.327)
Proportional*Population share	-10.17***	-8.291	-0.777***
	(3.397)	(5.132)	(0.162)
Proportional*Population share - squared	29.73**	23.65	2.077***
	(13.99)	(19.62)	(0.470)
Proportional	0.195**	-0.291*	0.00430
	(0.0716)	(0.157)	(0.00257)
$H_0 : \beta_1 + \beta_3 = 0$ (p-value)	.208	.345	.606
$H_0 : \beta_2 + \beta_4 = 0$ (p-value)	.962	.820	.702
Mean dependent	0.110	1.858	0.009
Observations	1,219	1,219	417
R-squared	0.819	0.757	0.898
Ethnicity-year controls	YES	YES	YES
Group-region-year FE	YES	YES	YES

Notes: Comparison of 21 ethnic groups in 40 countries. Column (3) compares 12 ethnic groups in 30 countries. The time period is 1946-2013. Standard errors are clustered at the country level and reported in parenthesis. *** p<0.01, ** p<0.05, * p<0.1.

respectively. The relations under MR and PR systems are also robust to using power rank as the dependent variable (column 4). We look at the relation of population share with development outcomes for an ethnic group using nightlight intensity in a group's settlement area as a proxy for development. We find that the effect of population share of the ethnicity on its political inclusion translates to development outcomes for the group. As we can see, column (5) shows an inverted U-shaped relation between population share and nighlight intensity in the group's settlement area under the MR system. Again we find that there is no statistically significant relation under the PR system. All these results are robust to taking population share as a fraction of population share of the largest ethnic group in the country-year of the ethnic group (columns 6 to 8).

Table 5: IV estimates

Panel A: Second stage

	Political inclusion		Nightlight intensity
	(1)	(2)	(3)
Population share	5.613***	10.11***	1.195
	(1.454)	(3.594)	(0.720)
Population share - squared	-10.91**	-28.01*	-2.303
	(4.620)	(15.65)	(1.719)
Proportional*Population share	-3.832**	-11.70**	-0.964
	(1.784)	(4.785)	(0.624)
Proportional*Population share - squared	9.468*	38.68*	1.868
	(5.638)	(22.92)	(1.561)
$H_0 : \beta_1 + \beta_3 = 0$ (p-value)	.091	.358	.087
$H_0 : \beta_2 + \beta_4 = 0$ (p-value)	.648	.269	.213
Mean Dependent	0.367	0.464	.021
Observations	8,712	5,199	2,306
R-squared	0.675	0.615	0.507
Country-year FE	YES	YES	YES
Kleibergen-Paap rk LM stats		3.50	2.80
Cragg-Donald Wald F stat		90.49	39.83
Stock and Yogo critical value		7.03	7.03
F stat (Proportional*Population share)		31.23	13.11
F stat (Proportional*Population share - squared)		19.80	10.41

Panel B: Country level

	Proportional
Colonialist proportional	0.463***
	(0.118)
Mean dependent	.450
Observations	1,309
R-squared	0.388
Region-year FE	YES

Notes: The data is at the ethnicity-country-year level. In panel A, column (2), we look at 267 ethno-country groups in 56 countries. 170 ethno-country groups in 44 countries in column (3). The time period is 1946-2013. Standard errors are clustered at the country level and reported in parenthesis. Results in panel B for 64 countries. Diagnostic tests for first stage regression. *** p<0.01, ** p<0.05, * p<0.1.

Within group comparison Our first identification strategy compares an ethnic group that is spread over more than one country within the same continent but has different population share in both the countries. Such a comparison eliminates the possible endogeneity that might arise due to unobserved characteristics of an ethnic group, for example, due to ethnicity specific historical shocks. Table 4 reports the coefficients with political inclusion (column 1), power rank (column 2) and nightlight intensity (column 3) as the dependent variables. Dimico (2016) shows in the African context that the partition of an ethnicity in two countries adversely affects their political representation when the resulting groups are small. However, we show that the effect of how an ethnic group is divided in two countries on the group's political representation and economic development depends on their electoral systems. The within group comparison reaffirms the inverted U-shaped effect of population share on political representation as well as on nightlight intensity under MR and no relation under the PR system.

Instrumental variable Our second identification strategy uses electoral system of primary colonial ruler as an instrument for a country's electoral system. Our country level analysis in table 5, panel B shows that the existence of proportional electoral system in a country is 46.3 percent more likely if the electoral system of its primary colonial ruler was also proportional in the colony's year of independence. The coefficient is statistically significant at 1% level. In line with this, Cragg-Donald Wald F statistic for the first stage regression is much larger than the Stock and Yogo critical value, alleviating concerns related to weak instrument. The Kleibergen-Paap rk LM statistic has a p-value .062, rejecting the null hypothesis that the model is underidentified. Column (2) in panel A is again consistent with a statistically significant inverted U-shaped effect of population share on political representation under the MR system. The within group comparison and IV regression indicate the peak of political representation at population shares of 20.9 percent and 18.0 percent respectively. There is no effect of population share on political representation under the PR system.

5 The Model

To understand the rationale behind our empirical results, we develop a probabilistic model of two party electoral competition with private transfers based on Persson and Tabellini (2002).

5.1 Basic Setup

There are three ethnic groups of voters. Each group is a continuum with population share n_j, where $\sum_{j=1}^{3} n_j = 1$. Voters have preferences over private transfers made by the government.[16] These transfers can be targeted at the ethnicity level but not at the individual level. In practice, targeting at the ethnic group level can be done when the ethnicities are geographically concentrated. However, even when they are not concentrated, their residential patterns are often segregated within regions which enables local leaders to deliver targeted benefits. Individual preferences can therefore be represented as:

$$w_j = U(f_j)$$

Where f_j denotes per capita private transfers to the group j. The utility function is strictly increasing and strictly concave i.e. $U'(f_j) > 0$ and $U''(f_j) < 0$. For interior solution, $U'(f_j) \to \infty$ as $f_j \to 0$. We assume that f_j is completely determined by the political processes of a country. In the pre-election stage, two political parties A and B simultaneously decide their candidates, who are representatives of various ethnic groups, for cabinet and senior administrative posts. An ethnic group's representation in the executive G_j^h promised by party h determines how much per capita transfers they will get:

$$f_j^h = f(G_j^h) \quad \text{or} \quad G_j^h = f^{-1}(f_j^h)$$

Where, $f'(G_j) > 0$. This is motivated by the fact that higher representation in executive would imply that the group will have more number of ministries or government departments under its indirect control, leading to preferential treatment received under the policies of the relevant department. Also, the policy decisions of the head of state will be subjected to more bargaining from that group. Since representation in executive determines the individual level payoff of the voters, the political parties commit to allocation of executive positions as their platforms during election. We use f_j^h directly as a choice variable in what follows instead of G_j^h. Any voter i belonging to group j votes for

[16] Targeting private transfers and benefits from pork-barrel spending to groups of voters is easier than targeting public goods (Lizzeri and Persico, 2001).

party A if:

$$U(f_j^A) > U(f_j^B) + \delta + \sigma_{i,j}$$

Where, $\delta \sim U[\frac{-1}{2\psi}, \frac{1}{2\psi}]$ and $\sigma_{i,j} \sim U[\frac{-1}{2\phi_j}, \frac{1}{2\phi_j}]$.

δ can be interpreted as population wide wave of support in favour of party B. $\sigma_{i,j}$ represents ideological bias of a member i of group j towards party B. ϕ_j is a measure of responsiveness of group j voters to private transfers determined through promised political representation by a party. Values of ψ and ϕ_j are known to both the parties. The government budget is constrained at S. Each party h maximizes the probability of winning p_h and chooses f_j^h subject to this budget constraint[17]:

$$\sum_{j=1}^{3} F_j^h \leq S \quad \text{or} \quad \sum_{j=1}^{3} n_j f_j^h \leq S$$

In majoritarian system all districts have equal population size and n_j^k is the size of group j relative to population in district k. We compare equilibrium political representation in single district PR system with that in K district MR voting system.

5.2 Equilibrium

Since the parties are symmetric, they will choose the same equilibrium policy.

Proposition 1 *Under a single district proportional representation voting system, ethnic group size n_j has no effect on equilibrium political representation G_j^*. In equilibrium:*

$$\phi_i U'(f_i^*) = \phi_l U'(f_l^*) \quad \forall i, l \tag{2}$$

Proof: In the Appendix. ■

An increase in group size does not affect f_j^*, and therefore G_j^* under proportional representation because increased importance of the group in national politics is offset by the greater amount of overall resources that need to be devoted to influence a given fraction of voters from the group to vote for a party. Also, ethnic groups that have higher

[17] The theoretical results will be identical if party objective is to maximize expected seat share across the country.

values of ϕ_j get higher equilibrium political representation G_j^* as they are easier to sway through electoral commitments and hence, parties compete more fiercely for their votes.

Proposition 2 *Under a majoritarian voting system with K districts, ethnic group size n_j has no effect on equilibrium political representation G_j^* for any group j if either of the following conditions is satisfied:*

$$(i) \quad \phi_j = \phi \quad \forall j$$
$$(ii) \quad n_j^k = n_j \quad \forall k, \forall j$$

The equilibrium in K district majoritarian voting system is given by:

$$\phi_i U'(f_i^*) \sum_{k=1}^{K} \frac{n_i^k / n_i}{\sum_{j=1}^{J} \phi_j n_j^k} = \phi_l U'(f_l^*) \sum_{k=1}^{K} \frac{n_l^k / n_l}{\sum_{j=1}^{J} \phi_j n_j^k} \quad \forall i, l \tag{3}$$

Proof: In the Appendix. ■

The above proposition shows that an ethnic group will get higher political representation and hence, private transfers in majoritarian system compared to proportional representation system if it is concentrated more in districts having a less responsive mass of voters relative to other districts. This is driven by the fact that in the majoritarian system political parties maximize the probability of winning more than 50% of votes within each constituency, and hence, compare a group's political responsiveness with the responsiveness of only those groups with which this group co-resides.

5.3 Comparative Statics

In this section as well, there are three groups with population shares n_1, n_2, and n_3 within a country. The K electoral districts are partitioned in a way such that each district has the same population size. Members of a given ethnic group can be dispersed across the entire country or they can be concentrated in a specific geographic region. Settlement area of a geographically concentrated group increases inelastically with its population share.[18] The total area of the country is normalized to 1. For mathematical simplicity, group population is uniformly distributed across its area of residence. Settlement area of a

[18]This might happen due to the existence of economic and social ties among group members or due to habit formation.

concentrated group i is $A(n_i) = n_i^\alpha$, where $\alpha \in (0, 1)$. Groups 1 and 2 are identical in their political responsiveness i.e. $\phi_1 = \phi_2 = \phi$. Group 3 is politically more responsive compared to the other groups i.e. $\phi_3 > \phi$. Group 1 is geographically concentrated and group 3 is dispersed across the country. We can think of group 1 and group 2 as minorities and group 3 as the majority group.[19] Though, as we show, the results hold for all $n_3 \in (0, 1)$. Consistent with the within country-year comparison of our empirical model, n_3 is constant and the entire increase in n_1 is compensated by a corresponding decrease in n_2.

When group 2 is concentrated. Settlement areas A_j of each group j are:

$$A_1 = n_1^\alpha \qquad A_2 = n_2^\alpha \qquad A_3 = 1$$

As is typically the case between concentrated minorities, there is some overlap between the settlement areas of group 1 and 2. This overlap is measured by the Szymkiewicz-Simpson coefficient:

$$O = \frac{A_{1\cap 2}}{\min(n_1^\alpha, n_2^\alpha)} \qquad O \in [0, 1]$$

This implies that the area of intersection between the settlement areas of group 1 and group 2 is:

$$A_{1\cap 2} = O \cdot min(n_1^\alpha, n_2^\alpha)$$

Proposition 3 *If group 1 is geographically concentrated and $\phi_3 > \phi$, then under the majoritarian voting system:*

1. *If group 2 is concentrated:*
   - *(a) When $n_1 < n_2$, equilibrium political representation G_1^* is increasing in n_1*
   - *(b) When $n_1 \geq n_2$, G_1^* is decreasing in n_1 if and only if $O > O^*$ for some $O^* \in (0, 1)$.*
2. *If group 2 is geographically dispersed, equilibrium political representation G_1^* follows*

[19] The minority groups will be less responsive to electoral promises made by the major political parties if members of minority groups have stronger ideological bias towards the party they support compared to other groups.

an inverted U-shaped relation with n_1 *with the peak of political representation at* $n_1^* = (1-\alpha)^{\frac{1}{\alpha}}$.

Proof: In the Appendix. ■

As discussed in proposition 2, concentration of an ethnic group in districts having a less responsive mass of voters improves their political representation. Since $\phi_3 > \phi$, it is beneficial for group 1 to be more concentrated in districts having a smaller proportion of group 3 members. The settlement area of a concentrated group increases inelastically with its population share. This means that an increase in population share of a group increases its population density in the area in which it resides. As n_1 increases, n_2 declines. The following three effects take place if group 2 is geographically concentrated:

1. In the region where only groups 1 and 3 reside, population density of group 1 increases. This leads to a decline in proportion of group 3 in the districts located in this region and increases G_1^*.

2. In the region where all groups 1, 2 and 3 reside, the population density of group 1 increases and the population density of group 2 declines. When $n_1 < n_2$, the combined density of group 1 and group 2 increases with an increase in n_1. This leads to a decline in population share of group 3 in districts in this region and increases G_1^*. The opposite happens when $n_1 \geq n_2$.

3. When $n_1 < n_2$ the ratio $\frac{A_{1\cap 2}}{A_1}$ remains constant at O. However when $n_1 \geq n_2$, an increase in n_1 decreases this ratio. As a result, a higher proportion of group 1 members now reside in the area where only group 1 and 3 reside. This area has a higher proportion of group 3 members. This has a negative effect on G_1^*.

Both the first and second factors improve G_1^* when $n_1 < n_2$. However, when $n_1 \geq n_2$, the first factor contributes to improvement in G_1^* and the second and third factor have a negative effect on G_1^*. When there is a sufficient overlap between the settlement areas of group 1 and 2, the second and third factor dominate and G_1^* decreases.

When group 2 is dispersed, the combined density of group 1 and 2 in the settlement area of group 1 initially increases and is maximized at n_1^*. When $n_1 < n_1^*$, an increase in n_1 decreases the proportion of more responsive group 3 members in districts in group 1's settlement area. This increases G_1^*. The opposite happens when $n_1 \geq n_1^*$.

6 Verification of the Model

In this section we first empirically verify one key parameter restriction of the model that we need for our main result. Proposition 3 required that a minority group's settlement area be inelastically related to its population share. To test this assumption we run the following specification

$$\ln S_{ict} = \alpha \ln n_{ict} + \gamma X_{ict} + \delta_{ct} + \epsilon_c \tag{4}$$

where S_{ict} is the settlement area of a group i which is geographically concentrated in country c in year t and n_{ict} is the population share that group. α therefore measures the elasticity of settlement area with respect to population share of a group, and therefore, is a direct estimate of the parameter α in the model. The EPR dataset provides information about the settlement area of groups which are geographically concentrated. Therefore, we can estimate the equation (4). The results are reported in Appendix table A6. Column (1) reports the main estimate of α to be 0.578. It is highly statistically significant and its magnitude is less than one. This confirms our hypothesis. Further, we estimate this parameter in two sub-samples - one where the minority groups' population shares are smaller than 0.25 (column (2)) and smaller than 0.1 (column (3)). Both estimates are close to each other and are similar to the main estimate. This shows that the elasticity of settlement area with respect to population share of a group is indeed stable, further confirming our model's assumption.

The primary aim of the model is to justify the empirical pattern established in the Section 4 of the paper. The model, however, generates some additional predictions regarding the exact nature of the relationship between group size and access to political power. It is, therefore, important to test if these additional comparative static results hold in order to verify if the proposed model is indeed valid. We now turn to that discussion in the following paragraphs.

Proposition 3 states that we should observe the inverted U-shaped relationship between group size and power status under the MR system only for groups which are geographically concentrated. Also, a group's geographic concentration should not matter for the result for the PR system. We verify this by running the following specification for the samples of MR and PR country-year observations separately:

$$Y_{ict} = \delta_{ct} + \eta_1 n_{ict} + \eta_2 n_{ict}^2 + \eta_3 C_{ict} * n_{ict} + \eta_4 C_{ict} * n_{ict}^2 + \gamma X_{ict} + \epsilon_c \tag{5}$$

Table 6: Geographical concentration

		Political inclusion	
	(1)	(2)	(3)
Population share	5.613***	1.754	1.972
	(1.454)	(2.242)	(1.220)
Population share - squared	-10.91**	-1.136	-6.604*
	(4.620)	(8.197)	(3.635)
Proportional*Population share	-3.832**		
	(1.784)		
Proportional*Population share - squared	9.468*		
	(5.638)		
Concentrated*population share		4.938**	0.608
		(2.200)	(1.172)
Concentrated*population share - squared		-12.35	2.595
		(7.830)	(4.120)
Mean inclusion	0.367	0.448	0.265
Observations	8,712	4,835	3,877
R-squared	0.675	0.652	0.769
Ethnicity-year controls	YES	YES	YES
Country-year FE	YES	YES	YES

Notes: The data is at the ethnicity-country-year level. Column (2) reports results for MR and column (3) reports results for PR. Standard errors are clustered at the country level and reported in parenthesis. *** p<0.01, ** p<0.05, * p<0.1.

where C_{ict} is a dummy indicating whether the group i is geographically concentrated in country c in year t. Proposition 3 implies that for the sample of MR countries, η_1 and η_2 should be zero and we should have $\eta_3 > 0$ and $\eta_4 < 0$. For the set of PR countries all the coefficients η_1-η_4 should be zero. Table 6 reports the results and the predictions largely are verified. Column (1) reproduces the main result, and columns (2) and (3) provides the estimates of η_1-η_4 for MR and PR countries, respectively. As is evident, for the MR countries the relationship is only true for geographically concentrated groups. For the PR countries only one second order term is significant at 10% level.

Proposition 3 further specifies that under the MR system, the peak political representation is achieved when the population share of the group equals $\frac{1-n_3}{2}$ when the group is geographically concentrated, where n_3 is the population share of the majority group. Therefore, for larger values of the majority group's share, the peak is achieved at lower values of the minority group's size. We test this prediction by running specification (1) on various sub-samples of the data where we vary the size of the majority group. The results are reported in table A7. Column (1) reproduces the main result and columns (2)-(6) report the results for sub-samples where the majority group's population share is larger than 0.3, 0.4, 0.5, 0.6 and 0.7, respectively. The table also reports the population shares where the peak inclusion is achieved. Barring the first case, in all other cases we see that the population share at which the peak inclusion is achieved is declining when we move to countries with larger majority groups.

Table 7: Varying largest group size

	Political inclusion					
	(1)	(2)	(3)	(4)	(5)	(6)
Population share	5.613***	4.106***	4.610***	5.428***	5.943***	7.695***
	(1.454)	(1.269)	(1.502)	(1.785)	(2.175)	(2.180)
Population share - squared	-10.91**	-6.005*	-7.354*	-8.200	-9.673	-18.79***
	(4.620)	(3.597)	(3.990)	(5.329)	(6.799)	(6.072)
Proportional*Population share	-3.832**	-2.474	-3.363*	-4.307**	-4.321	-7.534***
	(1.784)	(1.720)	(1.904)	(2.152)	(2.700)	(2.375)
Proportional*Population share - squared	9.468*	4.884	7.605	8.933	7.597	21.44***
	(5.638)	(5.036)	(5.313)	(6.503)	(9.537)	(6.986)
$H_0 : \beta_1 + \beta_3 = 0$ (p-value)	.091	.164	.263	.341	.285	.906
$H_0 : \beta_2 + \beta_4 = 0$ (p-value)	.648	.740	.935	.824	.718	.642
Peak inclusion	0.257	0.342	0.313	0.331	0.307	0.205
Mean inclusion	0.367	0.287	0.258	0.215	0.186	0.156
Observations	8,712	6,923	6,279	5,751	4,749	3,871
R-squared	0.675	0.679	0.699	0.671	0.706	0.730
Ethnicity-year controls	YES	YES	YES	YES	YES	YES
Country-year FE	YES	YES	YES	YES	YES	YES

Notes: The data is at ethnicity-country-year level for 102 countries and 438 ethno-country groups. Largest group size in Column (2) $\geq$ 0.3, in Column (3) $\geq$ 0.4, in Column (4) $\geq$ 0.5, in column (5) $\geq$ 0.6 and in column (6) $\geq$ 0.7. Standard errors are clustered at the country level. *** p<0.01, ** p<0.05, * p<0.1.

7 Conclusion

This paper explores how electoral systems influence the relation between size of a minority group and its access to power in the national executive. We find empirical evidence that in countries with the PR system, population share of a minority has no effect on its executive power. In countries with MR, there is an inverted U-shaped effect of population share on executive power. This suggests that electoral systems are important in determining inequality between groups of varying sizes. An implication is that under PR, group size inequality does not translate into inequality in the political representation of minorities. We further show that the inverted U-shaped relation under MR is driven by the way settlement area of a minority expands with size. This points to the importance of taking into account settlement patterns of groups in a comparative analysis of the two systems.

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A Tables

Table A1: Baseline specification

			Political inclusion			
	(1)	(2)	(3)	(4)	(5)	(6)
Population share	5.201***	5.027***	4.753***	4.475***	5.212***	5.613***
	(0.889)	(1.069)	(1.123)	(1.012)	(1.462)	(1.454)
Population share - squared	-8.511***	-8.433**	-7.648**	-6.340**	-9.535**	-10.91**
	(2.684)	(3.382)	(3.280)	(3.001)	(4.500)	(4.620)
Proportional*Population share	-0.715	-0.995	-1.356	-1.768	-3.217*	-3.832**
	(1.440)	(1.480)	(1.375)	(1.331)	(1.755)	(1.784)
Proportional*Population share - squared	0.506	1.319	2.013	2.524	7.395	9.468*
	(4.227)	(4.179)	(3.792)	(3.759)	(5.459)	(5.638)
Proportional	-0.127	-0.131	-0.0332	0.0624	0.260*	
	(0.0980)	(0.0902)	(0.0664)	(0.0652)	(0.149)	
Years peace		0.00134	0.00202	0.00356**	0.00501***	0.00511***
		(0.00114)	(0.00165)	(0.00143)	(0.00171)	(0.00168)
Statewide settlement		-0.170	-0.00666	0.0166	0.119	-0.159
		(0.327)	(0.297)	(0.242)	(0.377)	(0.386)
Concentrated settlement		-0.162	-0.0630	-0.0859	-0.0621	-0.195
		(0.123)	(0.122)	(0.115)	(0.0645)	(0.124)
Urban settlement		-0.334**	-0.170	-0.191	-0.113	-0.256*
		(0.150)	(0.149)	(0.142)	(0.0977)	(0.142)
Dispersed settlement		-0.261*	-0.0939	-0.158	-0.102	-0.234
		(0.143)	(0.147)	(0.140)	(0.0925)	(0.145)
Migrant settlement		-0.362**	-0.173	-0.333*	-0.312	-0.454
		(0.161)	(0.172)	(0.177)	(0.237)	(0.292)
Transethnic-kin inclusion		0.0744	0.0898*	0.0355	0.0276	0.0227
		(0.0623)	(0.0502)	(0.0521)	(0.0390)	(0.0421)
Transethnic-kin exclusion		0.0137	0.0380	0.0743*	0.0956***	0.109***
		(0.0671)	(0.0455)	(0.0417)	(0.0348)	(0.0336)
Fraction largest religion		-0.0743	-0.186	-0.228*	-0.0763	-0.0614
		(0.170)	(0.112)	(0.116)	(0.107)	(0.104)
Fraction largest language		0.307***	0.255***	0.223***	0.271***	0.289***
		(0.0931)	(0.0899)	(0.0829)	(0.0869)	(0.0884)
Ethnic fractionalization			0.0679***	0.0521**	0.0209	
			(0.0202)	(0.0201)	(0.0235)	
Number of relevant groups			0.00469	0.00233	0.0123	
			(0.00565)	(0.00534)	(0.0195)	
Competitiveness of participation			0.0671**	0.0856***	0.00949	
			(0.0274)	(0.0203)	(0.0172)	
Constraints chief executive			-0.00580	0.00228	-0.0189*	
			(0.0208)	(0.0210)	(0.0111)	
$H_0 : \beta_1 + \beta_3 = 0$ (p-value)	.000	.000	.001	.011	.060	.091
$H_0 : \beta_2 + \beta_4 = 0$ (p-value)	.016	.017	.046	.172	.508	.648
Mean inclusion	0.367	0.367	0.367	0.367	0.367	0.367
Observations	9,304	9,304	9,304	9,304	9,304	8,712
R-squared	0.240	0.282	0.412	0.450	0.640	0.675
Year FE	NO	NO	YES	YES	YES	NO
Region FE	NO	NO	NO	YES	NO	NO
Country FE	NO	NO	NO	NO	YES	NO
Country-year FE	NO	NO	NO	NO	NO	YES

Notes: The data is at ethnicity-country-year level for 102 countries and 438 ethno-country groups. There are 87 countries and 422 ethno-country groups in column (6). The time period is 1946-2013. Standard errors are clustered at the country level. *** p<0.01, ** p<0.05, * p<0.1.

Table A2: Robustness and development

	Political inclusion	Political inclusion (1946-1979)	Political inclusion (1980-2013)	Power rank	Nightlight intensity	Political inclusion	Power rank	Nightlight intensity
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Population share	5.613***	7.182**	5.175***	6.322***	1.485**			
	(1.454)	(2.679)	(1.089)	(1.880)	(0.641)			
Population share - squared	-10.91**	-16.17*	-9.550***	-9.478	-3.417**			
	(4.620)	(9.272)	(3.137)	(6.221)	(1.638)			
Proportional*Population share	-3.832**	-6.386**	-2.938*	-5.290*	-1.225*			
	(1.784)	(2.401)	(1.538)	(2.825)	(0.624)			
Proportional*Population share - squared	9.468*	16.38*	6.754	12.46	2.599			
	(5.638)	(8.478)	(4.589)	(9.451)	(1.654)			
Relative population share						2.699***	2.713***	0.715***
						(0.475)	(0.747)	(0.239)
Relative population share-squared						-2.372***	-1.725	-0.736***
						(0.527)	(1.063)	(0.238)
Proportional*relative population share						-1.675***	-2.054*	-0.527**
						(0.620)	(1.061)	(0.203)
Proportional*relative population share-squared						1.864**	2.398	0.452**
						(0.712)	(1.542)	(0.208)
Years peace	0.00511***	0.0168***	0.00485***	0.00686***	0.000340*	0.00439***	0.00606***	0.000199
	(0.00168)	(0.00303)	(0.00179)	(0.00211)	(0.000183)	(0.00130)	(0.00191)	(0.000214)
Statewide settlement	-0.159	-1.243***	0.114	0.344	0.173	-0.0741	0.386	0.185*
	(0.386)	(0.207)	(0.369)	(0.798)	(0.111)	(0.341)	(0.712)	(0.108)
Concentrated settlement	-0.195	-0.755***	-0.0886	0.166		-0.160	0.170	
	(0.124)	(0.0459)	(0.0817)	(0.208)		(0.125)	(0.195)	
Urban settlement	-0.256*	-0.814***	-0.143	-0.0588		-0.219	-0.0559	
	(0.142)	(0.0765)	(0.133)	(0.217)		(0.149)	(0.212)	
Dispersed settlement	-0.234	-0.685***	-0.140	-0.0108		-0.209	-0.0259	
	(0.145)	(0.0715)	(0.112)	(0.236)		(0.147)	(0.226)	
Migrant settlement	-0.454	-0.797***	-0.408	-0.434		-0.431	-0.445	
	(0.292)	(0.0837)	(0.262)	(0.363)		(0.290)	(0.340)	
Transethnic-kin inclusion	0.0227	0.0551	0.0138	0.00793	0.00568	0.0192	0.0175	-0.000283
	(0.0421)	(0.0900)	(0.0442)	(0.0659)	(0.00649)	(0.0463)	(0.0712)	(0.00780)
Transethnic-kin exclusion	0.109***	0.127**	0.0980***	0.0855	0.00385	0.0995***	0.0737	0.00185
	(0.0336)	(0.0544)	(0.0370)	(0.0540)	(0.00715)	(0.0331)	(0.0543)	(0.00725)
Fraction largest religion	-0.0614	0.0794	-0.0609	0.0610	0.0124	-0.0583	0.0800	0.00253
	(0.104)	(0.227)	(0.0925)	(0.195)	(0.0251)	(0.110)	(0.193)	(0.0187)
Fraction largest language	0.289***	0.349**	0.243***	0.256*	-0.00881	0.262***	0.250*	-0.0138
	(0.0884)	(0.135)	(0.0794)	(0.149)	(0.0320)	(0.0754)	(0.144)	(0.0321)
						(0.712)	(1.542)	(0.208)
$H_0: \beta_1 + \beta_3 = 0$ (p-value)	.091	.498	.048	.635	.185	.023	.394	.066
$H_0: \beta_2 + \beta_4 = 0$ (p-value)	.648	.942	.408	.684	.281	.344	.559	.108
Mean dependent	0.367	0.332	.379	2.277	0.026	0.367	2.227	0.026
Observations	8,712	2,296	6,416	8,712	3,469	8,712	8,712	3,469
R-squared	0.675	0.660	0.691	0.667	0.710	0.682	0.671	0.722
Country-year FE	YES	YES	YES	YES	YES	YES	YES	YES

Notes: The data is at ethnicity-country-year level. 70 countries and 278 ethno-country groups in column (4) and (8). Standard errors are clustered at the country level and reported in parenthesis. *** p<0.01, ** p<0.05, * p<0.1.

Table A3: Comparing ethnic group in countries in the same continent

	Political inclusion	Power rank	Nightlight intensity
	(1)	(2)	(3)
Population share	12.62***	10.69**	0.746***
	(2.545)	(4.254)	(0.118)
Population share - squared	-30.17***	-20.75*	-2.001***
	(6.620)	(10.11)	(0.327)
Proportional*Population share	-10.17***	-8.291	-0.777***
	(3.397)	(5.132)	(0.162)
Proportional*Population share - squared	29.73**	23.65	2.077***
	(13.99)	(19.62)	(0.470)
Proportional	0.195**	-0.291*	0.00430
	(0.0716)	(0.157)	(0.00257)
Years peace	0.000428	0.00108	0.000151*
	(0.00130)	(0.00235)	(7.48e-05)
Urban settlement	-0.185	-0.417*	
	(0.119)	(0.217)	
Dispersed settlement	-0.160*	-0.0948	
	(0.0898)	(0.193)	
Transethnic-kin inclusion	-0.902***	-1.179***	-0.0117**
	(0.122)	(0.261)	(0.00402)
Transethnic-kin exclusion	-0.0331	0.203*	0.0241***
	(0.0351)	(0.117)	(0.00220)
Fraction largest religion	-0.0557	0.0848	0.0182*
	(0.184)	(0.442)	(0.00943)
Fraction largest language	0.0301	-0.612	-0.0529***
	(0.212)	(0.423)	(0.00346)
Ethnic fractionalization	-0.0242	0.0348	-0.00255
	(0.0194)	(0.0393)	(0.00167)
Number of relevant groups	0.000899	-0.0133**	-0.00239**
	(0.00287)	(0.00629)	(0.000942)
$H_0 : \beta_1 + \beta_3 = 0$ (p-value)	.208	.345	.606
$H_0 : \beta_2 + \beta_4 = 0$ (p-value)	.962	.820	.702
Mean dependent	0.110	1.858	0.009
Observations	1,219	1,219	417
R-squared	0.819	0.757	0.898
Group-region-year FE	YES	YES	YES

Notes: Comparison of 21 ethnic groups in 40 countries. Column (3) compares 12 ethnic groups in 30 countries. The time period is 1946-2013. Standard errors are clustered at the country level and reported in parenthesis. *** p<0.01, ** p<0.05, * p<0.1.

Table A4: IV estimates

Panel A: Second stage

	Political inclusion		Nightlight intensity
	(1)	(2)	(3)
Population share	5.613***	10.11***	1.195
	(1.454)	(3.594)	(0.720)
Population share - squared	-10.91**	-28.01*	-2.303
	(4.620)	(15.65)	(1.719)
Proportional*Population share	-3.832**	-11.70**	-0.964
	(1.784)	(4.785)	(0.624)
Proportional*Population share - squared	9.468*	38.68*	1.868
	(5.638)	(22.92)	(1.561)
Years peace	0.00511***	0.00554***	0.000527**
	(0.00168)	(0.00147)	(0.000198)
Statewide settlement	-0.159	-0.377	0.0378
	(0.386)	(1.104)	(0.0349)
Concentrated settlement	-0.195	-0.153	
	(0.124)	(0.177)	
Urban settlement	-0.256*	-0.0825	
	(0.142)	(0.193)	
Dispersed settlement	-0.234	-0.130	
	(0.145)	(0.197)	
Migrant settlement	-0.454	0.0789	
	(0.292)	(0.206)	
Transethnic-kin inclusion	0.0227	0.0602	0.00104
	(0.0421)	(0.0587)	(0.00653)
Transethnic-kin exclusion	0.109***	0.0729	0.00620
	(0.0336)	(0.0508)	(0.00702)
Fraction largest religion	-0.0614	-0.0668	-0.00821
	(0.104)	(0.171)	(0.0365)
Fraction largest language	0.289***	0.394***	-0.0241
	(0.0884)	(0.144)	(0.0495)
$H_0 : \beta_1 + \beta_3 = 0$ (p-value)	.091	.358	.087
$H_0 : \beta_2 + \beta_4 = 0$ (p-value)	.648	.269	.213
Mean Dependent	0.367	0.464	.021
Observations	8,712	5,199	2,306
R-squared	0.675	0.615	0.507
Country-year FE	YES	YES	YES
Kleibergen-Paap rk LM stats		3.50	2.804
Cragg-Donald Wald F stat		90.49	39.83
Stock and Yogo critical value		7.03	7.03
F stat (Proportional*Population share)		31.23	13.11
F stat (Proportional*Population share - squared)		19.80	10.41

Panel B: Country level

	Proportional
Colonialist proportional	0.463***
	(0.118)
Mean dependent	.450
Observations	1,309
R-squared	0.388
Region-year FE	YES

Notes: The data is at the ethnicity-country-year level. In panel A, column (2), we look at 267 ethno-country groups in 56 countries. 170 ethno-country groups in 44 countries in column (3). The time period is 1946-2013. Standard errors are clustered at the country level and reported in parenthesis. Results in panel B for 64 countries. Diagnostic tests for first stage regression. *** p<0.01, ** p<0.05, * p<0.1.

Table A5: Geographical concentration

	Political inclusion		
	(1)	(2)	(3)
Population share	5.613***	1.754	1.972
	(1.454)	(2.242)	(1.220)
Population share - squared	-10.91**	-1.136	-6.604*
	(4.620)	(8.197)	(3.635)
Proportional*Population share	-3.832**		
	(1.784)		
Proportional*Population share - squared	9.468*		
	(5.638)		
Concentrated*population share		4.938**	0.608
		(2.200)	(1.172)
Concentrated*population share - squared		-12.35	2.595
		(7.830)	(4.120)
Years peace	0.00511***	0.00624***	0.00116
	(0.00168)	(0.00109)	(0.00215)
Statewide settlement	-0.159	-0.753***	0.496***
	(0.386)	(0.183)	(0.135)
Concentrated settlement	-0.195	-0.664***	-0.651***
	(0.124)	(0.163)	(0.0947)
Urban settlement	-0.256*	-0.669***	-0.584***
	(0.142)	(0.155)	(0.109)
Dispersed settlement	-0.234	-0.530***	-0.644***
	(0.145)	(0.142)	(0.0953)
Migrant settlement	-0.454	-0.787***	-0.863***
	(0.292)	(0.191)	(0.182)
Transethnic-kin inclusion	0.0227	-0.00318	0.0308
	(0.0421)	(0.0613)	(0.0544)
Transethnic-kin exclusion	0.109***	0.182***	-0.0188
	(0.0336)	(0.0472)	(0.0395)
Fraction largest religion	-0.0614	-0.0713	0.0756
	(0.104)	(0.178)	(0.106)
Fraction largest language	0.289***	0.409***	0.0496
	(0.0884)	(0.0994)	(0.0800)
Mean inclusion	0.367	0.448	0.265
Observations	8,712	4,835	3,877
R-squared	0.675	0.652	0.769
Country-year FE	YES	YES	YES

Notes: The data is at the ethnicity-country-year level. Column (2) reports results for MR and column (3) reports results for PR. Standard errors are clustered at the country level and reported in parenthesis. *** p<0.01, ** p<0.05, * p<0.1.

Table A6: Inelastic expansion of settlement area

	ln(Settlement area)		
	(1)	(2)	(3)
ln(Population share)	0.578***	0.602***	0.632***
	(0.135)	(0.149)	(0.130)
Years peace	0.00420	0.00287	0.000800
	(0.00731)	(0.00701)	(0.00676)
Statewide settlement	-0.527	-0.864	
	(1.043)	(0.990)	
Transethnic-kin inclusion	0.0759	0.165	0.299
	(0.156)	(0.152)	(0.192)
Transethnic-kin exclusion	0.326	0.277	0.0500
	(0.224)	(0.220)	(0.201)
Fraction largest religion	-0.158	0.0602	0.357
	(0.706)	(0.773)	(1.062)
Fraction largest language	-1.449***	-1.134***	-0.796*
	(0.419)	(0.415)	(0.471)
Mean dependent	10.140	10.006	9.783
Observations	6,665	5,946	4,357
R-squared	0.784	0.768	0.730
Country-year FE	YES	YES	YES

Notes: The data is at the ethnicity-country-year level for 75 countries and 68 years. All concentrated minorities in column (1). Minority population share in column (2) ≤ 0.25 and that in column(3) ≤ 0.10. Standard errors are clustered at the country level and reported in parenthesis. *** p<0.01, ** p<0.05, * p<0.1.

Table A7: Varying largest group size

	Political inclusion					
	(1)	(2)	(3)	(4)	(5)	(6)
Population share	5.613***	4.106***	4.610***	5.428***	5.943***	7.695***
	(1.454)	(1.269)	(1.502)	(1.785)	(2.175)	(2.180)
Population share - squared	-10.91**	-6.005*	-7.354*	-8.200	-9.673	-18.79***
	(4.620)	(3.597)	(3.990)	(5.329)	(6.799)	(6.072)
Proportional*Population share	-3.832**	-2.474	-3.363*	-4.307**	-4.321	-7.534***
	(1.784)	(1.720)	(1.904)	(2.152)	(2.700)	(2.375)
Proportional*Population share - squared	9.468*	4.884	7.605	8.933	7.597	21.44***
	(5.638)	(5.036)	(5.313)	(6.503)	(9.537)	(6.986)
Years peace	0.00511***	0.00278	0.00303	0.00306	0.00374*	0.00498**
	(0.00168)	(0.00185)	(0.00195)	(0.00193)	(0.00211)	(0.00215)
Statewide settlement	-0.159	-0.119	-0.386*	-0.967***	-1.120***	
	(0.386)	(0.370)	(0.220)	(0.185)	(0.163)	
Concentrated settlement	-0.195	-0.197	-0.155	-0.673***	-0.667***	0.669***
	(0.124)	(0.132)	(0.143)	(0.0734)	(0.0770)	(0.184)
Urban settlement	-0.256*	-0.289*	-0.245	-0.751***	-0.748***	0.517***
	(0.142)	(0.162)	(0.172)	(0.106)	(0.120)	(0.162)
Dispersed settlement	-0.234	-0.244	-0.180	-0.690***	-0.703***	0.628***
	(0.145)	(0.163)	(0.168)	(0.0787)	(0.0813)	(0.212)
Migrant settlement	-0.454	-0.458	-0.399	-0.918***	-0.926***	0.618***
	(0.292)	(0.288)	(0.302)	(0.140)	(0.133)	(0.201)
Transethnic-kin inclusion	0.0227	-0.00491	0.0286	0.0230	0.0132	0.0590
	(0.0421)	(0.0517)	(0.0540)	(0.0572)	(0.0693)	(0.0774)
Transethnic-kin exclusion	0.109***	0.0989**	0.0811**	0.0888**	0.101*	0.114**
	(0.0336)	(0.0422)	(0.0385)	(0.0422)	(0.0504)	(0.0510)
Fraction largest religion	-0.0614	-0.124	-0.0867	-0.0511	-0.0322	-0.0612
	(0.104)	(0.113)	(0.117)	(0.120)	(0.123)	(0.119)
Fraction largest language	0.289***	0.216***	0.223***	0.222**	0.181*	0.144
	(0.0884)	(0.0787)	(0.0837)	(0.0874)	(0.100)	(0.0960)
$H_0 : \beta_1 + \beta_3 = 0$ (p-value)	.091	.164	.263	.341	.285	.906
$H_0 : \beta_2 + \beta_4 = 0$ (p-value)	.648	.740	.935	.824	.718	.642
Peak inclusion	0.257	0.342	0.313	0.331	0.307	0.205
Mean inclusion	0.367	0.287	0.258	0.215	0.186	0.156
Observations	8,712	6,923	6,279	5,751	4,749	3,871
R-squared	0.675	0.679	0.699	0.671	0.706	0.730
Country-year FE	YES	YES	YES	YES	YES	YES

Notes: The data is at ethnicity-country-year level for 102 countries and 438 ethno-country groups. Largest group size in Column (2) ≥ 0.3, in Column (3) ≥ 0.4, in Column (4) ≥ 0.5, in column (5) ≥ 0.6 and in column (6) ≥ 0.7. Standard errors are clustered at the country level. *** p<0.01, ** p<0.05, * p<0.1.

B Proofs of Propositions

Proof of proposition 1 We give a general proof with J ethnic groups for propositions 1 and 2. Consider the case of party A. Vote share of party A among members of group j is given by:

$$\pi_{A,j} = Pr[U(f_j^A) > U(f_j^B) + \delta + \sigma_{i,j}]$$

Assuming that $\psi \geq \phi_j$ for all j, we get:

$$\pi_{A,j} = \frac{1}{2} + \phi_j[U(f_j^A) - U(f_j^B) - \delta]$$

Party A will win elections if more than half the population votes for it. Probability of winning for party A is given by:

$$p_A = Pr[\frac{\sum_{j=1}^{J} n_j \pi_{A,j}}{\sum_{j=1}^{J} n_j} > \frac{1}{2}]$$

This can simply be written as:

$$p_A = \frac{1}{2} + \frac{\psi \sum_{j=1}^{J} \phi_j n_j (U(f_j^A) - U(f_j^B))}{\sum_{j=1}^{J} \phi_j n_j}$$

Thus, party A solves:

$$\max_{f_j^A \geq 0} p_A = \frac{1}{2} + \frac{\psi \sum_{j=1}^{J} \phi_j n_j (U(f_j^A) - U(f_j^B))}{\sum_{j=1}^{J} \phi_j n_j}$$

$$s.t. \quad \sum_{j=1}^{J} n_j f_j^A \leq S$$

Solving the above optimization problem gives the equilibrium condition in (1).

Proof of proposition 2 In a K district majoritarian election, probability of winning for party A in constituency k, as can be seen from the result under proportional electoral system, is given by:

$$p_A^k = \frac{1}{2} + \frac{\psi \sum\limits_{j=1}^{J} \phi_j n_j^k (U(f_j^A) - U(f_j^B))}{\sum\limits_{j=1}^{J} \phi_j n_j^k}$$

Party A will win the election if it wins more than half the votes in more than half the districts. If both parties win in equal number of districts, then the winner will be chosen randomly. Party A solves the following optimization problem under majoritarian elections:

$$\max_{f_j^A \geq 0} p_A \quad s.t. \quad \sum_{j=1}^{J} n_j f_j^A \leq S$$

Since the parties are symmetric, in equilibrium, $p_A^k = \frac{1}{2}$ for all districts. Thus, given a district k, we denote the probability of winning in any other given district, with a slight abuse of notation, as p_A^{-k}. When K=2, Probability of winning can be written as:

$$p_A = p_A^k p_A^{-k} + \frac{1}{2}[p_A^k(1 - p_A^{-k}) + p_A^{-k}(1 - p_A^k)]$$

This can be simplified to:

$$= \frac{1}{2} p_A^k + \frac{1}{4}$$

And when K>2, probability of winning is:

$$\begin{aligned} p_A = & \sum_{i=\lfloor K/2 \rfloor}^{K-1} \binom{K-1}{i} p_A^k (p_A^{-k})^i (1 - p_A^{-k})^{K-1-i} \\ & + \sum_{i=\lfloor K/2 \rfloor + 1}^{K-1} \binom{K-1}{i} (1 - p_A^k)(p_A^{-k})^i (1 - p_A^{-k})^{K-1-i} \\ & + \frac{1}{2}[\frac{1 + (-1)^K}{2}][\binom{K-1}{\lfloor K/2 \rfloor - 1} p_A^k (p_A^{-k})^{(K/2)-1} (1 - p_A^{-k})^{K/2} \\ & + \binom{K-1}{\lfloor K/2 \rfloor} (p_A^{-k})^{K/2} (1 - p_A^{-k})^{(K/2)-1} (1 - p_A^k)] \end{aligned}$$

This can be simplified to:

$$\begin{aligned} p_A =& \frac{1}{2^{K-1}}[\binom{K-1}{\lfloor K/2 \rfloor} p_A^k + \sum_{i=\lfloor K/2 \rfloor+1}^{K-1} \binom{K-1}{i}] \\ &+ \frac{1}{2^K}[\frac{1+(-1)^K}{2}][(\binom{K-1}{\lfloor K/2 \rfloor - 1} - \binom{K-1}{\lfloor K/2 \rfloor}) p_A^k + \binom{K-1}{\lfloor K/2 \rfloor}] \end{aligned}$$

Using this, we calculate:

$$\frac{dp_A}{dp_A^k} = C(K) = (\frac{1+(-1)^{K-1}}{2}) \binom{K-1}{\lfloor K/2 \rfloor} \frac{1}{2^{K-1}} + (\frac{1+(-1)^K}{2}) \binom{K}{\lfloor K/2 \rfloor} \frac{1}{2^K}$$

For the first order condition to the optimization problem, we need to calculate:

$$\frac{dp_A}{df_j^A} = \sum_{k=1}^{K} \frac{dp_A}{dp_A^k} \frac{dp_A^k}{df_j^A}$$

Substituting the expression for dp_A/dp_A^k, we can write this as:

$$\frac{dp_A}{df_j^A} = C(K) \sum_{k=1}^{K} \frac{dp_A^k}{df_j^A}$$

We can now easily solve the optimization problem to give the equilibrium condition given in (2). Consider the case where all groups are equally responsive to electoral promises i.e. $\phi_j = \phi$ for all j. Since $\sum_{j=1}^{J} n_j^k = 1$ for all k and $\sum_{k=1}^{K} n_j^k / n_j = 1$ for all j, (2) can be simplified to:

$$U'(f_i^*) = U'(f_l^*) \quad \forall i, l$$

Now, consider the case where $n_j^k = n_j$ for all k. In this case, (2) can be simplified to:

$$\phi_i U'(f_i^*) = \phi_l U'(f_l^*) \quad \forall i, l$$

Both the above special cases indicate that when groups are evenly distributed across districts or when all groups are equally responsive to electoral promises, majoritarian elections give the same equilibrium political representation and per capita transfers as the proportional representation system.

Proof of proposition 3 (a) When group 2 is concentrated, we have four types of constituencies based on the identity of groups residing in them: (1) Only group 1 and 3 reside (2) Only group 2 and 3 reside (3) Group 1, 2 and 3 all reside (4) Only group 3 resides. Densities D^m of constituency type m are:

$$D^1 = n_1^{1-\alpha} + n_3 \qquad D^2 = n_2^{1-\alpha} + n_3 \qquad D^3 = n_1^{1-\alpha} + n_2^{1-\alpha} + n3 \qquad D^4 = n3$$

Since constituencies have equal populations:

$$D^m a^m = \frac{1}{K} \quad \forall m$$

Where a^m is the area per consituency for each type m. Using this we get:

$$a^1 = \frac{1}{K(n_1^{1-\alpha} + n_3)} \quad a^2 = \frac{1}{K(n_2^{1-\alpha} + n_3)} \quad a^3 = \frac{1}{K(n_1^{1-\alpha} + n_2^{1-\alpha} + n_3)} \quad a^4 = \frac{1}{K(n_3)}$$

Number of consituencies K^m of each type can be calculated by dividing total area of occupied by all constituencies of a given type by a^m:

$$\begin{aligned}
K^1 &= K(n_1^\alpha - O \cdot min(n_1, n_2)^\alpha)(n_1^{1-\alpha} + n_3) \\
K^2 &= K(n_2^\alpha - O \cdot min(n_1, n_2)^\alpha)(n_2^{1-\alpha} + n_3) \\
K^3 &= K(O \cdot min(n_1, n_2)^\alpha)(n_1^{1-\alpha} + n_2^{1-\alpha} + n_3) \\
K^4 &= K(1 - n_1^\alpha - n_2^\alpha + O \cdot min(n_1, n_2)^\alpha)(n_3)
\end{aligned}$$

Proportion of group i in constituency of type m n_i^m:

$$\begin{aligned}
&n_1^1 = \frac{n_1^{1-\alpha}}{n_1^{1-\alpha} + n_3} \quad n_1^2 = 0 \quad n_1^3 = \frac{n_1^{1-\alpha}}{n_1^{1-\alpha} + n_2^{1-\alpha} + n_3} \quad n_1^4 = 0 \\
&n_2^1 = 0 \quad n_2^2 = \frac{n_2^{1-\alpha}}{n_2^{1-\alpha} + n_3} \quad n_2^3 = \frac{n_2^{1-\alpha}}{n_1^{1-\alpha} + n_2^{1-\alpha} + n_3} \quad n_2^4 = 0 \\
&n_3^1 = \frac{n_3}{n_1^{1-\alpha} + n_3} \quad n_3^2 = \frac{n_3}{n_2^{1-\alpha} + n_3} \quad n_3^3 = \frac{n_3}{n_1^{1-\alpha} + n_2^{1-\alpha} + n_3} \quad n_3^4 = 1
\end{aligned}$$

For simplicity, let $U(f_j) = \log(f_j)$. Therefore, $U'(f_j) = \frac{1}{f_j}$. Similar to the proof of proposition 2, we can obtain the first order conditions at equilibrium as:

$$\begin{aligned}
\gamma f_1 =& K\phi(n_1^{\alpha} - O \cdot min(n_1, n_2)^{\alpha})(n_1^{1-\alpha} + n_3)(\frac{n_1^{-\alpha}}{\phi n_1^{1-\alpha} + \phi_3 n_3}) \\
&+ K\phi(O \cdot min(n_1, n_2)^{\alpha})(n_1^{1-\alpha} + n_2^{1-\alpha} + n_3)(\frac{n_1^{-\alpha}}{\phi(n_1^{1-\alpha} + n_2^{1-\alpha}) + \phi_3 n_3})
\end{aligned}$$

$$\begin{aligned}
\gamma f_2 =& K\phi(n_2^{\alpha} - O \cdot min(n_1, n_2)^{\alpha})(n_2^{1-\alpha} + n_3)(\frac{n_2^{-\alpha}}{\phi n_2^{1-\alpha} + \phi_3 n_3}) \\
&+ K\phi(O \cdot min(n_1, n_2)^{\alpha})(n_1^{1-\alpha} + n_2^{1-\alpha} + n_3)(\frac{n_2^{-\alpha}}{\phi(n_1^{1-\alpha} + n_2^{1-\alpha}) + \phi_3 n_3})
\end{aligned}$$

$$\begin{aligned}
\gamma f_3 =& K\phi_3(n_1^{\alpha} - O \cdot min(n_1, n_2)^{\alpha})(n_1^{1-\alpha} + n_3)(\frac{1}{\phi n_1^{1-\alpha} + \phi_3 n_3}) \\
&+ K\phi_3(n_2^{\alpha} - O \cdot min(n_1, n_2)^{\alpha})(n_2^{1-\alpha} + n_3)(\frac{1}{\phi n_2^{1-\alpha} + \phi_3 n_3}) \\
&+ K\phi_3(O \cdot min(n_1, n_2)^{\alpha})(n_1^{1-\alpha} + n_2^{1-\alpha} + n_3)(\frac{1}{\phi(n_1^{1-\alpha} + n_2^{1-\alpha}) + \phi_3 n_3}) \\
&+ K\phi_3(1 - n_1^{\alpha} - n_2^{\alpha} + O \cdot min(n_1, n_2)^{\alpha})(\frac{1}{\phi_3})
\end{aligned}$$

$$n_1 f_1 + n_2 f_2 + n_3 f_3 = S$$

The equilibrium value of per capita private transfers to group 1:

$$f_1 = \frac{S\gamma f_1}{n_1\gamma f_1 + n_2\gamma f_2 + n_3\gamma f_3}$$

Calculating the denominator of the above expression using the first order conditions we get:

$$\begin{aligned}
n_1\gamma f_1 + n_2\gamma f_2 + n_3\gamma f_3 =& K(n_1^\alpha - O\cdot min(n_1,n_2)^\alpha)(n_1^{1-\alpha}+n_3)(\frac{\phi n_1^{1-\alpha}+\phi_3 n_3}{\phi n_1^{1-\alpha}+\phi_3 n_3}) \\
&+ K(n_2^\alpha - O\cdot min(n_1,n_2)^\alpha)(n_2^{1-\alpha}+n_3)(\frac{\phi n_2^{1-\alpha}+\phi_3 n_3}{\phi n_2^{1-\alpha}+\phi_3 n_3}) \\
&+ K(O\cdot min(n_1,n_2)^\alpha)(n_1^{1-\alpha}+n_2^{1-\alpha}+n_3)(\frac{\phi(n_1^{1-\alpha}+n_2^{1-\alpha})+\phi_3 n_3}{\phi(n_1^{1-\alpha}+n_2^{1-\alpha})+\phi_3 n_3}) \\
&+ K(1 - n_1^\alpha - n_2^\alpha + O\cdot min(n_1,n_2)^\alpha)(n_3)(\frac{\phi_3 n_3}{\phi_3 n_3}) \\
=& K(n_1+n_2+n_3) = K
\end{aligned}$$

When $n_1 < n_2$, we get from first order condition:

$$\frac{f_1}{S\phi} = \frac{\gamma f_1}{K\phi} = \frac{1-O}{w_1} + \frac{O}{w3}$$

Where,

$$w_1 = \phi + \frac{(\phi_3-\phi)(n_3)}{n_1^{1-\alpha}+n_3} \qquad w_3 = \phi + \frac{(\phi_3-\phi)(n_3)}{n_1^{1-\alpha}+n_2^{1-\alpha}+n_3}$$

Derivative of w_1 and w_3 w.r.t. n_1:

$$w_1' = -\frac{(1-\alpha)(\phi_3-\phi)n_3 n_1^{-\alpha}}{(n_1^{1-\alpha}+n_3)^2} \qquad w_3' = -\frac{(1-\alpha)(\phi_3-\phi)n_3(n_1^{-\alpha}-n_2^{-\alpha})}{(n_1^{1-\alpha}+n_2^{1-\alpha}+n_3)^2}$$

As we can see $w_1' < 0$ and $w_3' < 0$ when $n_1 < n_2$. Therefore, $\frac{df_1}{dn_1} < 0$ in this case.

When $n_1 \geq n_2$, we can rewrite the first order condition as:

$$\frac{f_1}{S\phi} = \frac{\gamma f_1}{K\phi} = \frac{1-Or}{w_1} + \frac{Or}{w3}$$

Where,

$$r = (n_2/n_1)^\alpha, \quad r' = -\alpha r(\frac{1}{n_1}+\frac{1}{n_2}), \quad r\in[0,1]$$

Differentiating:

$$\frac{1}{S\phi}\frac{df_1}{dn_1} = \frac{-(1-Or)w_1'}{w_1^2} + Or'(\frac{1}{w_3} - \frac{1}{w_1}) + \frac{-(Or)w_3'}{w_3^2}$$

The first additive term on the R.H.S. is positive and the second and third terms are negative. It can be seen that $\frac{df_1}{dn_1}$ is strictly decreasing in O and is positive as O tends to 0. Therefore, to prove that the expression $\frac{df_1}{dn_1} < 0$ when $O > O^*$ for some $O^* \in (0,1)$, it is sufficient to show tha $\frac{df_1}{dn_1} < 0$ when $O = 1$. Substituting O =1 and rearranging the above expression, we need to show:

$$-\frac{(1-r)w_1'}{w_1^2} < -r'(\frac{1}{w_3} - \frac{1}{w_1}) + \frac{rw_3'}{w_3^2}$$

Substituting the values of w_1, w_2, w_1', w_3', r, r' and simplifying, our expression is reduced to:

$$z - \frac{1}{z} < \frac{\alpha(n_2/n_1 + 1)}{(1-\alpha)(1-(n_2/n_1)^\alpha)}$$

Where $z = 1 + \frac{\phi n_2^{1-\alpha}}{\phi n_1^{1-\alpha} + n_3}$

$$\implies \phi n_2^{1-\alpha}(2 + \frac{\phi n_2^{1-\alpha}}{\phi n_1^{1-\alpha} + \phi_3 n_3}) < \frac{\alpha(n_2/n_1 + 1)(\phi(n_1^{1-\alpha} + n_2^{1-\alpha}) + \phi_3 n_3)}{(1-\alpha)(1-(n_2/n_1)^\alpha)}$$

As the ratio $\frac{\phi_3}{\phi}$ increases, the above inequality will be satisfied more easily. Therefore, it is sufficient to show that weak inequality holds in the above expression when $\phi_3 = \phi$. Using this and rearranging, we now need to show:

$$(n_1^{1-\alpha} n_2^{1-\alpha})(2 + \frac{n_2^{1-\alpha}}{n_1^{1-\alpha} + n_3}) \leq \frac{\alpha(n_1 + n_2)(n_1^{1-\alpha} + n_2^{1-\alpha} + n_3)}{(1-\alpha)(n_1^\alpha - n_2^\alpha)}$$

This can be rearranged to give:

$$n_1^{3-2\alpha}X + n_1^{2-\alpha}n_3 Y \leq 0$$

Where,

$$X = (2-3\alpha)q^{1-\alpha} - (2-\alpha)q - \alpha - \alpha q^{2-\alpha}$$
$$Y = (2-3\alpha)q^{1-\alpha} - (2-\alpha)q - \alpha - \alpha q^{2-\alpha} - \alpha(1 + q + \frac{n_3}{n_1^{1-\alpha}}(1+q))$$
$$q = \frac{n_2}{n_1}, \qquad q \in [0,1]$$

As we can see, $Y < X$ and n_3 can take any value in $(0,1)$, therefore it is both necessary and sufficient to show that $X \leq 0$. In fact, it is sufficient to show that:

$$x(q,\alpha) = (2-3\alpha)q^{1-\alpha} - (2-\alpha)q - \alpha \leq 0 \qquad \forall q \in [0,1], \quad \alpha \in (0,1)$$

Since x is continuous in q, the above condition will hold if it can be shown to hold at the boundaries and at each critical point in (0,1). At the boundaries:

$$x(0,\alpha) = -\alpha < 0$$
$$x(1,\alpha) = -3\alpha < 0$$

At critical point q^*:

$$\frac{dx(q,\alpha)}{dq} = (1-\alpha)(2-3\alpha)q^{-\alpha} - 2 + \alpha = 0$$
$$\implies q^* = (\frac{(1-\alpha)(2-3\alpha)}{2-\alpha})$$

$\therefore q^* \in (0,1)$ only when $\alpha \in (0, \frac{2}{3})$. Substituting the value of q^* and simplifying we need to show:

$$x(q^*,\alpha) = \alpha((\frac{1-\alpha}{2-\alpha})^{\frac{1-\alpha}{\alpha}}(2-3\alpha)^{\frac{1}{\alpha}} - 1) \leq 0$$
$$\implies (\frac{2-\alpha}{1-\alpha})^{1-\alpha} \geq 2-3\alpha$$

Let $t = 1-\alpha$. Now we need to show:

$$y(t) = (1+\frac{1}{t})^t - 3t + 1 \geq 0 \qquad \forall t \in (\frac{1}{3}, 1)$$

Again, since y(t) is continuous in t, we only need to show that the above condition is true at the boundary points and at each critical point in $(\frac{1}{3}, 1)$. At the boundaries:

$$y(\frac{1}{3}) = 4^{\frac{1}{3}} > 0$$
$$y(1) = 0$$

At the critical point:

$$\frac{dy(t)}{dt} = (1+\frac{1}{t})^t(ln(1+\frac{1}{t}) - \frac{1}{1+t}) - 3 = 0$$

Substituting the value of $(1+\frac{1}{t})^t$ in $y(t)$ and rearranging sides, we now need to show:

$$(3t-1)(ln(1+\frac{1}{t}) - \frac{1}{1+t}) \leq 3$$

Since $t \in (\frac{1}{3}, 1)$, therefore:

$$3t - 1 < 2 \qquad ln(1+\frac{1}{t}) < ln(4) \qquad \frac{1}{1+t} > \frac{1}{2}$$
$$\therefore (3t-1)(ln(1+\frac{1}{t}) - \frac{1}{1+t}) < 2(ln(4) - \frac{1}{2}) = 1.77 < 3$$

This implies that $x(q^*, \alpha) \leq 0$. Thus, $x(q, t) \leq 0$. Therefore, when $n_1 \geq n_2$, $\frac{df_1}{dn_1} < 0$ if and only if $O > O^*$ for some $O^* \in (0, 1)$.

(b) When group 2 is dispersed, settlement areas of each group are:

$$A_1 = n_1^{\alpha} \qquad A_2 = 1 \qquad A_3 = 1$$

In this case, there are two types of constituencies: (1) Group 1, 2 and 3 all reside and (2) Only group 2 and 3 reside. Densities of constituencies are:

$$D^1 = n_1^{1-\alpha} + n_2 + n_3 \qquad D^2 = n_2 + n_3$$

Since the populations across the K constituency are equal, we can calculate area per constituency:

$$a^1 = \frac{1}{K(n_1^{1-\alpha} + n_2 + n_3)} \qquad a^2 = \frac{1}{K(n_1 + n_2)}$$

Number of constituencies of each type:

$$K^1 = Kn_1^{\alpha}(n_1^{1-\alpha} + n_2 + n_3) \qquad K^2 = K(1 - n_1^{\alpha})(n_2 + n_3)$$

Group proportions in each constituency type:

$$\begin{aligned}
n_1^1 &= \frac{n_1^{1-\alpha}}{n_1^{1-\alpha} + n_2 + n_3} & \qquad n_1^2 &= 0 \\
n_2^1 &= \frac{n_2}{n_1^{1-\alpha} + n_2 + n_3} & \qquad n_2^2 &= \frac{n_2}{n_2 + n_3} \\
n_3^1 &= \frac{n_3}{n_1^{1-\alpha} + n_2 + n_3} & \qquad n_3^2 &= \frac{n_3}{n_2 + n_3}
\end{aligned}$$

Again, taking $U(f_j) = ln(f_j)$, we get first order conditions. At equilibrium:

$$\gamma f_1 = K\phi(n_1^{\alpha})(n_1^{1-\alpha} + n_2 + n_3)\frac{n_1^{-\alpha}}{\phi(n_1^{1-\alpha} + n_2) + \phi_3 n_3}$$

$$\begin{aligned}
\gamma f_2 =& K\phi(n_1^{\alpha})(n_1^{1-\alpha} + n_2 + n_3)\frac{1}{\phi(n_1^{1-\alpha} + n_2) + \phi_3 n_3} \\
&+ K\phi(1 - n_1^{\alpha})(n_2 + n_3)\frac{1}{\phi n_2 + \phi_3 n_3}
\end{aligned}$$

$$\begin{aligned}
\gamma f_3 =& K\phi_3(n_1^{\alpha})(n_1^{1-\alpha} + n_2 + n_3)\frac{1}{\phi(n_1^{1-\alpha} + n_2) + \phi_3 n_3} \\
&+ K\phi_3(1 - n_1^{\alpha})(n_2 + n_3)\frac{1}{\phi n_2 + \phi_3 n_3}
\end{aligned}$$

$$n_1 f_1 + n_2 f_2 + n_3 f_3 = S$$

Similar to the proof of proposition 3, equilibrium per capita transfer to group 2 are:

$$f_1 = \frac{S\gamma f_1}{n_1\gamma f_1 + n_2\gamma f_2 + n_3\gamma f_3}$$

Calculating the denominator by substituting values from first order condition:

$$\begin{aligned}
n_1\gamma f_1 + n_2\gamma f_2 + n_3\gamma f_3 =& K(n_1^{\alpha})(n_1^{1-\alpha} + n_2 + n_3)\frac{\phi(n_1^{1-\alpha} + n_2) + \phi_3 n_3}{\phi(n_1^{1-\alpha} + n_2) + \phi_3 n_3} + \\
& K(1 - n_1^{\alpha})(n_2 + n_3)\frac{\phi n_2 + \phi_3 n_3}{\phi n_2 + \phi_3 n_3} \\
=& K(n_1 + n_2 + n_3) = K
\end{aligned}$$

Using this and the first order condition:

$$\frac{f_1}{S\phi} = \frac{\gamma f_1}{K\phi} = \frac{n_1^{1-\alpha} + n_2 + n_3}{\phi(n_1^{1-\alpha} + n_2) + \phi_3 n_3}$$

Differentiating and simplifying:

$$\frac{1}{S\phi}\frac{df_1}{dn_1} = \frac{(\phi_3 - \phi)n_3((1-\alpha)n_1^{-\alpha} - 1)}{(\phi(n_1^{1-\alpha} + n_2) + \phi_3 n_3)^2}$$

Since, $\phi_3 > \phi$, it follows:

$$\begin{aligned}
\frac{df_1}{dn_1} > 0 \quad \text{if} \quad n_1 < (1-\alpha)^{\frac{1}{\alpha}} \\
\frac{df_1}{dn_1} < 0 \quad \text{if} \quad n_1 > (1-\alpha)^{\frac{1}{\alpha}}
\end{aligned}$$

$\therefore$ There is an inverted U-shaped relation between n_1 and f_1^* and hence between n_1 and G_1^* with peak at $n_1^* = (1-\alpha)^{\frac{1}{\alpha}}$.