Price-Level Volatility and Optimal Taxation

Guido Cozzi
University of St.Gallen

Aditya Goenka
University of Birmingham

Minwook Kang
Nanyang Technological University

Karl Shell*
Cornell University

September 4, 2018

Abstract

We analyze a very simple economy in which taxes (employed purely for income redistribution) are denominated in money units (say, dollars). Volatility of the price-level is sunspot-driven. Some agents cannot participate in the market for hedging against fluctuations in the price level. The tax authority chooses money taxes to maximize Benthamite welfare, i.e., the sum of expected utilities. Optimization entails leveling the expected utilities among the group of consumers who have access to the hedging market. Aggregate welfare is decreasing in price-level volatility when some of the consumers are unable to hedge against this volatility. The money-taxation regime is compared to a commodity-taxation regime in which transfers suffer from (iceberg) spoilage. In the commodity-tax regime, optimization implies that all taxed consumers receive the same utility and that all subsidized consumers receive the same utility. The cost of money taxation is in volatility, while the cost of commodity taxation is the partial spoilage of commodity in the tax-transfer process.

*Corresponding author: Karl Shell, 402 Uris Hall, Cornell University, Ithaca, NY 14853-7601 USA, email: karl.shell@cornell.edu. Author email addresses: Cozzi guido.cozzi@unisg.ch, Goenka goenka.econ@gmail.com, Kang mwkang@ntu.edu.sg, Shell karl.shell@cornell.edu. Goenka would like to thank NUS Grant FY2012-FRC4-003 for research support.

1 Introduction

Finance plays a very important, and largely positive, role in advanced economies, but it can contribute to excess economic volatility. We build a simple model of taxation in terms of fiat money, our financial instrument. Price-level volatility is driven by sunspots. Only some agents can hedge against price-level volatility. Others cannot. In other words, some of the consumers are "hand-to-mouth" consumers. The friction can either be interpreted as in Cass and Shell (1983) as a restriction on market participation because (for example) some individuals are not alive while the security market is open. Alternatively, this is an information friction, i.e. is that this is a special case of asymmetric (or correlated) information.[1]

The tax authority is assumed to choose money taxes that maximize the sum of expected utilities. If there were no frictions, price-level volatility would not affect utilities or welfare. Otherwise, welfare is strictly decreasing in volatility. Our present model is an extension from exogenous money taxation to *endogenous* taxation. See Bhattacharya, Guzman, and Shell (1998) and Cozzi, Goenka, Kang, and Shell (2015). In these two exogenous tax papers, the tax authority's response to volatility is absent, since for these papers taxes are pre-determined.

We compare the financial money-taxation economy with the non-financial commodity taxation economy. The welfare cost of taxation in the financial economy is purely from volatility. The non-financial economy does not suffer from volatility,[2] but it does suffer from iceberg-style spoilage of net tax commodity transfers. We show that, for the commodity taxation case, optimization of welfare entails equalization of the utilities of the taxed consumers and equalization of the utilities of the subsidized consumers. We show, in terms of the volatility rate and the spoilage rate, which regime is chosen by the tax authority. The key cost of inflation volatility is that it reduces efficacy of the redistributive and hence, the insurance functions of nominal taxes when some agents cannot fully hedge against the inflationary uncer-

[1] See Aumann, Peck and Shell (1985), Aumann (1987), and Peck and Shell (1991).

[2] Goenka (1994) shows that an economy with real subsidies is immune to excess volatility, while one where the subsidies are denominated in value is susceptible to it.

tainty. The second moments matter. Just an increase in the average inflation rate will have no real effects.

The literature on taxation largely treats taxes as denominated in real terms. However, many taxes are determined in nominal terms - e.g. income tax that is determined on nominal income and this is settled with a lag in dollars. Similarly, many transfer payments are denominated in dollar terms, and these are typically not indexed fully. This paper investigates the effect of increase in inflation volatility on the efficacy of taxes denominated in money.

If there are complete markets, then the distinction between taxes denominated in real or nominal terms does not matter (Cass and Shell (1983)). In an earlier paper, Cozzi et al (2017) however, we showed in an economy with restricted market participation where some consumers can hedge against inflation while some are restricted from market participation (either due to demographic considerations as they are born after hedging takes place (Cass and Shell (1983)) or due to lack of information) i.e. they are "hand-to-mouth" consumers, that with fixed monetary taxes some consumers are winners and some are losers due to the increased inflation volatility.[3] The only source of uncertainty in the economy are (self-fulfilling) beliefs about price-level volatility with the price volatility arising as the price level is not anchored by fundamentals. The paper showed that as the hand-to-mouth cannot hedge against inflation volatility they are worse off. The effects on consumers who can hedge against the volatility is subtle and it depends on the nature of the tax scheme. This raises the question what will be the optimal monetary tax and transfer scheme and how it is affected by price-level volatility.

In this paper, we endogenize the taxes through a Ramsey planner who wishes to maximize weighted welfare. As some of the consumers do not hedge against inflation, the taxes and transfers act as an instrument to hedge against the inflationary risk. The planner takes expectations and their optimal decision rules while setting taxes and transfers (denominated in monetary terms) to maximize the weighted welfare. We study how increasing the second moments of inflation (through a mean-preserving spread) changes

[3] Goenka and Prechac (2006) have a similar result in an incomplete financial market setting

the taxes and transfers, and how this affects the aggregate and individual welfare.

The economy is a two-period economy which is parametrized with Cobb-Douglas preferences - not only to be able to obtain closed form solutions - but also to rule out multiplicity of equilibria as taxes are varied. The consumers differ in their endowments so we have rich, middle-class and poor consumers. We examine different configurations of market participation and whether this affects the welfare.

In the paper we show that increasing volatility decreases maximized welfare when there are some restricted or only restricted consumers. The intuition is that with higher inflation the real value of the tax or transfer decreases. Inflation benefits those taxed and deflation benefits those receiving transfers. Thus, with higher price-level volatility, the efficacy of the taxes and transfers to equalize marginal rates of substitution falls and there is an aggregate welfare loss. Note, the taxes and transfers are doing two things: first, holding price level constant, equalizing marginal rates of substitution which is a pure redistribution effect; and second, equalizing marginal utility of income across the states as all consumers do not have access to insurance markets, which is the insurance effect. Price volatility by affecting the insurance effect also affects the redistributive effect. If all consumers can hedge against inflation, then as there are complete market, increased volatility does not matter.

Real taxes - denominated in commodities - are not affected by price-level volatility and if used, the equilibria are also not affected by it. However, real taxes can be costly to administer and we show much these administrative costs have to be for monetary taxes to be used.

The consideration of the second moments of inflation on taxes, is to our knowledge, novel. When inflation is considered from a taxation perspective, it is in terms of the first moment - seignorage (see Phelps (1973)). There are some recent papers that look at the effect of increased first moment of inflation on welfare (see Auclert (2017), Kaplan, et al. (2018)) through different channels. Our paper identifies a new channel - changes in second moments affects ability of insurance markets that are denominated in money terms (here it is taxes) to hedge against the volatility.

Examples abound of governments which had committed to a stream of fiscal payments on nominal variables, without the ability of real indexation. Most topical in the news is Greek government debt - whose euro value is constantly increased by deflation - or the loss in oil proceeds, which afflicts the public finances of several oil-producing countries such as Russia, Nigeria, Norway, etc., following the oil price fall of the last year. Oil price excessive appreciation took place at the onset of the global financial crisis, which in turn inflated the governments' real liabilities to unexpected values. During these episodes, the nominal market value of commodities and other goods seem quite beyond the control of the main central banks, including the ECB and the Bank of Japan, which had to effectively drop their inflation target, due to a hard to overcome near zero-lower-bound constraint on nominal interest rates.[4]

Fully exploiting the possibility of negative nominal interest rates would, among other things, allow an approximate indexation of government debt, which would be pinned down in real term. However this has additional costs, which motivates the existence of large fractions of non-indexed government debt.

Finally, the "original sin" of developing economies, which cannot sign international debt contracts in their own currencies, renders the real value of within-country redistribution at the mercy of self-fulfilling currency fluctuations.

Despite the relevance of the well-known difficulties of real taxation and redistribution, this topics has not been explored theoretically: nearly all macroeconomic and political economy models assume that the real value of taxes and transfers is given and known. We are here trying a first analysis of the theoretical aspects of optimal policy in the presence of nominal redistribution and taxation. To make the analysis simple we assume a basic one period model, which is to be interpreted as long enough to realistically allow the

[4]While ECB and other Central Banks are experimenting with very small degrees of negative interest rates, these timid attempts are still far below (in absolute value) levels considered optimal for a full recovery of the global economy.

build-up of government claims and commitments and to require a fully balanced budget. Hence our analysis is effectively long-run. Being a first step in this important direction, we neglect to expand an underlying dynamic model which, for the moment, could render the main effects opaque to the reader. However, we hope this analysis will stimulate dynamic macroeconomic studies able to incorporate the important ingredients we highlight.

This is the first in a two-paper series on endogenous money taxation. The present paper is on optimal taxation. The next paper is on voting.

2 The Economy

There is 1 period and 1 consumption good (say, chocolate). There are 3 consumers, $h = 1, 2, 3$. The consumption of Mr. h is $x_h > 0$ and his endowment is $\omega_h > 0$. The consumers have identical logarithmic preferences given by the utility functions:

$$u_h(x_h) = \log(x_h) \quad \text{for} \quad h = 1, 2, 3.$$

These preferences (or, more generally, CRRA identical preferences) ensure that equilibrium is unique. We introduce sunspots (or, extrinsic uncertainty). There are two extrinsic states of nature $s = \alpha, \beta$, that occur with probabilities $\pi(\alpha), \pi(\beta), 0 < \pi(\alpha) < 1, \pi(\beta) = 1 - \pi(\alpha)$. We assume that Mr. h maximizes his expected expected utility

$$V_h = \pi(\alpha) \log(x_h(\alpha)) + \pi(\beta) \log(x_h(\beta)) \quad \text{for } h = 1, 2, 3.$$

The social policy instruments are lump-sum taxes $\tau = (\tau_1, \tau_2, \tau_3)$ denominated in units of money, say dollars. Each individual's tax is independent of the state of nature, i.e., $\tau_h(\alpha) = \tau_h(\beta) = \tau_h$ for $h = 1, 2, 3$. If τ_h is negative, $Mr.\ h$ is subsidized. If τ_h is zero, then he is neither taxed nor subsidized. The tax and transfer plan is balanced, i.e., $\tau_1 + \tau_2 + \tau_3 = 0$, else the goods price of money is zero.[5]

[5] See Balasko and Shell (1983).

Let $p(s)$ be the ex-ante (accounting) price of the good delivered in state $s = \alpha, \beta$ and $p^m(s)$ be ex-ante (accounting) price of money delivered in state s. Then $P^m(s) = p(s)/p^m(s)$ is the chocolate price of money in s, while $1/P^m(s)$ is the money price of chocolate in s, or the general price level in s. The set of equilibria is typically very large, but we focus on a sub-set in which volatilities can be ranked. We measure volatility by the mean-preserving spread parameter σ defined by

$$P^m(\alpha) = P^m - \frac{\sigma}{\pi(\alpha)}$$

$$P^m(\beta) = P^m + \frac{\sigma}{\pi(\beta)}$$

where P^m is the non-sunspot equilibrium chocolate price of dollars and σ belongs to $[0, \pi(\alpha) P^m)$. When $\sigma = 0$, the equilibrium allocations are not affected by sunspots (a non-sunspots economy). When $\sigma > 0$, the economy is a proper sunspots economy. State α is the inflationary state: a dollar buys less chocolate in state α than in state β. State β is the deflationary state: a dollar buys more chocolate in β than in α.

3 Money Taxation and Social Welfare

The social welfare function W is the sum of the individual expected utilities. The tax authority chooses the tax τ to maximize welfare $V_1 + V_2 + V_3$. Define the maximized value of welfare by

$$W = \max_{\tau} V_1 + V_2 + V_3.$$

Figure 1 is the time-line.[6]

There are three basic cases based on the pattern of the asset market re-

[6] We work in the traditional framework of economic policy formulation where consumers form price expectations, the policy maker then chooses the tax policy, and given these expectations an equilibrium outcome is realized. In equilibrium, the price expectations of consumers must be consistent with the equilibrium outcome: rational expectations must hold.

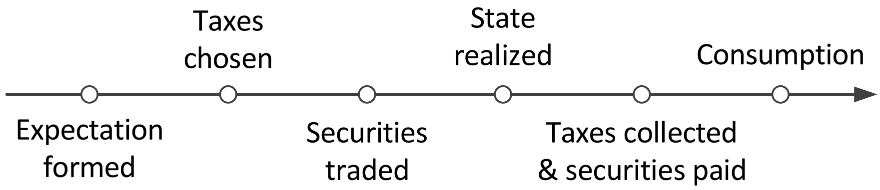


Figure 1: The time line

strictions: (U) Unrestricted security market participation, allowing for perfect risk-sharing among the 3 consumers, (I) Incomplete securities-market participation allowing for risk-sharing between 2 of the consumers but not the third, and (R) Fully restricted securities-market participation, in which none of the consumers can hedge against price-level fluctuations. Denote $W(U), W(I)$ and $W(R)$ as social welfare under perfect risk-sharing market, under partially restricted market and under fully restricted market, respectively.

In the case of perfect risk-sharing, sunspots do not matter and the first-best social welfare is achieved. The most interesting case is when some consumers are restricted and others are not: The case of Incomplete Participation I. Consider, for example, the case in which Mr 1 and Mr 2 have access to the security market and Mr. 3 does not:[7]

The problem of restricted consumer 3 is simple. He chooses $x_3(s) > 0$ to

$$\text{maximize } \log\left(x_3(s)\right)$$

subject to

$$p(s)x_3(s) = p(s)\omega_3 - p^m(s)\tau_3$$

for $s = \alpha, \beta$.

Define the tax-adjusted endowment $\widetilde{\omega}_h(s) = \omega_h - P^m(s)\tau_h$. Then, Mr

[7] The situation where only one consumer has access to security markets is not interesting as there is no counterpart to trade securities with.

3's budget constraints reduces to

$$x_3(s) = \widetilde{\omega}_3\,(s)$$

for $s = \alpha, \beta$. Mr 3 is passive: he consumes his tax-adjusted endowment in each state.

Mr 1 and Mr 2 trade in the securities market and the spot market. Each faces a single budget constraint. Mr h's problem is to choose $(x_h(\alpha), x_h(\beta)) > 0$ to

$$\text{maximize } V_h$$

subject to

$$p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) = (p(\alpha) + p(\beta))\,\omega_h - (p^m(\alpha) + p^m(\beta))\,\tau_h$$

for $h = 1, 2$. From the first-order conditions, we have

$$\frac{p(\beta)}{p(\alpha)} = \frac{\pi\,(\beta)\,x_1\,(\alpha)}{\pi\,(\alpha)\,x_1\,(\beta)} = \frac{\pi\,(\beta)\,x_2\,(\alpha)}{\pi\,(\alpha)\,x_2\,(\beta)}. \tag{1}$$

Market clearing implies

$$x_1(s) + x_2(s) + x_3(s) = \omega_1(s) + \omega_2(s) + \omega_3(s)$$

or simply

$$x_1(s) + x_2(s) + x_3(s) = \widetilde{\omega}_1(s) + \widetilde{\omega}_2(s) + \widetilde{\omega}_3(s) \tag{2}$$

for $s = \alpha, \beta$. But $x_3(s) = \widetilde{\omega}_3(s)$, so we have

$$x_1(s) + x_2(s) = \widetilde{\omega}_1\,(s) + \widetilde{\omega}_2\,(s) \quad \text{for } s = \alpha, \beta. \tag{3}$$

Equation (3) defines the relevant tax-adjusted Edgeworth box, which is typically a proper rectangular, the indication that sunspots will matter in equilibrium.[8]

[8]See Cass and Shell (1983, p. 212 or Section V).

4 Welfare and Restricted Market Participation

The first-best value of welfare W is

$$3\log\frac{\omega_1+\omega_2+\omega_3}{3}.$$

Welfare will be no smaller than its value in autarky,

$$\log\omega_1\omega_2\omega_3.$$

The following proposition shows that with strictly positive price-level volatility, asset market restrictions (i.e., the information frictions) negatively affect welfare:

Proposition 1 *If $\sigma > 0$, we have*

$$W(U) \geq W(I) \geq W(R).$$

Proof. Proposition 1 can be proven by Lemma 1, Proposition 2 and Lemma 2. ■

Proposition 1 indicates that as the asset market becomes more restricted, welfare declines.

Lemma 1 $W(U) = 3\log\frac{\omega_1+\omega_2+\omega_3}{3}$

Proof. When the 3 consumers do perfect risk sharing, $p(\alpha)$ and $p(\beta)$ are invariant in σ:

$$\frac{p(\beta)}{p(\alpha)} = \frac{\pi(\beta)}{\pi(\alpha)}.$$

Each consumer chooses $x_h(\alpha) = x_h(\beta)$, because we have

$$\frac{x_h(\alpha)}{x_h(\beta)} = \frac{p(\beta)/\pi(\beta)}{p(\alpha)/\pi(\alpha)} = 1.$$

With $x_h(\alpha) = x_h(\beta)$, the equilibrium V_h can be expressed as

$$V_h = \log\{\omega_h - (P^m(\alpha) + P^m(\beta))\,\tau_h\}$$

and social welfare can be expressed as

$$W(U) = \max_{\tau_1,\tau_2,\tau_3} \sum_{h\in H} \log\{\omega_h - (P^m(\alpha) + P^m(\beta))\,\tau_h\}$$

$$\text{subject to } \tau_1 + \tau_2 + \tau_3 = 0.$$

By the first order conditions, we have

$$\frac{-(P^m(\alpha) + P^m(\beta))}{\omega_1 - (P^m(\alpha) + P^m(\beta))\,\tau_1} = \frac{-(P^m(\alpha) + P^m(\beta))}{\omega_3 - (P^m(\alpha) + P^m(\beta))\,\tau_2} = \frac{-(P^m(\alpha) + P^m(\beta))}{\omega_3 - (P^m(\alpha) + P^m(\beta))\,\tau_3},$$

which implies that

$$x_1(\alpha) = x_1(\beta) = x_2(\alpha) = x_2(\beta) = x_3(\alpha) = x_3(\beta) = \frac{\omega_1 + \omega_2 + \omega_3}{3}.$$

Therefore, we have

$$W(U) = 3\log\frac{\omega_1 + \omega_2 + \omega_3}{3}.$$

■

Proposition 2 *In the partially restricted market (I), if Mr h and Mr h' trade in the securities market to share risk, we have $V_h = V_{h'}$.*

Proof. Without any loss of generality, let $h = 1$ and $h' = 2$. Because Mr 3 is restricted, he consumes his tax-adjusted endowment so that V_3 is affected only by τ_3. Therefore, the maximization problem can be re-written as

$$W(I) = \max_{\tau_3}\left\{\left(\max_{\tau_1,\tau_2|\tau_3} V_1 + V_2\right) + V_3\right\}$$

We need to show that for any given τ_3, we have $V_1 = V_2$ from the maximization problem, $\max_{\tau_1,\tau_2|\tau_3} V_1 + V_2$. Given τ_3, the aggregate tax-adjusted endowments of Mr 1 and Mr 2 are fixed as $\omega_1 + \omega_2 + P(\alpha)\tau_3$ in state α and

$\omega_1 + \omega_2 + P(\beta)\tau_3$ in state β. By the first welfare theorem applied in the tax-adjusted Edgeworth box, the trading equilibrium between Mr 1 and Mr 2 is Pareto optimal. Because they have identical homothetic vNM utility functions, the Pareto-optimal allocations satisfy the following:

$$\frac{x_1(\alpha)}{x_1(\beta)} = \frac{x_2(\alpha)}{x_2(\beta)} = \frac{\omega_1 + \omega_2 + P(\alpha)\tau_3}{\omega_1 + \omega_2 + P(\beta)\tau_3}$$

Define $t(\tau_3)$ as $(\omega_1 + \omega_2 + P(\alpha)\tau_3) / (\omega_1 + \omega_2 + P(\beta)\tau_3)$. Then, $V_1 + V_2$ can be expressed as

$$\begin{aligned} V_1 + V_2 &= \pi(\alpha)\log x_1(\alpha) + \pi(\beta)\log x_1(\beta) \\ &\quad +\pi(\alpha)\log x_2(\alpha) + \pi(\beta)\log x_2(\beta) \\ &= \pi(\alpha)\log x_1(\alpha) + \pi(\beta)\log t(\tau_3)\, x_1(\alpha) \\ &\quad +\pi(\alpha)\log x_2(\alpha) + \pi(\beta)\log t(\tau_3)\, x_2(\alpha) \\ &= \log x_1(\alpha)\, x_2(\alpha) + 2\pi(\beta)\log t(\tau_3)\,. \end{aligned} \tag{4}$$

By the Second Welfare Theorem applied in the tax-adjusted Edgeworth box, any Pareto optimal allocation can be achieved by lump-sum transfers between Mr 1 and Mr 2. This implies that we have τ_1 and τ_2 such that $\tau_1 + \tau_2 = -\tau_3$ and that they maximize $\log x_1(\alpha)\, x_2(\alpha)$ in equation (4). Because $x_1(\alpha) + x_2(\alpha)$ is fixed at $\omega_1 + \omega_2 + P(\alpha)\tau_3$, the maximizing $x_1(\alpha)$ and $x_2(\alpha)$ are

$$x_1(\alpha) = x_2(\alpha) = \frac{\omega_1 + \omega_2 + P(\alpha)\tau_3}{2}. \tag{5}$$

Equation (5) also implies that $x_1(\beta) = x_2(\beta)$. Because $x_1(\alpha) = x_2(\alpha)$ and $x_1(\beta) = x_2(\beta)$, we have $V_1 = V_2$. ∎

Lemma 2 $W(I) \geq W(R)$

Proof. From Lemma 2, welfare in the incomplete participation case can be expressed as

$$W(I) = \max_{\tau_3} \left\{ \left(\max_{\tau_1,\tau_2|\tau_3} V_1 + V_2 \right) + V_3 \right\}. \tag{6}$$

On the other hand, in a fully-restricted market each consumer consumes his endowment directly. Therefore, V_h is a function of only τ_h. Then, welfare in the fully restricted case R can be expressed as

$$W(R) = \max_{\tau_1} V_1 + \max_{\tau_2} V_2 + \max_{\tau_3} V_3,$$

which is equivalent to

$$W(R) = \max_{\tau_3} \left\{ \left(\max_{\tau_1|\tau_3} V_1 + \max_{\tau_2|\tau_3} V_2 \right) + V_3 \right\}. \tag{7}$$

For any given τ_3, we know that

$$\left(\max_{\tau_1,\tau_2|\tau_3} V_1 + V_2 \right) \geq \left(\max_{\tau_1|\tau_3} V_1 + \max_{\tau_2|\tau_3} V_2 \right). \tag{8}$$

From equations 6 and 7 and inequality 8, we have

$$\max_{\tau_3} \left\{ \left(\max_{\tau_1,\tau_2|\tau_3} V_1 + V_2 \right) + V_3 \right\} \geq \max_{\tau_3} \left\{ \left(\max_{\tau_1|\tau_3} V_1 + \max_{\tau_2|\tau_3} V_2 \right) + V_3 \right\}.$$

■

5 Volatility and Welfare

In the case of perfect risk-sharing, price volatility does not affect welfare. Welfare is at its maximum, independent of σ. However, in the cases where the securities market is not perfect and welfare is not at its maximum value, increased price volatility necessarily leads to decreased social welfare W.

Proposition 3 *$W(I)$ and $W(R)$ are strictly decreasing in σ.*

Proof. Proposition 3 can be proved by Lemmas 3-4 ■

Lemma 3 *$W(I)$ is strictly decreasing in σ.*

Proof. Let $W^{\sigma}(I)$ be welfare at volatility σ. We need to establish that $W^{\sigma'}(I) > W^{\sigma''}(I)$ if $\sigma' < \sigma''$. $W^{\sigma}(I)$ can be expressed as (See Lemma 2)

$$W^{\sigma}(I) = \max_{\tau_3} \left\{ \left(\max_{\tau_1,\tau_2|\tau_3} V_1^{\sigma} + V_2^{\sigma} \right) + V_3^{\sigma} \right\},$$

where V_h^σ is Mr h's utility value with volatility σ. Define $T^\sigma(\tau_3)$ as

$$T^\sigma(\tau_3) = \left(\max_{\tau_1,\tau_2|\tau_3} V_1^\sigma + V_2^\sigma\right) + V_3^\sigma.$$

We need to show that for any value of τ_3, the following is true:

$$T^{\sigma'}(\tau_3) > T^{\sigma''}(\tau_3). \tag{9}$$

For given τ_3, we have $V_3^{\sigma'} > V_3^{\sigma''}$ where

$$V_3^{\sigma'} = \pi(\alpha)\log(\widetilde{\omega}_3(\alpha)) + \pi(\beta)\log(\widetilde{\omega}_3(\beta)),$$

because the log function is strictly concave and the tax-adjusted endowment with σ'' is mean-preserving spread of that with σ'.

We have

$$\begin{aligned} & \max_{\tau_1,\tau_2|\tau_3} V_1^{\sigma'} + V_2^{\sigma'} \qquad (10) \\ = \;& 2\left\{\pi(\alpha)\log\left(\frac{\omega_1+\omega_2+P^m(\alpha)\tau_3}{2}\right) + \pi(\beta)\log\left(\frac{\omega_1+\omega_2+P^m(\beta)\tau_3}{2}\right)\right\} \end{aligned}$$

from Lemma 2. Since $P^m(s)$ is based on a mean-preserving spread, $\max_{\tau_1,\tau_2|\tau_3} V_1^{\sigma'} + V_2^{\sigma'}$ decreases because the log function in equation (10) is strictly concave Therefore, we have $T^{\sigma'}(\tau_3) > T^{\sigma''}(\tau_3)$ for all τ_3, which implies that

$$\max_{\tau_3} T^{\sigma'}(\tau_3) > \max_{\tau_3} T^{\sigma''}(\tau_3).$$

■

Lemma 4 *$W(R)$ is strictly decreasing in σ.*

Proof. For any given (τ_1, τ_2, τ_3), we know that each individual's expected utility strictly decreases in σ because (1) vNM utility is strictly concave and (2) each individual's consumption is a mean-preserving spread increasing in

σ. That is, for any balanced tax (τ_1, τ_2, τ_3) , we have

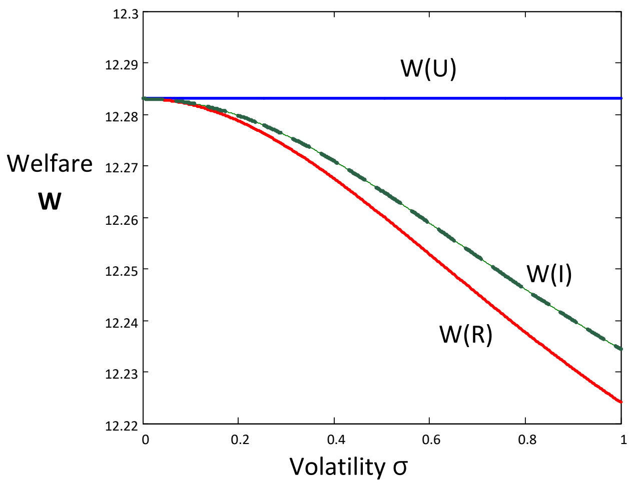
$$V_1^{\sigma'} + V_2^{\sigma'} + V_3^{\sigma'} > V_1^{\sigma''} + V_2^{\sigma''} + V_3^{\sigma''} \quad \text{if } \sigma' < \sigma''. \tag{11}$$

From equation (11), we have

$$\max_{\tau_1,\tau_2,\tau_3} V_1^{\sigma'} + V_2^{\sigma'} + V_3^{\sigma'} > \max_{\tau_1,\tau_2,\tau_3} V_1^{\sigma''} + V_2^{\sigma''} + V_3^{\sigma''}$$

if $\sigma' < \sigma''$. ■

In Figure 5, $W(U)$, $W(I)$ and $W(R)$ are plotted against σ. $W(U)$ is invariant in σ, but $W(I)$ and $W(R)$ are strictly decreasing in σ.



Simulation: $(\omega_1, \omega_2, \omega_3) =$
$(80, 60, 40), \pi(\alpha) = 0.5, P^m = 1$

6 Commodity Taxation

We assume that when the tax authority makes a transfer of x units of chocolate from one consumer to another, δx units of the transferred chocolate are lost to "melting". The melting rate is $0 < \delta < 1$. If Mr h is taxed, i.e., $\tau_h^c > 0$, he owes the tax authority τ_h^c in chocolate. If consumer h is subsidized, i.e., $\tau_h^c < 0$, he will receive $(1 - \delta)\tau_h^c$ units of chocolate from the tax authority. Mr h's consumption is then

$$\omega_h - \max(0, \tau_h^c) - \min(0, (1-\delta)\,\tau_h^c). \tag{12}$$

With the 3 consumers, maximized welfare is

$$W = \max_{\tau_1^c, \tau_2^c, \tau_3^c} \sum_{h=1,2,3} E \log\left[\omega_h - \max(0, \tau_h^c) - \min(0, (1-\delta)\,\tau_h^c)\right] \tag{13}$$

$$\text{subject to } \tau_1^c + \tau_2^c + \tau_3^c = 0.$$

First, we need to verify the conditions under which Mr 1 is taxed and Mr 3 is subsidized. Without loss of generality, we assume that $\omega_1 > \omega_2 > \omega_3$.

Lemma 5 *Mr 1 is taxed and Mr 3 is subsidized if and only if* $\delta < \frac{\omega_1 - \omega_3}{\omega_1}$*, i.e.,* $\omega_1 \geq \frac{\omega_3}{1-\delta}$.

Proof. Assume that Mr 1 is not taxed and Mr 3 is not subsidized. This implies that the tax authority cannot improve welfare through a transfer from Mr 1 to Mr 3. The condition for this is

$$\left[\frac{\partial \log(\omega_1 - \tau_1^c)}{\partial \tau_1^c} + \frac{\partial \log(\omega_3 + (1-\delta)\,\tau_1^c)}{\partial \tau_1^c}\right]_{\tau_1^c = 0} \leq 0,$$

which is equivalent to

$$\delta \geq 1 - \frac{\omega_3}{\omega_1} = \frac{\omega_1 - \omega_3}{\omega_1}.$$

Therefore, the condition that Mr 1 is taxed and Mr 3 is subsidized is

$$\delta < \frac{\omega_1 - \omega_3}{\omega_1}.$$

or,

$$\omega_1 \geq \frac{\omega_3}{1-\delta}.$$

■

Lemma 5 indicates that the tax authority taxes 1 chocolate from the rich to give $(1-\delta)$ chocolates to the poor until $\omega_3 = (1-\delta)\omega_1$. In the following Lemma, we verify the conditions under which Mr 2 is taxed or subsidized.

Lemma 6 *Mr 2 is subsidized if*

$$\omega_2 < \frac{1}{2}\left(\frac{\omega_1}{1-\delta} + \omega_3\right),$$

Mr 2 is taxed if

$$\omega_2 > \frac{1}{2}\left(\omega_1 + (1-\delta)\,\omega_3\right),$$

and Mr 2 is neither taxed nor subsidized if

$$\frac{1}{2}\left(\frac{\omega_1}{1-\delta} + \omega_3\right) \leq \omega_2 \leq \frac{1}{2}\left(\omega_1 + (1-\delta)\,\omega_3\right).$$

Proof. We assume that $\delta < (\omega_1 - \omega_3)\,\omega_1$. Then, Mr 1 is taxed and Mr 2 is subsidized by Lemma 5. Assuming that Mr 2 is neither taxed nor subsidized, we can derive the optimal tax θ_1^* for Mr. 1 from the following equation:

$$\frac{\partial \log\left(\omega_1 - \theta_1^*\right)}{\partial \theta_1} + \frac{\partial \log\left(\omega_3 + (1-\delta)\,\theta_1^*\right)}{\partial \theta_1} = 0,$$

which is equivalent to

$$\theta_1^* = \frac{1}{2}\frac{\omega_1 - \delta\omega_1 - \omega_3}{1-\delta} = \frac{1}{2}\left(\omega_1 - \frac{\omega_3}{1-\delta}\right).$$

Then, Mr 1's consumption x_1^* is

$$x_1^* = \omega_1 - \frac{1}{2}\left(\omega_1 - \frac{\omega_3}{1-\delta}\right) = \frac{1}{2}\omega_1 + \frac{1}{2}\frac{\omega_3}{1-\delta}.$$

Mr 3's consumption x_3^* is

$$x_3^* = \omega_3 + (1-\delta)\,\frac{1}{2}\left(\omega_1 - \frac{\omega_3}{1-\delta}\right) = \frac{1}{2}\left(\omega_1 + (1-\delta)\,\omega_3\right).$$

Because Mr 2 is not taxed, Mr 2's marginal cost of his commodity tax should be larger than the marginal benefit of Mr 3's additional subsidy:

$$\left[\frac{\partial \log\left(\omega_2 - \theta_2\right)}{\partial \theta_2} + \frac{\partial \log\left(\frac{1}{2}\left(\omega_1 + (1-\delta)\,\omega_3\right) + (1-\delta)\,\theta_2\right)}{\partial \theta_2}\right]_{\theta_2 = 0} < 0,$$

which is equivalent to

$$\omega_2 < \frac{\frac{1}{2}(\omega_1 + (1-\delta)\,\omega_3)}{(1-\delta)} = \frac{1}{2}\left(\frac{\omega_1}{1-\delta} + \omega_3\right) \tag{14}$$

This is the condition for when Mr 2 is not taxed. The condition where Mr 2 is not subsidized can be derived in the same way. Thus we have

$$\omega_2 > \frac{1}{2}(\omega_1 + (1-\delta)\,\omega_3). \tag{15}$$

From conditions (14) and (15), we can derive the condition for when Mr 2 is neither taxed nor subsidized. That is,

$$\frac{1}{2}(\omega_1 + (1-\delta)\,\omega_3) < \omega_2 < \frac{1}{2}\left(\frac{\omega_1}{1-\delta} + \omega_3\right) \tag{16}$$

■

One can interpret the tax authority's problem geometrically: To maximize welfare $x_1x_2x_3$ subject to the set of feasible allocations, the frontier of which has a kink at the endowment point.

7 Welfare and Commodity Taxation

Proposition 4 *Welfare is strictly decreasing in δ if $(\tau_1^c, \tau_2^c, \tau_3^c) \neq 0$.*

Proof. Define $T(\tau^c, \delta)$ by

$$T(\tau^c, \delta) = \sum_{h=1,2,3} E \log\left[\omega_h - \max(0, \tau_h^c) - \min(0, (1-\delta)\,\tau_h^c)\right]. \tag{17}$$

Then, maximized welfare is

$$W = \max_{\tau_1^c, \tau_2^c, \tau_3^c} T(\tau^c, \delta).$$

For any $(\tau_1^c, \tau_2^c, \tau_3^c)$, we have

$$T(\tau^c, \delta') > T(\tau^c, \delta'')\ if\ \delta' < \delta''$$

because $T(\tau^c, \delta)$ is decreasing in δ if $\tau_h^c < 0$ for some h in equation (17). Therefore, we have

$$\max_{\tau_1^c, \tau_2^c, \tau_3^c} T(\tau^c, \delta') > \max_{\tau_1^c, \tau_2^c, \tau_3^c} T(\tau^c, \delta'') \quad \text{if} \quad \delta' < \delta''.$$

■

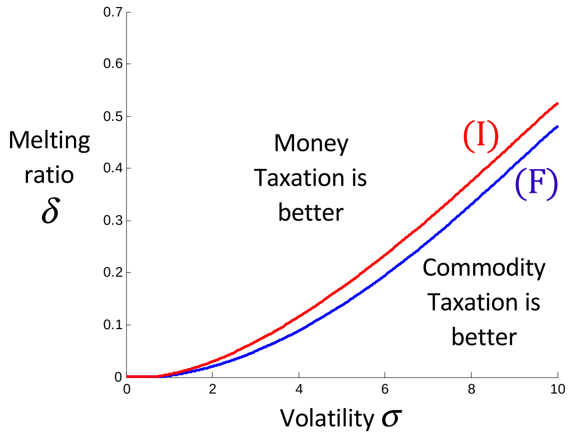
8 Money taxation vs. Commodity taxation

From Lemmas 5 and Proposition 4, for commodity taxation welfare reaches its first-best value $3\log\left(\frac{\omega_1+\omega_2+\omega_3}{3}\right)$ when $\delta = 0$, strictly decreases in δ, and reaches it minimum value of $\log \omega_1\omega_2\omega_3$ when $\delta = \frac{\omega_1-\omega_3}{\omega_1}$. On the other hand, with money taxation, welfare never falls below its minimum because even with high volatility σ, because near autarky (i.e., when $\tau = 0$) the marginal cost of taxation of the rich never exceeds the marginal benefit of subsidizing for the poor. Therefore, we have the following proposition:

Proposition 5 *If the tax authority can choose either money taxation and commodity taxation, for any given volatility level σ, there exists $\delta^* \in \left(0, \frac{\omega_1-\omega_3}{\omega_1}\right)$ such that welfare with money taxation (under partially or fully restricted markets) is higher (lower) than welfare with commodity taxation if $\delta > (<)\delta^*$. δ^* is strictly decreasing in σ. For any given volatility level σ, the value of δ^* under the partially restricted market is higher than that under the fully restricted market. δ^* is a different fraction depending on whether the money taxation economy is partially restricted or fully restricted.*

Proof. Directly from Propositions 1, 3 and 4 and Lemma 5. ■

In the plot in Figure 8, (δ, σ)-space is divided into a region in which dollar taxation is better and another region in which chocolate taxation is better. The region in which dollar taxation is better for partially restricted market participation is a subset of the corresponding set for fully restricted market participation.



Simulation: $(\omega_1, \omega_2, \omega_3) = (100, 50, 10), \pi(\alpha) = \pi(\beta) = 0.5, P^M = 10$

9 Concluding remarks

We weigh the advantages and disadvantages of a simple finance economy (the money-taxation regime) against those of the corresponding non-finance economy (the commodity-taxation regime). Taxes are endogenous. They are chosen optimally by the tax authority. The desirability of the money-taxation regime is declining in the volatility of the price level. The desirability of the commodity-taxation regime is declining in the iceberg-style costs of net tax transfers. In the money-taxation regime, the tax authority equalizes the expected utilities of all those with access to the security market. In the commodity-taxation regime, the tax authority equalizes the utilities of the taxed consumers and equalizes the utilities of the subsidized consumers.

The model allows for information frictions in which some or all of the consumers are restricted from participation on the securities market. When these restrictions are absent, the money-tax economy achieves the first-best allocation, in which all utilities are equalized. Otherwise, social welfare is strictly decreasing in price-level volatility.

The effects of volatility on individual expected utilities are more complicated and worthy of separate study. There are several effects (1) the direct effects on tax adjusted endowments, which become more volatile as price-level volatility increases, (2) the hedging effects through the securities market, (3) the effects of volatility on the tax authority's choice of tax regime and its choice of taxes. The third effect would not be present if — as in the existing literature — taxes were predetermined independently of volatility. Some individuals are harmed by volatility, but others might be made better off from volatility for at least two reasons: (1) Taxed individuals might benefit as the tax authority reduces taxation because of increased social costliness as increased volatility causes tax-adjusted endowments to become more volatile; (2) Some consumers might benefit from volatility by sharing through the market the increased risks of other consumers.

Our model is very simple, too simple to draw any direct policy implications except the obvious such as : Introducing financial instruments is more likely to be socially beneficial in economies with less volatile expectations. The hope is that this simple model might suggest similar work in richer models, ones that can be used to calculate actual social trade-offs between more sophisticated financial and money regimes versus less sophisticated ones. Our δ might seem to be a deus-ex-machina. The difference in transactions costs between the money-tax regime and the commodity-tax regime could be understood as reflecting the theory of monetary search as in Kiyotaki-Wright (1993), but this remains to be seen.

References

Auclert, A. (2017). Monetary policy and the redistribution channel. Technical report, National Bureau of Economic Research.

Aumann, R. J. (1987). Correlated equilibrium as an expression of bayesian rationality. *Econometrica 55*(1), 1–18.

Aumann, R. J., J. Peck, and K. Shell (1985, Revised 1988). *Asymmetric information and sunspot equilibria: a family of simple examples.* Working Paper 88-34, Center for Analytic Economics, Cornell University.

Balasko, Y. and K. Shell (1993). Lump-sum taxation: the static economy. In R. Becker, M. Boldrin, R. Jones, and W. Thomson (Eds.), *General equilibrium, growth and trade. Essays in honor of Lionel McKenzie*, Volume 2, pp. 168–180. San Diego, CA: Academic Press.

Bhattacharya, J., M. G. Guzman, and K. Shell (1998). Price level volatility: A simple model of money taxes and sunspots. *Journal of Economic Theory 81*(2), 401–430.

Cass, D. and K. Shell (1983). Do sunspots matter? *Journal of Political Economy 91*(2), 193–227.

Cozzi, G., A. Goenka, M. Kang, and K. Shell (2017). Winners and losers from price-level volatility: Money taxation and information frictions. In K. Nishimura, A. Venditti, and N. C. Yannelis (Eds.), *Sunspots and Non-Linear Dynamics: Essays in Honor of Jean-Michel Grandmont*, Volume 31 of *Studies in Economic Theory*, pp. 387–402. Springer International Publishing.

Goenka, A. (1994). Rationing and sunspot equilibria. *Journal of Economic Theory 64*(2), 424–442.

Goenka, A. and C. Préchac (2006). Stabilizing sunspots. *Journal of Mathematical Economics 42*(4-5), 544–555.

Kaplan, G., B. Moll, and G. L. Violante (2018). Monetary policy according to hank. *American Economic Review 108*(3), 697–743.

Kiyotaki, N. and R. Wright (1993). A search-theoretic approach to monetary economics. *American Economic Review 83*(1), 63–77.

Peck, J. and K. Shell (1991). Market uncertainty: correlated and sunspot equilibria in imperfectly competitive economies. *Review of Economic Studies 58*(5), 1011–1029.

Phelps, E. S. (1973). Inflation in the theory of public finance. *The Swedish Journal of Economics 75*(1), 67–82.