# Understanding the Short run Relationship between Stock market and Growth in Emerging Economies 

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#### Abstract

Correlation coefficients calculated from annual data of 35 countries for the period 1988-2012 show that emerging economies exhibit the highest contemporaneous correlations between per capita growth and market capitalization to GDP ratio. To explain this phenomenon, the paper extends the asset pricing framework of Lucas (1978) to accommodate production, capital accumulation and growth. Two additional features are separately added to the DSGE set up to capture two different aspects of globalized emerging economies. First, domestic firms are assumed to borrow capital in the international market to enhance their capital stocks. Second, along with domestic agents, foreign agents are assumed to hold domestic stocks. In the first scenario a growth enhancing productivity shock is shown to increase the market capitalization ratio in the short run. In the second scenario, a positive demand shock to the stock market is shown to increase short run growth. To remain as close as possible to the structure of an emerging economy, the model is calibrated using quarterly Indian data.


Keywords: Short Run, Market capitalization, Growth

## 1 Introduction

Financial development in an economy has long been recognized as an important determinant of economic development. There is an old literature, both theoretical and empirical, which emphasizes financial intermediation in general, and stock market developments in particular, as major factors behind long run growth. The theoretical literature goes as far back as Schumpeter (1912) and Hicks (1969) and culminates in more recent work of Jacklin (1987), Gorton and Penacchi (1990), Bencivenga and Smith (1991), Levine (1991), Japelli and Pagano (1994) and Bencivenga, Smith and Starr (1995) among others. In this literature, stock markets act as an efficient bridge between savers and investors, thereby facilitating capital accumulation and growth. The theory is supported by a host of empirical work which include King and Levine (1993), Levine and Zervos $(1996,1998)$ and Harris $(1997)$.

The question regarding a short run or contemporaneous relationship between the stock market and growth, however, remains somewhat unanswered. Curiously, it is the short run relationship which has attracted a lot of attention in the common parlance. The perception has often been extreme. While some perceive the stock market as a worldwide casino which has no effect on real income or its growth, others believe that movements in the stock market is a good indicator of movements in the real economy. To understand the short run relationship, the correlation coefficients between growth of per capita income on the one hand and the market capitalization to GDP ratio on the other are calculated from annual data for 35 countries for the period 1988-2012 are calculated. For 27 countries the coefficients turn out to be positive and significant, but the correlations are particularly strong (greater than +0.6 ) for all BRICS countries, except China, as well as for Sri Lanka and Bangladesh. This indicates that for emerging economies, which are opening up to the world financial markets, growth and stock market performance are highly connected.

The present paper provides a theoretical explanation as to why this may be so

Since correlations do not reveal the direction in which causality is flowing, the paper separately considers the possibilities of growth affecting the stock market and the stock market affecting growth. More specifically, the paper separately analyzes how an exogenous demand shock to the stock market might affect real per capita growth and how a growth enhancing exogenous productivity shock might affect the market capitalization to GDP ratio in a DSGE set up. For this purpose, the asset pricing framework of Lucas (1978) is extended to take into account production, capital accumulation and growth. However, in this basic model, an exogenous productivity shock does not generate the desired instantaneous correlation between growth and market capitalization ratio. Additional features, particularly attributable to emerging economies, need to be added to this basic model to get the desired outcome.

Two features are added separately to the basic model to capture two different aspects of globalized emerging economies. First, domestic firms are allowed to borrow from the international market at a given rate of interest to enhance their capital stocks. In this scenario of foreign borrowing, it is shown that a TFP shock leading directly to higher growth of output also leads to an increase in the market capitalization ratio. As we have already indicated, this outcome cannot be obtained without foreign borrowing.

To understand why the observed correlation cannot be obtained without foreign borrowing, but can be restored once foreign borrowing is introduced, first let us consider the model without borrowing. A TFP shock, increasing the productivity of the factor of production, increases output and growth instantaneously. On the other hand, it also increases profits. Part of this increase is invested and the remaining paid out as dividend. The increase in dividend income is partly spent on consumption and partly used to buy stocks. Since supply of stocks is constant in the Lucas model, in equilibrium, the price of stocks must rise to clear the market. With utility equal to logarithm of consumption, as assumed in the model, stock price is proportional to consumption. But because only a part of the rise in output goes
to consumption and the rest is retained by firms for investment, the proportionate rise in consumption is less than the proportionate rise in income. This, in turn, implies that the proportionate rise in the stock price is also less than that of income. Consequently, the market capitalization ratio falls after the TFP shock in the model without foreign borrowing.

Now consider the scenario where domestic firms are allowed to borrow in the international market. As pointed out above, a positive TFP shock, increases investment by firms and hence future capital stocks. Firms borrow in the international market by pledging their future capital stocks and hence their capacity to borrow depend on their future capital. Therefore, a TFP shock relaxes the borrowing constraint of the firms and increases the amounts they can borrow in the international market. The increased borrowing allows the firms to pay out more dividends than the increase in income and as a result, consumption and the stock price increase more than in proportion to current income. Consequently, the market capitalization ratio increases and this drives the positive and significant market capitalization-growth correlation

The second extension of the basic model involves a scenario where there is no foreign borrowing but domestic stocks are held by both domestic and foreign agents. We treat demand for domestic stocks by foreign agents as exogenous and stochastic. A positive demand shock by foreign agents has the immediate effect of raising the stock price and the market capitalization ratio. Now, it is assumed that domestic firms are managed by domestic agents, that is, foreign shareholding is not as large as to control the domestic firms. Hence domestic firms choose their capital stocks over time to maximize the expected future stream of domestic dividends. Consequently, when there is an increase in the foreign holding of stocks due to a positive demand shock, domestic firms choose to pay out less dividends. The increased retained profit is used for capital accumulation which increases output in the next period leading to a higher growth in the short run.

The paper, therefore, shows that for emerging economies, the positive correlation between growth and the stock market may imply causation from either direction a productivity shock primarily affecting growth causing a rise in the market capitalization ratio and a demand shock immediately affecting stock prices and market capitalization causing higher growth in the short run. These results are obtained only when special features are added to the model, namely, foreign borrowing or foreign holding of domestic stocks. A bit of explanation may be required as to why these features are particularly representative of emerging economies. First, emerging economies are growing rapidly, but they lack enough funds to sustain their growth. As a result, most of them borrow from the international market to supplement the gap between expenditure and income. This aspect of emerging economies is picked up in the model with foreign borrowing. Second, Though there is foreign participation in the stock markets of emerging economies in the form of foreign institutional investments, enough trust of foreign investors on these emerging stock markets has not yet been built up. This makes foreign demand for domestic stocks vulnerable and stochastic. This aspect of vulnerability is captured by stochastic foreign demand of domestic stocks. Finally, to remain as close as possible to the structure of an emerging economy, the model is calibtrated using Indian data.

In what follows, Section 2 deals with the asset pricing framework with production and investment, but without borrowing provision, Section 3 deals with the same framework with the borrowing provision for the firms,Section 4 discusses the asset pricing framework with home and foreign countries and Section 5 concludes.

## 2 Asset pricing framework with production and investment

I consider an economy consisting of infinitely many identical competitive firms and infinitely many identical households. The representative firm is indexed by $i \in$
$[0,1]$ and the representative household is indexed by $j \in[0,1]$. Thus the total size of households as well as the total size of firms are normalized to unity.Each firm produces a homogeneous output using capital and a linear production function. A firm invests a part of its output, thereby augmenting next period's capital stock and distributes the remaining as dividends to the households who are owners of the firm. For the representative household, dividend income is the only source of income, a part of which goes into consumption and the rest into buying new stocks.

### 2.1 Market capitalization ratio and growth

The representative household maximizes its expected utility over an infinite horizon. Time is discrete. The household's utility $u\left(c_{t}\right)$ is a function of its consumption $c_{t}$ alone. Taking $\beta$ as the household's discount factor, the household's objective function can be formally written as

$$
\begin{gather*}
M a x: E_{0} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right)  \tag{1}\\
\text { s.t. }: \int_{0}^{1} p_{t}^{z}(i)\left[z_{t+1}(i)-z_{t}(i)\right] d i+c_{t}=\int_{0}^{1} d_{t}(i) z_{t}(i) d i \tag{2}
\end{gather*}
$$

Equation (2) represents the household's resource constraint, where $p_{t}^{z}(i)$ and $d_{t}(i)$ are the price and dividend of the stock of the $i$ th firm at period $t$ and $z_{t}(i)$ is the quantity of the stock of the $i$ th firm held by the representative household at period $t$. Consequently, the right hand side of equation (2) represents the total (dividend) income of the representative household. On the left hand side of equation (2) the first term represents expenditure of the household at period $t$ to acquire additional assets. This, added with the consumption $c_{t}$ exhausts the household's total income.

The representative household maximizes (1) subject to (2) by choosing $c_{t}$ and $z_{t+1}(i)$. The first order condition with respect to these choice variables establish the following Euler equation:

$$
\begin{equation*}
u^{\prime}\left(c_{t}\right) p_{t}^{z}(i)=\beta E_{t} u^{\prime}\left(c_{t+1}\right)\left(d_{t+1}(i)+p_{t+1}^{z}(i)\right) \tag{3}
\end{equation*}
$$

Since firms are identical, dividends and share prices are the same across all firms, which means that $d_{t}(i)=d_{t}$ and $p_{t}^{z}(i)=p_{t}^{z}$ for all $i$. Using this, (3) can be written as

$$
\begin{equation*}
u^{\prime}\left(c_{t}\right) p_{t}^{z}=\beta E_{t} u^{\prime}\left(c_{t+1}\right)\left(d_{t+1}+p_{t+1}^{z}\right) \tag{4}
\end{equation*}
$$

Without any loss of generality, I assume that the total number of shares of a firm is unity and this remains unchanged over time. Since the measure of firms is unity, the total number of stocks in the economy is also unity. This is distributed equally among the households. Since the measure of households is also unity, each household has one unit of stock. This makes the right hand side of (2) equal to $d_{t}$. Again, since the total number of firms remain unchanged over time, the first term on the left hand side of equation (2) becomes zero which makes the left hand side of equation (2) equal to $c_{t}$. Therefore, equation (2) reduces to

$$
\begin{equation*}
c_{t}=d_{t} \tag{5}
\end{equation*}
$$

This is in line with Lucas (1978). In the Lucas asset pricing framework dividends are assumed to be fruits falling from a certain tree and have to be consumed entirely in equilibrium since no storage is possible within the economy. In the present scenario, the part of output that is not invested by firms has to be consumed and this is the part that is distributed as dividend. Hence total consumption is equal to the dividend income.

Next I assume that the utility function is logarithmic, i.e. $u\left(c_{t}\right)=\ln c_{t}$. The stock Euler equation in (3)then becomes

$$
\begin{equation*}
\frac{p_{t}^{z}}{c_{t}}=\beta E_{t}\left(\frac{c_{t+1}+p_{t+1}^{z}}{c_{t+1}}\right) \tag{6}
\end{equation*}
$$

Solving recursively, the above equation becomes

$$
\begin{equation*}
\frac{p_{t}^{z}}{c_{t}}=\frac{\beta}{1-\beta}+\beta E_{t}\left\{\lim _{n \rightarrow \infty} \beta^{n-1}\left(\frac{p_{t+n}^{z}}{c_{t+n}}\right)\right\} \tag{7}
\end{equation*}
$$

Assuming that the term $\frac{p_{t+n}^{z}}{c_{t+n}}$ is bounded above for all n , the limit term in equation (7) goes to zero. This means that the equilibrium asset price becomes

$$
\begin{equation*}
p_{t}^{z}=\frac{\beta}{1-\beta} c_{t} \tag{8}
\end{equation*}
$$

This price is determined in such a way that in equilibrium, each period, the representative household would not want either to increase or to decrease his holding of assets.This is guaranteed by the logarithmic utility function where income and substitution effects, which are of opposite signs, are of the same magnitude and offset each other.

In an economy described in Lucas (1978), dividend (described as fruit falling from trees in the Lucas asset pricing framework) arrives without any deliberate effort on the part of the consumers and is referred to as an endowment economy or exchange economy. There is no provision of storage and production in this kind of an economy. However, in the present theoretical framework, I allow for investment in physical capital and output production in each period by identical firms owned by the representative household.

The representative firm manufactures its product $\left(y_{t}\right)$ using capital $\left(k_{t}\right)$ as its only source of input, with the help of a linear production technology given by

$$
\begin{equation*}
y_{t}=\epsilon_{t} k_{t} \tag{9}
\end{equation*}
$$

where $\epsilon_{t}$ denotes the Total Factor Productivity (TFP) shock which influenced output production in time period $t$. In time period $t$ the firm invests a part of its produce and distributes the rest as dividend to the household. The firm invests an amount $i_{t}$ which gives rise to new accumulated capital for period $t+1$, given by $k_{t+1}$. The investment process for the firm is represented by the following equation.

$$
\begin{equation*}
k_{t+1}=(1-\delta) k_{t}+i_{t} \tag{10}
\end{equation*}
$$

where $(1-\delta) k_{t}$ stands for undepriciated capital stock at time period $t . \delta$ represents the rate of capital depriciation. The firm maximizes discounted stream of future
dividends, where dividend at time period $t$ is given by

$$
\begin{equation*}
d_{t}=y_{t}-i_{t} \tag{11}
\end{equation*}
$$

Thus the time $t$ objective function of the firm is given by

$$
\begin{equation*}
\operatorname{Max}: E_{t} \sum_{s=0}^{\infty} m_{t, t+s} d_{t+s}=E_{t} \sum_{s=0}^{\infty} m_{t, t+s}\left[\epsilon_{t+s} k_{t+s}-\left\{k_{t+s+1}-\left(1-\delta_{t}\right) k_{t+s}\right\}\right] \tag{12}
\end{equation*}
$$

with $k_{t+1}$ as the firm's choice variable. $m_{t+s}$ denotes the representative household's stochastic discount factor

$$
\begin{equation*}
m_{t, t+s}=\frac{\beta u^{\prime}\left(c_{t+s}\right)}{u^{\prime}\left(c_{t}\right)} \tag{13}
\end{equation*}
$$

As the firm maximizes its dividends on behalf of the household, it uses the latter's marginal rate of substitution or stochastic discount factor $m_{t+s}$ in its dividend maximization problem.

Since in equilibrium the household does not increase or decrease its holding of assets, the equilibrium resource constraint can be written as

$$
\begin{equation*}
c_{t}=\epsilon_{t} k_{t}-\left[k_{t+1}-\left(1-\delta_{t}\right) k_{t}\right] \tag{14}
\end{equation*}
$$

The left-hand-side of equation (14) represents the representative household's dividend income which is entirely consumed in equilibrium. Taking the first order condition of the firm's maximization problem w.r.t. $k_{t+1}$ and combining it with the equilibrium resource constraint, I can derive equilibrium consumption $\left(c_{t}\right)$ and capital accumulation $\left(k_{t+1}\right)$ expressions as

$$
\begin{equation*}
c_{t}=(1-\beta)\left(\epsilon_{t}+1-\delta_{t}\right) k_{t} \tag{15}
\end{equation*}
$$

and

$$
\begin{equation*}
k_{t+1}=\beta\left(\epsilon_{t}+1-\delta_{t}\right) k_{t} \tag{16}
\end{equation*}
$$

Utilizing the equilibrium consumption and capital accumulation expressions from the above equations in (15) and (16) along with the equilibrium asset price in (8),

I next derive equilibrium expressions for market capitalization as a ratio of output $\left(m k_{t}\right)$ and output growth $\left(y g_{t}\right)$.

Going by the usual definitions as specified in the previous framework, equilibrium market capitalization ratio and growth are solved as

$$
\begin{equation*}
m k_{t}=\beta\left[1+\frac{\left(1-\delta_{t}\right)}{\epsilon_{t}}\right] \tag{17}
\end{equation*}
$$

and

$$
\begin{equation*}
y g_{t}=\beta \epsilon_{t}\left[1+\frac{\left(1-\delta_{t-1}\right)}{\epsilon_{t-1}}\right] \tag{18}
\end{equation*}
$$

Detailed derivations of (15), (16), (17) and (18) are relegated to the appendix.

### 2.2 Short run quantitative analysis

Figure 1 represents the impulse response behaviours of market capitalization and growth along with a few other macroeconomic variables due to the realization of a TFP shock. The figure captures the short run dynamics of growth (denoted by yg), market capitalization (denoted by $m k$ ), consumption to capital ratio (denoted by $c k$ ), dividend to capital ratio (denoted by $d k$ ), dividend to output ratio (denoted by $d y$ ), investment to capital ratio (denoted by $i k$ ), investment to output ratio (denoted by $i y$ ) and expected capital growth (denoted by kg ). TFP shock is assumed to follow an $\operatorname{AR}(1)$ process given by:

$$
\begin{equation*}
\epsilon_{t}-\bar{\epsilon}=\rho_{\epsilon}\left(\epsilon_{t-1}-\bar{\epsilon}\right)+\zeta_{t}^{\epsilon} \tag{19}
\end{equation*}
$$

The steady state value of $\epsilon_{t}$ is $\bar{\epsilon}$. $\zeta_{t}^{\epsilon}$ is the disturbance term.
The household discount factor $\beta$ is fixed at 0.99 and the depreciation parameter $\bar{\delta}$ at 0.025 i.e. at the conventional levels consistent with quarterly calibration. In order to find an estimate of the productivity parameter $\bar{\epsilon}$ for emerging economies, the long run per capita quarterly real GDP growth rate for India at $1.41 \%$ for the sample period 1975-2014 is targeted to set the productivity parameter at $0.067 .{ }^{1}$. Without

[^0]any loss of generality, I fix the standard deviation of the exogenous component of the shock, i.e. $\sigma_{\epsilon}^{2}$ at unit level in order to normalize the impulse responses.

Figure 1 represents the effect of a TFP shock on the relevant macroeconomic variables.

A positive TFP shock at time period $t$ augments output, because of which there is an increase in $y g$, i.e. growth at time $t$. An increase in current production leads to increase in current investment by the firms, which is reflected in an increase in investment - capital ratio $i k$. Since the TFP shock follows an $A R(1)$ process, its persistence effect on output is reflected in future production as well, due to which firms find it worthwile to increase their investments. In fact, investment in physical capital increases more than proportionately compared to the increase in output, due to which a rise in investment - output ratio $i y$ is observed. A considerable rise in investment in physical capital is reflected in a rise in next period's expected capital growth kg . Also, since a positive TFP shock increases current output, an increase in firm dividends is also observed due to pure income effect, which is represented in Figure 1 by a rise in dividend to capital ratio $d k$. As dividend is entirely consumed in equilibrium, the rise in $d k$ is exactly proportional to the rise in consumption capital ratio $c k$. However, a fall in dividend - output ratio $d y$ signifies that dividends increase at a rate which is lower than the corresponding rise in output, which in effect implies that the relative rise in firm's dividend is less than that of firms' investment. As the increase in households' dividend income is slower than that of output, the increase in their demand for stocks is less than that of output, which gets manifested in a fall in the market capitalization to output ratio. Thus due to a positive TFP shock market capitalization ratio and growth move in opposite directions in the short run.

Figure 1: Impulse Response to TFP


## 3 Asset pricing framework with borrowing

In the last section, it was found that for reasonable values of the parameters the instantaneous response of the stock market to economic growth is not as positive as the rise in income itself as a result of which the market capitalization ratio has a tendency to go down after growth. Since this is not what was empirically observed, I need to modify the theoretical framework in order to get empirically justifiable results. It is well known that firms augment their capital not only through undistributed profits, but also by borrowing. In fact borrowing by firms is a natural phenomenon which I presently incorporate into the model. I consider a firm producing output with capital as the only input as in the previous framework. However, in the present framework, the firm is entitled to borrow an amount $b_{t}$ from an international bank or financial intermediary at a fixed gross rate of interest $r^{\prime}$, or net rate of interest $r$ where $r^{\prime}=1+r$

The firm being a price taker in the international market takes the international rate of interest as given. Also, only firms and not households have access to the international financial market. The firm's resource constraint can be written as

$$
\begin{equation*}
d_{t}+i_{t}+r^{\prime} b_{t-1}=\epsilon_{t} k_{t}+b_{t} \tag{20}
\end{equation*}
$$

with $i_{t}$ being represented by the same investment equation as in the previous framework, which is equation (10).

The right-hand-side of the firm's resource constraint represents total resources of the firm i.e. the total output produced at time period $t$ given by $\epsilon_{t} k_{t}$ plus the amount borrowed by the firm, which is $b_{t}$. The left-hand-side of the resource constraint equation shows that a part of the firm's total income goes into investment $i_{t}$, a part $r^{\prime} b_{t-1}$ is utilized to repay the amount borrowed at time period $t-1$ and the remainder $d_{t}$ is distributed to the household as dividends.

How much can the firm raise from the market through borrowing? Given that there is a repayment problem, that is, there is a possibility that the firm does not
repay its loan, it has to offer a collateral to its lender. All loans are assumed to be of one period. It is also assumed that there is no moral hazard problem in the sense that the lenders can observe how much the firm has invested in the current period when the loan is taken. Finally I assume that while the capital stock of the firm can be observed by the lenders, its output cannot be observed which means that the firm can only pledge its capital stock as collateral but not its output ${ }^{2}$. Under these assumptions, the lenders know next period's capital stock ${ }^{3}$, and the maximum the firm can pledge as collateral is the discounted value of its capital stock next period when the loan is to be repaid.

In this section, we assume that the firm can raise through borrowing an amount upto to the full value of its discounted capital stock of next period ${ }^{4}$

If $b_{t}$ is the amount of loan taken by the firm at period $t$ at rate of interest $r^{\prime}$, then at time period $t+1$ the firm is supposed to repay $r^{\prime} b_{t}$. But since the maximum amount that the lender can recover is $k_{t+1}$, the firm will face a borrowing constraint given by

$$
\begin{equation*}
r^{\prime} b_{t} \leq k_{t+1} \tag{21}
\end{equation*}
$$

### 3.1 Market capitalization and growth in a borrowing constrained equilibrium

Since firms are owned by households, the firm, on behalf of the representative household, will maximize discounted stream of future dividends subject to the borrowing constraint represented by equation (21). The maximization problem of the firm can be expressed more formally as:

[^1]\[

$$
\begin{align*}
M a x & : \quad E_{t} \sum_{s=0}^{\infty} m_{t, t+s}\left[\epsilon_{t+s} k_{t+s}+b_{t+s}+\left((1-\delta) k_{t+s}-k_{t+s+1}\right)-r^{\prime} b_{t+s-1}\right]  \tag{22}\\
\text { s.t. } & : \quad b_{t+s} \leq \frac{k_{t+s+1}}{r^{\prime}}, s=0, \ldots \infty \tag{23}
\end{align*}
$$
\]

where $m_{t, t+s}$ denotes the stochastic discount factor for the households as in the previous section and is given by equation (13). As before, since firms are owned by households and optimize dividends on behalf of the households, the latter's marginal rate of substitution enters the firm's maximization problem.

In order to solve the maximization problem given by (22), I set up the Lagrange function as:

$$
\begin{equation*}
L_{t}=E_{t} \sum_{s=0}^{\infty} m_{t, t+s}\left[\epsilon_{t+s} k_{t+s}+b_{t+s}+\left((1-\delta) k_{t+s}-k_{t+s+1}\right)-r^{\prime} b_{t+s-1}\right]+\sum_{s=0}^{\infty} \lambda_{t+s}\left(\frac{k_{t+s+1}}{r^{\prime}}-b_{t+s}\right) \tag{24}
\end{equation*}
$$

For the above problem in (24), $\lambda_{t+s}$ denotes the Lagrange multiplier at time period $t$. At time $t$, the choice variables of the firm are its investment $k_{t+1}$ and the amount it decides to borrow i.e. $b_{t}$. Let us assume for the moment that $\lambda_{t}>0$ or the borrowing constraint fully binds.

The first order conditions to the Lagrangian problem in (24) with respect to $k_{t+1}$ and $b_{t}$ yield

$$
\begin{equation*}
\frac{\lambda_{t}}{r^{\prime}}-1+E_{t} m_{t, t+1}\left[\epsilon_{t+1}+(1-\delta)\right]=0 \tag{25}
\end{equation*}
$$

and

$$
\begin{equation*}
1-\lambda_{t}-E_{t} m_{t, t+1} r^{\prime}=0 \tag{26}
\end{equation*}
$$

Combining the first order conditions and assuming $\lambda_{t}>0$ it implies

$$
r^{\prime}<\frac{u^{\prime}\left(c_{t}\right)}{\beta E_{t} u^{\prime}\left(c_{t+1}\right)}
$$

In what follows, I will assume that the above condition always holds. I have checked that for reasonable parameter values and range of standard errors, the above condition is indeed satisfied.

As in equilibrium, representative household consumes entire dividend earnings, with a logarithmic utility function, the equilibrium asset price, which follows from the household's utility maximization exercise, is given by equation (8) as

$$
p_{t}^{z}=\left(\frac{\beta}{1-\beta}\right) c_{t}
$$

Using the full borrowing equilibrium values of consumption and capital accumulation in the above asset pricing equation, I derive an expression for market capitalization - output ratio and output growth as

$$
\begin{equation*}
m k_{t}=\frac{p_{t}^{z}}{y_{t}}=\beta\left(\frac{\epsilon_{t}-\delta+1}{\epsilon_{t}}\right) \tag{28}
\end{equation*}
$$

and

$$
\begin{equation*}
y g_{t}=\frac{y_{t}}{y_{t-1}}=\beta\left(\frac{\epsilon_{t}}{\epsilon_{t-1}}\right)\left(\frac{r^{\prime}}{r^{\prime}-1}\right)\left(\epsilon_{t-1}-\delta\right) \tag{29}
\end{equation*}
$$

### 3.2 Short run quantitative analysis

I focus on the contemporaneous market capitalization - growth relationship within this framework. In order to understand this, I look into the short run dynamics of market capitalization and growth along with those of a few other relevant macroenomic variables in response to a TFP shock. ${ }^{5}$ The TFP shock is assumed to follow an $\mathrm{AR}(1)$ process, as in the previous section.

[^2]In order to carry out the necessary simulations, the household discount factor $\beta$ is fixed at 0.99 and the depriciation parameter $\bar{\delta}$ at 0.025 which are the conventional levels consistent with quarterly calibration. In order to find an estimate of the productivity parameter $\bar{\epsilon}$, the long run per capita quarterly real GDP growth rate for India is taken as a baseline measure. For the sample period 1975-2014 the Indian quarterly long run per capita growth rate is found to be $1.41 \%$ which is targeted to set the productivity parameter at 0.067 . As in the previous section, without any loss of generality, I fix the standard deviation of the exogenous component of the TFP shock i.e. $\sigma_{\epsilon}^{2}$, at unit level in order to normalize the impulse responses. The international borrowing rate is fixed at $1 \%$ such that $r^{\prime}=1.01$.

The correlation coefficient reproduced by the simulated model is positive and significant at 0.56 for the baseline parametric values calibrated for India.

Figure 2 demonstrates the effect of a TFP shock on the chief macroeconomic variables.

A positive TFP shock induces market capitalization ratio and growth to move in the same direction in the short run. A good TFP shock in time period $t$ increases production and hence growth $y g$ at time period $t$. Also, an increase in total output augments total investment by the firm, which is evident from an increase in $i k$ i.e. the investment to capital ratio. In fact, investment increases at a rate greater than the rise in output, due to which an increase in the investment - output ratio iy is observed. A rise in the total investments in physical capital lead to an increase in the total capital stock in the next period, which explains the rise in expected capital growth $k g$. This is why a spike in output growth is observed in time $t+1$, implying a further rise in growth from the current to the next period. A rise in next period capital stock increases the firms' borrowing limit. Also since TFP shock follows an AR(1) process, an anticipated rise in next period's production increases the firm's the world interest rate holds true in each period for which the borrowing constrained equilibrium is valid throughout the entire short run time path of the firm.

Figure 2: TFP impulse response in borrowing constrained model

ability to repay loans in the next period, which is why firms can afford to increase their optimal borrowing to capital ratio $b k$ in the current period. Now, although investment rises considerably, total dividends in time period $t$ also rise and that too at a rate higher than the rise in output, as is evident from a rise in $d y$. Thus both investments and dividends of firms increase at a rate higher than the increase in output, which is possible as a result of an increase in current borrowing by the firm.

Although the rise in dividend to capital ratio $d k$ is a bit lower than the rise in investment to capital ratio $i k$, as is evident from the above figure, an increase in $d y$ signifies that on the whole dividends increase at a rate higher than the increase in output. As income of the households increase at a rate greater than the increase in output, their rise in asset demand also exceeds the corresponding rise in output, subsequently leading a rise in the ratio of market capitalization to output. Since in equilibrium, household's consumption equals total dividends, a rise in consumption to capital ratio $c k$ in Figure 9 is reflected in a proportional rise in $d k$. In fact, in this case, a rise in total dividends increases households' total demand for assets on both counts; firstly because of the pure income effect of an increase in output getting translated into increased dividend income and secondly due to an increase in anticipated dividends as a result of a rise in expected production in the next period. An increased asset demand, in turn, contribute towards increase in the market capitalization ratio $m k$.

## 4 Asset pricing framework with home and foreign countries

There are two types of agents viz domestic and foreign who hold domestic shares. I consider the domestic economy consisting of infinitely many identical competitive firms and infinitely many identical households. The total size of households as well as the total size of firms are normalized to unity. Each firm produces a homogeneous
output using capital and a linear production function. The firm invests a part of its output, thereby augmenting next period's capital stock and distributes the remaining as dividends to the households who are owners of the firm. Let $Z_{t}$ and $F_{t}$ be the holdings of domestic and foreign agents respectively, with sum of domestic and foreign shares adding to unity.

Domestic agents solve the following problem by choosing $c_{t}$ and $Z_{t+1}$.

$$
\begin{gather*}
\operatorname{Max}: E_{0} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right)  \tag{30}\\
\text { s.t. }: c_{t}+\left(Z_{t+1}-Z_{t}\right) P_{t}=Z_{t} d_{t} \tag{31}
\end{gather*}
$$

Assuming a log utility function, the first order condition of the above maximization problem may be written as

$$
\begin{equation*}
\frac{p_{t}}{c_{t}}=\beta\left(\frac{d_{t+1}}{c_{t+1}}+\frac{p_{t+1}^{z}}{c_{t+1}}\right) \tag{32}
\end{equation*}
$$

Total dividends are defined as

$$
\begin{equation*}
d_{t}=\left[\epsilon k_{t}-\left[k_{t+1}-(1-\delta) k_{t}\right]\right] \tag{33}
\end{equation*}
$$

Firms maximize the domestic fraction of discounted sum of dividend payments using the household's stochastic discount rate by choosing $\mathrm{k}_{t+1}$ :

$$
\begin{equation*}
\operatorname{Max}: E_{0} \sum_{t=0}^{\infty}\left[\epsilon k_{t}-\left[k_{t+1}-(1-\delta) k_{t}\right]\right] Z_{t} \tag{34}
\end{equation*}
$$

The first order condition for this maximization is

$$
\begin{equation*}
Z_{t}=Z_{t+1}\left[\beta \frac{c_{t}}{c_{t+1}}(\epsilon+1-\delta)\right] \tag{35}
\end{equation*}
$$

Total number of shares is

$$
\begin{equation*}
1=Z_{t}+F_{t} \tag{36}
\end{equation*}
$$

### 4.1 Market capitalization ratio and growth

Market capitalization is the total valuation of outstanding shares calculated by multiplying current price of a share with the total outstanding shares. Indexing representative firm and representative household of the home country by $i \in[0,1]$ and $j \in[0,1]$ respectively, total value of stocks for the representative household is given by $\int_{0}^{1} p_{t}^{z}(i) z_{t}(i) d i=p_{t}^{z} z_{t} \int_{0}^{1} d i=p_{t}^{z}$. Integrating over all households in the economy, the total value of stock market capitalization is given by $p_{t}^{z} \int_{0}^{1} d j=p_{t}^{z}$, which means that domestic market capitalization to output ratio $m k_{t}$ is defined as

$$
\begin{equation*}
m k_{t}=\frac{p_{t}^{z}}{y_{t}} \tag{37}
\end{equation*}
$$

Also I define domestic growth at time period $t$ as

$$
\begin{equation*}
y g_{t}=\frac{y_{t}}{y_{t-1}} \tag{38}
\end{equation*}
$$

$F_{t}$ i.e. foreigners' share of domestic stocks is stochastic and is assumed to follow an $\operatorname{AR}(1)$ process.

### 4.2 Short run quantitative analysis

In order to carry out the necessary simulations, the household discount factor $\beta$ is fixed at 0.99 and the depreciation parameter $\bar{\delta}$ at 0.025 which are the conventional levels consistent with quarterly calibration. In order to find an estimate of the productivity parameter $\bar{\epsilon}$, the long run per capita quarterly real GDP growth rate for India is taken as a baseline measure. For the sample period 1975-2014 the Indian quarterly long run per capita growth rate is found to be $1.41 \%$ which is targeted to set the productivity parameter at 0.067 . Without any loss of generality, I fix the
standard deviation of the exogenous component of the shock at unit level in order to normalize the impulse responses.

The correlation coefficient reproduced by the simulated model is positive and significant at 0.66 for the baseline parametric values. The impulse response to a rise in $F_{t}$ is shown in the following figure. I focus on the effect on output growth in next period (yg) and market capitalization (mk) primarily, but in the process also look into the impulse responses of Tobin's q (q), capital growth (kg) dividend to capital ratio ( dk ) and consumption to capital ratio (ck).

When there is an increase in foreigners' share of domestic assets, domestic firms will typically reduce the amount distributed as dividends, which means a subsequent increase in investment of physical capital and an increase in growth in the next period (represented in figure 4). Also an increase in the worldwide demand for shares leads to a rise in stock prices and hence market capitalization ratio (represented in figure 4) and Tobin's q (represented in figure 3).

## 5 Conclusion

In this paper, production and investment are incorporated into the Lucas (1978) model, thereby making dividends endogenous, i.e. arising out of intertemporal optimization by firms. Within this framework, the effects of an asset market side shock and a production side shock on market capitalization ratio and growth are investigated. An increase in foreigners' share of domestic assets leads to reduction in the amount distributed as dividends, leading to an increase in investment of physical capital and a subsequent increase in growth in the next period. An increase in the worldwide demand for shares also leads to a rise in stock prices and hence market capitalization ratio. A positive productivity shock, on the other hand, can support the empirical findings only in presence of a borrowing constraint. When there is no

Figure 3: Impulse Response of Tobin's q, consumption to capital ratio and capital growth


Figure 4: Impulse Response of Dividend to capital ratio, output growth and market capitalization ratio

provision for borrowing, an increase in the productivity of the factor of production increases output and growth instantaneously and it also increases profits. Part of the increase in output is invested while the remaining paid out as dividend. The increase in dividend income is spent partly on consumption and the remaining is used to buy stocks driving up stock price. However, since a part of the rise in income is invested, the rise in consumption is less than the proportionate rise in income which means that the market capitalization as a ratio of income falls after the TFP shock. But when there is provision to borrow, a positive productivity shock, by means of augmenting investment and future capital stock, also increases the amount firms are allowed to borrow. As a result, dividends, consumption and stock price increase more than proportionally compared to the increase in current income, which drives the positive and significant market capitalization-growth correlation.

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[^0]:    ${ }^{1}$ Data Sorce: RBI

[^1]:    ${ }^{2}$ Output being a flow obtained through out the period is very costly to observe, but capital, being a stock, can be observed in one go.
    ${ }^{3} k_{t+1}=(1-\delta) k_{t}+i_{t}$ where the current capital stock and the realized values of the shocks are publicly known at the beginning of period $t$.
    ${ }^{4}$ This implicitly assumes that there is no recovery cost incurred by the lender.

[^2]:    ${ }^{5}$ Although I deal with a serially correlated shocks while investigating the short run dynamics, I assume that each period the borrowing constraint binding restriction on the world interest rate must be valid, such that the firm remains a net borrower in each period. For this reason, as mentioned earlier, I consider only admissible ranges of the values of the TFP shock for which the restriction on

