Patents and Growth in OLG Economy with Physical Capital

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July 2018

Abstract

We study the implications of patents in an overlapping generations model with horizontal innovation of differentiated physical capital. We show that within this demographic structure of finitely lived agents, weakening patent protection generates two contradicting effects on innovation and growth. Weakening patent protection lowers the (average) price of patented machines, thereby increasing machine utilization, output, aggregate saving, and investment. However, a higher demand for machines shifts investment away from the R&D activity aimed at inventing new machine varieties, toward the formation of physical capital. The growth-maximizing level of patent protection is incomplete. Shortening patent length is more effective than loosening patent breadth in spurring growth, due to an additional positive effect on growth, that is decreasing investment in old patents. Welfare can be improved by weakening patent protection beyond the growth-maximizing level.

JEL Classification: : L-16, O-30
Key-words: IPR, Patents, Physical Capital, Growth, OLG

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†This work integrates, corrects, and extends the analyses presented by Diwakar and Sorek in earlier working papers titled "Finite Lifetimes, Patents' Length and Breadth, and Growth" and "Dynamics of Human Capital Accumulation, IPR Policy, and Growth" (circulated as Auburn University Department of Economics Working Papers: AU-WP #2016-11 and #2016-7, respectively). We have benefitted from comments by seminar participant at Auburn University, the 2016 and 2017 Southern Economic Association conferences in Washington DC and Tampa FL, and the 2017 North American Econometric Society Summer Meeting in St. Louis MO.

1 Introduction

There is a relatively large literature on the role of patent policy in modern growth theory and the implications of patent strength to R&D-based growth and welfare. The current literature, however, is almost exclusively written about models with infinitely lived agents. This paper utilizes an overlapping generations model to highlight some unique implications of finite lifetimes to patent policy.

In an economy of finitely lived agents, the limited longevity sets a barrier to growth by inducing intergenerational trade in productive assets. This point was emphasized by Jones and Manuelli (1992) in a model of physical capital accumulation, and by Chou and Shy (1993) in an endogenous growth model of variety expansion with no physical capital. Both studies employed the canonical Overlapping Generations (OLG) model pioneered by Samuelson (1958) and Diamond (1965), where saving and investment are constrained by labor income.[1]

Jones and Manuelli (1992) showed that perpetual growth cannot prevail in the neoclassical OLG economy[2] due to the limited ability of the young to purchase capital held by the old. One of the remedies they consider to support sustained growth in such an economy is direct income transfers from old to young. Chou and Shy (1993) emphasized that inter-generational trade in old patents slows down growth as investment in old patents crowds out innovative (R&D) investment in new varieties. They showed that due to this crowding-out effect, which is not present in infinitely-lived agent economies, shortening patent length enhances growth.

To the best of our knowledge, Sorek (2011) is the only other work to study the growth implications of patents in the OLG framework. However, this work focuses on the effect of patents' breadth and length on quality growth (i.e. vertical innovation), where differentiated consumption goods are only produced with labor (i.e. there is no physical capital as in Chou and Shy 1993). In Sorek's (2011) setup, the effect of patent policy on growth depends crucially on the elasticity of inter-temporal substitution, through the effect of the interest rate on life-cycle saving in the OLG model. This effect plays no role in the current analysis (though it is considered in the Appendix).

The present work studies an OLG economy that incorporates variety expansion of specialized machines and physical capital accumulation, to highlight a unique mechanism through which loosening patents' strength spurs growth. Our analysis places the variety-expansion model proposed by Rivera-Batiz and Romer (1991)[3] into the canonical OLG demographic framework of Samuelson (1958) and Diamond (1965).

In order to isolate the main effect under study from the aforementioned crowding-out effect[4], we first show that under infinite patent length, growth is maximized with incomplete patent breadth.

[1] More generally, in economies with finitely lived agents the accumulation of assets is limited by the agent's consumption horizon (longevity).

[2] In other words, the perpetual accumulation of physical capital per-capita.

[3] Barro and Sala-i-Martin (2004) and Aghion and Howitt (2008) adopted this framework as the textbook variety-expansion model; See chapters 6 and 8, respectively.

[4] The weakening of breadth protection over all patents evenly (as considered here), does not reduce the crowding-out effect induced by intergenerational trade in old patents.

The mechanism at work behind this result involves the trade-off between the static and dynamic effects faced by the patents policy maker. Weakening patent breadth protection works to lower the price of patented machines (by weakening sellers' market power), which in turn increases demand for machines. With more machines being utilized, output and labor income are higher, thus increasing aggregate saving and investment. This is the positive effect of loosening patent breadth protection on growth.[5] However, higher demand for machines shifts investment away from patents and innovation toward physical capital. This is the negative effect of weakening patent breadth protection on growth. The growth-maximizing patent breadth is incomplete and depends negatively on the depreciation rate of capital.

The effect of patent policy on growth we are highlighting here is not present in the counterpart models of infinitely lived agents, where saving is not bounded by labor income. Previous works on Rivera-Batiz and Romer's (1991) model economy with infinitely lived agents concluded that growth is maximized with complete patent protection, that is, infinite patent length and complete patent breadth; See Iwaisako and Futagami (2003), Kwan and Lai (2003), Cysne and Turchick (2012, 2014), and Zeng et al. (2014).[6,7] The growth rate in the infinitely lived agents economy is determined by the familiar Euler condition, $\frac{\dot{c}}{c} = \frac{1}{\theta}(r-\rho)$.[8] Therefore, the effect of patent protection strength on growth works solely through its positive impact on the returns to innovation and, thereby, the interest rate.

Next, we show that, for any positive depreciation rate on physical capital, shortening patent length is more effective in spurring growth than loosening patent breadth protection. Shortening patent length triggers the mechanism presented above while mitigating the crowding out effect as in Chou and Shy (1993).[9] Shortening patent length induces the same effect as loosening patent breadth protection by lowering the average price of machine varieties. Patent expiration over a certain specialized machine results in competition among imitators of this specific variety, which brings its price down to marginal cost. Shorter patent length increases the fraction of competitive machine-industries, thus lowering average machines' price. Compared with Chou and Shy (1993) and Sorek (2011), who found that one-period patent length yields higher growth than infinite

[5] Since the old are the patent owners, this effect of weakening patent breadth protection is similar to income transfers from the old to the young considered by Jones and Manuelli (1992). Similarly, Uhlig and Yanagawa (1996) showed that reliance on capital-income taxation can also enhance growth.

[6] These studies differ mainly in their modelling approach of patent policy. All these works assume the differentiated inputs are intermediate goods that are formed in the same period they are being used, whereas we consider the differentiated inputs as investment goods (i.e. physical capital) that are formed one period ahead of utilization. Nonetheless, for the infinitely lived agents this assumption does not effect the implications of the main mechanism under study here.

[7] In another related work, Iwaisako and Futagami (2013) study the implications of patent policy for growth in a model of infinitely lived agents with physical capital. However, the role of physical capital is completely different than in the present analysis. They use homogenous (raw) physical capital, along with labor, as an input in the production of differentiated consumption goods - to which patent policy applies.

[8] Where c is per-capita consumption, θ is the inter-temporal elasticity of substitution, ρ is the time preference parameter and r is the interest rate. See for example equations (3),(14) and (15), in Zeng et al. (2014).

[9] This crowding-out reduction could be also achieved by weakening patent breadth protection gradually along patents' lifetime. Either way, the market value of an old patent will decrease, freeing investment resources for R&D activity.

patents protection in OLG economy with no physical capital, we also find that one-period patent length never maximizes growth in our model economy.

Our welfare analysis shows that loosening patent breadth protection beyond the growth-maximizing level can benefit all generations. Our welfare results do not differ qualitatively from the ones obtained in the counterpart studies of infinitely lived agents, where welfare is also maximized with incomplete patent protection; See Iwaisako and Futagami (2003), Cysne and Turchick (2012, 2014), and Zeng et al. (2014). Hence, both our OLG framework and the infinitely lived agents models exhibit the feature that welfare maximizing patent protection is weaker than growth-maximizing patent protection.

Finally, in the last section of the analysis, we present an implication of our main finding for patent policy and economic development. We show that when labor productivity increases relative to innovation cost, due to human capital accumulation, the growth-maximizing patent breadth protection adjusts to labor productivity. Hence, as the economy develops, the growth-maximizing patent strength is increasing as well. This result provides a normative case for the documented positive correlation between the strength of intellectual property rights (IPR) and economic development worldwide (See Eicher and Newiak 2013, and Chu et al. 2014).

Chu et al. (2014) presented the first analysis of stage-dependent optimal IPR, based on a trade-off between imitation from foreign direct investment (FDI) and reliance on domestic innovation. Our last result provides a complementary case for growth-enhancing stage-dependent IPR policy for a closed economy (which is independent of the imitation motive). In an earlier analysis of the topic, Diwakar and Sorek (2016) provide evidence that major developing economies strongly restrict (physical) capital inflows.

The paper proceeds in a straightforward manner. Section 2 presents the model. Section 3 studies the implications of alternative patent policies to growth and welfare. Lastly, Section 4 concludes.

2 Model

Our model incorporates the variety expansion model with lab-equipment innovation technology and differentiated capital goods proposed by Rivera-Batiz and Romer (1991), into Diamond's (1965) canonical OLG demographic structure. Each period two overlapping generations of measure L, the "young" and the "old, are economically active. Each agent is endowed with one unit of labor to be supplied inelastically when young. Old agents retire and consume their saving.

The benchmark model presented in this section assumes full patent protection (i.e., infinite patent duration and complete patent breadth protection), implying that in any period innovators can charge the unconstrained monopolistic price for their patented machines. We study the implications of incomplete patent protection in Section 3.

2.1 Production and innovation

The final good Y is produced by perfectly competitive firms with labor and differentiated capital goods, to which we refer also as specialized machines.

$$Y_t = AL^{1-\alpha} \int_0^{M_t} K_{i,t}^{\alpha} \,\mathrm{d}i \quad , \tag{1}$$

where $\alpha \in (0,1)$, A is a productivity factor, L is the constant labor supply, $K_{i,t}$ is the utilization level of machine-variety i in period t, respectively, and M_t measures the number of available machine-varieties.[10] Machines are subject to the depreciation rate $\delta \in (0,1)$ per usage-period, and the price of the final good is normalized to one. Under symmetric equilibrium, utilization level for all machines is the same, i.e. $K_{i,t} = K_t \; \forall \; i$, and thus total output is

$$Y_t = AM_t K_t^{\alpha} L^{1-\alpha}. \tag{1a}$$

The representative (perfectly-competitive) firm in the final-good production sector employs specialized machines at the rental price p_i and labor at the market wage w, in order to maximize the profit function

$$\pi_t = AL^{1-\alpha} \int_0^{M_t} K_{i,t}^{\alpha} \,\mathrm{d}i - \int_0^{M_t} p_{i,t} K_{i,t} \,\mathrm{d}i - w_t L \quad .$$

The labor market is perfectly competitive and the equilibrium wage and aggregate labor income are $w_t = (1-\alpha)\, AM_t K_t^{\alpha} L^{-\alpha}$ and $w_t L = A(1-\alpha) M_t K_t^{\alpha} L^{1-\alpha}$, respectively. The profit maximization with respect to each machine variety yields the familiar demand function: $K_{i,t}^{d} = A^{\frac{1}{1-\alpha}} L (\frac{\alpha}{p_{i,t}})^{\frac{1}{1-\alpha}}$. Assuming symmetric equilibrium prices and plugging the latter expression back into (1a) we obtain

$$Y_t = A^{\frac{1}{1-\alpha}} M_t L (\frac{\alpha}{p_t})^{\frac{\alpha}{1-\alpha}}. \tag{2}$$

We assume that innovation technology follows the "lab-equipment" specification proposed by Rivera-Batiz and Romer (1991). The cost of a new blue print, that is the cost of inventing a new machine variety, is η output units.[11] This cost is borne by the innovating firms. The innovation process takes one period, and then the machines of the newly invented variety can be rented to the producers of the final good under implemented patent protection policy.

[10] The elasticity of substitution between different varieties is $\frac{1}{1-\alpha}$.

[11] This innovation technology was assumed in all the counterpart models of infinitely lived agents mentioned in the introduction section.

2.2 Preferences

Lifetime utility of the representative agent born in period t is derived from consumption (denoted by c) over two periods, based on the logarithmic instantaneous-utility specification[12]

$$U_t = \ln c_t + \rho \ln c_{t+1}, \tag{3}$$

where $\rho \in (0,1)$ is the subjective discount factor. Young agents allocate their labor income between consumption and saving, denoted by s. The solution for the standard optimal saving problem is $s_t = \frac{w_t}{1+\rho^{-1}}$. Hence, aggregate saving by the young is $S_t = \frac{w_t L}{1+\rho^{-1}}$, which after substituting the explicit expressions for w_t becomes

$$S_t = \frac{(1-\alpha)A^{\frac{1}{1-\alpha}} M_t L \left(\frac{\alpha}{p}\right)^{\frac{\alpha}{1-\alpha}}}{1+\rho^{-1}}. \tag{4}$$

2.3 Equilibrium and growth

The patent owners of each machine variety borrow raw physical capital from savers/lenders at the net interest rate r_t. They then transform each unit of raw capital into one specialized machine, at no cost. This investment process of physical capital formation takes one period. In the following period, the specialized machines are rented to final output producers at the rate p. Hence, given the demand for each machine, as previously specified, the per-period surplus from each patented machine, denoted PS, is: $PS_{i,t} = \left[p_{i,t} - (\delta + r_t)\right] K^d_{i,t}$.

The surplus is maximized by the standard monopolistic price $p_{i,t} = \frac{\delta + r_t}{\alpha}$. Under infinite patent duration, all new and old varieties are priced equally and, therefore, share the same utilization level. As long as innovation takes place, the market value of old patents, that is patents over varieties that were invented in the past, equals the cost of inventing a new one, η. The gross rate of return on investment in patents is given by $1 + r_t = \frac{PS+\eta}{\eta}$. Notice that the numerator in the interest expression contains η because each and every period all patents held by old agents are sold to the young agents, that is the intergenerational trade in old patents.

Using the explicit term for the surplus and price of the specialized machines, we obtain the following implicit expression for the stationary equilibrium interest rate, r^*:

$$\forall t : 1 + r_t = \frac{\left[p_{i,t} - (\delta + r_t)\right] K^d_{i,t} + \eta}{\eta} \Longrightarrow 1 + r^* = \frac{(\delta + r^*)^{-\frac{\alpha}{1-\alpha}} \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}} + \widehat{\eta}}{\widehat{\eta}}, \tag{5}$$

where $\widehat{\eta} \equiv \frac{\eta}{A^{\frac{1}{1-\alpha}} L}$. Equation (5) also defines the no-arbitrage condition that equalizes the net rate of return on investment in patents and investment in physical capital.

Lemma 1 *There exists a unique stationary interest rate, r^*, which solves (5).*

[12] It is well known that under the assumed demographic structure, the logarithmic instantaneous utility implies that the saving (and investment) level is independent of the interest rate. In the Appendix, we consider the implications of the general CEIS preference form.

Proof. The left hand side of (5) is increasing linearly in r , from one (for $r = 0$) to infinity. The right hand side of (5) is decreasing in r from $\frac{\delta^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)\alpha^{\frac{2}{1-\alpha}}+\widehat{\eta}}{\widehat{\eta}} > 1$ (for $r = 0$) to one (for $r \to \infty$). Hence, by the intermediate value Theorem, there exists a positive stationary interest rate, r^*, that solves (5). ■

For the case $\delta = 0$, equation (5) yields an explicit solution for the stationary equilibrium interest rate:

$$\text{for } \delta = 0\text{: } r^* = \alpha^{1+\alpha}\left(\frac{1-\alpha}{\widehat{\eta}}\right)^{1-\alpha}. \tag{5a}$$

Under the stationary-equilibrium interest rate, aggregate saving of the young is allocated over investment in old and new patents, and in physical capital (i.e. specialized machines), where the investment in physical capital is set to meet the demand for specialized machines.

$$I_t = M_{t+1}\left[\eta + A^{\frac{1}{1-\alpha}} L \left(\frac{\alpha^2}{r^* + \delta}\right)^{\frac{1}{1-\alpha}}\right] \tag{6}$$

Equation (5) implies that under the stationary interest rate, machine prices are also stationary: $\forall t, i : p^* = \frac{\delta + r^*}{\alpha}$. Hence, the output growth rate, denoted $g_{Y,t+1} \equiv \frac{Y_{t+1}}{Y_t} - 1$, equals to the rate of machine-varieties expansion, i.e. $g_{Y,t+1} = g_{M,t+1}$. Imposing the equilibrium condition $S = I$, we equalize (4) and (6) to derive the stationary rate of variety expansion, g^*, which defines the output growth rate:

$$1 + g^* = \frac{\frac{1-\alpha}{1+\rho^{-1}}\left(\frac{\alpha^2}{r^*+\delta}\right)^{\frac{\alpha}{1-\alpha}}}{\widehat{\eta} + \left(\frac{\alpha^2}{r^*+\delta}\right)^{\frac{1}{1-\alpha}}} \tag{7}$$

Lemma 2 *For sufficiently low $\widehat{\eta}$ the growth rate defined in (7) is positive.*

Proof. Notice that as $\widehat{\eta}$ approaches zero, the right hand side in (7) approaches $\frac{1-\alpha}{1+\rho^{-1}}\frac{r^*+\delta}{\alpha^2}$. However, by (5), as $\widehat{\eta}$ approaches zero, the interest rate r approaches approaches infinity, and thus $\lim_{\widehat{\eta}\to 0}\frac{1-\alpha}{1+\rho^{-1}}\frac{r^*+\delta}{\alpha^2}$ approaches infinity as well. ■

Assumption 1 Based on Lemma 2, we assume hereafter that $\widehat{\eta}$ is sufficiently low, so that the growth rate under complete patent protection, defined in equation (7) is positive.

3 Patents

We are prepared now to explore the implications of patent policy for growth and welfare. The growth implications of incomplete patent breadth protection, under infinite patent length, are studied first. We then demonstrate the greater effectiveness of finite patent length in spurring economic growth. Lastly, we examine welfare enhancing stage-dependent patent policies.

3.1 Patent breadth and growth

We model patent breadth protection with the parameter λ, which limits the ability of patent holders to charge the unconstrained monopolistic price: $p^*(\lambda) = \frac{\lambda(\delta+r^*)}{\alpha}$ where $\lambda \in [\alpha, 1]$, and thus $p^*(\lambda) \in \left(\delta + r^*, \frac{\delta+r^*}{\alpha}\right)$. One can think of $p^*(\lambda)$ as the maximal price a patent holder can set and still deter competition by imitators. Weaker breadth protection lowers the cost of imitation, thereby imposing a lower deterrence price on patent holders.[13] When $\lambda = 1$, patent breadth protection is complete and patent holders can charge the unconstrained monopolistic price. With zero protection $\lambda = \alpha$, patent holders lose their market power completely and sell at marginal cost. Note that as patent breadth protection is weakened, machines' price is reduced and quantity demanded for each machine-variety increases. Under this patent breadth policy, the equilibrium stationary interest rate in equation (5) modifies to

$$1 + r^* = \frac{(\delta + r^*)^{-\frac{\alpha}{1-\alpha}} \left(\frac{\lambda}{\alpha} - 1\right) \left(\frac{\alpha^2}{\lambda}\right)^{\frac{1}{1-\alpha}} + \widehat{\eta}}{\widehat{\eta}}. \tag{8}$$

$$\text{For } \delta = 0 : r^* = \left[\frac{\left(\frac{\lambda}{\alpha} - 1\right)}{\widehat{\eta}}\right]^{1-\alpha} \left(\frac{\alpha^2}{\lambda}\right). \tag{8a}$$

Lemma 3 *The stationary equilibrium interest rate r^* is increasing with patent breadth protection and decreasing with the depreciation rate and innovation cost. That is $\frac{\partial r^*}{\partial \lambda} > 0$ and $\frac{\partial r^*}{\partial \delta}, \frac{\partial r^*}{\partial \eta} < 0$. Furthermore, $\frac{\partial (r^*+\delta)}{\partial \delta} > 0$.*

Proof. Differentiating the right hand of (8) side for λ yields a positive derivative for any $\alpha < \lambda < 1$. Hence, the value of r^*, which solves (8), is increasing with patent breadth protection λ. Similarly, as the right hand side of (8) is decreasing with the depreciation rate and the innovation cost, so does the value of interest rate that solves (8). Since r^* is a decreasing function of the depreciation rate, the left-hand side of equation (8) is decreasing in the depreciation rate. The term $(r^* + \delta)$ on the right-hand side of the equation must therefore be an increasing function of the depreciation rate, as the exponent is negative. Thus, $\frac{\partial (r+\delta)}{\partial \delta} > 0$. ∎

Lemma 3 implies that loosening patent breadth protection decreases machines' price, $p^*(\lambda)$, through capping the monopolistic markup and by decreasing the marginal cost (of capital) on which this markup builds. Thus, loosening patent breadth protection increases the demand for each machine variety. This increase in demand for machines has a positive effect on aggregate saving (4), for a given variety span:

$$S_t = \frac{(1-\alpha) A^{\frac{1}{1-\alpha}} M_t L \left(\frac{\alpha}{p_t(\lambda)}\right)^{\frac{\alpha}{1-\alpha}}}{1 + \rho^{-1}}.$$

[13] Similar modeling approach for patent breadth protection was used (among others) by Goh and Olivier (2002), Iwaisako and Futagami (2003,2013), and Chu et al. (2016). Zeng et al. (2014) interpret the same modeling approach as direct price regulation.

This is the positive effect of loosening patent breadth protection on aggregate saving (for a given variety span M_t) and, thereby, innovation and growth. However, for a given level of saving, the increased demand for machines works to shift investment toward physical capital and away from patents. This is the negative effect of loosening patent breadth protection on innovation and growth. From equation (6) we have:

$$I_t = M_{t+1}\left[\eta + A^{\frac{1}{1-\alpha}}L\left(\frac{\alpha}{p_{t+1}(\lambda)}\right)^{\frac{1}{1-\alpha}}\right].$$

Plugging $p^*(\lambda) = \frac{\lambda(\delta+r^*)}{\alpha}$ in the above saving and investment expressions and imposing the aggregate constraint $S = I$, we obtain

$$1 + g_y = \frac{1-\alpha}{1+\rho^{-1}}\frac{\psi^{\frac{\alpha}{1-\alpha}}}{\widehat{\eta} + \psi^{\frac{1}{1-\alpha}}}. \tag{9}$$

where $\widehat{\eta} = \frac{\eta}{A^{\frac{1}{1-\alpha}}L}$ (as before), and $\psi \equiv \frac{\alpha^2}{\lambda(\delta+r^*)}$. Finally, we denote the growth-maximizing patent breadth policy λ^{**}.

Proposition 1 *The growth-maximizing patent breadth protection policy, λ^{**}, is given by the solution to $\frac{\alpha^2}{\lambda^{**}(\delta+r^{**})} = \left(\frac{\alpha}{1-\alpha}\widehat{\eta}\right)^{1-\alpha}$. For any positive depreciation rate it is incomplete, and it is decreasing in the depreciation rate. That is, $\forall\delta > 0 : \alpha < \lambda^{**} < 1$ and $\frac{\partial\lambda^{**}}{\partial\delta} < 0$.*

Proof. Differentiating (9) for ψ reveals that the growth rate is increasing in ψ, if $\psi < \left(\frac{\alpha}{1-\alpha}\widehat{\eta}\right)^{1-\alpha}$, that is $\frac{\alpha^2}{\lambda(\delta+r^*)} < \left(\frac{\alpha}{1-\alpha}\widehat{\eta}\right)^{1-\alpha}$, and is maximized for $\frac{\alpha^2}{\lambda^{**}(\delta+r^*)} = \frac{\alpha}{1-\alpha}\widehat{\eta}^{1-\alpha}$. The interest rate equation (8) can be rearranged, to be written as: $\frac{\alpha}{1-\alpha}\widehat{\eta} = \frac{(\delta+r^*)\left(\frac{\lambda-\alpha}{1-\alpha}\right)}{r^*}\psi^{\frac{1}{1-\alpha}}$. Substituting the growth-maximizing condition into the latter expression yields $\frac{\lambda^{**}-\alpha}{1-\alpha} = \frac{r^*}{r^*+\delta}$. Therefore, under zero depreciation rate the growth-maximizing policy is $\lambda^{**} = 1$. And for any positive depreciation rate $\alpha < \lambda^{**} < 1$. Finally, by Lemma 3 we have $\frac{\partial r^*}{\partial\delta} < 0$ and $\frac{\partial(r^*+\delta)}{\partial\delta} > 0$. Hence, a high depreciation rate requires a lower λ^{**} to maintain the latter growth-maximizing condition. ■

The main mechanism behind Proposition 1 was already explained with the presentation of the aggregate saving and aggregate investment equations above. The negative relation between the growth-maximizing patent breadth and the physical depreciation rate, is due to the effect of the latter on machines' price. The lower the depreciation rate, the lower the price of physical capital and, therefore, the higher is the demand for physical capital. With initial lower machine prices, there is less potential for growth enhancement through further price decrease induced by loosening patent protection.

Proposition 1 shows that the growth-maximizing patent protection is negatively related to the depreciation rate of physical capital, δ.

Corollary 1 *The maximal growth rate, g_y^{**}, that corresponds to λ^{**} is not dependent on the depreciation rate. Under the growth-maximizing policy, which, by Proposition 1, satisfies $\psi^{**} \equiv$*

$\left(\frac{\alpha}{1-\alpha}\widehat{\eta}\right)^{1-\alpha}$*, the right hand side of (9) reduces to the following expression that is independent of* δ*:* $1+g_y^{**} = \frac{\alpha^{\alpha}(1-\alpha)^{2-\alpha}}{(1+\rho^{-1})\widehat{\eta}^{1-\alpha}}$.

3.2 Patent length and growth

We turn to study the implications of patent length for growth, under complete patent breadth protection. We study stochastic patent length, assuming that each period a fraction $1-\pi$ of the existing patents expire, where $\pi \in [0,1]$.[14,15] However, all new patents are certain to be granted with a patent for one period, which will expire with probability $1-\pi$ in the second period. In other words, imitation may take place only after the new variety was used for one period. This means that, at the beginning of each period, the expected lifetime of all patents (old and new), denoted T, is given by $E(T) = 1 + \frac{\pi}{1-\pi}$ Under this specification, the stationary fraction of patented industries, μ^*, is[16]

$$\mu^* = \frac{g^*}{1+g^*-\pi} \Rightarrow 1-\mu^* = \frac{1-\pi}{1+g^*-\pi}. \tag{10}$$

Applying (10) to (1) we write the modified output equation:

$$Y_t = A^{\frac{1}{1-\alpha}} M_t L \left[\frac{g^*}{1+g^*-\pi}\left(\frac{\alpha^2}{\delta+r^*}\right)^{\frac{\alpha}{1-\alpha}} + \frac{1-\pi}{1+g^*-\pi}\left(\frac{\alpha^2}{\delta+r^*}\right)^{\frac{\alpha}{1-\alpha}} \right]. \tag{11}$$

Aggregate saving is still a constant fraction of total output: $S_t = \frac{(1-\alpha)}{1+\rho^{-1}} Y_t$, and the modified investment equation is

$$I_t = M_{t+1}\left[\frac{g^*}{1+g^*-\pi}\eta + \frac{g^*}{1+g^*-\pi} A^{\frac{1}{1-\alpha}} L \left(\frac{\alpha^2}{\delta+r^*}\right)^{\frac{1}{1-\alpha}} + \frac{1-\pi}{1+g^*-\pi} A^{\frac{1}{1-\alpha}} L \left(\frac{\alpha}{\delta+r^*}\right)^{\frac{1}{1-\alpha}} \right]. \tag{12}$$

Imposing $I_t = S_t$ yields the following implicit equation for the stationary growth rate:

[14] This formulation of patent length is equivalent to the Blanchard's (1985) formulation of human longevity, in his classic "perpetual youth" model. This approach has two significant advantages. First, greatly enhance tractability by implying that in each and every period, all patents -old and new - have the same remaining expected lifetime. Therefore they have the same market value (price) as well. Secondly, this formulation implies a continuous policy instrument (which allows us using standard optimization techniques), although time in this model is discrete.

[15] Previous works used interpreted the same formulations more generally and as patent strength follows Helpman (1993), Kwan and Lai (2003), and Rubens and Turchick (2012). Their original interpretation was that a fraction π of the patented technologies is being imitated due to a lack of patent-protection enforcement. Rubens and Turchick (2014) interpret this formulation of patent policy as stochastic patent length, and demonstrate its equivalency to the deterministic patent length that was employed by Iwaisako and Futagami (2003) and Zeng et al. (2014).

[16] The number of patented industries is given by the sum of the renewed existing patents $\pi\mu_t M_t$ and the new patents $\triangle M_{t+t} \equiv M_{t+t} - M_t$. Hence, the fraction of patented industries in period $t+1$ is given by $\mu_{t+1} = \frac{\pi\mu_t M_t + M_{t+t} - M_t}{M_{t+1}} = \frac{\pi\mu_t}{1+g} + \frac{g}{1+g}$. This equation implies that under stationary growth rate the fraction of patented machine varieties converges to the stationary level given in equation (10).

$$1+g^{*}=\frac{\frac{1-\alpha}{1+\rho^{-1}}\left[\left(\frac{\alpha^{2}}{\delta+r^{*}}\right)^{\frac{\alpha}{1-\alpha}}+\frac{1-\pi}{g^{*}}\left(\frac{\alpha}{\delta+r^{*}}\right)^{\frac{\alpha}{1-\alpha}}\right]}{\widehat{\eta}+\left(\frac{\alpha^{2}}{\delta+r^{*}}\right)^{\frac{1}{1-\alpha}}+\frac{1-\pi}{g^{*}}\left(\frac{\alpha}{\delta+r^{*}}\right)^{\frac{1}{1-\alpha}}}=\frac{\frac{1-\alpha}{1+\rho^{-1}}\psi^{\frac{\alpha}{1-\alpha}}(1+\frac{1-\pi}{g^{*}}\alpha^{\frac{-\alpha}{1-\alpha}})}{\widehat{\eta}+\psi^{\frac{1}{1-\alpha}}\left(1+\frac{1-\pi}{g^{*}}\alpha^{\frac{-1}{1-\alpha}}\right)}. \tag{13}$$

where $\psi_t \equiv \frac{\alpha^2}{\delta+r_t}$, as before. Equation (13) has only one positive root, and for $\pi = 1$ it coincides with (7). The stationary interest rate under the current patent policy is given by

$$1+r^{*}=\frac{(\delta+r^{*})^{-\frac{\alpha}{1-\alpha}}\left(\frac{1}{\alpha}-1\right)\alpha^{\frac{2}{1-\alpha}}+\pi\widehat{\eta}}{\widehat{\eta}}. \tag{14}$$

The stationary equilibrium interest rate that satisfies (14), r^*, is increasing with the patent survival probability π, and thus ψ is decreasing with the patent survival probability, i.e. $\frac{\partial\psi}{\partial\pi} < 0$.

Remark 1 *Setting $\pi = (1-\delta)$ in (14) yields $\delta + r^* = \left(\frac{1-\alpha}{\alpha\widehat{\eta}}\right)^{1-\alpha}\alpha^2$. Thus, by Proposition 1, we have: $\psi\left(\pi = 1-\delta, \lambda = 1\right) = \psi^{**}\left(\pi = 1, \lambda^{**}\right) \equiv \left(\frac{\alpha}{1-\alpha}\widehat{\eta}\right)^{1-\alpha}$.*

Applying the implicit function theorem to (13) we obtain the following expression for $\frac{\partial g^*}{\partial\pi}$:

$$\frac{\frac{(1-\alpha)}{1+\rho^{-1}}\psi^{\frac{\alpha}{1-\alpha}}[\frac{\alpha\left|\frac{\partial\psi}{\partial\pi}\right|\left(1+\frac{1-\pi}{g^{*}}\alpha^{\frac{-\alpha}{1-\alpha}}\right)}{(1-\alpha)\psi}+\frac{\alpha^{\frac{-\alpha}{1-\alpha}}}{g^{*}}]-\psi^{\frac{1}{1-\alpha}}[\frac{\left|\frac{\partial\psi}{\partial\pi}\right|\left(1+\frac{1-\pi}{g^{*}}\alpha^{\frac{-1}{1-\alpha}}\right)}{(1-\alpha)\psi}+\frac{\alpha^{\frac{-1}{1-\alpha}}}{g^{*}}](1+g^{*})}{-\frac{(1-\pi)\psi^{\frac{\alpha}{1-\alpha}}\alpha^{\frac{-1}{1-\alpha}}}{(g^{*})^{2}}\left[\frac{(1-\alpha)\alpha}{1+\rho^{-1}}-\psi(1+g^{*})\right]-B} \tag{15}$$

Where B is the denominator in the right hand side of (13). Based on the above remark and equation (15), we obtain the following proposition.

Proposition 2 *For any positive depreciation rate, a finite expected patent length can yield a higher growth rate than incomplete patent breadth protection.*

Proof. Substituting $\psi = \psi^{**}$ into (13) yields the growth rate obtained in Corollary 1. That is for $\pi = (1-\delta)$ and $\lambda = 1$: $1 + g_y^{**} = \frac{\alpha^\alpha(1-\alpha)^{2-\alpha}}{(1+\rho^{-1})\widehat{\eta}^{1-\alpha}}$. Then, substituting $\psi = \psi^{**}$ and $g = g_y^{**}$ into (15) reveals that, for any positive depreciation rate, both the denominator and numerator are negative, and for zero depreciation rate the numerator equals zero (and the denominator remains negative). That is, $\forall\delta > 0 : \frac{\partial g^*}{\partial\pi}|_{\pi=1-\delta} > 0$, and for $\delta = 0 : \frac{\partial g^*}{\partial\pi}|_{\pi=1-\delta} = 0$. Hence, for any positive depreciation rate, growth under finite patent length can be enhanced beyond the maximal rate defined in Corollary 1 by marginal increase in expected patent length. Finally, for $\pi = 1$, the expression in (15) is negative for any positive depreciation rate. Therefore, $\forall\delta > 0 : \pi^{**} \in (1-\delta, 1)$, that is $E(T^{**}) \in (\ 1+\ \frac{1-\delta}{\delta}, \infty)$, and $g(\pi^{**}, \lambda = 1) > g(1, \lambda^{**})$. For zero depreciation on physical capital growth is maximized with infinite patent length. That is, for $\delta = 0$, $\pi^{**} = 1$, and $g(\pi^{**}, \lambda = 1) = g(1, \lambda^{**})$. ■

3.3 Patents and welfare

This section explores some welfare implications of patent policy in our model economy. To maintain tractability, we focus on patent breadth protection. The definition of a social welfare function for the OLG economy is not trivial, due to lack of a natural social discount factor.[17] Hence, we follow Chou and Shy (1993) in comparing the lifetime utility of all living generations under alternative, stationary, patent protection degrees.[18] That is, we are interested in characterizing patent breadth protection policy that is Pareto improving for all present and future generations. Substituting the explicit expressions for c_1 and c_2, (based on per-worker saving), into the lifetime utility function (3) yields the indirect lifetime utility of the representative consumer who was born in period t:

$$U_t = \ln\left[\frac{(1-\alpha)M_t A^{\frac{1}{1-\alpha}} L\left(\frac{\alpha}{p(\lambda)}\right)^{\frac{\alpha}{1-\alpha}}}{1+\rho}\right] + \rho\ln\left[\frac{\rho(1-\alpha)M_t A^{\frac{1}{1-\alpha}} L\left(\frac{\alpha}{p(\lambda)}\right)^{\frac{\alpha}{1-\alpha}}}{1+\rho}(1+r^*)\right]. \tag{16}$$

Equation (16) implies that $U_t = U_{t-1} + (1+\rho)\ln(1+g)$, and thus

$$U_t = U_0 + t(1+\rho)\ln(1+g), \tag{16a}$$

where U_0 is given by evaluating (16) for M_0. Equation (16) implies that, for every generation, loosening patent breadth protection involves a trade off between increasing first period consumption and saving (due to increased labor income), and a decrease in second-period consumption due to a lower interest rate (as $\frac{\partial r^*}{\partial \lambda} > 0$). In addition, equation (16a) implies that future generations benefit from a positive effect of loosening patent breadth protection on the growth rate, where this effect is stronger the more distant in the future the generation is born.

The derivative of the lifetime utility of generation t with respect to the patent breadth protection parameter is

$$\frac{\partial U_t}{\partial \lambda} = \frac{\partial U_0}{\partial \lambda}U_0 + t(1+\rho)\left(\frac{1}{1+g}\frac{\partial g}{\partial \lambda}\right). \tag{17}$$

The derivative $\frac{\partial U_0}{\partial \lambda}$ has the following expression:

$$\frac{\partial U_0}{\partial \lambda} = \left[\frac{\rho}{1+r} - \frac{\alpha}{1-\alpha}\frac{(1+\rho)}{(\delta+r^*)}\right]\frac{\partial r}{\partial \lambda} - \frac{\alpha}{1-\alpha}\frac{(1+\rho)}{\lambda}. \tag{17a}$$

We are interested in verifying whether the lifetime utility of all generations can be improved by weakening, or strengthening, patent protection further beyond the growth-maximizing policy. For this purpose, we need to evaluate the sign of (17a) under the growth-maximizing policy $\lambda = \lambda^{**}$. However, by definition, under the growth-maximizing policy the second addend in (17a) is zero.

[17] Which determines the weight that is given to the lifetime utility of different generations in the social objective function.

[18] They, however, only compare welfare under the two extreme policies - one period and infinite patent length. See Propositions 3-4 on page 310, there.

Therefore, we need only to determine the sign of $\frac{\partial U_0}{\partial \lambda}|_{\lambda=\lambda^{**}}$.

Applying the implicit function theorem to equation (8) we obtain: $\frac{\partial r^*}{\partial \lambda} = \frac{r^* \frac{\alpha}{\lambda-\alpha}\frac{1-\lambda}{1-\alpha}}{\frac{r^*}{r^*+\delta}\frac{\alpha}{1-\alpha}+1}$. Then, we use the growth-maximizing condition, $\frac{\lambda^{**}-\alpha}{1-\alpha} = \frac{r^*}{r^*+\delta}$ (from Proposition 1) to evaluate $\frac{\partial r^*}{\partial \lambda}$ for $\lambda = \lambda^{**}$: $\frac{\partial r^*}{\partial \lambda}|_{\lambda=\lambda^{**}} = \frac{\alpha(r^*+\delta)(1-\lambda)}{\lambda\alpha+1-2\alpha}$. Substituting the latter expression into (17a) yields the following condition $\frac{\partial U_0}{\partial \lambda}|_{\lambda=\lambda^{**}} < 0 \iff \frac{\delta+r^*}{1+r} < \frac{1+\rho}{\rho(1-\alpha)}\left[1 + \frac{\lambda\alpha+1-2\alpha}{\lambda(1-\lambda)}\right]$, for which the lifetime utility of all generations can be increased by weakening patent breadth protection further beyond the growth-maximizing level. The latter condition holds for all relevant parameter values, as the left hand side is never greater than one, but the right hand side is always greater than one. The next Proposition concludes this result.

Proposition 3 *Weakening patent breath protection further beyond the growth-maximizing level benefits all generations.*

Recall that, by equation (16a), a generation that is born more distant in the future would benefit more from growth enhancing policy. And, by derivation of the result presented in Proposition 1, the patent protection policy that maximizes the term U_0 (which is independent of the growth rate), is weaker than the growth-maximizing policy. Hence, the degree of patent protection that maximizes the lifetime utility of each generation depends positively on their birth period t. That is, the degree of patent protection that maximizes the lifetime utility of the generation born in period t is always lower than the one that maximizes the lifetime utility of generation $t + n$ (for any positive n).

Proposition 3 relies on a comparison between two alternative stationary policies. However, the direct transitional impact of loosening patent breadth policy at a certain period will not yield Pareto improvement even if the above proposition holds. At period zero, the amount of available machines is already pre-determined, and thus, decreasing their price can not increase their utilization level. Hence, the positive effect on aggregate saving will not prevail, and only the negative effect on second-period consumption (due to the lower interest rate) will be at work. Therefore, transfers from the next young generation (to be born in period one) to the current young generation will be required to maintain Pareto improvement. However, the complete analysis of this issue falls beyond the scope of the current study.

3.4 Stage-dependent patent policy

Proposition 1 implies that the stationary growth-maximizing patent policy depends positively on the value[19] of $\widehat{\eta} \equiv \frac{\eta}{A^{\frac{1}{1-\alpha}}L}$, which can be interpreted as innovation-cost, denoted η, per effective labor supply, denoted: $H \equiv A^{\frac{1}{1-\alpha}}L$.[20]

[19] Recall that the growth-maximizing policy defined in Proposition 1 satisfies $\left[\frac{\alpha^2}{\lambda^{**}(\delta+r^{**})}\right]^{\frac{1}{1-\alpha}} = \frac{\alpha}{1-\alpha}\widehat{\eta}$, and, by Lemma 3, the equilibrium interest rate is also increasing with the patent breadth protection λ.

[20] If $A^{\frac{1}{1-\alpha}}$ is interpreted as labor augmented productivity factor we can write (1) as : $Y = MK^{\alpha}\left(A^{\frac{1}{1-\alpha}}L\right)^{1-\alpha}$.

In this subsection, we attempt to extend this result for a transitional, non-stationary, trajectory which corresponds an economic development phase, along which the term $\widehat{\eta}$ is decreasing due to an increase in H. Indeed, labor productivity is typically increasing along the course of economic development through the accumulation of human capital.

Adding the time subscript to the relevant parameters, we re-write the output and growth equations

$$Y_t = M_t H_t \left[\frac{\alpha}{p_t(\lambda_t)}\right]^{\frac{\alpha}{1-\alpha}}, \tag{18}$$

$$1 + g_{M,t+1} = \frac{(1-\alpha)}{1+\rho^{-1}} \frac{H_t \left[\frac{\alpha}{p_t(\lambda_t)}\right]^{\frac{\alpha}{1-\alpha}}}{\eta + H_{t+1}\left[\frac{\alpha}{p_{t+1}(\lambda_{t+1})}\right]^{\frac{\alpha}{1-\alpha}}}, \tag{18a}$$

where $p_t(\lambda_t) = \frac{\lambda_t(\delta + r_t)}{\alpha}$, as before, and the interest rate follows the modified no-arbitrage condition

$$1 + r_{t+1} = \frac{(\delta + r_{t+1})^{\frac{-\alpha}{1-\alpha}} \left(\frac{\lambda_{t+1}}{\alpha} - 1\right) \lambda_{t+1}^{\frac{-1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} + \widehat{\eta}_{t+1}}{\widehat{\eta}_{t+t}}, \tag{18b}$$

where $\widehat{\eta}_{t+t} = \frac{\eta}{H_{t+1}}$. Note that as all the variables in (18b) are share same time index, Lemma 3 that characterized the properties of stationary interest rate still applies to the intra-temporal equilibrium for each and every period. Equation (18) implies the following growth rate of per-capita output

$$1 + g_{y,t+1} = (1 + g_{M,t+1})(1 + g_{H,t+1})(1 + g_{p(\lambda),t+1})^{\frac{-\alpha}{1-\alpha}}. \tag{19}$$

Combining equations (19) with (18a) yields

$$1 + g_{y,t+1} = \frac{(1-\alpha)}{1+\rho^{-1}} \frac{\psi_{t+1}^{\frac{\alpha}{1-\alpha}}}{\widehat{\eta}_{t+1} + \psi_{t+1}^{\frac{1}{1-\alpha}}}. \tag{19a}$$

Notice that the growth equation (19a) depends only on the patent policy expected to prevail in period $t+1$.

Proposition 4 *For any positive depreciation rate, the growth-maximizing patent breadth is increasing with effective labor supply. That is,* $\forall \delta > 0 : \frac{\partial \lambda^{**}}{\partial H} > 0$. *Hence, as effective labor supply increases along the phase of economic development, the growth-maximizing patent breadth protection is tightened.*

Proof. Proposition 1 implies that the growth rate in (19a) is maximized with $\underbrace{\frac{\alpha^2}{\lambda_{t+1}(\delta + r_{t+1})}}_{=\psi_{t+1}} = \left(\frac{\alpha}{1-\alpha}\widehat{\eta}_{t+1}\right)^{1-\alpha}$. As effective labor supply increases, $\widehat{\eta}_{t+1}$ decreases. Consequently, the right-hand side of the latter equation is decreasing and, by Lemma 3, the left-hand side is also decreasing (due to the effect of $\widehat{\eta}_{t+1}$ on r_{t+1}). Nevertheless, by the proof of Proposition 1, combining the latter condition with the interest-rate equation (18b), yields the following relation between the growth-maximizing policy and the equilibrium interest rate $\frac{\lambda^{**}_{t+1}-\alpha}{1-\alpha} = \frac{r^{*}_{t+1}}{r^{*}_{t+1}+\delta}$. Hence, as $\widehat{\eta}_{t+1}$ decreases (with the increase in effective labor supply), the right hand side of the latter equation increases for any positive depreciation rate (due to the increasing interest rate). Therefore, an increase in λ_{t+1}, that is tightening patent breadth protection, is required to maintain the latter growth-maximizing-policy equation. ■

4 Conclusion

This work proposes a contribution to the literature on patent policy and economic growth by exploring the implications of patent policy in an OLG framework with physical capital. We have highlighted a novel mechanism through which weakening patent protection can enhance growth. This result is unique to the OLG demographic structure of finitely lived agents, as complete patent protection maximizes growth in the counterpart model of infinitely lived agents. This mechanism involves a trade-off between the effect of patent strength on aggregate saving and investment and the allocation of total investment between patent ownership and physical capital.

The positive effect on growth can be induced by either shortening patent length or loosening patent breadth protection, in our framework. However, shortening patent length also mitigates the crowding out effect of trade in old patents on R&D investment. Hence, shortening patent length can be more effective at generating growth than loosening patent breadth protection. These effects are not present in similar models with infinitely lived agents. Consequently, growth in these models is maximized with eternal patent life and complete patent breadth protection.

Finally, we have also presented an important implication of the main mechanism under study to patent policy and economic development. A stage-dependent patent policy for which patent strength is increasing over the course of economic development may be growth maximizing. This result provides a normative case for the often observed positive correlation between patent strength and economic development around the world.

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Appendix: CEIS utility

We turn here to consider the implication of the general CEIS instantaneous utility to our previous result, considering the following lifetime utility form:

$$U = \frac{c_t^{1-\theta}}{1-\theta} + \rho \frac{c_{t+1}^{1-\theta}}{1-\theta} \tag{A.1}$$

where $\frac{1}{\theta}$ is the elasticity of inter-temporal substitution, and for $\theta = 1$ equation (A.1) falls back to the logarithmic form (3). The modified solution for the standard optimal saving problem is $s_t = \frac{w_t}{1+\rho^{-\frac{1}{\theta}}(1+r^*)^{\frac{\theta-1}{\theta}}}$. Hence, aggregate saving now is $S_t = \frac{w_t L}{1+\rho^{-\frac{1}{\theta}}(1+r^*)^{\frac{\theta-1}{\theta}}}$ .Substituting the explicit expressions for w_t into S_t and equalizing to aggregate investment, $I_t = M_{t+1}\left[\eta + A^{\frac{1}{1-\alpha}} L \left(\frac{\alpha}{p(\lambda)}\right)^{\frac{1}{1-\alpha}}\right]$, yields the growth equation

$$1 + g^* = \frac{1-\alpha}{1+\frac{1}{\rho}(1+r^*)^{1-\frac{1}{\theta}}} \frac{\left(\frac{\alpha^2}{\lambda(\delta+r^*)}\right)^{\frac{\alpha}{1-\alpha}}}{\widehat{\eta} + \left(\frac{\alpha^2}{\lambda(\delta+r^*)}\right)^{\frac{1}{1-\alpha}}} \tag{A.2}$$

As it is well known, in the standard OLG framework the effect of interest rate on saving depends on the inter-temporal elasticity of substitution: it is positive (negative) if $\theta < 1$ ($\theta > 1$). Hence, because the interest rate is increasing with patent protection, the positive impact of decreasing patent breadth on growth is diminishing with the inter-temporal elasticity of substitution. More specifically, for $\theta < 1$ all our results remain (and will hold for a larger set of parameters) as a decrease in the interest rate by itself stimulates saving and investment (this is an additional effect was not induced under the logarithmic utility form). However, as θ increases beyond one, the decrease in the interest rate due to loosening patent breadth protection will work to hinder growth, countering the positive effects that were defined in Proposition 1. For sufficiently high value of θ this direct interest effect may dominate the over all impact of loosening patent protection on innovation and growth. Nevertheless, the empirical literature commonly suggests that θ is lower than one, and thus supporting the relevance of our main findings.[21] The welfare analysis for $\theta \neq 1$ turns out being intractable.

[21] See for example Hall (1988), Beaudry and Wincoop (1996), Ogaki and Reinhart (1998), Engelhardt and Kumar (2009).