Bowerbirds' Mate-selection Contests: Analysis and an Application

Bharat Goel*

In the world of satin bower-birds, the male-birds engage in a contest among themselves to win their mates: they spend considerable time and effort in decorating their own bowers, and then attempt to destroy the decorations of their rivals' bowers. The female-birds, in turn, select their mates on the basis of the attractiveness of the best surviving bowers. We study a game-theoretic model of such a mating contest, where two male-birds of distinct strengths engage in competitive signaling with value-less signals followed by signal sabotage (in an environment where a female-bird infers a male-bird's strength by observing the quality of his surviving bower). We prove that sabotage possibilities can improve the outcomes for both male-birds – since anticipated threat of sabotage depresses each male-bird's incentive to engage in costly signaling, while harming the outcome for the female-bird – as sabotage introduces noise in the female-bird's selection process.

We go on to study a principal-agent model that is similar in structure to the bowerbirds' contest: A principal cares about the best output created between two agents, and can design a tournament with suitably chosen order-of-moves and prizes. In a scenario where (*a*) produced outputs need to be stress-tested to determine their durability (here, stress-testing is similar to mutual sabotage in its analytics, though not in its intent), and (*b*) outputs can only be evaluated in relation to each other and /or an exogenous benchmark, it is shown that if the two agents are not too asymmetric then a tournament is strictly better for the principal than individually contracting with a single agent; if no external benchmark is available then for limited agent-asymmetry, the optimal output-selection mechanism is a simultaneous tournament with maximal feasible winning prize.

Keywords: Bower-birds, mate-selection, contests, sabotage, output-selection.

1. Introduction

In the world of *satin bower-birds*, male-birds engage in a unique contest among themselves in order to win their mates. On the one hand, they spend considerable time and effort in elaborately decorating their individual bowers, and on the other hand, they attempt to destroy the decorations of their rivals' bowers. The female-birds, in turn, select their mates on the basis of the attractiveness of the best surviving bowers; presumably, they do so because a bower's surviving quality signals the inherent (and directly unobservable) *strength* of its maker. The bowerbirds'

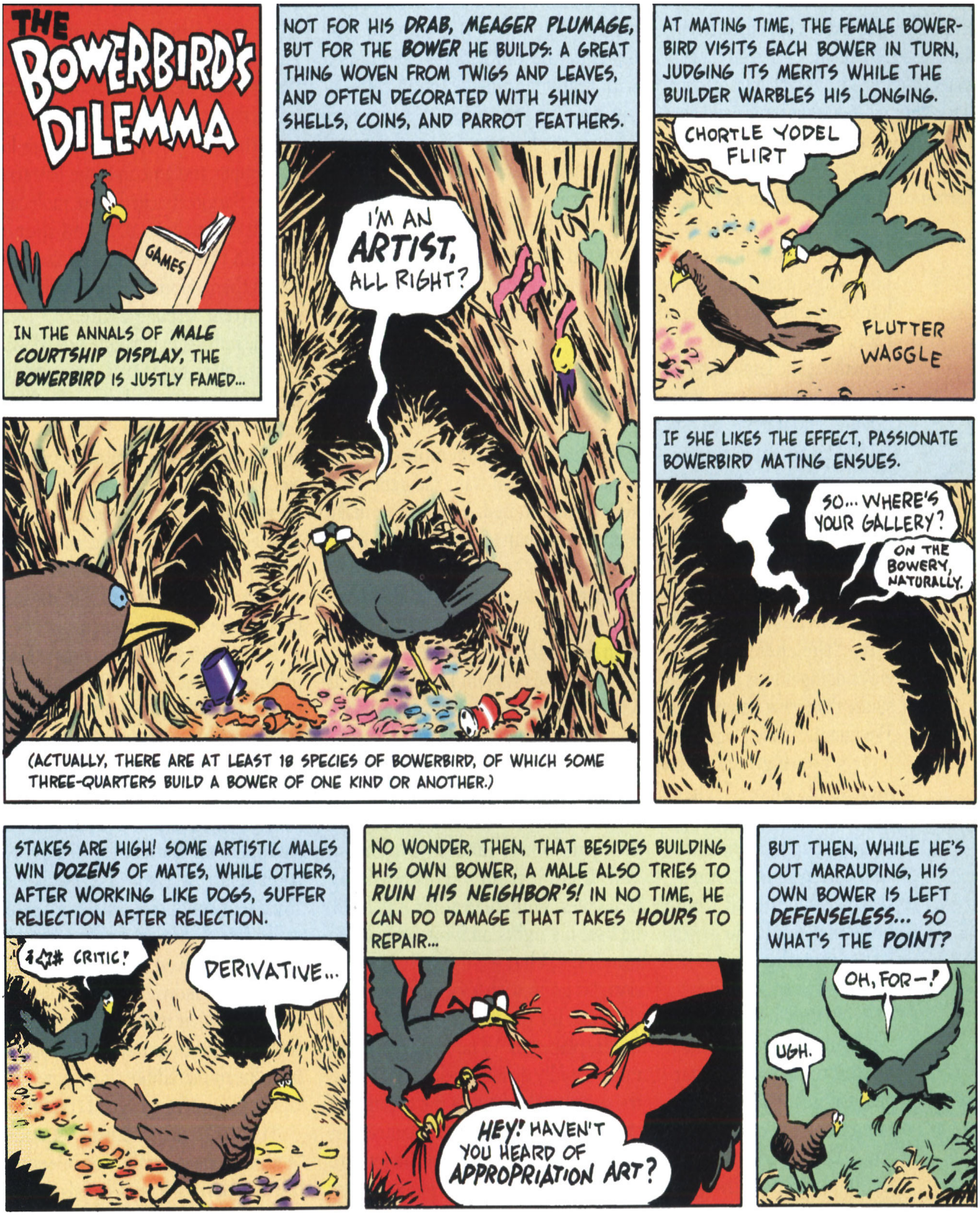
---

* *Author's affiliation*: Max Planck Institute for Tax Law and Public Finance, Munich 80539, GERMANY; bharat.goel@tax.mpg.de

Acknowledgements: The author is grateful for helpful comments from conference participants at the DSE Winter School 2017 and MPI for Tax Law and Public Finance, Munich.

mating contest has been extensively documented in the biology literature; for instance, see Borgia (1985*a*,*b*) and Doerr (2010).

The clearest description of this phenomenon might, however, be found in the *Science Classics* comic-strip created by Larry Gonick; the strip (some panels of which are presented below) was published in the *Discover* magazine.



Evolutionary biologists have attempted to understand the bower-birds' mate-selection process by studying it as an evolutionary game; see, for instance, the articles by Pruett-Jones and Pruett-Jones (1994) and Pruett-Jones and Heifetz (2012). However, most of the research in evolutionary biology has focused on the game of *mutual destruction of rival bowers* by male-birds of symmetric strength; this research has not studied the sequential decisions of male-birds of different abilities regarding their initial efforts in decorating their own bowers and their subsequent attempts to destroy rival bowers.

In contrast, we present a strategic model (in the rational choice paradigm) of the mating contest between two male bower-birds of asymmetric strengths, where the male-birds first decide on the effort to be invested in own bower decoration, and then decide on whether to defend own bower from rival attack (in order to preserve ones' decorations to the extent possible) or to attack rival bower (in order to destroy some of the rival's decorations). The single female-bird in our model follows the rule of choosing that male-bird who possesses the best surviving bower as her mate (given that she cannot directly observe the inherent strengths of the male-birds). Our contest model is thus one of competitive signaling between two differentially-strong male-birds, where each male-bird's (value-less) signal is the extent of his bower decorations and where each such signal is subject to subsequent sabotage.

Apart from throwing light on the logic of the bower-birds' mate-selection process, our analysis contributes to the growing literature on 'contests with sabotage'; see Konrad (2000), Chen (2003), and Gurtler and Munster (2010) for some early papers, and Chowdhury and Gurtler (2015) for a survey. We complement the extant literature by modeling a scenario (that is motivated by the biologists' description of the bower-birds' interactions) where:
(*a*) sabotage occurs only after each contestant has observed his rival's (intermediate) signal,
(*b*) sabotage involves a *discrete choice* of attacking or not attacking a rival, and
(*c*) each contestant has the opportunity to defend against rival attack.

Given the above-described sabotage possibilities, we address the following questions:

(1) Who gets sabotaged, and what is the equilibrium impact of sabotage activities?
(2) How do sabotage possibilities affect the initial signaling efforts of the contestants?
(3) What are the implications of *ex ante* signaling and *ex post* sabotage on the female-bird's ability to correctly identify the stronger male-bird?

We present our detailed results after our formal analysis of the bower-birds' mating contest (see Section 2.2). It is, however, worth stating our central conclusion up front: The possibility of sabotage (as compared to a scenario where the female-bird can select her mate directly on the basis of the male-birds' signals) can improve the welfare of *both* the male-birds – by reducing their *dissipative signaling efforts,* while concurrently worsening the expected utility of the female-bird – by making her selection process *more imprecise*.

Next, we turn our attention to a principal-agent model that has some *structural similarities* with the bowerbirds' mating contest. Specifically, we study the following 'output-selection problem' in a model with a single principal and two potential agents. We consider a scenario where a software developer (the principal) has to decide on an incentive contract / tournament to get a programmer to create a code for her. As the developer cares about the final surviving (i.e., bug-free) quality of the code, she has to engage a hacker to hack every code created by any programmer to determine its final surviving quality. [Note that the agency problem that we study applies to many situations where an agent's output has to be *stress-tested* to determine its *durability*.] As a code's final quality is not precisely verifiable by the courts (only specific quality rankings are verifiable), the principal has to consider the efficacy of holding a tournament between two programmers (the agents) to incentivize them to create the best codes possible (see Section 3.1 for the details of the model). In our model, the principal's payoff is posited to depend only on the final quality (i.e., durability) of the best surviving code; thus our model is closely related to a 'quality contest' studied by Serena (2017). The main issues that we study in this context are the following:

(i) Given that the developer cannot observe the realized efficiency of any programmer, when should she organize a tournament between two programmers in order to maximize the *ex ante expected quality* of the best surviving code?

(ii) When holding a tournament is optimal, should the developer structure the tournament as a 'simultaneous contest' (where each programmer has to create his code simultaneously), or as a 'sequential contest' (where one randomly-chosen programmer creates his code first, and then the other programmer creates his code after observing the first output)? [1]

We show that the analytics of this principal-agent problem is similar to that of the bower-birds' mate-selection game, even though the underlying stories / motivations are quite different. Specifically, the hacking (i.e., stress-testing) instigated by the developer in order to determine the best surviving code is analytically similar to the sabotage process among the male bower-birds; however, the developer's motivation to hack is entirely distinct from the sabotage incentives of the male-birds. [The similarities and dissimilarities of the bowerbirds' contest and the principal-agent interaction are further discussed in Section 3.] Our analysis of the output-selection problem proves that when the agents are not too '*ex ante* dissimilar' in

---

[1] For other studies of 'simultaneous *vs.* sequential contests', see Dixit (1987), Morgan (2003), and Serena (2017).

their efficiency levels, the principal's optimal mechanism is a *simultaneous-move tournament* between the agents, with the maximal feasible winning prize.

The rest of this paper is organized as follows. Section 2 presents the bower-birds' mate-selection game, and its equilibrium analysis. The principal-agent problem and its analysis are presented in Section 3. Section 4 contains some concluding remarks. An appendix contains the formal proofs of our results.

2. The Bowerbird's Mating Contest

A female bowerbird wants to select a single mate from among two male bowerbirds 1 and 2. The female-bird follows an exogenously specified *mate-selection rule* – the rule being based on the "decorative quality" of the bowers that the male-birds create (see footnote 2 for a justification of this rule). The selection rule induces a bower-decorating contest between the two male-birds, aimed at winning the "prize" of mating with the female-bird. We model this contest as follows.

The male-birds, $i = 1, 2$, differ in their *intrinsic abilities* θ_1 and θ_2. We assume that θ_1 and θ_2 are realizations of two independent and identical random variables, each with a continuous distribution function $H(.)$ over an interval $[\underline{\theta}, \bar{\theta}]$ with $0 < \underline{\theta} < \bar{\theta} < \infty$. In what follows, θ_- will refer to the lower realized ability parameter and θ_+ ($\geq \theta_-$) to the higher realized ability parameter; note that θ_1 and θ_2 will be distinct with probability one. We assume that while the female-bird cannot observe the realized θ_1 and θ_2, they are common knowledge between the male-birds. These parameters affect the male-birds' welfare in the following ways.

Each male-bird $i = 1, 2$ has $T > 0$ units of time to spend over two activities – foraging, and creating a bower of (decorative) quality $a_i \geq 0$. We will sometimes refer to bird i's bower as bower i. If male-bird i spends an amount of time $t_i \in [0, T]$ on foraging, then he earns a flow payoff $\{\phi(\theta_i).t\}$ from foraging, where $\phi(\theta_i) > 0$ represents i's *foraging effectiveness* and is assumed to be strictly increasing in ability. Further, in order to create a bower of quality $a_i \geq 0$, bird i requires the amount of time $[\tau(\theta_i).(a_i)^n]$, where the 'cost convexity parameter' n is no less than 1, and $\tau(\theta_i) > 0$ represents bird i's *time-efficiency* in bower-decoration (with a lower value of $\tau(.)$ representing greater efficiency). Note that if male-bird i with ability θ_i creates a bower of quality a_i, his foraging payoff will be $\{\phi(\theta_i).[T - \tau(\theta_i).(a_i)^n]\}$. We define male-bird i's *cost-efficiency* in bower-creation to be $c(\theta_i) \equiv [\phi(\theta_i)\times\tau(\theta_i)] > 0$ (with a lower value of $c(.)$ representing greater efficiency), and assume that $c(.)$ is strictly decreasing in ability (i.e., we assume that $\tau'(\theta) / \tau(\theta) < - \phi'(\theta) / \phi(\theta)$ for all θ). A higher-ability male-bird is thus more effective both in foraging and in bower-creation, and so we will refer to the higher-

θ male-bird as being the *stronger* one.

2.1 Alternative Mating Contests

We now define two mate-selection games for the bower-birds – Game *NS* and Game *S*. The former game does not permit sabotage – i.e., the opportunity of a male-bird to destroy (a part of) his rival's bower decorations, while the latter game does permit sabotage.

Game *NS* : The two male-birds 1 and 2 simultaneously choose their bower qualities a_1 and a_2 respectively (each male-bird is permitted to employ a *mixed strategy* and randomize over different quality choices). Given the realized bower qualities a_1 and a_2, each male-bird i spends $[\tau(\theta_i).a_i^n]$ amount of time decorating his bower, and $[T - \tau(\theta_i).a_i^n]$ amount of time foraging. Then, at the end of the time interval $[0, T]$, the female-bird enters the picture and executes the following mate-selection rule: She observes the realized bower qualities a_1 and a_2 and selects male-bird 1 (respectively, 2) as her mate if $a_1 > a_2$ (respectively, $a_1 < a_2$); if $a_1 = a_2$ then the female-bird selects either male-bird to be her mate with equal probability.[2]

Each male bird i is risk-neutral, and values winning the prize of mating with the female-bird at $W > 0$. As a result, if male-bird i creates a bower of quality a_i and then gets selected to be the female-bird's mate, then he obtains total utility of $\{W + \phi(\theta_i).T - c(\theta_i).a_i^n\}$, while if he does not get selected then he obtains total utility of $\{\phi(\theta_i).T - c(\theta_i).a_i^n\}$.[3] We assume that T is sufficiently large so that each male bird always spends a strictly positive amount of time on foraging; to be specific, we assume that $T > W / \phi(\underline{\theta})$.

The female bird F is also risk-neutral, and obtains utility of $V(\theta_i) > 0$ when she mates with a male bird i having ability parameter θ_i; we assume that $V(.)$ is strictly increasing in its argument. Recognize that this utility specification makes bower-decorating a completely dissipative activity, as neither the male-birds nor the female-bird get any direct utility from it. Thus, the bowerbird's mate-selection game is a game of competitive signaling *via* value-less

---

[2] We take the female-bird's selection rule as exogenously given, and do not study the case where she can pre-announce a strategically chosen selection rule. In this context, note the following point: Given that the female-bird prefers to select the stronger male-bird (see the female-bird's utility specification given subsequently), it is likely that an optimally-chosen selection rule by the female-bird will induce higher bower-creation effort by the stronger male-bird. In that case, it will be *sequentially optimal* for the female-bird to choose her mate according to the selection rule stated above. So, it is very likely that the stated mate-selection rule by the female-bird is optimal when she lacks commitment.

[3] We could incorporate a 'loser's prize' of $L < W$. We set $L = 0$ as that neither affects the strategic interaction between the male-birds, nor equilibrium characterization results in any substantive way.

signals.

Game *S* : As in Game *NS*, each male-bird i chooses his bower quality a_i and spends $[\tau(\theta_i).a_i^n]$ amount of time in bower-creation and $[T - \tau(\theta_i).a_i^n]$ amount of time in foraging. At the end of the time interval $[0, T]$, there is an additional time interval $[T, T + \Delta]$ (where $\Delta > 0$) in which each male-bird (after observing a_1 and a_2) chooses either to defend his own bower or to attack the rival bower. If a bower a_i is not attacked, it retains its intermediate quality as its final quality: $f_i = a_i$. When a bower a_i is attacked, its final quality f_i is the realization of a random variable whose distribution depends on whether the bower is defended or not: if the bower is defended then its final quality f_i is uniformly distributed on $[0, a_i]$, but if it is not defended then its final quality f_i is uniformly distributed on $[0, \delta.a_i]$, where the 'destruction parameter' $\delta \in (0, 1)$ represents the additional loss from leaving an attacked bower undefended (a smaller value of δ denotes a greater loss).[4] We assume that if a male-bird is indifferent between defending his bower and attacking rival bower, then he chooses to do the former.[5]

At the end of the time interval $[T, T + \Delta]$, the female-bird enters the picture and executes the same mate-selection rule as specified in Game *NS*, except that now she selects on the basis of the final bower qualities f_1 and f_2. The payoffs in the different contingencies to all the players are also identical to that in Game *NS* : if male-bird i creates a bower of quality a_i and then gets selected to be the female-bird's mate, then he obtains total utility $[W + \phi(\theta_i).T - c(\theta_i).a_i^n]$, while if he does not get selected then he obtains total utility $[\phi(\theta_i).T - c(\theta_i).a_i^n]$.[6] The female-bird obtains utility of $V(\theta_i) > 0$ when she mates with a male bird i having ability parameter θ_i; here, in keeping with the discussion in the evolutionary biology literature, we assume that the female-bird does not directly get utility from her mate's bower quality.

---

[4] There are other ways of modeling the benefit derived from defending a bower that will leave our main results unchanged (as long as the benefit from defending is not too high). For example, given $\delta \in (0, 1)$, we could posit that the final quality of a defended bower of intermediate quality a_i, when attacked, is: $f_i = a_i$ with probability $\chi \in [0, 0.5]$, while f_i is uniformly distributed on $[0, \delta.a_i]$ with probability $[1 - \chi]$.

[5] This assumption helps us pin down a unique continuation equilibrium in the sabotage subgame. However, in this bilateral contest, the continuation equilibrium win-probabilities of the two male-birds are uniquely determined even without this assumption (and our main results remain unaffected).

[6] We do not consider any disutility for the male-birds from attacking/defending bowers in the time interval $[T, T + \Delta]$. Assuming small fixed costs for these activities will not change our results.

2.2 Equilibrium Mating Outcomes

In this section, we characterize the equilibrium outcomes of the alternative mate-selection games, and delineate their comparative statics properties.

Game *NS* : Recognize that Game *NS* is essentially a first-price all-pay auction between the two male-birds under complete information. As is well understood from the auction literature, this game has a unique Nash equilibrium in which the contestants play mixed strategies.

PROPOSITION 1. In the unique Nash equilibrium of Game *NS*, when the realized ability parameters of the male-birds are $\{\theta_+, \theta_-\}$, they play as follows: Male-bird θ_+ chooses his bower quality according to the cumulative distribution function (*CDF*): $G_+^*(a_+) = \{[c(\theta_-)].(a_+)^n/(W)\}$ for $a_+ \in [0, (W/c(\theta_-))^{1/n}]$, while male-bird θ_- chooses his bower quality according to the *CDF*: $G_-^*(a_-) = \{1 - [c(\theta_+)/c(\theta_-)]\} + \{[c(\theta_+)](a_-)^n/(W)\}$ for $a_- \in [0, (W/c(\theta_-))^{1/n}]$. Resultantly, the male birds' equilibrium win-probabilities are: $P^*(\theta_+) = \{1 - 0.5[c(\theta_+)/c(\theta_-)]\}$ and $P^*(\theta_-) = [1 - P^*(\theta_+)]$; while their equilibrium payoffs are: $U^*(\theta_+) = \{1 - [c(\theta_+)/c(\theta_-)]\}.W + \phi(\theta_+).T$, and $U^*(\theta_-) = \phi(\theta_-).T$.

Proposition 1 establishes that even though the signaling contest embedded in Game *NS* generates value-less but costly signals, the contest is *not wholly dissipative* for the male-birds in the following sense: neither male-bird would be strictly better off if the possibility of mating with the female-bird was entirely eliminated. In fact, in terms of *ex ante* expected utility (i.e., before the ability parameters are realized), each male-bird is strictly better off from being able to participate in the contest because each male-bird has equal chance of being endowed with higher ability. For some ability level θ_+, the stronger male-bird's *ex post* expected utility (post realization of abilities) increases in *cost inequality* – as measured by the ratio $[c(\theta_-)/c(\theta_+)] \geq 1$ (a higher ratio implying higher cost inequality) precisely because his win-probability increases in cost inequality. In contrast, the *ex post* utility of the weaker male-bird depends only on his absolute ability level. [7]

As far as the female-bird is concerned, the randomized strategies pursued in equilibrium by the male-birds prevent her from being always *ex post correct* in identifying the stronger male-bird. Recognize that the magnitude of $P^*(\theta_+)$ is an obvious measure of *precision in selection*

---

[7] For the weaker male-bird with a given ability level θ_-, an increase in the ability of the rival male-bird dampens his effort choice for bower decoration *and* his win-probability in a way that the net effect is zero and his *ex post* expected utility remains $\phi(\theta_-).T$.

by the female bird, and the fact that $P^*(\theta_+) > 0.5$ implies that the female-bird's chance of being *ex post* correct in her selection is always strictly greater than her chance of being *ex post* incorrect. Further, $P^*(\theta_+)$ rises in cost inequality. This implies the following: Consider two distinct realized ability vectors (θ'_+, θ_-) and (θ''_+, θ_-) with $\theta'_+ > \theta''_+$. Then the female-bird's expected utility under the former realization $\{P^*(\theta'_+).V(\theta'_+) + P^*(\theta_-).V(\theta_-)\}$ will be strictly greater than her expected utility under the latter realization $\{P^*(\theta''_+).V(\theta''_+) + P^*(\theta_-).V(\theta_-)\}$ for two reasons – her likelihood of being correct in her selection will be higher in the former case, and when she is correct in selection she will get a higher *ex post* payoff.

Finally, note that the magnitude of the destruction parameter δ, and that of the cost convexity parameter n, have no impact on the equilibrium win-probabilities or on any player's equilibrium payoffs in Game *NS*. Next, we turn to the mate-selection game with sabotage (Game *S*), where the former property will continue to hold in equilibrium, but the latter property will not.

Game *S* : In this game, we restrict attention to 'subgame-perfect Nash equilibria in pure strategies (*P-SPNE*)'. As we will show below, this is not a restriction in the sabotage subgame, since there exists a unique continuation Nash equilibrium in that subgame that is in pure strategies. In contrast, while there exists a unique *P-SPNE* in the overall game, it is an open question whether there exist other *SPNE*s in the overall game in which the male-birds randomize in their bower quality selection (our intuition suggests that there do not).

Our first result for Game *S* relates to the sabotage subgame between the two male-birds during the time interval $[T, T + \Delta]$. This subgame starts after the intermediate bower qualities of the two-male-birds $\{a_1, a_2\}$ are realized at the end of the time interval $[0, T]$.

LEMMA 2. In the unique Nash equilibrium of the sabotage subgame, each male-bird attacks his rival's bower as long as the realized intermediate bower qualities a_1 and a_2 are strictly positive. If at least one of the realized bower qualities is zero, each male-bird defends his own bower.

In our model, the realized abilities of the two male-birds can be very different, and as a result there can be significant difference in their chosen bower qualities. However, in our posited environment where sabotage possibilities arise only after intermediate bower qualities are revealed, and where sabotage consists of a {0-1} attack/defense choice, Lemma 2 establishes that whenever both bower qualities are positive, each male-bird will attack the other irrespective of the inequality in the bower qualities. Given the unique Nash equilibrium in the sabotage subgame, we are able to completely characterize the unique *P-SPNE* in the overall Game *S*.

PROPOSITION 3. There exists a unique *P-SPNE* in Game *S*. In this equilibrium, when the realized ability parameters of the male-birds are $\{\theta_+, \theta_-\}$, they play as follows: Male-bird θ_+ chooses his bower quality to be $a^*(\theta_+) = [(0.5W/n)]^{1/n}.[1/(c(\theta_-).c(\theta_+)^{n-1})]^{1/(n_{\times}n)}$, while male-bird θ_- chooses $a^*(\theta_-) = [(0.5W/n)]^{1/n}.[\, c(\theta_+)/c(\theta_-)^{n+1}]^{1/(n_{\times}n)}$. The equilibrium win-probabilities of male-birds θ_+ and θ_- are $P^*(\theta_+) = \{1 - 0.5[c(\theta_+)/c(\theta_-)]^{1/n}\} > 0.5$ and $P^*(\theta_-) = [1 - P^*(\theta_+)]$, and their equilibrium payoffs are $U^*(\theta_+) = \{1 - 0.5[(n+1)/n].\, [c(\theta_+)/c(\theta_-)]^{1/n}\}.W + \phi(\theta_+).T$, and $U^*(\theta_-) = \{0.5[(n-1)/n].\, [c(\theta_+)/c(\theta_-)]^{1/n}\}.W + \phi(\theta_-)T$.

In what follows, we present a set of remarks delineating the properties of the unique *P-SPNE* outcome in the bower-birds' mate-selection game with sabotage possibilities, and comparing these properties with those of the unique Nash equilibrium in the game without sabotage.

Remark 1. The equilibrium outcomes – win-probabilities and each players' payoffs – are identical for Game *NS* with *any* convex structure of signaling effort costs (i.e., for any $n \geq 1$) and for Game *S* with *linear* signaling effort costs ($n = 1$). This result is to be understood as follows. In the all-pay auction in Game *NS*, each male-bird wants to 'bid a little more' against any 'small enough' deterministic bid by the rival. This forces the contestants to randomize in their bids in equilibrium, and while the mixed strategy distribution of each player does depend on cost convexity (n), the 'expected outcome' does not. In contrast, the randomness in final payoffs induced by the equilibrium sabotage strategies in Game *S* permits a unique set of pure-strategies to constitute a signaling equilibrium in the first stage. When signaling costs are linear, the equilibrium signaling and attack/defense choices in Game *S* generate the same outcome as when sabotage is not permitted.[8]

Remark 2. In the unique *P-SPNE* in Game *S*, the two male-birds signal in such a way that the individual signaling costs are equalized between them: for any realized $\{\theta_+, \theta_-\}$ with $\theta_+ > \theta_-$, $a^*(\theta_+) > a^*(\theta_-)$ such that $c(\theta_+).[a^*(\theta_+)]^n = c(\theta_-).[a^*(\theta_-)]^n$. This implies that for a given structure of signaling cost convexity (n), the male-birds' bower quality asymmetry $[a^*(\theta_+)/a^*(\theta_-)]$ increases in the realized cost inequality $[c(\theta_-)/c(\theta_+)]$, while for a given level of realized cost inequality, bower quality asymmetry decreases in signaling cost convexity.

Remark 3. In the unique *P-SPNE* in Game *S*, the male-birds signal in such a way that for any

---

[8] For an analogous result, see Hart (2016), especially his Theorem 6, for the equivalence between an all-pay auction and an "expenditure game followed by a *General Lotto* game" when bid-costs are linear in bids.

realized $\{\theta_+, \theta_-\}$, the *total* equilibrium signaling costs $\{c(\theta_+).[a^*(\theta_+)]^n + c(\theta_-).[a^*(\theta_-)]^n\}$ equal $\{[W/n].[\, c(\theta_+)/c(\theta_-)]^{1/n}]\}$, and are thus affected by the realized cost inequality and the structure of signaling cost convexity in the following ways: Given n, the total signaling costs fall with an increase in cost inequality $[c(\theta_-)/c(\theta_+)]$. Further, as long as realized cost inequality is 'small' so that $[c(\theta_-)\,/c(\theta_+)] \in [1, e]$, the total signaling costs fall with an increase in n (see the proof of Proposition 3).[9]

Remark 4. For any realized $\{\theta_+, \theta_-\}$ in the unique *P-SPNE* in Game *S*, the win-probability of the 'stronger' and the 'weaker' birds – $P^*(\theta_+)$ and $P^*(\theta_-)$ – change with the realized cost inequality and the structure of signaling cost convexity in the following ways: given n, $P^*(\theta_+)$ increases and $P^*(\theta_-)$ decreases in cost inequality $[c(\theta_-)/c(\theta_+)]$; and given cost inequality, $P^*(\theta_+)$ decreases and $P^*(\theta_-)$ increases in n. The win-probabilities are unaffected by the 'prize value' W and the magnitude of the destruction parameter δ.[10]

Remark 5. In the unique *P-SPNE* in Game *S*, for any $n \geq 1$, the equilibrium outcome is "not wholly dissipative" for the male-birds in exactly the same sense as in the unique equilibrium outcome of Game *NS*. Further, the following results hold for the equilibrium payoffs to the two male-birds in Game *S*: (i) given n, $U^*(\theta_+)$ increases and $U^*(\theta_-)$ decreases in cost inequality; (ii) for any cost inequality $[c(\theta_-)/c(\theta_+)]$, $U^*(\theta_-)$ strictly increases in n; and (iii) as long as cost inequality is 'close' to 1 such that $[c(\theta_-)/c(\theta_+)] \in [1, e^{0.5}]$, $U^*(\theta_+)$ strictly increases in n (see the proof of Proposition 3).[11] The impact of signaling cost convexity on the equilibrium payoffs of the male-birds is to be understood as follows. As n increases, 'incremental increase' in signaling becomes more costly for each male-bird. Given the expected loss of value from subsequent sabotage, (as long as cost inequality is low) this depresses the male-birds' signaling efforts in a way that reduces bower quality asymmetry (see Remarks 2 and 3) and thus the *gap* between the win and loss probabilities. All this is good news for the weaker male-bird, whose equilibrium payoff increases unambiguously in n. On the other hand, there are conflicting impacts of an increase in n on the welfare of the stronger male-bird – while he saves on signaling costs his win-probability goes down. The

---

[9] Here, e is the e is the base of the *natural* logarithm. For $[c(\theta_-)/c(\theta_+)] > e$, the total signaling costs increases in n for $n \in (1, log[c(\theta_-)/c(\theta_+)])$ and decreases in n for $n > log[c(\theta_-)/c(\theta_+)]$.

[10] Note that in the in the unique *P-SPNE* in Game *S*, all outcomes are independent of δ. On the other hand, an increase in the prize value W improves the equilibrium welfare of both male-birds in spite of inducing an increase in the intermediate signals and thus an increase in individual signaling costs.

[11] For $[c(\theta_-)/c(\theta_+)] \in (e^{0.5}, e)$, $U^*(\theta_+)$ falls in n for $n \in (1, log(c(\theta_-)/c(\theta_+))/[1 - log(c(\theta_-)/c(\theta_+))])$ and increases in n for $n > log(c(\theta_-)/c(\theta_+))/[1 - log(c(\theta_-)/c(\theta_+))]$. For $[c(\theta_-)/c(\theta_+)] \geq e$, $U^*(\theta_+)$ falls in n.

net effect is positive for the stronger male-bird as long as the cost inequality between the contestants is not too great.

Remark 6. In the unique *P-SPNE* in Game *S*, the following results hold regarding the 'selection precision' of the female-bird – as measured by $P^*(\theta_+)$, and her expected payoff given the realized abilities of the male-birds $\{\theta_+, \theta_-\}$. As in Game *S*, it is also true in Game *NS* that since $P^*(\theta_+) > 0.5$, the female-bird's chance of correct selection *ex post* is always greater than her chance of being incorrect *ex post*. Further, as $P^*(\theta_+)$ rises in cost inequality, the female-bird's expected utility under the ability realization (θ'_+, θ_-) is greater than that under the ability realization (θ''_+, θ_-) whenever $\theta'_+ > \theta''_+$. However, for any ability realization (θ_+, θ_-), an increase in n reduces the expected equilibrium payoff of the female-bird in Game *S*. This is in contrast to game *NS* (where n does not affect the female-bird's equilibrium payoff), and occurs due to the following reason. An increase in n raises incremental signaling costs and reduces bower-quality asymmetry, impacting adversely on the female-bird's selection precision by lowering $P^*(\theta_+)$.

We conclude our analysis of the bowerbirds' mate-selection game by re-emphasizing the role that sabotage plays in affecting selection precision and the birds' welfare. Sabotage possibilities make the selection process more *imprecise*, and this has the following asymmetric impacts on the male and the female birds for every set of male-birds' ability realizations: the female-bird is unambiguously worse off while the weaker male-bird is unambiguously better off; further, when there is limited ability asymmetry between the two male-birds, the stronger male-bird is also better off; when the signaling cost structure becomes more convex, these impacts are reinforced. Recognize that these results depend critically on the posited payoff structures of the bowerbirds. As mentioned before, we follow the evolutionary biology literature in positing that the female-bird's utility depends only on her chosen mate's efficiency. To understand the implications of this assumption, consider the polar opposite case where her utility is assumed to depend only on her mate's final bower quality. When it is the case that $V(.)$ depends only on f_i, it can be proved that the female-bird will obtain a higher payoff in Game *S* than in *NS* in the following scenario: $V(.)$ is highly concave in f_i, and the two male-birds are *ex ante* not too asymmetric.[12] This indicates the benefit that a 'principal' can obtain from a 'second stage' after initial output-production, in which stress-testing can reveal some *hidden quality / durability* of the output. In the agency model studied in the next section, where the principal's utility is posited to depend *only* on

---

[12] To see this, note that when both male-birds are identical and $V(.)$ is linear in f_i, then *S* and *NS* generate exactly the same expected utility for the female-bird.

the final output quality of her selected agent, the hacking in the second stage to find the durability of the produced output(s) will be essential for meeting the principal's goal.

3. Output-Selection Tournament in a Principal-Agent Model

In this section, we consider the problem regarding the design of an output-selection mechanism by a principal. As the following description will clarify, the induced game among two agents has *structural similarities* with the male-birds' contest in the bower-birds' mate-selection game; the similarities (and dissimilarities) will be clarified below.

3.1 The Principal-Agent Model

A software-developer (the principal) needs a customized code to be developed by a specialist programmer. The developer's utility depends on the final surviving quality of the code. In order to uncover this attribute of a code, the developer has to take any 'intermediate code' as written by a programmer and then get her 'hacker' to hack it. The code quality that survives after the hacking process represents the final (bug-free) quality of the code. The *hacking* considered in this model is analytically equivalent to *stress-testing* a product to determine its durability.

The basic problem faced by the developer is that while she can observe the final quality of a code, this quality is not third-party verifiable. This precludes the developer from hiring a programmer and offering him a compensation contract conditioned on the final quality of the code. Instead, the developer can only take advantage of the fact that the courts can verify two kinds of 'ranking outcomes': (*a*) the courts can verify whether the final surviving quality of a code is above or below an exogenous quality threshold, and (*b*) given two surviving codes to compare, the courts can correctly identify which code has the better final quality.

There exist two specialist programmers (agents) who can do the coding, but by incurring different costs. Specifically, each programmer i, for $i = 1, 2$, incurs a cost of $[c_i.a_i]$ to produce a code of intermediate quality a_i. When such a code is hacked (i.e., stress-tested) its final bug-free (i.e., durable) quality f_i will be the realization of a random variable that is uniformly distributed on $[0, a_i]$. The *unit effort cost* c_i of each programmer i is drawn independently from an identical distribution $H(.)$ over an interval $[\underline{c}, \bar{c}\,]$ with $0 < \underline{c} < \bar{c} < \infty$. While the developer is only aware of this fact (and does not get to observe the agents' realized unit costs), the realized c_1 and c_2 are common knowledge between the programmers (and *this* fact is also known by the developer). In what follows, c_- (resp., $c_+ \geq c_-$) will refer to the lower (resp., higher) realized unit cost; note that the realizations c_1 and c_2 will be distinct with probability one.

In this environment, the developer – who has a budget of B – can choose between a set of distinct *output selection schemes*, where a subset of the schemes will involve a 'tournament' between the two agents. We will denote this set of output-selection schemes to be **K**, and posit that **K** contains three distinct schemes – K[1], K[2.1], and K[2.2] – that are described below (note that K[2.1] and K[2.2] are specific tournament schemes).

- Under scheme K[1], the developer has to hire a randomly-selected programmer and offer him a contract K_1: "After you produce the intermediate code I will have it hacked, and if the final quality is above a (court-verifiable) threshold f_0 (> 0) I will pay you $W \in (0, B]$, otherwise I will pay you $L \in [0, W)$."
- Under schemes K[2.1] and K[2.2], the developer has to hire both programmers 1 and 2 and offer each of them (publicly) the following contract K_2: "After you produce the intermediate code I will have it hacked, and if your final quality is better than the rival quality (this event being court-verifiable) I will pay you $W > 0$, otherwise I will pay you $L \in [0, W)$; if the final surviving qualities are identical then I will pay you W and your rival L with probability ½, and *vice-versa* with probability ½."[13, 14] In each scheme, it must be that $W + L \leq B$.
- The distinction between the schemes K[2.1] and K[2.2] is as follows: Under the former scheme, the developer has to ask both programmers to code simultaneously, and then hack both the codes; while under the latter scheme, the developer has to pick one programmer to write his code first, then require the other programmer to write his code after observing the intermediate quality of the first code, and then finally hack both the codes.

As the following analysis will demonstrate, in this principal-agent model, hacking (i.e., stress-testing) of the codes created by the programmers corresponds to the sabotage of rival bowers by the male bower-birds in their mating contest. But, of course, while the hacking of both codes is similar to mutual sabotage in its analytics, it is not similar in its intent; in the principal-agent model, the hacking is conducted by the principal in order to determine the durability of the codes (which is something that he directly cares about). The two models are

---

[13] Note that given the model specification, a tie in final qualities will be a zero probability event.

[14] A more sophisticated version of the tournament schemes {K[2.1], K[2.2]} can include the following additional condition imposed by the principal on each agent: "After you produce the intermediate code, I will have it hacked, and if your final quality is better than the rival quality as well as the exogenous threshold f_0 (this event being court-verifiable) I will pay you $W > 0$, otherwise I will pay you $L \in [0, W)$." Our central result that a "tournament is better" – i.e., that scheme K[1] is *dominated* either by K[2.1] or K[2.2] – when agent-asymmetry is low (see Proposition 4) will hold even for this generalization; see footnote 17.

structurally similar in the sense that 'individual effort incentives' are similar in the following sub-models – in game S for the male bower-birds, and under contract K[2.1] (and, to some extent, under contract K[2.2]) for the two programmers.

The 'outside options' of the two programmers and of the developer are (normalized to) zero. Under any contract in **K**, if programmer i produces a code of intermediate quality a_i and receives remuneration R from the developer, his payoff will be $\{R - [c_i.a_i]\}$. In contrast, the developer's payoff is assumed to be a function only of the quality of the best surviving code. Specifically, if the developer employs selection scheme K[1] and the final surviving code quality is f_i, the developer's payoff will be $V(f_i) = f_i$; while if the developer employs either selection scheme K[2.1] or K[2.2] and the final surviving code qualities are f_1 and f_2 , the developer's payoff will be $V(f_1, f_2) = max\{f_1, f_2\}$.[15]

3.2 The Optimal Output-Selection Mechanism

We begin by stating the optimal selection mechanism for the software-developer (the principal), when the two programmers (the agents) are not very asymmetric *ex ante*. We then present a set of results that prove the optimality of the stated mechanism. Recall that the unit effort cost c_i of each programmer i is drawn independently and identically from $[\underline{c}, \bar{c}]$ with $0 < \underline{c} < \bar{c} < \infty$.

PROPOSITION 4. There exists $k > 1$ such that whenever $(\bar{c} / \underline{c}) < k$, the following result holds: {Selection scheme K[2.1] with $W^* = B$ and $L^* = 0$} is the optimal scheme in the set **K**.[16, 17]

The above proposition asserts the optimality of the *simultaneous-move tournament with*

---

[15] As indicated at the beginning of Section 3.1, the principal's utility is posited to depend only on the agents' final outputs, and not on their inherent efficiency levels. Also note that our payoff specifications incorporate the following assumptions: (i) the developer incurs no costs of hacking, and (ii) he gets no additional benefit if a part of the budget B is saved. Regarding (i), it is easy to check that introducing a small lump-sum cost of hacking will not change our results. As for (ii), if the benefit from saving a part X of the budget is $\eta.X$, our results will hold as long as η is 'sufficiently small'.

[16] This result is valid under the assumption that in any induced contest, the agents play a *P-SPNE*.

[17] Consider the more sophisticated version the tournament schemes {K[2.1], K[2.2]} described in Footnote 14. If the developer can choose whether to introduce the additional exogenous threshold benchmark in {K[2.1], K[2.2]}, then his expected payoff can only (weakly) improve upon the payoff that he obtains from the scheme-set {K[2.1], K[2.2]} [where the threshold can be thought to be set at 0]. Then, given that K[1] is dominated by K[2.1] *vide* Proposition 4, *some* version of the tournament scheme will still dominate K[1] for the developer when *ex ante* agent-asymmetry is low.

maximal winning prize when the realized efficiencies of the two agents are *a priori* not very dissimilar. The following results prove the veracity of Proposition 4.

LEMMA 5. Consider the case where a randomly-picked programmer i with unit effort cost c_i is hired under scheme K[1]. Then for any compensation vector $\{W \in (0, B], L \in [0, W)\}$ and for any exogenous final quality threshold f_0 (> 0) (that the courts can verify), the developer's expected payoff from the mechanism will be no greater than $[(0.25B) / (c_i)]$.[18]

LEMMA 6. For any realized $\{c_-, c_+\}$ with $c_- \leq c_+$, if the developer employs scheme K[2.1], then in the unique *P-SPNE*, the programmer c_- creates a code of intermediate quality $a_-^* = [0.5(W - L)/c_+]$ while the programmer c_+ creates a code $a_+^* = 0.5(W - L)[(c_-)/(c_+)^2]$. The 'better' programmer c_- wins the tournament with probability $[1 - 0.5(c_-)/(c_+)]$, and the 'worse' programmer c_+ wins with the complementary probability. Consequently, the developer's expected payoff is $\{0.25(W - L).[1 + (c_-/c_+)^2/3] /(c_+)\}$.

LEMMA 7. For any realized $\{c_-, c_+\}$ with $c_- \leq c_+$, if the developer employs scheme K[2.2] and the better programmer c_- gets to move first, then in the unique *P-SPNE*, the programmer c_- creates a code of intermediate quality $a_-^* = \{0.5(W - L)/c_+\}$ while the programmer c_+ creates a code $a_+^* = 0$. The better programmer c_- wins the tournament for sure, and the developer's expected payoff is $\{0.25(W - L) /c_+\}$.

LEMMA 8. For any realized $\{c_-, c_+\}$, if the developer employs scheme K[2.2] and the worse programmer c_+ gets to move first, then the unique *P-SPNE* outcome depends on the magnitude of the cost inequality $[c_+ /c_-]$: (i) if $[c_+ /c_-] < \{1 + 3^{0.5}/2\}$ then the programmer c_+ creates a code $a_+^* = \{0.5(W - L)/c_-\}$ following which the programmer c_- creates a code of intermediate quality $a_-^* = 0$. The worse programmer c_+ wins the tournament for sure, and the developer's expected payoff is $\{0.25(W - L) /c_-\}$; and (ii) if $[c_+ /c_-] > \{1 + 3^{0.5}/2\}$ then the programmer c_+ creates a code $a_+^* = \{(c_-)/(c_+)^2.[W - L]/8\}$ following which the programmer c_- creates a code of intermediate quality $a_-^* = \{0.25(W - L)/c_+\}$. The better programmer c_- wins the tournament with probability $[1 - 0.5(c_-)/(c_+)]$, and the developer's expected payoff is $\{0.125(W - L).[1 + (c_-/c_+)^2/12] / c_+\}$.

Straightforward calculations on the basis of Lemmas 5–8 generate the following results, which in turn immediately prove Proposition 4:

---

[18] Specifically, when $f_0 \leq 0.25B/c_i$, the developer's optimal expected payoff will be $0.5[f_0B/c_i]^{0.5}$, while if $f_0 > 0.25B/c_i$ the developer's payoff will be zero as she cannot induce the programmer to exert any effort for any feasible compensation scheme.

(1) Under *any* selection scheme, it is optimal for the developer to set $W^* = B$ and $L^* = 0$.

(2) Given that, as long as realized $[c_+/c_-]$ is not much greater than unity, the developer's expected utility given $\{c_-, c_+\}$ is maximal under the scheme K[2.1] in the set **K**.

4. Concluding Remarks

This paper focuses on a remarkable competitive interaction in the animal world, and analyzes this behaviour through the lens of 'rational choice theory'. With respect to the mating contests of the satin bowerbirds, evolutionary biologists have developed formal models of the male-birds' competition. But their research has been confined to studying evolutionary games focused on 'bower sabotage' by male-birds of identical strengths; they have not studied the sequential decisions of male-birds of different abilities regarding initial investments in bower-decoration and subsequent attempts in rival-bower-sabotage. In that regard, this paper complements the extant evolutionary biology literature.

In this context, a central finding of the current analysis is the following: Sabotage possibilities improve the welfare of both the male-birds, while lowering the female-bird's utility; the former happens because the anticipated threat of sabotage depresses each male-bird's incentive to engage in costly (and dissipative) signaling, while the latter happens because sabotage inserts *noise* into the female-bird's selection process. This result also complements the existing results in the 'contest theory' literature and incorporates sabotage possibilities; this is achieved in this paper by considering a contest model where sabotage – which is modeled as a {0-1} choice of attacking or not attacking a rival – occurs *after* each contestant has observed his rival's (intermediate) output.

Subsequently, this paper uses the *bowerbirds' dilemma* as an allegory and studies a principal-agent problem with a similar incentive structure for the contestants. In the agency model, a principal cares about the best among the outputs created by two agents, and is in a position to design a tournament with suitably chosen order-of-moves and prizes. In a setting where (*a*) produced outputs need to be *stress-tested* to determine durability (where stress-testing is analytically similar to mutual sabotage, though not in intent) and (*b*) outputs can only be evaluated in relation to other outputs or an exogenous benchmark, our analysis shows that if the agents are not too asymmetric then a tournament is strictly better for the principal than individually contracting with a single agent; further, if no exogenous benchmark is available, then for limited agent-asymmetry, the optimal output-selection mechanism is a *simultaneous tournament* with the maximal feasible winning prize.

Appendix

PROOF OF PROPOSITION 1: Let each of the two male-birds be represented by the index $i = 1$, 2. In the single-stage game, i.e., Game *NS*, between the two male-birds, bird *i*'s choice variable is $a_i \geq 0$. However, a_i^n is a strictly monotonic function of a_i. So, we can transform the game and assume that the choice variable for bird *i* is a_i^n: this transformation does not affect the equilibrium set. The transformed game is now a standard all-pay auction with two bidders, symmetric prizes and asymmetric linear costs of bidding. It is well known that this game has a unique equilibrium; further, it is easily checked that the potential equilibrium provided in the proposition is actually an equilibrium of the game, and hence it is the unique equilibrium of the game. The equilibrium win-probabilities and equilibrium payoffs are then determined by straightforward calculations. ■

PROOF OF LEMMA 2: Let the two male-birds be represented by the indices $i, j = 1, 2$, where $i \neq j$. Now, given bower qualities $a_1 \geq 0$ and $a_2 \geq 0$ produced in Stage 1 of Game *S*, we want to establish that there exists a unique continuation equilibrium at Stage 2 of the game. As we have assumed that there are no costs of any actions in Stage 2, each male-bird is simply interested in maximizing own win-probability (however, see footnote 6). Note that if $a_i > a_j > 0$, then the dominant strategy for bird *j* is to sabotage bird *i*'s bower since otherwise *j*'s win-probability will be 0 (which it can strictly improve upon by sabotaging *i*'s bower, irrespective of bird *i*'s action). Given that bird *j* sabotages *i*'s bower, what is bird *i*'s best response? Bird *i*'s win-probability from defending own bower is $[1 - a_j/a_i]$, while that from sabotaging *j*'s bower is strictly higher: $[1 - 0.5a_j/a_i]$. Hence, if $a_i > a_j > 0$ then there is a unique continuation equilibrium in which each male-bird sabotages the rival's bower. It is easy to check that this is also the unique continuation equilibrium when $a_i = a_j > 0$. Finally, when $[a_1 \times a_2] = 0$ then a male-bird's win-probability is the same across all possible action profiles, hence each bird will simply choose to defend own bower in the unique continuation equilibrium. ■

PROOF OF PROPOSITION 3: Let the two male-birds be represented by the indices $i, j = 1, 2$, where $i \neq j$. Let c_i represent bird *i*'s *cost-efficiency* in bower-creation and ϕ_i represent his *foraging effectiveness*. Without any loss of generality, let us assume that bird 1 possesses the higher realized ability and hence we have: $\phi_1 \geq \phi_2$ and $c_1 \leq c_2$. We provide here the details of the proof for the case with $n > 1$; the details for the case with $n = 1$ are almost identical. Having established in Lemma 2 the unique continuation equilibrium at Stage 2 of Game *S*, we obtain the first-stage expected payoff $U_i(a_i, a_j)$ of male-bird *i* as:

$$U_i(a_i, a_j) = \begin{cases} 0.5.W + \phi_i.T & \text{if } a_i = a_j = 0, \\ [1 - 0.5a_j/a_i].W + \phi_i.T - c_i.a_i^n & \text{if } a_i > a_j, \\ [0.5a_i/a_j].W + \phi_i.T - c_i.a_i^n & \text{if } a_i \leq a_j \ (\textit{excluding } a_i = a_j = 0). \end{cases}$$

We will next check whether $U_i(a_i, a_j)$ is concave in a_i:

$\partial U_i(.)/\partial a_i = W.a_j/[2a_i^2] - n.c_i a_i^{n-1}$, if $a_i > a_j$;

$\partial U_i(.)/\partial a_i = W/[2a_j] - n.c_i a_i^{n-1}$, if $a_i \leq a_j$ (*excluding* $a_i = a_j = 0$).

Note that neither $U_i(.)$ nor $\partial U_i(.)/\partial a_i$ is discontinuous at $a_i = a_j > 0$. Now, given $a_j > 0$, it is directly observed that $\partial U_i(.)/\partial a_i$ is decreasing in $a_i \geq 0$ and hence $U_i(.)$ is indeed concave in a_i (given that $a_j > 0$). Now, it is obvious that we cannot have an equilibrium (a_1*, a_2*) with [a_1*×a_2*] = 0, so let us assume that there exists an equilibrium (a_i*, a_j*) such that $a_i^* \geq a_j^* > 0$. Since the payoff function of each of the two male-birds is concave in own bower-building effort (given that the rival's bower-building effort is strictly positive), it must be the case that (a_i*, a_j*) solves the *First Order Conditions* (i.e., the *FOC*s) of the two birds simultaneously. Given the assumption that $a_i^* \geq a_j^* > 0$, the two birds' *FOC*s are given by:

$W.a_j/[2a_i^2] - n.c_i a_i^{n-1} = 0$ [male-bird *i*'s *FOC*],

$W/[2a_i] - n.c_j a_j^{n-1} = 0$ [male-bird *j*'s *FOC*].

Solving the two *FOC*s simultaneously gives: $c_i.(a_i^*)^n = c_j.(a_j^*)^n$. Since we have taken $c_1 \leq c_2$, so $a_1^* \geq a_2^*$ and hence we get that $i = 1$ and $j = 2$. Now, solving for a_1* and a_2* we get:

$a_1^* = [0.5W/n]^{1/n}.[1/(c_2.c_1^{n-1})]^{1/(n_\times n)}$,

$a_2^* = [0.5W/n]^{1/n}.[c_1/c_2^{n+1}]^{1/(n_\times n)}$.

Hence, there is a unique *SPNE* in pure strategies of Game *S* in which the two male-birds exert strictly positive bower-building efforts in the first-stage (provided by the above expressions); then, in the continuation subgame, each male-bird attacks his rival's bower. Straightforward calculations give us the expressions for equilibrium win-probabilities and equilibrium payoffs.

We also include in this proof the following analysis that will help establish the validity of the different remarks that we make following Proposition 3.

Firstly, direct use of the expressions for a_1* and a_2* given in Proposition 3 gives us the *total* equilibrium signaling costs, $R \equiv \{c_1.[a_1^*]^n + c_2.[a_2^*]^n\} = \{[W/n].[c_1/c_2]^{1/n}\}$. Since $[1/n] \leq 1$ (as $n \geq 1$), so R increases in $[c_1/c_2]$ and hence it decreases in $[c_2/c_1] \geq 1$. Now, $\partial R/\partial n = -[W/n^2].[c_1/c_2]^{1/n} + [W/n^3].[c_1/c_2]^{1/n}.ln[c_2/c_1]$. Hence, for $c_2/c_1 \in [1, e]$, where e is the exponential constant, $\partial R/\partial n < 0$ for all $n > 1$. Further, for $c_2/c_1 > e$, $\partial R/\partial n > 0$ for all

$n \in [1, ln[c_2/c_1]]$ and $\partial R/\partial n < 0$ for larger n. These arguments prove the claims made in Remark 3.

Secondly, from the expression for $U^*(\theta_1)$ we obtain: $\partial U^*(\theta_1)/\partial n = [0.5/n^2].[c_1/c_2]^{1/n}.\{1 - [(n+1)/n].ln[c_2/c_1]\}$. Hence, for $c_2/c_1 \in [1, e^{0.5}]$ we have $\partial U^*(\theta_1)/\partial n > 0$ for all $n > 1$; for $c_2/c_1 \geq e$ we have $\partial U^*(\theta_1)/\partial n < 0$ for all $n > 1$. Finally, for $c_2/c_1 \in (e^{0.5}, e)$, we have $\partial U^*(\theta_1)/\partial n < 0$ for $n \in (1, ln[c_2/c_1] / [1 - ln[c_2/c_1]])$ while $\partial U^*(\theta_1)/\partial n > 0$ for $n > ln[c_2/c_1] / [1 - ln[c_2/c_1]]$. These arguments prove the claims made in Remark 5. ■

PROOF OF LEMMA 5: Let $W \in (0, B]$, $L \in [0, W)$ such that $W + L \leq B$, and let the exogenous final quality threshold be $f_0 > 0$. For $a_i < f_0$, the optimal effort by the programmer is $a_i = 0$ giving him a payoff of L. Suppose $a_i \geq f_0$, then the programmer's expected payoff is: $U_i(a_i) = [(a_i - f_0)/a_i].W + [f_0/a_i].L - c_i.a_i = W - [f_0/a_i].(W - L) - c_i.a_i \Rightarrow \partial U_i(a_i)/\partial a_i = [f_0/a_i^2].(W - L) - c_i$. Since $\partial U_i(a_i)/\partial a_i$ is decreasing in a_i, so $U_i(a_i)$ is concave in $a_i \geq f_0$. We want to determine the optimal effort by the programmer from among the efforts that are weakly greater than f_0. Set $\partial U_i(a_i^\#)/\partial a_i = 0 \Rightarrow a_i^\# = [f_0.(W - L)/c_i]^{1/2}$. For $a_i = a_i^\#$ to be the optimal effort by the programmer in the interval $a_i \geq f_0$, we need: $a_i^\# \geq f_0 \Rightarrow f_0 \leq (W - L)/c_i$. For $f_0 > (W - L)/c_i$, the optimal effort for the programmer from among the efforts that are weakly greater than f_0 will be f_0; however, the payoff obtained by the programmer with this effort will be strictly less than L and so he will strictly prefer to exert zero effort over this effort, and hence the developer will obtain a payoff of 0 (if $f_0 > (W - L)/c_i$). Given the way the programmer will behave when facing the contract, it is straightforward that from the developer's viewpoint the optimal L is 0 and the optimal f_0 is some value less than $(W - L)/c_i$.

Henceforth, we take $L = 0$, $W \in (0, B]$ and $f_0 \in (0, W/c_i]$. Given such a contract, $a_i^\# = [f_0.W/c_i]^{1/2}$. Since $U_i(0) = 0$, in order to induce the programmer to exert a strictly positive effort we require that $U_i(a_i^\#) \geq 0 \Rightarrow f_0 \leq 0.25W/c_i$. Since $a_i^\#$ strictly increases in both W and f_0, we get the developer's optimal contract as one with $W = B$ and $f_0 = 0.25B/c_i$. Facing the optimal contract with $W^* = B$ and $f_0^* = 0.25B/c_i$, the effort exerted by the programmer i will be $a_i^* = 0.5B/c_i$. Then, the developer's expected payoff with the optimal contract is $0.25B/c_i$. ■

PROOF OF LEMMA 6: The simultaneous game of coding between the two programmers is analogous to Game S between the two male-birds that we studied in the previous section, with the difference that there is a 'loser's prize' of L here (and effectively we have $T = \infty$), we have taken $n = 1$ here, and mutual sabotage is externally imposed here in the 'second stage' (in the form of hacking by the developer). We immediately get the equilibrium of this game by adapting the solution of the earlier game. Given that the final code quality f_i of each programmer i is uniformly distributed between 0 and a_i^*, the developer's expected payoff (which is equal to the expected value of $max\{f_1, f_2\}$) is obtained through straightforward calculations. ■

PROOF OF LEMMA 7: Let the two programmers be represented by the indices $i, j = 1, 2$, where $i \neq j$. Let c_i represent programmer i's realized *unit effort cost*. Without any loss of generality, let us assume that programmer 1 is the more efficient programmer and hence we have: $c_1 \leq c_2$. In Lemma 7 we are considering the sequential game where the better programmer 1 (with unit effort cost c_1) moves first and deploys effort a_1; after the better programmer deploys his effort, the worse programmer 2 (with unit effort cost c_2) will respond by deploying effort a_2. Let $U_i(a_i, a_j)$ represent programmer i's expected payoff. Then, using the solution of Game S provided in the proof of Proposition 3 (in the previous section) we get:

$$U_i(a_i, a_j) = \begin{cases} 0.5(W-L)+L & \text{if } a_i = a_j = 0, \\ [1-0.5a_j/a_i].(W-L)+L-c_ia_i & \text{if } a_i > a_j, \\ [0.5a_i/a_j].(W-L)+L-c_ia_i & \text{if } a_i \leq a_j \ (\textit{excluding } a_i = a_j = 0). \end{cases}$$

Then, we have:

$$\partial U_i(.)/\partial a_i = (W-L).a_j/[2a_i^2] - c_i, \text{ if } a_i > a_j;$$

$$\partial U_i(.)/\partial a_i = (W-L)/[2a_j] - c_i, \text{ if } a_i \leq a_j \ (\textit{excluding } a_i = a_j = 0).$$

Note that neither $U_i(.)$ nor $\partial U_i(.)/\partial a_i$ is discontinuous at $a_i = a_j > 0$. Taking programmer j to be the first-mover and programmer i to be the second-mover, we can easily solve for the optimal $a_i = a_i^{\mathrm{BR}}(a_j)$:

$$a_i^{\mathrm{BR}}(a_j) = \begin{cases} [0.5(W-L).a_j/c_i]^{1/2} & \text{if } a_j \in (0, 0.5(W-L)/c_i], \\ m \in [0, 0.5(W-L)/c_i] & \text{if } a_j = 0.5(W-L)/c_i, \\ 0 & \text{if } a_j > 0.5(W-L)/c_i. \end{cases}$$

Replacing a_i with $a_i^{\mathrm{BR}}(a_j)$ in $U_j(a_j, a_i)$ gives us $U_j(a_j, a_i^{\mathrm{BR}}(a_j))$. From the interval $a_j \geq 0.5(W-L)/c_i$, j's optimal effort choice is $0.5(W-L)/c_i$, giving him an expected payoff of $(W-L).[1-0.5c_j/c_i]$. For $a_j < 0.5(W-L)/c_i$, we have: $U_j(a_j, a_i^{\mathrm{BR}}(a_j)) = [0.5(W-L).a_j.c_i]^{1/2} + L - c_ja_j$. Finding the optimal $a_j = a_j^{\#} < 0.5(W-L)/c_i$ by straightforward differentiation gives us: $a_j^{\#} = 0.125(W-L).c_i/c_j^2$. Since the better programmer 1 moves first in the sequential game being considered here, so we have $j = 1$ and $i = 2$. Straightforward calculations from here on establish that $a_1^* = 0.5(W-L)/c_2$ is the optimal effort choice for the better programmer 1 for all parameter values, following which the worse programmer 2 deploys zero effort, i.e., $a_2^* = 0$. Then, the developer's expected payoff is simply half the effort exerted by the better programmer, i.e., $0.25(W-L)/c_2$. ■

PROOF OF LEMMA 8: The proof of this result is just like that of Lemma 7 with programmer j = 2 moving first and programmer i = 1 moving second, and we suppress the algebraic details here. ■

References

Borgia, G. (1985 *b*) "Bower Destruction and Sexual Competition in the Satin Bowerbirds," *Behavioral Ecology and Sociobiology* 18, 91–100.

Borgia, G. (1985 *a*) "Bower Quality, Number of Decorations, and Mating Success of Male Satin Bowerbirds: An Experimental Analysis," *Animal Behavior* 33, 266–71.

Chen, K. (2003) "Sabotage in Promotion Tournaments," *Journal of Law, Economics and Organization* 19, 119–40.

Chowdhury, S. and O. Gurtler. (2015) "Sabotage in Contests: A Survey," *Public Choice* 164, 135–55.

Dixit, A. (1987) "Strategic Behavior in Contests," *American Economic Review* 77, 891–98.

Doerr, N. (2010) "Does Decoration Theft Lead to an Honest Relationship between Male Quality and Signal Size in Great Bowerbirds?" *Animal Behavior* 79, 747–55.

Gurtler, O. and J. Munster. (2010) "Sabotage in Dynamic Tournaments," *Journal of Mathematical Economics* 46, 179–90.

Hart, S. (2016) "Allocation games with caps: from Captain Lotto to all-pay auctions," *International Journal of Game Theory* 45(1–2), 37–61.

Konrad, K. (2000) "Sabotage in Rent-Seeking Contests," *Journal of Law, Economics and Organization* 16, 155–65.

Morgan, J. (2003) "Sequential Contests," *Public Choice* 116(1), 1–18.

Pruett-Jones, S. and A. Heifetz (2012) "Optimal Marauding in Bowerbirds," *Behavioral Ecology* 23, 607–14.

Pruett-Jones, S. and M. Pruett-Jones (1994) "Sexual Competition and Courtship Disruptions: Why do Male Bowerbirds Destroy each other's Bowers?" *Animal Behavior* 47, 607–20.

Serena, M. (2017) "Quality Contests," *European Journal of Political Economy* 46, 15–25.

Serena, M. (2017) "Sequential Contests Revisited," *Public Choice* 173 (1–2), 131–144.