Employment Targeting in a Frictional Labor Market*

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Abstract

Governments in both developing and developed economies play an active role in labor markets in the form of providing both formal public sector jobs and employment through public workfare programs. We refer to this as employment targeting. In the context of a simple search and matching friction model, we show that the propensity for the public sector to target more employment can increase the unemployment rate in the economy and lead to an increase in the size of the informal sector. Employment targeting can therefore have perverse effects on labor market outcomes.

Keywords : Search and Matching Frictions, Labor Markets, Employment, Informal Sector, Public Sector.

JEL Codes : J46, D83, O17, O20

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1 Introduction

Governments in both developed and developing economies play an active role in labor markets to meet their growth and development objectives. In the case of India, the twin phenomenon of jobless growth and the growing casualization of the work-force has led to a vibrant debate about the role of government policy in stimulating employment (see Kapoor (2017) and Abraham (2017)). One particular intervention takes the form of the public sector being the provider of jobs. We refer to this as employment targeting. For instance, public workfare programs are amongst the most common forms of anti-poverty programs in developing countries. NREGS, the flagship workfare government scheme in India employs several hundred million people. In the US, the Works Projects Administration (WPA) started in 1935 was initiated in response to the Great Depression, and hired unemployed workers directly. Large scale poverty reduction is a central policy objective of developing countries in Latin America, Africa, and Asia, where employment guaranteed schemes have been at the centre of an employment oriented approach to anti-poverty policy-making (Basu et al, 2009). More recently, the aggressive response of fiscal policy in the financial crisis of 2008 by developed economies has sparked a burgeoning literature on the merits of counter-cyclical government spending (see Rendahl (2016)).

In each of these cases, the general equilibrium effects of policies that target employment on overall unemployment remains a key research question. In the context of employment guarantee schemes, like NREGS, a question that arises is that by leading to an increase in wages, do employment guarantee schemes crowd out private sector employment ? In a recent paper, Muralidharan, Niehaus, and Sukhtankar (2018) study the policy-relevant general-equilibrium estimates of the total effect on wages, employment, income, and assets of increasing the effective presence of NREGS. They show that a public employment guarantee, by improving the outside option for workers, puts upward pressure on labor markets that drives up wages and earnings. Basu et. al (2009) develop a formal model of an employment guarantee scheme and show that such schemes introduce contestability in labor hiring, and raise the reservation wage. Gomes (2015) characterizes a government's acyclical wage policy that protects workers from business cycle fluctuations. He argues that very high public sector wages can create disincentives to private players for posting vacancies and can reduce overall employment. In this context, he proposes an optimum level of the public sector wage which maximizes welfare.

What is less understood in the literature however, is the impact of employment targeting on the size of the informal sector in developing economies. We fill this gap in the literature.[1]

[1] There are only a handful of papers that use search and matching frameworks to study informal labor

We build a simple model of a developing country labor markets characterized by search and matching frictions. We show that public sector intervention in the labor market can lead to an increase in the size of the informal sector. Because the informal sector is characterized by a high firing rate and lower unemployment benefits, employment targeting leads to an perverse effects on labor market outcomes. This is our main result. We also show that, under certain parametric restrictions, an increase in the public sector hiring rate can increase employment unambiguously. In particular, we find it is possible that the private sector wage falls as a result of an increase in the public sector hiring rate which leads to more job creation in the private sector. This reverses the consensus findings in the search and matching literature which shows that an increase in public sector employment disincentivizes private sector vacancy postings, as in the paper by Gomes (2015).

2 The Model

The economy is comprised of three infinitely lived agents: firms, agents or workers, and the government. Heterogeneous individuals are uniformly distributed according to their abilities. Each individual's ability is indexed as $i \in (0,1)$ where 0 is the lowest ability and 1 is the highest ability. Since agents do not have any other distinguishing features, they are indexed as i. Firms present in the economy produce a single final good which is consumed by agents. We call a private firm's production unit as the "private sector", denoted by P. The government's production unit is termed as "public sector", denoted by G. Unemployed agents are denoted by U. Agents are risk neutral and their utility comes only from consuming the final good.

Each agent has one unit of labour endowment, which he supplies inelastically in each point of time. However, the labour market is characterized by frictions. Private sector firms and agents face search and matching friction before commencing production activity. Unemployed agents search for jobs irrespective of their abilities and can search for both private sector and public sector jobs. Vacant firms looking for workers post a vacancy by paying a vacancy posting cost, $d > 0$. Private sector firms and job seekers are matched according to a Pissarides style matching function: $m = m(u, v)$, where u is the number of unemployed, and v is the number of vacant firms (Pissarides 2000). The function, m, is homogeneous of degree one, concave, and increasing in each of its arguments. Hence, $m/u = m(1, \theta)$,where $\theta \equiv v/u$, denotes the job finding rate, while $m/v = m(\theta^{-1}, 1)$ is the

markets. See Albrecht et. al. (2009), Castillo and Montoro (2010), Maarek (2012), and Charlot et. al. (2013). None of these papers however focus on the effects on employment targeting.

vacancy matching rate.[2] Production starts in the private sector once a firm and a worker are matched. Production follows a constant returns to scale (CRS) technology in the economy: i.e., the i^{th} ability agent produces i units of output. Firms get to know about their workers' ability once they are matched.

Unemployed agents get an amount, $b > 0$, which is an unemployment benefit from the government. Workers who are employed in the private sector get a per period wage, w_i, according to their ability. The firing rate in the private sector is given by $\lambda > 0$. The rate at which an unemployed agent finding a public sector job is given by $\gamma > 0$. The parameter γ can be considered as the hiring rate of public sector. We assume that the government pays a fixed wage to its employees, $\bar{w}$, irrespective of their ability. The firing rate in the public sector is given by, $\tilde{\lambda}$. Therefore, in a small time span, Δt, an unemployed agent can get a public sector job with a probability, $\gamma * \Delta t$, while a public sector worker can be fired with the probability, $\tilde{\lambda} * \Delta t$. Similarly, a private sector job match can break with probability, $\lambda \Delta t$, within Δt. r is the discount rate in the economy. Finally, we assume that a job seeker cannot get a net surplus from a public sector job and a private sector job simultaneously. All the public/private job creation and job destruction rates follow a Poission process as in Pissarides (2000).

We formalize the public sector's employment policy by the policy-tuple, $\{\bar{w}, b, \gamma\}$ and call this the *employment targeting policy* of the government. Our main focus in this paper, however, is on the parameter, γ, and its effect on unemployment and informalization.

2.1 Steady state

In this paper, we focus on characterizing the steady state. Let V_j^i denote the infinite income stream of the i^{th} worker, where the state $j = P, G, U$. This implies that

$$rV_P^i = w_i - \lambda(V_P^i - V_U^i) \tag{1}$$

This implies that the flow value of a private sector job (or a filled vacancy), rV_P^i, equals the wage from the private sector job (w_i) plus the expected net surplus from being unemployed if the private sector job is destroyed $(\lambda(V_U^i - V_P^i))$. Analogously, the flow value of being employed in the public sector is given by

$$rV_G^i = \bar{w} - \tilde{\lambda}\,(V_G^i - V_U^i), \tag{2}$$

[2] $m(1, \theta) * \Delta t$ and $m(\theta^{-1}, 1) * \Delta t$ are the transition probabilities from being unemployed to employed and vacant to a filled post, respectively, in the private sector, at a very small time interval Δt.

and lends it to a similar interpretation to equation (1), except now, the wage in the public sector is given by $\bar{w}$, with the job destruction rate in the public sector given by $\tilde{\lambda}$. The flow value of being unemployed is given by,

$$rV_U^i = b + m(1,\theta)\,(V_P^i - V_U^i) + \gamma\,(V_G^i - V_U^i). \tag{3}$$

which equates the flow value of being unemployed, rV_U^i, to the level of the unemployment benefit, b, plus the net surplus from finding a job in either the private sector or public sector. Since workers cannot work in both sectors simultaneously, there is no net surplus associated with joint employment in both sectors.

Subtracting equation (3)from (1) yields

$$(r + \lambda + m(1,\theta))\,(V_P^i - V_U^i) = w_i - b - \gamma\,(V_G^i - V_U^i). \tag{4}$$

Likewise, subtracting equation (3) from (2), and solving for $V_G^i - V_U^i$ yields,

$$V_G^i - V_U^i = \frac{1}{r + \tilde{\lambda} + \gamma}\,[\bar{w} - b - m(1,\theta)]\,(V_P^i - V_U^i). \tag{5}$$

Equation (5) gives the net surplus of being employed in the public sector relative to the net surplus of being employed in the private sector. Likewise, substituting equation (5) into equation (4) and manipulating terms yields,

$$V_P^i - V_U^i = \frac{\bar{w} - b}{m(1,\theta)} + \frac{r + \tilde{\lambda} + \gamma}{m(1,\theta)}\left[\frac{(w_i - b)m(1,\theta) - (\bar{w} - b)(r + \lambda + m(1,\theta))}{(r+\lambda)(r + \tilde{\lambda} + \gamma) + m(1,\theta)(r + \tilde{\lambda})}\right]. \tag{6}$$

Equation (6) expresses the net return of a productive matching to a worker. After a productive matching, workers receive V_P^i but at the cost of sacrificing V_U^i.

We denote the value functions of infinitely lived private firms as J_P^i and J_V^i, where P stands for productive matching and v stands for a vacancy, respectively. The flow value of a productively matched private firm is given by

$$rJ_P^i = (i - w_i) - \lambda(J_P^i - J_V), \tag{7}$$

and for a firm with a vacancy,

$$rJ_V = -d + m(\theta^{-1},1)\,(E(J_P^i) - J_V). \tag{8}$$

Equation (8) contains the term $E(J_P^i)$. A vacant firm does not know about a worker's ability prior to a successful match and therefore, does not know the exact return before the firm gets matched with a worker. Instead, vacant firms use the information about expected returns from a filled job, $E(J_P^i)$, to take a vacancy posting decision.

In equilibrium firms entry and exit freely in the market such that

$$J_V = 0. \tag{9}$$

Equation (8) therefore implies that

$$E(J_P^i) = \frac{d}{m(\theta^{-1}, 1)}. \tag{10}$$

Likewise, substituting $J_V = 0$ into equation (7) and solving for J_P^i yields

$$J_P^i = \frac{i - w_i}{\lambda + r} \tag{11}$$

which is increasing in the ability of the i^{th} worker. Notice that for a private sector firm, the net return from a productive matching is given by, $(J_P^i - J_V)$.

2.2 Wage Bargaining

The Nash bargaining solution is the w_i that satisfies

$$w_i = argmax_{w_i}(V_P^i - V_U^i)^{\beta}\,(J_P^i - J_V)^{1-\beta}, \tag{12}$$

where $\beta \in (0, 1)$ represents worker bargaining power. It is imperative to understand the effect of heterogeneous agents in the bargaining process. Since, each individual has an unique ability, his corresponding wage is also unique. This has an important implication in wage bargaining. If the workers were homogeneous then one individual could not affect the wage rate which is available outside ones particular job match, because there would be a large number of similar agents participating in the labour market. One agent would be too small to affect the rest of the market. However, in the present set up with heterogeneous ability, this argument does not hold. A matched worker knows that, ceteris paribus, any wage decision in a particular matching is going to replicate in all possible productive matchings because each agent is unique in their ability, i. In other words, a change in w_i also changes the agent's outside option, V_U^i. This implies that, $\frac{\partial V_U^i}{\partial w_i} \neq 0$.

The first order maximization condition is given by

$$\beta\left[\frac{\partial V_P^i}{\partial w_i}-\frac{\partial V_U^i}{\partial w_i}\right]J_P^i+(1-\beta)\,\left[V_P^i-V_U^i\right]\frac{\partial J_P^i}{\partial w_i}=0. \tag{13}$$

To obtain an expression for $\frac{\partial V_P^i}{\partial w_i}-\frac{\partial V_U^i}{\partial w_i}$, we differentiate equation (6) to get

$$\frac{\partial V_P^i}{\partial w_i}-\frac{\partial V_U^i}{\partial w_i}=\frac{r+\tilde{\lambda}+\gamma}{(r+\lambda)(r+\tilde{\lambda}+\gamma)+m(1,\theta)(r+\tilde{\lambda})}. \tag{14}$$

Substituting equation (14) and $\frac{\partial J_P^i}{\partial w_i}$ from (11) and putting these into equation (13), we obtain an expression for w_i :

$$w_i=[i\beta+b(1-\beta)]+\frac{(\bar{w}-b)(1-\beta)}{m(1,\theta)}\left[\lambda-\gamma+\frac{\gamma\, m(1,\theta)}{r+\tilde{\lambda}+\gamma}\right] \tag{15}$$

Equation w_i is increasing in the ability of the i^{th} worker, although since our focus is on employment targeting, we would like to know how an increase in γ, the hiring rate of the public sector, affects the optimal wage. To see this, recall equation (13). Using equations (1), (7), and (9), we can re-write (13) as

$$\begin{aligned}(1-\beta)\left[V_P^i-V_U^i\right]&=\beta(1-r\frac{\partial V_U^i}{\partial w_i})(\frac{i-w_i}{\lambda+r})\\ w_i-rV_U^i&=\frac{\beta}{1-\beta}(i-w_i)(1-r\frac{\partial V_U^i}{\partial w_i})\\ w_i\left[1+\frac{\beta}{1-\beta}(1-r\frac{\partial V_U^i}{\partial w_i})\right]&=rV_U^i+\frac{\beta}{1-\beta}i(1-r\frac{\partial V_U^i}{\partial w_i}).\end{aligned}$$

Using equations (1), (2), and (3), it is easy to show that, $r\frac{\partial V_U^i}{\partial w_i}=\frac{m(1,\theta)}{1+m(1,\theta)+\gamma}$. Using this, and after a few algebraic manipulations, we obtain

$$w_i=rV_U^i+(i-rV_U^i)\left[\frac{\beta\frac{1+\gamma}{1+\gamma+m(1,\theta)}}{(1-\beta)+\beta\frac{1+\gamma}{1+\gamma+m(1,\theta)}}\right]. \tag{16}$$

The first term on the right hand side, rV_U^i, is the minimum compensation a worker requires to give up search (Pissarides, 2000). On top of this, the worker requires a fraction of the rent, or net surplus, that a productive match generates. It can be shown that if γ increases, then both rV_U^i (because a public sector job serves as an outside option for a private sector worker) and the square bracketed term on the right hand side are increasing. However, due to an increase in rV_U^i, the term, $i-rV_U^i$, is falling, or the surplus itself is less. Since the proportionate share of the surplus accruing to the worker is more (because of the monopoly power of the i^{th} worker), the effect of the fall in net surplus pulls the wage down, and gets amplified. This means that an increase in γ creates an ambiguous effect on the wage.

2.3 Equilibrium

Recall that agents are distributed uniformly over the interval $[0,1]$. Therefore, from equation (11), we have

$$E(J_P^i) = \int_0^1 J_P^i di = \int_0^1 \frac{i - w_i}{\lambda + r} di. \tag{17}$$

Substitute out for w_i in equation (17) using equation (16). Solving the integration makes equation (17) free of i and $w_{i.}$ The only remaining endogenous variable in (17) is θ. Hence,

$$E(J_P^i) = \frac{1}{2(\lambda + r)} - \frac{1}{(\lambda + r)}\left[b(1-\beta) + \frac{\beta}{2}\right] - \frac{(\bar{w} - b)(1-\beta)}{m(1,\theta)}\left[\lambda - \gamma + \frac{\gamma\, m(1,\theta)}{r + \tilde{\lambda} + \gamma}\right] \tag{18}$$

Equating equation (10) and (18) implies,

$$\frac{d}{m(\theta^{-1},1)} + \frac{(\bar{w} - b)(1-\beta)}{m(1,\theta)(\lambda + r)}(\lambda - \gamma) = \frac{1}{2(\lambda + r)} - \frac{1}{(\lambda + r)}\left[b(1-\beta) + \frac{\beta}{2}\right] - \frac{(\bar{w} - b)(1-\beta)\gamma}{r + \tilde{\lambda} + \gamma} \tag{19}$$

which implicitly solves for the value of θ.

Steady state unemployment happens when the flow out of unemployment equals the flow into unemployment, i.e., $u\left[m(1,\theta) + \gamma\right] = (1-u)\left(\lambda + \tilde{\lambda}\right)$. This implies,

$$u^* = \frac{\left(\lambda + \tilde{\lambda}\right)}{m(1,\theta) + \gamma + \lambda + \tilde{\lambda}} \tag{20}$$

2.4 Comparative Statics

We are interested in the impact of employment targeting, or the public sector's hiring objectives on the overall level of unemployment. To obtain this, we totally differentiate both sides of equation (19) with respect to γ to obtain

$$\frac{d\theta^*}{d\gamma} = -\left[\frac{(\bar{w} - b)(1-\beta)}{\left(r + \tilde{\lambda} + \gamma\right)^2}\right]\left[\frac{m(1,\theta^*)}{(d - \varepsilon_m(1,\theta^*)) - (\bar{w} - b)(1-\beta)(1 + (\lambda - \gamma)\frac{\varepsilon_m(1,\theta^*)}{\theta^*})}\right], \tag{21}$$

where $\varepsilon_m(1,\theta^*)$ is the elasticity of the matching function with respect to θ, i.e., $\frac{\partial m(1,\theta^*)}{\partial \theta^*}\frac{\theta^*}{m(1,\theta^*)}$. The condition for $\frac{d\theta^*}{d\gamma} > 0$ is given by

$$\frac{\theta^*}{\lambda-\gamma} > \varepsilon_m(1,\theta^*)\left[\frac{d-\varepsilon_m(1,\theta^*)}{(\bar{w}-b)(1-\beta)}-1\right]^{-1}. \tag{22}$$

We can interpret the above condition more precisely if we consider the class of matching functions with constant elasticity. In this case, the right hand side of equation (22) will be a constant in terms of d, $\bar{w}$, b, β, and ε_m, which we denote by κ. Equation (22) can be written as

$$\theta^* + \kappa(\gamma-\lambda) > 0. \tag{23}$$

Figure 1a and Figure 1b below shows that if the equilibrium value of θ, or θ^*, lies to the *right hand side* or above (respectively) of the line given in (23), then $\frac{d\theta^*}{d\gamma} > 0$. Conversely, if θ^* lies to the left or below, then $\frac{d\theta^*}{d\gamma} < 0$. This leads to our first proposition.

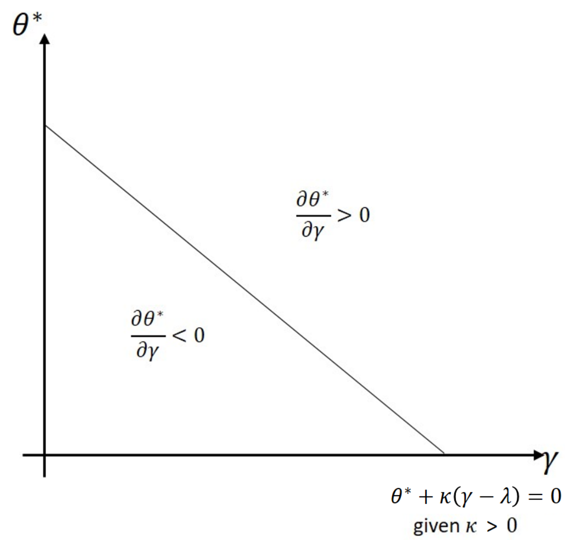

Figure 1a: θ^* versus γ when $\kappa > 0$

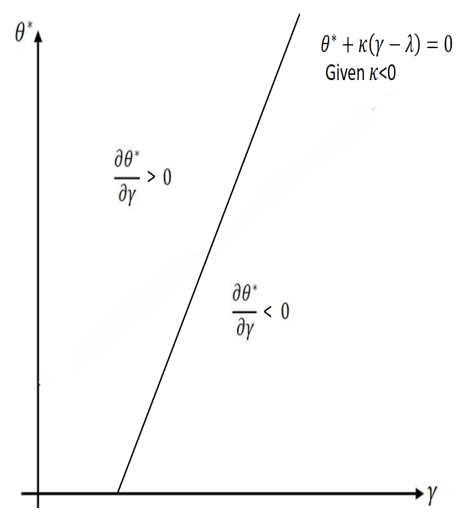

Figure 1b: θ^* versus γ when $\kappa < 0$

Proposition 1 *Consider a value γ such that the equilibrium value of $\theta^*(= \frac{v}{u})$ lies above the straight line, $\theta^* + \kappa(\gamma - \lambda) = 0$. Employment targeting, or an increase in hiring by the public sector (increase in γ), increases θ^*, or reduces equilibrium unemployment, u^*. If θ^* lies below the straight line, then an increase in γ leads to a fall in θ^*, or an increase in equilibrium unemployment, u^*, if $\varepsilon_m(1, \theta^*)$ is sufficiently large.*

The intuition behind Proposition 1 is as follows. Recall that the impact of γ on w_i is ambiguous. Suppose a rise in γ increases w_i, then the return from a vacant post for a firm falls. Hence, firms start leaving the market and the number of vacancies, v, falls, since in equilibrium, $J_v = 0$. This leads to a fall in $m(1, \theta)$. If the fall in $m(1, \theta)$ is large enough to off-set the rise in γ, then from equation (20), u^* can rise. On the other hand, if a rise in γ makes w_i fall, then the return from vacancies rise, and more firms enter the market and more vacancies are created. Both γ and $m(1, \theta)$ increase, and u^* falls. Equation (23) is the sufficiency condition for the fall in u^*.

There is an important corollary to Proposition 1, which relates to the case when $(\bar{w} - b) \to 0$. In this case, the public sector wage is so low, that it is close to the per-period unemployment benefit, b. It is easily seen from equation (19) that the equation is independent of γ. This implies that changes in γ have no impact on θ, or on the rate of getting a private sector job and a private sector wage. This implies that an increase in γ unambiguously reduces u^*. Intuitively, $(\bar{w} - b)$ is the net surplus from working in the public sector relative to being unemployed. As the net surplus falls, the outside option (the public sector job) facing a worker in the bargaining process to determine his wage is negligible. This is true for a firm too. So the private sector offers more vacancies. There is more matching. And this leads to lower unemployment.

3 Informal Sector

In this section we extend the baseline model above to include an informal sector. Our main goal is to derive conditions under which employment targeting by the public sector can lead to an increase in the size of the informal sector. We assume that labor is divided into two categories: formal and informal. As before, within the formal sector, there is a public sector and a private sector, and their characterization remains the same.

The description of the informal sector is as follows. Private sector firms operate in both the informal and formal sector (example, textiles, or leather goods). If they operate in the informal sector, they pay a training cost, c, once they are matched with a worker. After receiving the training, the productivity of all matched workers (in the informal sector) becomes the same, and workers get a wage corresponding to their new productivity. Hence, the heterogeneity in ability of the worker is not reflected in the wage that they receive in the informal sector. We assume that the firing rate is higher in the informal sector than in the formal sector. For simplicity, we assume that the firing rate of the informal sector is 1. Firms post vacancies unless the returns to posting vacancies becomes zero. When the returns from posting a vacancy becomes zero, there is no incentive for firms to enter into the market. In the informal sector, firms and job seekers match through the typical matching function used in the previous section, except that here the outside option is, by assumption, $b_I < b$.

In the formal sector, individual ability is uniformly distributed over $[i^*, 1]$, while in the informal sector, individual ability is distributed over $[0, i^*]$.[3]We solve for all endogenous variables in the steady state. In addition, we also characterize the problem for the *pivotal* worker, who is indifferent between working in the informal and formal sectors.

3.1 Labor market in the informal sector

Let V_U^I denote the value function corresponding to the infinite income stream of an unemployed worker in the informal sector (I). The value function does not include the subscript i which corresponds to individual ability; as mentioned before, workers get a homogenous return. Similarly, V_E^I is the value function corresponding to the infinite income stream of an employed worker in the informal sector. The flow values are given by

$$rV_U^I = b_I + m(1, \theta_I)(V_E^I - V_U^I) \tag{24}$$

and

$$rV_E^I = w_I - (V_E^I - V_U^I) \tag{25}$$

where θ_I is the market tightness in the informal sector, and w_I is the wage rate in the informal sector.

Let J_E^I be the value function of matched firm, while J_V^I denotes the value function of a vacant firm in the informal sector, i.e.,

$$rJ_E^I = (p - w_I - c) - (J_E^I - J_V^I) \tag{26}$$

and

$$rJ_V^I = -d + m(\theta_I^{-1}, 1)(J_E^I - J_V^I) \tag{27}$$

where $p > 0$ is the constant productivity from a productive matching in the informal sector. After a productive matching, firms pay the wage, w_I, and the training cost, c.

As before, in equilibrium $J_V = 0$ due to the free entry condition. The wage in the informal sector, like the private sector wage, is determined by Nash bargaining. However, the difference relative to the previous section is that in case of the informal sector, an individual's differential ability is not reflected in their productivity. Hence, the wage in the informal sector is the same for all workers. For the same reason, in this bargaining problem, the assumption that one individual worker's decision cannot change the outside option is a valid one.[4]

[3]In the previous section, individual ability was uniformly distributed over $[0, 1]$

[4]This is a commonly made assumption in the literature on Pissarides type search and matching. However, in the case of the formal private sector wage bargaining problem in the previous section, this assumption

3.2 Wage Bargaining

The Nash bargaining solution is the w_I that satisfies

$$w_I = \arg\max_{w_I}(V_E^I - V_U^I)^{\beta}(J_E^I - J_V^I)^{1-\beta}.$$

The maximization exercise yields

$$\left(V_E^I - V_U^I\right) = \beta\left(V_E^I - V_U^I + J_I^E\right) \tag{28}$$

which implies

$$w_I - rV_U^I = \beta(p-c) - \beta rV_U^I$$

or,

$$w_I = \beta(p-c) + (1-\beta)rV_U^I. \tag{29}$$

Equation (28) can be also be written as

$$V_E^I - V_U^I. = \frac{\beta}{1-\beta}J_E^I. \tag{30}$$

Substituting $\left(V_E^I - V_U^I\right)$ in equation (30) into equation (24), we obtain

$$rV_U^I = b_I + m(1,\theta_I)\frac{\beta}{1-\beta}J_E^I. \tag{31}$$

Since the free entry condition requires that $J_V^I = 0$, from equation (27), we obtain

$$J_E^I = \frac{d}{m(\theta_I^{-1},1)} \tag{32}$$

Substituting the value of J_E^I from (32) into (31) yields

$$rV_U^I = b_I + \frac{\beta}{1-\beta}\theta_I d. \tag{33}$$

Putting this back into (29) yields,

$$w_I = (1-\beta)b_I + \beta(p-c+\theta_I d). \tag{34}$$

Hence, the optimal wage in the informal sector is a positive function of labor market tightness

was not valid.

in the informal sector, θ_I. What is noteworthy is that for a given θ_I, a rise in the training cost leads to a fall in the informal sector wage. This is because a rise in training costs reduces the surplus accruing to the informal sector firm, which responds by reducing its wage rate.

From equation (26), setting $J_V^I = 0$ implies $J_E^I = \frac{(p-w_I-c)}{1+r}$. Setting this equal to the value of J_E^I in (32) implies

$$\frac{(p - w_I - c)}{1+r} = \frac{d}{m(\theta_I^{-1}, 1)} \tag{35}$$

Equation (35) depicts a negative relationship between θ_I and w_I. On the other hand, equation (34) depicts a positive relationship between θ_I and w_I. Figure 2 below depicts the two equations. Their intersection yields the equilibrium values of w_I and θ_I. An interesting implication is that as the training costs facing informal sector firms increases, as shown in Figure 3, both curves shift. In particular, equation (35) shifts down/out, while equation (34) shifts in. Hence, both w_I^* and θ_I^* fall. Intuitively, as c increases, effective output from a productive matching, $p - c$, falls in the informal sector. Since both firms and workers are sharing their returns from the surplus, $p - c$, both their returns fall. Hence, facing $J_V^I < 0$, firms exit the market, to ensure that $J_V^I = 0$ in equilibrium. As a result, both θ_I^* and w_I^* decrease.

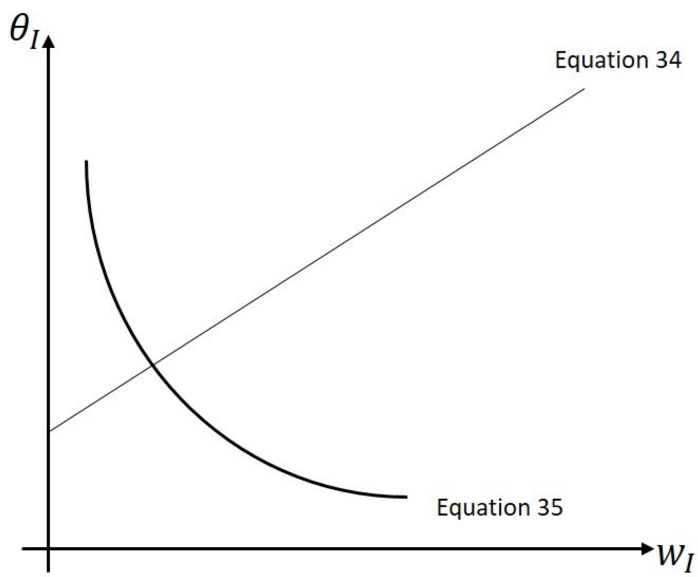

Figure 2: Labor Market Equilibrium in the Informal Sector

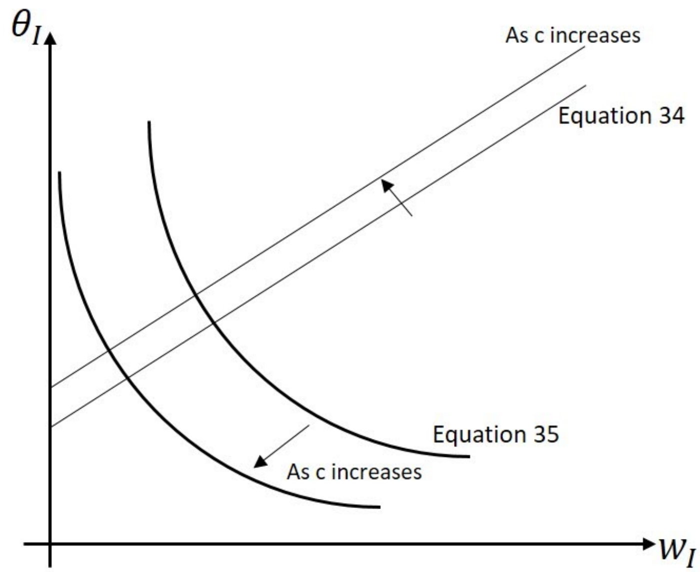

Figure 3: Impact of Training Costs on θ_I and w_I

3.3 The Formal Sector

Individuals from $[i^*, 1]$ work in the formal sector. We determine i^* endogenously in equilibrium. As mentioned in the previous section, the wage in the formal sector is an increasing function of an individual's ability (see equation (16)). Since the return from the informal sector is independent of the ability of the worker (i.e., fixed), an individual with higher ability is incentivized to work harder in the formal sector. In essence, the formal sector here is not different from the previous section, apart from the fact that the formal sector corresponds to individuals with ability distributed over $[i^*, 1]$. As a result, equation (17) becomes

$$E(J_P^i) = \int_{i^*}^{1} \frac{J_P^i}{1-i^*} di = \int_{i^*}^{1} \frac{i - w_i}{(\lambda + r)(1 - i^*)} di. \tag{36}$$

Recall that the expression for w_i in the formal sector is given by (15). We proceed in steps. First,

$$\int_{i^*}^{1} w_i di = \frac{\beta}{2}(1 - i^*) + (1 - i^*)\left[b(1-\beta) + \frac{(\bar{w} - b)(1-\beta)}{m(1,\theta)}\left[\lambda - \gamma + \frac{\gamma\, m(1,\theta)}{r + \tilde{\lambda} + \gamma}\right]\right].$$

Therefore,

$$\int_{i^*}^{1} (i - w_i) di = \frac{1-\beta}{2}(1 - i^*)^2 - (1 - i^*)\left[b(1-\beta) + \frac{(\bar{w} - b)(1-\beta)}{m(1,\theta)}\left[\lambda - \gamma + \frac{\gamma\, m(1,\theta)}{r + \tilde{\lambda} + \gamma}\right]\right]$$

Substituting the value of $\int_{i^*}^{1}(i - w_i)di$ above into equation (36) and simplifying yields,

$$E(J_P^i) = \frac{1}{(\lambda + r)} \left[\frac{1-\beta}{2}(1 + i^*) - \left[b(1-\beta) + \frac{(\bar{w} - b)(1-\beta)}{m(1,\theta)} \left(\lambda - \gamma + \frac{\gamma\, m(1,\theta)}{r + \tilde{\lambda} + \gamma} \right) \right] \right] \tag{37}$$

Equating the value of $E(J_P^i) = \frac{d}{m(\theta^{-1},1)}$ from (10) with the expression given above in equation (37), we obtain

$$\frac{d}{m(\theta^{-1},1)} = \frac{1}{(\lambda + r)} \left[\frac{1-\beta}{2}(1 + i^*) - \left[b(1-\beta) + \frac{(\bar{w} - b)(1-\beta)}{m(1,\theta)} \left(\lambda - \gamma + \frac{\gamma\, m(1,\theta)}{r + \tilde{\lambda} + \gamma} \right) \right] \right]$$

or,

$$\frac{d}{m(\theta^{-1},1)} + \frac{(\bar{w} - b)(1-\beta)\,(\lambda - \gamma)}{m(1,\theta)\,(\lambda + r)} = \frac{1}{(\lambda + r)} \left[\frac{1-\beta}{2}(1 + i^*) - b(1-\beta) - \frac{(\bar{w} - b)(1-\beta)\gamma}{r + \tilde{\lambda} + \gamma} \right]. \tag{38}$$

Equation (38) depicts the equilibrium relationship between θ and i^* which guarantees a firms' free entry and exit. Here, θ and i^* are positively related, as long as $\gamma > \lambda$. If i^* increases, to clear the labor market, more firms enter and increase the number of vacancies. This is because a firms' entry decision is based on the expected return from a filled post. Since i^*, increases, and the upper bound of ability is 1, the average productivity in the formal sector must rise. In other words, more able individuals are left, and therefore average productivity must be higher.

Since we have two endogenous variables (θ and i^*), we need another equation to pin down both variables. We turn to this in the next section.

3.4 Equivalence of Formal and Informal Sectors

In the previous sub-section, we assumed the existence of an interior solution where the work force could be partitioned between the formal and informal sectors. Therefore, there must be a marginal worker who is indifferent between joining the informal and formal sectors. We denote the marginal worker as i^*. Since the ability of every individual in the population is indexed by i, the marginal worker's ability is indexed by i^*. Therefore, the flow value of search for a job in the formal sector for the marginal worker is $rV_U^{i^*}$. Likewise, in the informal sector, it is given by rV_U^I. Since the individual with i^* ability is indifferent between joining both the informal sector and formal sector, it follows that

$$V_U^{i^*} = V_U^I. \tag{39}$$

Using equation (3) and equation (5), we can determine $rV_U^{i^*}$ as a function of $(V_P^{i^*} - V_U^{i^*})$:

$$rV_U^{i^*} = b + \frac{(\bar{w}-b)\gamma}{r+\tilde{\lambda}+\gamma} + \frac{m(1,\theta)(r+\tilde{\lambda})}{r+\tilde{\lambda}+\gamma}(V_P^{i^*} - V_U^{i^*}). \tag{40}$$

Wage determination in the formal sector is determined from: $(V_P^{i^*} - V_U^{i^*}) = \frac{\beta}{1-\beta}(i^* - w_{i^*})(\frac{\partial V_P^i}{\partial w_i} - \frac{\partial V_U^i}{\partial w_i})$. Using equation (14) in this expression yields

$$(V_P^{i^*} - V_U^{i^*}) = \frac{\beta}{1-\beta}(i^* - w_{i^*})\left[\frac{r+\tilde{\lambda}+\gamma}{(r+\lambda)(r+\tilde{\lambda}+\gamma)+m(1,\theta)(r+\tilde{\lambda})}\right]. \tag{41}$$

We now have $(V_P^{i^*} - V_U^{i^*})$ in terms of $(i^* - w_{i^*})$. Equation (15) already solves for the optimal w_i, and therefore w_i^*. So we can get an expression for $(i^* - w_i^*)$. Using equation (15) and equation (40), $rV_U^{i^*}$ is determined by

$$rV_U^{i^*} = b + \frac{(\bar{w}-b)\gamma}{r+\tilde{\lambda}+\gamma} + \frac{\beta m(1,\theta)(r+\tilde{\lambda})}{(r+\lambda)(r+\tilde{\lambda}+\gamma)+m(1,\theta)(r+\tilde{\lambda})}\left[(i^*-b) + \frac{(\bar{w}-b)(\gamma-\lambda)}{m(1,\theta)} - \frac{\gamma(\bar{w}-b)}{\left(r+\tilde{\lambda}+\gamma\right)}\right]. \tag{42}$$

Equation (33) determines V_U^I. Therefore, both the right hand side and left hand side in the equivalence equation, (39), are now a function of θ and i^*. Using equation (33) and (42), we obtain

$$\frac{\beta}{1+\frac{(r+\lambda)(r+\tilde{\lambda}+\gamma)}{m(1,\theta)(r+\tilde{\lambda})}}\left[(i^*-b) + \frac{(\gamma-\lambda)(\bar{w}-b)}{m(1,\theta)}\right] + \frac{\gamma(\bar{w}-b)}{(r+\tilde{\lambda}+\gamma)}\left[1 - \frac{\beta}{1+\frac{(r+\lambda)(r+\tilde{\lambda}+\gamma)}{m(1,\theta)(r+\tilde{\lambda})}}\right] = \frac{\beta}{1-\beta}\theta_I d \tag{43}$$

3.5 Equilibrium

Equations (38) and (43) denote the labor market equilibrium and equivalence equations, respectively. The solution of these two equations solve for i^* and θ endogenously. However, equation (43) depicts an ambiguous relationship between θ and i^*. This makes the conclusion unclear.

3.6 Comparative Statics

We focus on an analytical special case to find whether employment targeting can have an impact on the composition of the workforce between the informal and formal sectors. Later, we consider a numerical example that shows that our result is more general. We consider the special case where $(\bar{w}-b) \to 0$. Note that θ_I has already been solved in equation (34) and (35). Equation (43) now shows a negative relationship between i^* and θ. Equation (38) has a positive intercept in the i^* and θ plane, for $(\bar{w}-b) \to 0$. This ensures an interior equilibrium for i^* and θ, as shown in Figure 4. In Figure 4, if the government decides to increase its hiring rate (increase γ), or target a higher employment rate (when $(\bar{w}-b) \to 0$), equation (38) remains unchanged, but (43) shifts upward. In this case, market tightness in the formal sector, and the size of the informal sector - i^* and $\theta-$ respectively, both rise. This is because an increase in the market tightness of the formal sector results in a rise in the rate of obtaining a job in the formal sector. We summarize this result in terms of the following proposition.

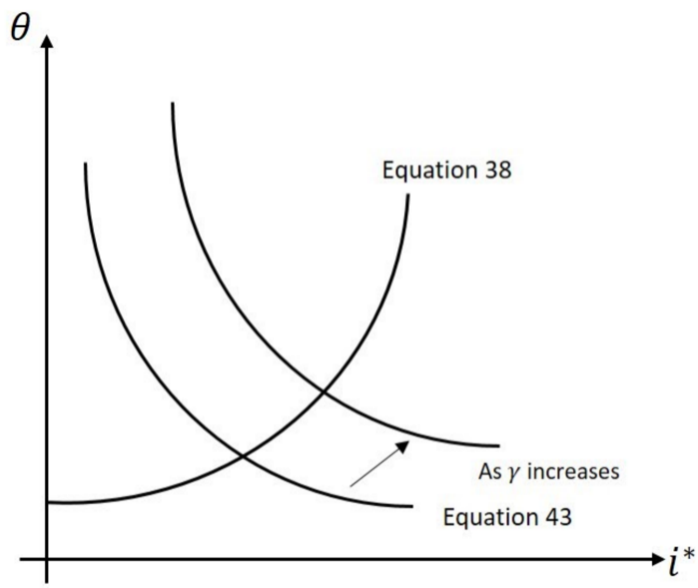

Figure 4: Impact of Employment Targeting on Size of the Informal Sector.

Proposition 2 *Suppose $(\bar{w}-b) \to 0$. Then an increase in γ, or more public sector hiring, increases 1) market tightness in the formal sector (θ^*) and 2) the size of the informal sector (i^*).*

The intuition is as follows. When $(\bar{w}-b) \to 0$, the per-period (net) return to public sector employment tends to zero. If the public sector expands, the marginal job seeker, i^*, who was originally getting the same return as if he was in the informal sector finds it detrimental to stay in the formal sector, since staying in this sector is not remunerative. However, once i^* increases, θ^* starts increasing to clear the market because the average productivity in the formal sector is higher, and more firms enter into the market. This creates more vacancies, which means θ^* increases. Hence, as γ increases, provided that $(\bar{w} - b) \rightarrow 0$, both θ^* and i^* increase. Thus, the size of the informal sector increases.

There is an interesting implication with training costs. As c increases, the opposite happens (the size of the informal sector falls). This is because θ_I^* falls and this shifts equation (43) backwards although equation (38) remains unchanged. As θ_I^* falls staying in the informal sector becomes less remunerative because the rate of getting a job is lower. So i^* falls. To clear the labor market, θ^* also falls.

3.7 Numerical Exercise

The assumption of $(\bar{w} - b) \rightarrow 0$ is a special case. What happens if $(\bar{w} - b)$ is sufficiently small but non-zero ? We show that the results of Proposition 2 go through, at least locally, using arbitrary parameters that allows for a sufficiently small $\bar{w} - b > 0$. [5] We utilize a matching function of Cobb-Douglas form: $au^{a_1}v^{(1-a_1)}$. Table 1 below summarizes the parameter values. Figures 5, and 6 characterize the equilibrium in informal and formal markets respectively. Figure 7 examines the effect of change in γ on labor market outcomes.

No.	Parameters	Values
1	d	0.2
2	a	0.5
3	a_1	0.65
4	b	0.39
5	b_I	0.1
6	β	0.35
7	$\bar{w}$	0.4
8	c	0.1
9	λ	0.15
10	$\tilde{\lambda}$	0.05
11	r	0.15

Table 1: Parameter Values

Figure 5, generated using equations (34) and (35), shows an interior solution corresponding to the parameters for the informal sector where $\bar{w} > b$. We assume $\gamma = 0.5$ in the baseline case. The numerical solution of θ_I is 0.14. While this number is arbitrary, it says that of

[5] We are unable to check whether Proposition 2 holds for large values of $\bar{w} - b > 0$. We plan to address this in a future draft of the paper.

all the job seekers in the informal sector, at most only 14% of them can be matched with vacancies in the informal sector.

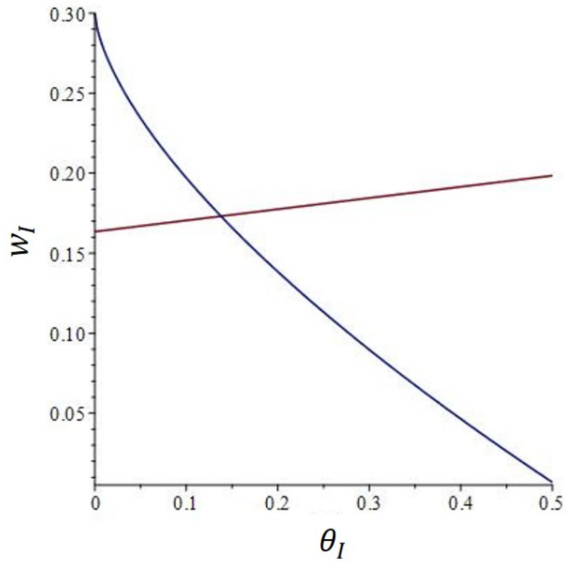

Figure 5: Equilibrium in the Informal Sector

Figure 6, generated using equations (38) and (43), characterizes equilibrium in the formal market. For $\gamma = 0.5$, the solution for θ^* and i^* are shown to approximately be $\theta^* = .5$ and $i^* = .5$. This means that approximately half the population works in the informal sector, and the other half works in the formal sector.

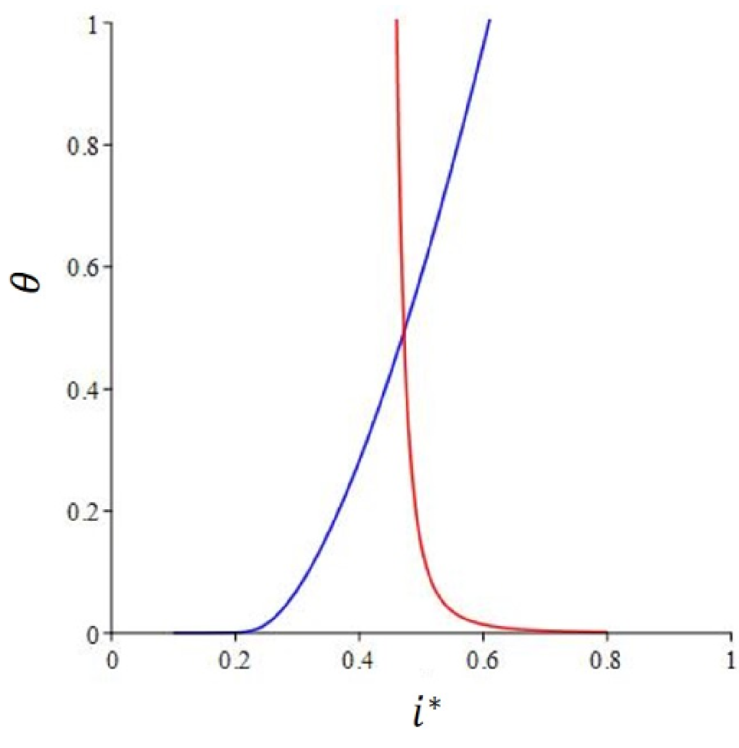

Figure 6: Equilibrium in the Formal Sector

Now, we increase the government hiring rate, γ, to 0.8. Figure 7 below shows that for a small but non zero $(\bar{w} - b)$ a higher γ leads to an increase in both θ^*, i^* consistent with the result in Proposition 2. As i^* increases, the size of the informal sector increases. This increases θ,which means compared to the earlier case (where earlier roughly half of the job seekers could get a job in the formal sector), now more than half can get a job in the formal sector since the return from posting a vacancy in the formal sector has increased. However, since the informal sector is characterized by a higher firing rate (1), and lower unemployment benefits, the rise in γ leads to a perverse labor market outcome.

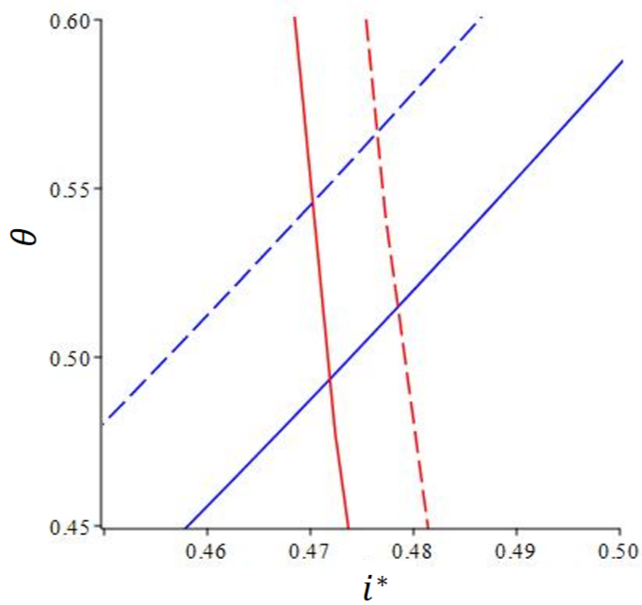

Figure 7: Effect of Change in the Public Sector Hiring Rate

4 Conclusion and Policy Implications

Many governments as part of their growth and development objectives, play an active role in labor markets. Such interventions come in the form of setting a minimum wage, providing unemployment benefits, and directly hiring workers. We refer to this as employment targeting. In the context of a simple search and matching friction model with heterogenous agents, we show that the propensity for the public sector to target more employment can increase the unemployment rate in the economy and leads to an increase in the size of the informal sector. Employment targeting can therefore have perverse effects on labor market outcomes. We also find it is possible that the private sector wage falls as a result of an increase in the public sector hiring rate which leads to more job creation in the private sector. This reverses the consensus findings in the search and matching literature which shows that an increase in public sector employment disincentivizes private sector vacancy postings.

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