Bundling in platform markets in the presence of data advantage

Gaurav Jakhu[a,*]

[a] *Economics and Planning Unit, Indian Statistical Institute, New Delhi, India 110016*

Abstract

This paper examines the private and social incentive to bundle when one platform has data advantage from another market. There are two platforms competing over users and advertisers. Endogenizing the choice of business models, symmetric (both platforms are ad financed or user financed) business models or strategic differentiation (platforms charging opposite sides) can emerge in an equilibrium. Next, it is shown that the profitability of bundling and its welfare impact depends on the strength of advertiser network benefits. In markets with large advertiser network benefits, bundling may be profitable but reduces social welfare. This result is in contrast to previous work. Moreover, the impact of mandatory unbundling depends on the welfare standard considered. In markets with large advertiser benefits and low nuisance cost of advertisements, mandatory unbundling increases social welfare at the cost of reduced user welfare.

Keywords: Platforms, Bundling, Data Advantage

JEL classification: D21, D42, L12, L13, L42, L51

1. Introduction

Recent antitrust cases have focussed on tying practices employed by a dominant platform. There are many high profile tying cases under scrutiny across

*Corresponding author, Tel + 91 9873033405

Email address: `gaurav14r@isid.ac.in` (Gaurav Jakhu)

jurisdictions. Google has been under investigation for its alleged anti competitive practices in search and mobile operating system. EU launched a formal investigation against Google for analysing the claim that it discriminates against other comparison shopping websites by providing a favourable position to its own comparison shopping product in its general search results pages in the European Economic Area (EEA)[1]. If this holds, then Google's practice can artificially divert traffic from other websites to its own hindering growth of other platforms. Another sphere of Google's dominance is mobile operating system where it leads the market with over 80 percent market share. Other leading Google applications on mobiles are Google Maps, Google Search, Youtube etc. The antitrust complaint against Google is based on the business strategy used by it to promote its own applications on mobile devices. Google requires mobile device manufacturers to sign "Mobile Application Development Agreement (MADA) " among other agreements [2]. This paper aims to understand the bundling strategy employed by a platform to extend its dominance from one market to another.

A key feature highlighted in this paper is the role of user information as a strategic asset and its effect on price competition. A platform can collect information about users which is known as big data in policy jargon. It relates to personal information (user IP address, location), demographics information and behavioral information (online browsing, interests etc). An online platform can use this information to target ads catering to consumer needs and interests. This can improve the probability that a user would buy the ad product. For example, a search engine can target ads based on search queries entered by the users. Facebook shows ads which can be targeted based on user's characteristics. Thus,

[1]Statement of Objections of the EU Commission published on April 14, 2015

[2]See Edelman and Geradin (2016), Android and Competition Law: Exploring and Assessing Google's Practices in Mobile, Harvard Business School. It is observed that, first, manufacturers must "pre-install" "all Google applications" that Google specifies. Second, Google requires that these pre-installed apps be placed prominently on mobile devices. Third, Google requires that Google Search "must be set as the default search engine for all Web search access points," ruling out the possibility of any other search engine being the default.

targeted advertising is a rationale, profit maximizing behaviour of an online platform. In addition to generating benefits for advertisers, a user can also benefit from big data through improved services. A search engine can modify the answers to user queries based on user information. A social networking site can highlight the news feed which a user would be most interested in. How can this big data be collected? A platform can understand user behaviour from the user history on its interface. Alternatively, they could indirectly make a predictive analysis about a user through information that a user would leave on other platforms. For example, a user location data from mobile devices can be used by Google to recommend specific restaurants in its vicinity. They can utilize the acquired data advantage from such moves to entrench its positions in the core search sector.

Combining these two empirical facts about the internet platforms, in this paper, the focus is on the use of data advantage to bundle products and services across markets. A firm with dominance in market 1 sells its good in market 2 with valuable data sets created in market 1. This is the mechanism underlying targeted advertising where advertising slots are accompanied with valuable information about consumers to increase ad effectiveness. Improvement in user services due to better information available about them will not be considered. Only improvement in predictive power of user attitudes and its impact on advertising technology will be part of our analysis. I examine how cross usage of data across two markets on advertising side affects the private and social incentive to bundle services on user side. Finally, implications for competition policy and regulation are drawn.

This paper develops a simple model in which there are two markets - market 1 and platform market 2. Firm G is a monopolist in market 1 and a duopolist in market 2 with firm S as its rival. In market 2, users singlehome and advertiser multihome. Users dislike advertisements and have identical intrinsic value for two platforms. However, platform G has a data advantage from its presence in market 1. This provides it with user information relevant to functioning in core platform market 2. On the advertising side, advertising technology determines the probability of informing a user when an advertisement is placed on a platform. It is asymmetric across two platforms with data advantage platform G having a higher probability/more efficient technology.

The contribution of this paper to the literature is to show how the presence of user data advantage affects bundling in platform markets and analyse its welfare implications. The main results are as follows. First, endogenizing the choice of business model choice by platforms, symmetric business model (both platforms are ad financed or user financed) or strategic differentiation (platform charge opposite sides) can emerge in an equilibrium. Bundling expands the parameter space over which strategic differentiation emerges as an equilibrium. Next, it is shown that bundling can be profitable only when the platform adopts an advertising financed model. In this case, collection of user data makes possible additional surplus available in the tied market i.e market 2. When a platform bundle its monopolized product with user services in other market it increases the value of user base to the advertisers making bundling profitable. Whereas, it losses on the profits from the tying market. The balance between gains and loss depends on the parameter values. Bundling is profitable for i) when nuisance cost of advertisement is not very high ii) when advertiser network benefits are not very strong. This result is in contrast to the previous result in Choi and Jeon (2016) paper on bundling in platform market. They consider leveraging of monopoly power from one market to another. According to them, profitability of bundling increases with strength of advertiser network benefits in the tied market. However, their model focuses on the role of monopoly power in extending dominance to another market. This paper considers the role of data advantage from one market as an instrument to create dominance in another market. In the absence of data advantage, the result of monopoly advantage as considered in Choi and Jeon (2016) holds in our paper as a special case. However, in presence of data advantage the results are different. Thus, this paper develops and extends the Choi and Jeon (2016) framework on leverage theory of tying in two sided markets.

Next, I assess the social welfare change as a result of bundling. The change in social welfare depends on the equilibrium business model. In general, there are two opposing effects that work on social welfare. When the platform has adopted ad financed model then bundling increases the nuisance cost of ads (if ads are the source of revenue) and transportation cost; reduces advertising revenue from rival platforms (if it has chosen ad financed model); and may reduce user surplus in tying market. On the other hand, it improves advertising revenue on the dominant platform G. The net effect depends on the interplay of these forces.

A careful examination of the parameter region shows that there are regions where private and social incentives diverge and converge. In markets with strong advertiser network benefits if bundling is profitable then it leads to fall in social welfare. Whereas, in markets with small advertiser network benefits profitable bundling can increase social welfare. Lastly, analysing the components of total welfare (user surplus and advertiser surplus), it is shown that improvement in total welfare can entail i) increase in advertiser surplus and decrease in user surplus and vice versa ii) increase in both advertiser and user surplus. So, bundling can have opposite effects on social and user welfare. It depends on the strength of advertiser network benefits. In this model, depending on the parameter values, antitrust intervention requiring mandatory unbundling of goods may increase social welfare but reduce user welfare.

2. Related literature

This paper is related to many strands of literature. First, it is related to literature on strategic business model choice by competing firms. Casadesus-Masanell and Zhu (2009) examines the interaction between an incumbent and a free ad-sponsored entrant and allows the incumbent to respond with different business models. Casadesus-Masanell and Zhu (2011) allow both competing firms to decide their business models and look at the role of competitive imitation in choice of business models. Calvano and Polo (2014) studies the role of advertising technology effectiveness and value of informed viewers in influencing business model choice by media platforms. Our paper differentiates itself from the previous work by studying how user attitude toward online ads affects the choice of business models.

Second, this study contributes to the understanding of bundling incentives in platform markets. The leverage theory of bundling has a well established intellectual history and many papers have studied bundling as an entry deterence device (eg. Whinston, 1990; Choi and Stefanadis, 2001; Carlton and Waldman, 2002; Nalebuf, 2004.). In addition a few papers have focussed on bundling in multi sided markets. Amelio and Jullien (2012) and Choi and Jeon (2016) consider models with platforms that are unable to charge negative prices. They examine incentives of a monopolist to tie its monopolized product with product facing competition in two sided markets and derive its welfare implications. The novel mechanism which makes bundling profitable in these papers is the ability to overcome non negative price constraints. Since rival is constrained to set non negtaive prices it limits aggressive response by rival and additional profits are generated. Choi (2010) studied tying in two sided markets when each platform has some exclusive content to offer to consumers. It shows that tying can improve social welfare if multihoming is allowed on the content provider side. Corniere and Taylor (2017) set up a slightly different model in which platforms can set negative prices. There are application developers and users on two sides interacting through a platform - smartphone manufacturers. Applications derive benefits for their developers, and developers can offer payments to the device manufacturers in exchange for being installed. They show that bundling reduces rival application developers' willingness to pay manufacturers for inclusion on their devices, and allows a multi application developer to capture a larger share of industry profit.

The policy stand on tying in two sided markets is divided. It has defenders and opponents arguing their case for whether bundling should be allowed or prohibited in platform markets. They look at Google bundling practices in search and mobile operating. Defenders [3] argue that bundling is not anti competitive for two main reasons i) there are no restrictions on multihoming on user side. ii) bundling helps firm to innovate and it is a product improvement. Opponents [4] of bundling argue that i) users face increased advertisements as well as cost of processing information. ii) bundling imposes restrictions on advertiser side iii) bundling allows the platform to gather a huge amount of data from complementary markets and reinforce its dominant position in the core market.

This paper formalizes the argument substantiated in Newman (2014) on control of user data in platform markets. From a theoretical standpoint, despite the importance of user level data in affecting market outcomes, none of the studies mentioned above consider the role of data in strategic decision making. To fill the gap, this paper explicitly considers the role of data advantage in bundling decisions and analyse its welfare implications. This would help in exploring the market conditions under which a platform, present in multiple markets, can use user level data for leveraging market power. The elimination of competition via bundling of a monopolized product with one side of the platform market can expand the set of users to whom the tying firm can sell on the other side of the market. This increases the user data set and ad targeting is possible over a larger user base. Thus, additional advertising revenues can be captured through bundling. Next, I set up the model and derive the main results.

3. The Model

There are two markets (market 1 and two sided market 2) and two firms/platforms (Firm G and Firm S). Firm G is a monopolist in market 1 and both firms compete in market 2.

[3] Refer Bork and Sidak (2012) and Manne and Wright (2011)

[4] Refer Newman(2014) and Edelman and Geradin (2016)

Users: The population of users is identical in market 1 and market 2. A user payoff in market 1 is

$$U_{G1} = v - q_{G1} \tag{1}$$

Where v is the standalone utility. For tractability of the model, we assume that v is heterogenous across users and uniformly distributed over [0,1]. The other term q_{G1} is the price charged to users. In market 1, N_{G1} users have purchased good 1 and $1 - N_{G1}$ did not. Whenever a user purchases good 1 it provides information to Firm G. So, this N_{G1} works as an installed base advantage for Firm G. It can use this data on N_{G1} users to improve quality and effectiveness of targeted ads in market 2.

In market 2, users are uniformly located on a horizontal line with density 1. Firm G is located at point 0 and firm S is located at point 1. There is no instrinsic difference between the two platforms and both have same quality X [5]. A user singlehomes and chooses a single platform. Its payoff are

$$U_{G2} = X - tm_{G2} - q_{G2} - c : \text{ if it joins firm G} \tag{2a}$$

$$U_{S2} = X - tm_{S2} - q_{S2} - (1 - c) : \text{ if it joins firm S} \tag{2b}$$

Where parameter t > 0 measures aversion for ads; c is the transportation cost parameter uniformly distributed over [0,1]; q_{i2} is the price charged by platform i; i = G,S and m_{i2} is number of ads on platform i; i = G,S.

An important assumption taken for the rest of analysis is that location of a user in market 1 is independent of its location in market 2. Let v and c be location of a consumer in market 1 and market 2. Then, v and c are independently distributed with support [0,1]. Consumers dislike ads on a platform. This has been

[5]In the baseline model, there is no data advantage on user side. However, the analysis remains unaffected qualitatively even if we include improvement in user utility from access to data

empirically validated in few media studies which found that advertising reduces a user's utility (Wilbur 2008; Depken and Wilson 2004). Theoretical work has also characterised advertising as a nuisance to users (e.g., Anderson and Coate 2005). Hence, t > 0 reflects users' attitude toward advertisements. The functional form tm_{i2} shows that marginal disutility of ads is constant. No firm can offer a subsidy to a user which implies prices are non negative in the model. The value to the user from an outside option is assumed to be zero.

Advertiser: In market 2, there is a single advertiser and it decides the number of advertisements (m_{G2}, m_{S2}) to send through two platforms . Its expected revenue from sending m_{i2} advertisements on platform i is
Return from informing a single user × Probability of informing a single user × No. of users on platform i.
For simplicity, return from informing a single user is taken to be unity. Let $N_{i2}N_{G1}$ be the proportion of users on platform i who have purchased good 1 and $N_{i2}(1 - N_{G1})$ be the proportion of users on platform i who didn't purchase good 1. Firm G has data about N_{G1} users which it can share with the advertiser. This data can be behavioural, location based data, demographic information etc about the users. Firm G can use this data to improve the effectiveness of ads on its platform for $N_{G1}N_{G2}$ users.
Let the probability of informing a user in market 2 be given by the function

$$I * k + (1 - k)\beta : \text{ where I is an indicator function such that} \tag{3a}$$

$$I = \begin{cases} 1 & \text{if platform has data over user} \\ \beta & \text{if platform has no data over user} \end{cases} \tag{3b}$$

In the equation above, β is the benchmark probability that a user who views an ad product would purchase it and k $\epsilon[0, 1]$ is the targeting technology efficiency. If a platform has information about user characteristics the probability of match is assumed to be exactly equal to 1. Platform G has better information over the user set $N_{G1}N_{G2}$ but platform S has no such information[6]. Neither platform has any information over the users who didn't purchase good 1. So under this scenario, on platform G, an advertiser has a higher match probability over those users who had purchased good 1. This is given by $k + (1 - k)\beta$, the improved probability that a users who had purchased good 1 would also purchase an ad product on platform G. Whereas for users who didn't purchase good 1 the probability of match remains equal to β.

Now, advertiser's expected profit function from sending messages on two platforms can be obtained. Expected ad revenues on platform G is

$$\underbrace{[k + (1 - k)\beta]N_{G1}N_{G2}m_{G2}}_{\text{Exp. ad revenues from users who purchased good 1}} \quad + \quad \underbrace{\beta(1 - N_{G1})N_{G2}m_{G2}}_{\text{Exp. ad revenues from users who didn't purchase good 1}} \tag{4}$$

which can be written as

$$([k + (1 - k)\beta]N_{G1} + \beta(1 - N_{G1}))m_{G2}N_{G2} = [\beta + k(1 - \beta)N_{G1}]m_{G2}N_{G2} \tag{5}$$

where m_{G2} is the number of advertisements on platform G and N_{G2} is the number of users on platform G.

Similarly, it can be shown that expected ad revenues on platform S is $\beta N_{S2}m_{S2}$ where m_{S2} is the number of advertisements on platform S and N_{S2} is the number of users on platform S. The total ad profits from the two platforms is

$$[\beta + k(1 - \beta)N_{G1}]m_{G2}N_{G2} + \beta m_{S2}N_{S2} - p_{G2}m_{G2} - p_{S2}m_{S2} \tag{6}$$

where p_{i2} is the price paid by advertiser to send a message on platform i = G,S. It can be shown that the advertiser would demand advertisements on platform i

[6] In the baseline model there is no data sharing between the two platforms

= G,S such that price charged equals marginal benefit of an ad [7]. The simple linear functional form would imply that it earns zero profit in an equilibrium and platforms would siphon off the entire surplus from the advertiser.

Profit Functions: In a general framework, firm G maximizes profit by setting q_{G1}, q_{G2} and p_{G2} and firm S maximizes profit by setting q_{S2} and p_{S2}. Each of these strategic variable is non negative. It is to be noted that a firm can maximize profits with respect to quantity of advertisements or prices. In this model, there exists a negative relationship between number of ad quantities and price charged. For simplification, profits are optimized with respect to ad quantities.

$$\pi_G = q_{G1}N_{G1} + q_{G2}N_{G2} + p_{G2}m_{G2} : \text{ Firm G's profit} \tag{7a}$$

$$\pi_{S2} = q_{S2}N_{S2} + p_{S2}m_{S2} : \text{ Firm S's profit} \tag{7b}$$

Social Welfare: It is defined as the sum of consumer surplus (CS), advertiser surplus (AS) and platforms' profits. Since platforms' profits are transfers from other agents in the model, social welfare is simply the sum of consumer surplus (CS) and advertiser surplus (AS) i.e. SW = CS + AS.

Timing: The timing of the game is as follows:

- Stage 1: Firm G chooses price in market 1 (q_{G1}). Firm G and firm S compete in market 2 over users and advertisers. They simultaneously choose prices to be charged to users (q_{G2} and q_{S2}) and quantity of advertisements to be given (m_{G2} and m_{S2})[8].

- Stage 2: Users decide i) whether to buy good 1 or not in market 1. ii)

[7] More generally, there can be diminishing returns to ads on each platform i.e. the advertising revenues would fall with number of ad messages. In this paper, I have simplified the setting by assuming there constant returns to advertising and expected ad revenues stay constant with number of ad messages.

[8] In this model, the quantity of advertisements and demand for advertisements by the advertiser would be the same.

Which platform to join in market 2. Advertiser decides how many messages to send through each platform.

The solution concept used is subgame perfect nash equilibrium. A strategy for firm G is $[q_{G1}, q_{G2}, p_{G2}]\epsilon[0,\infty)\times[0,\infty)\times[0,\infty)$ and for firm S is $[q_{S2}, p_{S2}]\epsilon[0,\infty)\times[0,\infty)$. A strategy for a user is choice of product $\subseteq G, S$ and a strategy for an advertiser is number of advertisements to be sent $[m_{G2}, m_{S2}]\epsilon[0,\infty)\times[0,\infty)$.

4. Independent Pricing

In this section, market equilibrium is characterized. A user is defined by a pair (v,c) where v is its valuation for good 1 and c defines its location in market 2. Its choice set consists of four different options i.e.

- G1G2: Buy good 1 in market 1 and join platform G in market 2.
- G1S2: Buy good 1 in market 1 and join platform S in market 2.
- G2: Do not buy good 1 and join platform G in market 2.
- S2: Do not buy good 1 and join platform S in market 2.

When deriving the demand for two platforms we look for an interior scenario when both have positive demands. Using these, demand configuration for each option can be obtained. The total demand for the platforms in two markets can be derived as

$$N_{G1} = 1 - q_{G1} : \quad \text{good 1 in market 1} \tag{8a}$$

$$N_{G2} = \frac{1}{2} + + \frac{tm_{S2} - tm_{G2}}{2} + \frac{q_{S2} - q_{G2}}{2} : \quad \text{Platform G in market 2} \tag{8b}$$

$$N_{S2} = \frac{1}{2} + \frac{tm_{G2} - tm_{S2}}{2} + \frac{q_{G2} - q_{S2}}{2} : \quad \text{Platform S in market 2} \tag{8c}$$

It can be seen that in market 2, demand for a platform is decreasing in its own price and advertising level, and increasing in those of rivals.
On advertiser side, there is a single advertiser and its profit is

$$[\beta + k(1-\beta)N_{G1}]m_{G2}N_{G2} + \beta m_{S2}N_{S2} - p_{G2}m_{G2} - p_{S2}m_{S2} \tag{9}$$

There exists an inverse relationship between the demand for advertising and price charged to an advertiser. The inverse demand functions for the two platforms can be obtained using $\partial A/\partial p_{G2} = 0$ and $\partial A/\partial p_{S2} = 0$. This gives

$$p_{G2} = [\beta + k(1-\beta)N_{G1}]N_{G2} \text{ and } p_{S2} = \beta N_{S2}$$

In this paper, the choice of business model refers to the pricing regime that a platform can adopt to compete. In market 2, four different kinds of business models can emerge

- Ad financed: Both platform charge only advertiser side in market 2
- User financed: Both platform charge only user side in market 2.
- Mixed Model: Both platform charge the two sides.
- Strategic Differentiation: Platform i charges one side and platform j charges the other side.

Now, we derive the main result of this section i.e. the choice of business models by the two platform. We characterize the optimal strategies of two platforms through a series of lemmas and propositions.

Lemma 1: There does not exist a SPNE in which Firm G or/and Firm S chooses a mixed business model.

So, in an equilibrium, a platform would rely on single price instrument in market 2. The following proposition shows the optimal choice of price and ad slots under different business models.

Proposition 1: The optimal price and ad slots under each business model are:-
a) **Ad financed model**: $q^*_{G1} = Max[\frac{1}{2} - \frac{k(1-\beta)}{4t}, 0]$; $m^*_{G2} = \frac{1}{t}$ and $m^*_{S2} = \frac{1}{t}$.
b) **User financed model**: $q^*_{G1} = \frac{1}{2}$; $q^*_{G2} = 1$ and $q^*_{S2} = 1$
c) **Strategic Differentiation**: $q^*_{G1} = \frac{1}{2} - \frac{k(1-\beta)}{4t}$; $m^*_{G2} = \frac{1}{t}$ and $q^*_{S2} = 1$
d) There does not exist any other kind of equilibrium.

In the ad financed model or the strategic differentiation model, platform G can employ a loss leader strategy if the nuisance cost of ads is sufficiently low for users or data advantage measured by the difference between probability of match over $N_{G1}N_{G2}$ and $(1 - N_{G1})N_{G2}$ is sufficiently high in market 2. It would set a zero price for good 1. This strategy has been observed in many other industries.

Proposition 2: Suppose both platforms choose advertising revenue model in an equilibrium. Firm G might operate as a loss leader when either i) marginal disutility of ads is very low or/and ii) data advantage is high.

The intuition stems from ability to improve ad targeting on its platform. When either of the two conditions hold then firm G use zero pricing in market 1 to attract maximum number of users. The data on these users would lead to a higher marginal revenue of ads which lead to higher overall advertising revenues. This can be used to cover up losses incurred in market 1. This strategy conforms to findings in previous literature on loss leader pricing. Li et al (2013) shows that when firms have high cross selling abilities then they would reduce prices on one product to attract a larger set of customers for other products.

In a user financed model, the linkage between market 1 and market 2 through premium per click on ads is not present. So, firm G would look at two markets separately. Since it is a monopolist in market 1 and demand function is linear, the optimal price in market 1 is half the reservation price. In all business models, as user aversion toward ads increases, the optimal number of ad quantities in market 2 reduces.

Now, we look at the platforms' strategy i.e how they choose to compete.

Proposition 3: There exists t_0, t_1, t_2 such that

a) **Ad Supported Platforms**: For $0 < t < \frac{k(1-\beta)}{2}$; SPNE is ($q^*_{G1} = 0; q^*_{G2} = 0; m^*_{G2} > 0$) and ($q^*_{S2} = 0$; $m^*_{S2} > 0$. For $\frac{k(1-\beta)}{2} < t < t_0$; SPNE is ($q^*_{G1} > 0; q^*_{G2} = 0; m^*_{G2} > 0$) and ($q^*_{S2} = 0$; $m^*_{S2} > 0$).

b) **Strategic Differentiation**: For $t_0 < t < \frac{k(1-\beta)}{2}$; SPNE is ($q^*_{G1} = 0; q^*_{G2} = 0; m^*_{G2} > 0$) and ($q^*_{S2} > 0$; $m^*_{S2} = 0$). For $\frac{k(1-\beta)}{2} < t < t_1$; SPNE is ($q^*_{G1} > 0; q^*_{G2} = 0; m^*_{G2} > 0$) and ($q^*_{S2} > 0$; $m^*_{S2} = 0$).

c) **Strategic Differentiation**: For $t_1 < t < t_2$; Two SPNE are

ci) ($q^*_{G1} \geq 0; q^*_{G2} = 0; m^*_{G2} > 0$) and ($q^*_{S2} > 0$; $m^*_{S2} = 0$) and

cii) ($q^*_{G1} > 0; q^*_{G2} > 0; m^*_{G2} = 0$) and ($q^*_{S2} > 0$; $m^*_{S2} = 0$)

d) **User based pricing**: For $t > t_2$; SPNE is ($q^*_{G1} > 0; q^*_{G2} > 0; m^*_{G2} = 0$) and ($q^*_{S2} > 0$; $m^*_{S2} = 0$)

In order to understand the intuition behind proposition 4, we need to compare the marginal effect of ads on user utility i.e. t and marginal revenue of ads which equals β for firm S and $\beta + k(1-\beta)N_{G1}$ for firm G. It is clear from above that a firm's choice of revenue model would depend on the marginal effect of ads on two sides. For very low marginal disutility of ads each firm would find it optimal to use advertising revenue model. The marginal revenue of ads can compensate for any negative effect on firm's profit. Whereas, when marginal disutility of ads is very high, relying on ad revenues is not an optimal strategy.

The role of data advantage comes in when it leads to higher marginal revenue of ads for firm G. This brings in a difference between the marginal trade offs for the two firms. The strength of data advantage can be measured as difference between probability of match over two user sets which is given by k(1 - β). Due to better ad targeting possible on its platform, firm G can use advertising revenue model for a higher range of t. Figure 1 below clearly describes the choice of business models in $t - \beta$ space. It shows the region in which different types of business models can arise in an equilibrium.

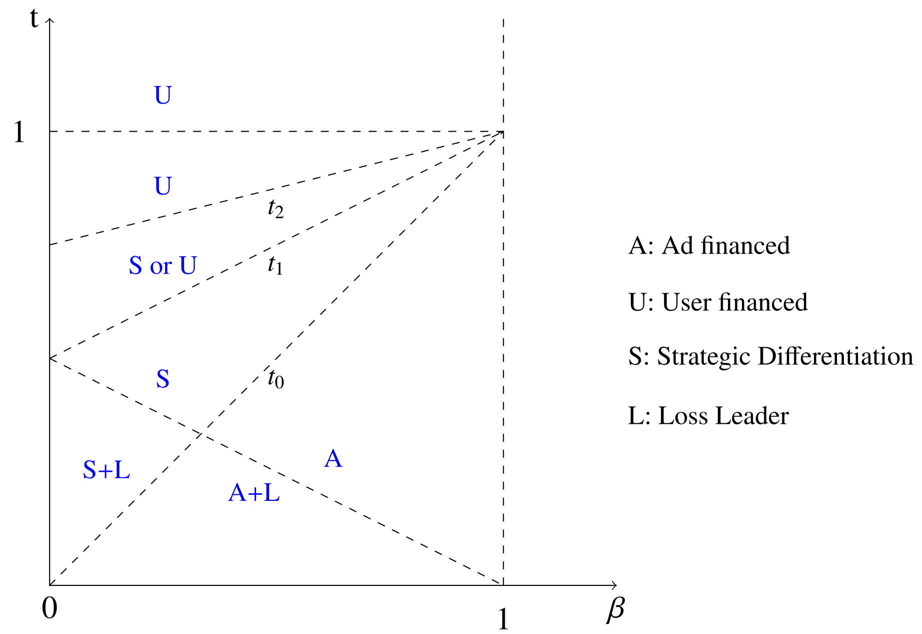


Figure 1: Independent Pricing: Choice of Business Model

Example involves k = 0.75.

5. Bundling

In this section we analyze the bundling decision of platform G. The objective is to understand how incentive to bundle changes under different business mod-

els and strength of data advantage. At the outset, we will consider pure bundling decision [9]. So good 1 and platform G are served as a pure bundle to the users. Any user purchasing good 1 joins platform G.
Let q_G be the bundled price of good 1 and platform G to the users. Rest of the notations are same as under no bundling case. A user choice set is now reduced to

- Bundled good: G1G2
- Platform S: S2

Now the decision is to choose one of the above choice. A user's utility from consumption of two goods is

$$v + X - tm_{G2} - c - q_G : \text{ Bundled Good} \tag{10a}$$

$$X - tm_{S2} - q_{S2} - (1 - c) : \text{ Platform S} \tag{10b}$$

An indifferent user is defined by a pair (v,c) satisfying the following equality

$$v + X - tm_{G2} - c - q_G = X - tm_{S2} - q_{S2} - (1 - c) \tag{11}$$

Using 11 we can draw the region in v-c space to describe the demand for bundled good and platform S.

For the rest of analysis, it is assumed that some users with no valuation for good 1 and with highest valuation for good 1 prefer platform S over G. In other words, $0 < x_0 < 1$ and $0 < x_1 < 1$. Using figure 2, demand functions can be written as

[9]Under pure bundling two goods are not available in isolation. Whereas under tying, the tied good is available on a stand alone basis.

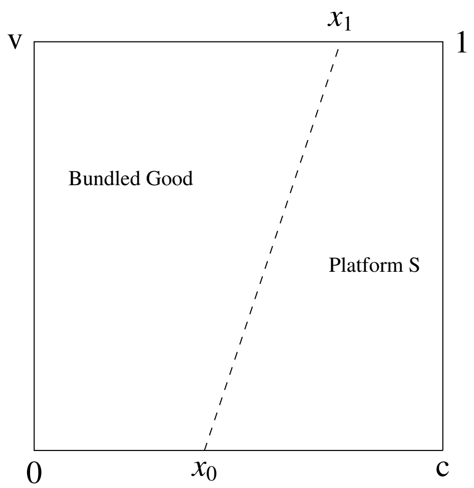


Figure 2: Demand under bundling case

$$N_{G2} = N_{G1} = \frac{3}{4} + \frac{tm_{S2} - tm_{G2}}{2} + \frac{q_{S2} - q_G}{2} : \text{ Bundled Good} \tag{12a}$$

$$N_{S2} = \frac{1}{4} + \frac{tm_{G2} - tm_{S2}}{2} + \frac{q_G - q_{S2}}{2} : \text{ Platform S} \tag{12b}$$

The crucial difference from no bundling case is the change in platform G's profit from advertisers. Earlier, the two kind of users joined platform G - those who have purchased good 1 and those who didn't. Now, in presence of bundling, only set of users on platform G are those who have purchased good 1 i.e. $N_{G1} = N_{G2}$. Thus, platform G has access to information over all users on its platform. In other words, it can earn premium per click over all users. The total revenue of an advertiser on platform G is $[\beta + k(1-\beta)]m_{G2}N_{G2}$ which will be taken away by G. On platform S, advertiser revenue is given by the same function i.e. $\beta m_{S2}N_{S2}$. The total profits of each platform are given by

$$\pi_{G2} = q_G N_{G1} + [\beta + k(1-\beta)]m_{G2}N_{G2} : \text{ Platform G's Profits} \tag{13a}$$

$$\pi_{S2} = q_{S2}N_{S2} + \beta m_{S2}N_{S2} : \text{ Platform S's Profits} \tag{13b}$$

Using these demand and profit functions, we can analyze the platform strategies and choice of business models. Through following propositions we show how bundling equilibrium departs from the independent pricing equilibrium.

Proposition 4: The optimal prices and ad slots under different business models are

1.**Ad Financed Model**: $\tilde{q_G} = 0; \tilde{m_{G2}} = \frac{7}{6t}$ and $\tilde{q_{S2}} = 0; \tilde{m_{S2}} = \frac{5}{6t}$

2. **User Financed Model**: $\tilde{q_G} = \frac{7}{6}; \tilde{m_{G2}} = 0$ and $\tilde{q_{S2}} = \frac{5}{6}; \tilde{m_{S2}} = 0$

3. **Strategic Differentiation**: $\tilde{q_G} = 0; \tilde{m_{G2}} = \frac{7}{6t}$ and $\tilde{q_{S2}} = \frac{5}{6}; \tilde{m_{S2}} = 0$

Like previous section, bundling could lead to three different business models - Ad Financed Model, User Financed Model and Strategic Differentiation. However, there exists important differences with respect to no bundling case. When firm G could bundle two goods, it would always employ loss leader strategy under ad financed model and set optimal bundled price $q_G = 0$. Since, marginal costs are zero in our model, firm G set $q_G = 0$. More generally, it would set price below cost for good 1 and use the demand generated to maximize ad revenues in market 2. This strategy has empirical validation in internet markets. One such example is Google's use of loss leader strategy in one market to maximize ad revenues from other markets. Android handsets are sold at below costs and Google hardly makes any profit from its sales. However, android handsets has a set of pre installed Google apps. Now, when an app is already installed on a phone, a user would be reluctant to install a new app unless the installed is of poor quality. Using the set of users, Google gets access to them through Android handsets, it can earn revenues from them by selling the attention span on apps like search engine to advertisers. In this way, Google earns profits mainly through advertising side. Apart from this, other crucial difference under bundling is characterized in the following proposition.

Proposition 5: Compared to independent pricing equilibrium, the parameter space over which strategic differentiation would emerge as an equilibrium becomes large under bundling.

Figure 3 below illustrates the above result. It shows the parameter space over which different business models would emerge under two scenarios. In the range $t_1 < t < t_2$ a user financed model or strategic differentiation can emerge as an equilibrium under independent pricing. But, under bundling, only strategic differentiation emerges as an equilibrium. The intuition behind this result is that platform G's best response would change when it could bundle two goods. Now, it can resort to advertising space as the source of revenue for a larger range of t. A user utility is interlinked under bundling case i.e addition of utilities from two goods. So, some users with high v who were earlier not joining platform G due to high t can now join it. The total demand for platform G is now higher. Thus, G can differentiate and still rely on ad revenues even when other platform is user subscription based (the shaded region).

5.1. Private Incentive to Bundle

Now, we look at Firm G's incentive to bundle under different business models. The result is explained through a series of lemmas and illustrated in figure 4.

Ad Financed Model

Lemma 2: Consider the case when both platforms are ad financed under pure bundling (i.e. $0 < t < t_0$). Then there exists a threshold t_{p1} such that bundling

i) Improves Platform G's profits for $0 < t < t_{p1}$

ii) Reduces Platform G's profits for $t_{p1} < t < t_0$

iii) Reduces Platform S's profits for all values of t.

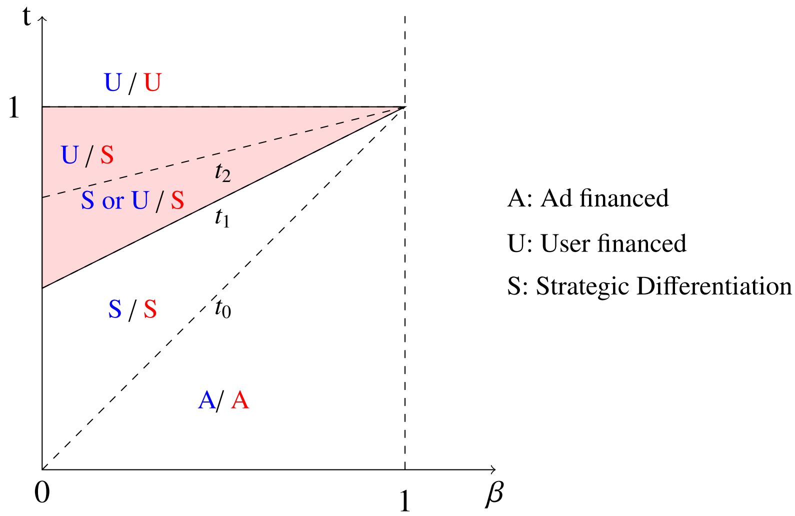


Figure 3: Independent Pricing vs Bundling: Choice of Business Model

Example involves k = 0.75. *Blue* letter represents IP regime equilibrium and *Red* letter represents bundling equilibrium.

The intuition stems from the data advantage and premium price per click that G can get from advertisers. When the data advantage i.e. $k(1-\beta)$ is high and t is low then, through bundling, platform G can earn higher premium for all users. Thus, $\tilde{p}_{G2} > p^*_{G2}$ and overall ad revenues improves for platform G. However, for higher t, users' ad aversion tends to dampen demand generated through bundling and profits decline as a result.

Strategic Differentiation

Lemma 3: Consider the case when platforms choose opposite business models under pure bundling (i.e $t_0 < t < 1$). Now two cases can be differentiated depending on which equilibrium occurs under no bundling case.

A) If equilibrium is user financed model for $t_1 < t < t_2$ under independent pricing then there exists t_{p1} and t_{p2} such that

i) Pure bundling improves firm G's profits if ia) $0 < t < min(t_1, t_{p1})$; and ib) $t_1 < t < t_{p2}$.
ii) Pure Bundling reduces firm G's profits if iia) $t_{p1} < t < t_1$ and iib) $t_{p2} < t$.
iii) Firm S's profits reduces under pure bundling.

B) If equilibrium is strategic differentiation for $t_1 < t < t_2$ under independent pricing then
i) Pure bundling improves firm G's profits for ia) $0 < t < min(t_2, t_{p1})$ and ib) $t_2 < t < t_{p2}$.
ii) Pure bundling reduces firm G's profits for iia) $t_{p1} < t < t_2$ and iib) $t_{p2} < t$.
iii) Firm S's profits reduces under pure bundling.

So, for intermediate values of t, bundling can improve platform G's profits if independent pricing equilibrium is user financed model or if platform G has high data advantage i.e. $k(1 - \beta)$ is high. In the former case, when platform G shifts best reply to ad slots under bundling and switch to ad financed model, then business model effect comes into play. Since, G strategically differentiates now, and advertisers solely advertise on G, it could improve profits under bundling scenario. Whereas, when G has initially chosen strategic differentiation, no business model effect works then. Platform G already is the sole platform for advertisers to advertise. Bundling does not bring any change in that. Now, since platform G charges no price for good 1, a part of revenue source is lost and ad revenues cannot compensate for that due to higher t. Hence, Platform G's profit reduces. For platform S, $\tilde{q_{S2}} < q^*_{S2}$ and number of users who join it also reduces. So, its profits reduces irrespective of the equilibrium business model.

User Financed Model

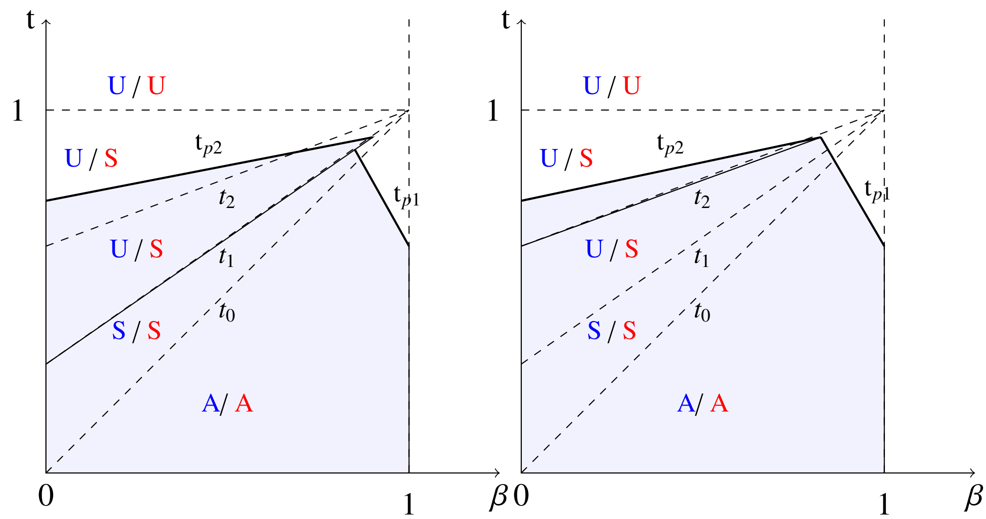
Lemma 4: When both platforms choose user financed model under bundling (i.e. $t > 1$) then profits of both platforms are reduced.

The sole effect of bundling here is reduction in profits. This happens because bundling effect on ad revenues is not present. It gives no data advantage to such platforms as they are user subscription based.
These results can be summarised in the form of proposition below:-

Proposition 6: In the baseline model with k sufficiently large
i) Pure bundling can improve platform G's profits for sufficiently low t and β.
ii) Pure bundling can reduce platform G's profits for sufficiently high t and high β.
iii) Pure Bundling always reduces platform S's profits.

This result is in contrast to Choi and Jeon (2016) paper. According to them, incentive to tie in advertising markets increases with increase in degree of two sidedness of tied market B and decrease in degree of two sidedness of market A. This is based on assumption of zero nuisance costs. In my model, the degree of two sidedness of tied market can be measured from the value of parameter β. It measures how much advertiser values the presence of other side i.e advertiser network benefits. Interpreting this way, Choi and Jeon (2016) result would have implied that "Profitability of bundling increases with increase in advertiser's network benefits in the tied market". Here, it is important to highlight the difference from Choi and Jeon result.
1. In the benchmark case in my model, when there is no connection between the two markets through user data, profitability of bundling increases with degree of two sidedness of market 2. This result is similar in spirit to Choi (2016).
2. In presence of data advantage i.e when markets are connected through user data , this result breaks down. Infact, the opposite might hold for some value of t i.e. profitability of tying decreases with two sidedness of market (high β).

The intuition of this result stems from the trade off which affects incentive to tie. The underlying economic forces are different in Choi and Jeon paper and my paper. In their paper, it was how advertising revenues are affected in two markets. But the two markets are not connected. In my paper, the trade off is between forgone user revenues in market 1 and higher advertising revenues in market 2. User revenues are affected by the degree of data advantage or in other words, degree of two sidedness of market 2 and, also, nuisance costs. So, the lower the degree of two sidedness (small β), the more aggressive pricing in market 1 and less is the loss in user revenue from bundling and higher are the gains from targeted advertising under bundling. Hence, bundling is profitable.



(a) User Financed Model under IP (b) Strategic Differentiation under IP

Figure 4: Private Incentive to Bundle

Example involves k = 0.75. *Blue* letter represents IP regime equilibrium and *Red* letter represents bundling equilibrium. Shaded region represents the area in which bundling improves Platform G's profits.

6. Social Welfare

In this section we derive the change in social welfare as a result of bundling. Clearly, the overall change would depend on the equilibrium price configurations. Here, we specify the change is social surplus under alternative business model choice.

6.1. *When Ad financed model is the equilibrium configuration under IP and bundling*

Since payments collected by platforms are simply transfers from users and advertiser, the change in social welfare (SW) is sum of change in gross user surplus (CS) and advertiser surplus (AS) i.e.

$$\begin{aligned}\Delta SW = & \underbrace{\int_0^1 vf(v)dv - \int_{q^*_{G1}}^1 vdv}_{\text{Change in user surplus in market 1}} - \\ & \underbrace{\int_0^1 [t\tilde{m_{G2}} + t\tilde{m_{S2}}] + \int_0^1 [tm^*_{G2}N^*_{G2} + tm^*_{S2}(1-N^*_{S2})]}_{\text{Change in nuisance costs}} - \\ & \underbrace{\int_0^1 [f(v)^2 - f(v)] + \int_0^1 [(N^*_{G2})^2 - N^*_{G2}]}_{\text{Change in Transportation costs}} + \\ & \underbrace{[\beta + k(1-\beta)]\tilde{m_{G2}}\tilde{N_{G2}} - [\beta + k(1-\beta)N_{G1}]m^*_{G2}N^*_{G2}}_{\text{Change in Advertiser Surplus on platform G}} + \\ & \underbrace{\beta\tilde{m_{S2}}\tilde{N_{S2}} - \beta m^*_{S2}N^*_{S2}}_{\text{Change in Advertiser Surplus on Platform S}}\end{aligned} \tag{14}$$

Where $\int_0^1 f(v) = \tilde{N_{G2}}$ and rest of the notations are same as defined in earlier sections. The over channel change in social welfare would depend on the sign of each change. Here, we outline the predicted signs:-

1. Change in consumer welfare in market 1: It may increase or decrease depending on parameter values

2. Change in nuisance cost: negative

3. Change in Transportation cost: negative

4. Change in Advertiser Surplus: It may increase or decrease depending on parameter values

The net effect on welfare would depend on how surplus changes in market 1 monopolized by platform G and advertising market. The interplay depends on parameter t and β. The first term increases with t and falls with β. Whereas, the last term, rises with β. For low values of t or β, social welfare would increase whereas for high values of t or high β it falls. In the former case, low β would improve advertiser surplus from platform G sufficiently to overcome distortions arising from rise in nuisance and transportation costs and fall in advertising revenues on platform S. When t is low, user surplus in market 1 may fall. But advertiser gains from data advantage would be sufficient to improve welfare overall.

Proposition 7: When both platforms adopt advertising business models under bundling i.e. $0 < t < t_0$ then there exists a threshold t_{s1} such that social welfare rises for $t < t_{s1}$ and falls for $t > t_{s1}$.

6.2. When strategic differentiation is the equilibrium business model under IP and bundling

Social welfare in this case can be written as

$$\Delta SW = \underbrace{\int_0^1 vf(v)dv - \int_{q^*_{G1}}^1 vdv}_{\text{Change in user surplus in market 1}} - \underbrace{\int_0^1 t\tilde{m_{G2}} + \int_0^1 tm^*_{G2}N^*_{G2}}_{\text{Change in nuisance costs}} - \underbrace{\int_0^1 [f(v)^2 - f(v)] + \int_0^1 [(N^*_{G2})^2 - N^*_{G2}]}_{\text{Change in Transportation costs}} + \underbrace{[\beta + k(1-\beta)]\tilde{m_{G2}}\tilde{N_{G2}} - [\beta + k(1-\beta)N_{G1}]m^*_{G2}N^*_{G2}}_{\text{Change in Advertiser Surplus}} \quad (15)$$

The crucial difference from previous case is that now advertiser surplus always rises with bundling. Similar to last case, we can identify a threshold t'' below which welfare rises. It can be stated as a proposition:

Proposition 8: When strategic differentiation is the equilibrium under both regimes i.e $t_0 < t < t_1$ then there exists a threshold t_{s2} such that for $t_0 < t < t_{s2}$ social welfare rises with bundling and falls for $t_{s2} < t < t_1$.

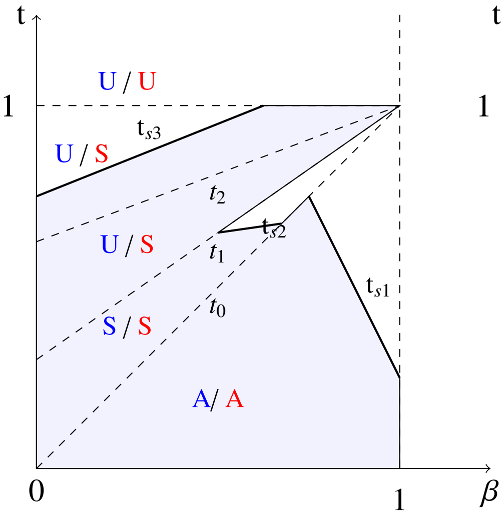
6.3. When user financed is the equilibrium under IP and strategic differentiation is the equilibrium under bundling

In this case the change in social welfare can be written as

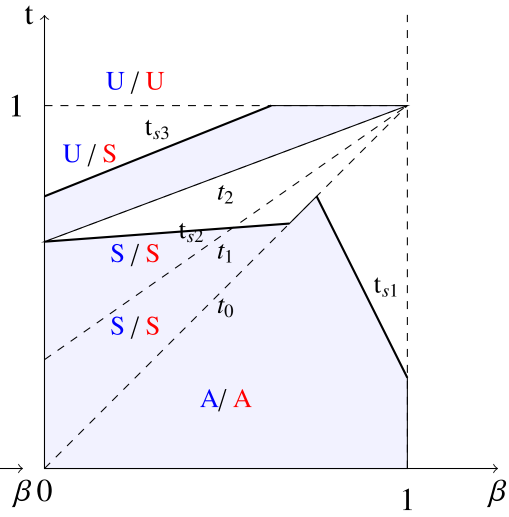
$$\Delta SW = \underbrace{\int_0^1 vf(v)dv - \int_{1/2}^1 vdv}_{\text{Change in user surplus in market 1}} - \underbrace{\int_0^1 t\tilde{m_{G2}}]}_{\text{Change in nuisance costs}} - \underbrace{\int_0^1 [f(v)^2 - f(v)] + \int_0^1 [(N^*_{G2})^2 - N^*_{G2} +}_{\text{Change in Transportation costs}} \underbrace{[\beta + k(1-\beta)]\tilde{m_{G2}}\tilde{N_{G2}}}_{\text{Change in Advertiser Surplus}} \tag{16}$$

In this case, user surplus in market 1 always fall with bundling. The only gain from bundling is the advertiser revenues from platform G net of nuisance costs.

Proposition 9: For $t_1 < t < 1$ and user financed is the equilibrium under IP for $t_1 < t < t_2$, then there exists t_{s3} such that social welfare rises for $t_1 < t < min[t_{s3}, 1]$ and falls otherwise.



(a) User Financed Model under IP



(b) Strategic Differentiation under IP

Figure 5: Change in Social Welfare
Example involves k = 0.75. *Blue* letter represents IP regime equilibrium and *Red* letter represents bundling equilibrium. Shaded region represents the area in which bundling improves Social Welfare.

6.4. *When user financed model is the equilibrium under both IP and bundling*

Social welfare can be written as

$$\Delta SW = \underbrace{\int_0^1 vf(v)dv - \int_{1/2}^1 vdv}_{\text{Change in user surplus in market 1}} - \underbrace{\int_0^1 [f(v)^2 - f(v)] + \int_0^1 [(N_{G2}^*)^2 - N_{G2}^*]}_{\text{Change in Transportation costs}} \tag{17}$$

In this case user surplus in market 1 falls and in market 2 also user surplus decreases. There is no revenue gain that can occur from advertising side through bundling. Hence, social welfare always falls.

Proposition 10: When user financed is the equilibrium business model under both regimes i.e. $t > 1$ then social welfare always fall.

One implication of above result is that social and private incentives are not the same over the parameter region. The divergence between the two comes from how user surplus in market 2 changes with bundling. The above welfare analysis can help us to state the following result.

Corollary 1: For $t < min[t_{s1}, t_{s2}]$ and $t_2 < t < t_{p2}$ bundling increases platform G's profit and social welfare. Otherwise, there can be a divergence between the two.

To illustrate this result graphically, look at figure 6. There exist region (blue region), where social and private incentive converge i.e bundling increases both G's profit and social welfare. In grey region, there is excess private incentive to bundle i.e bundling increases G's profits but reduces social welfare. In green region, bundling will decreases G's profits but increase social welfare.

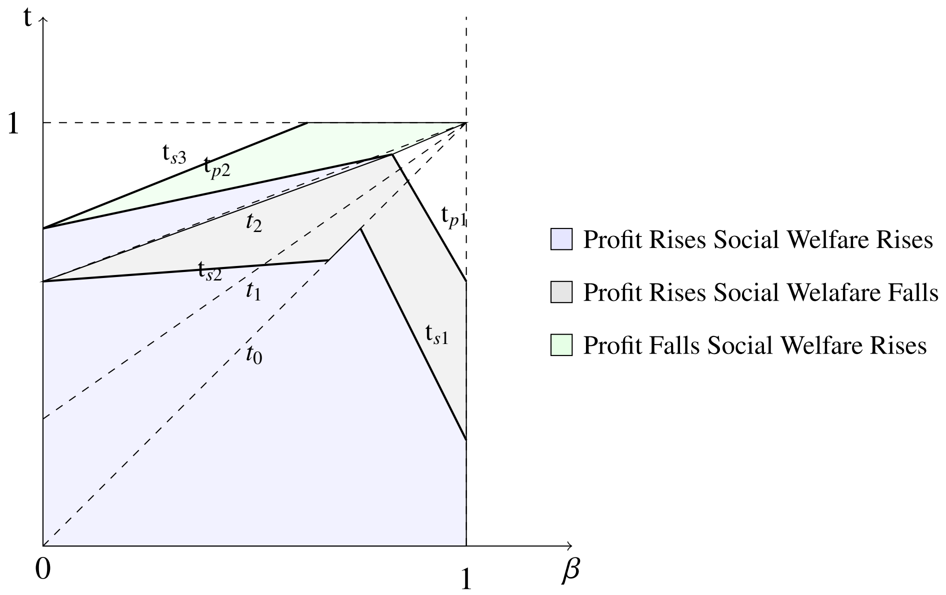


Figure 6: Private vs Social Incentive to Bundle
Example involves k = 0.75.

From figure 6, it is clear that there can be regions where private and social in-

centives converge or diverge. Thus, in presence of data advantage, markets with different parameter values of t or β can have different implication for welfare. In markets with small advertiser network benefits i.e low β, social welfare and G's profit rises. This is due to the data advantage and better predictability of user behaviour that bundling allows which helps advertiser to obtain better returns from their investments. Whereas, in markets with large β bundling is profitable at the expense of reduced social welfare.
Finally, I analyse user surplus and how it changes with bundling. A comparison with social welfare shows that there can be parameter region where bundling improves social welfare but at the cost of user welfare.

Proposition 11: When platform G adopts pure bundling then there exist thresholds $t_c, t_{p1}, t_{p2}, t_{s1}, t_{s2}$ and t_{s3} such that
i) Bundling reduces user welfare but improves platform G's profit and social welfare when $t < min[t_{s2}, t_c]$
ii) Bundling improves platform's profits, user welfare and social welfare for $t_c < t < t_{s1}$ and $t_2 < t < t_{p2}$.
iii) Bundling improves platform G's profits and user welfare but reduces social welfare for $t_{s1} < t < t_{p1}$.
iv) Bundling improves user welfare but reduces platform's profit and social welfare for $t_{p1} < t < t_2$ and $max[t_{s3}, 1] < t$.
v) Bundling improves user welfare and social welfare but reduces G's profit for $t_{p2} < t < min[t_{s3}, 1]$.
vi) Bundling reduces user welfare and social welfare but improves G's profit for $t_{s2} < t < min[t_c, t_2]$.

The above proposition is illustrated in figure 7. For reference, the sign of platform G's profit, user welfare and social welfare are presented in the table 1 below.

Table 1: Change in Platform G's profit, User Welfare and Social Welfare

Region	**Change in G's profit**	**Change in CS**	**Change in SW**
Region I (Blue)	+	-	+
Region II(Red)	+	+	+
Region III(Grey)	+	+	-
Region IV(White)	-	+	-
Region V(Green)	-	+	+
Region VI(Yellow)	+	-	-

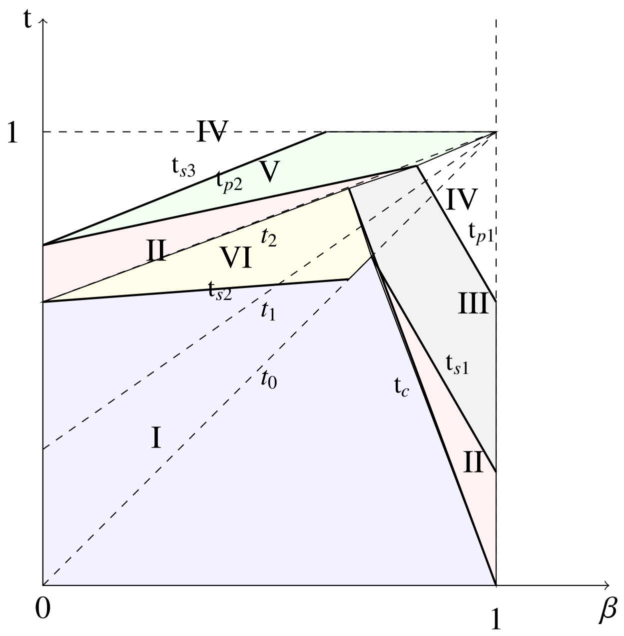


Figure 7: Platform G's profit, User Welfare and Social Welfare
Example involves k = 0.75.

In Region I (refer figure 7), social welfare and platform G's profit rises but user welfare falls with bundling. The reason could be higher nuisance cost which is not compensated by higher surplus in market 1. In region II, platform profits, user welfare and social welfare increases. In this region, user surplus generated in market 1 is sufficient to cover up the rise in nuisance cost and transportation cost. In region III, social welfare falls but platform G's profit and user welfare increases with bundling. Here, nuisance cost parameter is high enough to generate sufficient gains from bundling in market 1 but advertiser incur loss reducing overall welfare. In region IV, either data advantage is very small or nuisance cost of ads is very high such not both platform G and society looses but user welfare improves. In region V, user welfare increases and sufficient to compensate for any fall in advertiser revenues due to bundling leading to rise in social welfare. In region VI, user and social welfare falls under strategic differentiation.
From figure 7, it is clear that the parameter region where firm and society would disagree depends on the welfare standard and also the equilibrium business model. A useful case study can be when the dominant platform is ad financed in equilibrium. This can happen if either there is strategic differentiation or ad financed equilibrium under both regimes. In this case, if competition authorities follow a total welfare standard, bundling would be prohibited in region III, but users would loose. On the contrary, in region I, if bundling was allowed, users would loose because gains from bundled discounts are not sufficient.

7. Policy Implication

From the above analysis, it is clear that how an antitrust intervention requiring mandatory unbundling affects market would depend on the eqilibrium business model and the degree of two sidedness of the tied market or in other words, the strength of advertiser network benefit. Here we summarise the implications for antitrust derived from last two sections for the case when the dominant platform

G is ad financed.

Corollary 2: The first implication for antitrust is the following (Refer Figure 6)

1. In markets with small β, incentive to bundle based on data advantage is very high and social welfare can increase.
2. In market with large β, incentive to bundle may or may not exist. If it exists, then social welfare falls.

Corollary 3: The second implication for antitrust is (Refer Figure 7)

1. In markets with small β, bundling is privately optimal. But user welfare decreases and social welfare may increase (region I) or decrease (region VI).
2. In markets with large β, bundling may or may not be profitable. If it is then user welfare increases and social welfare may increase (region II) or decrease (region III). If it is not then user welfare increase but Social welfare falls (region IV).

8. Application and Conclusion

In this paper, a multi product platform has to decide whether to sell two goods independently or as a bundle. A useful application of the results presented is Google's strategy to bundle Android operating system with its other apps. It can be argued that through MADA requirements Google is able to leverage its dominance in mobile sector to maintain and strengthen its dominance in search advertising sector. From the begining, Google has offered Android to hardware manufacturers at no cost. It intends to make no profit from sale of android phones to users. Instead it is used an indirect tool to attract as much attention as possible from users on other platforms such as Google search, Maps, Youtube etc. It can use this attention to amass advertising revenues. The mechanism can be explained as follows. Through Android phones, Google has access to critical location data. It can decipher the location where people were when they made the searches. This information, in complement to other information collected, gives Google a big data advantage over rivals. It not only marginally increases users' search results but, more importantly, raises the willingness to pay by advertisers. This gives Google a premium per click on ads compared to any rival in keyword based advertising sector. It controls 85 percent of search ad revenues and over 90 percent of mobile advertising revenues [10]. This surge in advertsing revenues is not only a result of rising user market share but, more importantly, from premium price on each click these users make on an advertisement. Thus, in presence of cross market data advantage, bundling Android with other apps is a profitable strategy for Google.

Based on a simple model, it was shown that the profitability of pure bundling depends on the interplay of nuisance cost and strength of network benefits on the advertiser side. Profitability of bundling for an ad financed multi product platform falls with strength of advertiser network benefits. Moreover, bundling is never profitable when user financed model is adopted. This is because the synergies that exist across markets on advertiser side does not exist in this case and bundling only generates losses for the platform.

Next, how bundling affects social welfare was evaluated. The parameter regions where private and social incentive to bundle coincide or diverge were clearly specified. The platform can have excess incentives to bundle vis a vis a social planner. Lastly, individual welfare components behave differently with bundling. There exist parametric regions where user surplus and advertiser surplus move in opposite directions. This can happen for instance when nuisance cost and advertiser network benefits are small.

[10]See Nathan Newman (2015), How Mobile Supports Google's Online Adverting Dominance: Why the European Union Competition Authority Should Take Action, Data Justice Policy Brief.

9. References

Amelio, Andrea and Bruno Jullien (2012). “Tying and Freebies in Two-Sided Markets”. International Journal of Industrial Organization, 30.5, pp. 436446.

Armstrong, Mark (2006). “Two-Sided Markets, Competitive Bottlenecks and Exclusive Contracts”. Economic Theory, 32: 353-380.

Bork, Robert.H J. Gregory Sidak (2012) “What Does the Chicago School Teach About Internet Search and the Antitrust Treatment of Google?”. Journal of Competition Law and Economics, 8(4): 663–700.

Calvano, Emilio and Michele Polo (2016). “Strategic Differentiation by business models: Free-to-Air and Pay-TV”. CSEF Working Paper 438.

Carlton, Dennis W. and Michael Waldman (2002). “The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries”. RAND Journal of Economics 33.2, pp. 194-220.

Casadesus-Masanell and Feng Zhu (2010). “ Strategies to fight ad-sponsored rivals”. Management Science, 56(7): 1484–1499.

Casadesus-Masanell, Ramon and Feng Zhu (2011). “Business Model Innovation and Competitive Imitation: The Case of Sponsor-Based Business Models”. HBS Working Paper 11-003.

Choi, Jay Pil and Christodoulos Stefanadis (2001). “Tying, investment, and the dynamic leverage theory”. RAND Journal of Economics 32.1, pp. 52-71.

Choi, Jay-Pil (2010). “Tying in Two-Sided Markets with Multi-Homing”. Journal of Industrial Economics LVIII.3, pp. 607-626.

Choi, Jay-Pil and Doh-Shin Jeon (2016). “A Leverage Theory of Tying in Two-Sided Markets”. Working Paper.

Corniere, Alexandre de and Greg Taylor (2017) “Application Bundling in Platform Markets”. TSE Working Paper, n. 17-827, July 2017.

Edelman, Benjamin and Damien Geradin (2016) “Android and Competition Law: Exploring and Assessing Google’s Practices in Mobile”. Harvard Business School working paper, 17-018.

Manne, Geoffrey Joshua Wright (2011) “Google and the Limits of Antitrust: The Case Against the Case Against Google”, Harvard Journal of Law Public Policy,34, 171- 244.

Nalebuff, Barry (2004) “Bundling as an Entry Barrier”. Quarterly Journal of Economics, 119.1, pp. 159-187.

Nathan Newman, Nathan (2014) “Search, Antitrust, and the Economics of the Control of User Data”. Yale Journal of Regulation, Volume 31(2): 402-452.

Whinston, Michael. D (1990). “Tying, Foreclosure, and Exclusion”. American Economic Review 80.4, pp. 837-859.

10. Appendix

Proof of Lemma 1

The first order necessary conditions of platforms' optimization problem are

$$\frac{\partial \pi_G}{\partial q_{G1}} = 1 - 2q_{G1} - k(1-\beta)N_{G2}m_{G2} \leq 0 \tag{18a}$$

$$\frac{\partial \pi_G}{\partial q_{G2}} = 1 + tm_{S2} - tm_{G2} + q_{S2} - 2q_{G2} - [\beta + k(1-\beta)N_{G1}]m_{G2} \leq 0 \tag{18b}$$

$$\frac{\partial \pi_S}{\partial q_{S2}} = 1 + tm_{G2} - tm_{S2} + q_{G2} - 2q_{S2} - \beta m_{S2} \leq 0 \tag{18c}$$

$$\frac{\partial \pi_G}{\partial m_{G2}} = -tq_{G2} + [\beta + k(1-\beta)N_{G1}][1 + tm_{S2} - 2tm_{G2} + q_{S2} - q_{G2}] \leq 0 \tag{18d}$$

$$\frac{\partial \pi_S}{\partial m_{S2}} = -tq_{S2} + \beta[1 + tm_{G2} - 2tm_{S2} + q_{G2} - q_{S2}] \leq 0 \tag{18e}$$

Where, strict inequality holds if the corresponding variable takes value zero.

Claim 1: There does not exist any solution with $q_{G2} > 0; q_{S2} > 0; m_{G2} > 0; m_{S2} > 0$.

Proof: Suppose $q_{G1} = 0$. Then, for above claim to hold all F.O.Cs corresponding to the variables $q_{G2}; q_{S2}; m_{G2}$ and m_{S2} must be equal to zero. Since, it becomes a system of linear equations, It can be shown that the rank of matrix corresponding to these four F.O.Cs (17b-17e) is less than four. Hence, no solution will exist with all four F.O.Cs equal to zero.

If $q_{G1} > 0$ then it is a system of non linear equations. It can be shown by substitution method that there does not exist a solution with all variables greater than 0. For example. suppose $q_{G1} > 0$ and $q_{S2} > 0$. Then, substituting for values of q_{G2} and q_{S2} from the F.O.Cs 17b and 17c into the F.O.Cs 17d and 17e, it becomes

$$3 + [\beta - t + k(1-\beta)N_{G1}]m_{G2} - (\beta - t)m_{S2} = 0$$

$$3 - [\beta - t + k(1-\beta)N_{G1}]m_{G2} + (\beta - t)m_{S2} = 0$$

These two equations cannot be explicitly solved for any $m_{G2} > 0$ and $m_{S2} > 0$. Hence, no solution exists. A contradiction.

Claim 2: There does not exist any solution in which platform i relies on both strategic variables i.e $q_{i2} > 0$ and $m_{i2} > 0$ while platform j uses a single price variable i.e $q_{i2} > 0$ or $m_{i2} > 0$.
Proof by contradiction: Suppose there exists such a solution. Using, F.O.Cs equal to zero corresponding to these variables, it can be shown that one of the strategic variables will be negative.

Proof of Proposition 3

The only possible candidates for a solution to the system of F.O.Cs are

1. $q_{G1} \geq 0; q_{G2} = 0; q_{S2} = 0; m_{G2} > 0; m_{S2} > 0.$
2. $q_{G1} \geq 0; q_{G2} > 0; q_{S2} > 0; m_{G2} = 0; m_{S2} = 0.$
3. $q_{G1} \geq 0; q_{G2} > 0; q_{S2} = 0; m_{G2} = 0; m_{S2} > 0.$
4. $q_{G1} \geq 0; q_{G2} = 0; q_{S2} > 0; m_{G2} > 0; m_{S2} = 0.$

The next step is to check whether F.O.Cs hold for the candidate solutions. It can be shown that (refer figure 1)
1. For $t < t_0$, candidate 1 satisfies F.O.Cs, where $t_0 = \beta$.

In equilibrium $q^*_{G1} = Max[\frac{1}{2} - \frac{k(1-\beta)}{4t}, 0]$. If $t < \frac{k(1-\beta)}{2}$, then $q^*_{G1} = 0$. Otherwise $q^*_{G1} > 0$. Hence

1a) For $t < t_0$ and $t < \frac{k(1-\beta)}{2}$, solution is $q^*_{G1} = 0; q^*_{G2} = 0; q^*_{S2} = 0; m^*_{G2} = \frac{1}{t} > 0; m^*_{S2} = \frac{1}{t} > 0.$

1b) For $\frac{k(1-\beta)}{2} < t < t_0$, solution is $q^*_{G1} > 0; q^*_{G2} = 0; q^*_{S2} = 0; m^*_{G2} = \frac{1}{t} > 0; m^*_{S2} = \frac{1}{t} > 0$.

2. For $\beta < t < t_2$, candidate 4 satisfies F.O.Cs, where

$$t_2 = \frac{(k(1-\beta)+2\beta)+[(k(1-\beta)+2\beta)^2+4k^2(1-\beta)^2]^{1/2}}{4}$$

Hence we have

2a) For $\beta < t < \frac{k(1-\beta)}{2}$ solution is $q^*_{G1} = 0; q^*_{G2} = 0; q^*_{S2} = 1 > 0; m^*_{G2} = \frac{1}{t} > 0; m^*_{S2} = 0$.

2b) For $\frac{k(1-\beta)}{2} < t < t_2$, solution is $q^*_{G1} = \frac{1}{2} - \frac{k(1-\beta)}{4t} > 0; q^*_{G2} = 0; q^*_{S2} = 1 > 0; m^*_{G2} = \frac{1}{t} > 0; m^*_{S2} = 0$.

3. For $t > t_1$, candidate 2 satisfies F.O.Cs, where

$$t_1 = \frac{k(1-\beta)+2\beta}{2}$$

Hence, solution is $q^*_{G1} = \frac{1}{2} > 0; q^*_{G2} = 1 > 0; q^*_{S2} = 1 > 0; m^*_{G2} = 0; m^*_{S2} = 0$.

Candidate 3 does not satisfy F.O.Cs for any parameter values. Also, since $t_1 < t_2$, there exist multiple equilibria with candidate 4 and candidate 2 as the solutions to F.O.Cs.
No deviation constraint: In order for these set of candidate solutions to be a nash equilibrium, the last part remaining is to show that there dose not exist any incentive to deviate for each platform from these candidate solutions 1, 2 and 4. Simple computations will show that, given the parametric restrictions, no deviation constraints are satisfied for them.

Proof of Proposition 5

The first order necessary conditions of platforms' optimization problem under bundling are

$$\frac{\partial \pi_G}{\partial q_G} = 1.5 + tm_{S2} - tm_{G2} + q_{S2} - 2q_G - [\beta + k(1-\beta)]m_{G2} \leq 0 \tag{19a}$$

$$\frac{\partial \pi_S}{\partial q_{S2}} = 0.5 + tm_{G2} - tm_{S2} + q_G - 2q_{S2} - \beta m_{S2} \leq 0 \tag{19b}$$

$$\frac{\partial \pi_G}{\partial m_{G2}} = -tq_G + [\beta + k(1-\beta)][1.5 + tm_{S2} - 2tm_{G2} + q_{S2} - q_G] \leq 0 \tag{19c}$$

$$\frac{\partial \pi_S}{\partial m_{S2}} = -tq_{S2} + \beta[0.5 + tm_{G2} - 2tm_{S2} + q_G - q_{S2}] \leq 0 \tag{19d}$$

It is a system of linear equations. Similar to the proof of lemma 1, it can be shown that

Claim 1: There does not exist any solution with $q_{G2} > 0; q_{S2} > 0; m_{G2} > 0; m_{S2} > 0$.

Claim 2: There does not exist any solution in which platform i relies on both strategic variables i.e $q_{i2} > 0$ and $m_{i2} > 0$ while platform j uses a single price variable i.e $q_{i2} > 0$ or $m_{i2} > 0$.

Since each platform will find it optimal to charge one side, the possible candidates for solution to the system of F.O.Cs are

1. $q_G = 0; q_{S2} = 0; m_{G2} > 0; m_{S2} > 0$.
2. $q_G > 0; q_{S2} > 0; m_{G2} = 0; m_{S2} = 0$.
3. $q_G > 0; q_{S2} = 0; m_{G2} = 0; m_{S2} > 0$.
4. $q_G = 0; q_{S2} > 0; m_{G2} > 0; m_{S2} = 0$.

The next step is to check the parametric conditions under which F.O.Cs hold for each candidate solution.

1. For $t < t_0 = \beta$, candidate 1 is the solution with $q^*_G = 0; q^*_{S2} = 0; m^*_{G2} = \frac{7}{6t} > 0; m^*_{S2} = \frac{5}{7t} > 0.$.

2. For $t_0 < t < 1$, candidate 4 is the solution $q^*_G = 0; q^*_{S2} = \frac{5}{6}0; m^*_{G2} = \frac{7}{6t} > 0; m^*_{S2} = 0.$.

3. For $t > 1$, candidate 2 is the solution $q^*_G = \frac{1}{2}; q^*_{S2} = \frac{1}{2}; m^*_{G2} = 0; m^*_{S2} = 0.$.

No deviation constraint: At each of the candidate solution 1,2 and 4, no platform has an incentive to deviate.
Hence, the SPNE are candidate 1,2 and 4.
There does not exist multiple equilibria and candidate 3 does not satisfy F.O.Cs for any parameter range. Now, since $t_2 < 1$, the parameter range over which strategic differentiation is the solution expands under bundling.

Proof of Lemma 2
For $0 < t < t_0$, we are in the case when ad financed model is the equilibrium business model under IP and bundling.

Profits under independent pricing are
$\pi^*_G = q^*_{G1}N^*_{G1} + [\beta + k(1-\beta)N_{G1}]N^*_{G2}m^*_{G2}$ if $q^*_{G1} > 0$ and

$\pi^*_G = [\beta + k(1-\beta)]N^*_{G2}m^*_{G2}$ if $q^*_{G1} = 0$
Putting in the values for price variables under two cases gives

$$\pi^*_G = \frac{1}{4} + \frac{\beta}{4t} + \frac{[\beta + k(1-\beta)]}{4t} + \frac{k^2(1-\beta)^2}{16t^2} \ : \ \text{if} q^*_{G1} > 0 \tag{20}$$

$$\pi^*_{G1} = \frac{[\beta + k(1-\beta)]}{2t} \ : \ \text{if} q^*_{G1} = 0 \tag{21}$$

Profit of Firm G under bundling is

$$\tilde{\pi_G} = [\beta + k(1-\beta)]\tilde{N_G}\tilde{m_G} = \frac{49[\beta + k(1-\beta)]}{72t}$$

Now, when $q^*_{G1} = 0 \Rightarrow \tilde{\pi_G} > \pi^*_G$ for all $0 < t < \frac{k[1-\beta]}{2}$.

When $q^*_{G1} > 0$, then $\tilde{\pi_G} = \pi^*_G$ at $t_{p1} > \frac{k[1-\beta]}{2}$. Where

$$t_{p1} = \frac{[13\beta + 31k(1-\beta)] + [(13\beta + 31k(1-\beta))^2 - 162k^2(1-\beta)^2]^{1/2}}{36}$$

This implies

i) $\frac{k[1-\beta]}{2} < t < t_{p1} \Rightarrow \tilde{\pi_G} > \pi^*_G$ and

ii) For $t_{p1} < t < t_0 \Rightarrow \tilde{\pi_G} < \pi^*_G$.
Platform S's profit are

$$\pi^*_S = \beta N^*_S m^*_S = \frac{\beta}{2t} : \text{ Independent Pricing}$$

$$\tilde{\pi_S} = \beta\tilde{N_S}\tilde{m_S} = \frac{25\beta}{72t} : \text{ Bundling}$$

For all t and $\beta \Rightarrow \tilde{\pi_S} < \pi^*_S$.
Hence proved.

Proof of Lemma 3

For $t_0 < t < 1$, table 2 shows the the equilibrium business models that can occur under IP and bundling.

Table 2: Equilibrium business models

Regime	$t_0 < t < t_1$	$t_1 < t < t_2$	$t_2 < t < 1$
IP	S	S and U	U
Bundling	S	S	S

Case I) The analysis for case $t_0 < t < t_1$ follows the same as under ad financed model for firm G. For platform S, the profits under two regimes are

$$\pi_S^* = \frac{1}{2} : \text{ Independent Pricing and } \tilde{\pi_S} = \frac{25}{72} \text{ under Bundling}$$

For all t and $\beta \Rightarrow \tilde{\pi_S} < \pi_S^*$.

Case IIA) If, for $t_1 < t < t_2$, equilibrium is strategic differentiation under independent pricing then analysis for platform G will follow the same way as under lemma 2 proof.

Case IIB) If, for $t_1 < t < t_2$, equilibrium is user financed model under independent pricing.
Platform G's profit is

$$\pi_G^* = \frac{3}{4} \text{ under IP and } \tilde{\pi_G} = \frac{49[\beta + k(1-\beta)]}{72t} \text{ under bundling.}$$

This gives a threshold $t_{p2} = \frac{49[\beta+k(1-\beta)]}{54}$ as shown in figure 4 such that

i) For $t_1 < t < max[t_2, t_{p2}] \Rightarrow \tilde{\pi_G} > \pi_G^*$

ii) For $t_{p2} < t < t_2 \Rightarrow \tilde{\pi_G} < \pi_G^*$

Case III) For the case, $t_2 < t < 1$, equiibrium is user financed model under IP and analysis follows the same way as in case IIB and platform G's profit rises for $t_2 < t < t_{p2}$ and falls for $t_{p2} < t < 1$. Platform S' profit falls.
Hence Proved.

Proof of Lemma 4

Platform G's profits are

$$\pi_G^* = \frac{3}{4} \text{ under IP } > \tilde{\pi_G} = \frac{49}{72} \text{ under bundling.}$$

Hence, bundling is unprofitable for platform G.
For platform S, bundling would have reduced its profit like in previous cases.

Proof of Proposition 7

There can be two cases depending whether platform G act as a loss leader under IP regime or not.
Case IA) $t < \frac{k[1-\beta]}{2}$: $q_{G1}^* = 0$. Then social welfare is

$$SW^* = X - \frac{3}{4} + \frac{[\beta + k(1-\beta)]}{2t} + \frac{\beta}{2t} : \text{ IP}$$

$$S\tilde{W} = X - \frac{22}{36} + \frac{49[\beta + k(1-\beta)]}{72t} + \frac{25\beta}{72t} : \text{ Bundling}$$

It is easily seen that $S\tilde{W} - SW^* > 0$ for all t $< \frac{k[1-\beta]}{2}$.

Case IB) $\frac{k[1-\beta]}{2} < t < t_0$: $q_{G1}^* > 0$. Then Social welfare is

$$SW^* = X - \frac{(q_{G1}^*)^2}{2} - \frac{3}{4} + \frac{[\beta + k(1-\beta)N_{G1}]}{2t} + \frac{\beta}{2t} : \text{ IP where } q_{G1}^* = \frac{1}{2} - \frac{k(1-\beta)}{4t}$$

$$S\tilde{W} = X - \frac{35}{36} + \frac{49[\beta + k(1 = \beta)]}{72t} + \frac{25\beta}{72t} : \text{ Bundling}$$

Comparison of social welfare under two regimes gives a threshold t_{S1} such that

$S\tilde{W} - SW^* > 0$ for $t < t_{S1}$ where

$$t_{s1} = \frac{[2\beta + 22k(1-\beta)] + [(2\beta + 22k(1-\beta))^2 - 189k^2(1-\beta)^2]^{1/2}}{14}$$

Hence proved.

Proof of proposition 8

There can be many cases depending on equilibrium business model under IP as shown in table 2.

I. Consider the case when $t_0 < t < t_1$. Similar to previous proof two cases can be differentiated depending on the value of $q^*_{G1} = 0$ under IP.

Case IA) When $t_0 < t < \frac{k[1-\beta]}{2} \Rightarrow q^*_{G1} = 0$ under IP. Then, social welfare is

$$SW^* = X - \frac{1}{4} + \frac{[\beta + k(1-\beta)]}{2t} \; : \; \text{IP}$$
$$S\tilde{W} = X - \frac{45}{72} + \frac{49[\beta + k(1-\beta)]}{72t} \; : \; \text{Bundling}$$

Straightforward computation shows that $S\tilde{W} - SW^* > 0$.

Case IB) $\frac{k[1-\beta]}{2} < t < t_1 \Rightarrow q^*_{G1} > 0$. Then social welfare is

$$SW^* = X - \frac{(q^*_{G1})^2}{2} - \frac{1}{4} + \frac{[\beta + k(1-\beta)N^*_{G1}]}{2t} \; : \; \text{IP where } q^*_{G1} = \frac{1}{2} - \frac{k(1-\beta)}{4t}$$
$$S\tilde{W} = X - \frac{35}{72} + \frac{49[\beta + k(1-\beta)]}{72t} \; : \; \text{Bundling}$$

Comparison of social welfare gives a threshold t_{s2} such that for $\frac{k[1-\beta]}{2} < t < t_{s2}$ social welfare rises and falls for $t_{s2} < t < t_1$, where

$$t_{s2} = \frac{[13\beta + 22k(1-\beta)] + [[13\beta + 22k(1-\beta)]^2 - 486k^2(1-\beta)^2]^{1/2}}{36}$$

II. Consider the case when $t_1 < t < t_2$. In this case also, two further cases can be differentiated based on whether we have strategic Differentiation or User financed as the equilibrium under independent pricing.
If Strategic differentiation is equilibrium under independent pricing. In this, the analysis would follow the same as in case IB).

III. Consider the case when $t_2 < t < 1$. Then Social welfare is

$$SW^* = X + \frac{1}{8} : \text{ IP}$$
$$S\tilde{W} = X - \frac{35}{72} + \frac{49[\beta + k(1-\beta)]}{72t} : \text{ Bundling}$$

Comparison of social welfare gives a threshold $t_{s3} = \frac{49[\beta+k(1-\beta)]}{44} > t_2$. Thus, social welfare falls.
Hence Proved.

Proof of proposition 9
Rest of the remains the same as in last proof except that user financed is the equilibrium under IP for $t_1 < t < t_2$. Then Social welfare is as given in case IIIB) in last proof. Comparison of social welfare gives a threshold $t_{s3} > t_2$ such that for it rises for $t_1 < t < t_{s3}$ and falls for $t_{s3} < t < 1$.
Proof of Proposition 10
For $t > 1$, user financed is the equilibrium business model under IP and Bundling. Then, social welfare is

$$SW^* = X + \frac{1}{8} : \text{ IP}$$
$$S\tilde{W} = X + \frac{4}{72} : \text{ Bundling}$$

Therefore, social welfare falls with bundling.

Proof of Proposition 11

The thesholds $t_{p1}, t_{p2}, t_{s1}, t_{s2} and t_{s3}$ are as derived in previous proofs. Only threshold t_c is left to be calculated. For the change in user welfare with bundling, just like previous proofs, various cases need to be considered.

Case I) When $t < \beta$. Ad financed is the business model under IP and bundling. This case can be further subdivided depending on the value of q^*_{G1} under IP.

Case IA) When $0 < t < \frac{k[1-\beta]}{2}$ then $q^*_{G1} = 0$. Then user welfare is

$$UW^* = X - \frac{3}{4} : \text{ IP}$$

$$\tilde{UW} = X - \frac{35}{36} : \text{ Bundling}$$

Clearly, $\tilde{UW}$ - $UW^* < 0$.

Case IB) when $\frac{k[1-\beta]}{2} < t < t_0$ then $q^*_{G1} = \frac{1}{2} - \frac{k(1-\beta)}{4t}$. User welfare is

$$UW^* = X - \frac{3}{4} + \frac{(q^*_{G1})^2}{2} - q^*_{G1} \text{ IP}$$

$$\tilde{UW} = X - \frac{35}{36} : \text{ Bundling}$$

There exists a threshold $t_c = 2.53k(1-\beta)$ such that for $\frac{k[1-\beta]}{2} < t < 2.53k(1-\beta)$ UW falls and rises for $2.53k(1-\beta) < t < t_0$.

Case II) When $t_0 < t < t_1$, strategic differentiation is the equilibrium business model under both IP and bundling. The value of UW under two regimes is the same as for case I and analysis remains the same.

Case III) When $t_1 < t < t_2$ and strategic differentiation is the equilibrium business model under IP. The analysis remains the same as for case IB. When user financed is the equilibrium business model under IP, user welfare is

$$UW^{*} = X - \frac{9}{8} : \text{ IP}$$

$$\tilde{UW} = X - \frac{35}{36} : \text{ Bundling}$$

Clearly,user welfare rises.

Case IV) When $t > 1$, the analysis remains the same as in previous case and user welfare rises.

Hence Proved.