How to Deter Crimes?*

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Abstract

Controlling crimes of different types to minimize social harm requires carefully targeted enforcement measures depending on whether the crimes are vertically linked or unrelated. Equally important is the consideration whether the crime data can be documented or law enforcement must rely only on detection data. Crime data is a sufficient statistic for enforcement initiatives, whereas detection data lacks this quality. Crime deterrence budget allocation as part of an incentive design is thus a 2×2 combinatorial exercise.

Our main observations are as follows. For linked crimes the State should target to block 'ground zero', the origin on the feeder side of crimes, as a priority, thereby denying their downstream operations ('Say's Law'). An example would be that the State focuses on foiling drug smuggling at various ports of entry than catching isolated drug transactions. But given that ground zero enforcement does not always succeed, the State might not be able to set an absolute priority of upstream deterrence and neglect downstream operations. In fact, on two occasions downstream enforcement might be granted higher priorities: (i) downstream detections have a secondary trace-back effect in discouraging upstream crime, (ii) without the culmination of downstream crime the social harm of upstream crime is negligible.

For independent crimes, the optimal budget allocation depends critically on the availability of crime data. Detection-based enforcement makes compliance beyond a certain level impossible, whereas incentives based on a verifiable measure of crime enables the State to increase compliance at first-best cost. This means the State may allocate more resources to prevention of burglaries and breaking of bank tills, which only transfer values from rich to the poor, than for the detection of drug smuggling and peddling that may destroy many young lives.

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Key Words: Decentralized crimes, crime chain, externalities, drug supply, burglaries, illegal immigration, forced prostitution; budget allocation, crime data, sufficient statistic, first-order reward; missing information, detection data, secondary discipline, moral hazard rent, limits of enforcement, Say's Law.

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1 Introduction

Crime is extremely diverse by its harm, nature, links and organization. Some, like shoplifting and cybercrime, are unrelated, whereas large-scale drug smuggling across the border and disparate small-scale drug peddling on street corners form an input-output chain. Selling drugs raises the benefit from other crimes as well like theft or other violent crimes due to the intoxicating effects of drugs. This diversity defies a unified approach to the budgeting and motivation of law enforcement activities.

While it is unclear whether to set deterrence targets for independent crimes separately, curtailing one of two interrelated crimes should clearly affect the deterrence of the other. A relevant issue facing any government therefore is how to structure enforcement incentives for independent law enforcement branches – through individualized incentives or a coordinated (or joint) incentives design? Does coordination imply the performance targets of one law enforcement division also depend on the performance of another division? That is, should the divisions be encouraged to compete or cooperate in controlling crimes?

Every fiscal year federal and state governments branch out enormous sums to local and specialized law enforcement units, who in turn justify their budget demands by formulating verifiable performance targets. For the central management the question is, what apparent results should (or can) enforcement units deliver? The moral hazard problem in law enforcement has led governments to seek and develop verifiable correlates of performance, which are increasingly common and tied to agency budgets−a trend that is expected only to grow.[1] Though many of these indicators are primarily related to detections or apprehensions, at the level of state or metropolitan police departments one can find others based on the crime level.[2] It is thus important to understand the power and limits of enforcement incentives depending on alternative indices of crime that can be applied in any given context.

We consider a model with two independent law enforcement agencies each responsible for controlling one of two crimes. The output of each agency is an intensity of enforcement, i.e., probability of detection, produced by unobservable effort in combination with other resources. The objective of the State is to minimize total harm from crimes by allocating a budget of fixed size and designing incentives that include rewards to ensure that the agencies exert appropriate efforts and use their

[1] See Sherman (2013), who traces the first important initiatives of statistical management in the UK in the Thatcher era. Police agencies were ranked and assessed on whether they had met specific goals on the basis of key performance indicators, on national and local levels. In the U.S., the Department of Justice Fiscal Years 2016-2017 priority goals include, besides specific national and cyber security targets (disruption of 400 terrorist groups or threats, dismantling of 1,000 cyber threat actors), five percent increases over Fiscal Year 2015 targets in the numbers of investigations concerning sexual exploitation of children and human trafficking by September 30, 2017; see U.S. Department of Justice (2016). In the fight against health care fraud, the department formulates its success (in collaboration with other agencies) in monetary terms, by \$7.70 to \$1 return on investment for law enforcement and detection efforts in Fiscal Year 2014."

[2] For example, the Metropolitan Police Department of DC mentions ten measures: Percentage changes in the number of homicides, violent crime and property crime, besides clearance rates for homicides, forcible rape, robbery, aggravated assault, burglary, larceny-theft and motor vehicle theft. Data to be used consists of reported crimes. Relevant reference to be added.

budgets most effectively.[3] The analysis distinguishes between independent crimes and two cases of interlinked crimes. For the latter, in one variant commission of a root (cause) crime leads to an increase in the potential criminal population of another (effect) crime, and in a second variant the upstream crime supplies an indispensable input to a downstream crime. In this last environment, upstream criminals may be detected before matching with their downstream partners and hence before realizing their benefits, whereas downstream criminals commit the crime, and thus can be detected, after matching with an upstream partner. Deterrence of one of these crimes may create shortages on one side, reducing the matching prospect and the expected benefits for the other side. This creates positive enforcement externalities.

The need to motivate law enforcement brings in a second layer of distinction, according to the type of observable statistic available for that purpose, which depends on the crime. Some crimes, like open-air drug markets, are more visible than others like sexual crimes or corruption; in the case of the latter victims may not report the crime (e.g., in domestic abuse) for fear of retaliation. When a crime is not directly observable and its occurrence or non-occurrence can be ascertained by law enforcers only, the State has to rely on detection or apprehension data alone to motivate the enforcement agency.[4] On the other hand, the combat against observable or predominantly reported crimes can admit both the crime data and the detection data.

Intuition may favor crime data over detection data in the provision of enforcement incentives for its apparent congruence with the harm-minimization objective of the State. Our analysis confirms the choice, if not the intuition: Crime-based incentive systems should be preferred because they satisfy a fundamental property for implementation of crime levels at first-best cost, namely, *monotonicity in enforcement effort*. We identify crime environments in which none of the detection-related data satisfy the effort monotonicity property. For the crime environments we consider in this paper, crime-based performance indicators weakly dominate those that are detection-based.

The potential limitation of detection-based incentives can be understood by visualizing the relation between total detected criminals and enforcement intensity (probability of detection), for independent crimes. The measure of detected criminals reaches a maximum in the probability of detection, beyond which it falls, to a minimal level in the limit if all criminals are sought to be detected with probability one. It follows that high enforcement efforts and the corresponding high levels of deterrence cannot be induced by setting a detection target because the agency can achieve the same detection target with a lower effort. Nor can the State prop up deterrence by relying on other operational supports subject to less stringent or no moral hazard constraints, for any such

[3] Sherman (2013) notes the need and inclination among enforcement experts to create an aggregate macro-level measure of crime, akin to the State's objective in our model, an index made up of harm-weighted averages of different crimes.

[4] Alternatively the State could use victimization surveys that indicate the extent of the unreported crime (in the United States, the FBI's Uniform Crime Reports and the National Crime Victimization Survey). It goes without saying, and we acknowledge, that the performance indicators could be subject to influence, even outright manipulation including flexibility in crime definitions, by the enforcement agencies themselves. However, safeguards against abuse seem to be operating with integrity, at least in the case of computerized statistics COMPSTAT (developed by William Bratton and Jack Maple in use in NYPD since 1996), as noted by Sherman (2013), in some instances leading to arrests of local commanders charged with manipulating crime statistics.

attempt will be continue to be upset by the agent further adjusting efforts downwards consistent with target detections (due to substitutability between efforts and other operational supports). This highlights a feasibility problem in the combat against unobservable crimes for which the only data that can be produced would be detections. In the case of an observable independent crime, if the State has a large budget with an ambitious crime reduction objective, performance targets of the agencies should be formulated in terms of crime levels, not detections. High deterrence levels are not compatible with detection-based incentives, due to moral hazard, though low deterrence levels can be induced just as effectively as under crime-based incentives.

Crime-based incentives continue to implement harm minimizing crime levels for crimes linked by unidirectional causality or those forming a vertical input-output chain. These types of crime environments, however, also present a richer set of implementation possibilities for detection-based incentive systems. We allow for the possibility of tracing back criminals of root or upstream crimes from detection of downstream crimes, in addition to other cross-detection effects of enforcement that operate even without the possibility of backtracing. In the case of crimes that form input-output chain, we show that first-best enforcement incentives can be restored under detection-based systems, provided upstream-crime detection data can be decomposed into its components as those purely owing to downstream enforcement and those due to upstream enforcement. In the case of causal links, i.e., when one crime leads to an increase in the potential criminals of another crime, root crime enforcement costs are first-best because, as we show, detections of the effect crime are monotonic in root crime enforcement. However, none of the detection measures are monotonic in downstream effect-crime enforcement effort and therefore detection-based incentive systems become more costly for the effect crime.

The optimal budget allocation depends primarily on enforcement costs, hence on the incentive systems and whether crime and/or detection data are available, the relative harms from crimes and the criminal benefit distributions. In the reference case of independent crimes, it is optimal to allocate a larger enforcement budget to the crime that causes a larger harm, as expected, provided crime data is available for both crimes. In the interlinked case of vertical crime chains, we identify a structural mechanism which favors concentration of enforcement efforts on the upstream/root crime. *Undeterred* downstream criminals seek to match with *undetected* upstream criminals who supply the instrument they need to complete the crime. Under conditions of symmetry, an equal distribution of the enforcement budget between the two crimes produces an excess demand for the instrument needed by *undeterred* downstream criminals. A fraction of these undeterred criminals will not be able to commit the crime. This *indirect deterrence* effect creates a tendency for the State to allocate a smaller budget to the downstream crime unless its social harm is substantially larger than the upstream crime. One possible exception to the above prescription is when the detection of downstream crime has an adversarial trace-back effect on the incentives of committing upstream crime. There, spending a bigger proportion of the crime deterrence budget on downstream enforcement gains grounds.[5]

[5] The exact breakdown of the enforcement budget and how law enforcers' rewards should be designed will depend on

The tendency to favor the upstream root crime persists in the presence of a causal link between the crimes, qualified by the trace-back effect mentioned above. However, if crime data is not available for incentive provision, under detection-based incentives the State has another reason to allocate a larger budget to the root crime. Unless the enforcement budget and the target crime rate is small, detection-based incentives for the effect crime are subject to the same feasibility problem that plagues implementation of independent crimes.

■ **Related Literature.** The economics literature of prime relevance to our paper, formal analysis of incentives and budgeting in law enforcement, is small. The efficiency of crime-based incentives in coping with moral hazard was first pointed out by Harris and Raviv (1978) in a single crime context as an application of their agency model. Their analysis has not been extended any further than a few applications to specific enforcement contexts. Graetz et al. (1986) is the first formal treatment of the incentive problem in a moral hazard framework with an explicit objective for the agency enforcing tax compliance. Besanko and Spulber (1989) study the effort commitment problem in a game between the law enforcer and a representative criminal. A common ground of these models and ours is inclusion of the law enforcement agency as a separate decision maker. We follow their approach in taking criminal sanctions as exogenously given, to keep the focus on incentives and the allocation of enforcement resources.[6] Also related to our paper is Polinsky and Shavell (2001) who study optimal incentives for corruptible law enforcers. Rewards for crime detections are determined by the tradeoff the State faces from the possibility that crimes can be concealed in return for bribes and the possibility of framing innocent individuals. They do not address the question as to how detections can be used to cope with moral hazard in enforcement efforts, which we address in this paper.

While research on the efficiency of public enforcement resources, predominantly by criminologists (e.g., Sherman, 2013, and the references therein), concentrated on allocations according to activity type such as imprisonment, policing and prevention, the impact of crime-based re-allocation of expenditures has been studied in one instance. Benson et al. (1995) discuss the response of law enforcement system to incentives in a specific instance, the Comprehensive Crime Act of 1984 which allows police agencies to keep the proceeds of assets forfeited as a result of drug enforcement activities. They argue that the resource shift which the policy entails from non-drug crime such as burglary to drug crimes has induced an increase in drug enforcement efforts, but with a pessimistic

whether overall detection of upstream crimes can be verifiably attributed to upstream and downstream initiatives or such a distinction is not feasible. In either environment, the externalities that exist between upstream and downstream deterrence can get aggravated or mitigated, if not completely internalized, based on how swiftly downstream detections may lead to its upstream root, from zero link to a complete capture. The details of the strategic interactions between the two layers' enforcement efforts will be discussed in Section 4.

[6]Sanctions determined by the legislative branch of government can by and large be taken as exogenous for the executive branch and its law enforcement agency. Sanctions are bounded from above by fairness principles and constitutional requirements, but also efficiency considerations such as minimizing the costs of errors in adjudication, coping with corruption and issues of marginal deterrence (see Polinsky and Shavell, 2000), leaving the intensity of enforcement as the main policy instrument for the executive branch in the control of crimes. The optimal combination of sanctions and detection probabilities has been the subject of a large literature starting with Becker (1968), followed by Becker and Stigler (1974) and, to cite a few, Shavell (1991) and Mookherjee and Png (1992). See Shavell (1992) and Mookherjee and Png (1994) for the analysis of marginal deterrence.

view of the results on crime. Their results are refined in Baicker and Jacobson (2007). Anti-drug enforcement effort has increased to the detriment of other petty crimes, leading to higher drug prices due to shortened supply, which is consistent with the predictions of our vertical crime chain model.

One of the environments we analyze, input-output crime chains, can be a fertile ground for criminal organizations. In many crime chains organized and decentralized segments co-exist, for example, specialized gangs and individual burglars who sell the guns they stole to other potential criminals or to middlemen. The growing literature on organized crime has a branch that studies quasi-governmental models of gangs (Garoupa, 2000, Mansour et al., 2006; Chang et al., 2013). This branch aims at explaining the optimal enforcement policy when a criminal organization responds by modifying its size and defensive strategy. The other branch models organized crime as networks, with implications on its members' detection probabilities (Ballester, Calvo-Armengol and Zenou, 2006; Baccara and Bar-Isaac, 2008; Goyal and Vigier, 2014).[7] Generally, if a criminal is caught then anyone connected to the criminal also risks being caught with a high probability in a follow-up investigation. In contrast to these integrated crime organizations, in our decentralized crime setup upstream criminals can be traced back with positive probability from detection of the downstream criminal with whom they interacted whereas interception of an upstream criminal (before any transaction) does not change the detection probability for downstream criminals. This asymmetry, we show, has implications on budget allocation as well as the choice of the incentive system.

The core model is presented in the next section. The case of independent crimes is analyzed in Section 3, and the interlinked crimes appear in Sections 4 and 5, followed by Conclusion.

2 Model and preliminaries

Consider two crimes, crime A and crime B. There are four actors, the State acting as the principal, two law enforcement units, and the citizens. The State, in its executive capacity, decides on the budgets, rules and incentives for law enforcement. Crime A and crime B are targeted by separate units. Each unit would be composed of police staffs managed by a chief law enforcement officer, whom we call '*the agent*'. Agent i is delegated the task of controlling crime $i = A, B$. The social harm from crime i is denoted by $h_i > 0$.

A fixed measure of population make up the potential criminals, with one-half prone to commit-

[7] Ballester, Calvo-Armengol and Zenou (2006) define a *key player* based on a notion of intercentrality measure reflecting network payoff externalities, whose removal should weaken significantly the network's functionality. Baccara and Bar-Isaac (2008) study repeated interactions between an external authority and a criminal organization. The authority's objective is to restrain cooperation between members, to which the organization responds by modifying its internal information exchange structure. They show that the organization should arrange either isolate binary cells (allowing mutual exchanges) or unlinked agents or at most one hierarchy with a central agent, whereas the State may employ a symmetric or hierarchical scrutiny (i.e., some agents monitored more than others). In Goyal and Vigier (2014), an attack/defense contest is framed between a single Designer and a single Attacker, with the former considering what type of network to form and how to allocate resources between defending different parts of the network. The Attacker then decides which subset of nodes or links to attack. The main result is that in most situations the Designer should organize a 'star' network with all defence resources concentrated on the central node.

ting crime A only and the other half, crime B only. Each group size is normalized to one.

■ **Crimes**. Crimes will be classified according to two criteria.

(i) Observability/measurability. We say that a crime is (ex-post) *observable* if it leaves a physical mark of its occurrence behind, such as a victim, a witness or a property damage. Burglary, assault, robbery or hate speech are examples. A crime is (ex-post) non-observable otherwise; the occurrence or non-occurrence of this kind of crimes is almost unidentifiable unless detected by law enforcers. Bribery, smuggling, drug trafficking ensuring steady supply of drugs or taking drugs making up the demand side, are examples. Admittedly, crimes may differ in observability along a continuum (say, because victims are more likely to report some crimes than others); here we consider a binary classification for simplicity.

(ii) Interdependence. The second criterion relates to whether or not the two crimes form an activity chain. Crime A may be an indispensable input for crime B, as is transportation of illicit drugs from production sites, including smuggling through border controls, to city ghettos for sales to final users. Trade in unlicensed or illegal guns feeding other crimes in which they would be used, say, in assault or murder attempt, and human traffickers who supply labor to the informal black markets such as under-age labor, prostitution are other examples.

■ **Criminals**. Utility of not committing a crime is normalized to zero. Potential criminals of crime i derive a positive private benefit, b, from committing crime i only, distributed according to a continuous cdf $F_i(.)$ with support $[0, \bar{b}]$ and a continuous density function $f_i(.)$, strictly positive in this domain. We denote by s_i the sanction on crime i and assume it can be administered costlessly.

■ **The Agents**. Agent i, acting as the head of the enforcement unit specializing in crime i, determines an effort e to manage and organize his own unit. We assume that this effort is unobservable, hence not contractible. The cost of effort, $z(e)$, is increasing, differentiable and strictly convex in e, with $z(0) = z'(0) = 0$.

The agents' outside options are zero. An incentive system $r \in \{C, D\}$ in enforcement remunerates the agent according to a verifiable indicator, which in system $r = C$ is the measure of crime (or crime rate) and in system $r = D$ the level of detections (or, apprehensions). Denoting agent i's rewards under system r by w_i^r, his final utility is

$$w_i^r - z(e).$$

Although w_i^r cannot be conditioned on e, it will depend on the number of detections or crime data that reflects the agent's effort.

Data availability makes system D a more feasible option than system C, which can be used only for (ex-post) observable crimes if all incidences are reported and recorded; alternatively, the State may conduct periodic surveys to generate information about the level of crime and use it as a statistic in compensating its law enforcement agent.

■ **The State**. The State will allocate an exogenous budget R between the two enforcement agents,

$R = R_A + R_B$. It also chooses an incentive system $r \in \{C, D\}$ under which each agent's expected reward payments Ew_i^r, together with the cost of other operational enforcement resources R_i^X, cannot exceed R_i. The State's objective is to minimize total expected harm from crimes.

■ **Enforcement technology**. Agent i's law enforcement efforts together with the budget R_i^X for all other enforcement resources, e.g., infrastructure, personnel, operational inputs produce an unverifiable probability of detection per criminal, $\mu_i = \mu(e, R_i^X)$, also referred as the "intensity of (crime i) enforcement".

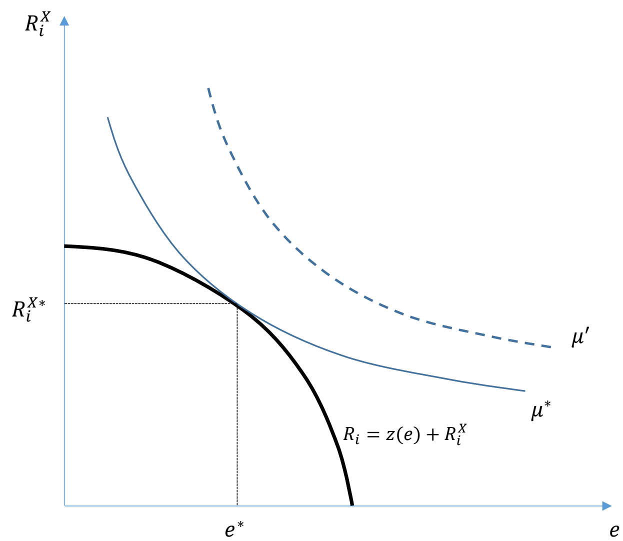

Figure 1. The optimal (R_i^{X*}, e) combination given budget R_i, inducing the maximal feasible detection probability μ^*.

Assumption 1. *The detection probability, $\mu_i(e, R_i^X)$, is increasing in each of its arguments and differentiable, with $\mu_i(0, 0) = 0$, $\frac{\partial \mu_i(0, R_i^X)}{\partial e} = \infty$ and $\frac{\partial \mu_i(e, 0)}{\partial R_i^X} = \infty$. Moreover, $\mu_i(e, R_i^X)$ is strictly concave, i.e., $\mu_i(\alpha * (e, R_i^X) + (1-\alpha) * (\tilde{e}, \tilde{R}_i^X)) > \alpha \cdot \mu_i(e, R_i^X) + (1-\alpha) \cdot \mu_i(\tilde{e}, \tilde{R}_i^X)$ for any $(e, R_i^X) \neq (\tilde{e}, \tilde{R}_i^X)$ and $\alpha \in (0, 1)$.*

As illustrated in Fig. 1, the upper-contour sets defined by μ_i-levels are strictly convex. In the hypothetical scenario of efforts perfectly observable and contractible, for any given enforcement budget R_i for unit i, *feasible combinations* (e, R_i^X) are defined by the set $\mathcal{F}(R_i) = \{e \geq 0, R_i^X \geq 0 | z(e) + R_i^X \leq R_i\}$. Because the effort cost function $z(.)$ is strictly convex, the boundary of this set defined by $z(e) + R_i^X = R_i$ is strictly concave.

The optimal input combination (e^*, R_i^{X*}) maximizing μ_i in the feasible set $\mathcal{F}(R_i)$ will be unique. Note also that by duality, R_i is the minimum budget required to induce the detection probability $\mu_i^* = \mu(e^*, R_i^{X*})$, so $R_i = c(\mu_i^*)$ $(= z(e^*) + R_i^{X*})$. Now using strict concavity of $\mu(e, R_i)$ (Assumption 1) and $z(0) = z'(0) = 0$, the following result is immediate (see, for example, Proposition 6.11, part (ix) of Manove (2005) Lecture Notes):

Lemma 1 (First-best enforcement)**.** *Under effort contractibility, first-best enforcement effort minimizing expected harm from crimes will be $e^* > 0$ for all $R_i > 0$. The enforcement cost function*

$$c(\mu_i) = z(e^*) + R_i^{X*} \tag{1}$$

is increasing and strictly convex in μ_i, with $c(0) = 0$ and $c'(0) = 0$.

3 Independent crimes

Committing crime i yields the benefit b if undetected, $b - s_i$ if detected and punished. Thus, a potential criminal commits the crime under incentive system $r \in \{C, D\}$ if

$$b > \mu_i s_i = b_i^r. \tag{2}$$

There are two ways to incentivize law enforcement. Under crime-based incentives, agent i's reward can be made contingent on the ***level of (crime) deterrence***:

$$\sigma_i = F_i(\mu_i s_i). \tag{3}$$

Under detection-based incentives, the reward is contingent on the ***level of (crime) detections***:

$$d_i = \mu_i(1 - F_i(\mu_i s_i)). \tag{4}$$

The difference between (3) and (4) is that in the former crimes can be controlled directly by linking higher rewards to higher μ_i whereas in the latter such direct linking to μ_i might not be possible due to the non-monotonicity of the $d_i(\mu_i)$ function. The $d_i(\mu_i)$ curve can be of any shape depending on the benefit distribution function $F_i(\cdot)$;[8] see the top panel of Fig. 2a. But by Weistrass Theorem there will be a maximal detection $d_i^{\max}$, although uniqueness of a corresponding maximizer μ_i cannot be guaranteed. Similarly, for any arbitrary level of detection $0 < d < d^{\max}$ there will be at least one μ_i if not multiple μ_i's associated with it, by the Intermediate Value Theorem.

Definition 1. *For any $d \in [0, d_i^{\max}]$, let*

$$\mu_i(d) = \min\{\mu_i | d_i(\mu_i) = d\}$$

under the incentive system D. Further, for $d = d_i^{\max}$, denote

$$\widehat{\mu}_i = \mu_i\big(d_i^{\max}\big).$$

[8] One can see that for F_i uniform, $d_i(\mu_i)$ is increasing with a slope $1 - \frac{1}{\overline{\mathrm{b}} - \underline{\mathrm{b}}} < 1$ assuming $\overline{\mathrm{b}} - \underline{\mathrm{b}} > 1$. A non-monotonic d_i-curve can be constructed for an appropriately chosen density. See also the discussion in the Appendix on the property of the $d_i(\cdot)$ function.

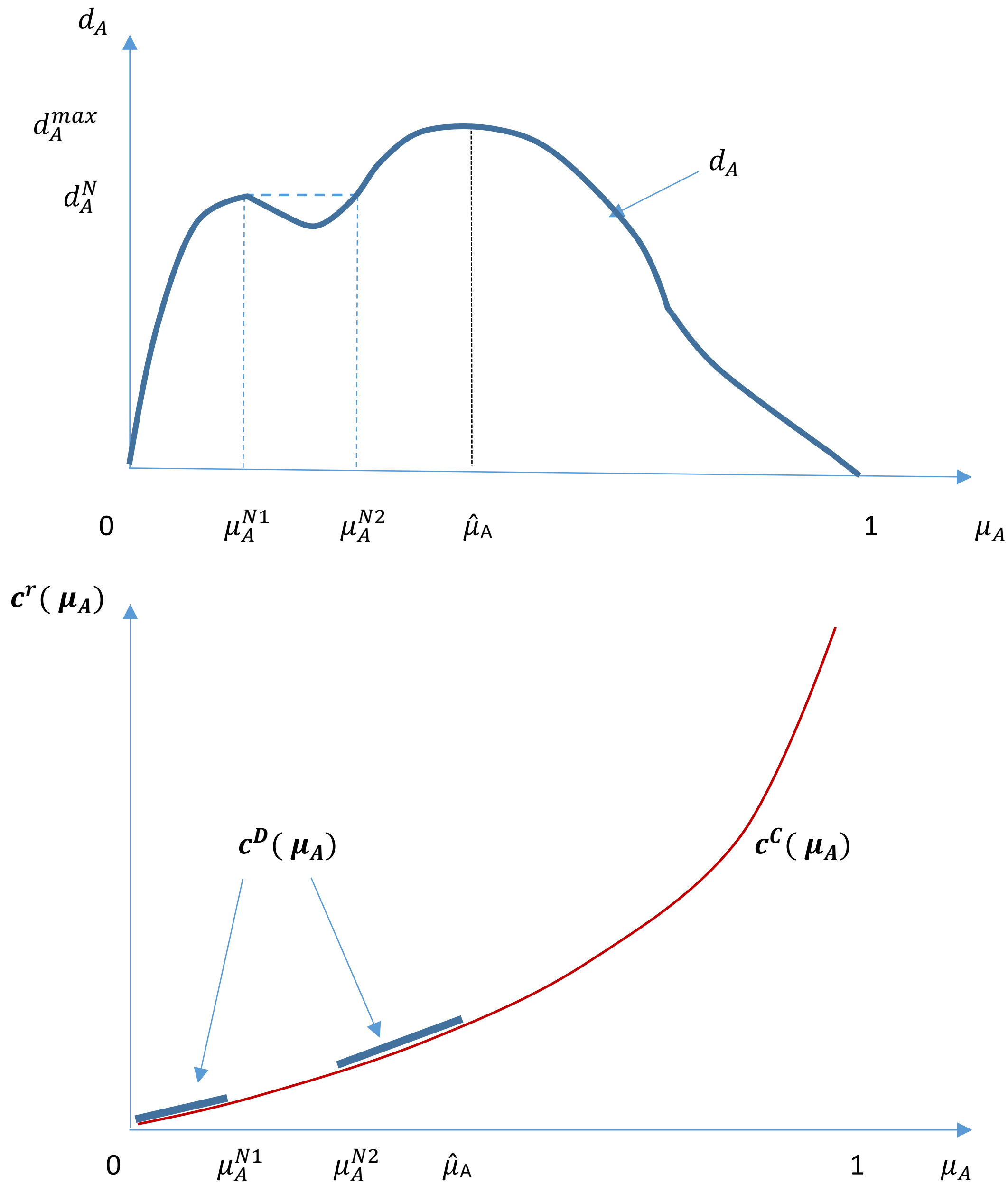

Figure 2a. Impact of multiple local maximands of d_A on incentives and costs. The red continuous curve $c^C(\mu_A)$ represents the cost under crime-based incentives. The blue broken curve intervals describe the implementable ranges $[0, \mu_A^{N1}]$ and $[\mu_A^{N1}, \mu_A^{N2}]$ and their costs.

Finally, define

$$\widehat{b}_i = \widehat{\mu}_i s_i, \quad \textit{and} \quad \sigma_i^{\max} = F_i(\widehat{b}_i).$$

Note that under detection-based incentives the State should never target implementing any μ_i in excess of $\widehat{\mu}_i$, for cost efficiency reasons.

System C. Agent i is rewarded according to some crime deterrence target set. Let us return to Fig. 1. We already know that for any μ_i the cost-minimizing solution under effort contractibility is given by the unique (e^*, R_i^{X*}) pair, resulting in cost $c(\mu_i)$ as in (1). The following simple *all-or-nothing* reward mechanism will induce the agent to exert e^* when effort is unobservable (using (3)) :

$$w_i^C(\sigma_i) = \begin{cases} z(e^*), & \text{if } \sigma_i \geq F_i(\mu_i s_i) \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Failing to meet the target crime level and receiving zero reward can be interpreted as the agent being replaced or denied a promotion. We assume the agent will break the indifference in favor of effort e^* in accordance with the preference of the State.

Lemma 2. *Under crime-based incentives, $c^C(\mu_i) = c(\mu_i)$ for all μ_i as in (1).*

Principal does not concede any moral hazard rent, and incentives based on verifiable crime deterrence is first-best efficient. This is a standard result in contract theory – no efficiency loss in the presence of a sufficient statistic capturing hidden agent effort (e.g., Harris and Raviv, 1978).

System D. Law enforcement officer is rewarded according to the achieved level of detections. Unlike in system C, in system D determining implementation cost in terms of μ_i (for comparability with the cost in system C) is problematic for the simple reason that $d_i(\mu_i)$ is not necessarily monotonic as illustrated in the top panel of Fig. 2a: none of the μ_A's in the interval $[\mu_A^{N1}, \mu_A^{N1}]$ are implementable; see also the unmarked part of the cost curve in the lower panel (explanation for this panel to come later).

Given the above observation, we first need to determine the implementable μ_i's in the form of $\mu_i(d)$ for all $d \in [0, d_i^{\max}]$ using Definition 1. **The remainder of the analysis of this section is currently being rewritten.**

Proposition 1 summarizes the analysis of incentives under the two systems.

Proposition 1 (Limitation of detection-based system)**.** *Consider an independent crime. There exists a deterrence level $\sigma_i^{\max}$ and a corresponding detection probability $\widehat{\mu}_i$ such that:*

(i) *Under system C all deterrence targets σ_i up to $\min\{1, F_i(s_i)\}$, and under system D deterrence targets $\sigma_i \leq \sigma_i^{\max}$, can be implemented at the first-best cost $c(\mu_i)$ as in (1).*

(ii) *Deterrence targets $\sigma_i > \sigma_i^{\max}$ cannot be implemented under system D.*

(iii) *If the sanction s_i is so small that $F_i(s_i) < 1$, then deterrence targets $\sigma_i > F_i(s_i)$ cannot be implemented even under system C.*

Part (i) identifies when systems C and D are interchangeable – for low enforcement budgets inducing high crime targets only. For large enforcement budgets and high deterrence targets as in part (ii), the corresponding large detection probabilities cannot be induced through detection-based incentives: giving it a hard try makes the actual criminal pool and, hence detections, thinner, but a lower effort also generates the same level of detections, so conversion into actual rewards hits a roadblock (Fig. 2b). Part (iii) makes the obvious point that if the sanction for committing a crime is not sufficiently high, criminals with large benefits can never be deterred. Overall, a simple rule implied by Proposition 1 is that crime-based incentives should be used to motivate law enforcement, whenever feasible.

4 Interlinked crimes: crime A an input into crime B

"Supply creates its own demand" – the famous quote that goes under the heading called 'Say's Law' (Baumol, 1999). For crimes that form an input-output chain, it is unclear where should enforcement start: at the top or bottom of the chain? An abundant supply of guns, one might argue, fosters the demand for guns and gun crimes. Illegal immigrants are forced into prostitution and slavery or working at black market wages in hazardous jobs without adequate training, or resort to street crimes.[9] Smuggling of substantial amount of banned drugs (heroin, cocaine etc.) eventually find their ways to underground drug users and fuel drug addiction. Illegal wildlife products such as ivory and rhino horn are sold in markets in south-east Asia, to feed which elephants and rhinos are routinely killed by poachers.[10]

When one crime supplies the instrument to potential criminals of another crime, enforcement policies become interdependent even if each crime deterrence strategy may be overseen by different law enforcement departments, for example, the border-control department and the city police anti-crime branch. Suppose crime A precedes crime B and, if undetected, provides the instrument for undeterred B-criminals to act. As such there is no guarantee that a successful implementation of crime A would culminate into a match with a B-criminal. A B-criminal must find criminal A to execute his plan. The probability of matching, as in any decentralized market, will depend primarily on the relative sizes of the two populations to be matched.

Apportioning social harms $h_A > 0$ and $h_B > 0$ separately to crime A and crime B is a way to reflect the seriousness with which the authorities might view the distinct parts of the crime production process. Alternatively, we could assign a single harm $h > 0$ on committing crime B

[9]See the reports on trafficking (illegal immigration) of women in the UK and women and children in the USA; https://www.theguardian.com/uk/2005/nov/02/immigration.ukcrime and https://www.oas.org/en/cim/docs/Trafficking-Paper%5BEN%5D.pdf (a study by *Inter-American Commission of Women*, an inter-governmental agency).

[10]BBC report, 26 August 2018, https://www.bbc.com/news/world-africa-45288429.

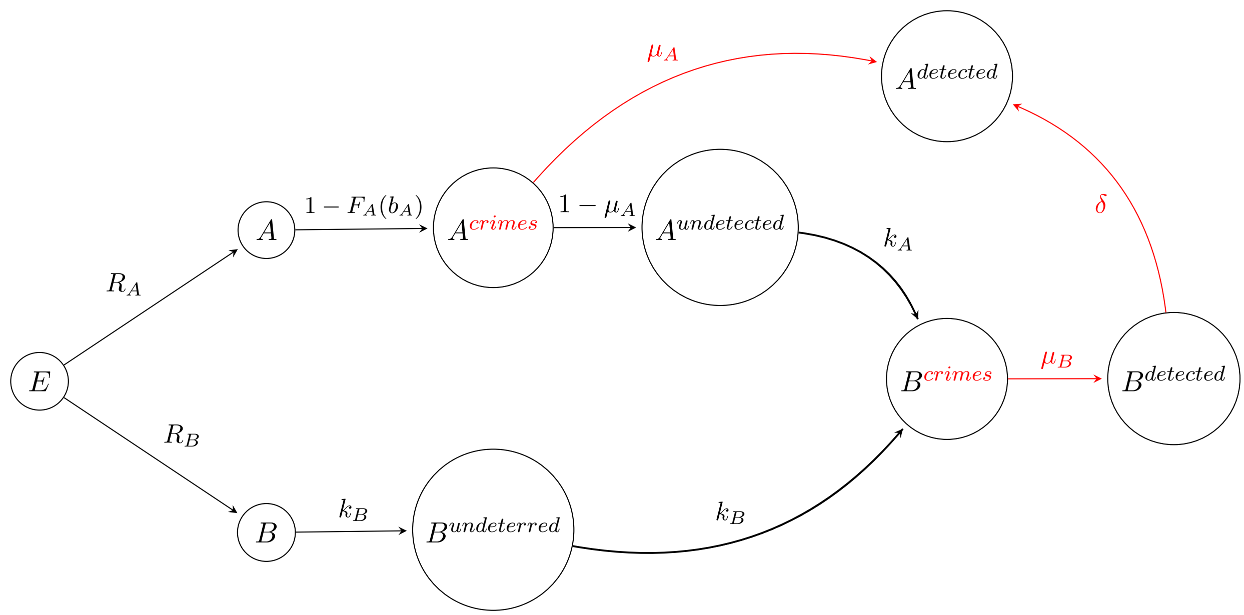

Figure 4: Crime chain; $k_A < k_B$ (A on short side), $k_A > k_B$ (B on short side) resulting in $\min\{k_A, k_B\}$-measure of the complete chain of (AB)-crimes. Given that it takes the two types of criminals to complete the final crime, B, either end may be considered as pivotal, although crime initiatives start with A-crime. This is particularly true in the decentralized, demand-supply, environment with observable allocation of enforcement efforts, (R_A, R_B), as the only means of coordination. B's detection leads to A's detection with probability $0 \leq \delta \leq 1$.

(or A) and treat the other crime as an indispensable step to crime B (or A).[11] For cross-border crimes, law enforcement has to invest in monitoring at check points, intelligence and cooperating with the country where the supply originates, all of which we lump together under *enforcement A*. Then a separate law enforcement division monitoring inside the country will detect illegal residents and the related crimes such as prostitution, extortionary employment, street drug selling, sale of contraband goods (ivory, brand name cigarettes, wines) etc that are undocumented. This latter we label as *enforcement B*.

Before proceeding to the formal analysis we should emphasize that the approach here is very different from the crime networks literature (e.g., Ballester, Calvo-Armengol and Zenou, 2006; Baccara and Bar-Isaac, 2008; Goyal and Vigier, 2014). Ours is a decentralized matching (or market) mechanism between the perpetrators of crime, whereas the network approach is predominantly one of bilateral/far-sighted coordination among criminals. The question of optimal enforcement response in our formulation should thus be of independent interest.[12]

[11] For instance, transporting drugs from one location to another may not per se cause much harm, except facilitating the sales to final users, which is quite harmful. Or, if selling a rhino horn is not harmful, killing a rhino for its horn is quite harmful.

[12] The distinction between decentralized matching and bilateral/far-sighted coordination can be understood in the following example. In the latter, a player may connect to another player seeing the benefits of both the direct link with the player and the secondary benefits derived from links to other players the connected player generates; see, for example, Jackson and Wolinksy (1996). This way many players may be connected only to a star player, in a 'star' network. So enforcement by taking out the star criminal can weaken/destroy the entire criminal gang. In contrast, in our decentralized environment taking out a single (or small fraction of) A-criminal(s) or B-criminal(s) does not destabilize the matching. Instead, enforcements have to be at aggregate levels in different parts of the crime possibilities,

■ **Sequence of events.** Following the State's decisions on budget allocation and enforcement incentives that are publicly announced, various parties choose their actions in the following order (see Fig. 4):

(i) Agent i determines effort e_i, hence μ_i, while potential criminals choose between compliance (deterred) and taking action to commit crime i (not deterred).

(ii) • Hatching a plan to deliver to an eventual crime down the chain (called crime B) or preparing for it is in itself a crime, called crime A. Perpetrators of A-crime, on detection, are removed from the crime chain and sanctioned.

• Potential B-criminals may choose to stay away from the crime. The rest are undeterred B-criminals.

(iii) Only undetected A-criminals of measure k_A and undeterred B-criminals of measure k_B search for each other to match.

(iv) Undetected A-criminals who find a B-partner realize their benefits.

(v) Undeterred B-criminals who find an A-partner commit the crime and realize their benefits. If detected, they are sanctioned.

(vi) Detection of a B-criminal leads to a second shot at detection of the partner A-criminal with probability $0 \leq \delta \leq 1$.

Two remarks: First, the sequence of events excludes the possibility of detection at the moment of matching, when A-criminals and potential B-criminals meet to transfer the crime instrument (or, sometimes pass on valuable information). A-criminals are detected in one of two stages – (i) after committing the crime but prior to matching, (ii) after crime B has been completed – whereas B-criminals can be detected only after a match with A-criminals and thereupon successful execution of the crime. Second, as an alternative to the imperfect interim matching process, one could also posit an interim crime-instrument market where the price would reflect shortages of demand by B-criminals or of supply by A-criminals. Raising enforcement to combat crime B, for example, would deter B-criminals and reduce the demand for the instrument and its price, thereby, have a deterrent effect on A-criminals. This price mechanism should generate a similar qualitative relationship between enforcement efforts and their cross-deterrence effects, as the present approach.

■ **Deterrence and matching.** Denote by p_i the probability that an i-criminal finds a j-partner, $i \neq j$, $i, j = A, B$. These probabilities will later be determined in the overall equilibrium following enforcement decisions by the State.

The benefit from crime A will be realized with probability

$$(1 - \mu_A)p_A,$$

as opposed to thinking about the interaction between enforcement directed at certain links of the criminal network and what equilibrium implications it might have for the emerging network. An analysis of enforcement capturing strategic interactions between the criminal network and multiple enforcement departments with an overall budget is certainly an interesting problem but also likely to be very challenging. The work of Ballester et al. (2006) offers a background structure that needs to be further extended to bring in multilateral enforcements as additional players.

and thus a potential A-criminal with benefit b will commit the crime if

$$(1-\mu_A)p_A b - [\mu_A + (1-\mu_A)p_A \cdot (\mu_B \delta)]s_A > 0. \tag{6}$$

An A-criminal may be detected either in the preliminary crime-A stage or in a follow-up investigation on detection of the partner criminal B. If $\delta = 0$, A-criminals will escape untraced after transacting with B-criminals, which can be attributed to the largeness and non-anonymity of the decentralized, uncoordinated crime market. On the other hand if $\delta > 0$, the detection mirrors part of the story of a crime chain in networks. The only difference is that detection of an A-criminal in the early stage does not lead to an apprehension of a corresponding B-criminal because the former hasn't yet met the latter. In the analysis to follow, we will consider δ unrestricted.

Given μ_A and μ_B, the measure of crime A is

$$1 - F_A(b_A), \quad \text{where} \quad b_A = \frac{[\mu_A + (1-\mu_A)p_A \cdot (\mu_B \delta)]s_A}{(1-\mu_A)p_A}. \tag{7}$$

On the other hand, undeterred B-criminals realize their benefits only if they find an A-partner, when they complete the crime, thus, with probability p_B. They will be detected and punished with probability $p_B \mu_B$, so, a potential B-criminal will commit the crime if

$$p_B b - \mu_B p_B s_B > 0. \tag{8}$$

Thus, the measure of crime B is[13]

$$[1 - F_B(b_B)]p_B, \quad \text{where} \quad b_B = \mu_B s_B. \tag{9}$$

We now turn to the determination of p_i, which we relate below to the endogenous variables k_A, the measure of undetected A-criminals, and k_B, the measure of undeterred B-criminals, where

$$k_A = (1-\mu_A)(1 - F_A(b_A)) \quad \text{and} \quad k_B = 1 - F_B(b_B). \tag{10}$$

Assumption 2.

$$p_i = \rho(k_j/k_i) \begin{cases} = \pi & \text{if } k_j/k_i \geq 1, \\ \in [0, \pi) & \text{if } k_j/k_i < 1, \end{cases} \tag{11}$$

where $0 < \pi < 1$ *and* $\rho(k_j/k_i)$ *is strictly concave and increasing, with* $1 > \rho'(0) > 0$.

The matching technology stated in (11) involves frictions. The probability p_i is smaller than k_j/k_i; it is increasing in k_j/k_i at a decreasing rate until $k_j = k_i$, whereafter it remains constant at π. Thus, $p_i < 1 \Rightarrow p_j = \pi$, $i \neq j$: A criminal on the short side of the market will find a partner

[13] Notice the difference between (6) and (8). In (6), criminal A gets sanctioned for "killing the rhino" even if he fails to "deliver the horn" to (match with) criminal B. In contrast, in (8), criminal B is sanctioned only when executing his crime (marketing the horn) after *matching* with criminal A. The difference gets reflected in the cutoff benefits b_A and b_B, with the former dependent on the matching probability p_A whereas the latter is independent of p_B.

with maximal probability, π. To illustrate, if $k_B > k_A$, undetected A-criminals match and get their private benefit with probability $p_A = \pi$, whereas undeterred B-criminals complete their crime with probability $p_B < \pi$. If smaller than π, p_B is increasing in k_A and decreasing in k_B.

Frictionless matching is obtained as a special case of Assumption 2 by setting $\pi = 1$ and $\rho(k_j/k_i) = k_j/k_i$ for $k_j/k_i < 1$. If the population of undetected A-criminals is 40 percent of the population of undeterred B-criminals, under frictionless matching each undeterred B-criminal expects to match with probability 0.4. With frictions, this probability is smaller than 0.4.

■ **Equilibrium analysis.** Given the cost function $c_i(\mu_i) = R_i$ and its inverse $\mu_i(R_i)$, an allocation of the total enforcement budget induces a pair of crime outcomes, as formally stated in the following lemma.

Lemma 3. *Given a feasible budget allocation (R_A, R_B) hence a pair of enforcement intensities $\big(\mu_A = \mu_A(R_A), \mu_B = \mu_A(R_B)\big)$, an induced crime equilibrium consists of a pair of cutoff criminal types $(\widetilde{b}_A, \widetilde{b}_B)$ satisfying (7), (9) and (10), that is,*

$$\begin{aligned} b_A &= \left[\frac{\mu_A(R_A)}{(1-\mu(R_A)).\rho\Big(\frac{1-F_B(b_B)}{(1-\mu_A(R_A))(1-F_A(b_A))}\Big)} + (\mu_B(R_B)\delta)\right] s_A, \\ \textit{and} \qquad b_B &= \mu_B(R_B) s_B. \end{aligned} \tag{12}$$

A solution pair is easily guaranteed. The proof is integrated in the proof of the next proposition. The first equation in (12) is obtained by using (10) in the definition of p_A and (7). Note that the potential cross-deterrence effect of enforcement is unidirectional, whereas the cross-crime effects work in both directions. Deterrence of A-criminals does not marginally affect deterrence of crime B though it can affect the measure of undeterred B-criminals who complete the crime. On the other hand, enforcement effort by agent B will affect crime A deterrence via two potential channels – one through p_A, the probability that an A-criminal realizes his benefit, the other through δ, the likely detection of A-criminal following criminal B's detection.

Our first result on vertical crime chains is on the existence and uniqueness of induced crime outcomes and the related comparative statics.

Proposition 2 (Decentralized coordination). *(a) In the decentralized environment of linked crimes, a feasible budget allocation (R_A, R_B) induces a unique coordinated crime equilibrium $(\widetilde{b}_A, \widetilde{b}_B)$.*

(b) In any induced equilibrium, an increase in enforcement spending on crime i lowers crime i whereas an improvement in A's detection through the follow-up investigation, δ, only lowers crime A without any cross-deterrence effect on crime B:

(i) $\frac{d\widetilde{b}_A}{d\delta} > 0, \quad \frac{d\widetilde{b}_B}{d\delta} = 0,$

(ii) $\frac{d\widetilde{b}_A}{d\mu_A} > 0, \quad \frac{d\widetilde{b}_B}{d\mu_B} > 0, \quad \frac{d[p_B(1-F_B(\widetilde{b}_B))]}{d\mu_B} < 0.$

Uniqueness of equilibrium crime levels may be surprising, especially in our decentralized matching environment where greater crimes at either end enhance the attractiveness of crimes at the other end. The uniqueness is due to two reasons: (i) in segment B, the crime can be committed and its benefits and potential costs be realized only after successful matching, so the matching probability p_B has no impact on the deterrence level ($\widetilde{b}_B$) and the measure of undeterred B-criminals (k_B) is uniquely determined by μ_B; (ii) with k_B thus determined, moving back the chain only a unique pair of (p_A, b_A) can satisfy (7), (10) and (11), pinning down $\widetilde{b}_A$. This holds with and without frictions in matching.

Let us now look at the comparative statics. If the induced crime equilibrium involves $p_B = \pi$, i.e., undeterred B population is on the short side, raising enforcement intensity μ_B should lower crime B by driving out the marginal criminal. However if $p_B < \pi$, those still undeterred will see their chances of completing the crime increased, which in turn tends to dilute deterrence of B-crimes. The net impact on the measure of crime $p_B(1 - F(\widetilde{b}_B))$ should be negative, that is, the initial impetus on $\widetilde{b}_B$ should dominate the opposing feedback effect through p_B. Similarly, raising the intensity of enforcement should lower crime A, but if $p_A < \pi$ this will also enhance the incentives for crime A as those who escape initial detection foresee an increase in their probability of completing the crime. The negative feedback effect, however, should not offset the initial inductor effect, so that $\widetilde{b}_A$ increases if μ_A increases. Since $\mu_i = 0 \rightarrow \widetilde{b}_i = 0$, Proposition 2(b)-(ii) implies that $\widetilde{b}_i > 0$ for all $\mu_i > 0$. Finally, we note that crime B deterrence is neutral to δ, but the measure of completed crime B, $p_B(1 - F_B(\widetilde{b}_B))$, will depend on δ through p_B.

4.1 Incentives

• **Crime-based incentives**. Suppose data is available to set separate verifiable crime A and crime B targets, $1 - F_A(b_A)$ and $(1 - F_B(b_B))p_B$. We know from Proposition 2(b) that an increase in the probability of detection μ_i lowers crime $i = A, B$. The monotonicity chain from enforcement inputs e_i and R_i^X to deterrence, coupled with verifiability of the crime level, implies that rewards conditional on crime targets as exposed in Section 3 can successfully implement any crime level, budget permitting, at first-best cost.

Proposition 3. *Suppose crime data is available. Then any μ_i, $i = A, B$, can be implemented at first-best cost, that is, $c_i^C(\mu_i) = c(\mu_i)$, with corresponding deterrence levels determined by (12).*

However, the picture is quite different from independent crimes under detections-based incentives.

• **Detection-based incentives**. Contrary to the case of independent crimes where detections of crime i depend solely on the enforcement effort by agent i, in a crime chain the agents' efforts produce cross-detection effects which the authorities can explore in designing incentives. Supplementary detection measures become available as indicators of each agent's enforcement effort. It turns out that all crime levels can be implemented via first-best enforcement incentives by setting targets based on cross-detection indicators, provided at least one of these indicators is monotonic in the agent's effort. We identify the indicators below and check their monotonicity in enforcement efforts.

Detections of crime A and crime B are now given by

$$d_A = (\mu_A + (1-\mu_A)p_A\mu_B\delta)(1-F_A(\widetilde{b}_A)), \quad \text{and} \quad d_B = \mu_B p_B(1-F_B(\widetilde{b}_B)). \tag{13}$$

The prime supplementary indicator of enforcement effort is detections of the other crime. Observe that in (13) upstream enforcement intensity μ_A affects downstream detections d_B through the probability p_B whereas downstream enforcement intensity μ_B affects upstream detections d_A through both p_A and δ. Thanks to this δ effect, two additional sets of detection data can be obtained by decomposing d_A according to its source, as $d_A = d_{AA} + d_{AB}$, where

$$d_{AA} = \mu_A(1-F_A(\widetilde{b}_A)), \quad d_{AB} = (1-\mu_A)p_A\mu_B\delta(1-F_A(\widetilde{b}_A)). \tag{14}$$

The measure d_{AA} is owed to agent A's enforcement, d_{AB} to agent B's enforcement and follow up investigations. Let us now focus on the relationship between these detection measures and enforcement intensities.

Lemma 4.

The signs of $\dfrac{d[d_A]}{d\mu_A}$, $\dfrac{d[d_B]}{d\mu_B}$, $\dfrac{d[d_{AA}]}{d\mu_A}$, $\dfrac{d[d_{AB}]}{d\mu_B}$ *and hence,* $\dfrac{d[d_A]}{d\mu_B}$, *are ambiguous.*

However, $\dfrac{d[d_{AB}]}{d\mu_A} < 0$, $\dfrac{d[d_{AA}]}{d\mu_B} < 0$, *and* $\dfrac{d[d_B]}{d\mu_A} \begin{cases} > 0 & \textit{if } k_A/k_B < 1 \ (p_B < \pi) \\ = 0 & \textit{if } k_A/k_B \geq 1 \ (p_B = \pi). \end{cases}$

As in the independent crimes case, d_i is not monotonic in μ_i, hence, not monotonic in own enforcement effort, e_i. We know that $\widetilde{b}_i > 0$ for all $\mu_i > 0$, given fixed μ_j. Holding thus μ_B constant in (13) it is easy to verify that detections of crime A are positive and vary between the two limits,

$$lim_{\mu_A \to 0} d_A \equiv d_A^0 = p_A^0\delta\mu_B(1-F_A(\delta\mu_B s_A)) \quad \text{and} \quad lim_{\mu_A \to 1} d_A = 0. \tag{15}$$

Detections of crime B, on the other hand, converge to the same value $d_B = 0$ as $\mu_B \to 0$ and $\mu_B \to 1$, taking strictly positive values in between.

Fig. 5 illustrates the shapes of d_{AA} and d_A, which are not monotonic, and d_{AB}, which is monotonic in μ_i. Crime A detections have a maximum, $d_A^{\max}$, at the enforcement intensity $\widehat{\mu}_A = argmax[d_A]$. Because enforcement effort is not contractible, the agent would exert the lowest effort compatible with any target measure of detections. If d_A data were available but not its decomposition, it would be impossible to implement $\mu_A > \widehat{\mu}_A$ through detection targets $d_A < d_A^{\max}$, for the agent can generate the same d_A by exerting lower effort.[14]

Regarding the cross-detection effects of enforcement, $d[d_B]/d\mu_A$ is strictly negative when A-criminals are at the short side of the crime chain so that $p_B < \pi$. An increase in μ_A deters crime A and reduces p_B, which in turn deters more of crime B and reduces detections of crime B, for

[14]Exceptions could be the μ_A levels that are so high that the corresponding detections d_A are smaller than even d_A^0, the measure of detections owning solely to B's enforcement while agent A exerts zero effort.

given enforcement intensity by agent B. When p_B is maximal and equal to π (which corresponds to $k_B \leq k_A$), however, this impact vanishes.

On the other hand, d_{AA} is unambiguously decreasing in μ_B. When agent B intensifies enforcement and raises μ_B, at constant μ_A agent A will see his own detections fall. This happens thanks to the increase in the measure of A-criminals backtraced through their B-partners detected by agent B, which deters more of crime A and reduces d_{AA}. There is another, reinforcing, effect through p_A when B-criminals are at the short side of the crime chain: the rise in μ_B will reduce p_A, hence the prospective benefit from crime A, deterring crime A and reducing the measure of detections by agent A.

The third monotonic relationship is between d_{AB} and μ_A. Detections of A-criminals attributed to B's enforcement is unambiguously decreasing in A's enforcement intensity, μ_A. Given μ_B, a higher enforcement effort by agent A reduces the measure of A-criminals who can be detected in follow-up investigations through their detected B-partners.[15]

The impact of an increase in μ_B on d_{AB}, however, is ambiguous. Intuition may suggest that a larger measure of A-criminals should be backtraced (the δ effect) when agent B raises μ_B, at constant μ_A. The ambiguity arises because, first, the δ effect is dampened by rise in deterrence of crime A, and second, the combined effect of these changes on the matching probability p_A is ambiguous.

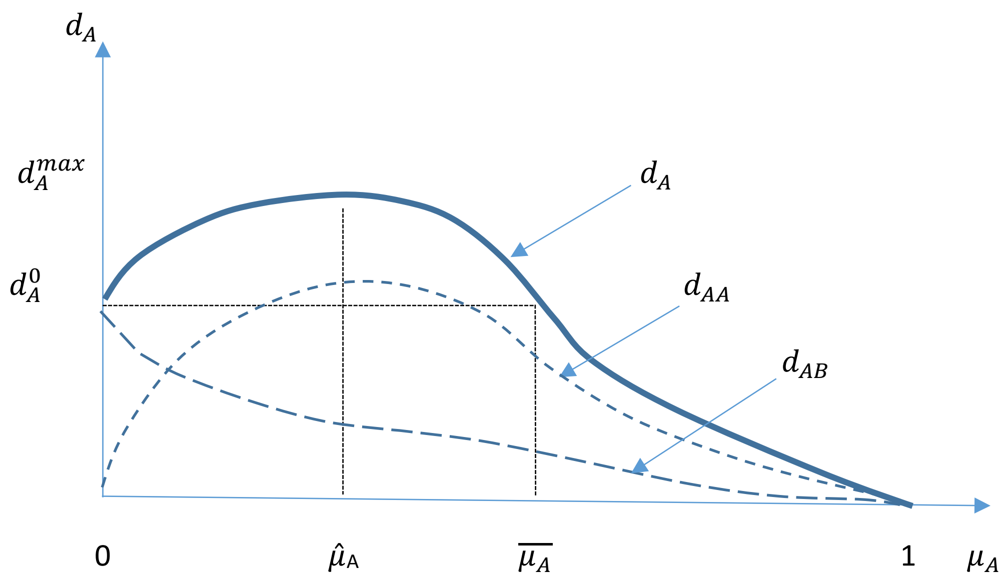

Figure 5. The measure of detections of crime A decomposed, $d_A = d_{AA} + d_{AB}$, as a function of the probability of detection, for $\delta > 0$. The same measure d_A^0 of detections is induced by the detection probabilities $\overline{\mu_A}$ and $\mu_A = 0$.

[15]The rise in μ_A can lead to an increase in p_A and partially offset the fall in crime A and own detections by agent A. This indirect effect cannot dominate as long as the matching probability p_A is concave in k_B/k_A, as assumed.

To recapitulate, the detection measures d_B and d_{AB} can be used as indicators of agent A's enforcement effort, and d_{AA} can be used for agent B's enforcement effort, provided these measures are verifiable. The State can set appropriate detection targets d_B or d_{AB} for agent A and d_{AA} for agent B, and implement *all* deterrence targets at first-best cost, as under crime-based incentives, thanks to the monotonicity properties highlighted in Lemma 4. It is important to note that these detection measures constitute the minimal tool set for first-best incentive provision. Other detection measures which in general are not guaranteed to be monotonic in μ_i may be monotonic for specific parameter constellations or sub-intervals of detection probabilities, thus, may also be used as such.

Proposition 4 (Costs and feasibility under system D)**.** *If crime A detection data is available in decomposed form as d_{AA} and d_{AB}, given any budget allocation with $R_i > 0$, the equilibrium detection probabilities $\{\mu_A, \mu_B\}$ and deterrence levels $\{\widetilde{b}_A, \widetilde{b}_B\}$ satisfying (12) can be implemented via detection-based incentives at first-best cost, $c_i^D(\mu_i) = c(\mu_i)$.*

The number of criminals A traced and detected thanks to agent B's efforts can be used as a contractual target to motivate effort by agent A, because as shown in Fig. 5, d_{AB} is monotonically decreasing in μ_A. So, if a large fraction of A criminals are detected thanks to agent B's follow up investigations, this is taken as a signal of poor enforcement by agent A. An alternative indicator to motive agent A's enforcement activity is the measure of detected B-criminals, except when A is combinations of deterrence levels such that $k_A < k_B$, i.e., $p_A < \pi$. Basically, given agent B's enforcement effort, any deviation by agent A from the desired effort will be revealed by these detection data.

Agent B's detection target, on the other hand, should be based on d_{AA}, agent A's own detection of A-criminals. Because d_{AA} is monotonically decreasing in μ_B, a lower effort by agent B will dilute crime A deterrence and raise crime A detections by agent A, given his own effort and μ_A. So, if A's detection of A-criminals exceeds the target this is taken as a signal that B is withholding his enforcement effort.

Combining Propositions 3 and 4, we conclude that any combination of deterrence levels $(\widetilde{b}_A, \widetilde{b}_B)$ for interlinked crimes can be implemented at the same, first-best, cost $c(\mu_i)$ through crime- and detection-based systems. The implementation problem that plagues the control of an independent crime via detection-based incentives disappears thanks to a positive δ and the cross-enforcement effects through the crime chain, the probability p_A or p_B, if the detection indicators d_{AA} and d_{AB} are available. The monotonicity requirement is met and all levels of the two crimes in the chain are implementable at first-best enforcement costs. If d_{AA} and d_{AB} data are not separately available, system D can be used only for agent A based on the measure of crime B detections, to implement crime targets that leave A criminals at the short side of the crime chain ($k_A < k_B$).

4.2 Budget allocation

Given the incentive system for each agent and hence the enforcement costs $c(\mu_i)$, the optimal budget allocation $(R_A^*.R_B^*)$ minimizes the expected social harm

$$\begin{aligned} &\min_{\{R_A,R_B\}}\ SH = [1 - F_A(b_A)]h_A + [1 - F_B(b_B)]p_B h_B, \\ &\text{subject to} \quad R_A + R_B \leq R, \qquad \text{and (12).} \end{aligned} \tag{16}$$

Below we consider the symmetric case to highlight the fundamental factor that would favor an asymmetric budget allocation, rooted in the vertical chain structure of the crimes.

■ **Symmetric crime environments**. We say that two crimes form a *symmetric crime environment* if the sanctions and the criminal benefit distributions are identical, i.e., if $s_A = s_B$, and $F_A(.) = F_B(.)$. Denote by s the common sanction and by $F(.)$, the common distribution function.

For any budget allocation such that $R_A \geq \frac{R}{2} \geq R_B$, the induced crime equilibrium satisfies $\widetilde{b}_A \geq \widetilde{b}_B$, by (12):

$$\widetilde{b}_A = \left[\frac{\mu(R_A)}{(1-\mu(R_A)).p_A} + (\mu(R_B)\delta)\right] s > \mu(R_B)s = \widetilde{b}_B \text{ for any } p_A \leq \pi.$$

It follows that $1 - F(\widetilde{b}_A) < 1 - F(\widetilde{b}_B)$, hence, $k_A \equiv (1-\mu(R_A))(1-F(\widetilde{b}_A)) < 1 - F(\widetilde{b}_B) \equiv k_B$. We have a crime equilibrium with $p_A = \pi$ and $p_B < \pi$. An equal budget allocation in a symmetric crime environment generates a stronger deterrence on crime A than crime B, $\widetilde{b}_A > \widetilde{b}_B$, even if $\delta = 0$, because A criminals face the risk of apprehension before matching with B partners, thus, before realizing their criminal benefits.

We study below the impact of a small *balanced* budget adjustment, such that $dR_A = -dR_B$, on the expected social harm in (16). For a clear picture, in Proposition 5 we take δ sufficiently small.

Proposition 5 (Priority: upstream)**.** *Consider a symmetric crime environment in which crime A is an input to crime B, with δ sufficiently small. The budget allocated to upstream enforcement should be larger than downstream enforcement, $R_A^* > R_B^*$, if*

$$h_A > h_B(1-\mu(\frac{R}{2}))^2\left[\frac{f(\widetilde{b}_B)\pi}{f(\widetilde{b}_A)}(p_B - \rho'(.)\frac{k_A}{k_B}) - \frac{\rho'(.)}{1-\mu(\frac{R}{2})}(1+\frac{\pi k_A.\mu(\frac{R}{2})}{sf(\widetilde{b}_A)\mu'(\frac{R}{2})})\right], \tag{17}$$

that is, unless crime B is sufficiently more harmful than crime A.

There are two reasons for allotting a larger budget to the upstream enforcement agent A. The first reason bears on the net harm-weighted deterrence impact at constant matching probabilities. Starting from an equal budget allocation, a marginal balanced-budget shift from downstream to upstream enforcement will raise deterrence of crime A more than it dilutes deterrence of crime B, unless $f(\widetilde{b}_A)$ is too small relative to $f(\widetilde{b}_B)$. This is so, because raising the probability of detection of crime A reduces the probability that A-criminals realize their benefit by delivering to a B-

partner, besides of course raising their probability of punishment, whereas deterrence of B criminals depends on the punishment probability alone. The marginal enforcement dollar thus has a stronger deterrence effect in the hand of agent A than agent B. Add to this the fact that the harm h_A is proportional to the measure of undeterred A criminals (upstream, the harm occurs when the rhino is killed), whereas downstream undeterred B-criminals will complete their crime and inflict the harm h_B with probability p_B (if they find an A-criminal who delivers a rhino horn). Even if $f(\widetilde{b}_A) = \pi f(\widetilde{b}_B)$ so that the net increase in overall deterrence is zero, a rise in A-deterrence coupled with an equal fall in B-deterrence is not neutral for the objective of the state.

The second reason favoring a budget transfer to agent A relates to the impact of the transfer on the probability that crime B will be completed. The balanced budget shift will impact on $p_B = \rho(\frac{k_A}{k_B})$ through three variables: the two deterrence levels $\widetilde{b}_A$ and $\widetilde{b}_B$ and the detection probability of crime A, $\mu_A = \mu(R_A)$. All three induced impacts contribute to the fall in p_B as captured by the negative terms at the right hand side of (17); the rise in R_A does it by increasing $\widetilde{b}_A$ and μ_A and hence by reducing k_A, and the fall in R_B, by raising $\widetilde{b}_B$ and hence k_B as well.

Thus, contrary to the case of independent crimes, even if the upstream and downstream crime environments are identical in every respect, upstream law enforcement should receive a larger budget for reasons primarily related to the vertical structure of the crime chain, unless the harm from downstream crime is sufficiently larger than the harm from upstream crime. Raising the budget to fight the latter will reduce expected harm from crimes.

It is worth noting that Proposition 5 can hold in vertical crime chains where h_A is very small relative to h_B, or even $h_A = 0$, if by deterring crime A the state can induce a large fall in B-criminals' matching probability p_B. The larger is the cross-deterrence effects on p_B (represented by the negative terms in the coefficient of h_B in (17)), the more likely it is that the State will allocate a larger enforcement budget to the upstream crime even if the social harm from the upstream crime is very small or zero.

In the symmetric opposite case h_B is very small relative to h_A, or in the limit, $h_B = 0$, (17) suggests $R_B = 0$. Crime B, marketing the fur of a polar bear is itself harmless and crime A, killing a polar bear for its fur, produces all the harm along the chain. The practice of enforcement in such crime chains does not involve a corner solution; positive amount of resources are devoted to deter clandestine fur sales contrary to the implication of condition (17) for $\delta = 0$. It would be optimal to spare positive enforcement effort to detect the harmless downstream activity only if $\delta > 0$ so that these B-criminals can help law enforcers trace their upstream harmful A-partners.

When δ is large, the positive externality from agent B's enforcement activity can lead to a large measure of A-detections and raise crime A deterrence. The level of crime B, however, is not affected by changes in δ, by Proposition 2(b). Expected harm from crime will be smaller thanks to the fall in crime A and its potential consequence, the reduction in p_B.

In a symmetric crime environment with large δ and an equal budget allocation, the equilibrium conditions in (12) continue to imply $\widetilde{b}_A > \widetilde{b}_B$. It is easy to verify that $k_B > k_A$ holds even stronger under $\delta > 0$, hence, $p_A = \pi$, $p_B < \pi$. Now the condition for $dSH < 0$ in (17) is modified as

$$h_A\left[\frac{1}{(1-\mu(\frac{R}{2}))^2\pi} - \delta\right]$$
$$> h_B\left[\frac{f(\widetilde{b}_B)}{f(\widetilde{b}_A)}(p_B - \rho'(.)\frac{k_A}{k_B}) - \rho'(.)\Big\{\frac{\mu(\frac{R}{2})(1-F(\widetilde{b}_A))}{\mu'(\frac{R}{2})sf(\widetilde{b}_A)} + (1-\mu(\frac{R}{2}))(\frac{1}{(1-\mu(\frac{R}{2}))^2\pi} - \delta)\Big\}\right]. \quad (18)$$

The impact of $\delta > 0$ is apparent at the left and right hand sides of (18). A marginal balanced-budget transfer from agent B to agent A, on top of those mentioned in Proposition 5, dilutes deterrence of crime A by reducing the backtracing effect. This is captured by the negative δ term in the coefficient of h_A at the left-hand side of (18). Second, the larger population of A-criminals seeking a B-partner will increase p_B and feed crime B. This indirect effect appears at the right hand side with the additional δ term. Both of these effects will lead to an increase in crime B and reduce the expected benefit from marginally shifting a budget from agent B to A. If strong enough, these new effects can lead the State to prioritize the downstream crime.

5 Interlinked crimes: crime A causes crime B

A crime could fertilize the ground for another crime through a multitude of mechanisms. In this section we consider a predominantly unidirectional and probabilistic causality; we say that crime A causes crime B if it leads to an increase in the population of *potential* B-criminals. Crime B does not cause, but is in part an effect of, crime A. Crime A could be child abuse and crime B, theft, assault, or juvenile vandalism. We know that children subject to parental abuse of any form are more likely to commit crimes than children that are not. Similarly there is ample evidence that illegal drug sales lead to an increase in the population of potential violent criminals.

The difference between Section 4 and the present section is not confined to replacement of a matching process by a probabilistic link. In Section 4, neither the upstream input crime A nor the downstream output crime B can be identified as the cause of the other, but each crime needs the other: Without crime B there would be no crime A and without crime A there would be no crime B. What links the two crimes is an imperfect matching process through which changes in the supply of one crime affects the supply of the other. In this section, undetected criminals don't need any partner to realize their benefits, so, each of the two crimes would still be committed if the other is eradicated. However, crime A increases the pool of potential B-criminals and this creates a positive externality from agent A's enforcement to crime B outcome. We ask if the cause-effect relation favors the root crime A over crime B in the allocation of enforcement resources and we address the incentive problem in enforcement.

Consider two groups of individuals, potential A-criminals and a B-population, each of measure one. Potential B-criminal population is partly endogenous, for crime A "affects" a fraction α of the B-population and increases their probability of becoming potential B-criminals. Depending on the context, the α fraction of the B-population could be victims of, deal with, or happen to interact with, some A-criminals. In the case of parental child abuse α could be taken equal to one (there is a parent for each child, the B-person). If A-criminals are drug sellers and crime B is theft or shoplifting, α would be much smaller than one. Some, but not all, shoplifters would be acting under drug influence.

Formally, a B-person not affected by crime A becomes a potential B-criminal with probability p_L, but crime A increases this probability to p_H for a fraction α of the B-population. So, when the measure of crime A grows from zero to $1 - F_A(b_A)$, the size of *potential* B-criminals grows from p_L to $[(1-\alpha)+\alpha F_A(b_A)]p_L + (1-F_A(b_A))\alpha p_H$. The two crimes become fully independent if $p_L = p_H$, with equal measures of potential A- and B-criminals if $p_L = p_H = 1$.

The sequence of events is a variant of Section 4 but for the matching process. Given the budget allocation and enforcement incentives set by the state, agent i determines effort, hence, μ_i. Then,

- Undeterred potential A-criminals commit crime A, realize their benefits and, if detected, are sanctioned.
- The measure of potential B-criminals thus determined, undeterred potential B-criminals commit the crime, realize their benefits and, if detected, are sanctioned.
- Detection of a B-crime leads to detection of its affector A-criminal with probability $0 \leq \delta \leq 1$.

The relevance of the possibility of backtracking, that originally undetected A-criminals are detected through detection of B-criminals whom they have affected, should also be contextual. It may be difficult to prove parental abuse by detecting the child in shoplifting but it is possible to trace the drug seller from a criminal acting under the influence of the drug.

■ **Crime equilibrium.** Fix a pair of enforcement intensities μ_A and μ_B and consider first a potential B-criminal with benefit b. Committing the crime yields the expected utility $b - \mu_B s_B$ whereas the utility from compliance is zero. Thus a critical criminal benefit can be defined by

$$b_B = \mu_B s_B, \tag{19}$$

such that a potential B-criminal commits the crime if $b > b_B$ and complies otherwise.

Crime A, on the other hand, is detected with probability μ_A owing to enforcement by agent A and possibly also by follow up investigations of affected B-criminals detected by agent B. Each A-criminal has a B-criminal whom he has affected with probability $\alpha p_H(1 - F(b_B))$. Thus, crime A is detected by agent B's enforcement with probability $\delta(1-\mu_A)\mu_B\alpha p_H(1-F_B(b_B))$. Given this, a potential A-criminal will commit the crime if his benefit exceeds

$$b_A = [\mu_A + \delta(1-\mu_A)\mu_B\alpha p_H(1-F_B(b_B))]\, s_A. \tag{20}$$

A *crime equilibrium* consists of a pair of deterrence levels $(\widetilde{b}_A, \widetilde{b}_B)$ that satisfy (19) and (20), given the enforcement intensities (μ_A, μ_B). It is easy to verify that the crime equilibrium is unique: μ_B uniquely determines $\widetilde{b}_B$ through (19), and $\widetilde{b}_A$ is determined through (20) given (μ_A, μ_B) and $\widetilde{b}_B$. Moreover, $\frac{d\widetilde{b}_i}{d\mu_i} > 0$, $\frac{d\widetilde{b}_B}{d\mu_A} = 0$, whereas the sign of $\frac{d\widetilde{b}_A}{d\mu_B}$ is in general ambiguous (except for sufficiently small μ_B where it is positive.) Also, note that $\frac{d\widetilde{b}_A}{d\mu_B} \to 0$ as $\delta \to 0$.

5.1 Incentives

The measures of the two crimes are

$$\text{crime A: } 1 - F_A(\widetilde{b}_A); \quad \text{crime B: } \left[p_L + \alpha(1 - F_A(\widetilde{b}_A)).(p_H - p_L)\right](1 - F_B(\widetilde{b}_B)). \tag{21}$$

Since $\frac{d\widetilde{b}_i}{d\mu_i} > 0$, crime i is monotonically decreasing in μ_i. It follows that any deterrence target can be induced at first-best cost, budget permitting, through crime-based incentives. Proposition 3 continues to hold in the case of causally linked crimes.

Consider now detections by agent A and agent B:

$$\begin{aligned} d_A &= \left[\mu_A + \delta(1-\mu_A)\mu_B \alpha p_H (1 - F_B(\widetilde{b}_B))\right](1 - F_A(\widetilde{b}_A)), \\ d_B &= \mu_B \left[p_L + \alpha(1 - F_A(\widetilde{b}_A)).(p_H - p_L)\right](1 - F_B(\widetilde{b}_B)). \end{aligned} \tag{22}$$

An increase in μ_B has two conflicting direct effects on d_B. Detections of crime B will rise at constant level of deterrence $\widetilde{b}_B$, but deterrence will also increase and lead to a fall in crime B, hence detections. The net change in crime B detections, positive or negative, will then impact on crime A deterrence and detections through δ, the backtracking effect. The change in crime A will cause a change the measure of potential B-criminals by the α effect, thereby, feed back on crime B and its deterrence. Even in the absence of these feedback effects, the two conflicting direct effects would suffice to jeopardize monotonicity of d_B in μ_B.

Crime A detections d_A can be decomposed into its components as in the previous section, $d_{AA} = \mu_A(1 - F_A(\widetilde{b}_A))$ and $d_{AB} = \delta(1-\mu_A)\mu_B \alpha p_H(1 - F_B(\widetilde{b}_B))(1 - F_A(\widetilde{b}_A))$. Recall that monotonicity of the measure of detections in μ_i is necessary for implementation of any level of deterrence with first-best combination of enforcement inputs. Lemma 5 clarifies this issue.

Lemma 5. *The measures of Crime A and crime B detections are not monotonic in μ_i, except d_{AB} and d_B which are monotonically decreasing in μ_A. That is, the signs of*

$$\frac{d[d_A]}{d\mu_A}, \frac{d[d_B]}{d\mu_B}, \frac{d[d_{AA}]}{d\mu_A}, \frac{d[d_A]}{d\mu_B}, \frac{d[d_{AB}]}{d\mu_B}, \frac{d[d_{AA}]}{d\mu_B} \text{ are ambiguous. However, } \frac{d[d_{AB}]}{d\mu_A} < 0, \ \frac{d[d_B]}{d\mu_A} < 0.$$

d_{AA}, d_{AB} and d_B are monotonic in μ_B at sufficiently low values of μ_B. There exists $\hat{\mu}_B(X) > 0$, $X \in \{AA, AB, B\}$, such that the detection measure d_X is monotonic in μ_B for $\mu_B \in [0, \hat{\mu}_B(X)]$.

Crime B detection data, or crime A detection data owing purely to agent B's enforcement, can be used as indicators to motivate agent A's enforcement effort. Even if agent A's detections are not available in decomposed form d_{AA} and d_{AB}, enforcement incentives for agent A can be set in terms of total detections by agent B. Given fixed μ_B, higher number of B-detections indicate lower effort by agent A. Therefore any crime A deterrence target can be implemented at first-best cost through detection-based incentives.

However, because the detection measures are not monotonic in μ_B, the moral hazard problem in agent B's enforcement raises implementation costs above the first-best level, except for low deterrence targets $\widetilde{b}_B$ which can be induced with relatively low enforcement intensities $\mu_B \in [0, \hat{\mu}_B(X)]$, hence, small enforcement budgets R_B. For μ_B in that range, the detection measures are monotonic in μ_B, restoring first-best enforcement incentives based on detections data.

Let $\mu_B^{\max} = max\{\hat{\mu}_B(AA), \hat{\mu}_B(AB), \hat{\mu}_B(B)\}$. Proposition 6 summarizes the analysis of enforcement costs under crime- and detection-based incentives.

Proposition 6 (Cost advantage of crime-based incentives)**.** *Any deterrence target for the cause crime A can be implemented through crime- or detection-based incentives at first-best cost. Whereas crime-based systems implement at first-best cost all crime B deterrence targets, detection-based systems implement at first-best cost only the deterrence levels compatible with enforcement intensities in $[0, \mu_B^{\max}]$. As for $\mu_B > \mu_B^{\max}$, $c^D(\mu_B) > c^C(\mu_B)$.*

5.2 Budget allocation

The objective of the State in determining a budget allocation (R_A, R_B) under incentive system $r = C, D$ given the total enforcement budget R is to minimize

$$SH = (1 - F_A(\widetilde{b}_A)).h_A + \left[p_L + \alpha(1 - F_A(\widetilde{b}_A)).(p_H - p_L)\right](1 - F_B(\widetilde{b}_B)).h_B \tag{23}$$

subject to (19) and (20), the budget constraint $R = R_A + R_B$ and the enforcement cost functions $R_i = c^r(\mu_i)$.

Denote the solution to this problem by (R_A^*, R_B^*). Under the optimal budget allocation the (endogenous) measure of potential B-criminals is $k_B^* = p_L + \alpha(1 - F_A(\widetilde{b}_A^*)).(p_H - p_L)$ where $\widetilde{b}_A^*$ satisfies (20), whereas the measure of potential A-criminals is $k_A = 1$. If the two crimes were independent as in Section (3) with $k_A = 1$ and $k_B = k_B^*$, the social harm from crime would be $SH_I = (1 - F_A(\widetilde{b}_A)).h_A + k_B^*(1 - F_B(\widetilde{b}_B)).h_B$. Under the optimal budget allocation (R_A^I, R_B^I) the impact of a marginal balanced budget adjustment on SH_I must be zero. Thus, (R_A^I, R_B^I) satisfies the analogue of the first-order condition (**??**), adjusted for the differential measures of potential criminals:

$$\frac{h_A}{h_B} = \left[p_L + \alpha(p_H - p_L)(1 - F_A(\widetilde{b}_A^*))\right] \frac{s_B f_B(\widetilde{b}_B^I)\mu'(R_B^I)}{s_A f_A(\widetilde{b}_A^I)\mu'(R_A^I)}. \tag{24}$$

Turning to the case where crime A causes crime B, for δ sufficiently small the optimal budget allocation (R_A^*, R_B^*) satisfies the first-order condition

$$\frac{h_A}{h_B} = \left[p_L + \alpha(p_H - p_L)(1 - F_A(\widetilde{b}_A^*))\right] \frac{s_B f_B(\widetilde{b}_B^*)\mu'(R_B^*)}{s_A f_A(\widetilde{b}_A^*)\mu'(R_A^*)} - \alpha(p_H - p_L)(1 - F_B(\widetilde{b}_B^*)). \tag{25}$$

If the ratio of harms $\frac{h_A}{h_B}$ is larger than the right hand side of of (25), shifting some budget from agent B to agent A will reduce the harm from crime. The last term, $\alpha(p_H - p_L)(1 - F_B(\widetilde{b}_B^*))$, represents the impact of agent A's enforcement on the measure of potential B criminals. Absent this external effect, the two optimality conditions (24) and (25), hence their solutions, would be identical.

It is possible to compare the optimal budget allocations (R_A^I, R_B^I) and (R_A^*, R_B^*) by imposing an intuitive regularity condition on the benefit distribution functions. Namely, we shall assume that $F_i(b)$ is not too convex at any b, more precisely, $\frac{F_i''(.)}{F_i'(.)} < -\frac{\mu''(.)}{s_i \mu'(.)^2}$, so that the term $f_i(\widetilde{b}_i).\mu'(R_i)$ is decreasing in R_i. In words, the marginal deterrence from an extra dollar in enforcement should be falling.

Proposition 7 (Priority to the cause)**.** *Assume that δ is sufficiently small and that enforcement costs are identical across the cases and crimes.*

(i) The optimal budget allocations when the crimes are independent and when crime A causes crime B compare as follows: $R_A^I < R_A^*$, $R_B^I > R_B^*$;

(ii) In a symmetric crime environment, $R_A^* = R_B^* \Rightarrow \frac{h_A}{h_B} = p_L$ and $R_A^I = R_B^I \Rightarrow \frac{h_A}{h_B} = k_B^* > p_L$.

The assumptions of small δ and identical enforcement costs serve to highlight the pure effect of causal links on the budget allocation. Part (i) states that agent A receives a larger budget when crime A causes crime B than in the case of independent crimes purely because his enforcement impacts on the size of potential B-criminals, hence on the level of crime B. Part (ii) states an implication of causality in symmetric crime environments. When an equal budget allocation is optimal, the harm ratio h_A/h_B must be smaller when crime A causes crime B than if the crimes are independent. Thus, if the State allocates equal budgets to the independent crimes, it should be allocating more resources to the cause crime than the effect crime. When δ is very small, $R_A^* = R_B^*$ induces $\widetilde{b}_A^* \approx \widetilde{b}_B^*$ and hence $F(\widetilde{b}_A^*) \approx F(\widetilde{b}_B^*)$. With equal budgets and same deterrence on crime A and crime B, however, the harm from crime A must be smaller, $h_A = p_L h_B$. The enforcement budget per potential A-criminal is then a fraction $k_B^* = p_L + \alpha(1 - F_A(\widetilde{b}_A^*)).(p_H - p_L) > p_L$ of the enforcement budget per potential B-criminal. On the other hand, for for independent crimes, if an equal budget allocation is optimal the ratio of per potential criminal enforcement budgets must be equal to the ratio of harms.

We can now relax the assumption of identical enforcement costs and incorporate the case of large δ. The first assumption does not hold for large deterrence targets and detection-based incentives in the case of independent crimes and, for the effect crime B in the case of causal links. In those cases, enforcement costs are larger than first-best. As for the cause crime A, we know from Proposition 6 that all deterrence targets are induced at first-best cost through crime- and detection-based systems. Thus, the fact that crime B enforcement costs are larger than crime A for any same, large, deterrence target favors an increase in the budget for agent A, which reinforces the asymmetric allocation result in Proposition 7.

In contrast, the case of large δ favors the effect crime B: Now that agent B's enforcement becomes more productive, the marginal benefit from a balanced budget shift from A to B is larger. But the possibility of a large backtracking effect through detections of the effect crime depends on the context. It may not cause a shift of budget when the cause crime is parental child abuse where δ is close to zero, but it should shift some budget from deterrence of drug sales to deterrence of its various effect crimes.

6 Conclusion

In this paper we address two issues in law enforcement, motivating enforcement units to exert effort under moral hazard, and the related issue as to the optimal allocation of an enforcement budget between the units. We study the moral hazard problem in various environments with multiple, related or independent, crimes. We consider two indicators that strongly correlate with the performance of law enforcement units, crime levels and the number of detections/apprehensions, to motivate effort in enforcement.

The key property that determines whether an indicator can be used to implement crime deterrence targets at first-best cost is monotonicity of the indicator in the enforcement effort of the units. In this respect, the analysis reveals that crime-based incentives weakly dominate detection-based incentives. The crime level is monotonic in enforcement effort whereas detection measures in some environments and crimes are not. If available and reliable, crime-based incentives should be used, for they implement deterrence targets at first-best cost given any enforcement budget.

Whereas crime data may not always be available, as for unobservable crimes which can be known only if detected by law enforcers, or measures of crime may not be of reasonable accuracy, the number of detected/apprehended crime suspects should, in principle, be available for any crime. However, we show that detection-based incentive systems are not as effective in coping with moral hazard because detections are not, in general, monotonic in enforcement effort. The issue is relevant particularly for independent crimes than interlinked crimes. In crimes interlinked through causality or an input-output relation, cross-detections data can be explored for incentive provision. Moreover, if upstream criminals can be backtracked through detection of their downstream partners, two sets of upstream detection data become available, one owing purely to upstream enforcement, the other owing to the unit fighting the downstream crime. Including these cross-detection measures we get a rich set of detection measures, some of which are monotonic in downstream or upstream enforcement effort.

For each of the crimes in an input-output chain, effort-monotonic detection measures can be found to implement any level of deterrence at first-best cost. Thus, for such crimes, crime- and detection-based systems are equally effective. This conclusion does not hold for the downstream (effect) crime partially caused by an upstream crime, that is, in the case of causally linked crimes. None of the detection measures is monotonic in downstream unit's effort. Under detection-based incentives, moral hazard in enforcement generates agency costs for the downstream unit, which offers an additional reason for favoring the upstream unit in budget allocation.

We identify structural mechanisms that favor larger budget allocations to the upstream (input or cause) crime. For interlinked crimes forming an input-output chain, raising deterrence of the upstream crime reduces the benefit from downstream crime. In addition, because undeterred upstream criminals complete their crimes with probability one whereas for downstream criminals this probability is less than one, the upstream crime has some priority in the budget allocation. This is possible even if the upstream crime generates little or no social harm. The backtracking effect, if present and powerful enough, can shift the balance in the opposite direction, to the downstream (output or effect) crime.

An interesting question about enforcement incentives is whether crime- and detection-based systems can be used in conjunction, rather than separately as implied in our set-up where the only concern is moral hazard in law enforcement. Concerns about measurement errors, relative manipulability and lags in availability of crime and detections data, if added on top of moral hazard, we conjecture, could lead the State to use both crime and detections data in designing incentives for its law enforcement units.

A Appendix

■ **Property of d_i function.** Note that $\frac{\partial d_i}{\partial \mu_i} = 1 - F_i(.) + \mu_i\{-\frac{\partial F_i}{\partial \mu_i}\}$, which is positive at low μ_i values and negative at high μ_i values. So $\widehat{\mu}_i$ is bounded away from 1. Further, $\frac{\partial^2 d_i}{\partial \mu_i^2} = -2\frac{\partial F_i}{\partial \mu_i} - \mu_i \frac{\partial^2 F_i}{\partial \mu_i^2}$, where $\frac{\partial^2 F_i}{\partial \mu_i^2} = f_i'(.)\frac{s_i^2}{(1-\mu_i)^4} + f_i(.)\frac{-2s_i}{(1-\mu_i)^3}$. So long as $\frac{\partial^2 F_i}{\partial \mu_i^2} \geq 0$,[16] $d_i(.)$ will be strictly concave in μ_i, because $\frac{\partial F_i}{\partial \mu_i} > 0$ (of course it is possible that $\frac{\partial^2 F_i}{\partial \mu_i^2} < 0$ and yet $d_i(.)$ is strictly concave). This would guarantee a unique global maximum $d_i^{\max} = \widehat{\mu}_i \left(1 - F_i(\frac{\widehat{\mu}_i s_i}{1-\widehat{\mu}_i})\right)$ at a positive $\widehat{\mu}_i$. Marginal detections decline and become negative at μ_i levels above $\widehat{\mu}_i$. ||

Proof of Proposition 2.

(a) Given R_B, $\widetilde{b}_B$ is uniquely determined by the second equation in (12). Consider the first equation in (12), given R_B, $\widetilde{b}_B$ and R_A. The right-hand side is continuous and monotonically decreasing in b_A for $\rho(.) < \pi$, constant for $\rho(.) = \pi$, with limits (under the assumption that the upperbound $\bar{b}$ of benefits is sufficiently large):

$$\left[\frac{\mu_A}{(1-\mu_A)\rho\left(\frac{1-F_B(b_B)}{1-\mu_A}\right)} + \mu_B\delta\right] s_A > 0 \text{ as } b_A \to 0; \qquad \left[\frac{\mu_A}{1-\mu_A} + \mu_B\delta\right] s_A < \bar{b} \text{ as } b_A \to \bar{b},$$

where $\mu_A = \mu(R_A)$ is bounded away from 1. The limit at $b_A \to 0$ is larger than the limit at $b_A \to \bar{b}$. Applying the intermediate value theorem to the difference $LHS - RHS$ of this equation and using monotonicity, a unique fixed-point solution, $\widetilde{b}_A$, is guaranteed.

[16] This will be guaranteed by (weak) log-convexity of the density $f_i(b)$.

(b) The following signs are immediate from (12):

$$\frac{d\widetilde{b}_B}{d\mu_B} = s_B > 0, \qquad \frac{d\widetilde{b}_B}{d\delta} = 0. \quad \text{For } p_A = \pi, \quad \frac{d\widetilde{b}_A}{d\mu_A} = \frac{s_A}{(1-\mu_A)^2\pi} > 0, \qquad \frac{d\widetilde{b}_A}{d\delta} = \mu_B s_A > 0.$$

Suppose $p_B < \pi$. We have

$$\frac{d[p_B(1-F_B(\widetilde{b}_B))]}{d\mu_B} = [\rho'(.)\frac{k_A}{k_B^2}(1-F_B(\widetilde{b}_B))f_B(\widetilde{b}_B) - p_B f_B(\widetilde{b}_B)]\frac{d\widetilde{b}_B}{d\mu_B}.$$

This expression is negative if $\rho'(.)\frac{k_A}{k_B^2}(1-F_B(\widetilde{b}_B))-p_B < 0$ or, using $1-F_B(\widetilde{b}_B) = k_B$, if $\rho'(.)\frac{k_A}{k_B} < p_B$, which is equivalent to strict concavity of $p_B = \rho(.)$ for $p_B < \pi$, stated in Assumption 2.

Consider equilibria with $p_A < \pi$. We differentiate the first equilibrium condition in (12) combined with (11):

$$d\widetilde{b}_A = \frac{s_A}{(1-\mu_A)^2 p_A}d\mu_A + \delta s_A d\mu_B - \frac{\mu_A s_A}{(1-\mu_A)p_A^2}dp_A; \tag{26}$$

$$dp_A = \rho'(\frac{k_B}{k_A})\Big[\frac{(1-F_A(\widetilde{b}_A))k_B}{k_A^2}d\mu_A + \frac{f_A(\widetilde{b}_A))(1-\mu_A)k_B}{k_A^2}d\widetilde{b}_A - \frac{f_B(\widetilde{b}_B)}{k_A}d\widetilde{b}_B\Big]. \tag{27}$$

Setting $d\mu_A = 0$ and rearranging terms yields $\frac{d\widetilde{b}_A}{d\delta} > 0$, unambiguously. Setting $d\delta = 0$ and using $k_A = (1-F_A(\widetilde{b}_A))(1-\mu_A)$, we get $\frac{d\widetilde{b}_A}{d\mu_A} > 0$ if $p_A - \mu_A\rho'(.)\frac{k_B}{k_A} > 0$, which holds by strict concavity of $\rho(.)$, i.e., $p_A > \rho'(.)\frac{k_B}{k_A}$, and the fact that $\mu_A \leq 1$. **Q.E.D.**

Proof of Lemma 4. We begin by differentiating the second equilibrium condition in (12) and the expression for p_B (thus completing the set, coupled with (26) and (27)):

$$d\widetilde{b}_B = s_B d\mu_B, \tag{28}$$

$$dp_B = \rho'(\frac{k_A}{k_B})\left[-\frac{(1-F_A(\widetilde{b}_A))}{k_B}d\mu_A - \frac{f_A(\widetilde{b}_A))(1-\mu_A)}{k_B}d\widetilde{b}_A + \frac{k_A}{k_B^2}f_B(\widetilde{b}_B)d\widetilde{b}_B\right]. \tag{29}$$

Now consider the expressions in (13) and (14), beginning with d_B.

Set $d\mu_B = 0$, hence by (28), $d\widetilde{b}_B = 0$. Clearly, if $p_B = \pi$, then $\frac{d[d_B]}{d\mu_A} = 0$. Suppose $p_B < \pi$ and thus $p_A = \pi$, hence $dp_A = 0$. Using $d\widetilde{b}_A = \frac{s_A}{(1-\mu_A)^2 p_A}d\mu_A > 0$ from (26) in the expression for dp_B in (29), we get

$$\frac{d[d_B]}{d\mu_A} = \mu_B(1-F_B(\widetilde{b}_B))\frac{dp_B}{d\mu_A} < 0 \quad \text{because} \quad \frac{dp_B}{d\mu_A} < 0.$$

Set $d\mu_A = 0$ and consider

$$\frac{d[d_B]}{d\mu_B} = \mu_B(1-F_B(\widetilde{b}_B))\frac{dp_B}{d\mu_B} + p_B(1-F_B(\widetilde{b}_B)) - p_B\mu_B f_B(\widetilde{b}_B)\frac{d\widetilde{b}_B}{d\mu_B}.$$

The second term is positive but the third is negative, because $\frac{d\widetilde{b}_B}{d\mu_B} > 0$. Therefore, the sign of $\frac{d[d_B]}{d\mu_B}$ is ambiguous regardless the sign of the first term (which is equal to zero if $p_B = \pi$, non-zero if

$p_B < \pi$).

Now consider the components of d_A, beginning with

$$\frac{d[d_{AA}]}{d\mu_A} = 1 - F_A(\widetilde{b}_A) - \mu_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A}.$$

By Proposition 2(b), $\frac{d\widetilde{b}_A}{d\mu_A} > 0$, but the sign of the expression above is ambiguous because it depends on the magnitude of $\frac{d\widetilde{b}_A}{d\mu_A}$. Therefore, the sign of $\frac{d[d_A]}{d\mu_A}$ is also ambiguous.

Set $d\mu_A = 0$ and consider the expression

$$\frac{d[d_{AA}]}{d\mu_B} = -\mu_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_B}.$$

In the case $p_A = \pi$ we have $dp_A = 0$ and thus from (26) we get $\frac{d\widetilde{b}_A}{d\mu_B} = \delta s_A > 0$. If $p_A < \pi$ and so $dp_A \neq 0$, using (27) in (26) it is easy to verify that $\frac{d\widetilde{b}_A}{d\mu_B} > 0$. Thus, $\frac{d[d_{AA}]}{d\mu_B} < 0$, unambiguously.

The last detection measure is d_{AB}. Holding μ_B constant and differentiating the corresponding expression in (14) yields

$$\frac{d[d_{AB}]}{d\mu_A} = -\mu_B\delta\left[p_A(1 - F_A(\widetilde{b}_A)) - (1-\mu_A)(1 - F_A(\widetilde{b}_A))\frac{dp_A}{d\mu_A} + (1-\mu_A)p_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A}\right]. \tag{30}$$

If $p_A = \pi$, the second term vanishes and thus, given $\frac{\partial\widetilde{b}_A}{\partial\mu_A} > 0$, the expression of $\frac{d[d_{AB}]}{d\mu_A}$ in (30) is negative. Suppose $p_A < \pi$ and thus $dp_B = 0$. Using (26) in (27) we can express the second term in the squared brackets in (30) as

$$-k_A\frac{d\widetilde{p}_A}{d\mu_A} = -\frac{k_B}{k_A}\rho'(\frac{k_B}{k_A})\left[(1 - F_A(\widetilde{b}_A)) + f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A})\right].$$

By substitution, the term in the squared brackets in (30) can be written as

$$p_A(1 - F_A(\widetilde{b}_A)) - \frac{k_B}{k_A}\rho'(\frac{k_B}{k_A})\left[(1 - F_A(\widetilde{b}_A)) + (1-\mu_A)f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A})\right] + (1-\mu_A)p_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A},$$

or, grouping the terms, as

$$(1 - F_A(\widetilde{b}_A))[p_A - \frac{k_B}{k_A}\rho'(\frac{k_B}{k_A})] + f_A(\widetilde{b}_A)(1-\mu_A)\frac{d\widetilde{b}_A}{d\mu_A}[p_A - \frac{k_B}{k_A}\rho'(\frac{k_B}{k_A})],$$

which is positive because $p_A > \frac{k_B}{k_A}\rho'(\frac{k_B}{k_A})$ by strict concavity of $\rho(.)$ and $\frac{d\widetilde{b}_A}{d\mu_A} > 0$ by Proposition 2(b). Hence, $\frac{d[d_{AB}]}{d\mu_A} < 0$.

Finally, consider

$$\frac{d[d_{AB}]}{d\mu_B} = (1-\mu_A)\delta\left[p_A(1-F_A(\widetilde{b}_A)) + \mu_B(1-F_A(\widetilde{b}_A))\frac{dp_A}{d\mu_B} - \mu_B p_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_B}\right]. \quad (31)$$

If $p_A = \pi$ and thus $\frac{dp_A}{d\mu_B} = 0$, $\frac{d\widetilde{b}_A}{d\mu_B} = \delta s_A > 0$, implying $\frac{d[d_{AB}]}{d\mu_B} = (1-\mu_A)\delta\pi[(1-F_A(\widetilde{b}_A)) - \mu_B f_A(\widetilde{b}_A)\delta s_A]$, whose sign is ambiguous. If $p_A < \pi$ and thus $\frac{dp_A}{d\mu_B} \neq 0$, again no clear statement can be made about the sign of $\frac{d[d_{AB}]}{d\mu_B}$ because (see (26) and (27)) the signs of $\frac{d\widetilde{b}_A}{d\mu_B}$ when p_A can adjust and the sign of $\frac{dp_A}{d\mu_B}$ when $\widetilde{b}_A$ can adjust are ambiguous. **Q.E.D.**

Proof of Proposition 4. Fix an allocation (R_A, R_B) such that $R_i > 0$, $i = A, B$. Fix also $\mu_B > 0$ and let $\widetilde{e}(R_A)$ denote the first-best effort input under budget R_A, producing the detection probability $\mu_A(R_A)$ and the pair $\{\widetilde{b}_A, \widetilde{b}_B\}$ through (12). Denote the resulting d_{AB} detections by $d^D_{AB}(R_A|\mu_B)$ as determined by (14). Consider agent A's incentives based on d_{AB}, as follows:

$$w^D_A(d_{AB}|\mu_B) = \begin{cases} z(\widetilde{e}(R_A)), & \text{if } \; d_{AB} \leq d^D_{AB}(R_A|\mu_B) \\ 0, & \text{otherwise.} \end{cases} \quad (32)$$

Agent A has no incentive to raise effort above $\widetilde{e}(R_A)$ because effort is costly and the reward is the same, whereas decreasing effort results in $d_{AB} > d^D_{AB}(R_A|\mu_B)$ and reduces the reward to zero. Given the functional relationship between $\{\mu_A, \mu_B\}$ and $\{\widetilde{b}_A, \widetilde{b}_B\}$ through (12), any crime A level can be implemented by appropriately adjusting R_A and adjusting $d^D_{AB}(R_A|\mu_B)$ in (32).

Similar arguments apply for Agent B's incentives, based on d_{AA}. **Q.E.D.**

Proof of Proposition 5. We know that under an equal budget allocation in a symmetric crime equilibrium, $p_A = \pi$, $p_B < \pi$, and $\widetilde{b}_A > \widetilde{b}_B$.

Total differentiation of the state's objective function at the induced crime equilibrium yields

$$dSH = -h_A f_A(\widetilde{b}_A)[d\widetilde{b}_A] - p_B h_B f_B(\widetilde{b}_B)[d\widetilde{b}_B] + h_B(1-F_B(\widetilde{b}_B))[dp_B]. \quad (33)$$

Since $p_A = \pi$ and hence $dp_A = 0$, equations (26)-(29) become:

$$\begin{aligned} d\widetilde{b}_A &= \frac{\mu'(R_A)s_A}{(1-\mu(R_A))^2\pi}dR_A + \mu'(R_B)\delta s_A dR_B, \qquad d\widetilde{b}_B = \mu'(R_B)s_B dR_B, \\ dp_B &= \rho'(.)\left[-\frac{\mu'(R_A)(1-F_A(\widetilde{b}_A))}{(1-F_B(\widetilde{b}_B))}dR_A - \frac{f_A(\widetilde{b}_A))(1-\mu(R_A))}{(1-F_B(\widetilde{b}_B))}d\widetilde{b}_A + \frac{(1-\mu(R_A))(1-F_A(\widetilde{b}_A))}{(1-F_B(\widetilde{b}_B))^2}f_B(\widetilde{b}_B)d\widetilde{b}_B\right]. \end{aligned}$$

Substituting the expressions for $d\widetilde{b}_A$, $d\widetilde{b}_B$ and dp_B in (33) we get

$$\begin{aligned} dSH &= -h_A f_A(\widetilde{b}_A)[\frac{\mu'(R_A)s_A}{(1-\mu(R_A))^2\pi}dR_A + \mu'(R_B)\delta s_A dR_B] - p_B h_B f_B(\widetilde{b}_B)\mu'(R_B)s_B dR_B \\ &\quad + h_B\rho'(.)\Bigg[-\mu'(R_A)(1-F_A(\widetilde{b}_A))dR_A + \frac{k_A}{k_B}f_B(\widetilde{b}_B)\mu'(R_B)s_B dR_B \\ &\quad - f_A(\widetilde{b}_A)(1-\mu(R_A))\left(\frac{\mu'(R_A)s_A}{(1-\mu(R_A))^2\pi}dR_A + \mu'(R_B)\delta s_A dR_B\right)\Bigg]. \end{aligned} \tag{34}$$

Set $R_A = R_B$, $dR_A = -dR_B$ and let $F_i(.) = F(.)$, $s_i = s$, $i = A, B$ (symmetric crime environment) in (34). Rearranging the terms and simplifying (34), as $\delta \to 0$ we have $dSH < 0$ if

$$h_A > h_B(1-\mu(\frac{R}{2}))^2\left[\frac{f(\widetilde{b}_B)\pi}{f(\widetilde{b}_A)}(p_B - \rho'(.)\frac{k_A}{k_B}) - \frac{\rho'(.)}{1-\mu(\frac{R}{2})}(1+\frac{\pi k_A.\mu(\frac{R}{2})}{sf(\widetilde{b}_A)\mu'(\frac{R}{2})})\right],$$

which reproduces (17). If $f(\widetilde{b}_B)\pi \le f(\widetilde{b}_A)$, the coefficient of h_B in the squared brackets is definitely smaller than one because $\widetilde{p}_B < 1$ and $\rho'(x) > 0$ for $x < 1$. Then, (17) holds and the adjustment $dR_A = -dR_B > 0$ at $R_i = R/2$ reduces SH unless h_B is sufficiently larger than h_A. **Q.E.D.**

Proof of Lemma 5. Consider first the impact of μ_A on the detection measures for crime A. Because $\frac{d\widetilde{b}_A}{d\mu_A} > 0$,

$$\begin{aligned} \frac{d[d_{AA}]}{d\mu_A} &= (1-F_A(\widetilde{b}_A)) - \mu_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A}, \quad \text{sign ambiguous,} \\ \frac{d[d_{AB}]}{d\mu_A} &= -\delta\mu_B\alpha p_H(1-F_B(\widetilde{b}_B))\left[(1-F_A(\widetilde{b}_A)) + (1-\mu_A)f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A}\right] < 0. \end{aligned}$$

On the other hand, The sign of $\frac{d[d_{AA}]}{d\mu_B}$ is ambiguous because $\frac{d\widetilde{b}_A}{d\mu_B}$ has an ambiguous sign.

$$\begin{aligned} \frac{d[d_{AA}]}{d\mu_B} &= -\mu_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_B}; \\ \frac{d[d_{AB}]}{d\mu_B} &= (1-\mu_A)\delta p_H\alpha\left[(1-F_A(\widetilde{b}_A))\left[1-F_B(\widetilde{b}_B) - \mu_B f_B(\widetilde{b}_B)\frac{d\widetilde{b}_B}{d\mu_B}\right] - \mu_B(1-F_B(\widetilde{b}_B))f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_B}\right]. \end{aligned}$$

Using $\frac{d\widetilde{b}_B}{d\mu_B} = s_B$ and $\frac{d\widetilde{b}_A}{d\mu_B} = (1-\mu_A)\delta\alpha p_H[1-F_B(\widetilde{b}_B) - \mu_B s_B f_B(\widetilde{b}_B)]s_A$ in the expression above reveals that the sign of $\frac{d[d_{AB}]}{d\mu_B}$ depends on the sign of $1-F_A(\widetilde{b}_A) - \mu_B f_B(\widetilde{b}_B)f_A(\widetilde{b}_A)(1-\mu_A)\delta p_H s_A$, which is ambiguous.

The sign of $\frac{d[d_B]}{d\mu_B}$ is also ambiguous; d_B is in the form $\mu_B X_B(1-F_B(\widetilde{b}_B))$ where X_B is the measure of potential B-criminals and $\widetilde{b}_B$ is increasing in μ_B. However, $\frac{d[d_B]}{d\mu_A} = -\alpha\mu_A f_A(\widetilde{b}_A)\frac{d\widetilde{b}_A}{d\mu_A} < 0$.

For the proof of the claim that d_{AA}, d_{AB} and d_B are monotonic in μ_B in a range of small μ_B levels, it suffices to verify that the signs of the expressions of $\frac{d[d_{AA}]}{d\mu_B}$, $\frac{d[d_{AB}]}{d\mu_B}$ and $\frac{d[d_B]}{d\mu_B}$ become definite as $\mu_B \to 0$. **Q.E.D.**

Proof of Proposition 7. Differentiation of (23) with respect to the endogenous variables yields

$$dSH = -f_A(\widetilde{b}_A)\left[h_A + \alpha(p_H - p_L)(1 - F_B(\widetilde{b}_B))h_B\right] d\widetilde{b}_A - \left[p_L + \alpha(1 - F_A(\widetilde{b}_A)).(p_H - p_L)\right] f_B(\widetilde{b}_B))h_B d\widetilde{b}_B. \tag{35}$$

Assume δ is sufficiently small, so that $\widetilde{b}_B = s_B d\mu_B$ and, $\widetilde{b}_A \approx s_A d\mu_A$. In a symmetric crime environment and under an equal budget allocation $R_A = R_B$, $\mu_A \approx \mu_B$ and thus, $\widetilde{b}_A \approx \widetilde{b}_B$, $f_A(\widetilde{b}_A) \approx f_B(\widetilde{b}_B)$ and $F_A(\widetilde{b}_A) \approx F_B(\widetilde{b}_B)$. Using these facts in (35) with a negative sign for $d\mu_B$ and arranging the terms yields, $dSH < 0$ if

$$f(\widetilde{b}_A)s\frac{h_A}{h_B} > \left[p_L + \alpha(1 - F(\widetilde{b}_A)).(p_H - p_L)\right] f(\widetilde{b}_B))s - f(\widetilde{b}_A)\alpha(1 - F(\widetilde{b}_B)).(p_H - p_L)s.$$

Thus, $dSH < 0$ if $h_A > p_L.h_B$. **Q.E.D.**

$$\begin{aligned} dSH &= -f_A(\widetilde{b}_A)\left[h_A + h_B\rho'(.)(1-\mu(R_A))\right] d\widetilde{b}_A - h_B\rho'(.)\mu'(R_A)(1 - F_A(\widetilde{b}_A))\, dR_A \\ &\quad - h_B f_B(\widetilde{b}_B)\left[\widetilde{p}_B - \rho'(.)\frac{(1-\mu(R_A))(1-F_A(\widetilde{b}_A))}{(1-F_B(\widetilde{b}_B))}\right] d\widetilde{b}_B \\ &= \left[-f(\widetilde{b}_A)\left[h_A + h_B\rho'(.)(1-\mu(\frac{R}{2}))\right]\frac{\mu'(\frac{R}{2})s}{(1-\mu(\frac{R}{2}))^2\pi} - \mu'(\frac{R}{2})(1-F(\widetilde{b}_A))h_B\rho'(.)\right] dR_A \\ &\quad - \left\{ f(\widetilde{b}_A)[h_A + h_B\rho'(.)(1-\mu(\frac{R}{2}))]\mu'(\frac{R}{2})\delta s \right. \\ &\quad \left. + h_B f(\widetilde{b}_B)\left[\widetilde{p}_B - \rho'(.)\frac{(1-\mu(\frac{R}{2}))(1-F(\widetilde{b}_A))}{(1-F(\widetilde{b}_B))}\right]\frac{\mu'(\frac{R}{2})s}{(1-\mu(\frac{R}{2}))^2}\right\} dR_B. \end{aligned}$$

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