Happiness in the finite[*]

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Abstract

The idea that 'more is better' has retained its axiomatic stature in economics. Psychologists however, have contested this idea that too many choices translate to greater welfare, by pointing out that the cognitive costs of evaluating available choices against each other often take away from agent-satisfaction. We formalize this idea and apply the same in the context of markets with welfare implications that contradict the idea that competition maximizes surplus. We demonstrate that welfare is maximized with a *strictly* finite number of firms even when they are heterogeneous, thereby mimicking an monopolistic structure. An immediate implication for regulatory authorities is that maximizing welfare is not the same as maximizing competition.

Keywords: welfare, market structure, choice

JEL classifications: L11, L13, L15, D43, D91

[*]This paper has received valuable feedback from Prabal Roy Chowdhury, Arunava Sen, Amit Kumar Goyal, Lionel Page, Dipanwita Sarkar, Jayanta Sarkar, and Uwe Dulleck.

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1. Introduction

We re-examine one of the central and important results of welfare economics, that competition maximizes welfare. We argue that the validity of this known result implicitly rests on the assumption that consumers have infinite cognitive capacity. We show that in a real world of consumers with limited cognition, a welfare-maximizing market has a *strictly* oligopolistic structure. In the process, we introduce an elegant new approach to incorporate product-heterogeneity in an oligopoly, that unifies the known attempts to that end in the industrial organization literature (Eaton and Lipsey, 1989, Häckner, JET 2000. Tirole, 1988, Tremblay and Tremblay, 2011).

We model decision-makers' cognitive limitations directly from the biology of the human brain (Sapolsky, 2017; Eagleman 2015) and study the welfare implications of cognitively limited agents in markets. This immediately relates to the limited mental bandwidth problem of Mullainathan and Shafir, 2013. To the best of our knowledge, this is a first attempt at the endeavor of uncovering the welfare implications of Mullainathan and Shafir's '*science in the making*'.

The idea that the availability of greater choices can only improve welfare, is embedded in economic theory.[1] The empirical truth that the Western world is flooded with choices, is however, seen as the *official dogma* of the industrialized world by psychologists (Schwartz, 2004; Iyengar and Lepper, 2000). Specifically, what economists call *variety* is formally termed as *choice overload* by psychologists.

[1] The intuition is that any additional item in the choice basket will be consumed only if it enhances utility, otherwise that additional choice will be redundant. Thus, additional choices can only be welfare improving ... even if the number of choices is infinite.

If we accept that utility is derived not only from final consumption but also from the process of determining what to consume (called respectively as 'outcome satisfaction' and 'process satisfaction' - see Reutskaja and Hogarth, 2009), then the cognitively-taxing process of evaluating too many choices against each other may very well take away from satisfaction. On the aggregate, therefore, welfare will reduce beyond the number of products where the consumers' love for variety and the crunch/pressure/tax on cognitive capacity balance each other out. Thus, we show that a finite variety of *products*, and therefore a finite number of *producers*, is consistent with welfare maximization.[2] Additionally, firms benefit from reduced competition - thus, the additions to both consumers' and producers' surplus contribute to welfare maximization.

From economic theory we know that it is sufficient for a consumer to be able to make pair-wise comparisons in order to arrive at the complete preference ordering between a certain number n of commodities/bundles. The number of such pair-wise comparisons increases at a much higher rate than n itself (specifically it is $n(n - 1)/2$). The cognitive costs of comparison therefore, increase according to the number of possible comparisons rather than just on the basis of the number of options. Even binary ($n = 2$) choices that involve only one (= $n(n - 1)/2$) possible comparison (between the two available alternatives) could be mentally taxing in some cases. For example, a diabetic who decides whether or not to consume the cake in front of him, experiences an internal conflict. The amygdala (or the 'reptilian brain' governing individual instincts) gives in to the temptation of immediate (tasty) benefits from consumption, but the pre-fontal cortex (PFC hereafter, also the most recently evolved part of the brain that is broadly responsible for reasoning), interferes with this instinct by making the individual aware of the threat of future health-related costs. This 'tug of war' between the two brains consumes a sufficient amount of

[2] The exact finite number will, among other things, depend on the nature and the strength of the cognitive costs of comparison.

energy, leaving the agent mentally exhausted at the end of this decision-making process (Sapolsky, 2017). Even if the amygdala wins, the still-active PFC takes away from the utility from immediate consumption.

For a sufficiently large number of available choices, Schwartz (2004) provides two (additional) psychological channels through which, decision-making becomes costly (in utility terms). First, there is an automatic upward revision of expectations from the very availability of too many alternatives. More specifically, it is easy to imagine that a better choice was possible whenever one is made. The psychological comparison (between the choice made and the imagined alternative) that follows, reduces the utility from the choice made. Second, if a bad choice is made, then the blame is on the 'world' when the alternatives to choose from are very few (in the extreme case with $n = 1$, the decision-maker can reason to himself that he could do nothing about what was available), but there is an element of *self-blame* when there are too many alternatives to choose from. Further, if the consumable in question is durable, then the memory of this *self-blame* is triggered (in the hippocampus - the region responsible for memory storage) every time the good is used, which further reduces utility. Therefore, in addition to the biological cognitive costs of making a choice between too many alternatives, there are additional psychological costs that take away from utility.

The comparison of too many alternatives directly translates to the processing of too much information. The human brain has evolved to avoid the time-costs of too much processing and focuses on only a subset of information that is deemed most important at any given point of time (Sapolsky, 2017). This is the reason why one may forget his wallet when in a hurry to make it to an important meeting. The mind creates a *tunnel* of focus on the key objective at hand (in this case, to reach the meeting on time) and all the mental bandwidth is used up to think about booking a cab, organizing the meeting briefs, and so on. Therefore, even basic (and often important) things that remain out of the tunnel (in this case, remembering to carry wallet) are immediately ignored by the brain — the sense of urgency created in the limited time leads an agent to subliminally economize on his brain capacity (Mullainathan and Shafir, 2013). Since cognition is a scarce resource, in order to facilitate quicker decision making when desirable, any agent's actions are frequently dependent on the information that his senses can immediately access (Sapolsky, 2017 and Eagleman, 2015). This information frequently translates to focal or reference points, thereby explaining why the *decoy effect* works in marketing (Ariely, 2008), or why economic agents are frequently *present-biased* (see Camerer, 2003; Bardsley et al 2009; Frey and Stutzer, 2007), or why they rely on expert opinion (Smith, 2007; Beattie et al, 1994), or even why they can be *nudged* to achieve greater levels of welfare (Thaler and Sunstein, 2008) with an external influence on their choice architecture. In a nutshell, limited cognition against too many alternatives has well-established empirical consequences, and is therefore worthy of a theoretical examination from a welfare perspective.

Regulatory authorities frequently try to increase welfare by promoting competition to counter the market power possessed by monopolies. This gives any consumer, a sense of freedom of choice accompanied with autonomy, thereby making the experience of shopping in the presence of variety very pleasant. So long as the individual evaluation of each variety is costless, more choices can only increase welfare (if not leave the same unaltered). Therefore, any regulation that promotes variety in choices is expected to be welfare enhancing. We argue that this traditional view of welfare only holds when consumers have infinite cognitive capacity to arrive at the best decisions, after meticulously comparing all the available alternatives.

Section 2 presents our multi-agent model and establishes the presently known welfare results. In section 3, we incorporate the cognitive costs of decision making in our framework, and establish new welfare results. In section 4,we demonstrate the robustness of our results with the introduction of product-heterogeneity, and we conclude with section 5.

2. The model

To begin with, we assume that there are k identical utility-maximizing (and perfectly rational) consumers, and n identical profit maximizing firms who engage in Cournot competition. Our choice of Cournot competition is simply based on the flexibility to look at the extreme cases of monopoly ($n = 1$) and competitive ($n = \infty$) scenarios and show the established welfare implications of each. This also requires us *for now*, to assume perfect product homogeneity so that our results are immediately comparable (and consistent) with the known welfare implications of perfect competition.

2.1. The consumers' problem

We are interested in the market for a given commodity, the quantity (consumed) of which is represented by x. Each identical consumer must decide how to allocate his/her given income M, between x units of this commodity and y units of all other goods. The price for the latter is normalized to unity. We assume that utility U has a quasi-linear specification and is specifically additive in the functional components of x and y. The representative consumer faces the following (simple) utility maximization problem:

$$
\begin{aligned}
&Maximize: \quad U = ax - bx^2 + y \\
&Subject\ to: \quad px + y = M
\end{aligned}
\tag{1}
$$

with the latter being the budget constraint: $px + y = M$, where p is the price of the commodity of interest, and therefore, px is the total expenditure on the commodity. We look at a quadratic specification in x (with $a > 0$ and $b > 0$) particularly for two reasons. First, quadratic expressions can uniquely approximate any other well behaved utility specification remarkably well.[3] Second, and more importantly, we naturally arrive at a *linear* demand curve to keep things tractable when we progressively introduce complications in sections 3 (consumers with limited cognition) and 4 (dropping the assumption of strict product homogeneity).

The solution to (1) involves the substitution of y from the budget constraint into the objective function and equating the derivative (of U w.r.t. x) to zero.[4] This gives us the following (individual) demand curve for our representative consumer.

$$x^* = \frac{a-p}{2b}$$

Finally, the aggregate market demand for k consumers $(X = kx^*)$ is attained from the horizontal summation of k identical individual demand curves. Thus, the market demand X is given by:

$$X = kx^* = k\left(\frac{a-p}{2b}\right) = A - Bp$$

with $A = (ak/2b)$, and $B = (k/2b)$.

We now invert the above and write the inverse demand curve as

[3] For example, the function $f(x) = ln(ax + b)$ can be approximated remarkably well by the quadratic expression $g(x) = (-a^2/8b^2)x^2 + (3a/4b)x + [ln(2) + ln(b) - (5/8)]$ in the vicinity of $x = b/a$. Additionally see Small (2007) and Ball (2003) for beautiful discussions on quadratic functions and approximations.
[4] Note that $U = ax - bx^2 + (M - px)$, after substitution. Maximization is guaranteed from the global concavity of U.

$$p = \frac{A - X}{B} = \alpha - \beta X \quad (2)$$

with $\alpha = (A/B)$, and $\beta = (1/B)$. This is the demand curve faced by our n identical profit-maximizing firms. We now turn to the supply-side of our story.

2.2. The producers' problem

Just like X represents the total market demand, we define the total market supply as $Q = \sum_{i=1}^{n} q_i$, where q_i is the quantity produced and supplied by *Firm i* ($i \epsilon \{1, \ldots , n\}$). Using the fact that market supply Q will exactly equal market demand X in the equilibrium, so that the market clearing condition ($X = Q$) can be incorporated in the inverse demand function (2) as

$$p = \alpha - \beta Q = \alpha - \beta \sum_{i=1}^{n} q_i \quad (3)$$

The profit function of a typical firm i is given by $\pi_i = (p - c)q_i$. Assuming zero costs ($c = 0$) in the interest of simplicity, the problem of *Firm i* is to maximize its own profit shown below.[5]

$$Maximise\colon\ \pi_i = pq_i = (\alpha - \beta Q)q_i \quad (4)$$

This leads us to the first order condition $\alpha - \beta Q - \beta q_i = 0$. Finally, we use symmetry (our firms are identical) and use $q_i = (Q/n)$ to get

$$Q^* = \frac{n}{(n+1)}\frac{\alpha}{\beta}\ ;\ q_i^* = \frac{1}{(n+1)}\frac{\alpha}{\beta}\ ; and\ p^* = \frac{\alpha}{(n+1)} \quad (5)$$

[5] Even with zero costs, we will be dealing with exceedingly complex expressions in Sections 3 and 4. The results presented here easily extend to general (upward sloping or constant) cost structures.

Which is the standard *n-firm* Cournot outcome. We now come to a discussion of total utility received by k consumers and n firms in the following subsection.

2.3. Welfare implications

Using (5), we can immediately work out individual profits ($\pi^* = p^*q^*$) and the total producers' surplus ($n\pi^* = np^*q^*$) as follows:

$$\pi^* = \frac{1}{(n+1)^2}\frac{\alpha^2}{\beta} \; ; \; n\pi^* = \frac{n}{(n+1)^2}\frac{\alpha^2}{\beta}$$

Finally, replacing α and β above by *(A/B)* and *(1/B)* respectively, and replacing A and B in turn, by *(ak/2b),* and *(k/2b)* respectively (i.e. working back the transformations introduced in subsection 2.1.), we re-write the (market and individual) quantities, the price, and the total producers' surplus above as:

$$Q^* = \frac{akn}{2b(n+1)} \; ; \; q_i^* = \frac{ak}{2b(n+1)} \; ; \; p^* = \frac{a}{(n+1)} \; ; n\pi^* = \frac{ka^2n}{2b(n+1)^2} \tag{6}$$

We now use the market clearing condition to replace Q^* by X, (so that $Q^*/k = X/k$), to work out individual consumption as follows:

$$x^* = \frac{an}{2b(n+1)} \tag{7}$$

To explain this equilibrium with an example, suppose eighty units of output are produced and traded in a market comprising twenty consumers and five producers so that $k = 20$ and $n = 5$, with $X = Q = 80$, then, each consumer must consume four units and each producer must produce (and sell) sixteen units, so that $x^* = 4$, and $q^* = 16$. The equilibrium level of utility for each consumer is given by

$$U(x^*) = \frac{a^2 n}{2b(n+1)} - b\left(\frac{an}{2b(n+1)}\right)^2 + y$$

Now, we replace $y = (M - px) = \left(M - \frac{a^2 n}{2b(n+1)^2}\right)$ in the above expression to get

$$U(x^*) = U^* = \frac{a^2 n}{2b(n+1)} - b\left(\frac{an}{2b(n+1)}\right)^2 + \left(M - \frac{a^2 n}{2b(n+1)^2}\right) \quad (8)$$

Finally, we sum up the individual utilities for *k* consumers, and add to that, the total profits of the firms to define the total surplus $S(n) = kU^* + n\pi^*$ (which is our measure of total welfare) as follows:

$$S(n) = \frac{ka^2 n}{2b(n+1)} - bk\left(\frac{an}{2b(n+1)}\right)^2 + k\left(M - \frac{a^2 n}{2b(n+1)^2}\right) + \frac{ka^2 n}{2b(n+1)^2}$$

which in turn, can be further simplified (after accounting for the fact that $kp^*x^* = n\pi^*$, i.e. what consumers pay to producers is the latter's earning, and thus gets cancelled out from the last two terms above) to:

$$S(n) = \frac{ka^2 n(n+2)}{4b(n+1)^2} + kM \quad (9)$$

It is immediately clear that welfare increases with more firms, since $S(n + 1) - S(n)$ is always positive as shown below:

$$S(n+1) - S(n) = \frac{ka^2(2n+3)}{4b(n+1)^2(n+2)^2} > 0 \; for \; \{n > 0\} \quad (10)$$

Thus, it always benefits to accommodate more and more firms, thereby encouraging the competitive removal of barriers to entry. Welfare in this world, has a maximum possible value of $ka^2/4b$ (with an infinitely large number n of firms), and our model structure is consistent with the established welfare results. We now introduce cognitive costs in our model while retaining the current feature of product-homogeneity.

3. Markets with consumers of limited cognition

We now introduce bounds on perfect reasoning (i.e. perfect cognition is not costless) in a manner, systematically different from those discussed in Spiegler (2011).[6] More specifically we model a (cognitive/biological and psychological) 'cost' of deciding what choice to make, the basis for which has already been presented in the introductory section.

It may well be argued that if products are indeed homogeneous, then there is no reason for consumers to 'choose between' what different firms make available in the market. In other words, the consumers can simply buy the product without incurring any psychological/biological costs (for example, without having to worry about comparing what they have bought against what they have not, since the products are homogeneous). Therefore, with homogeneous products, the need for comparison may not arise (thereby associating welfare maximization with the perfectly competitive result as in the previous section). It is here, however, where we stress on a realism that we intend to capture in our model. In the real world, consumers learn about the homogeneity between two or more brands (even if there is any) only after they have already incurred the cognitive cost of comparison. In other words, people need to first compare two things even to

[6] More specifically, we distinguish Spiegler's (2011) bounded rational agents from our agents of limited cognition. It is the former that implies the latter, and the latter does not systematically rule out rationality.

realise that they are indeed identical. Therefore, the cognitive costs of comparison (and therefore deciding what to buy) are not necessarily lost even when agents deal with (comparing two or more) homogeneous products. If anything, it is much easier to rank two items when one is distinctly (and significantly) better than the other. Things become difficult (i.e. more cognitive resources are required) when one has to choose between two very similar items (e.g. choosing between two very similar jobs). The second reason why we want to incorporate cognitive costs here is that we want our welfare results to be directly comparable with those in the previous section (with homogeneous products). In any case, we will drop the assumption of product homogeneity in the next section. The last reason why we incorporate cognitive costs here is because we will use some of the results established here in the key derivations of the next section.

To model the cost $M(n)$ of comparing n brands, we propose an idea that is roughly similar to the notion of *polynomial time-complexity* in the field of computer science. Since the consumer has a task of (pair-wise) comparing n 'brands' (to completely specify the ordering required by economic theory), the *information size* (in the computing nomenclature) is n. Since the task of pair-wise comparison involves '*choice within choices'* (i.e. a *choice* between any two items within/out of n available *choices*), the complexity of M must necessarily rule out an involution (i.e. $M(M(n)) = M \circ M(n)$ must have a strictly greater complexity than $M(n)$ itself). Finally, we recognize that a naturally minimum degree of polynomial that must necessarily rule out an involution, is two (see Small, 2007).[7] Since, the combinatorial number $C(n, 2) = n(n - 1)/2$ is naturally a degree-two polynomial in n, we choose $M(n) = \mu C(n, 2)$ as the simplest specification that captures the cognitive (Mullainathan and Shafir, 2013), biological (Sapolsky, 2017; and

[7] Simply put, the *degree-n* polynomial $P(x) = \sum_{i=0}^{n} a_i x^i$, can only be an involution if $n = 1$ (for example, $P(x) = a - x$). For $n > 1$, $P(P(x))$ will necessarily be of a higher degree/complexity.

Eagleman, 2015) and psychological (Schwartz, 2004) costs of evaluating and comparing *n* objects (with $\mu > 0$, which can be thought of as the *degree or measure of cognitive paralysis* of an agent). This composite *cost* of processing *n* 'information items' is also sufficiently in line with economic theory that in general requires costs to be increasing and convex. We now look at the modified consumers' and producers' problems in the subsection that follows.

3.1. Agent behaviour in the market

We borrow from psychology that the cognitive costs involving too many choices, *take away* from the utility from consumption in the following modified specification of the representative consumer's problem.[8]

$$\begin{aligned} &Maximize: \quad U = ax - bx^2 + y - \mu\left(\frac{n(n-1)}{2}\right) \\ &Subject\ to: \quad px + y = M \end{aligned} \tag{11}$$

The solution to the above problem gives us the exact demand structure and market outcomes of the previous section specified from equations (2) to (7). With this additional negative utility component though, the welfare conditions change. We turn to this now.

3.2. Analysis of welfare

The equilibrium utility level for our representative consumer is now given as follows

$$U(x^*) = U^* = \frac{a^2 n}{2b(n+1)} - b\left(\frac{an}{2b(n+1)}\right)^2 + y - \mu\left(\frac{n(n-1)}{2}\right)$$

[8] Minssky (1986) gives compelling argument that support the structure of the cognitive cost that we propose.

Just as in the previous section, the total utility of k consumers and the total producers' surplus amounts to $S(n) = kU^* + n\pi^*$, and results in the following specification (after some steps).

$$S(n) = \frac{ka^2 n(n+2)}{4b(n+1)^2} + kM - \frac{k\mu n(n-1)}{2} \qquad (12)$$

The steps to arrive at the above expression are deferred to the Appendix. It is clear that

$$S(1) = \frac{3ka^2}{16b} + kM > 0; and \lim_{n\to\infty} S(n) = -\infty < 0$$

It is therefore, easily seen that the combined costs of cognition for k consumers must eventually outweigh the welfare benefits of competition. We conclude that social welfare must necessarily be at a positive maximum at a finite value of n, which in turn, will depend on the value of μ. We now progressively examine the restrictions on μ for a given number of firms n, such that the entry of the next firm (i.e. the $(n+1)^{th}$ firm) is necessarily welfare improving. More specifically, we see (after a few complex algebraic steps deferred to the Appendix) that

$$S(n+1) - S(n) = k\left(\frac{a^2(2n+3)}{4b(n+1)^2(n+2)^2} - \mu n\right) \qquad (13)$$

Thus, $S(n+1) - S(n) > 0$ means that the cognitive costs should be lower than a critical value. More precisely, μ should not exceed $a^2(2n+3)/4bn(n+1)^2(n+2)^2$. This means that the latter should be a strict lower bound for μ for maximum welfare to be attained with exactly n firms. Thus, we have the following result.

Proposition 1: If exactly n firms maximize social welfare, then it must be the case that μ is at least $a^2(2n+3)/4bn(n+1)^2(n+2)^2$.

Proof: Trivial. It follows directly from (13).

In Figure 1, we plot the socially most desirable number of firms against these lower bounds on μ after normalizing $a^2/b = 100$ and fixing $k = M = 10$.

Figure 1: Socially desirable number of firms against the measure of cognitive paralysis μ

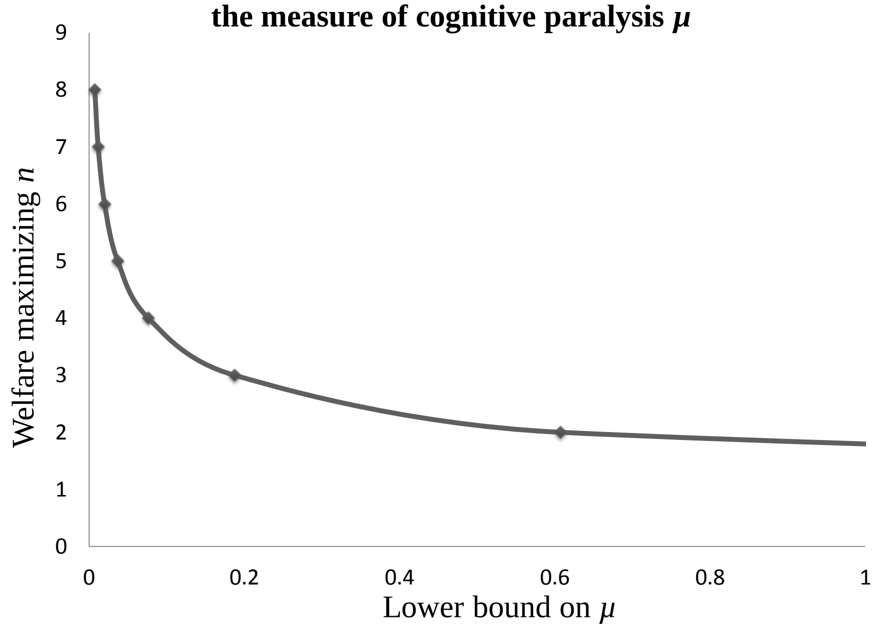


Figure 1 shows that as μ increases beyond each given critical bound, the number of firms that maximize welfare progressively diminishes. Indeed, in Figure 2, we see that welfare is maximized for a *strictly finite* value of n, for carefully chosen values of μ between the critical bounds. The panels (a), (b), (c) and (d) show how $S(n)$ is maximized at $n = 3, 4, 5$ *and* 6 respectively for different values of μ (*0.20, 0.10, 0.05* and *0.02* in that order).

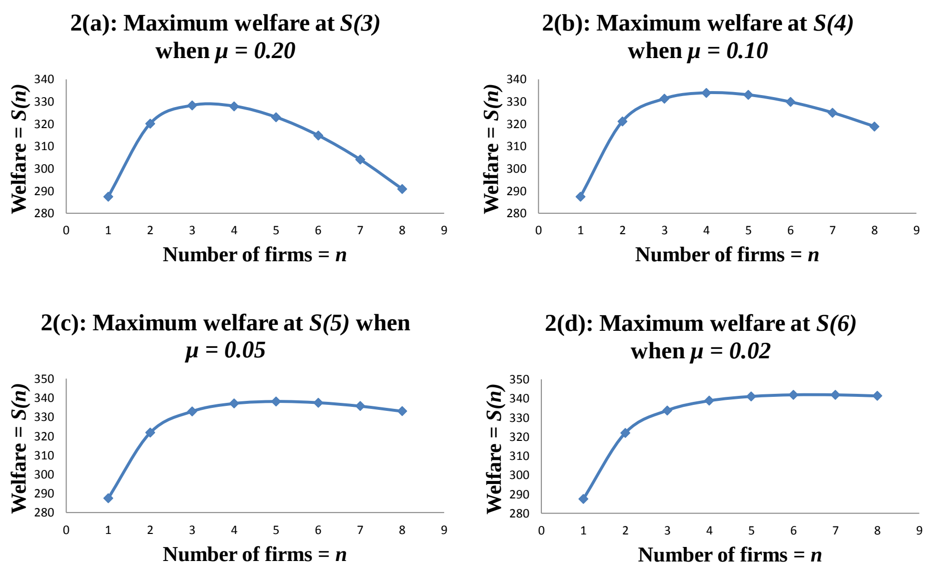
We now come to the final result of this section.

Proposition 2: If consumers have infinite cognitive capacity ($\mu = 0$), then welfare is the maximum with an infinite number of firms.

Proof: Trivial. It follows directly from the previous section where μ is indeed zero.

The importance of the above proposition becomes clear here where we have discussed cognitive costs of comparison. Since there is no *variety* (i.e. all the products in this market are homogenous - an assumption that we will relax in the next section), the fact that utility is strictly increasing in n, gives an insight into the consumers' implicit love for the *flexibility* to choose his optimal consumption amount from *any* of the n available firms. This is because U is strictly dependent on the *amount* (x) of consumption of the commodity in question, and there is no *a priori* reason for U to be strictly increasing in n as well. The latter just happens to be a consequence of equilibrium. We bring in *variety* in the section that follows.

Figure 2: Finite number of firms maximize welfare



Finally, before concluding this section we present one final result that we intend to use in the section that immediately follows.

Theorem 1: The consumption component of utility is strictly positive in equilibrium. It is also *strictly* increasing in consumption x.

Proof: We use the above equilibrium condition above and observe that

$$x^* = \frac{an}{2b(n+1)} < \frac{a}{2b} < \frac{a}{b}$$

which gives us $bx^* < a$, or $a - bx^* > 0$, which on multiplying by x^*, further gives us $ax^* - b(x^*)^2 > 0$, thereby completing the proof. The latter part of the theorem follows directly from recognizing that $x^* < a/2b$.

4. Markets with finitely cognitive consumers who value variety

It may be argued that cognitive costs of undertaking a consumption decision only make sense when there is *variety* in choice. To the extent that consumers actually love variety (i.e. they get a positive utility just from the fact that there is variety), the welfare results established in the previous section are expected to weaken. In this section we examine to what extent consumers invite more and more variety, before eventually succumbing to the cognitive costs of *evaluating* them to make their consumption decisions. Since such consumers must necessarily undertake buying in a *heterogeneous-product* market, we model the latter first to then back out a utility specification that is consistent with the established demand structure. It is clear that a demand curve that captures a love for variety must necessarily come from a utility specification that incorporates the same. We now come to the formulation of our heterogeneous-product market.

4.1. The heterogeneous-product market

Our model of the 'market with variety' unifies many key features that are well-established in various known models of industrial organization theory (for a detailed discussion on each of the following features, see the models covered in Tirole, 1988; Carlton and Perloff, 2000; and Spiegler, 2011).

(a) Demand is relatively inelastic with product differentiation.

(b) Product-differentiation leads to market-power.

(c) Market-power dilutes with greater competition and more product homogeneity.

(d) Superior quality products are priced higher.

The final point brings about a realism that we want to capture. In keeping with a market with a unique price, we capture this fundamental empirical reality in an indirect way by recognizing that any given quantity of a higher brand is worth a greater quantity of a lesser brand. In other words, a given amount of money either buys more of an inferior quality or less of a superior quality.

A key feature of Cournot competition, is a unique price for all the players. We retain this feature of a unique equilibrium price p but replace the quantity variable q (of the previous sections) by $q' = \gamma q$, for some *quality* parameter γ (≥ 1), so that for any two firms, i and j, with exogenous quality parameters γ_i and γ_j, the equilibrium price p of $\gamma_i q_i$ will be the same as that of $\gamma_j q_j$. Now, since $\gamma_i q_i$ and $\gamma_j q_j$ have the same value and are therefore worth the same price, $\gamma_i > \gamma_j$ implies that, $q_j < q_i$. For example, if one can buy two shirts of 'brand j' ($q_j = 2$) for the amount (p) that it takes to buy just one shirt of 'brand i' ($q_i = 1$), then it must be the case that the latter brand is of a superior quality since clearly $\gamma_i = 2\gamma_j > \gamma_j$ (which follows from $\gamma_i q_i = \gamma_j q_j$ and $q_j = 2q_i > q_i$). We have shown that, for any given price, one can only afford a lesser quantity of a greater quality (or 'brand'). Thus, the parameter γ for each firm can be thought of the 'rate at which a given quantity q, of a product has been *embellished* to charge a higher price'. We therefore call the quality parameter 'γ' as the *rate of product-embellishment*. We use the following reduced-form demand specification that captures this heterogeneity of brands.

$$p = \alpha - \beta \sum_{i=1}^{n} \gamma_i q_i = \alpha - \beta Q' \, ; \left(Q' = \sum_{i=1}^{n} \gamma_i q_i \, ; and \; Q = \sum_{i=1}^{n} q_i \right) \tag{14}$$

The above demand specification, apart from immediately satisfying feature (a), also implicitly makes a simplifying assumption that each firm produces and sells a unique brand. We will later show that features (b), (c), and (d), will be a consequence of the market equilibrium. We now turn to the producers' problem.

4.2. The heterogeneous producers' problem

We retain the 'zero production-costs' assumption, so that *Firm i* solves the following problem:

$$Maximise\colon \ \pi_i = pq_i = (\alpha - \beta Q')q_i = \left(\alpha - \beta \sum_{i=1}^{n} \gamma_i q_i \right) q_i \tag{15}$$

We define for simplicity, $q_i' = \gamma_i q_i$, so that $Q' = \sum_{i=1}^{n} q_i'$, and the profit function can be written below as follows:

$$Maximise\colon \ \pi_i = \left(\alpha - \beta \sum_{i=1}^{n} q_i' \right) \frac{q_i'}{\gamma_i} = \frac{1}{\gamma_i} \left(\alpha - \beta \sum_{i=1}^{n} q_i' \right) q_i' \tag{16}$$

It is clear that since for each firm i, γ_i is a strictly exogenous quality parameter, and $q_i' = \gamma_i q_i$ is strictly linear in q_i, maximising π_i with respect to q_i in (15) is the same as maximising π_i with respect to q_i' in (16). The latter in turn is the same as maximizing $\gamma_i \pi_i$ with respect to q_i'. Thus, the problem reduces to

$$Maximize\colon \left(\alpha - \beta \sum_{i=1}^{n} q_i'\right) q_i'$$

Thus, each firm must choose q_i' as a best response to other firms' augmented q'. This problem is in fact, identical to the producers' problem discussed in Section 2 with the only difference being that q_i has been replaced by q_i'. Finally, since this problem is symmetric in q_i', we can simply borrow the equilibrium results of Section 2 after suitably replacing q_i by q_i'. The market equilibrium is described as under:

$$q_i'^{*} = \frac{1}{(n+1)}\left(\frac{\alpha}{\beta}\right);\ p^{*} = \frac{\alpha}{(n+1)};\ q_i^{*} = \frac{q_i'^{*}}{\gamma_i} = \frac{1}{(n+1)\gamma_i}\left(\frac{\alpha}{\beta}\right) \tag{17}$$

Clearly, the greater the γ_i, the lesser will be the q_i for the given equilibrium price. We now use these above equilibrium results to back out the representative consumer's utility specification.

4.3. The variety-loving consumers' problem

We are in the process of constructing an equilibrium where market demand $X = kx^*$ and market supply $Q = \sum_{i=1}^{n} q_i = \sum_{i=1}^{n} (q_i'/\gamma_i)$ balance each other out in the equilibrium. We use this market clearing condition to back out the optimal x^*.

$$kx = X = \sum_{i=1}^{n} q_i = \sum_{i=1}^{n} \frac{q_i{}'}{\gamma_i}$$

Now from equation (17), we know that in equilibrium, $q_i' = \frac{1}{(n+1)}\left(\frac{\alpha}{\beta}\right)$, we get

$$x^* = \frac{1}{k}\sum_{i=1}^{n}\left(\frac{1}{(n+1)}\right)\left(\frac{\alpha}{\beta}\right)\left(\frac{1}{\gamma_i}\right) = \frac{\alpha}{k\beta(n+1)}\sum_{i=1}^{n}\frac{1}{\gamma_i} = \frac{\alpha n}{\bar{\gamma}k\beta(n+1)} \qquad (18)$$

where, $\bar{\gamma}$ is the *average rate of product embellishment*. In particular, $\bar{\gamma}$ is the *harmonic* average defined as under.

$$\bar{\gamma} = \frac{n}{\sum_{i=1}^{n}\frac{1}{\gamma_i}}$$

It is clear that in our framework, the higher this average quality, the higher will be the price, and therefore the equilibrium consumption will be lower. In order to work out the exact utility specification that is compatible with the demand function that yields the above equilibrium conditions, let us consider the following utility maximization problem for our typical consumer:

$$\begin{aligned} &Maximize: && U = a\bar{\gamma}x - b(\bar{\gamma}x)^2 + y - \frac{\mu n(n-1)}{2} \\ &Subject\ to: && p\bar{\gamma}x + y = M \end{aligned} \qquad (19)$$

Note that the above problem is identical to the original problem (1), only with the exception that *x* in the original problem (1) is replaced by $\bar{\gamma}x$ here. If the consumer values quality, he must pay for the same (as shown in the budget constraint). Now, just like in the previous problem (more specifically *Theorem 1*), so long as $\bar{\gamma}x < a/2b$, (the consumption component of) utility is strictly concave and *strictly* increasing in both *x* and $\bar{\gamma}$. Further, if we explicitly replace $\bar{\gamma}$ by $\frac{n}{\sum_{i=1}^{n}\frac{1}{\gamma_i}}$, then it is clear that (the consumption component of) utility remains *strictly* increasing in n, thereby implicitly capturing the love for variety.

The solution to problem (19) gives us the following market demand function:

$$X = kx^* = k\left(\frac{a-p}{2b\bar{\gamma}}\right) = A - Bp$$

where, $A = ak/2b\bar{\gamma}$, and $B = k/2b\bar{\gamma}$. Thus, the inverse demand function is

$$p = \frac{A}{B} - \frac{X}{B}$$

Finally, in order to work out the 'correct' values of α and β, we just equate the optimal equilibrium demand and supply values of x^* to each other.

$$x^* = \frac{\alpha n}{\bar{\gamma}k\beta(n+1)} = \left(\frac{a-p}{2b\bar{\gamma}}\right)$$

Finally, recognizing that, $p^* = \alpha/(n+1)$, and suitably comparing coefficients, we get $\alpha = a$ and $\beta = 2b/k$. This can be readily verified from the following (which is arrived at, by multiplying both sides of the above equation by $\bar{\gamma}$).

$$\frac{n\alpha}{k(n+1)\beta} = \frac{a}{2b} - \frac{\alpha}{2(n+1)b}$$

Thus, the consumers' optimal choice of quantity is given by:

$$x^* = \frac{an}{2b\bar{\gamma}(n+1)} \tag{20}$$

Interestingly, it can also be shown that at this optimal quantity, each consumer also chooses optimal $\bar{\gamma}$. The proof for this is deferred to the Appendix. Thus, in this section, we show through the consumer's problem that our demand function must necessarily stem/originate from and be implicit in a utility function that captures a love for variety. Indeed large variety is captured by large *n*. We finally come to the implications on welfare.

4.4. A discussion on welfare

Before we begin, we re-write (17) after replacing α by *a*, and β by *2b/k* as below.

$$q_i'^* = \frac{ka}{2b(n+1)};\ p^* = \frac{a}{(n+1)};\ \ q_i^* = \frac{q_i'^*}{\gamma_i} = \frac{ka}{2b(n+1)\gamma_i} \tag{21}$$

In order to work out welfare in the heterogeneous market $S_H(n)$, we first evaluate the producers' surplus as follows:

$$\sum_{i=1}^{n} \pi_i^* = \sum_{i=1}^{n} p^* q_i^* = \sum_{i=1}^{n} \frac{ka^2}{2b(n+1)^2\gamma_i} = \frac{ka^2}{2b(n+1)^2} \sum_{i=1}^{n} \frac{1}{\gamma_i} = \frac{ka^2 n}{2b\bar{\gamma}(n+1)^2} \tag{22}$$

We now work out U^* below

$$U(x^*) = \frac{a^2 n}{2b(n+1)} - b\left(\frac{an}{2b(n+1)}\right)^2 + M - \frac{a^2 n}{2b(n+1)^2} - \mu\left(\frac{n(n-1)}{2}\right)$$

We multiply the above by *k* to obtain kU^* and add the same to the expression for total producers' surplus in (22) to get the total surplus $S_H(n)$ as below

$$S_H(n) = \frac{ka^2 n}{2b(n+1)} - bk\left(\frac{an}{2b(n+1)}\right)^2 + Mk - \frac{ka^2 n}{2b(n+1)^2} - \mu k\left(\frac{n(n-1)}{2}\right) + \frac{ka^2 n}{2b\bar{\gamma}(n+1)^2}$$

Which, on further simplification gives us

$$S_H(n) = \frac{ka^2 n(n+2)}{4b(n+1)^2} + Mk - \frac{ka^2 n}{2b(n+1)^2}\left(\frac{\bar{\gamma}-1}{\bar{\gamma}}\right) - \mu k\left(\frac{n(n-1)}{2}\right) \tag{23}$$

Now, on comparing the above with the expression for *S(n)* in (12), we immediately see that

$$S(n) - S_H(n) = \frac{ka^2 n}{2b(n+1)^2}\left(\frac{\bar{\gamma}-1}{\bar{\gamma}}\right) > 0$$

This can be interpreted as the 'deadweight loss' from the introduction of product-heterogeneity. Firms get a certain degree of market power due to product differentiation and as a result charge higher prices for lower quantities because they know that consumers value quality and are willing to pay for it. On the whole, therefore, there is a welfare loss form this imperfection. Clearly, this deadweight loss vanishes if each firm has $\gamma_i = 1$ (in which case, there is no heterogeneity), or when the number of firms is infinitely large.

It is clear that since our representative consumer of this section values variety, the critical values that the costs of cognition must exceed (for a finite socially optimal *n*) should be higher in comparison to the previous section. We formally state this in the following theorem.

Theorem 2: The costs of cognition must be higher for the variety loving consumer for any given welfare maximizing (finite) number of firms.

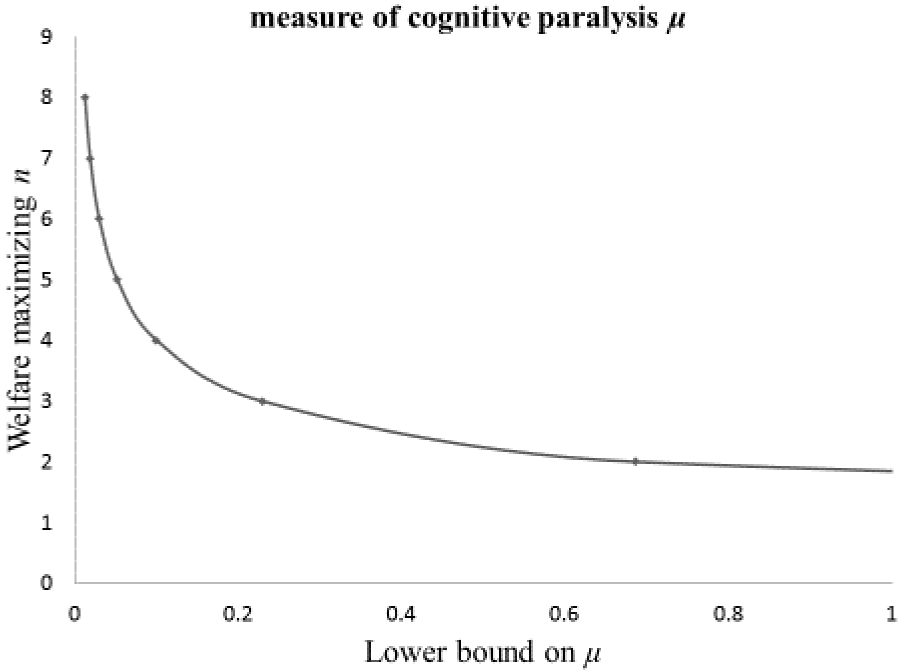
Proof: Trivial. On comparing the values of $S_H(n+1)$ and $S_H(n)$¸it is clear that welfare is maximized at exactly n firms (i.e. the (n + 1)th firm can only reduce welfare) when

$$\mu > \frac{a^2}{2b(n+1)^2(n+2)^2}\left(\left(\frac{2n+3}{2}\right) + \left(\frac{\bar{\gamma}-1}{\bar{\gamma}}\right)(n^2+n-1)\right)$$

Which strictly exceeds the critical value for the consumer who does not necessarily value variety.

We show these corresponding lower bounds on μ against the welfare maximizing n, in Figure 3 (analogous to Figure 1 of the previous section) below.

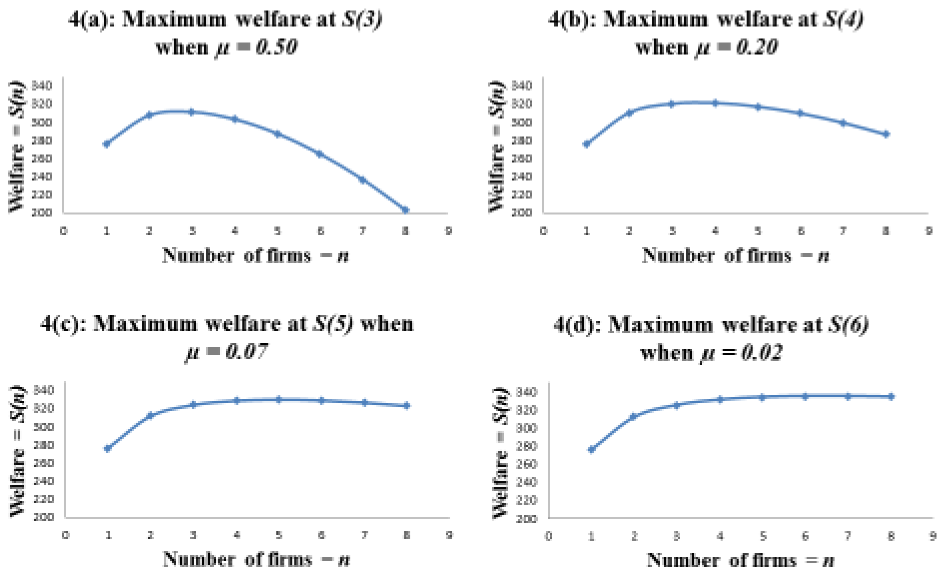
Figure 3: Socially desirable number of firms against the measure of cognitive paralysis μ



The above figure assumes the previously used values of $a^2/b = 100$ and $k = M = 10$, in addition to our choice of $\bar{\gamma} = 10$. We immediately see that this curve is only marginally to the left of that in Figure 1, implying higher costs of cognition for a given value of welfare maximizing n. In Figure 4 (analogous to Figure 2), we choose values of μ equal to *0.50, 0.20, 0.07,* and *0.02* in panels (a), (b), (c), and (d) respectively, where welfare is maximized (again respectively) at n = *3, 4, 5* and *6.*

We immediately see that the shape and structure of *S(n)* against *n* is remarkably the same as it is in the case of the previous section. Each curve however, sits on a strictly lower value in comparison to Figure 2 because of the associated deadweight loss, the magnitude of which increases with *n* in each case.

Figure 4: Finite number of firms maximize welfare



5. Conclusion

In this paper we have suggested a general theoretical approach to incorporate the cognitive cost faced by consumers (as they have limited cognitive ability) in market structure. We consider both heterogeneous and homogeneous products. We show that in presence of cognitive cost and love for variety welfare is maximised with finite number of firms. This also suggests that policy makers should think of both the aspects, as increasing competition need not be the best way to increase welfare, especially if consumers find variety as choice overload.

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