Communication and Commitment with Resource Constraints

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Abstract

I study strategic information transmission between an informed Sender and an uninformed Receiver when *(i)* both players take actions that are substitutable and, *(ii)* players face resource constraints. When actions are simultaneous and in the absence of resource constraints, there is completely truthful information revelation and both players achieve full efficiency. The presence of resource constraints restricts communication, resulting in partial revelation of information. The most informative equilibrium is ex-ante pareto dominant for both Sender and Receiver, and ex-post efficient only for the sender. When the Receiver is allowed to commit to an action ex-post communication (sequential protocol), welfare of both players is higher compared to the simultaneous protocol. Finally, I characterize the optimal (ex-ante) commitment mechanism for the Receiver. It exhibits two key features: maximal resource extraction from the Sender and capping of contributions by the Receiver. The full commitment protocol improves information revelation and provides higher welfare for both players. This provides a novel rationale for the existence of commitment in organizations and government bureaucracies.

Keywords: strategic communication, substitutes, resource constraints, team theory, organizational design

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1 Introduction

A number of interactions in economics and political science typically involve the sharing of private information by agents and strategic interdependencies in their actions. In organizations, multiple functional teams share information regarding a project and exert effort together to implement them. Countries work with each other by sharing intelligence information on security related issues; and governments at the federal and state level often exchange information and implement public projects by pooling resources together. An important feature in these examples is that the players contribute resources that are *strategic substitutes* in order to achieve a common objective and they typically face some form of *resource constraints* – e.g. fiscal, human capital, or time – that affects their capacity to contribute effectively. As a result, when there are informational asymmetries, an informed agent has incentives to misrepresent her private information to attract more contributions from the uninformed player. Therefore, the presence of resource constraints imposes natural limits on truthful information revelation, resulting in loss of information (transparency) and ensuing welfare inefficiencies.

The purpose of this paper is to precisely characterize the nature of communication and the resulting inefficiencies in the presence of such constraints. An important question that my paper seeks to address is how to mitigate some of these –informational and welfare– inefficiencies. Specifically, I analyze and compare different decision-making protocols in terms of their effect on transparency and welfare. In doing so, my paper provides two fundamental contributions. First, no theoretical work before has studied the relationship between strategic communication and action substitutability, which remains the primary focus of this paper. Second, to the best of my knowledge, this paper provides the first intuitive characterization of the *relationship between resource constraints and information revelation* in the literature.

I study the problem of information transmission between an informed Sender and uninformed Receiver[1] with action substitutability and resource constraints. The nature of private information is *soft* and communication takes the form of cheap talk –costless and non-verifiable– messages, à la Crawford and Sobel (1982) (hereafter CS.) The baseline *simultaneous protocol* proceeds as follows: the informed Sender observes the state of the world and sends a message to the Receiver. After the communication stage both players simultaneously take actions that are substitutable.

In this setting, I completely characterize the set of pure strategy equilibria (henceforth equilibria) in the Sender's messaging strategy and the subsequent actions of both players. The first important finding is that in the absence of resource constraints, captured by the domain of

[1]Though I use the terminology of Sender-Receiver, the paper captures interactions between divisions within a firm, different ministries within a cabinet, countries in an alliance, and so on.

permissible actions, there is *full information revelation* by the Sender. This is in sharp contrast to most results in the strategic communication literature, with exceptions (Kartik, Ottaviani, and Squintani (2007)). In the presence of resource constraints, full-revelation breaks down and there is some inefficiency in communication, leading to *threshold* (partial-pooling) equilibria - the Sender reveals the true state up to a cutoff state, and pools beyond this by sending the highest possible message. This resembles equilibria characterized by Kartik (2009), Ottaviani and Squintani (2006) with a crucial difference. There is no inflated messaging on the separation interval. Instead, the Sender is able to truthfully reveal the true state without having to worry about being mimicked by lower types on the interval.

To understand the intuition behind these results, consider the incentives of the Sender to reveal information. Since the Sender is also a decision maker along with the Receiver,she can anticipate the posterior beliefs induced by the message (in equilibrium) and precisely predict the Receiver's best response. As a result, what matters for truthful communication is whether the Sender, given the permissible set of actions, is able to best respond to the Receiver. That is, the resources behave like incentive constraints on truth-telling for the Sender. When resource constraints are not binding, the Sender achieves first best by truthfully revealing information and coordinating her contributions with the Receiver. Since there is no information loss from communication, the Receiver also achieves her first best leading to *full efficiency*. On the other hand, with binding resource constraints, the Sender is unable to best respond with truthful messaging for every possible state. Therefore, there is an incentive to inflate her message and induce a greater contribution from the Receiver. As a consequence, the Sender's message loses credibility beyond some threshold resulting in partial transparency during communication. This information loss leads to further *welfare inefficiencies* for both players.

My model generates multiplicity of threshold equilibria. Due to the non-verifiability of cheap talk messages, the Sender can choose how much information to reveal in equilibrium. I characterize the *most informative equilibrium* as the one with the highest threshold. Beyond this threshold, the resource constraint is binding and no information is credibly revealed by the Sender. That is, all types beyond the highest threshold *always* pool together in every equilibria of the communication game leading to a loss of transparency.

My analysis yields other equilibria that combines both separation and partitioning of information within the most informative threshold. Under such a *hybrid* equilibria, the Sender reveals certain types truthfully and pools some other types. The hybrid equilibria bears some semblance in structure to the *central pooling equilibria* in the work of Bernheim and Severinov (2003).[2] Instead of central pooling, however, the hybrid equilibrium exhibits pooling on (possi-

[2]They characterize signaling equilibria in which there is pooling towards the middle of the spectrum of types and separation on either ends of the type space.

bly) both extremes, and induces separation in the middle of the type space.[3]

Next, I consider the efficiency properties of the equilibria. I show that they exhibit an intuitive pareto ordering - the most informative equilibrium is ex-ante efficient among the set of threshold equilibria. This contrasts findings in the literature that argue for the welfare benefits of non-transparency (Jehiel (2014) and Prat (2005).) Instead, in the presence of action substitutability and resource constraints, the ex-ante welfare of both players is monotone increasing in transparency. Further, I establish that this equilibrium is ex-post efficient for the Sender, but not so for the Receiver. In contrast, the most informative partition of the original CS setup ceases to be ex-post efficient for the Sender. For example, there is always a low type Sender that prefers the babbling equilibrium over the more informational partition. Hence, even though the most informative partitional equilibrium is ex-ante efficient, they are never ex-post efficient.

In Section 7, I consider a sequential decision-making mechanism, namely a *sequential protocol* in which the informed Sender communicates her private information and the receiver proceeds to take an action (Stackelberg leader), followed by the Sender. I find that the most informative equilibrium is the same as in the simultaneous protocol and hence, there is *no improvement in transparency* from switching to this protocol. However, interestingly, the sequential protocol pareto dominates the simultaneous one, providing both players a higher ex-ante welfare. This stems from the fact that in the sequential protocol, once the Receiver's action is sunk post communication, the Sender can observe this action and correspondingly moderate her decisions. As a result of this moderating effect, the Receiver takes a higher action compared to the simultaneous protocol for the same pooling message. This translates into higher ex-ante welfare for the players.

Finally, in Section 8, I propose an alternate mechanism - the *commitment protocol*. In this, the Receiver commits to a communication dependent incentive compatible decision rule (Melumad and Shibano (1991)) that maximizes her ex-ante expected welfare. The optimal commitment mechanism[4] exhibits two key features: maximal resource extraction from the Sender and capping of contributions by the Receiver (Alonso, Brocas, and Carrillo (2013).) The rule takes an intuitive form. The Receiver mimics the actions of the simultaneous protocol on the separating (informative) interval. On the pooling interval, the Receiver follows a simple *resource extraction rule* wherein the Sender always contributes the maximum resources available and Receiver only contributes the residual required to satisfy the IC constraint of the Sender.

[3]This also gives rise to the possibility of discontinuous (possibly multiple) intervals of pooling and separating types.

[4] Alonso and Matouschek (2008) show that the commitment problem is equivalent to a delegation problem that involves *interval delegation* in which the uninformed player provides a delegation set to the informed one, instead of using a commitment rule. However, the commitment rule is a more realistic way to capture scenarios involving action substitutability.

This way, the Receiver commits to contributing resources that provide the Sender with first best levels up to a threshold, and beyond this, the Receiver *caps her contributions*.[5] This threshold is higher compared to the previous two protocols, implying that there is *greater transparency under commitment*. Further, the commitment rule also improves the welfare of both players, in direct contrast to Melumad and Shibano (1991). In their work, only the uninformed Receiver benefits while the Sender suffers a welfare loss as a result of commitment. The welfare improvement is driven by the maximal resource extraction rule on the pooling interval. This rule ensures that of all contribution pairs that satisfy Sender's IC for truth telling, the one that also maximizes the Receiver's utility is the one in which her own contribution is minimized. This precisely happens when the Sender contributes all resources at her disposal.

Taken together, these results have important implications for the design of hierarchy and transparency within organizations. When multiple teams/agents are required to interact and take decisions that are interdependent, there is a direct impact on transparency and welfare. Specifically, when an informed agent fails to communicate information truthfully it hurts the ability of the uninformed agent to coordinate and take decisions in the most efficient way. As a result, there is some welfare loss generated by this lack of transparency. One possible way to mitigate this inefficiency is to allow teams to act sequentially. However, my result shows that even though this increases welfare for both players, it does not improve transparency between the teams. The *commitment-to-resources mechanism* is a way to address these twin issues of loss of transparency and welfare inefficiency. My analysis therefore provides an intuitive informational rationale for why in organizations with strict hierarchical structures there are ex-ante commitments by uninformed parties to mitigate incentive problems associated with information asymmetry and resource constraints.

Related Literature

This paper extends and contributes to the vast literature of theoretical and applied models on strategic communication. For example, the role of strategic communication with complementarity in actions of players (*coordination incentives*) have been widely studied and applied to varied settings (e.g. Alonso, Dessein, and Matouschek (2008); Baliga and Morris (2002); Hagenbach and Koessler (2010); Rantakari, 2008). Barring Alonso (2007), who considers a principal-agent setting in which an uninformed principal controls the decision rights and actions of the two players are either strategic complements or substitutes, none of the other papers have looked into the relationship between strategic communication and action substitutability.

[5]Without this capping, the Receiver would end up providing the first best levels for the Sender in all possible states, which is equivalent to full delegation as in Dessein (2002).

The threshold equilibria of my paper are closely related to those derived by Kartik et al. (2007), Kartik (2009), and Ottaviani and Squintani (2006). Kartik et al. (2007) derive a fully separating equilibrium with lying costs and the possibility of a naive Receiver. The key condition driving their result is the unboundedness of the domain of private information. On the other hand, in Kartik (2009), truthful communication is restricted by lying costs and a bounded state space, leading to incomplete separation. Finally, Ottaviani and Squintani (2006) construct cheap talk equilibria with naive receivers and a bounded state space in which communication is truthful (but inflated) up to a threshold, and partitional beyond.

The key difference in my results is that it is driven by substitutability and resource constraints on the Sender. Resource constraints indirectly affect the capacity of the Sender to best respond post truthful messaging, and this in turn affects the credibility of messages in equilibrium, resulting in incomplete separation.

The work of Melumad and Shibano (1991) characterize the optimal commitment rule for the sender in the standard cheap talk game. Alonso and Matouschek (2008) similarly solve for the optimal delegation problem that subsumes the earlier paper of Melumad and Shibano (1991) by converting the problem of commitment into one where the uninformed principal delegates the decision rights to the informed agent by providing a set of actions. In contrast, since both players take interdependent actions in my paper, the role of delegation is not as pertinent. My paper uses the former approach and characterizes a novel commitment mechanism for the receiver that is welfare improving for both players, compared to the two decision making protocols.

Finally, the optimal commitment rule characterized in my paper has similarities to the capping of resources mechanism derived by Alonso et al. (2013). Specifically, they study the problem of resource allocation over a set of tasks performed simultaneously in the brain. The optimal mechanism in their model exhibits the resource capping feature similar to the one in this paper. However, my work is different in two important aspects. First, I consider strategic interdependencies in the actions of agents involved. Second, there are informational asymmetries with respect to the state but not on the resource requirements. This changes the results in an interesting way in that the optimal commitment rule also has an additional resource extraction feature that is missing in Alonso et al. (2013).

The rest of the paper proceeds as follows. In Section 2, I present a simple example to show the main intuition driving my results. Section 3 outlines the basic model and Section 4 presents conditions for full information revelation equilibrium. Section 5 contains the results pertaining to partial revelation threshold equilibria. In Section 5, I present efficiency properties of the partial revelation equilibrium and analysis of sequential decision-making protocol follows in Section 6. Finally, Section 7 contains concluding remarks.

2 Leading Example

Consider a variant of the basic Crawford-Sobel set-up with action substitutability. An informed player, S, receives a perfectly observable signal about the state of the world θ, drawn from an uniform distribution $[0,1]$ and communicates this information through a cheap talk message $m(\theta)$ to an uninformed player, R. Upon communication, both R and S take actions in a way that affects both their payoffs. Let the modified utility function be the following:

$$U^R = -\left[\left(\frac{x_R + \eta x_S}{1+\eta}\right) - \theta\right]^2$$

$$U^S = -\left[\left(\frac{x_S + \eta x_R}{1+\eta}\right) - \theta - b\right]^2$$

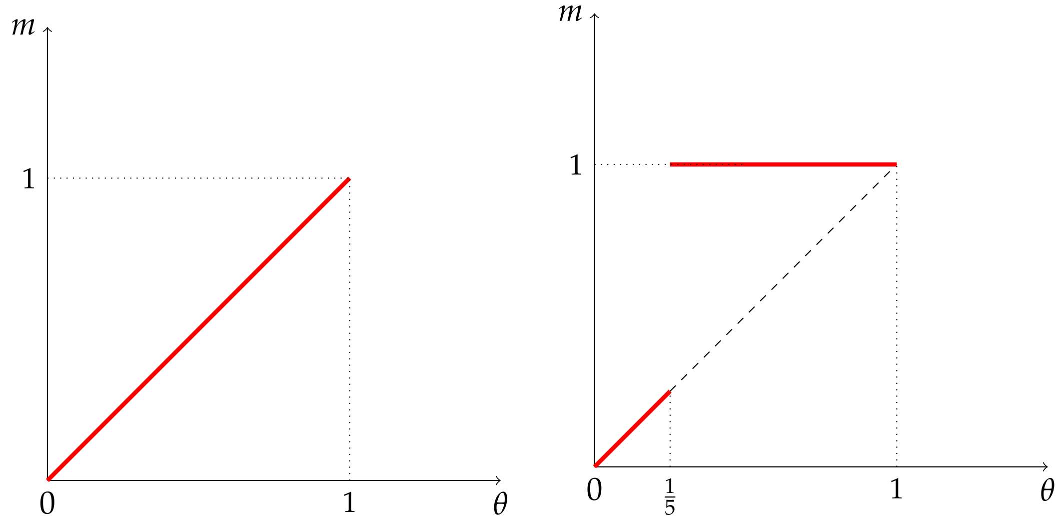
Observe the small departure from the CS set-up. Both players now are allowed to take actions after communication, and actions are substitutes in that $\frac{\partial^2 U^i}{\partial x_R \partial x_S} < 0$, where $\eta \in (0,1)$ captures the degree of substitutability. Further, let the actions of players x_i have a domain $[-a, a]$. Given this structure, when S truthfully reveals the true state of the world through her message, $m(\theta) = \theta$, the two players solve the following best responses:

$$R : x_R = (1+\eta)\theta - \eta x_S$$

$$S : x_S = (1+\eta)(\theta + b) - \eta x_R$$

To simplify the exposition, let $b = \frac{2}{5}$ and $\eta = \frac{1}{2}$. Equilibrium actions after (truthful) messaging are given by: $x_R^* = \theta - \frac{2}{5}$, $x_S^* = \theta + \frac{4}{5}$. Notice immediately that full information revelation is possible if $a \geqslant \frac{9}{5}$. This is so because S is able to compensate precisely even for the highest type, $\theta = 1$. At the other extreme, if $a < \frac{4}{5}$, no information can be credibly revealed by S, since irrespective of what the true state is, reporting the truth is never optimal for S. This stems from her inability to sufficiently compensate even for the lowest type signal.

For example, when $a = \frac{2}{5}$, the equilibrium action of the sender under truthful communication is $x_S^* = \frac{2}{5}$, irrespective of the state. However, S can inflate her signal in order to make R play a higher action. To see this, suppose instead of $m(0) = 0$, S inflates and sends a message $m(0) = \frac{2}{5}$. Then, R best responds by taking an action $x_R^* = \frac{2}{5}$. But notice that S can fully anticipate this response by R and suitably adjust her optimal action. In particular, S takes an action $x_S^* = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$. Though S's action has not changed, she has managed to push R's action upwards, and thereby achieves a payoff of 0. But this incentive to misrepresent means that R would never believe any message from S, and therefore communication is rendered ineffective



(a) $a \geq \frac{9}{5}$ (b) $a = 1$

Figure 1: When $a \geq \frac{9}{5}$, there are no resource constraints for the Sender, resulting in full information revelation. On the other hand, when $a \in (\frac{4}{5}, \frac{9}{5})$ there is only partial revelation of information. Specifically, for all states above $\frac{1}{5}$, the Sender pools and sends a message $m = 1$.

in equilibrium.

Finally, when $\frac{4}{5} < a < \frac{9}{5}$, S has an incentive to reveal some information. To see this, let $a = 1$. Then, for any $\theta \in [0, \frac{1}{5}]$, S reveals the state truthfully since her optimal action is within the domain of available actions (in this case $x_S^*(\frac{1}{5}) = 1$). But, for any $\theta > \frac{1}{5}$, S cannot sustain a truthful messaging strategy. To see this, suppose $\theta > \frac{1}{5}$, and S reports truthfully. Then the optimal action for S is bounded by $x_S = 1$, while R provides the residual as demanded by her best response function, which is $x_R = \frac{3}{2}\theta - \frac{1}{2}$. This cannot be an equilibrium since there is under-provision as far as S is concerned: S gets a payoff of $U_S = -\left(\frac{1+\frac{1}{2}(\frac{3}{2}.\theta-\frac{1}{2})}{\frac{3}{2}} - \theta - \frac{2}{5}\right)^2 \neq 0$ for $\theta > \frac{1}{5}$. Therefore, S has an incentive to exaggerate her information in order to induce R to contribute more. This precludes separation beyond $\theta = \frac{1}{5}$.

In fact, all types above this cutoff must pool and send the highest message, $m = 1$. This is primarily because the signals are (imperfectly) invertible in this environment. Any pure message $m < 1$ could be interpreted as coming from one of the many possible (weakly lower) types. For instance, when $\theta = \frac{2}{5}$, S would want to exaggerate and send a message of at least $m \geq \frac{3}{5}$, since this would ensure that S's action is within the bound $a = 1$. Say S sends $m = \frac{3}{5}$. But this message could possibly come from any of the types $\theta \in (\frac{1}{5}, \frac{2}{5}]$, each of whom have incentives to deviate and send $m = \frac{3}{5}$. Hence, R could invert the message and form beliefs accordingly[6]. But if this is the case, every type in the interval $(\frac{1}{5}, 1]$ would find it optimal to send the highest pooling message possible, $m = 1$. Therefore, there is at most a partially revealing equilibrium in which S is truthful (separates) in the range $\theta \in [0, \frac{1}{5}]$ and pools her messages for $\theta \in (\frac{1}{5}, 1]$ by sending the message $m = 1$.

The example suggests a novel trade-off for information transmission with action substitutability. The ability to truthfully reveal information depends on the domain of the set of resources available to the informed player. The informed Sender is able to provide more information regardless of the extent of the biases between the two players.

3 The Model

Consider two players, a receiver R and sender S, who decide on contributions to a joint project. The payoff from the project is contingent on an unknown state $\theta \in \Theta \equiv [0, 1]$, distributed according to the density function $f(.)$. The sender receives a perfectly observable private signal about the state θ, while the receiver has no information.

Each player's utility is given by $U(\phi^i(x_i, x_{-i}), \theta, b_i)$, where $\phi^i(.)$ is the player-specific (symmetric) joint contribution function.[7] The contribution function $\phi^i(.)$ depends on player i's action x_i, as well as the contribution of the other player, x_{-i}. Actions of players are such that $x_i \in V \subseteq \mathbb{R}$, where the set V is closed and compact. The contribution function is therefore a mapping $\phi^i : V \times V \to \mathbb{R}$. The bias parameter b_i measures the conflict of interest between the two players.

The standard Crawford-Sobel assumptions on the utility function of players hold. Specifically, $U : V^2 \times [0, 1] \times \mathbb{R} \to \mathbb{R}$ is twice continuously differentiable, $U_1(.) = 0$ for some ϕ^i, $U_{12}(.) > 0$, $U_{13}(.) > 0$ and $U_{11}(.) < 0$ so that U has a unique maxima for any given pair (θ, b_i). This implies that there is an unique joint contribution function ϕ^i for each player that satisfies their maximization problem. Consequently, let $\bar{\phi}^S_\theta \equiv \arg\max_{\phi^S} U\left(\phi^S, \theta, b\right)$ and $\bar{\phi}^R_\theta \equiv \arg\max_{\phi^R} U\left(\phi^R, \theta\right)$ be the first best levels of contribution for the sender and receiver for any given θ, respectively.

The utility functions of the players satisfy the condition $\frac{\partial^2 U}{\partial x_i x_j} < 0$, implying that actions of the two players are strategic substitutes. For sake of exposition, I normalize the bias of receiver to

[6]For precisely a similar argument, partition equilibria of the kind developed by CS are also ruled out on the interval $(\frac{1}{5}, 1]$. Again, this is because there would not exist an indifference type in this interval, since there is a natural propensity to inflate information. This incentive to exaggerate ensures that if there are two partitions, the high types in the lower partition would find it profitable to deviate to the higher partition, precluding the existence of an indifference type in the first place.

[7]The assumption of symmetry is not important in order to generate the main findings. However, it aides comprehension of the same.

0 and that of sender to $b > 0$[8]. The two players therefore maximize their payoff functions given by $U(\phi^R(x_R, x_S), \theta)$ and $U(\phi^S(x_S, x_R), \theta, b)$. Players have action interdependence in the sense

that each players' action x_i affects the contribution function of the other player $\phi^{-i}(.)$. Since $b > 0$ and $U_{13}(.) > 0$, it implies that $\phi^S(.) > \phi^R(.)$ for every θ. I make the following further assumptions on the functional form of $\phi^i(.)$ to ensure an interior solution to the contribution decision of the players:

Assumption 1 *Increasing marginal contribution:* $\frac{\partial \phi^i(.)}{\partial x_i} > 0$

Assumption 2 *Positive spillover:* $\forall i, j \neq i : \frac{\partial \phi^i(.)}{\partial x_j} > 0$

Assumption 3 *Imperfect substitutability:* $\forall i, j \neq i : \frac{\left(\frac{\partial \phi^i}{\partial x_i}\right)}{\left(\frac{\partial \phi^i}{\partial x_j}\right)} > 1$

Assumption 1 ensures that the contribution function is non-decreasing in the player's own action, while the second assumption implies that a player's contribution function is non-decreasing in the other player's action. Assumption 3 implies that the *marginal contribution effect* dominates the *spillover effect*. Further, it rules out perfectly substitutable actions.[9]

Timing - Simultaneous decision-making:

Following Kartik (2009), let $M = \bigcup_\theta M_\theta$ be a Borel space of messages available to the Sender such that $\forall \theta, \theta' \in [0,1] : M_\theta \bigcap M_{\theta'} = \emptyset$. The game proceeds in two stages.

- In the first stage, the sender observes the true state $\theta \in [0,1]$ and sends a message $m \in M$ to the receiver. Let this messaging strategy be defined by a mapping $\mu : [0,1] \to M$ and the message $m = \mu(\theta)$.

- In the second stage, both players simultaneously decide on contributions $\alpha_S : [0,1] \times M \to V$ and $\alpha_R : M \to V$.

[8]The biases could be a function of θ in that $b(\theta)$ may be the extent of conflict of interest, instead of a constant b. This, however, does not change the main results of the paper as long as single-crossing property holds, meaning $U_{13} > 0$.

[9]When actions are perfect substitutes, notice that there is no guarantee of an interior equilibrium. Take the example presented in Section 2 and substitute $\eta = 1$. The best responses are such that there is no equilibrium in pure strategies. For this reason, I focus on imperfect substitutability of actions.

Equilibrium

An equilibrium of the *simultaneous protocol* game is a Perfect Bayesian Equilibrium in pure strategies that satisfies the following properties:

- R and S simultaneously choose actions $\left(x_R^*(m), x_S^*(\theta, m)\right)$ that maximizes their expected utility according to the dual optimization problem:

$$x_R^*(m) \equiv \arg\max_{x_R \in V} \mathbb{E}_{\theta|m}[U(\phi^R(x_R, x_S^*(\theta, m)), \theta)] \text{ subject to } x_R \in V \tag{1}$$

$$x_S^*(\theta, m) \equiv \arg\max_{x_S \in V} [U(\phi^S(x_S, x_R^*(m)), \theta, b)] \text{ subject to } x_S \in V \tag{2}$$

- contribution function maximizes each players' expected utility conditional on their information, ie, $\phi^{R^*}(x_R^*(m), x_S^*(\theta, m)) \equiv \arg\max_{\phi^R} U(\phi^R(x_R, x_S), \theta)$ and $\phi^{S^*}(x_S^*(\theta, m), x_R^*(m)) \equiv \arg\max_{\phi^S} U(\phi^S(x_S, x_R), \theta, b)$
- the posterior beliefs, given by a cdf $P(\theta \mid m)$, are updated using Bayes' rule whenever possible, given the messaging rule $\mu^*(\theta)$
- given the beliefs and second stage contributions $x_R(m)$ and $x_S(\theta, m)$, S chooses a messaging strategy that maximizes expected payoff in the first stage,

$$\mu^*(\theta) \in \arg\max_{m \in M} \mathbb{E}_{P(.|m)} \left[U\left(\phi^S(x_S(\theta, m), x_R(m)), \theta, b\right)\right]$$

A PBE always exists in games with cheap talk. This is a babbling equilibrium in which the sender's message is ignored and the receiver acts based on her prior information, while the sender anticipates this and acts accordingly. In this paper, I try to identify conditions under which more informative equilibria emerge.

4 Full Information Revelation

In a full revelation equilibrium, the private information of the sender is completely revealed to the receiver, meaning $\mu(\theta) = \theta$ for all $\theta \in [0, 1]$. To see if a full revelation equilibrium[10] exists, it is important to understand the trade-offs for the sender. For truthful messaging to be an equilibrium, the sender must be able to choose an action in the set V such that $\phi^{i*}(x_S(\theta), x_R^*(\theta))$

[10] Any messaging function $\mu : [0, 1] \to [0, 1]$ that is one-to-one and onto is a fully revealing messaging strategy. I will, however, concentrate on the most intuitive one in which if the state is θ, the sender sends a message that is equivalent to the statement - "The state is θ".

is the unique maximum for the state θ. Since sender is constrained by the lower and upper bound on permissible actions, given by $\inf V = \underline{k}$ and $\sup V = \bar{k}$ respectively, the size of these bounds directly affects her ability to reveal information. If the sender's action after truthful communication is within the bounds, then it precludes her incentive to misrepresent. Therefore, in some sense, the set of actions V acts as an *incentive compatibility constraint* for truth-telling.

Given this intuition, it is convenient to reformulate the second stage problem when the sender has an *unrestricted domain* to choose her action from. The following definition does precisely that.

Definition 1 ***Unconstrained best-response****: Let $\tilde{x}_S(\theta, m)$ be the optimal action of the sender when i) unrestricted domain is satisfied ($x_S \in \mathbb{R}$); and ii) the sender's message m (truthful or otherwise) is believed by the receiver to be the true state. That is, $\tilde{x}_S(\theta, m)$ is the solution to the unconstrained optimization problem of the sender when her message is believed, i.e.,*

$$\tilde{x}_S(\theta, m) \text{ solves } \max_{x_S \in \mathbb{R}} U\left(\phi^S(x_S, \tilde{x}_R(m)), \theta, b\right) \text{ subject to}$$

$$\tilde{x}_R(m) \equiv \arg\max_{x_R \in V} U\left(\phi^R(x_R, \tilde{x}_S(\theta, m)), m\right)$$

Further, when communication is truthful ($m = \theta$), let the optimal action of players under the unconstrained optimization problem be $\tilde{x}_R(\theta)$ and $\tilde{x}_S(\theta) = \tilde{x}_S(\theta, \theta)$.

Assumption 4 $\underline{k} \leq \tilde{x}_S(0) \leq \bar{k}$

Note that Definition 1 does not necessarily prescribe the action of the sender in equilibrium, $x_S^*(.)$. Instead, $\tilde{x}_S(\theta, m)$ allows us to intuitively characterize the response of the sender when her message is believed to be true by a naive receiver (Kartik et al. (2007), Ottaviani and Squintani (2006)), and her actions have an unrestricted domain $\mathbb{R}$. Assumption 4 ensures that the optimal response of the sender for the lowest state is within the bounds. If not, the communication game has no meaningful information revelation trade offs.[11] Finally, the following definition helps characterize the full information revelation equilibrium.

Definition 2 *Highest type incentive compatibility (**HTIC**)*[12] *: $\tilde{x}_S(1) \leq \bar{k}$*

Definition 2 implies that the best response of the sender, after revealing the highest state $\theta = 1$, is within the domain of actions V. That is, the unconstrained best-response corresponds

[11] $\tilde{x}_S(0) \leq \bar{k}$ provides an intuitive condition for any information transmission with action substitutability. When this fails, no information can be credibly revealed by the sender, since the receiver always believes that the sender is exaggerating her information.

[12] HTIC has no relation the *No incentive to separate* (NITS) condition proposed by Chen, Kartik, and Sobel (2008).

with the equilibrium action. Because single crossing property $U_{12} > 0$, it must be that $\tilde{x}_S(\theta) \leq \bar{k}$ for every $\theta < 1$.

Proposition 1 *A full information revelation equilibrium exists if and only if HTIC condition is satisfied.*

Proof. See Appendix A.1 ■

The *HTIC* condition provides an *IC* constraint for full revelation. This is because the solution to the sender's constrained best response coincides with that of the unconstrained optimization problem, implying that $x^*_S(\theta) = \tilde{x}_S(\theta)$ for all $\theta \in [0, 1]$. This ensures that there is no incentive for S to deviate from truth-telling and full revelation is achieved as an equilibrium.

5 Partial Revelation under Resource Constraints

Clearly, HTIC is a strong requirement in the context of real world examples. Typically, divisions within organizations, governments, and even private individuals who try to work jointly in projects face constraints in terms of time or fiscal and human capital. Such resource constraints prevent them from acting optimally and increases their incentive to misrepresent their private information beyond some thresholds to extract more resources from others in the project. This section focuses on the nature of equilibria that emerge in the presence of such binding resource constraints. The following assumption ensures an intuitive characterization of equilibria with resource constraints.

Assumption 5 $\underline{k} \leq \tilde{x}_S(0, 1) \leq \bar{k}$

Assumption 5 makes upward deviations by the sender[13] possible in the sense that they would never induce an action that goes below the lower bound of the feasible action set, meaning $\tilde{x}_S(0, m) \in V$ for all $m < 1$.

The starting point of the analysis is to formulate the sender's incentive to misrepresent her information. This happens precisely, as argued in the previous section, when the *HTIC* condition fails and their are states for which truthful messages can never be credible. Let $G = \{\theta : \tilde{x}_S(\theta) > \bar{k}\}$ be the set of states for which truthful revelation is not possible for the sender.

Given this set of types, observe that there must then exist a cutoff $\bar{\theta}$ such that $\tilde{x}_S(\bar{\theta}) = \bar{k}$ and $\bar{\theta} = \sup\{[0, 1] \backslash G\}$. The set $G = (\bar{\theta}, 1]$ represents the types for which there are incentives to misrepresent for the sender. This is because for all types in the set G, reporting the truth

[13] Note that this is a stronger version of assumption 4, which concerns the feasibility of truthful communication for the lowest type information.

implies that the sender's resource constraint is binding. Therefore, by misreporting her private information θ, say by reporting $m > \theta$, the sender can induce the receiver to take a higher action. As a result, none of the messages in this interval are credible and will never be believed in equilibrium.[14]

Lemma 1 *When HTIC is violated, all types in set G pool on the same message in every equilibrium of the communication game.*

Proof. See Appendix A.2 ■

The intuition behind Lemma 1 is the following. Suppose it was possible for the sender to partition the set G into two - $G_1 = (\bar{\theta}, \bar{\theta}_g]$ and $G_2 = (\bar{\theta}_g, 1]$. Then, there are always types that are pooled in the first partition for whom the optimal action of the sender is constrained by the bound $\bar{k}$. This implies the sender would have an incentive to exaggerate her message and pool with the higher types in G_2, precluding the possibility of such a partition in equilibrium. Therefore, in the presence of resource constraints, two things hold: *i)* at most, there is only partial revelation of information; and *ii)* no credible information is conveyed beyond $\bar{\theta}$. The next proposition characterizes the set of all partially revealing threshold equilibria.

Proposition 2 *Under assumptions 1-5, when HTIC is violated, there are Partially Revealing Threshold Equilibria (PRTE) such that, $\forall \theta^* \in [0, \bar{\theta}]$: $m = \theta$ if $\theta \in [0, \theta^*]$ and $m = 1$ if $\theta \in (\theta^*, 1]$.*

Proof. See Appendix A.3 ■

Proposition 2 suggests that inflated messaging occurs above the cutoff state, while every message within the cutoff is truthful.[15]Further, the sender could possibly choose how much information to reveal in equilibrium. Though the PRTE $\theta^* = \bar{\theta}$ is the *most informative equilibrium*, it does not necessarily restrict the sender from providing less information to the receiver. In the next section, I will establish some intuitive welfare properties of the different threshold equilibria.

In fact, the sender could choose to partition the information within the interval $[0, \theta^*]$, instead of revealing them truthfully. This is so because, under the PRTE, the resource constraints

[14]This resembles the credibility notion of *self-signaling*, identified by Aumann (1990), and Farrell and Rabin (1996). When the unconstrained action is above the bound, it implies that the equilibrium action of the sender is $x_S^*(\theta) = \bar{k}$. Given imperfect substitutability, the receiver's action has a *positive spillover* implying that $U_1(\phi^{S^*}(\bar{k}, x_R^*(\theta)), \theta, b) > 0$ when $\tilde{x}_S(\theta) > \bar{k}$. This *'positive spillover effect'* implies that communication ceases to be credible, since the sender (weakly) prefers to induce a higher action from the receiver, by inflating her private information. See Baliga and Morris (2002) for more on this point.

[15]This is in contrast with the result of Ottaviani and Squintani (2006). They construct a cutoff equilibrium in which messages are revealing (albeit inflated) below the threshold, and for states above the cutoff, information transmission is partitional in nature. The PRE expressed above is also similar to the cut-off equilibria obtained by Kartik (2009) in which the exaggeration in communication is driven by lying costs. See also Morgan and Stocken (2003).

are satisfied with slack for any type in this interval. As a result, there is always a possibility for the sender to pool any type $\theta \in [0, \theta^*]$ with lower types within the interval such that the incentive compatibility with respect to the resource constraints is still satisfied.

As a result, there are other *partitional type* equilibria that emerge. The sender chooses to reveal some types truthfully in the interval $[0, \theta^*]$ and pools some others, giving rise to multiple discontinuous partitions (Bernheim and Severinov (2003)). Notice however that in all such equilibria, the types belonging to $(\bar{\theta}, 1]$ are always pooled together. What is different is that instead of threshold separation and pooling beyond, there are intervals of pooling and separating types. The following proposition characterizes all such *hybrid equilibria*.

Proposition 3 *Hybrid equilibria: Fix a PRTE with threshold θ^*. For every such θ^* equilibrium, there exists partitions in the separating interval $[0, \theta^*]$ such that sender pools some types and separates on other types.*

Proof. Appendix A.4 ■

The intuition for the existence of hybrid equilibria is straightforward. The only incentive constraint that requires to be satisfied to sustain pooling of types is that the marginal type has no incentive to deviate. The IC requires that the resource constraint is not binding for the highest type in the pooling message. For any such type $\theta \leq \theta^*$, it is true that $x_S(\theta) < \bar{k}$. As a result, there is always a $\delta > 0$ such that instead of revealing $m = \theta$, if the sender sends a pooling message $m_{pool} = (\theta - \delta, \theta]$, the optimal action of the sender still satisfies the resource constraint, implying $\tilde{x}_S(m_{pool}) \leq \bar{k}$. This ensures that the marginal type θ does not have any incentive to deviate, thereby sustaining the pooling message in equilibrium.

6 Efficiency

As is the case with cheap talk models, there is a multiplicity of equilibria in this setup. The previous section establishes that the sender may choose to reveal any threshold of information, starting from a cutoff $\theta^* = 0$ up to a $\theta^* = \bar{\theta}$. An important question that arises is whether the sender would find it in her interest to convey more information. To answer this, I first look at the receiver's best response to a pooling message beyond any generic threshold θ^*.

Lemma 2 *The receiver's optimal response on receiving the pooling message $m^{\theta^*}_{pool} = (\theta^*, 1]$ is given by $x^{sim}_R(m^{\theta^*}_{pool})$ that solves,*

$$\arg\max_{x_R \in V} \int_{\theta^*}^{\theta^*_{sim}} U(\phi^R(x_R, x^*_S(t, m^{\theta^*}_{pool})), t) dP(t \mid m^{\theta^*}_{pool}) + \int_{\theta^*_{sim}}^{1} U(\phi^R(x_R, \bar{k}), t) dP(t \mid m^{\theta^*}_{pool})$$

Proof. See Appendix A.5 ■

The above lemma states that the receiver's best response entails an important trade off. Since the sender has an informational advantage in that she knows the true state within the interval $m_{pool}^{\theta^*}$, the receiver takes this into account while best responding. This implies that the receiver's action is such that, $x_R^*(\theta^*) < x_R^*(m_{pool}^{\theta^*}) < x_R^*(1)$. But if this were true, then it follows that there is a measure of sender types $(\theta^*, \theta_{sim}^*]$ such that $\forall \theta \in (\theta^*, \theta_{sim}^*] : x_S^*(\theta, m_{pool}^{\theta^*}) \leq \bar{k}$ and for all other types $(\theta_{sim}^*, 1]$, $x_S^*(\theta, m_{pool}^{\theta^*}) = \bar{k}$.

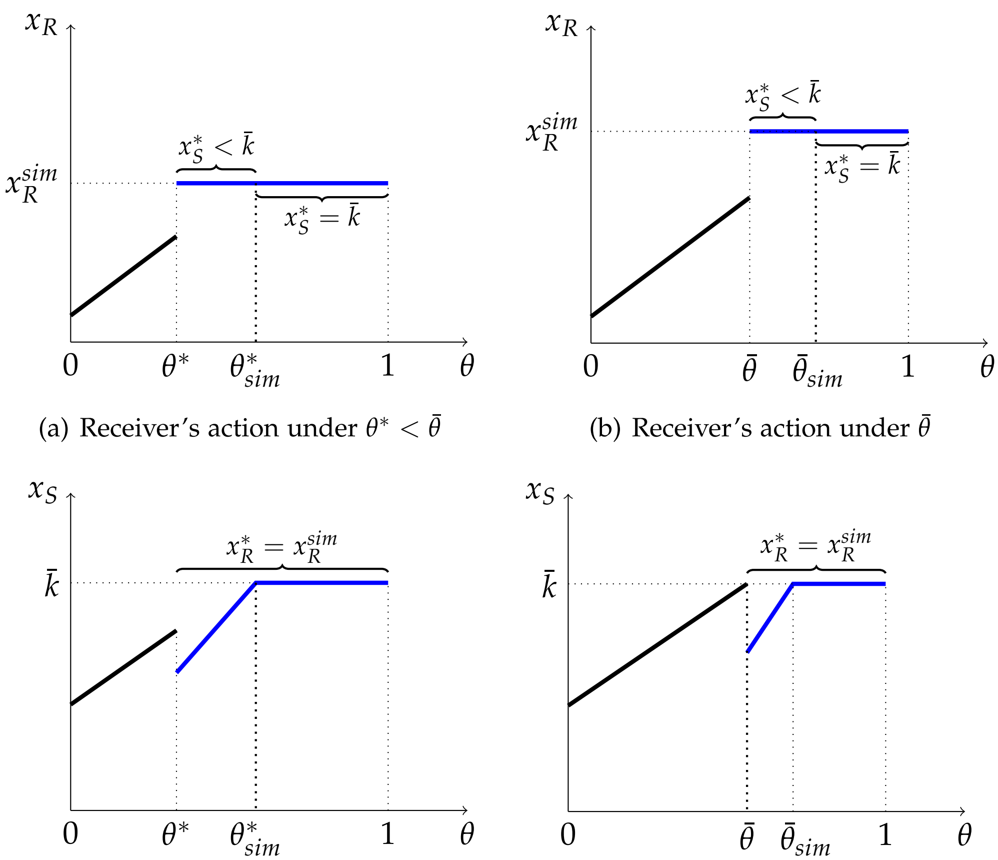
Figure 2 illustrates this point. Notice that there is non-monotonicity in the sender's action at θ^* because of the discontinuity in receiver's response upon receiving the pooling message. Since the receiver's action has a discontinuous jump at θ^*, the sender readjusts her action so as to achieve first best levels of contribution, by decreasing her action just to the right of θ^*. Further, since the receiver's action is not high enough, there is always an interval of types —$(\theta_{sim}^*, 1]$ —for which the sender does not achieve first best and is constrained by the resource constraint.

Lemma 2 clearly illustrates the benefit for sender from revealing more information. First, it increases her welfare on the interval $[0, \theta_{sim}^*]$ since the resource constraints are not binding. Second, as θ^* increases, the receiver's best response $x_R^{sim} = x_R^*(m_{pool}^{\theta^*})$ also increases. This further implies that on the interval $(\theta_{sim}^*, 1]$, where the sender is resource constrained, extracting greater resources (a higher action) from the receiver is welfare improving for the sender.

Proposition 4 *The most informative equilibrium, $\theta^* = \bar{\theta}$, is ex-ante efficient for both sender and receiver.*

Proof. See Appendix A.6 ■

The receiver's welfare is increasing in the amount of information revealed. This immediately implies that the welfare of the receiver is increasing in the threshold of partial revelation. For the sender, compare two thresholds θ^1 and θ^2 $(\theta^1 < \theta^2)$. Then, it follows from previous arguments that $\theta_{sim}^1 < \theta_{sim}^2$ and $x_R^{sim}(m_{pool}^1) < x_R^{sim}(m_{pool}^2)$. Therefore, a greater threshold of information means the resource constraint is not binding (increases efficiency) for a greater measure of types for the sender and also entails a higher action from the receiver on the pooling interval. Both of these effects provide the sender with a greater ex-ante welfare from revealing more information. Figure 3 shows these trade offs. On the left, under a less informative threshold, the receivers pooling action is lower and this directly affects the extent to which the sender can achieve first best levels of joint contributions.



(a) Receiver's action under $\theta^* < \bar{\theta}$

(b) Receiver's action under $\bar{\theta}$

(c) Sender's action under $\theta^* < \bar{\theta}$

(d) Sender's action under $\bar{\theta}$

Figure 2: i) — *interval of separation*: $m(\theta) = \theta$; ii) — *interval of pooling*: $m_{pool} = 1$

Proposition 5 ***Ex-post efficiency:***
i) Every sender type weakly prefers a PRE with $\theta^* = \bar{\theta}$.
ii) The $\bar{\theta}$ PRTE is not interim efficient for the receiver.

Proof. See Appendix A.7 ■

The intuition is an extension of the arguments made in the case of Proposition 4. Specifically, every sender type on the interval $[0, \bar{\theta}_{sim}]$ weakly prefers the $\bar{\theta}$ PRTE. For types in $(\bar{\theta}_{sim}, 1]$, the resource constraint is binding, $\forall \theta \in (\bar{\theta}_{sim}, 1] : x_S^*(\theta, m_{pool}^{\bar{\theta}}) = \bar{k}$. However, by revealing more information, the sender ensures that the receiver's optimal best response on the pooling interval is also increasing. Given there is a positive spillover effect on this interval, it directly follows that every sender type on this interval is better off inducing a higher action from the receiver. For the receiver, on the other hand, for any cutoff θ', there exists some $\theta \in (\theta'_{sim}, 1]$ for which the equilibrium response $x_R^{sim}(m_{pool}^{\theta'})$ results in first best levels of contribution. But if this were true, then for this particular θ, $U(\phi^R(x_R^{sim}(m_{pool}^{\theta'}), \bar{k}), \theta) > U(\phi^R(x_R^{sim}(m_{pool}^{\bar{\theta}}), \bar{k}), \theta)$. This implies

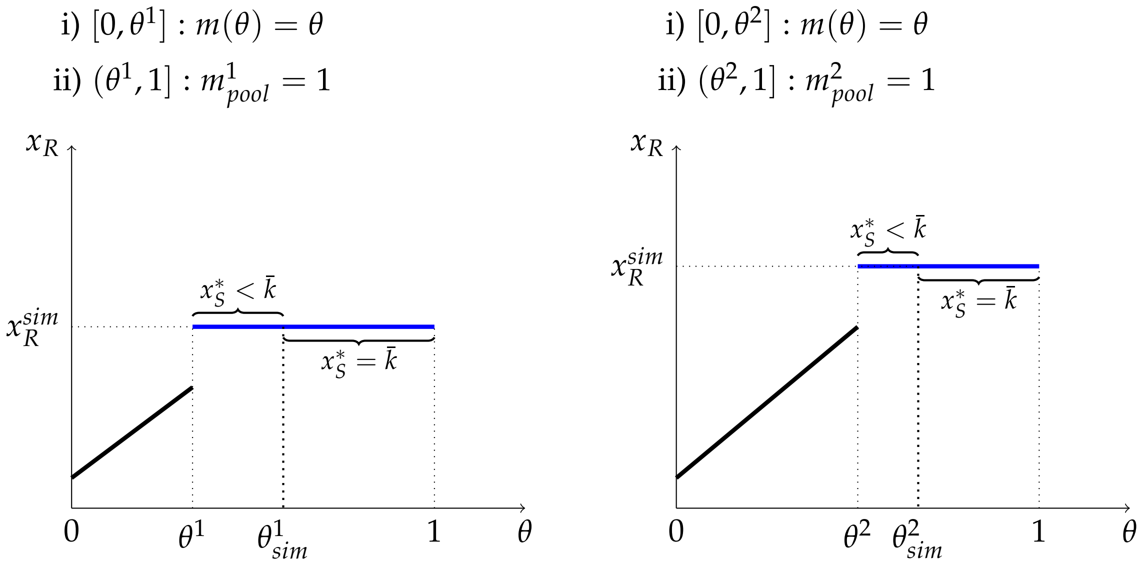


Figure 3: a) $\theta^1 < \theta^2$; b) $\theta^1_{sim} < \theta^2_{sim}$; c) $x_R^{sim}(m^1_{pool}) < x_R^{sim}(m^2_{pool})$

that the most informative equilibrium can never be ex-post efficient for the receiver.

7 Sequential Decision-making Protocol

In a number of scenarios, it may be required of the uninformed receiver to move first (*stackelberg leader*). Organizations, for example, typically have functional units that complete their task before another downstream unit takes over the project (e.g. product development and design teams usually move first, followed by the marketing/sales team, even though project relevant information is held by the marketing team). In other cases, it may be required for the sender to communicate and also make *binding commitments* to joint projects (e.g. federal governments may typically commit to budget resources for infrastructure projects before the states decides). Sequential decisions therefore enables the sender to observe the receiver's actions and then decide on contributions.

The sequential protocol proceeds as follows:

- The sender observes the true state $\theta \in [0,1]$, sends a message $m \in M$ st $\mu : [0,1] \to M$ to the receiver
- The receiver observes the message m takes an action $\alpha_R : M \to V$.
- The sender observes the action choice of the receiver and decides on contribution $\alpha_S : [0,1] \times V \to V$

Notice the critical difference in the sequential protocol. By moving first and revealing her action, the receiver provides additional information to the sender. Further, the presence of resource constraints implies that the sender is unable to credibly convey the true state beyond a threshold. This is driven by the earlier observations under simultaneous protocol. The messaging strategy of the sender therefore in all the possible threshold equilibria mirrors the simultaneous protocol. The following proposition lays out this result.

Proposition 6 *Every PRTE under simultaneous protocol is also an equilibrium under the sequential protocol.*

Proof. See Appendix A.8 ■

Simultaneous vs Sequential Protocol

Given a set of resources, under the most informative threshold equilibrium, both protocols provide the same (ex-ante) welfare to the sender and receiver on the interval $[0, \bar{\theta})$. The crucial difference between the two protocols arises on the uninformative domain of the state space, when a pooling message $m_{pool} = (\bar{\theta}, 1]$ is sent by the sender. Since the receiver moves first, the equilibrium action solves the following,

$$x_R^{seq} = x_R^{seq}(m_{pool}) \equiv \arg\max_{x_R \in V} \int_{\bar{\theta}}^{\bar{\theta}_{seq}} U\left(\phi^R\left(x_R, x_S^{seq}(t, x_R)\right), t\right) dF + \int_{\bar{\theta}_{seq}}^{1} U\left(\phi^R\left(x_R, \bar{k}\right), t\right) dF \quad (3)$$

Notice the difference between the two protocols. In the simultaneous move game, the sender reacts to the receiver without observing her actions. This changes the incentives of the receiver. In the sequential protocol, when the receiver knows that the sender observes her action, there is an additional *undoing effect* ($\frac{dx_S}{dx_R} < 0$) in that the sender moderates any action of the receiver by contributing lesser in order to achieve her first best $\bar{\phi}^S$. This undoing effect that is present in the sequential protocol implies that the receiver can now play a higher action on the pooling interval.

Lemma 3 $x_R^{seq} > x_R^{sim}$

For the sender, the welfare improvement in the sequential protocol directly follows from the above lemma. Specifically, the sender's resource constraint is now binding for a smaller interval of types $[\bar{\theta}_{seq}, 1]$ (see Figure 4), and over this interval, $U_1(.) > 0$ because there is under-provision

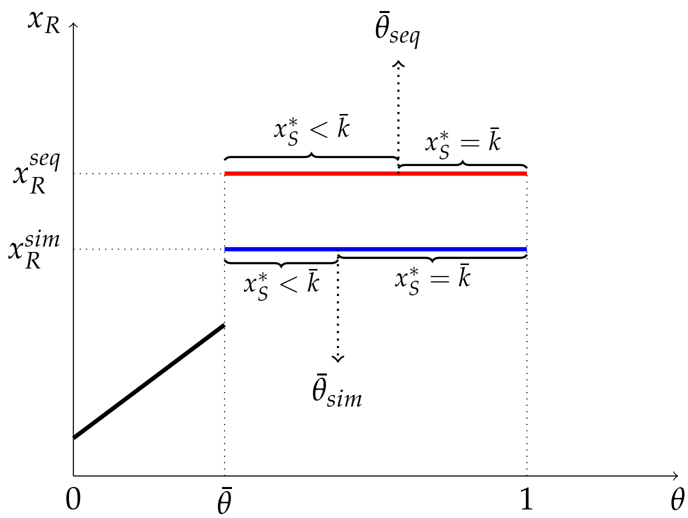


Figure 4: i) $[0, \bar{\theta}]$: *interval of separation*; ii) $(\bar{\theta}, 1]$: *interval of pooling*

as far as the sender is concerned. Since $x_R^{seq} > x_R^{sim}$, it follows that the expected utility of the sender is greater on this interval under the simultaneous protocol. For the receiver, the reason is intuitive. Suppose, under the sequential protocol, the receiver were to mimic the action x_R^{sim}. Then, since the actions are sequential, there is an additional *moderating influence* of the sender's action upon observing x_R^{sim}. To put it differently, if the receiver plays a slightly higher action $x_R^{sim} + \epsilon$, the sender observes this and readjusts her action downwards under the sequential protocol, but not so in the simultaneous one since the action of the receiver are not observed. This readjustment is akin to an undoing effect since by moderating her action downwards, the receiver's payoff also increases since the over-provision is lesser, resulting in increased welfare for the receiver.

Proposition 7 *The sequential protocol provides a higher ex-ante welfare to both the sender and receiver compared to simultaneous protocol.*

Proof. See Appendix A.9 ■

8 Optimal Commitment

From the previous section, it is clear why an uninformed receiver would rather prefer to be a Stackelberg leader. This may be interpreted as a mandate for transparency between project teams. That is, the decisions of one team are revealed and shared with the other under sequential decision making (transparency), while they are not under simultaneous protocol. Such transparency is usually mandated in organizations when multiple teams work together on a single project. Clearly, Proposition 7 provides an efficiency rationale for the requirement of transparency and how it improves the welfare of both players (Jehiel (2014), Prat (2005)), even though the amount of information communicated remains the same under either protocols.

However, there is still a potential source of inefficiency for the receiver. At the beginning of the pooling interval the sender has a non-monotonic response (see Figure 2). As a result, the receiver fails to extract the maximum contributions possible from the sender, $\bar{k}$. This implies that the receiver is over-contributing in order to satisfy the sender's first best $\bar{\phi}^S_\theta$ on the interval $(\bar{\theta}, \bar{\theta}_{seq}]$.[16] By instead committing to a decision rule ex-ante, the receiver can mitigate some of this inefficiency. This is equivalent to the receiver choosing an optimal ex-ante *commitment rule* (Melumad and Shibano (1991)) contingent on the information communicated by the sender.

Let me begin by first defining the optimal commitment rule problem:

$$\begin{aligned}
&\underset{x^c_R(\theta)\in V}{\operatorname{argmax}} \int_0^1 U\left(\phi^R\left(x^c_R(\theta), x^c_S(\theta, x^c_R(\theta))\right), \theta\right) dF \text{ such that } \forall \theta', \theta'' \in [0,1]: \\
&U\left(\phi^S\left(x^c_S(\theta', x^c_R(\theta')), x^c_R(\theta')\right), \theta', b\right) \geq U\left(\phi^S\left(x^c_S(\theta', x^c_R(\theta'')), x^c_R(\theta'')\right), \theta', b\right) \\
&x^c_S(\theta, x^c_R(\theta)) \equiv \underset{x_S\in V}{\operatorname{argmax}}\, U\left(\phi^S(x_S, x^c_R(\theta)), \theta, b\right)
\end{aligned}$$

From Revelation Principle, the problem for the receiver boils down to choosing a sequence of commitments for every state $\theta \in [0,1]$ such that they maximize expected utility of the receiver conditional on the IC constraint that ensures truthful revelation for all types of sender's private information. Clearly, the receiver can mimic the sequential protocol actions over the type space and guarantee an expected utility at least as much as in the sequential protocol. Such a mimicking strategy would be incentive compatible since the sender on the separating interval achieves first best, and on the pooling interval, cannot do any better than merely revealing her type as the action of the receiver is fixed at x^{seq}_R.

However, as previously discussed, the receiver can do better. Instead of taking a single action on the pooling interval and allowing the sender to adjust her actions, the receiver can commit to a message contingent resource contribution rule $x^c_R(\theta)$ for every $\theta \in [0,1]$. I will now state a series of claims that must be valid for an optimal commitment rule.

Claim 1 *On the separating interval, the commitment rule mirrors the simultaneous (and sequential) protocol, i.e.,* $\forall \theta \in [0, \bar{\theta}] : x^c_R(\theta) = \tilde{x}_R(\theta)$.

This follows directly from noting that the receiver and sender achieve their first best levels

[16] For that matter, on $(\bar{\theta}, \bar{\theta}_{seq}]$ under the simultaneous protocol.

of contribution $\bar{\phi}^i_\theta$ on this interval. To see this, the best response of the sender to $\tilde{x}_R(\theta)$ is $\tilde{x}_S(\theta)$. Further, the pair of actions $(\tilde{x}_S(\theta), \tilde{x}_R(\theta))$ achieve first best for the sender and therefore is incentive compatible. Therefore, there is no reason for the receiver to commit to some other levels of contribution.

Claim 2 *On the pooling interval m_{pool}, there is no single flat segment such that $\forall \theta \in m_{pool} : x^c_R(\theta) = z \geq \tilde{x}_R(\bar{\theta})$.*

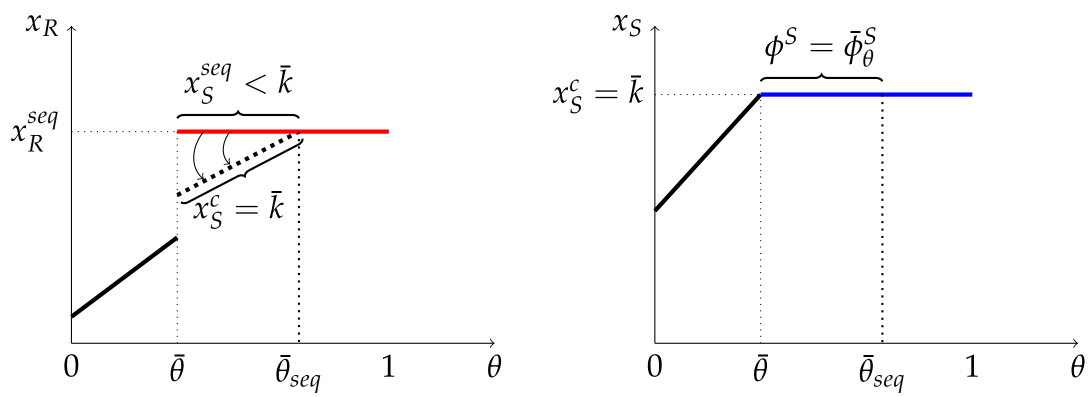
Suppose $x^c_R(\theta) = \tilde{x}_R(\bar{\theta})$. Then $\forall \theta \in m_{pool} : x_S(\theta) = \bar{k}$. This cannot be optimal for the receiver since the receiver can always do better by committing a bit more and satisfying the sender's IC. Instead, suppose $x^c_R(\theta) = z > \tilde{x}_R(\bar{\theta})$. Say, for the sake of argument that $z = x^{seq}_R$, i.e. the receiver mimics the sequential protocol action. This cannot be optimal for the receiver since the sender's action is less than $\bar{k}$ on the interval $(\bar{\theta}, \bar{\theta}_{seq})$. The receiver can always contribute something lesser and induce the sender to contribute all her resources, whilst still satisfying her IC. Given the imperfect substitutability property of actions, this increases the payoff to the receiver by minimizing the extent of over-provision in this interval.

Claim 3 *If $x^c_R(\theta)$ is strictly increasing in any interval (θ_1, θ_2) within m_{pool}, then sender must contribute $\bar{k}$ for all types in this interval.*

Claim 3 follows from the previous arguments. Again, there are number of different ways in which the sender's IC can be satisfied on the increasing interval, i.e. multiple pairs (x_R, x_S) satisfy $\forall \theta \in (\theta_1, \theta_2) : \phi^S(x_S, x_R) = \bar{\phi}^S_\theta$. However, of all these pairs that satisfy first best for the sender, the one that minimizes the receiver's over-provision is the one in which the sender contributes all her resources, meaning $x_S = \bar{k}$. If this weren't true, the receiver could increase her expected utility by decreasing her contributions and extracting more resources from the sender.

Claim 4 *On m_{pool}, there cannot be a flat segment followed by a strictly increasing interval.*

Claim 4 is true since on a flat segment, where the receiver's decision is independent of communication, either the sender's IC is satisfied for all types in that interval or there is always inefficiency for some sender types. If it is the former, then the receiver can improve her payoff by decreasing contributions in that interval and extracting more resources from the sender. If it is the latter, on the other hand, the sender types that do not achieve first best can always deviate to the strictly increasing interval and benefit from greater contributions by the receiver, thereby violating incentive compatibility.



(a) Receiver's actions (b) Sender's actions

Figure 5: The receiver can commit to an action that is strictly lower than x_R^{seq} on the interval $(\bar{\theta}, \bar{\theta}_{seq})$. Notice this is possible since the sender can always increase her contributions to $\bar{k}$ and still achieve first best $\tilde{\phi}^S_\theta$.

Figure 5 illustrates the consequence of the above claims. Specifically, the receiver, instead of playing a flat action on the pooling interval $(\bar{\theta}, \bar{\theta}_{seq})$, pivots and provides lesser contributions thereby extracting the most resources possible from the sender. While doing so, the sender still achieves first best levels $\phi^S(\bar{k}, x_R^c(\theta)) = \bar{\phi}^S_\theta$. For the receiver, $\bar{\phi}^R_\theta < \phi^R(x_R^c(\theta), \bar{k}) < \phi^R(x_R^{seq}, x_S^{seq}(\theta, x_R^{seq}))$ and this implies that over-provision is now minimized, leading to an increase in expected utility for the receiver on this interval. Given the above arguments, the commitment problem can be reformulated as the following:

$$\begin{aligned}
&\underset{x_R^c(\theta)\in V}{\text{argmax}} \int_{\bar{\theta}}^{\bar{\theta}_c} U\left(\phi^R\left(x_R^c(\theta), \bar{k}\right), \theta\right) dF + \int_{\bar{\theta}_c}^{1} U\left(\phi^R\left(x_R^c(\bar{\theta}_c), \bar{k}\right), \theta\right) dF \text{ such that} \\
&\forall \theta', \theta'' \in [0,1] : U\left(\phi^S\left(\bar{k}, x_R^c(\theta')\right), \theta', b\right) \geq U\left(\phi^S\left(x_S^c(\theta', x_R^c(\theta'')), x_R^c(\theta'')\right), \theta', b\right) \\
&x_R^c(\bar{\theta}_c) \equiv \underset{x_R\in V}{\text{argmax}}\, U\left(\phi^S(\bar{k}, x_R), \theta, b\right)
\end{aligned}$$

Two important properties of the optimal commitment rule becomes clear from the above reformulation. First, there is *maximal resource extraction* from the sender on the interval m_{pool} compared to the two protocols. Second, the receiver *caps resource allocation* at $\bar{\theta}_c$ by contributing $x_R^c(\bar{\theta}_c)$, but no more on the interval $(\bar{\theta}_c, 1]$. Together, they determine the precise nature of the optimal commitment rule, given by the following proposition.

Proposition 8 *The optimal commitment rule for the receiver is given by the following:*

1. $\forall \theta \in [0, \bar{\theta}] : x_R^c(\theta) = \tilde{x}_R(\theta)$

2. $\forall\theta \in (\bar{\theta}, \bar{\theta}_c] : x_R^c(\theta) \equiv \arg\max_{x_R} U(\phi^S(\bar{k}, x_R), \theta, b)$

3. $\forall\theta \in (\bar{\theta}_c, 1] : x_R^c(\theta) = x_R^c(\bar{\theta}_c)$

Proof. See Appendix A.10 ■

The optimal decision rule mimics the unrestricted domain decisions on the separating interval $[0, \bar{\theta}]$. On the pooling interval, the decision rule maintains IC by providing the first best levels for the sender up to some higher threshold $\bar{\theta}_c (< 1)$ and then is unchanged beyond. The optimal rule exhibits two key features. First, it is discontinuous at exactly $\bar{\theta}$ and nowhere else. Second, on the interval $(\bar{\theta}, \bar{\theta}_c]$ where the receiver's actions are strictly increasing, the sender's action is constant and fixed at $\bar{k}$. This is driven by the imperfect substitutability of players' contribution. Out of all possible incentive compatible commitment rules, the one that maximizes the receiver's utility is the one that extracts the most resources from the sender. That is, $\forall\theta \in (\bar{\theta}, \bar{\theta}_c]$, $\phi^S(\bar{k}, x_R) = \phi^S(\bar{k} - \epsilon, x_R + \gamma) = \bar{\phi}_\theta^S$, implies that $U(\phi^R(x_R, \bar{k}), \theta) > U(\phi^R(x_R + \gamma, \bar{k} - \epsilon), \theta)$.

Proposition 9 *The optimal commitment rule provides a higher ex-ante welfare to both the sender and receiver, compared to the sequential protocol.*

Proof. See Appendix A.11 ■

The intuition is the following. Notice that apart from choosing a sequence of contributions $x_R^c(\theta)$, the receiver must also decide the threshold $\bar{\theta}_c$ up to which there is strictly increasing contributions. In other words, the receiver's problem is equivalent to choosing a cutoff $\bar{\theta}_c$ and a corresponding cap on contributions $x_R^c(\bar{\theta}_c)$. The receiver could always choose a cutoff $\bar{\theta}_{seq}$ and a cap $x_R^c(\bar{\theta}_{seq})$ that mimics the sequential protocol action on the separating interval and satisfies maximal resource extraction on $(\bar{\theta}, \bar{\theta}_{seq}]$. This guarantees the receiver at least the same payoff as the sequential protocol (see Figure 5). Since the receiver is able to extract $\bar{k}$ on the pooling interval from the sender, the marginal utility for the receiver is strictly increasing at $(\bar{\theta}_{seq}, x_R^c(\bar{\theta}_{seq}))$. This implies that the receiver is able to provide first best for the sender up to a threshold greater than $\bar{\theta}_{seq}$, and ipso facto, the cap on contributions with commitment is also higher. That is, $\bar{\theta}_c > \bar{\theta}_{seq}$ and $x_R^c(\bar{\theta}_c) > x_R^c(\bar{\theta}_{seq})$ (see Figure 6).

As argued earlier, since the sender's welfare is strictly increasing in the cutoff threshold and therefore on the size of the cap, the optimal commitment rule is also welfare improving for the sender since she achieves first best for the types $[0, \bar{\theta}_c]$ and on the interval $(\bar{\theta}_c, 1]$ the contributions of the receiver under commitment is higher than in the sequential case. These two effects lead to a greater overall welfare for the sender under commitment.

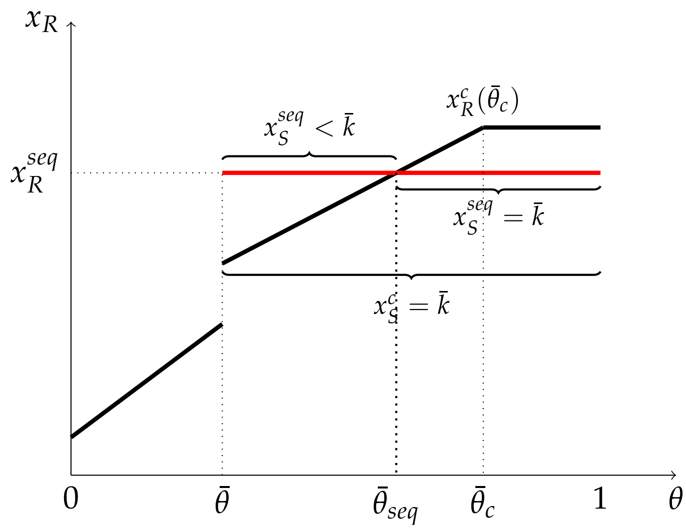


Figure 6: The optimal mechanism exhibits two key features. On the interval $(\bar{\theta}, 1]$, there is maximal resource extraction from the sender ($x_S^c = \bar{k}$). Further, the receiver places a cap on the contributions given by $x_R^c(\bar{\theta}_c)$.

9 Discussion

The optimal commitment rule has features that are prevalent in many real world environments. Proposition 8 could be seen as a form of binding commitment to resources by the informed agent, prior to communication. Proposition 9 suggests a combination of transparency and ex ante commitment in organizations can be welfare improving. This could arise in political economy settings where governments jointly share resources required to implement a public project (infrastructure, say). Governments typically announce budgetary allocations with a cap on the maximum contributions to be made for the (joint) project. Alternatively, in organizations, such commitments are usually written into contractual agreements before the start of a project. This could be a commitment from an upstream division indicating a time schedule with the amount of human capital resources that the division is willing to commit to, contingent on the progress made in the project. This way, the uninformed division can achieve first best when the quality of project is fairly low, and extract the most resources from the downstream division when the project is of high quality.

Extensions

Lying costs

The equilibrium in both protocols exhibits some level of lying by the informed sender. Experimental evidence suggests that there is an intrinsic propensity to say the truth even when the information conveyed is *soft* (Gneezy (2005), Hurkens and Kartik (2009), and Sánchez-Pagés and Vorsatz (2007)), suggesting an aversion to lying. The presence of lying costs (Kartik (2009)) seems relevant in the examples described earlier. For example, within organizations, misrepresentation of information by a department to another could lead to distrust among them. Alternatively, it is possible for the uninformed functional unit to learn about the true project type ex-post, leading to a loss of reputation.

Introducing lying costs in the presence of action substitutability and resource constraints changes the incentives of the informed sender drastically. Suppose, for sake of exposition, lying costs are minimized when the messages are truthful (i.e. $\mu(\theta) = \theta$). Then, the presence of lying costs eliminates all but the most informative equilibrium under either protocols. The intuition is that there is now a lying cost associated with wrongful reporting for no marginal benefit in utility. (On the interval $[0, \bar{\theta}]$, $U_1\left(\phi^S(x_S(\theta,\theta), x_R(\theta)), \theta, b\right) = 0$ implying that truthful reporting is indeed a solution.) That is, by lying, the sender does no better than under truthful reporting but incurs a *wasteful cost* by pooling with the other types and sending $\tilde{m} = 1$. This implies that there is an unique separating equilibrium on $[0, \bar{\theta}]$ such that $\mu(\theta) = \theta$.

Proposition 10 *With lying costs, there is an unique separating interval such that all the information is revealed up to the threshold $\bar{\theta}$, under either protocols.*

What is left to consider is the equilibrium messaging on the pooling interval, $m_{pool} = (\bar{\theta}, 1]$. One way to look at my earlier results is by taking them as the limit case of lying costs. As the intensity of lying costs goes to zero, the equilibrium messaging on the pooling interval is such that all types send the message $m = 1$. Specifically, when the intensity of lying is very small, there is an incentive to (almost) costlessly exaggerate the information, resulting in the maximal message at the limit, $\lim_{\theta \downarrow \bar{\theta}} \mu(\theta) = 1$. On the other hand, when the lying costs are sufficiently high, there is full separation as the incentives to exaggerate are counteracted by the lying costs.

The interesting case is when the lying costs are sufficiently high but not prohibitively so. It is then possible for alternate equilibrium messaging strategies to emerge. For example, the sender could partition the states and send the same inflated message for every type in this partition, resulting in clustering of sender's private information (Chen (2011)) on the interval m_{pool}.

Let me construct a simple two cluster example. Suppose the sender partitions the interval into two, $m^1_{pool} = [\bar{\theta}, \bar{\theta}_1]$ and $m^2_{pool} = (\bar{\theta}_1, 1]$, such that $\forall \theta \in m^1_{pool} : m_1 = \mu(\theta) = \bar{\theta}_1$ and $\forall \theta \in m^2_{pool} : m_2 = \mu(\theta) = 1$. Given this messaging strategy, the receiver correctly anticipates the sender type given the message m_i and takes an action $x_R(m_i)$ that solves a problem similar to Lemma 2. Finally, for this strategy to be incentive compatible for the sender, the marginal type $\bar{\theta}_1$ must be indifferent between sending m_1 and m_2. This gives a simple trade off between marginal benefits of sending a higher message m_2 and marginal (lying) costs of doing so. By sending the message m_1, there are no lying costs but the receiver's equilibrium response is sub-optimal for the sender. By deviating and clustering with the higher types, the sender can guarantee a higher payoff (since $U_1 > 0$ at $x_S = \bar{k}, x_R = x_R(m_1)$) but it also comes with a positive lying cost. Depending on the intensity of cost parameter, the bias and extent of substitutability, the indifference condition would precisely characterize the message clustering.[17]

Role of Delegation

The role of delegation in organizations has been widely studied since the seminal work of Holmstrom (1978). The work of Dessein (2002) finds that delegation provides a higher welfare (to the uninformed player) over communication when the conflicts of interest is low enough.[18] When both players take substitutable actions, however, full delegation by the receiver would imply that the sender replicates the same outcomes as in the most informative equilibrium with communication in the interval $[0, \bar{\theta}]$, and additionally also achieves first best ($\bar{\phi}^S_\theta$) in the pooling interval $m_{pool} = (\bar{\theta}, 1]$ by borrowing the extra resources required from the receivers contribution set. This results in over-provision on this interval from the perspective of the receiver.

However, private firms with multiple functional teams that work together do so under the premise of *functional autonomy*. For example, a marketing and sales team cannot appropriate the decision rights from the product development team and allocate resources on their behalf. Even in the scenario where one team is told to delegate its decision to another, the team delegating its decision rights can do so optimally.

From Section 8, we know that the optimal commitment rule has a cap on resources given by $x^c_R(\bar{\theta}_c)$. However, Alonso and Matouschek (2008) have shown that the commitment rule problem is equivalent to a delegation one in which the uninformed player provides a delegation set to the informed sender. That is, instead of following the commitment rule characterized in Proposition 8, the receiver can instead allow the sender to choose a contribution from the set $D = [\underline{k}, x^c_R(\bar{\theta}_c)]$. This form of interval delegation would provide the receiver with the same expected utility if the sender mimics her contributions according to the commitment rule. That is, on the pooling interval, the sender always chooses to contribute $\bar{k}$ and borrows the rest from the delegation set D.

However, in the case where both players make decisions that are substitutable, there are multiple combinations through which to achieve the first best levels of contribution for the

[17]Chen (2011) finds clustering and inflated messaging in a completely different setup. In Chen's work, there is a small prior probability that the sender is honest (always reports truthfully) and the receiver is naive (always believes the message). This leads to message inflation and clustering at the top end of the message spectrum.

[18]$b < \frac{1}{4}$ to be specific in a quadratic utility framework.

sender. To see this, fix a $\theta \in m_{pool}$. Let the first best levels of contribution for the sender be $\bar{\phi}^S_\theta$. Since there is positive spillover, decreasing x_S could be compensated by a corresponding increase in x_R. As a result, there are multiple pairs of (x_S, x_R) such that $\phi^S(x_S, x_R) = \bar{\phi}^S_\theta$. If there was an earlier stage where both players invest in resources at some marginal cost c, then the delegation problem is not anymore equivalent to the commitment rule problem since the receiver has incentives to strategically *under-invest* in resources under delegation in order avoid free riding by the sender. The investment in resources problem introduces an additional layer of incentive issues that are worth investigating for future research.

Verifiable Information Disclosure

So far, the analysis has focused mainly on transmission of *soft information* by the sender. In many projects the nature of information is verifiable (Grossman (1981); Milgrom (1981)) by the uninformed receiver. Project quality can be verified by the uninformed team by acquiring information from outside sources, for example. Alternatively, the project contract might specify evidence provision as a requirement for the informed party. When information can be verified costlessly, the incentives for communication change completely. There is unraveling in the sense that the sender would always find it optimal to reveal every state truthfully, leading to full information transmission even in the presence of resource constraints. This is straightforward to observe. On the pooling interval, for the highest state $\theta = 1$, the sender is better off revealing. This way, $x_R(1) > x_R(m_{pool})$ and since there is under-provision for the sender ($U_1(\phi^S(\bar{k}, x_R(m_{pool})), 1, b) > 0$), it follows that revealing the highest state improves the sender's utility. However, this argument holds for all states below as well and there is complete unraveling (see Milgrom (1981)). Obviously, in case of verifiable disclosure, both sequential and simultaneous protocols lead to full information revelation resulting in the same ex-ante welfare for the receiver and sender.

10 Conclusion

This paper investigates the nature of cheap talk communication with (one sided) incomplete information when actions of both players are strategic substitutes. Under simultaneous decision making protocol, with pure messaging strategies, I show that information is fully revealed when the informed sender does not face resource constraints — defined by the permissible domain of actions that the players choose from. Consequently, when resource constraints are binding, communication deteriorates and there is only partial information revelation in equilibrium. I establish an intuitive monotonic relationship between information transmission and resource constraints of players.

Next, I consider an alternate protocol – sequential decision making – in which the uninformed receiver takes an action first followed by the sender, post the communication stage. While the total amount of information conveyed remains unchanged in the most informative equilibrium, the two players' ex ante welfare improves under the sequential protocol. This result elucidates the importance of transparency in decision making within organization when projects involve multiple teams that jointly make (substitutable) contributions.

Finally, I characterize an intuitive commitment mechanism for the receiver that displays three features: *i)* the receiver mimics the simultaneous protocol actions up to the informative threshold; *ii)* beyond this, the receiver extracts maximal resources possible from the sender while contributing the residual required to satisfy the sender's incentive constraints; *iii)* the receiver caps her contributions beyond a threshold. The commitment rule, interestingly, is welfare improving for both players, compared to the two decision making protocols. This result provides a rationale for the efficiency of ex ante commitments in the presence of action substitutability and resource constraints.

The paper makes an important contribution in uncovering the informational inefficiencies driven by the presence of resource constraints. There are other incentive problems associated with resource constraints that are worth exploring. For example, when there is two sided incomplete information, resource constraints would still affect the ability of players to share information. In fact, as information is more dispersed, the inefficiencies emerging from resource constraints might worsen leading to decreased welfare. Also, even though resource constraints were assumed to be exogenous in this paper, it could very well be that players invest in resources ex-ante at some marginal cost. This investment decision might be affected by the decision-making protocol. Alternatively, when players instead have a coordination motive with strategic complementarity in actions, resource constraints might play a similar role in constraining the credibility of information. All such scenarios require a more detailed analysis, and are left for future work.

A Appendix

A.1 Proof of Proposition 1

Sufficiency

Let $\bar{\phi}^S_\theta = \phi^S(\tilde{x}_S(\theta), \tilde{x}_R(\theta))$ be the first best levels of contribution for the sender when the state is θ. When HTIC condition is satisfied, it implies that for every other $\theta \in [0,1)$, $\tilde{x}_S(\theta) < \bar{k}$ by single crossing property of the utility function ($U_{12} > 0$). But if this is the case, when the sender sends a truthful message $m = \theta$, the optimal action under both constrained optimization and unconstrained optimization coincide for the sender. This means that for every $\theta \in [0,1]$, $x^*_S(\theta) = \tilde{x}_S(\theta)$. Since the receiver does not face any resource constraints, it also implies that $x^*_R(\theta) = \tilde{x}_R(\theta)$ and $\bar{\phi}^S_\theta = \phi^S(x^*_S(\theta), x^*_R(\theta))$ This ensures there is no inefficiency in terms of contributions and sender always first best levels of contribution for every θ. Hence, there exists an equilibrium in which the sender reveals her all the information truthfully.

Necessity

Suppose HTIC is violated ($\tilde{x}_S(1) > \bar{k}$) but there is full information revelation by sender. Then, by definition, there exists a non-empty set $G = \{\theta : \tilde{x}_S(\theta) > \bar{k}\}$. When HTIC is violated, the unconstrained actions does not coincide with the equilibrium actions that are bounded by the resource constraint, i.e $\forall \theta \in G : x^*_S(\theta) = \bar{k}$ under truthful revelation.
Now take a $\theta' \in G$. If S reports θ', the optimal actions are $x^*_S(\theta') = \bar{k}$ and $x^*_R(\theta')$ solves $\max_{x_R \in V} U\left(\phi^R(x_R, \bar{k}), \theta'\right)$. However, given imperfect substitutability, $\phi^R(x^*_R(\theta'), \bar{k}) < \phi^S(\bar{k}, x^*_R(\theta'))$ $< \phi^S(\tilde{x}_S(\theta'), \tilde{x}_R(\theta')) \equiv \bar{\phi}^S_{\theta'}$. But, because HTIC is violated, the contribution function under truth-telling is $\phi^S(\bar{k}, x^*_R(\theta'))$ which is clearly not optimal in the sense that $U_1\left(\phi^S(\bar{k}, x^*_R(\theta')), \theta', b\right)$ > 0.[19] From continuity, there exists an ϵ such that the sender by deviating to $\theta' + \epsilon$, induces equilibrium actions that are $x^*_R(\theta' + \epsilon) > x^*_R(\theta')$ and $x^*_S(\theta', \theta' + \epsilon) = \bar{k}$. This way the sender can guarantee a higher payoff since $\phi^S(\bar{k}, x^*_R(\theta' + \epsilon)) > \phi^S(\bar{k}, x^*_R(\theta'))$ and,

$$U(\phi^S(\bar{k}, x^*_R(\theta' + \epsilon)), \theta', b) > U(\phi^{S^*}(\bar{k}, x^*_R(\theta')), \theta', b)$$

However, this means that the sender type θ' has an incentive to deviate and pretend to be a higher type $\theta' + \epsilon$, precluding truthful communication. This is a contradiction. **QED**

[19] This condition corresponds with the *positive spillover effect at the bound*.

A.2 Proof of Lemma 1

Suppose there exists an equilibrium in messaging strategy such that some types in $G = (\bar{\theta}, 1]$ send a different message. Since I consider monotonic messaging equilibria, wlog, $\exists \theta' \in G$ such that types in $(\bar{\theta}, \theta']$ send a message m' and types in $(\theta', 1]$ send a message m'', where $m' < m''$. Then, from single crossing ($U_{12} > 0$) it must be that $x_R^*(m') \equiv \underset{x_R \in V}{\text{argmax}}\, \mathbb{E}_\theta U\left(\phi^R(x_R, x_S^*(\theta, m')), \theta\right)$ and $x_R^*(m'') \equiv \underset{x_R \in V}{\text{argmax}}\, \mathbb{E}_\theta U\left(\phi^R(x_R, x_S^*(\theta, m'')), \theta\right)$ are such that $x_R^*(m') < x_R^*(m'')$. By a similar argument, $x_R^*(\theta') \equiv \underset{x_R \in V}{\text{argmax}}\, U\left(\phi^R(x_R, x_S^*(\theta')), \theta'\right)$ must be such that $x_R^*(m') > x_R^*(\theta') > x_R^*(m'')$. $x_R^*(\theta')$ is simply the receiver's equilibrium action when the sender's message is truthful and is believed by the receiver to be so (i.e. $m = \theta', p(\theta'|m) = 1$).

But if this were the case, at $m = \theta'$, $\tilde{x}_S(\theta') > \bar{k} \implies x_S^*(\theta') = \bar{k}$. Further, for the sender, the utility is increasing at $m = \theta'$, i.e. $U_1\left(\phi^S(\bar{k}, x_R^*(\theta')), \theta', b\right) > 0$. This is driven by Assumption 3, since the receiver chooses x_R to achieve $\bar{\phi}^R_{\theta'} < \bar{\phi}^S_{\theta'}$. However, if $U_1\left(\phi^S(\bar{k}, x_R^*(\theta')), \theta', b\right) > 0$ and $U_{11} < 0$, it implies that the following holds:

$$U\left(\phi^S(\bar{k}, x_R^*(\theta')), \theta', b\right) > U\left(\phi^S(\bar{k}, x_R^*(m')), \theta', b\right)$$

The utility for the sender from sending a truthful message at θ' is greater than from pooling with some lower types and sending the message m'. Given Assumption 5, it holds that $\underline{k} < x_S^*(\theta', m'')$. The sender's equilibrium action from sending a pooling message m'' when the true state is θ' is always within the available domain of actions. But if this is true, then there are two possibilities.

If $x_S^*(\theta', m'') = \bar{k}$, then it holds that

$$U\left(\phi^S(\bar{k}, x_R^*(m'')), \theta', b\right) > U\left(\phi^S(\bar{k}, x_R^*(m')), \theta', b\right)$$

If $x_S^*(\theta', m'') < \bar{k}$, then $\phi^S(x_S^*(\theta', m''), x_R^*(m'')) = \bar{\phi}^S_{\theta'}$ meaning that the sender achieves her first best levels of contribution in which case,

$$U\left(\bar{\phi}^S_{\theta'}, \theta', b\right) > U\left(\phi^S(\bar{k}, x_R^*(m')), \theta', b\right)$$

As a result, the sender type θ' would always deviate and send the higher pooling message m''. This argument holds for any $\theta \in G$ and for any two pooling messages of the above form. This completes the proof. **QED**

A.3 Proof of Proposition 2

Consider the following construction of PRTE for a threshold θ^*:

- If $\theta \leq \theta^*$, $m = \theta$; if $\theta > \theta^*$, $m = 1$.
- If $m \leq \theta^*$, $p(\theta \mid m = \theta) = 1$; if $m = 1$, $p(\theta \mid m) = f(\theta)$
- When $m \leq \theta^*$: $x_S^*(m) = \tilde{x}_S(m)$ and $x_R^*(m) = \tilde{x}_R(m)$
- When $m = 1$:

  $x_S^*(\theta, m) \equiv \arg\max_{x_S \in V} U\left(\phi^S(x_S, x_R^*(m)), \theta, b\right)$

  $x_R^*(m) \equiv \arg\max_{x_R \in V} \int_{\theta^*}^{1} U\left(\phi^R(x_R, x_S^*(\theta, m)), \theta\right) f(\theta) d\theta$
- When $m \in (\theta^*, 1)$: $p(\theta^* | m) = 1$.

The first condition says that for all states in $[0, \theta^*]$, the sender communicates truthfully, and for any state above, pools by sending an exaggerated message $m = 1$. The second condition describes the formation of posterior beliefs. For any message on $[0, \theta^*]$, receiver believes it to be truthful and for messages $m = 1$, the posterior is just the conditional prior on the state space. The third and fourth statements indicate the equilibrium actions conditional on the message and posterior beliefs of the receiver. The final condition rules out any profitable off-equilibrium path deviations. For off-equilibrium path messages $m \in (\theta^*, 1)$, the receiver assigns the belief $\theta = \theta^*$, that is the deviation comes from the highest possible truth-telling type.

Then, for an equilibrium with cutoff θ^* to exist, there should be no profitable deviations for any sender types. To check this, consider the types in $(0, \theta^*]$ and $(\theta^*, 1]$. For any $\theta \in (0, \theta^*]$, sender does not have an incentive to deviate from truth telling since the sender achieves first best levels $\bar{\phi}_\theta^S$ because the resource constraints are not binding, $x_S^*(\theta) = \tilde{x}_S(\theta) \leq \bar{k}$.

For types $\theta \in (\theta^*, 1]$, the payoff from sending $m = 1$ is still higher than sending any other off-equilibrium path message. There are two cases possible for the sender.

Case i): $x_S^*(\theta, m) < \bar{k}$

In this case, the sender achieves first best in that she can do no better than under $m = 1$.

Case ii): $x_S^*(\theta, m) = \bar{k}$

Here, the sender is constrained by the bound meaning there is some under-provision for the sender type θ (meaning $U_1 > 0$). Notice that $x_R^*(m) > x_R^*(\theta^*)$ which means that the receiver takes an higher action upon receiving the pooling message $m = 1$ resulting in a discontinuity at θ^*. However, since $\phi_2^S(\bar{k}, x_R) > 0$ and $U_1 > 0$ for the sender at the bound, a higher contribution from the receiver reduces this under-provision. Given that $x_R^*(m) > x_R^*(\theta^*)$, it follows that $U\left(\phi^S(\bar{k}, x_R^*(m)), \theta, b\right) > U\left(\phi^S(\bar{k}, x_R^*(\theta^*)), \theta, b\right)$ for all such θ. This concludes the proof. **QED**

A.4 Proof of Proposition 3

Take any PRTE with threshold θ^*. I make the following claim.

Claim: $\forall \theta^{'} \in (0, \theta^*), \exists\, \epsilon > 0 : \; \forall \theta \in (\theta^{'} - \epsilon, \theta^{'}]$,

$$U\left(\phi^S\left(x_S^*(\theta), x_R^*(\theta)\right), \theta, b\right) = U\left(\phi^S\left(x_S^*(\theta, m_{(\theta^{'}-\epsilon,\theta^{'}]}), x_R^*(m_{(\theta^{'}-\epsilon,\theta^{'}]})\right), \theta, b\right)$$

Where the message $m_{(\theta^{'}-\epsilon,\theta^{'}]}$ simply implies that the type is in the interval $(\theta^{'}-\epsilon, \theta^{'}]$. The claim just states that for any separating type $\theta^{'}$, it is possible to find a pooling interval of types $m_{pool} = m_{(\theta^{'}-\epsilon,\theta^{'}]}$ such that the indifference condition holds for all types within this interval, i.e. each of the types in the pooling interval is indifferent between the separating message and the pooling one. The indifference (IC) condition merely requires that the sender is able to achieve her first best levels $\bar{\phi}_\theta^S$ which is possible as long as her best responses are within the resource constraints.

To show this, all we need to check for are the indifference conditions of the boundary types $\theta^{'} - \epsilon$ and $\theta^{'}$,

$$U\left(\phi^S\left(x_S^*(\theta^{'}), x_R^*(\theta^{'})\right), \theta^{'}, b\right) = U\left(\phi^S\left(x_S^*(\theta^{'}, m_{(\theta^{'}-\epsilon,\theta^{'}]}), x_R^*(m_{(\theta^{'}-\epsilon,\theta^{'}]})\right), \theta^{'}, b\right)$$
$$U\left(\phi^S\left(x_S^*(\theta^{'}-\epsilon), x_R^*(\theta^{'}-\epsilon)\right), \theta^{'}-\epsilon, b\right) = U\left(\phi^S\left(x_S^*(\theta^{'}-\epsilon, m_{(\theta^{'}-\epsilon,\theta^{'}]}), x_R^*(m_{(\theta^{'}-\epsilon,\theta^{'}]})\right), \theta^{'}-\epsilon, b\right)$$

The latter condition follows from noting that any upward deviation is always within the domain of available actions (from Assumption 5). That is, $x_R^*(\theta^{'}-\epsilon) > x_R^*(m_{(\theta^{'}-\epsilon,\theta^{'}]})$ from single crossing ($U_{12} > 0$) and $x_S^*(\theta^{'}-\epsilon) < x_S^*(\theta^{'}-\epsilon, m_{(\theta^{'}-\epsilon,\theta^{'}]})$ due to imperfect substitutability. However, $\phi^S\left(x_S^*(\theta^{'}-\epsilon), x_R^*(\theta^{'}-\epsilon)\right) = \phi^S\left(x_S^*(\theta^{'}-\epsilon, m_{(\theta^{'}-\epsilon,\theta^{'}]}), x_R^*(m_{(\theta^{'}-\epsilon,\theta^{'}]})\right) = \bar{\phi}^S_{\theta^{'}-\epsilon}$ meaning that the sender achieves first best levels of contributions for the type $\theta^{'}-\epsilon$ irrespective of whether the message is a separating or pooling one.

The former condition states that the type $\theta^{'}$ would pool with lower types and be indifferent from separating. To see this, notice that $x_S^*(\theta^{'}) = k^{'} < \bar{k}$ under a separating (truthful) message. By continuity, there must exist a ϵ-deviation such that the $x_S^*(\theta^{'}, m_{(\theta^{'}-\epsilon,\theta^{'}]}) \in (k^{'}, \bar{k}]$. If this were not true, then $\lim_{\epsilon \to 0} x_S^*(\theta^{'}, m_{(\theta^{'}-\epsilon,\theta^{'}]}) = k^{'} < \bar{k}$, a contradiction. As before, since $x_S^*(\theta^{'}) < x_S^*(\theta^{'}, m_{(\theta^{'}-\epsilon,\theta^{'}]})$ it follows (from Assumption 3 and SC) that $x_R^*(\theta^{'}) > x_R^*(m_{(\theta^{'}-\epsilon,\theta^{'}]})$ but $\phi^S\left(x_S^*(\theta^{'}), x_R^*(\theta^{'})\right) = \phi^S\left(x_S^*(\theta^{'}, m_{(\theta^{'}-\epsilon,\theta^{'}]}), x_R^*(m_{(\theta^{'}-\epsilon,\theta^{'}]})\right) = \bar{\phi}^S_{\theta^{'}}$. If not, the sender can always increase her contributions up to the point where she achieves first best. Therefore, there is always the possibility of pooling within any PRTE. This completes the proof. **QED**

A.5 Proof of Lemma 2

Take a pooling message $m_{pool} = (\theta^*, 1]$ associated with the PRTE θ^*. Suppose, the receivers response $x_R^*(m_{pool})$ is such that $\forall \theta \in (\theta^*, 1) : x_S^*(\theta, m_{pool}) < \bar{k}$ and $x_S^*(1, m_{pool}) = \bar{k}$. This means that the sender achieves her first best levels $\bar{\phi}_\theta^S$ for every type in the interval m_{pool}. Then evaluating the FOC of the receiver gives us,

$$\int_{\theta^*}^{1} U_1\left(\phi^R\left(x_R^*(m_{pool}), x_S^*(\theta, m_{pool})\right), \theta\right)\phi_1^R f(\theta)d\theta \tag{4}$$

When the sender achieves first best levels, it must be that $\phi^S\left(x_S^*(\theta, m_{pool}), x_R^*(m_{pool})\right) = \bar{\phi}_\theta^S$. But this implies that there is over-provision for the receiver in that $\phi^R\left(x_R^*(m_{pool}), x_S^*(\theta, m_{pool})\right) > \bar{\phi}_\theta^R$. This further entails that $U_1\left(\phi^R\left(x_R^*(m_{pool}), x_S^*(\theta, m_{pool})\right), \theta\right) < 0$ on the interval $(\theta^*, 1]$. From this, it follows that equation 4 is less than zero. This means that the receiver's action cannot be such that the sender's response is within the bound for all types in m_{pool}. Since $x_S^*(\theta^*) \leq \bar{k}$, from continuity property, it follows that $\exists \theta_{sim}^* \in (\theta^*, 1] : \forall \theta \in (\theta^*, \theta_{sim}^*), x_S^*(\theta) \leq \bar{k}$ and $\forall \theta \in [\theta_{sim}^*, 1], x_S^*(\theta) = \bar{k}$. This completes the proof. **QED**

A.6 Proof of Proposition 4

Let $W_R(\theta^*)$ and $W_S(\theta^*)$ be the ex-ante welfare of the receiver and sender respectively, represented purely in terms of the cutoff threshold θ^*.

Receiver's ex-ante utility:

$$W_R(\theta^*) = \int_0^{\theta^*} U\left(\phi^R\left(x_R^*(t), x_S^*(t)\right), t\right)f(t)dt + \int_{\theta^*}^{1} U\left(\phi^R\left(x_R^*(m_{pool}^{\theta^*}), x_S^*(t, m_{pool}^{\theta^*})\right), t\right)f(t)dt$$

Taking the derivative of receiver's welfare with respect to θ^*,

$$\frac{dW_R(\theta^*)}{d\theta^*} = \left[U\left(\phi^R\left(x_R^*(\theta^*), x_S^*(\theta^*)\right), \theta^*\right) - U\left(\phi^R\left(x_R^*(m_{pool}^{\theta^*}), x_S^*(\theta^*, m_{pool}^{\theta^*})\right), \theta^*\right)\right]f(\theta^*) > 0$$

for any $\theta^* \leq \bar{\theta}$ since $\phi^R\left(x_R^*(\theta^*), x_S^*(\theta^*)\right) = \bar{\phi}_{\theta^*}^R$,the first best levels of contribution. Further, there is a discontinuous jump at θ^* following a pooling message, implying that

$$\left|\phi^R\left(x_R^*(\theta^*), x_S^*(\theta^*)\right) - \phi^R\left(x_R^*(m_{pool}^{\theta^*}), x_S^*(\theta^*, m_{pool}^{\theta^*})\right)\right| > 0 \text{ at } \theta^*.$$

Sender's ex-ante utility:

Take any two cutoff equilibria $\theta^1, \theta^2 \leq \bar{\theta}$, call them $PRTE_1$ and $PRTE_2$, such that $\theta^1 < \theta^2$ (wlog). Let the corresponding pooling messages associated with the PRTE be $m_{pool}^1 = (\theta^1, 1]$ and $m_{pool}^2 = (\theta^2, 1]$ respectively. I will establish that sender is better off with the more informative equilibrium θ^2. Similar to arguments made in Lemma 2, for cutoff equilibria θ^1, θ^2 there exists a corresponding θ_{sim}^1 and θ_{sim}^2 such that $x_S^*(\theta_{sim}^1, m_{pool}^1) = x_S^*(\theta_{sim}^2, m_{pool}^2) = \bar{k}$.
From SC property, the receiver's action must be higher for the pooling message m_{pool}^2 corresponding to the threshold θ^2, i.e. $x_R^*(m_{pool}^2) > x_R^*(m_{pool}^1)$. If this is true, then $\theta_{sim}^1 < \theta_{sim}^2$. Suppose not, and $\theta_{sim}^1 > \theta_{sim}^2$. Then, $x_S^*(\theta_{sim}^2, m_{pool}^1) < x_S^*(\theta_{sim}^1, m_{pool}^1) = \bar{k}$. But $x_S^*(\theta_{sim}^2, m_{pool}^1) \geq x_S^*(\theta_{sim}^2, m_{pool}^2) = \bar{k}$. This is a contradiction. Therefore the claim holds. In order to prove the result for the sender, I consider two possible scenarios.

Scenario (a): *When* $\theta_{sim}^1 < \theta^2$. That is, $\theta^1 < \theta_{sim}^1 < \theta^2 < \theta_{sim}^2$. The sender's utility under the two PRTE's is given by,

$$W_S(\theta^1) = \int_0^{\theta^1} U\left(\phi^S\left(x_S^*(t), x_R^*(t)\right), t, b\right) f(t)dt + \int_{\theta^1}^{1} U\left(\phi^S\left(x_S^*(t, m_{pool}^1), x_R^*(m_{pool}^1)\right), t, b\right) f(t)dt$$

$$W_S(\theta^2) = \int_0^{\theta^2} U\left(\phi^S\left(x_S^*(t), x_R^*(t)\right), t, b\right) f(t)dt + \int_{\theta^2}^{1} U\left(\phi^S\left(x_S^*(t, m_{pool}^2), x_R^*(m_{pool}^2)\right), t, b\right) f(t)dt$$

Under $PRTE_1$ the sender's equilibrium action is within the bound for the interval $(0, \theta_{sim}^1]$. Since $\theta_{sim}^1 < \theta_{sim}^2$, the sender's action is also within the bound over the interval $(0, \theta_{sim}^1$ under $PRTE_2$. Therefore, what is left to be checked are those states in which the resource constraints are binding for the sender. In $PRTE_1$, this corresponds to the interval $(\theta_{sim}^1, 1]$. On the same interval, I compare the utility (ex-ante) achieved under $PRTE_2$. I will refer to this utility as the residual welfare that results from inefficiency, $W_S^{RES}(\theta^1)$ and $W_S^{RES}(\theta^1)$.

$$W_S^{RES}(\theta^1) = \int_{\theta^1_{sim}}^{\theta^2_{sim}} U\left(\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right), t, b\right) f(t)dt + \int_{\theta^2_{sim}}^{1} U\left(\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right), t, b\right) f(t)dt$$

$$W_S^{RES}(\theta^2) = \int_{\theta^1_{sim}}^{\theta^2} U\left(\phi^S\left(x_S^*(t), x_R^*(t)\right), t, b\right) f(t)dt + \int_{\theta^2}^{\theta^2_{sim}} U\left(\phi^S\left(x_S^*(t, m^2_{pool}), x_R^*(m^2_{pool})\right), t, b\right) f(t)dt$$
$$+ \int_{\theta^2_{sim}}^{1} U\left(\phi^S\left(\bar{k}, x_R^*(m^2_{pool})\right), t, b\right) f(t)dt$$

Taking the expression $W_S^{RES}(\theta^1)$ and expanding the first term, we get,

$$\int_{\theta^1_{sim}}^{\theta^2} U\left(\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right), t, b\right) f(t)dt + \int_{\theta^2}^{\theta^2_{sim}} U\left(\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right), t, b\right) f(t)dt$$

Comparing the above expression with the first two terms of $W_S^{RES}(\theta^2)$,

$$\int_{\theta^1_{sim}}^{\theta^2} U\left(\phi^S\left(x_S^*(t), x_R^*(t)\right), t, b\right) f(t)dt + \int_{\theta^2}^{\theta^2_{sim}} U\left(\phi^S\left(x_S^*(t, m^2_{pool}), x_R^*(m^2_{pool})\right), t, b\right) f(t)dt >$$
$$\int_{\theta^1_{sim}}^{\theta^2} U\left(\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right), t, b\right) f(t)dt + \int_{\theta^2}^{\theta^2_{sim}} U\left(\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right), t, b\right) f(t)dt$$

This follows from pair-wise comparison of the terms,

$$\int_{\theta^1_{sim}}^{\theta^2} U\left(\phi^S\left(x_S^*(t), x_R^*(t)\right), t, b\right) f(t)dt > \int_{\theta^1_{sim}}^{\theta^2} U\left(\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right), t, b\right) f(t)dt \tag{5}$$

$$\int_{\theta^2}^{\theta^2_{sim}} U\left(\phi^S\left(x_S^*(t, m^2_{pool}), x_R^*(m^2_{pool})\right), t, b\right) f(t)dt > \int_{\theta^2}^{\theta^2_{sim}} U\left(\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right), t, b\right) f(t)dt \tag{6}$$

Similarly comparing the last term of $W_S^{RES}(\theta^1)$ and $W_S^{RES}(\theta^2)$,

$$\int_{\theta^2_{sim}}^{1} U\left(\phi^S\left(\bar{k}, x_R^*(m^2_{pool})\right), t, b\right) f(t)dt > \int_{\theta^2_{sim}}^{1} U\left(\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right), t, b\right) f(t)dt \tag{7}$$

The inequality 5 follows from noting that on the interval $(\theta^1_{sim}, \theta^2]$, the sender achieves the first best levels of contribution $\bar{\phi}^S_t$ under the higher threshold equilibrium.

$$\forall t \in (\theta^1_{sim}, \theta^2] : U\left(\phi^S\left(x_S^*(t), x_R^*(t)\right), t, b\right) > U\left(\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right), t, b\right)$$

Similarly, inequality 6 is true since on the interval $(\theta^2, \theta^2_{sim}]$, the sender induces the receiver to play a higher action with message m^2_{pool} and changes her action to achieve first best contributions $\bar{\phi}^S_t$.

$$\forall t \in (\theta^2, \theta^2_{sim}] : U\left(\phi^S\left(x_S^*(t, m^2_{pool}), x_R^*(m^2_{pool})\right), t, b\right) > U\left(\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right), t, b\right)$$

The last inequality 7 follows from noting that since $x_R^*(m^1_{pool}) < x_R^*(m^2_{pool})$, it is valid that $\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right) < \phi^S\left(\bar{k}, x_R^*(m^2_{pool})\right)$ and because there is a positive spillover at the bound for the sender, i.e. $U_1 \mid_{t \in (\theta^2_{sim}, 1]} > 0$,

$$\forall t \in (\theta^2_{sim}, 1] : U\left(\phi^S\left(\bar{k}, x_R^*(m^2_{pool})\right), t, b\right) > U\left(\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right), t, b\right)$$

Comparing the terms pairwise therefore yields the required result, $W_S^{RES}(\theta^2) > W_S^{RES}(\theta^1)$.

Scenario (b): *When* $\theta^1_{sim} > \theta^2$. That is, $\theta^1 < \theta^2 < \theta^1_{sim} < \theta^2_{sim}$.
In this case, as earlier, I will look at states in which there is inefficiency generated by information pooling and compare the residual welfare.

$$W_S^{RES}(\theta^1) = \int_{\theta^1_{sim}}^{\theta^2_{sim}} U\left(\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right), t, b\right) f(t)dt + \int_{\theta^2_{sim}}^{1} U\left(\phi^S\left(\bar{k}, x_R^*(m^1_{pool})\right), t, b\right) f(t)dt$$

$$W_S^{RES}(\theta^2) = \int_{\theta^1_{sim}}^{\theta^2_{sim}} U\left(\phi^S\left(x_S^*(t, m^2_{pool}), x_R^*(m^2_{pool})\right), t, b\right) f(t)dt + \int_{\theta^2_{sim}}^{1} U\left(\phi^S\left(\bar{k}, x_R^*(m^2_{pool})\right), t, b\right) f(t)dt$$

Pairwise comparison yields,

$$\int_{\theta^1_{sim}}^{\theta^2_{sim}} U\left(\phi^S\left(x^*_S(t,m^2_{pool}),x^*_R(m^2_{pool})\right),t,b\right)f(t)dt > \int_{\theta^1_{sim}}^{\theta^2_{sim}} U\left(\phi^S\left(\bar{k},x^*_R(m^1_{pool})\right),t,b\right)f(t)dt \tag{8}$$

$$\int_{\theta^2_{sim}}^{1} U\left(\phi^S\left(\bar{k},x^*_R(m^2_{pool})\right),t,b\right)f(t)dt > \int_{\theta^2_{sim}}^{1} U\left(\phi^S\left(\bar{k},x^*_R(m^1_{pool})\right),t,b\right)f(t)dt \tag{9}$$

The inequalities 8 and 9 follow from arguments made earlier. Specifically, on $(\theta^1_{sim},\theta^2_{sim}]$ the sender is able to achieve $\bar{\phi}^S_t$ with the cutoff equilibrium θ^2 and is therefore strictly better off compared to the equilibrium threshold θ^1. In the interval $(\theta^2_{sim},1]$, there is inefficiency (under-provision) in that $\phi^S(.) < \bar{\phi}^S_t$. However, since the sender induces a higher action from the receiver under θ^2 equilibrium, $x^*_R(m^2_{pool}) > x^*_R(m^1_{pool})$, it follows that $\phi^S\left(\bar{k},x^*_R(m^1_{pool})\right) < \phi^S\left(\bar{k},x^*_R(m^2_{pool})\right) < \bar{\phi}^S_t$ and given $U_1 > 0$ on this interval,

$$\forall t \in (\theta^2_{sim},1] : U\left(\phi^S\left(\bar{k},x^*_R(m^2_{pool})\right),t,b\right) > U\left(\phi^S\left(\bar{k},x^*_R(m^1_{pool})\right),t,b\right)$$

Therefore, $W^{RES}_S(\theta^2) > W^{RES}_S(\theta^1)$. This completes the proof. **QED**

A.7 Proof of Proposition 5

Sender's ex-post efficiency:

For the sender, I will show ex-post efficiency by making pairwise comparison between two thresholds $\bar{\theta}$ and any $\theta' < \bar{\theta}$ (wlog). From Proposition 4, we know that $\theta'_{sim} < \bar{\theta}_{sim}$. As before, there are two scenarios to consider.

Scenario (a): $\theta' < \theta'_{sim} < \bar{\theta} < \bar{\theta}_{sim}$.
In this case, every type $\theta \in [0,\theta'_{sim}]$, the sender is indifferent between the two threshold equilibria, since she achieves first best $\bar{\phi}^S_\theta$ on this interval under either equilibria. However, every $\theta \in (\theta'_{sim},1]$ strictly prefers the $\bar{\theta}$ threshold equilibrium. To see this, let us further divide the interval to $(\theta'_{sim},\bar{\theta}_{sim}]$ and $(\bar{\theta}_{sim},1]$. Now, every $\theta \in (\theta'_{sim},\bar{\theta}_{sim}]$ prefers the threshold $\bar{\theta}$ since $x^*_S(\theta,\bar{m}_{pool}) \leq \bar{k}$, which implies that $\bar{\phi}^S\left(x^*_S(\theta,\bar{m}_{pool}),x^*_R(\bar{m}_{pool})\right) = \bar{\phi}^S_\theta$, whereas with threshold θ', $x^*_S(\theta,m'_{pool}) = \bar{k}$. Therefore, $U\left(\phi^S\left(x^*_S(t,\bar{m}_{pool}),x^*_R(\bar{m}_{pool})\right),t,b\right) > U\left(\phi^S\left(\bar{k},x^*_R(m'_{pool})\right),t,b\right)$.
Lastly, for types $\theta \in (\bar{\theta}_{sim},1]$, the sender's action is bounded by the resource constraint $\bar{k}$. However, since $\bar{m}_{pool}$ induces a higher action from the receiver ($x^*_R(\bar{m}_{pool}) > x^*_R(m'_{pool})$), this

means that $\phi^S\left(\bar{k}, x_R^*(\bar{m}_{pool})\right) > \phi^S\left(\bar{k}, x_R^*(m_{pool}^{'})\right)$. Since the resource constraints are binding, $U_1 > 0$ and from the positive spillover property, it follows that $U\left(\phi^S\left(\bar{k}, x_R^*(\bar{m}_{pool})\right), t, b\right) > U\left(\phi^S\left(\bar{k}, x_R^*(m_{pool}^{'})\right), t, b\right)$ for all $\theta \in (\bar{\theta}_{sim}, 1]$.

Scenario (b): $\theta' < \bar{\theta} < \theta'_{sim} < \bar{\theta}_{sim}$.
A analogous set of arguments hold true for this case. In particular, every type $\theta \in [0, \theta'_{sim}]$ is indifferent between the thresholds $\bar{\theta}$ and θ'. Every type $\theta \in (\theta'_{sim}, \bar{\theta}_{sim}]$ is strictly better off under threshold $\bar{\theta}$ because the resource constraints are not binding in this interval and therefore the sender is able to achieve first best $\bar{\phi}_\theta^S$. Types $\theta \in (\bar{\theta}_{sim}, 1]$ are also strictly better off under threshold $\bar{\theta}$ because of the positive spillover argument made earlier. This completes the proof.

Receiver's ex-post efficiency

For the receiver, pick any two thresholds as before, (θ^1, θ^2) such that $\theta^1 \leq \theta^2 \leq \bar{\theta}$. Let m_{pool}^1 and m_{pool}^2 be the respective pooling messages and, θ_{sim}^1 and θ_{sim}^2 be the corresponding cutoffs (see proof of Proposition 5) such that the sender's action is bounded by the resource constraint. Now consider the interval $(\theta_{sim}^1, 1]$. I make the following claim:

$$\exists \theta_R^1 \in (\theta_{sim}^1, 1] : x_R^*(m_{pool}^1) \equiv \underset{x_R \in V}{\operatorname{argmax}}\, U\left(\phi^R\left(x_R, \bar{k}\right), \theta_R^1\right) \implies \phi^R\left(x_R^*(m_{pool}^1), \bar{k}\right) = \bar{\phi}_{\theta_R^1}^R$$

Suppose not. Then, we know that at θ_{sim}^1, $\phi^R\left(x_R^*(m_{pool}^1), \bar{k}\right) > \bar{\phi}_{\theta_{sim}^1}^R$ implying that $U_1 \mid_{\theta \in (\theta^1, \theta_{sim}^1]} < 0$ for the receiver. If $\phi^R\left(x_R^*(m_{pool}^1), \bar{k}\right) \geq \bar{\phi}_1^R$, then it means that the marginal utility of the receiver from the action $x_R^*(m_{pool}^1)$ is less than zero. That is, it cannot be an equilibrium action, which is a contradiction. On the other hand, if $\phi^R\left(x_R^*(m_{pool}^1), \bar{k}\right) < \bar{\phi}_1^R$, then by continuity, there must exist a type t such that $\phi^R\left(x_R^*(m_{pool}^1), \bar{k}\right) = \bar{\phi}_t^R$. Therefore, the claim holds.
If there is such a θ_R^1 under the threshold equilibrium θ^1, there must exist one similarly, θ_R^2 corresponding to the threshold θ^2. Further, since $x_R^*(m_{pool}^1) < x_R^*(m_{pool}^2)$, it holds that $\phi^R\left(x_R^*(m_{pool}^1), \bar{k}\right) < \phi^R\left(x_R^*(m_{pool}^2), \bar{k}\right) \implies \theta_R^1 < \theta_R^2$. Finally, notice that at $\theta = \theta_R^1$,

$$U\left(\phi^R\left(x_R^*(m_{pool}^1), \bar{k}\right), \theta_R^1\right) > U\left(\phi^R\left(x_R^*(m_{pool}^2), \bar{k}\right), \theta_R^1\right)$$

Therefore the more informative threshold θ^2 is not ex post efficient. **QED**

A.8 Proof of Proposition 6

Consider the following set of messaging strategies, beliefs and action profiles under the sequential protocol:

1. If $\theta \leq \theta^*$, $m = \theta$; if $\theta > \theta^*$, $m = 1$.
2. If $m \leq \theta^*$, $p(\theta \mid m = \theta) = 1$; if $m = 1$, $p(\theta \mid m) = f(\theta)$
3. When $m \leq \theta^*$: $x_R^*(m) = \tilde{x}_R(m)$ and $x_S^*(\theta, x_R^*(m)) = \underset{x_S \in V}{\text{argmax}}\, U\left(\phi^S\left(x_S, x_R^*(m)\right), \theta, b\right) \equiv \tilde{x}_S(m)$
4. When $m = 1$:
   - $x_R^*(m) \equiv \arg\max_{x_R \in V} \int_{\theta^*}^{1} U\left(\phi^R(x_R, x_S^*(\theta, x_R)), \theta\right) f(\theta) d\theta$
   - $x_S^*(\theta, x_R^*(m)) \equiv \arg\max_{x_S \in V} U\left(\phi^S(x_S, x_R^*(m)), \theta, b\right)$
5. When $m \in (\theta^*, 1)$: $p(\theta^* | m) = 1$.

Notice that the main point of departure from the simultaneous protocol arises from the sender's equilibrium response $x_S^*(\theta, x_R^*(m))$ that takes into account the receiver's action in the second stage, post communication. Clearly, on the interval of separation $[0, \theta^*]$ the receiver can do no better than play $\tilde{x}_S(\theta)$. This is driven by the concavity of $U(.)$ in that there is an unique $\bar{\phi}_\theta^R$ for every θ and this corresponds to the pair of actions $(\tilde{x}_S(\theta), \tilde{x}_R(\theta))$. Now, on the pooling interval the receiver takes into account that the sender can now observe her actions and best respond to them. Jointly, $(x_S^*(\theta, x_R^*(m)), x_R^*(m)))$ must maximize the expected payoffs of the player.

Finally, I check to see if the sender would want to deviate from the equilibrium messaging strategy. Suppose the sender deviates and sends an out-of-equilibrium message $m \in (\theta^*, 1)$. Then, the receiver assigns the belief that it comes from the type θ^* and plays the corresponding action $x_R^*(m) = \tilde{x}_R(\theta^*)$. The sender types $m_{pool}^* = (\theta^*, 1]$ are at least as better off sending the pooling message $m = 1$. To see this, if $x_R^*(m) = \tilde{x}_R(\theta^*)$, then there exists a threshold, say $\theta_{out} \leq \bar{\theta}$, such that $\phi^S\left(\bar{k}, \tilde{x}_R(\theta^*)\right) = \bar{\phi}_{\theta_{out}}^S$. This is true since $\phi^S\left(\bar{k}, \tilde{x}_R(\bar{\theta})\right) = \bar{\phi}_{\bar{\theta}}^S$ and by continuity there should exist such a type θ_{out}. Given this, every type in $(\theta^*, \theta_{out}]$ cannot do any better from deviating, since under the pooling message, they induce a higher action from the receiver. This means every type $t \in (\theta^*, \theta_{out}]$ achieves first best levels $(\bar{\phi}_t^S)$ under the pooling message. That is, $\phi^S\left(x_S^*(t, x_R^*(m_{pool}^*)), x_R^*(m_{pool}^*)\right) = \phi^S\left(\bar{k}, x_R^*(\theta^*)\right) = \bar{\phi}_t^S$. But notice that every type in $(\theta_{out}, 1]$ would prefer sending the message $m = 1$ instead of the out-of-equilibrium one. This is driven by the under-provision concerns that manifest as a result of the resource constraints. More specifically,

$$\forall \theta \in (\theta_{out}, 1] : \phi^S\left(\bar{k}, x_R^*(\theta^*)\right) < \phi^S\left(x_S^*(\theta, x_R^*(m_{pool}^*)), x_R^*(m_{pool}^*)\right) \leq \bar{\phi}_{\bar{\theta}}^S$$

Therefore, all sender types in $(\theta_{out}, 1]$ are strictly worse off by deviating and the types $\bar{\phi}_t^S$ are indifferent between deviating and playing the equilibrium strategy. Given that the sender can always induce a higher action from the receiver by sending the pooling message and subsequently moderate her own actions implies that the sender types can never do better by deviating, therefore precluding any deviation. This completes the proof. **QED**

A.9 Proof of Proposition 7

I will continue to focus on the efficient equilibrium ($\bar{\theta}$) under the two protocols. Again, on the separating interval $[0, \bar{\theta}]$, both the protocols provide the same ex ante welfare to both the sender and receiver. So it is sufficient to focus on the pooling interval, henceforth $m_{pool} = (\bar{\theta}, 1]$. Let the receiver's action after m_{pool} be x_R^{sim} and x_R^{seq} under simultaneous and sequential protocols respectively. To compare equilibrium welfare, it is essential to prove Lemma 3.

Lemma 3: $x_R^{seq} > x_R^{sim}$

Under simultaneous protocol, the receiver's equilibrium response x_R^{sim} is given by the following FOC (from Lemma 2),

$$\int_{\bar{\theta}}^{\bar{\theta}_{sim}(x_R^{sim})} U_1\left(\phi^R\left(x_R^{sim}, x_S^{sim}(t, m_{pool})\right), t\right)\phi_1^R dF + \int_{\bar{\theta}_{sim}(x_R^{sim})}^{1} U_1\left(\phi^R\left(x_R^{sim}, \bar{k}\right), t\right)\phi_1^R dF = 0 \quad (10)$$

The the receiver's equilibrium action under the sequential protocol is given by the FOC from differentiating equation 3. That is, x_R^{seq} solves,

$$\int_{\bar{\theta}}^{\bar{\theta}_{seq}(x_R)} U_1\left(\phi^R\left(x_R, x_S^{seq}(t, x_R)\right), t\right) \cdot \left[\phi_1^R + \phi_2^R . \frac{dx_S}{dx_R}\right] dF + \int_{\bar{\theta}_{seq}(x_R)}^{1} U_1\left(\phi^R\left(x_R, \bar{k}\right), t\right)\phi_1^R dF = 0 \quad (11)$$

Evaluating the above equation 11 at x_R^{sim} gives,

$$\int_{\bar{\theta}}^{\bar{\theta}_{seq}(x_R^{sim})} U_1\left(\phi^R\left(x_R^{sim}, x_S^{seq}(t, x_R^{sim})\right), t\right) \cdot \left[\phi_1^R + \phi_2^R . \frac{dx_S}{dx_R}\right]\Bigg|_{x_R = x_R^{sim}} dF + \int_{\bar{\theta}_{seq}(x_R^{sim})}^{1} U_1\left(\phi^R\left(x_R^{sim}, \bar{k}\right), t\right) \phi_1^R dF \tag{12}$$

But at $x_R = x_R^{sim}$, it holds that $\bar{\theta}_{seq}(x_R^{sim}) = \bar{\theta}_{sim}(x_R^{sim})$ and $x_S^{seq}(t, x_R^{sim}) = x_S^{sim}(t, m_{pool})$. The second expression follows from the fact that the sender's equilibrium action mimics the simultaneous protocol action $x_S^{sim}(t, m_{pool})$ as there is an unique type θ for which $\phi^S(\bar{k}, x_R^{sim}) = \bar{\phi}_\theta$. Further, this implies that when $x_R = x_R^{sim}$ under the sequential protocol, the cutoff after which the sender always contributes $\bar{k}$ corresponds with $\bar{\theta}_{sim}(x_R^{sim})$, resulting in the first equality. Substituting these expressions into equation 12 and rearranging gives,

$$\int_{\bar{\theta}}^{\bar{\theta}_{sim}(x_R^{sim})} U_1\left(\phi^R\left(x_R^{sim}, x_S^{seq}(t, x_R^{sim})\right), t\right) \phi_1^R dF + \int_{\bar{\theta}_{sim}(x_R^{sim})}^{1} U_1\left(\phi^R\left(x_R^{sim}, \bar{k}\right), t\right) \phi_1^R dF$$
$$+ \int_{\bar{\theta}}^{\bar{\theta}_{sim}(x_R^{sim})} U_1\left(\phi^R\left(x_R^{sim}, x_S^{seq}(t, x_R^{sim})\right), t\right) .\phi_2^R . \frac{dx_S^{seq}}{dx_R}\Bigg|_{x_R = x_R^{sim}} dF$$

However, the first two expressions are equal to the LHS of equation 10, and therefore equal to zero. The only expression left is the last one given by,

$$\int_{\bar{\theta}}^{\bar{\theta}_{sim}(x_R^{sim})} U_1\left(\phi^R\left(x_R^{sim}, x_S^{seq}(t, x_R^{sim})\right), t\right) .\phi_2^R . \frac{dx_S^{seq}}{dx_R}\Bigg|_{x_R = x_R^{sim}} dF$$

Notice that $U_1\left(\phi^R\left(x_R^{sim}, x_S^{seq}(t, x_R^{sim})\right), t\right) < 0$ for the receiver on this interval since the sender always contributes moderates her action in order to achieve first best $\bar{\phi}_t^S$, but this results in over-provision for the receiver, $\phi^R\left(x_R^{sim}, x_S^{seq}(t, x_R^{sim})\right) > \bar{\phi}_t^R$. Given Assumption 2, $\phi_2^R > 0$ and from Assumption 3, $\frac{dx_S}{dx_R} < 0$ implying that the above integral is always positive.

$$\int_{\bar{\theta}}^{\bar{\theta}_{sim}(x_R^{sim})} U_1\left(\phi^R\left(x_R^{sim}, x_S^{seq}(t, x_R^{sim})\right), t\right) .\phi_2^R . \frac{dx_S^{seq}}{dx_R}\Bigg|_{x_R = x_R^{sim}} dF > 0 \tag{13}$$

Since the expected utility for the receiver in the sequential protocol is increasing at $x_R = x_R^{sim}$ and $U_{11} < 0$, it follows that $x_R^{seq} > x_R^{sim}$. This completes the proof of the lemma.

Given $x_R^{seq} > x_R^{sim}$, it is straightforward to see that equilibrium welfare is higher under the sequential protocol. By mimicking x_R^{sim}, the receiver's expected utility is the same as in the simultaneous protocol, on the pooling interval. However, from equation 13, we have established that the expected utility is increasing at $x_R = x_R^{sim}$. More formally, the following equations hold:

$$\mathbb{E}_\theta \left[U \left(\phi^R \left(x_R, x_S^{seq}(\theta, x_R) \right), \theta \right) \right] \Big|_{x_R = x_R^{sim}} = \mathbb{E}_\theta \left[U \left(\phi^R \left(x_R^{sim}, x_S^{sim}(\theta, m_{pool}) \right), \theta \right) \right]$$

$$\left. \frac{d\mathbb{E}_\theta \left[U \left(\phi^R \left(x_R, x_S^{seq}(\theta, x_R) \right), \theta \right) \right]}{dx_R} \right|_{x_R = x_R^{sim}} > 0$$

These above two equations guarantee that the receiver, by mimicking the simultaneous protocol action can guarantee an expected payoff equal to that under the simultaneous protocol and therefore does better by increasing her action such that $x_R^{seq} = x_R^{sim}$.

For the sender, $x_R^{seq} > x_R^{sim}$ implies that $\bar{\theta}_{seq} > \bar{\theta}_{sim}$. That is, for a greater measure of types on the pooling interval, the sender's resource constraint is not binding, $\forall t \in (\bar{\theta}, \bar{\theta}_{seq}] : x_S^{seq}(t, x_R^{seq}) \leq \bar{k} \implies \phi^S \left(x_S^{seq}(t, x_R^{seq}), x_R^{seq} \right) = \bar{\phi}_t^S$. As before, I will write down the residual welfare on the interval $(\bar{\theta}_{sim}, 1]$ under both protocols.

$$W_S^{sim}(\bar{\theta}) = \int_{\bar{\theta}_{sim}}^{\bar{\theta}_{seq}} U \left(\phi^S \left(\bar{k}, x_R^{sim} \right), t, b \right) f(t)dt + \int_{\bar{\theta}_{seq}}^{1} U \left(\phi^S \left(\bar{k}, x_R^{sim} \right), t, b \right) f(t)dt$$

$$W_S^{seq}(\bar{\theta}) = \int_{\bar{\theta}_{sim}}^{\bar{\theta}_{seq}} U \left(\bar{\phi}_t^S, t, b \right) f(t)dt + \int_{\bar{\theta}_{seq}}^{1} U \left(\phi^S \left(\bar{k}, x_R^{seq} \right), t, b \right) f(t)dt$$

Pairwise comparison yields,

$$\int_{\bar{\theta}_{sim}}^{\bar{\theta}_{seq}} U \left(\bar{\phi}_t^S, t, b \right) f(t)dt > \int_{\bar{\theta}_{sim}}^{\bar{\theta}_{seq}} U \left(\phi^S \left(\bar{k}, x_R^{sim} \right), t, b \right) f(t)dt \tag{14}$$

$$\int_{\bar{\theta}_{seq}}^{1} U \left(\phi^S \left(\bar{k}, x_R^{seq} \right), t, b \right) f(t)dt > \int_{\bar{\theta}_{seq}}^{1} U \left(\phi^S \left(\bar{k}, x_R^{sim} \right), t, b \right) f(t)dt \tag{15}$$

Clearly, equation 14 follows from the fact that the sender achieves the unique maximum level of contribution $\bar{\phi}_t^S$ under the sequential protocol on the interval $(\bar{\theta}_{sim}, \bar{\theta}_{seq}]$ and therefore cannot do better. Equation 15 holds because on the interval where there is under-provision, i.e. $(\bar{\theta}_{seq}, 1]$, $\phi^S(\bar{k}, x_R) < \bar{\phi}_t^S \implies U_1(.) > 0$ for the sender. Since $x_R^{seq} > x_R^{sim}$, from Assumption 2 it follows that the sender is better off under the sequential protocol for all types in $(\bar{\theta}_{seq}, 1]$. Therefore, $W_S^{seq}(\bar{\theta}) > W_S^{sim}(\bar{\theta})$. This completes the proof. **QED**

A.10 Proof of Proposition 8

The key to proving this is to look at the multiple pairs of contributions that achieve the first best for the sender, in order to satisfy her IC constraint. Given Assumption 1 and Assumption 2, for any $\theta \in [0,1]$, there are different contribution pairs (x_R, x_S) such that $\phi^S(x_S, x_R) = \bar{\phi}_\theta^S$. I proceed by constructing the set of ϕ^R that corresponds with all admissible pairs (x_R, x_S) such that for any θ, $\phi^S(x_S, x_R) = \bar{\phi}_\theta^S$. The following defines this admissible set:

$$\forall \theta \in [0,1], (x_S, x_R) \in V : \mathcal{A}_\theta = \left\{ \phi^R(x_R, x_S) : \phi^S(x_S, x_R) = \bar{\phi}_\theta^S \right\}$$

Therefore, the commitment rule for the receiver becomes one of choosing an appropriate pair from $\mathcal{A}_\theta$ such that it maximizes the expected utility of the receiver.

Claim 1: From previous arguments, on the interval $[0, \bar{\theta}]$ the incentive compatible decision rule that maximizes the receiver's expected utility is the one that mimics the unconstrained action $\tilde{x}_R(\theta)$. Specifically, the contribution pair $(\tilde{x}_R(\theta), \tilde{x}_S(\theta))$ is such that $\phi^R(\tilde{x}_R(\theta), \tilde{x}_S(\theta)) \in \mathcal{A}_\theta$ and $\phi^R(\tilde{x}_R(\theta), \tilde{x}_S(\theta)) = \bar{\phi}_\theta^R \equiv \underset{\phi^R}{\operatorname{argmax}}\, U(\phi^R, \theta)$. This proves Claim 1.

To show claims 2,3 and 4, I will impose further structure on the set $\mathcal{A}_\theta$ for the interval m_{pool}. From continuity property of $\phi^R(.)$ and $\phi^S(.)$, the set $\mathcal{A}_\theta$ is compact. Further, let $\sup \mathcal{A}_\theta = \phi_{sup}^R(\theta)$ and $\inf \mathcal{A}_\theta = \phi_{inf}^R(\theta)$.

Definition 3 *Let $x_R^{inf}(\theta)$ be such that $\phi^S\left(\bar{k}, x_R^{inf}(\theta)\right) = \bar{\phi}_\theta^S$.*

Lemma 4 $\forall \theta \in m_{pool} : \phi^R(x_R^{inf}(\theta), \bar{k}) = \phi_{inf}^R(\theta)$

Proof. Note that x_S varies from $\underline{k}$ to $\bar{k}$ and x_R is just the residual contribution that ensures $\phi^S(.) = \bar{\phi}_\theta^S$. Applying total differentiation to ϕ^S, we get the following:

$$d\phi^S = \frac{\partial \phi^S}{\partial x_S}.dx_S + \frac{\partial \phi^S}{\partial x_R}.dx_R$$

Since $\phi^S(.) = \bar{\phi}^S_\theta$, a constant in $\mathcal{A}_\theta$, $d\phi^S = 0$. Substituting this in the above equation and rearranging,

$$\left|\frac{dx_R}{dx_S}\right| = \frac{\frac{\partial\phi^S}{\partial x_S}}{\frac{\partial\phi^S}{\partial x_R}} > 1$$

Similarly,

$$d\phi^R = \frac{\partial\phi^R}{\partial x_S}.dx_S + \frac{\partial\phi^R}{\partial x_R}.dx_R$$

$$\frac{d\phi^R}{dx_S} = \frac{\partial\phi^R}{\partial x_S} + \frac{\partial\phi^R}{\partial x_R}.\frac{dx_R}{dx_S} = \frac{\partial\phi^R}{\partial x_S} - \left|\frac{dx_R}{dx_S}\right|.\frac{\partial\phi^R}{\partial x_R} \tag{16}$$

$$\implies \frac{d\phi^R}{dx_S} < \left[\frac{\partial\phi^R}{\partial x_R} - \left|\frac{dx_R}{dx_S}\right|.\frac{\partial\phi^R}{\partial x_R}\right] = \frac{\partial\phi^R}{\partial x_R}.\left[1 - \left|\frac{dx_R}{dx_S}\right|\right] < 0 \tag{17}$$

Equation 17 follows from imperfect substitutability in that $\frac{\partial\phi^R}{\partial x_R} > \frac{\partial\phi^R}{\partial x_S}$. Lemma 4 establishes that ϕ^R is decreasing in the contribution of the sender. This implies that the infimum of the set $\mathcal{A}_\theta$ corresponds with the pair in which the sender contributes all her resources $\bar{k}$ and the receiver, the residual $x_R^{inf}(\theta)$. ■

Lemma 5 $\forall\theta \in m_{pool} : \phi^R_{inf}(\theta) > \bar{\phi}^R_\theta$

Proof. From lemma 4 it is clear there is a ordering over ϕ^R. Specifically, $\phi^R_{sup}(\theta) > ...... > \phi^R_{inf}(\theta)$. Suppose $\phi^R_{inf}(\theta) > \bar{\phi}^R_\theta$ were not true. Then, either $\phi^R_{sup}(\theta) > ..... > \bar{\phi}^R_\theta > ... > \phi^R_{inf}(\theta)$ or $\bar{\phi}^R_\theta > \phi^R_{sup}(\theta) > ..... > \phi^R_{inf}(\theta)$. If the former was true, then the receiver can achieve first best by revealing the state θ under truthful communication. That is, the sender could have revealed truthfully up to some higher threshold $\bar{\theta}$, which violates the most informative threshold equilibrium $\bar{\theta}$. The latter cannot be true because of the imperfect substitutability assumption and a positive conflict of interest. Therefore it must hold that $\phi^R_{sup}(\theta) > ...... > \phi^R_{inf}(\theta) > \bar{\phi}^R_\theta$. ■

From Lemma 5, it is clear that on the interval m_{pool}, there is over-provision for the receiver as long as the sender achieves her first best levels of contribution. However, this implies that $U_1 < 0$ for the receiver and therefore, the following holds:

$$\forall\theta \in m_{pool} : \phi^R_{inf}(\theta) \equiv \underset{\phi^R \in \mathcal{A}_\theta}{\operatorname{argmax}}\, U(\phi^R, \theta) \tag{18}$$

That is, of all contribution pairs (x_R, x_S) that satisfy the sender's IC constraint, the ones that maximize the receiver's utility is the one that minimizes the over-provision, which coincides with $x_S = \bar{k}$. I proceed now to prove Claim 2,3 and 4.

Claim 2: Suppose the claim weren't true and say the receiver, wlog, takes an action $x^c_R(\theta) =$

$z, \forall\theta \in m_{pool}$. There are two possible cases to consider.

Case i) $z = x_R^c(\bar{\theta}) = \tilde{x}_R(\bar{\theta})$

In this case, the sender contributes $x_S = \bar{k}$ for every possible type in m_{pool}. If this is so, then $\forall\theta \in m_{pool} : \phi^R(\tilde{x}_R(\bar{\theta}), \bar{k}) = \bar{\phi}^R_{\bar{\theta}} < \bar{\phi}^R_{\theta}$. This implies that the expected marginal utility of the receiver is less than zero and given $U_{11} < 0$, there is an incentive for the receiver to increase her contribution. Therefore, $z \neq x_R^c(\bar{\theta})$.

Case ii) $z > x_R^c(\bar{\theta})$

If this were true, then there exists some types such that the sender contributes less than $\bar{k}$ and still achieves first best. That is,

$$\exists T \subset m_{pool}, \forall t \in T : x_S^c(t, z) < \bar{k}$$

Such a set T must exist from the continuity property of $U(.)$ and $\phi^i(.)$. Specifically, when $z > x_R^c(\bar{\theta})$, then there is always a cutoff type θ_z (from Lemma 2) such that $x_S^c(\theta_z, z) = \bar{k}$. However, this implies that for all types $t \in (\bar{\theta}, \theta_z)$, it must be that $x_S^c(t, z) < \bar{k}$. That is $T = (\bar{\theta}, \theta_z)$ exists. But if this set exists, then the receiver is not maximizing her expected utility since she can always reduce her contribution and make the sender contribute more resources $\bar{k}$. To see this, consider the following alternate decision rule:

$$\forall t \in T : x_R^c(t) = x_R^{inf}(t) \text{ such that } \phi^R(x_R^{inf}(t), \bar{k}) = \phi^R_{inf}(t) \in \mathcal{A}_t$$

$$\forall t \in m_{pool} \setminus T : x_R^c(t) = z$$

Clearly, on the interval subset T, the receiver now achieves a greater expected utility since $\forall t \in T, U\left(\phi^R(x_R^{inf}(t), \bar{k}), t\right) > U\left(\phi^R(z, x_S^c(t, z)), t\right)$. Further, this decision rule is also incentive compatible in that the sender cannot do better by misreporting. Therefore, there cannot be a flat segment on m_{pool} such that the receiver commits to a communication independent decision. This proves Claim 2.

Claim 3: Suppose instead there was a strictly increasing interval $(\theta_1, \theta_2) \in m_{pool}$ such that $\exists t \in (\theta_1, \theta_2) : x_S^c(t, x_R^c(t)) < \bar{k}$. Then, given IC must be satisfied, $\phi^R\left(x_R^c(t), x_S^c(t, x_R^c(t))\right) \in \mathcal{A}_t$. But clearly, from Lemma 4, Lemma 5 and Equation 18 the receiver can always instead choose to contribute $x_R^{inf}(t)$ such that $x_S^c(t, x_R^{inf}(t)) = \bar{k}$. This satisfies IC of the sender since $\phi^R(x_R^{inf}(t), \bar{k}) \in \mathcal{A}_t$ and increases the payoff to the receiver since $U\left(\phi^R(x_R^{inf}(t), \bar{k}), t\right) > U\left(\phi^R(x_R^c(t), x_S^c(t, x_R^c(t))), t\right)$.

This proves Claim 3.

Claim 4: Suppose, instead there exists a flat segment followed by a strictly increasing segment in m_{pool}. Say, wlog, the flat segment is on $(\theta_1, \theta_2]$ such that $\forall t \in (\theta_1, \theta_2] : x_R^c(t) = z$, and let the strictly increasing segment be on (θ_2, θ_3). From Claim 3, it holds that the sender must contribute all her resources on this interval and further, her IC must be satisfied in that $\forall t \in (\theta_2, \theta_3) : \phi^S(\bar{k}, x_R^{inf}(t)) = \bar{\phi}_t^S$. Take the type θ_2. For this type it must be that the IC is satisfied on the flat segment, i.e. $\phi^S(x_S^c(\theta_2, z), z) = \bar{\phi}_{\theta_2}^S$. If not, the sender can always deviate and report $t \in (\theta_2, \theta_3)$ and increase her payoff. This implies that z must be such that $x_S^c(\theta_2, z) = \bar{k}$, since otherwise the receiver is not payoff maximizing, again from previous arguments. But when the sender contributes $\bar{k}$, the receiver's residual contribution must be in the set $\mathcal{A}_{\theta_2}$ and equal to,

$$z = x_R^{inf}(\theta_2) \text{ such that } \phi^R(x_R^{inf}(\theta_2), \bar{k}) = \phi_{inf}^R(\theta_2) \in \mathcal{A}_{\theta_2}$$

But clearly, if $z = x_R^{inf}(\theta_2)$, then for all types $t \in (\theta_1, \theta_2)$, it must also hold that $x_S^c(t, z) < \bar{k}$, from single crossing condition. However, if $x_S^c(t, z) < \bar{k}$, then the receiver can always decrease her contributions on this interval, and extract more resources from the sender whilst satisfying her IC (Lemma 4 and Lemma 5). Therefore, this violates expected utility maximization of the receiver. This proves Claim 4.

Together, the four claims imply the following rules hold:

1. Claim 1 $\implies$ On the separating interval $[0, \bar{\theta}]$, the optimal rule mimics the simultaneous/sequential protocol actions, $\tilde{x}_R(\theta)$.
2. Claim 2 and Claim 3 $\implies$ There is an interval $(\bar{\theta}, \bar{\theta}_c) \in m_{pool}$ in which the receiver's decisions are dependent on communication and given by $x_S^c(\theta) = \bar{k}$ and $x_R^c(\theta) = x_R^{inf}(\theta)$ such that $\phi^S(\bar{k}, x_R^{inf}(\theta)) = \bar{\phi}_\theta^S$ and $\phi^R(x_R^{inf}(\theta), \bar{k}) \in \mathcal{A}_\theta$.
3. Finally, Claim 4 $\implies$ On the interval $[\bar{\theta}_c, 1]$, the receiver's contribution is independent of communication and is equal to $x_R = x_R^{inf}(\bar{\theta}_c) = x_R^c(\bar{\theta}_c)$.

This completes the proof. **QED**

A.11 Proof of Proposition 9

Consider the following commitment strategy. The receiver commits to a decision rule on m_{pool} such that:

$$\forall t \in (\bar{\theta}, \bar{\theta}_{seq}) : x_R^c(t) = x_R^{inf}(t)$$

$$\forall t \in [\bar{\theta}_{seq}, 1] : x_R^c(t) = x_R^{seq}(m_{pool}) \equiv x_R^{seq}$$

This decision rule exactly replicates the sequential protocol in that it provides the sender first best contribution levels $\bar{\phi}_t^S$ on the interval $(\bar{\theta}, \bar{\theta}_{seq}]$. Clearly, this decision rule is IC for the sender and provides the same expected welfare compared to the sequential protocol. Further, on the interval $[\bar{\theta}_{seq}, 1]$, the receiver's welfare is also the same as under the sequential protocol.
However, $\forall t \in (\bar{\theta}, \bar{\theta}_{seq})$, the receiver actually does better since the sender now contributes all her resources on this interval and this minimizes the over-provision for the receiver, as shown in Lemma 4 and Lemma 5 in the proof of Proposition 8. That is,

$$\forall t \in (\bar{\theta}, \bar{\theta}_{seq}) : U\left(\phi^R\left(x_R^{inf}(t), \bar{k}\right), t\right) > U\left(\phi^R\left(x_R^{seq}, x_S(t, x_R^{seq})\right), t\right)$$

Therefore by following a IC commitment rule that is strictly increasing on $(\bar{\theta}, \bar{\theta}_{seq})$ and flat on $[\bar{\theta}_{seq}, 1]$, the receiver achieves a higher ex-ante welfare and the sender is indifferent, compared to the sequential protocol.
Now consider the sequence of contributions $\left\{x_R^{inf}(t)\right\}_{t\in(\bar{\theta},\bar{\theta}_{seq})}$ and checking the marginal utility of the receiver for each type t,

$$U_1\left(\phi^R\left(x_R^{seq}, x_S(t, x_R^{seq})\right), t\right) < U_1\left(\phi^R\left(x_R^{inf}(t), \bar{k}\right), t\right) \tag{19}$$

Equation 19 follows from noting that utility of the receiver is decreasing in ϕ^R on this interval and since $U_{11} < 0$ and $\phi^R\left(x_R^{seq}, x_S(t, x_R^{seq})\right) < \phi^R\left(x_R^{inf}(t), \bar{k}\right) = \phi_{inf}^R(t)$. Now, on the interval $[\bar{\theta}_{seq}, 1]$, since $x_R^c(t) = x_R^{seq}$, the commitment rule provides the same marginal utility as the sequential protocol for the receiver. Summing the marginal utilities under the commitment rule with the sequence $\left\{x_R^{inf}(t)\right\}_{t\in(\bar{\theta},\bar{\theta}_{seq})}$ and $\left\{x_R^{seq}\right\}_{t\in[\bar{\theta}_{seq},1]}$, it is clear that,

$$\int_{\bar{\theta}}^{\bar{\theta}_{seq}} U_1\left(\phi^R\left(x_R^{inf}(t), \bar{k}\right), t\right) . \left[\phi_1^R + \phi_2^R . \frac{dx_S}{dx_R}\right] dF + \int_{\bar{\theta}_{seq}}^{1} U_1\left(\phi^R\left(x_R^{seq}, \bar{k}\right), t\right) \phi_1^R dF \tag{20}$$

Since $x_S^c(t) = \bar{k}$, it follows that $\frac{dx_S}{dx_R} = 0$ under the commitment rule. The above equation simplifies to,

$$\int_{\bar{\theta}}^{\bar{\theta}_{seq}} U_1\left(\phi^R\left(x_R^{inf}(t), \bar{k}\right), t\right) . \phi_1^R dF + \int_{\bar{\theta}_{seq}}^{1} U_1\left(\phi^R\left(x_R^{seq}, \bar{k}\right), t\right) \phi_1^R dF > 0 \tag{21}$$

The above inequality follows from Equation 19. Therefore, the marginal utility of the receiver from following the above commitment rule that satisfies the sender's first best up to $\bar{\theta}_{seq}$ is below zero, and given $U_{11} < 0$, this further implies that under commitment protocol, the receiver can satisfy the sender's first best above this threshold. From Equation 21, it therefore follows that:

$$\bar{\theta}_c > \bar{\theta}_{seq}$$

Receiver's welfare

The increase in receiver's welfare is driven by the fact that the mimicking strategy provided the receiver a higher expected utility under commitment and further, the marginal utility is increasing at $\bar{\theta}^{seq}$ (from Equation 21). Therefore, the receiver's expected utility from a commitment sequence such that $\bar{\theta}_c > \bar{\theta}_{seq}$ is higher compared to the sequential protocol.

Sender's welfare

On the interval $[0, \bar{\theta}_c]$, the sender achieves first best levels of joint contribution in that $\forall t \in [0, \bar{\theta}_c] : \phi^S(.) = \bar{\phi}^S_t$ under the equilibrium commitment rule. Further, on $t \in (\bar{\theta}_c, 1]$, it must be that $x^c_R(t) = x^c_R(\bar{\theta}_c) > x^c_R(\bar{\theta}_{seq})$, since $x^c_R(\bar{\theta}_{seq}) = x^{seq}_R$ and $\bar{\theta}_c > \bar{\theta}_{seq}$. However, on this interval, there is under-provision for the sender ($U_1 > 0$) and therefore, it must hold that $U\left(\phi^S\left(\bar{k}, x^c_R(\bar{\theta}_c), t\right), t\right) > U\left(\phi^S\left(\bar{k}, x^{seq}_R, t\right), t\right)$. Therefore, the overall expected ex-ante welfare is greater under the commitment protocol for the sender. This completes the proof. **QED**

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