Double Auction Mechanisms for Land Assembly

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Abstract

A land assembly problem is modeled as an environment with many buyers on one side, each of whom is interested in a fixed massive area of land whose ownership is dispersed among many landowners. Landowners have an incentive to overstate their valuation leading to holdout while buyers have an incentive to understate theirs. A double auction mechanism based on the idea that complementary land parcels have an integrated ownership is proposed and shown to satisfy a number of market design desiderata- strategy-proofness in the large, approximate asymptotic efficiency, budget balance, buyers' free will and a notion of collective property rights. However, it violates individual property rights of some landowners. The double auction, therefore, needs external enforcement. It is shown that there is no feasible mechanism that satisfies all the desiderata at once. A different but related double auction is suggested for environments with very few buyers.

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1 Introduction

Governments, private developers and industries routinely face the problem of land assembly when acquiring land to construct railroads, pipelines, housing projects, energy infrastructure, special economic zones, manufacturing plants and airports etc. A characteristic of these environments is that massive amounts of land is needed for these projects; however, ownership of land is dispersed among many landowners. A fundamental problem in trade involving land assembly is the holdout problem- each owner, knowing that he is pivotal for trade,

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tries to extract all the gains from trade by demanding a very high price without taking into account the adverse impact that everyone taking such a trading posture will lead to virtually no trade. Bergstrom (1978) studied the holdout problem in the context of Cournot model of perfectly complementary monopolies. Legal scholars like Heller (1998) have coined the phrase 'tragedy of the anti-commons' to describe it as a problem symmetrical (cf. Buchanan and Yoon (2000)) to the more commonly known tragedy of the commons; the former resulting from fragmented property rights while the latter stemming from common property rights.

Historically, governments around the world have legislated what is referred to as 'Eminent Domain' powers under which they can coercively take the private land of any owner, ostensibly for 'public purpose'. In the United States, the Fifth Amendment confers this power on the government while in India, it is conferred by "The Right to Fair Compensation and Transparency in Land Acquisition, Rehabilitation and Resettlement Act (RFCTLARR) 2013". The ambit of public purpose, however, is expanding progressively and often, private industries lobby government to use their eminent domain powers to give them the land they need. Benson (2008) gives an account of the evolution of eminent domain powers in the English property law and subsequently in colonial America while Bhattacharyya (2015) and Sampat (2013) give an account of its history and application in India. However, the application of eminent domain to resolve holdouts in land assembly is beset with problems of wasteful assembly, inadequate compensation and government high-handedness. In fact, the Indian legislation referenced above was a culmination of a string of popular protests against forcible acquisition and unfair compensation in many states of India.

In this paper, we suggest a market design solution for the land assembly problem which we model in Section 3 as follows- there are many landowners on one side, each of them having property rights over their own land parcel of a certain area; and a bunch of interested buyers on the other side, each of them demanding the entire land that is the sum total of all these land parcels. In other words, for each buyer, all the land parcels are perfect complements. That the landowners have incentives to overstate their valuation is well known. Perhaps less well appreciated is the incentive of the buyers to understate their valuations. If the buyer is a producer, then land is an input for him. The imperatives to maximize his profits give him incentive to minimize the cost of land acquisition, thereby, to understate his valuation. A good market design solution must take these incentive problems on both sides of the market into account. Each trader has his valuation in per unit area terms.

Our solution is a fairly simple prior-free double auction mechanism that we describe in Section 4 [1]. The principle behind the mechanism reflects the insight from the literature on contract theory (Hart and Holmstrom (2010), Segal and Whinston (2012), Bresnahan and Levin (2012)) that property rights should be allocated in a way that ensures efficient ex-post decisions. That insight leads to our design principle that trading decisions in a land assembly environment be made *as if* all the complementary land parcels were owned by a single landowner. In a first step, suppose that there is a single interested buyer. The

[1]We encourage the reader, at this point, to take a look at the formal description of the mechanism in Section 4

mechanism treats the aggregate of all the complementary land parcels as a single resource owned entirely by a fictitious landowner whose total valuation of the aggregate landmass is the aggregate of total valuations of all the landowners of their individual land parcels. This converts the original multilateral trade problem to a simple bilateral trade problem. Our mechanism is then simply the 1/2- double auction of Chatterjee and Samuelson (1983) with the proviso that the total ask price of the fictitious landowner is the aggregate of total ask prices of all the original landowners. Extension to the case of multiple buyers then simply introduces an auction element on the buyer side which is resolved by declaring the highest bidder as the winner.

We assess our double auction mechanism on several desiderata for market design. Azevedo and Budish (2018), in recent work, propose strategy-proofness in the large as a notion of approximate strategy-proofness in markets with a meaningful number of participants. We discuss it in greater detail in the main body of the paper and therefore contend ourselves here with a brief description. It tests whether players who are 'price-takers' in large markets have an incentive to manipulate the mechanism. It is a weaker notion than strategy-proofness but stronger and more robust than Bayes-Nash incentive compatibility. Our double auction is strategy-proof in the large because both the allocation and the compensation rule depend only on the sample statistics of traders' reports which, as the market gets larger, converge to population aggregates that are independent of any trader's reports. We also take an alternative route to show that our double auction is strategy-proof in the large. This involves showing that it has certain other properties- the outcomes for landowners are envy-free and no buyer is asymptotically pivotal. Efficiency in the land assembly environment has two aspects- efficiency in trading volume and efficiency in allocation conditional on trade. We show that our double auction is asymptotically approximately efficient. It is also budget-balanced by construction. We then define the robust analogues of bayesian participation constraints. A mechanism respects buyers' free will if the total compensation paid by the winner does not exceed the total compensation he bid. It respects individual property rights if no landowner gets a compensation less than his ask price in the event of trade. While our double auction respects buyers' free will by construction, it does not respect individual property rights by design. More precisely, it respects some landowner's property rights while violating others'. We finally assess our mechanism on the criterion of whether it respects collective property rights. A mechanism respects collective property rights if trade never occurs at a total price that is less than the total ask price quoted by the community of landowners. Our double auction respects this notion of collective property rights. In summary, Proposition 1 in Section 5 proves the proposed double auction stands good on several desirable criteria- strategy-proofness in the large, asymptotic approximate efficiency, budget balance, respect for buyers' free will and respect for collective property rights. It only violates individual property rights of some landowners. The proposed double auction, thus, needs external enforcement for its implementation.

We next explore whether there is a feasible mechanism that satisfies the conjunction of all the desiderata. Proposition 2 in Section 5 proves that this is impossible. We regard Proposition 2 as a counterpart of Myerson and Satterthwaite (1983) for the land assembly context that is our focus. We devote our attention in Section 6 to environments where there are few buyers. In environments with very few buyers, which is often the empirical reality, demanding strategy-proofness as a criterion to judge buyers' incentives seems theoretically unsound. We suggest a different double auction in which any buyer's incentives are similar to those in a second price auction. Proposition 3 is the analogue of Proposition 1 for land assembly environments with very few buyers.

Section 2 discusses the related literature. In Section 7, we simulate the two double auctions to see how some interesting distributional parameters impact the probability of trade and the extent of individual property rights protection. Finally we conclude in Section 8.

2 Related Literature

Kominers and Weyl (2011) consider the holdout problem in an environment with one buyer and propose a solution inspired by Cournot (1838). Their 'concordance mechanism' asks the buyer to make the monopsonist-optimal offer to the aggregate community of sellers which is accepted when it exceeds the total reported valuation of sellers. Sellers have an option to exert influence or not. Those that do not exert an influence get an exogenously determined share of the buyer's offer while those that do have to pay a Pigouvian tax. A primitive of their model is planner's beliefs about sellers' ex-ante expected shares of the total value and the extent to which individual property rights are respected in their model rests on the accuracy of these beliefs. Such an assumption is vulnerable to Wilson's critique (Wilson (1987)) that a good mechanism be robust to assumption on participants' beliefs. Moreover, as Kominers and Weyl (2012) point out, lack of robustness is a limitation for the revelation mechanisms they consider. Our mechanism addresses these concerns in two ways. First, our proposed double auction is prior-free. Second, we rely on the idea that quite distinctly from the rules of the game, market size can ease incentive problems. Roberts and Postlewaite (1976) and Hammond (1979) are early expositions of this idea. In this paper, we rely on a new approach to incentive compatibility in large markets developed by Azevedo and Budish (2018) which retains the advantages of strategy-proof market design, we show that traders who are 'price-takers' in large markets have no incentive to manipulate the mechanism, thus making it strategically simple, safe and fair (Roth (2008)) to play it.

It is well known in the contract theory literature (Segal and Whinston (2012)) that the allocation of property rights affects individual owners' incentives to trade and invest. We recognize that fragmented ownership of complementary assets is the source of the holdout problem in a land assembly context. Our approach is to treat the disparate complementary assets as an integrated asset with a single fictitious owner. That integration can be beneficial for incentives is well recognized by Williamson (1971), Hart and Holmstrom (2010) and Bresnahan and Levin (2012) among others. The coordination of incentives is then achieved by a trading institution that, for the case of a single buyer, treats the problem as a bilateral trade problem between the buyer and the fictitious landowner.

For the case of a single buyer and a single landowner, our proposed double auction reduces to the bilateral 1/2- double auction of Chatterjee and Samuelson (1983) which satisfies all the desiderata considered in this paper except strategy-proofness. The objective of that paper was to show that traders can use private information strategically in bargaining. Rustichini, Satterthwaite and Williams (1994) show that gains from strategic manipulation in a simple model of double auction are a consequence of thinness of the market- such gains vanish fast as number of traders on both sides increases. The message of Cripps and Swinkels (2006) in a more general model of double auction and Swinkels (2001) in a uniform price auction is much the same. Gresik and Satterthwaite (1989) study the same question in a mechanism design framework. In terms of this larger message that gains from strategic manipulation get smaller as the market grows bigger, our paper conforms with this literature. In terms of analysis, though, we do not rely on characterizing Bayes-Nash equilibria. Our approach relies on Azevedo and Budish (2018) who develop a notion of thinking about approximate strategy-proofness from an interim perspective in large markets.

In related literature in the mould of mechanism design, Grossman, Pincus and Shapiro (2010) suggest a variant of second-price auction with secret reserves that is sufficient to compensate every seller to his satisfaction as an assembly mechanism. In favoring property rights over efficiency, their mechanism sacrifices some efficient sales and performs poorly in solving the fundamental holdout problem as the number of sellers grow. Sarkar (2017) incorporates a contiguity structure in modeling the land acquisition setting and explores conditions on that structure under which Myerson-Satterthwaite Impossibility Result does not hold for a set of priors and ex-post efficient Bayesian incentive compatible mechanisms do exist.

There is a different but related literature that models holdout as delays in decentralized bargaining protocols among one buyer and multiple sellers. Menezes and Pitchford (2004) find that severity of holdout is increasing in the complementarity in buyer's assembly technology. Chowdhury and Sengupta (2012) show that holdout severity depends on transparency of bargaining protocol, buyer's outside option and the marginal contribution of the last seller. Miceli and Segerson (2007) explicitly model how eminent domain changes the bargaining game and the ensuing expectations of sellers. This paper, in contrast, is in the mould of market design i.e. we seek centralized trading mechanism with good robust properties that perform well in incomplete information environments.

3 Economic Environment

There is a set $M = \{1, \ldots, m\}$ of landowners, indexed by j such that each landowner j owns a land parcel of surface area A_j and $A = \sum_{j=1}^{m} A_j$. There is another set $N = \{1, \ldots, n\}$ of interested buyers, indexed by i, each of who demands the entire landmass of total area A. In other words, the land parcels of all the landowners are perfect complements for trade.

The attribute of any buyer i is his valuation per unit area which we denote by w_i and is private information to him. The attribute of any landowner j is first, the area A_j of the land parcel he owns; and second, his valuation per unit area of his land parcel which we denote by v_j. While the area attribute is common knowledge, the valuation attribute is private information to the landowner.

There are two elements that any resource allocation outcome has to describe- which buyer gets the land, if at all; and what is the compensation paid to each landowner. Let $\mathbf{0}$ denote the no-trade outcome in which there is no change in property rights assignment. Let e_i, the i-th basis vector in $\mathbb{R}^n$, denote that buyer i is allocated the land. Let $p \in \mathbb{R}^m$ denote the vector of compensation per unit area paid to the landowners. Then the set of outcomes can be described as

$$\Omega = \mathbf{0} \cup \{(e_i, p) : i \in N, e_i \in \mathbb{R}^n, p \in \mathbb{R}^m\}$$

We now describe the preferences of players over outcomes. In the no-trade outcome, the buyers enjoy zero payoffs while each landowner j enjoys his valuation v_j. In any other outcome, the payoffs are

$$u_j(e_i, p) = p_j A_j$$
$$u_i(e_i, p) = w_i A - \sum_{j=1}^{m} p_j A_j$$

4 Double Auction Mechanism

A sealed bid double auction is a direct mechanism in which all landowners and all buyers simultaneously and confidentially report their valuations to a coordinator, who then determines the allocation as a function of the reports received. We will label our double auction mechanism as $DA1$. Let $b = (b_1, \dots, b_n) \in \mathbb{R}^n$ be the profile of buyers' bid prices and $a = (a_1, \dots, a_m) \in \mathbb{R}^m$ be the profile of landowners' ask prices. The rules of $DA1$ then specify that trade take place if and only if the highest total compensation bid exceeds the total compensation ask. In that case, the highest bidder is the winner and the uniform per-unit area compensation is the simple average of the highest bid and the weighted average of asks, where the weight of ask a_j of any landowner j is the proportion of the total area that j controls. To describe the rules formally, define the order statistics $b_{(1)} \geq \dots \geq b_{(n)}$. Formally, the outcome of our double auction $DA1$ is that trade occurs if and only if $b_{(1)}A > \sum_{j \in M} a_j A_j$. If trade occurs, the highest bidder is allocated the land (ties among bidders at the highest bid are resolved by a lottery) and the uniform compensation (per unit area) paid to the landowners is

$$p = \frac{b_{(1)}A + \sum_{j \in M} a_j A_j}{2A}$$

As discussed in the introduction, for the case of single buyer, this double auction can be identified with the 1/2- double auction of Chatterjee and Samuelson (1983) once we think of the aggregate land as a single resource owned by a fictitious landowner whose total ask price is $\sum_{j \in M} a_j A_j$, the aggregate of the total ask prices of all the real landowners. Trade happens if and only if total bid price of the buyer is higher than the total ask price of the fictitious landowner. Trade occurs at a total compensation that is the simple average of the two.

$DA1$ has several attractive properties. The rule that trade happen only if the highest bid exceeds the weighted average in the sample of asks mitigates incentives of buyers to understate their valuations. An implication of the same rule is that the highest bid need not exceed all the asks for the trade to be executed. This mitigates incentive for the landowners to hold up trade by exercising a veto. We will show in the next subsection that though our double auction is not strategy proof, it is approximately strategy proof in a large market.

4.1 Strategy-proofness in the Large

Strategy-proofness or Dominant Strategy Incentive Compatibility is the strongest criteria of incentive compatibility in markets. It requires that playing the mechanism truthfully is a dominant strategy for everyone. Hence strategy-proof mechanisms are robust in a natural sense. They are also strategically simple for players. However, it is also a very strong requirement. Azevedo and Budish (2018), in recent work, propose strategy-proofness in the large as a notion of approximate strategy-proofness. In order to precisely define it for the land assembly environment, we will need a discrete formulation for the type space and several definitions.

Let us formulate our environment in a discrete way. Each trader belongs to one of two groups- (b)uyers and (s)ellers. Let $G = \{b, s\}$. Let $\mathbb{V} = \{1, 2, \dots, \bar{v}\}$ be a finite set of positive integers. Let $\{A_{\min}, \dots, A_{\max}\}$ be the subset of positive integers in which areas of various land parcels live. We define the type space of the buyers to be $T_b = \mathbb{V}$ and that of the sellers to be $T_s = \{A_{\min}, \dots, A_{\max}\} \times \mathbb{V}$. The outcome space for any seller is $X_s = \{0, 1\} \times \mathbb{R}_+$, the first coordinate is a binary variable that describes whether trade happens or not and the second coordinate describes the compensation received. The outcome space for any buyer is $X_b = \{0, 1\} \times \mathbb{R}^m_+$, where the first coordinate is again a binary variable that describes whether trade happens or not and the second coordinate describes the vector of compensation paid. Define $X_S = \Delta(\{0, 1\}) \times \mathbb{R}_+$ and $X_B = \Delta(\{0, 1\}) \times \mathbb{R}^m_+$ to denote the respective outcome sets when randomization over whether trade happens or not is allowed. The utility functions are now easy to define. Of course, we continue to maintain that every buyer demands the total area of all the landowners. This can be built as a feasibility constraint that the first coordinate of the outcome vector for all landowners is the same, that is, either all of them trade or none of them do. Since the area of landowners is public information, they cannot lie about it.

Definition 1. A direct mechanism $\Big((\Phi^{m,n})_{m \in \mathbb{N}, n \in \mathbb{N}}, A_b, A_s\Big)$ consists of finite sets of actions $A_b = T_b$ and $A_s = T_s$ for the buyers and the sellers respectively; and a sequence of a pair of allocation functions

$$\Phi^{m,n}_b : A^n_b \times A^m_s \mapsto X^n_B$$
$$\Phi^{m,n}_s : A^n_b \times A^m_s \mapsto X^m_S$$

such that they satisfy a feasibility constraint which reflects that trade is of all or none variety.

$$\forall m \in \mathbb{N}, \forall n \in \mathbb{N}, \forall b \in A_b^n, \forall(\vec{A}, a) \in A_s^m, \forall j \in M \quad \Phi_{s,j}^{m,n}[b, \vec{A}, a](p) = \sum_{i \in N} \Phi_{b,i}^{m,n}[b, \vec{A}, a](p)$$

where where $\vec{A}$ is the vector of areas of land parcels, $\Phi_{s,j}^{m,n}[b, \vec{A}, a](p)$ denotes the probability of trade for the j-th landowner and $\Phi_{b,i}^{m,n}[b, \vec{A}, a](p)$ denotes the probability of trade for the i-th buyer; when the size of the market is (m, n).

We will need the following concept due to Kalai (2004). For our purposes, we use Azevedo and Budish (2018)'s version.

Definition 2. Semi-anonymous Mechanism (Kalai (2004), Azevedo and Budish (2018)). A mechanism is semi-anonymous if agents are divided into a finite set of groups and an agent's outcome depends only on her own action, her group and the distribution of actions within each group.

Since we will be concerned with the incentive properties of our double auction in large markets, it is essential to have a notion of a large market. Azevedo and Budish (2018) base their notion of how the market grows large on the approach taken in Kojima and Pathak (2009) and Immorlica and Mahdian (2005). We adapt their definition for our purposes.

Definition 3. Given a direct mechanism $\Big((\Phi^{m,n})_{m \in \mathbb{N}, n \in \mathbb{N}}, A_b, A_s\Big)$, define for each market size (m, n), the functions $\phi_b^{m,n} : A_b \times \Delta(A_b) \times \Delta(A_s) \mapsto X_B$ and $\phi_s^{m,n} : A_s \times \Delta(A_s) \times \Delta(A_b) \mapsto X_S$ by

$$\phi_b^{m,n}(b_i, f, g) = \sum_{b_{-i} \in A_b^{n-1}} \sum_{(\vec{A}, a) \in A_s^m} \Phi_{b,i}^{m,n}(b_i, b_{-i}, \vec{A}, a) Pr\big(b_{-i} | b_{-i} \sim iid(f)\big) Pr\big(\vec{A}, a | \vec{A}, a \sim iid(g)\big)$$

$$\phi_s^{m,n}(A_j, a_j, g, f) = \sum_{(A_{-j}, a_{-j}) \in A_s^{m-1}} \sum_{b \in A_b^n} \Phi_{s,j}^{m,n}(A_j, a_j, \vec{A}_{-j}, a_{-j}, b) Pr\big(\vec{A}_{-j}, a_{-j} | \vec{A}_{-j}, a_{-j} \sim iid(g)\big) Pr\big(b | b \sim iid(f)\big)$$

where $\vec{A}$ is the vector of areas of land parcels that the landowners control, f is distribution of actions among the buyers, g is the distribution of actions among the landowners, $Pr\big(b_{-i} | b_{-i} \sim iid(f)\big)$ denotes the probability the action (bid) vector b_{-i} is realized given $n-1$ i.i.d. draws from the action distribution $f \in \Delta(A_b)$. The other probability expressions are analogously defined.

The notion of the large market defined above via interim allocation rules can be used to define the large market limit by letting traders in both groups grow large and drawing types from both groups in an i.i.d. fashion.

Definition 4. The large market limit of mechanism $\Big((\Phi^{m,n})_{m \in \mathbb{N}, n \in \mathbb{N}}, A_b, A_s\Big)$ is the pair of functions $\phi_b^\infty : A_b \times \Delta(A_b) \times \Delta(A_s) \mapsto X_B$ and $\phi_s^\infty : A_s \times \Delta(A_s) \times \Delta(A_b) \mapsto X_S$ given by

$$\phi_b^\infty(b_i, f, g) = \lim_{m\to\infty, n\to\infty} \phi_b^{m,n}(b_i, f, g)$$
$$\phi_s^\infty(A_j, a_j, g, f) = \lim_{m\to\infty, n\to\infty} \phi_s^{m,n}(A_j, a_j, g, f)$$

if the limits exist.

Following Azevedo and Budish (2018), a direct mechanism is strategy-proof in the large (SP-L) if for any full support i.i.d. distribution of opponents' reports, the mechanism is approximately strategy proof for both landowners and buyers in a large enough market. If the mechanism has a large market limit, this is equivalent to the requirement that reporting truthfully is optimal in the limit mechanism. We will need one additional bit of notation to state the definitions formally. For any finite set T, let $\bar{\Delta}(T)$ denote the set of distributions on T with full support.

Definition 5. A direct mechanism is strategy-proof in the large (SP-L) for landowners if $\forall f \in \bar{\Delta}(T_b)$, $\forall g \in \bar{\Delta}(T_s)$ and $\forall \epsilon > 0$, $\exists \hat{m}$ such that $\forall m \geq \hat{m}$ and $\forall (A_j, a_j) \in T_s, \forall a_j' \in \mathbb{V}$, we have

$$\mathbb{E}u_j[\phi_s^{m,n}(A_j, a_j, g, f)|A_j, a_j] \geq \mathbb{E}u_j[\phi_s^{m,n}(A_j, a_j', g, f)|A_j, a_j] - \epsilon$$

If the mechanism has a large market limit, the above inequality is equivalent to

$$\mathbb{E}u_j[\phi_s^\infty(A_j, a_j, g, f)|A_j, a_j] \geq \mathbb{E}u_j[\phi_s^\infty(A_j, a_j', g, f)|A_j, a_j]$$

The corresponding definition for buyers is as follows.

Definition 6. A direct mechanism is strategy-proof in the large (SP-L) for buyers if $\forall f \in \bar{\Delta}(T_b)$, $\forall g \in \bar{\Delta}(T_s)$ and $\forall \epsilon > 0$, $\exists \hat{n}$ such that $\forall n \geq \hat{n}$ and $\forall b_i \in T_b, \forall b_i' \in \mathbb{V}$, we have

$$\mathbb{E}u_i[\phi_b^{m,n}(b_i, f, g)|b_i] \geq \mathbb{E}u_i[\phi_b^{m,n}(b_i', f, g)|b_i] - \epsilon$$

If the mechanism has a large market limit, the above inequality is equivalent to

$$\mathbb{E}u_i[\phi_b^\infty(b_i, f, g)|b_i] \geq \mathbb{E}u_i[\phi_b^\infty(b_i', f, g)|b_i]$$

A direct mechanism is then SP-L if no trader wants to misreport as a different type within the same group. SP-L is weaker than the traditional notion of strategy-proofness but stronger than the notion of bayesian incentive compatibility. To show that our double auction is strategy-proof in the large, we will need two lemmas.

Lemma 1. *Suppose $\mathbb{V} = \{1, 2, \ldots, \bar{v}\}$ with $\bar{v}$ finite. Consider a pdf f on $\mathbb{V}$ that has full support. Suppose $X_1, \ldots, X_n$ is a sample of n independent draws from f and $Y_1^{(n)} = \max(X_1, \ldots, X_n)$ is the highest-order statistic of the sample. Then $\lim_{n\to\infty} Y_1^{(n)} = \bar{v}$ with probability 1.*

Proof. By Glivenko-Cantelli's Fundamental Theorem of Mathematical Statistics (Billingsley (1995), p268), the empirical density f_n of the sample of size n converges uniformly with probability 1 to the population density f as $n \to \infty$. This implies $\lim_{n\to\infty} f_n(\bar{v}) = f(\bar{v}) > 0$. That is, for all large enough sample sizes, $\bar{v}$ appears in the sample at least once with the consequence that the highest order statistic of that sample is $\bar{v}$. Therefore, the same must be true of the limit, thereby proving the lemma. Q.E.D.

Lemma 2. *Suppose* $(A_1, a_1), \ldots, (A_m, a_m)$ *is a sample of* m *independent draws of a random vector* $(\mathcal{A}, \mathfrak{a})$ *which is distributed as density* g *with full support on* $\{A_{min}, \ldots, A_{max}\} \times \mathbb{V}$, *where* $\{A_{min}, \ldots, A_{max}\}$ *is a subset of positive integers. Let* $S(m) = \frac{\sum_{j=1}^m A_j a_j}{\sum_{j=1}^m A_j}$. *Then* $\lim_{m\to\infty} S(m) = \frac{\mathbb{E}[\mathcal{A}\mathfrak{a}]}{\mathbb{E}[\mathcal{A}]}$ *with probability 1.*

Proof. Observe that $S(m) = \frac{\sum_{j=1}^m A_j a_j/m}{\sum_{j=1}^m A_j/m} \to \frac{\mathbb{E}[\mathcal{A}\mathfrak{a}]}{\mathbb{E}[\mathcal{A}]}$ as $m \to \infty$ by the Strong Law of Large Numbers (Billingsley (1995), p282). Q.E.D.

We are now in a position to show that our double auction $DA1$ is SP-L. Let f denote the full support density over $\mathbb{V}$ from which buyers' bids are drawn in an i.i.d. fashion; g denote the full support density over $\{A_{\min}, \ldots, A_{\max}\} \times \mathbb{V}$ of the random vector $(\mathcal{A}, \mathfrak{a})$ from which landowners' areas and asks are drawn in an i.i.d. fashion. Then we have the following result.

Lemma 3. *The double auction* $DA1$ *is strategy-proof in the large.*

Proof. By Lemma 1 and Lemma 2, the large market limit of our double auction prescribes that trade happens if and only if $\bar{v} \geq \frac{\mathbb{E}[\mathcal{A}\mathfrak{a}]}{\mathbb{E}[\mathcal{A}]}$. If trade occurs, the highest bidder is allocated the land and the uniform compensation paid to the landowners is

$$p = \frac{\bar{v} + \frac{\mathbb{E}[\mathcal{A}\mathfrak{a}]}{\mathbb{E}[\mathcal{A}]}}{2}$$

Since both the volume of trade and the terms of trade are independent of any individual buyer's or seller's report, reporting truthfully is an optimal strategy for each trader in the limit double auction. This proves the lemma. Q.E.D.

4.1.1 Alternative Route to Strategy-Proofness in the Large

In this section, we exploit certain other properties of our double auction $DA1$ to show that it is SP-L. The fact that $DA1$ satisfies these properties is of independent interest in itself. In addition, it yields insights into what makes our double auction SP-L.

We first need to define a notion of closeness in the outcome space $X_B = \Delta(\{0,1\}) \times \mathbb{R}^m_+$ for the buyers. Note that we do not allow for randomization over compensation conditional on trade. The only randomization permissible is over whether trade occurs or not. So a buyer may face an outcome lottery like $([q]\mathbf{0}+[1-q]\mathbf{1}, p)$, where $\mathbf{0}$ denotes no trade, $\mathbf{1}$ denotes trade with compensation vector p, q denotes the probability of no-trade outcome and the addition is symbolic merely meaning that the buyer faces the outcome 'no trade' with probability q and the outcome 'trade with compensation vector p' with probability $1-q$. In order to measure distance between two lotteries, an outcome lottery can be mapped to a point in $[0,1] \times \mathbb{R}^m_+$ via the mapping $([q]\mathbf{0} + [1-q]\mathbf{1}, p) \mapsto (q, p)$. Since we have the usual Euclidean metric over $[0,1] \times \mathbb{R}^m_+$, we can measure the distance between two outcome lotteries.

$$d\big(([q]\mathbf{0} + [1-q]\mathbf{1}, p), ([q']\mathbf{0} + [1-q']\mathbf{1}, p')\big) := d\big((q,p),(q',p')\big) = \sqrt{(q-q')^2 + \sum_{j=1}^{m}(p_j - p'_j)^2}$$

In order to measure the distance between $\mathbf{0}$ and $([q]\mathbf{0} + [1-q]\mathbf{1}, p)$, map $\mathbf{0} \mapsto (1, p)$ so that $d\big(\mathbf{0}, ([q]\mathbf{0} + [1-q]\mathbf{1}, p)\big) = 1-q$. A buyer i of type w_i when facing the outcome lottery $([q]\mathbf{0} + [1-q]\mathbf{1}, p)$ expects the payoff

$$u_i[(q,p); w_i] = (1-q)(w_i A - \sum_{j=1}^{m} p_j A_j)$$

which is continuous in (q, p).

We want to formalize the notion that a buyer may have the power to influence the outcome of the double auction by his actions. Definition 7 and Definition 8 formalize progressively weaker notions of when a buyer is 'pivotal' in the double auction. We then establish that such influence of a buyer is limited in a large enough double auction and actually vanishes in the limit. Lemma 4 and Lemma 5 establish that no buyer is pivotal asymptotically.

Definition 7. A buyer i is asymptotically pivotal in large double auction if there is a full support i.i.d. distribution of other buyers' reports $f \in \bar{\Delta}(\mathbb{V})$, a full support i.i.d. distribution of sellers' reports $g \in \bar{\Delta}(\{A_{\min}, A_{\max}\} \times \mathbb{V})$ and a constant $K > 0$ such that for every pair $(b_i, b'_i) \in \mathbb{V} \times \mathbb{V}$ for which $b_i \neq b'_i$, $d\big(x_i^n(b_i, f, g), x_i^n(b'_i, f, g)\big) > K$ for all n large enough; where $x_i^n(b_i, f, g)$ is the outcome lottery that i gets in a market with n buyers when he reports b_i and faces the distribution f of other buyers' reports and g of sellers' reports.

Lemma 4. *No buyer is asymptotically pivotal in any sufficiently large double auction $DA1$.*

Proof. Fix f and g. Choose $(b_i, b'_i) \in \mathbb{V} \times \mathbb{V}$ such that $b_i \neq \bar{v}$, $b'_i \neq \bar{v}$ and $b_i \neq b'_i$. The probability that buyer i wins with a bid of b_i against $n-1$ other buyers whose bids are drawn i.i.d. from f is $F(b_i)^{n-1}$ which converges to 0 as $n \to \infty$. The previous assertion remains true if we replace b_i with b'_i. Therefore, as n gets larger and larger, the outcome

i receives by reporting b_i and the outcome he gets by reporting b_i' get closer and closer i.e. $d\big(x_i^n(b_i, f, g), x_i^n(b_i', f, g)\big) \to 0$ as $n \to \infty$. This proves the lemma. Q.E.D.

Definition 8. A buyer i is asymptotically weakly pivotal in large double auction if there is a full support i.i.d. distribution of other buyers' reports $f \in \bar{\Delta}(\mathbb{V})$, a full support i.i.d. distribution of sellers' reports $g \in \bar{\Delta}(\{A_{\min}, A_{\max}\} \times \mathbb{V})$, a pair $(b_i, b_i') \in \mathbb{V} \times \mathbb{V}$ for which $b_i \neq b_i'$, and a constant $K > 0$ such that for $d\big(x_i^n(b_i, f, g), x_i^n(b_i', f, g)\big) > K$ for all n large enough.

Lemma 5. *No buyer is asymptotically weakly pivotal in any sufficiently large double auction $DA1$.*

Proof. Fix f and g. Choose $(b_i, b_i') \in \mathbb{V} \times \mathbb{V}$ such that $b_i \neq b_i'$. There are two cases.

Case 1. Suppose $b_i \neq \bar{v}$ and $b_i' \neq \bar{v}$. Then the argument is the same as in the proof of Lemma 4.

Case 2. Suppose $b_i \neq \bar{v}$ and $b_i' = \bar{v}$. Then the probability that buyer i wins with a bid of b_i against $n-1$ other buyers whose bids are drawn i.i.d. from f is $F(b_i)^{n-1}$ which converges to 0 as $n \to \infty$. Following Kalai (2004), define the empirical distribution induced by a bid vector $b = (b_1, \ldots, b_n)$ on $\mathbb{V}$ by $emp[b](v) = \frac{\#\{i : b_i = v\}}{n}$. Then, the probability that buyer i wins with a bid of $b_i' = \bar{v}$ against $n-1$ other buyers whose bids are drawn i.i.d. from f is $\frac{1}{emp[b](\bar{v})n}$ which converges to 0 as $n \to \infty$ since by Glivenko-Cantelli Theorem, $emp[b](\bar{v}) \to f(\bar{v}) > 0$ as $n \to \infty$. Therefore, as n gets larger and larger, the outcome i receives by reporting b_i and the outcome he gets by reporting b_i' get closer and closer i.e. $d\big(x_i^n(b_i, f, g), x_i^n(b_i', f, g)\big) \to 0$ as $n \to \infty$. This proves the lemma. Q.E.D.

Azevedo and Budish (2018) identify envy-freeness of a mechanism as a sufficient condition for SP-L. The uniform compensation feature makes the double auction $DA1$ envy-free among the landowners. However, this is not the case for the buyers. But since a buyer's report cannot affect the outcome by much, he cannot gain much by misreporting. This makes the double auction SP-L even for the buyers.

Lemma 6. *The double auction $DA1$ is strategy-proof in the large.*

Proof. Note that our double auction is envy-free for the landowners. This is because of two features- either every landowner trades or no one does; and the compensation is uniform for all of them. By Theorem 1 in Azevedo and Budish (2018), the double auction is SP-L in the large for the landowners. By contrast, the double auction is not envy-free for the buyers. In particular, losers may envy the winner. However, Lemma 5 and the continuity of buyer i's payoff function in (q, p) implies that the double auction is SP-L even for the buyers. Q.E.D.

4.2 Efficiency

In this subsection, we want to assess the double auction $DA1$ on grounds of efficiency. There are two aspects to efficiency in our environment. One, efficiency in trading volume; and second, efficiency in allocation conditional on trade. Definition 9 defines a notion of approximate efficiency in an asymptotic sense that expresses the idea that the degree of inefficiency in trades gets smaller as the market grows larger. Lemma 7 shows that the proposed double auction is indeed efficient in this sense.

Definition 9. A direct mechanism is asymptotically approximately efficient (AAE) if $\forall \epsilon > 0$, $\exists \hat{m} \in \mathbb{N}, \exists \hat{n} \in \mathbb{N}$ such that $\forall m \geq \hat{m}$ and $\forall n \geq \hat{n}$, (i) the mechanism prescribes trade if $w_{(1)}A \geq \sum_{j=1}^{m} v_j A_j + \epsilon$ and only if $w_{(1)}A \geq \sum_{j=1}^{m} v_j A_j - \epsilon$; and (ii) the land is allocated to a buyer who has the highest valuation.

Lemma 7. *The double auction $DA1$ is asymptotically approximately efficient.*

Proof. Fix $\epsilon > 0$ and suppose trade happens. By Lemma 3, the double auction is SP-L. So choose $\hat{m}$ such that $\forall m \geq \hat{m}$, $0 \leq \sum_{j=1}^{m}(a_j - v_j)A_j < \frac{\epsilon}{2}$. Let $w_{(1)}$ denote the highest valuation in any given sample population of buyers. Let the symbol $b_{(1)}$ denote the bid submitted by a buyer with the highest valuation $w_{(1)}$. Choose $\hat{n}$ such that $\forall n \geq \hat{n}$, $0 \leq (w_{(1)} - b_{(1)})A < \frac{\epsilon}{2}$ and $b_{(1)}$ can be interpreted as the highest bid in the sample of bids. Then $\forall m \geq \hat{m}$ and $\forall n \geq \hat{n}$

$$
\begin{aligned}
w_{(1)}A - \sum_{j=1}^{m} v_j A_j &= \sum_{j=1}^{m}(a_j - v_j)A_j + (w_{(1)} - b_{(1)})A + \underbrace{\big(b_{(1)}A - \sum_{j=1}^{m} a_j A_j\big)}_{\text{>0 under trade}} \qquad (1) \\
&> \sum_{j=1}^{m}(a_j - v_j)A_j + (w_{(1)} - b_{(1)})A \\
&\geq 0 > -\epsilon
\end{aligned}
$$

Now suppose $w_{(1)}A \geq \sum_{j=1}^{m} v_j A_j + \epsilon$. Then Equation (1) implies

$$
\begin{aligned}
b_{(1)}A - \sum_{j=1}^{m} a_j A_j &= \big[w_{(1)}A - \sum_{j=1}^{m} v_j A_j\big] - \big[\sum_{j=1}^{m}(a_j - v_j)A_j\big] - \big[(w_{(1)} - b_{(1)})A\big] \\
&> \epsilon - \frac{\epsilon}{2} - \frac{\epsilon}{2} \\
&= 0
\end{aligned}
$$

Thus the double auction prescribes trade.

Since $b_{(1)}$ and $w_{(1)}$ get arbitrarily close as the double auction gets larger, the second requirement of Definition 9 is satisfied as well. This proves the lemma. Q.E.D.

4.3 Budget-Balance, Buyers' Free Will and Property Rights

We first define when a mechanism satisfies budget balance.

Definition 10. Budget-Balanced Mechanism. A direct mechanism is budget-balanced (BB) if, whenever trade ocuurs, the total compensation paid by the buyers is equal to the total compensation received by the sellers. Formally, $\forall m \in \mathbb{N}, \forall n \in \mathbb{N}, \forall b \in T_b, \forall(\vec{A}, a) \in T_s$

$$\forall i = 1, \ldots, n \qquad \Phi_{b,i}^{m,n}[b, \vec{A}, a](p) > 0 \implies \sum_{j=1}^{m} \Phi_{b,i}^{m,n}[b, \vec{A}, a](j) A_j = \sum_{j=1}^{m} \Phi_{s,j}^{m,n}[b, \vec{A}, a](2) A_j$$

where $\Phi_{b,i}^{m,n}[b, \vec{A}, a](p)$ is the probability with which buyer i gets the land, $\Phi_{b,i}^{m,n}[b, \vec{A}, a](j)$ is the compensation (per unit area) paid by buyer i to landowner j and $\Phi_{s,j}^{m,n}[b, \vec{A}, a](2)$ is the compensation (per unit area) received by landowner j.

The double auction $DA1$ is budget-balanced by construction. Next, we define when a mechanism respects buyers' free will.

Definition 11. A direct mechanism respects buyers' free will (BFW) if, whenever trade occurs, the total compensation paid by a winner does not exceed the total compensation that he bid. Formally, $\forall m \in \mathbb{N}, \forall n \in \mathbb{N}, \forall b \in T_b, \forall(\vec{A}, a) \in T_s$

$$\forall i = 1, \ldots, n \qquad \Phi_{b,i}^{m,n}[b, \vec{A}, a](p) > 0 \implies \sum_{j=1}^{m} \Phi_{b,i}^{m,n}[b, \vec{A}, a](j) A_j \leq b_i A$$

where $\Phi_{b,i}^{m,n}[b, \vec{A}, a](p)$ is the probability with which buyer i gets the land while $\Phi_{b,i}^{m,n}[b, \vec{A}, a](j)$ is the compensation paid by buyer i to landowner j.

A mechanism respects buyers' free will if the terms of the trade are acceptable to all buyers. In this sense, it is clear the double auction $DA1$ respects buyers' free will. The next two definitions enunciate notions of individual and collective property rights.

Definition 12. A direct mechanism respects individual property rights (IPR) if, whenever trade occurs, every landowner j gets a compensation (per unit area) no less than a_j. Formally, $\forall m \in \mathbb{N}, \forall n \in \mathbb{N}, \forall b \in T_b, \forall(\vec{A}, a) \in T_s$,

$$\forall j = 1, \ldots, m \quad \Phi_{s,j}^{m,n}[b, \vec{A}, a](2) \geq a_j$$

We show, by example, that the double auction $DA1$ may not respect the property rights of all the landowners.

Example 2. Suppose $n = 1, m = 3, A = 100$ and for every $j \in \{1, 2, 3\}$, $A_j = \frac{A}{3}$. Suppose also that the bid-ask profile is $b = 0.69$ and $a = (0.9, 0.7, 0.4)$. Then the area-weighted average of asks is just a simple average which is approximately 0.67. Since $0.69 > 0.67$, trade occurs at a uniform compensation of 0.68. The double auction $DA1$ thus violates the property rights of two of the three landowners.

However, there is a sense in which the double auction $DA1$ respects the collective property rights of the landowners- it never forces trade at a compensation less than the total valuation quoted by the landowners. Kominers and Weyl (2011) also use this notion when discussing the property rights aspect of their mechanism. In other words, $DA1$ respects the individual property rights of our fictitious landowner who owns all the land. The following definition formalizes the idea.

Definition 13. A direct mechanism respects collective property rights (CPR) if, whenever trade occurs, the total compensation received by all the landowners is no less than the total ask price. Formally, $\forall m \in \mathbb{N}, \forall n \in \mathbb{N}, \forall b \in T_b, \forall(\vec{A}, a) \in T_s$,

$$\sum_{j=1}^{m} \Phi_{s,j}^{m,n}[b, \vec{A}, a](2) A_j \geq \sum_{j=1}^{m} a_j A_j$$

Note the double auction $DA1$, by construction, respects collective property rights. In this sense, although the double auction $DA1$ is not a market mechanism with respect to the original fragmented property rights allocation that is the feature of land assembly environment, it is, nevertheless, a market mechanism with respect to our fictitious property rights allocation. This means the collective of landowners certainly have a veto over trade, but individually, they do not. Although Kominers and Weyl (2011) define this property somewhat differently, the basic idea of defining this notion, as they note, is to give some collective mechanism to the landowners to prevent wasteful and frivolous assembly.

5 Results

Our main result is the following.

Proposition 1. *The double auction $DA1$ is budget balanced, strategy-proof in the large, asymptotically approximately efficient, respects buyers' free will, respects collective property rights but does not respect individual property rights.*

Proof. It is budget balanced, respects buyers' free will and respects collective property rights by construction, SP-L by Lemma 3, asymptotically approximately efficient by Lemma 7. Example 2 shows that it does not respect individual property rights. Q.E.D.

In the next example, we present a natural double auction mechanism that respects every landowner's property rights. However, it turns out to violate SP-L as well as efficiency even in an approximate and asymptotic sense.

Example 3. Consider the following double auction mechanism. For any bid-ask profile (b, a), trade happens if and only if $b_{(1)}A > \sum_{j \in M} a_j A_j$. If trade occurs, the highest bidder is allocated the land (ties among bidders at the highest bid are resolved by a lottery) and the discriminatory compensation rule is that for every landowner j, $p_j = a_j$. This double auction, by construction, respects every landowner's property rights. Clearly, the compensation rule is the same in the large market limit of this double auction. As such, any individual landowner of type $v_j < \bar{v}$ has an incentive to overstate his valuation in the limit mechanism because that is what determines his compensation. The failure of this double auction to be SP-L implies that it is not asymptotically approximately efficient either.

Example 3 suggests an impossibility of simultaneous satisfaction of the desiderata that we have been considering. Proposition 2 makes this precise.

Proposition 2. *Any direct mechanism (with a well defined large market limit) that is budget-balanced, respects buyers' free will, is strategy-proof in the large and asymptotically approximately efficient must violate individual property rights of some landowners.*

Proof. First, restrict attention to direct mechanisms that prescribe uniform compensation to landowners whenever they prescribe trade. Consider any such mechanism and let $p^{m,n}(b, \vec{A}, a)$ be the uniform compensation prescribed in the event of trade when the market size is (m, n). By BB, $p^{m,n}(b, \vec{A}, a)$ is both the compensation paid by a winning buyer and the compensation received by every landowner. Define $v_{(1)}$ to be the highest valuation among the landowners and $w_{(1)}$ to be the highest valuation among the buyers. Similarly, define $a_{(1)}$ as the highest ask price and $b_{(1)}$ as the highest bid price. Consider the event $a_{(1)} > b_{(1)} > \sum_{j \in M} \frac{A_j}{A} a_j$. Since the mechanism is SP-L, this event is almost the same as the event $v_{(1)} > w_{(1)} > \sum_{j \in M} \frac{A_j}{A} v_j$ for all markets large enough. Since the mechanism is AAE, it prescribes trade in this event for all markets large enough and the highest valuation buyer (also the highest bidder by SP-L) is the winner. But since the mechanism respects BFW, $p^{m,n}(b, \vec{A}, a) \leq b_{(1)}$. So there exists a landowner j, the one who submitted the highest ask $a_{(1)}$, whose IPR is violated under the mechanism.

Next, consider any discriminatory compensation mechanism. For this mechanism to be SP-L, any landowner j's compensation in the large market limit mechanism must be independent of a_j and may only depend on the full support distributions f of bids and g of asks. So lets denote j's compensation in the limit mechanism as $p_j^\infty(g, f)$. Then there exists a landowner k for whom $p_k^\infty(g, f) < \bar{v}$; otherwise, the mechanism is not discriminatory. By AAE, the mechanism always prescribes trade in the limit market as the inequality $\bar{v} > \frac{\mathbb{E}[\mathcal{A}\mathfrak{a}]}{\mathbb{E}[\mathcal{A}]}$ holds because $(\mathcal{A}, \mathfrak{a})$ is distributed as g with full support on $\{A_{\min}, \ldots, A_{\max}\} \times \{1, \ldots, \bar{v}\}$. Consider the event $a_k > p_k^\infty(g, f)$ in the limit market. In this event, trade occurs but violates the IPR of landowner k in the limit market and therefore in nearby large markets. Q.E.D.

6 The Case of Few Buyers

The double auction proposed in Section 4 has good incentive properties when there are a meaningful number of traders both on the buyer and the seller side. Indeed, we know from Bulow and Klemperer (1996) that increasing competition on the buyer side is a valuable goal in itself. However, one is often faced with the empirical reality of a market with very few buyers. In this case, the choice of SP-L as a desiderata for judging buyers' incentives is problematic and theoretically unappealing.

We propose a modified double auction, $DA2$, for the environment with few buyers. Trade occurs if and only if $b_{(1)}A > \sum_{j\in M} a_j A_j$. If trade occurs, the highest bidder is allocated the land (ties among bidders at the highest bid are resolved by a lottery) and the uniform compensation paid to the landowners is

$$p = \max\left(b_{(2)}, \sum_{j\in M}\left(\frac{A_j}{A}\right)a_j\right)$$

When we want to assess the incentive properties of $DA2$ as the number of landowners grow but the number of buyers do not, we need to define the large market limit and notion of SP-L a bit differently from Section 4. The large market limit is now defined by letting traders on the landowners side grow large by drawing types in an i.i.d. fashion.

Definition 14. The large market limit of mechanism $\left((\Phi^{m,n})_{m\in\mathbb{N},n\in\mathbb{N}}, A_b, A_s\right)$ as the number of landowners grow is the function $\phi_s^{\infty} : A_s \times \Delta(A_s) \times A_b^n \mapsto X_S$ given by

$$\phi_s^{\infty,n}(A_j, a_j, g, b) = \lim_{m\to\infty} \phi_s^{m,n}(A_j, a_j, g, b)$$

if the limit exists.

The definition of strategy-proofness in the large (SP-L) for landowners also needs to be modified in view of our insistence on doing asymptotics only on the landowners side of the market.

Definition 15. A direct mechanism is strategy-proof in the large (SP-L) for landowners if $\forall b \in A_b^n$, $\forall g \in \bar{\Delta}(T_s)$ and $\forall \epsilon > 0$, $\exists \hat{m}$ such that $\forall m \geq \hat{m}$ and $\forall (A_j, a_j) \in T_s, \forall a_j' \in \mathbb{V}$, we have

$$\mathbb{E}u_j[\phi_s^{m,n}(A_j, a_j, g, b)|A_j, a_j] \geq \mathbb{E}u_j[\phi_s^{m,n}(A_j, a_j', g, b)|A_j, a_j] - \epsilon$$

If the mechanism has a large market limit, the above inequality is equivalent to

$$\mathbb{E}u_j[\phi_s^{\infty,n}(A_j, a_j, g, b)|A_j, a_j] \geq \mathbb{E}u_j[\phi_s^{\infty,n}(A_j, a_j', g, b)|A_j, a_j]$$

A similar modification is required in the definition of approximate efficiency.

Definition 16. A direct mechanism is asymptotically approximately efficient (AAE) if $\forall \epsilon > 0$, $\exists \hat{m} \in \mathbb{N}$ such that $\forall m \geq \hat{m}$, (i) the mechanism prescribes trade if $w_{(1)}A \geq \sum_{j=1}^{m} v_j A_j + \epsilon$ and only if $w_{(1)}A \geq \sum_{j=1}^{m} v_j A_j - \epsilon$; and (ii) the land is allocated to the buyer who has the highest valuation.

With the required definitions in place, Proposition 3 is our summary result about the performance of double auction $DA2$.

Proposition 3. *The double auction $DA2$ is budget-balanced, strategy-proof in the large for landowners, asymptotically approximately efficient, respects buyers' free will, respects collective property rights but does not respect individual property rights. Moreover, truth telling is a weakly dominant strategy for any buyer.*

Proof. The double auction $DA2$ is budget-balanced, respects buyers' free will and respects collective property rights by construction. Note that the large market limit of our double auction prescribes that trade happens if and only if $b_{(1)} \geq \frac{\mathbb{E}[\mathcal{A}\mathfrak{a}]}{\mathbb{E}[\mathcal{A}]}$. If trade occurs, the highest bidder is allocated the land and the uniform compensation paid to the landowners is

$$\lim_{m \to \infty} p = \max\left(b_{(2)}, \frac{\mathbb{E}[\mathcal{A}\mathfrak{a}]}{\mathbb{E}[\mathcal{A}]}\right)$$

Since both the occurrence of trade and the terms of trade are independent of any individual landowner's report, reporting truthfully is an optimal strategy for each landowner in the limit double auction. Hence the double auction $DA2$ is strategy-proof in the large for landowners. The argument for the assertion that truth telling is a weakly dominant strategy for any buyer in $DA2$ is analogous to the argument that truth telling is a weakly dominant strategy for any buyer in a second price auction and is therefore omitted. Example 2 with the observation that trade in $DA2$ occurs at a uniform compensation of 0.67 serves as an example that the $DA2$ does not respect individual property rights.

Since truth telling is weakly dominant strategy for any buyer in $DA2$, we have $b_{(1)} = w_{(1)}$. Choose $\hat{m}$ such that $\forall m \geq \hat{m}$, $0 \leq \sum_{j=1}^{m}(a_j - v_j)A_j < \epsilon$. The argument in the proof of Lemma 7 goes through. This shows that $DA2$ is asymptotically approximately efficient as well. Q.E.D.

7 Simulations

It would be interesting to see in some instances of the model, what is the probability of trade under the mechanisms considered in the paper. Moreover, even though the proposed mechanisms violate individual property rights by design, it would still be informative to look at the percentage of landowners whose property rights are respected, conditional on trade. In this section, we simulate the double auction mechanisms $DA1$ and $DA2$ proposed in the paper to get some insight into the above two variables of interest. The simulation parameters will be the distributional parameters.

Suppose each buyer's bid price is independently and identically drawn (i.i.d.) from density $f(b)$, each landowner's ask price and area of his land parcel are i.i.d. from joint density $g(a, A)$. We specify both f and g as truncated normal distributions where the respective truncation bounds are determined by the bounds on bid prices, ask prices and areas. More specifically,

$$g = \mathbf{N}_{[\underline{a},\overline{a};\underline{A},\overline{A}]}\left(\left(\begin{array}{c}\mu_a\\ \mu_A\end{array}\right),\left(\begin{array}{cc}\sigma_a^2 & \sigma_{aA}\\ \sigma_{aA} & \sigma_A^2\end{array}\right)\right) \quad \text{and} \quad f = \mathbf{N}_{[\underline{b},\overline{b}]}(\mu_b,\sigma_b^2)$$

where μ's denote the means, σ^2's the variances and σ_{aA} the covariance of the underlying distributions. We set the default values of parameters as follows: $\underline{a} = \underline{b} = 1$, $\overline{a} = \overline{b} = 10$, $\underline{A} = 1$, $\overline{A} = 4$, $\mu_a = \mu_b = 3$, $\sigma_b^2 = 1$, $\sigma_a^2 = 2$, $\sigma_A^2 = 2$ and $\sigma_{aA} = -0.2$. The other parameters are the size of the trading problem (m, n) where m is the number of landowners and n is the number of interested buyers.

The simulations are done as follows. We first draw an i.i.d. sample of n bid values $(b_1, \ldots, b_n)$ from density f and m ask-parcel area pairs $((a_1, A_1), \ldots, (a_m, A_m))$ from density g with the densities as specified above. For this sample, we evaluate whether or not trade happens according to the double auction trading rules. If trade happens in this sample, we compute the proportion of landowners whose individual property rights are respected i.e. $\frac{\#\{j \in M : p > a_j\}}{m}$. We repeat the aforementioned steps for $k = 200$ independent samples. The probability of trade, denoted $Pr(trade)$, is calculated as the proportion of k samples in which trade happened. The individual property rights (IPR) performance, measured by proportion of landowners whose individual property rights are satisfied and denoted $\%IPR$, is calculated as the average across the samples where trade did happen.

The simulation results (the top left panel in both figures) show that the probability of trade increases as the difference $\mu_b - \mu_a$ between mean bid price and mean ask price increases. This is expected because it increases the opportunities for gains from trade. However, the IPR performance of $DA1$ on the improves sharply with the increased spread and contrasts with that of $DA2$ which does not improve by much. This is attributable to the different compensation rules in these auctions.

It is interesting to look at the impact of heterogeneity of landowners in terms of their ask prices i.e. σ_a^2 and their parcel areas i.e. σ_A^2 on our two variables of interest. Simulations results displayed in top right panels of Figure 1 and 2 indicate that as the variance of landowners' ask price increases, the probability of trade decreases and the IPR performance decreases as well. This is true for both $DA1$ and $DA2$. However, the heterogeneity in terms of parcel areas does not show a clear relation with either the probability of trade or the IPR performance. This is manifest from the bottom left panels in Figure 1 and 2.

Lastly, it is of interest to look at how the covariance σ_{aA} of ask price and parcel area impact the two variables. The bottom right panels in Figure 1 and 2 show that as the covariance increases, both the probability of trade and the IPR performance decreases. This makes intuitive sense as an environment where landowners who own larger land parcels quote smaller asks and those with smaller land parcels quote higher asks is more conducive for trade and individual property rights protection than an environment in which the opposite relation holds.

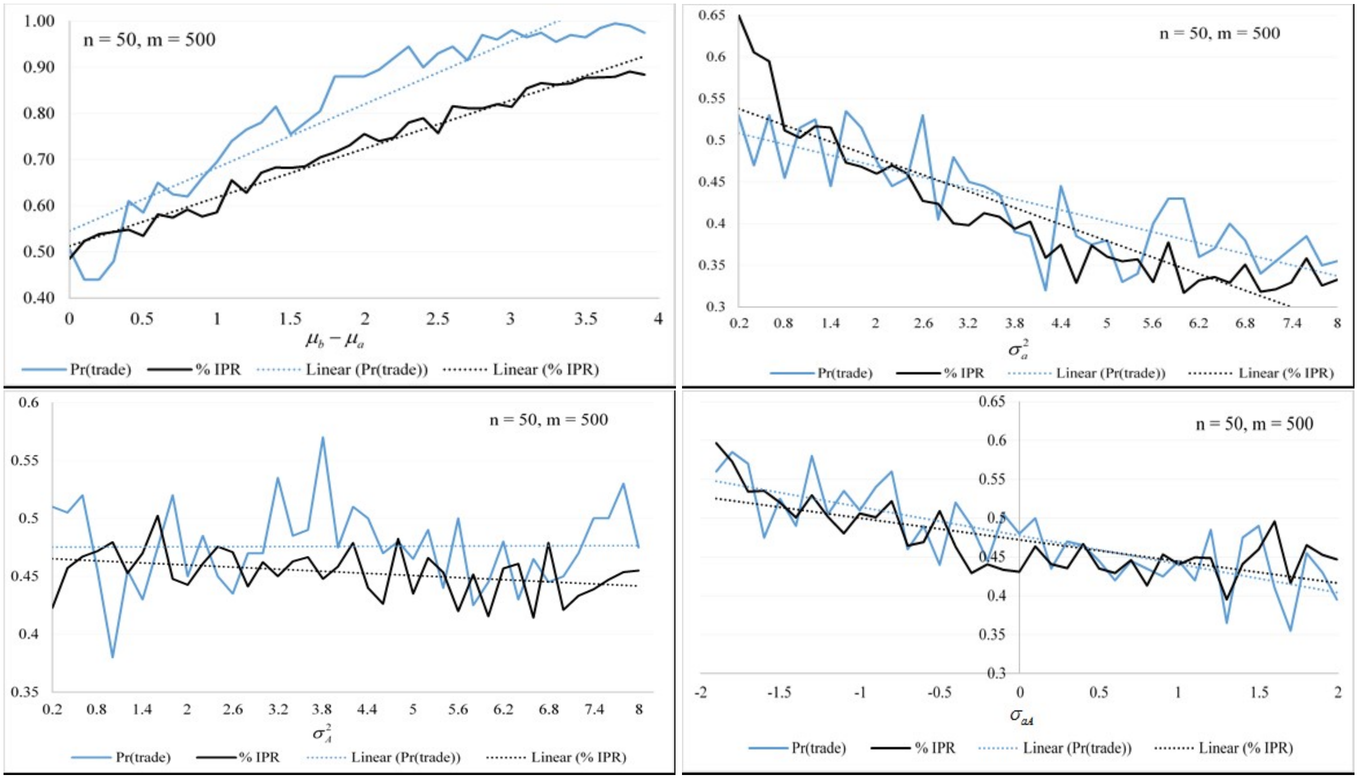

Figure 1: Simulations for Double Auction $DA1$. The solid line plot is actual simulation plot while the dashed line is a linear trend line. Simulation parameter in (a) top left panel is $\mu_b - \mu_a$; (b) in top right panel is variance σ_a^2 of asks; (c) in bottom left panel is variance σ_A^2 of parcel areas; and (d) in bottom right panel is the covariance σ_{aA} of ask and parcel areas. In top left panel, the origin denotes $\mu_b = \mu_a = 3$.

8 Concluding Remarks

Land assembly environments are settings with a large expanse of land but with fragmented ownership and therefore with a large number of landowners. In this paper, we proposed and studied double auction mechanisms for two kinds of land assembly environments. One, in which, the number of buyers is meaningfully large and the other, in which that is not the case. The double auction $DA1$ was proposed for the first kind of environment and the double auction $DA2$ for the second kind. The mechanisms we suggested are based on the idea that complementary assets have unified ownership for efficient trading decisions. These double auctions were judged on a number of market design criteria. We also show that satisfying all these criteria at once is impossible.

A number of questions suggest themselves for further inquiry. We think it would be interesting to do Bayesian equilibrium analysis of the double auction mechanisms suggested in this paper to quantify the inefficiency and scope for strategic manipulation in case of small markets. Further research is needed to study land assembly problem in other interesting environments. One setting is when the buyer is content with getting less than the full expanse of land. This also introduces the possibility of collusion which was absent in the formulation

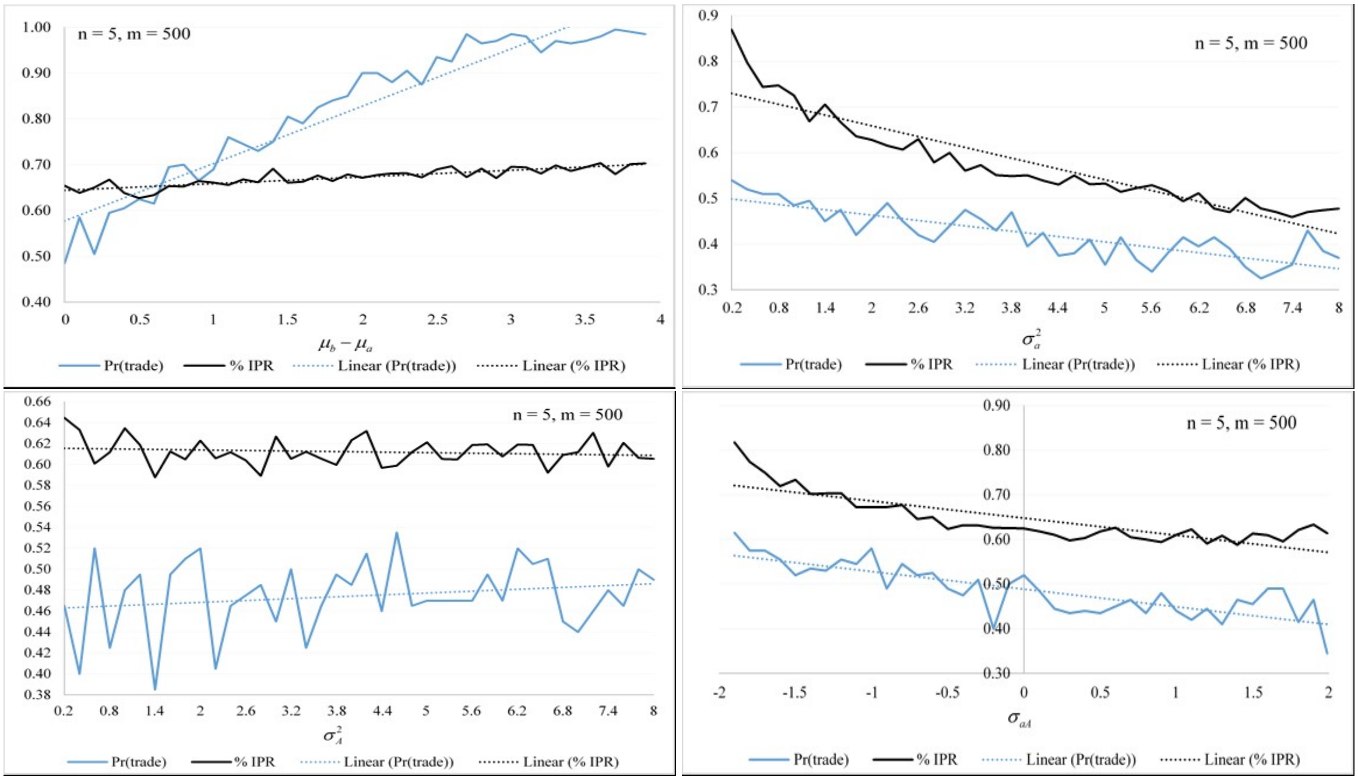

Figure 2: Simulations for Double Auction $DA2$. The solid line plot is actual simulation plot while the dashed line is a linear trend line. Simulation parameter (a) in top left panel is $\mu_b - \mu_a$; (b) in top right panel is variance σ_a^2 of asks; (c) in bottom left panel is variance σ_A^2 of parcel areas; and (d) in bottom right panel is the covariance σ_{aA} of ask and parcel areas. In top left panel, the origin denotes $\mu_b = \mu_a = 3$.

of this paper since anything less than the full stretch of land was of no value to the buyer. The design of collusion-proof mechanism would be an important challenge in that model. Another interesting setting is one in which no assembly is not an option. Such settings arise when considering land assembly for developing public transportation systems and other critical infrastructure in which there is little flexibility in the choice of land and it has been decided that it is welfare optimal for the larger society that assembly take place.

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