Coalition Formation in Public Good Games

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Abstract

Commitment devices such as coalitions can increase outcome efficiency in public goods provision. This paper incorporates social preference into a two stage public good game where heterogeneous agents first choose whether or not to join a coalition, and in the next stage the coalition votes on whether its members will contribute. The findings show that individuals who place more weight on social preference are more likely to join the coalition and vote for the coalition to contribute to the public good. Increasing benefits of cooperation/contribution leads to more people joining the coalition and contributing to the public good.These results hold true for various kind of voting rules.

Keywords: Coalition, Social Preference, Public Good

JEL Classification: H4, C7

1 Introduction

Market produces efficient outcomes for private goods due to excludability and rivalry. Public goods and common resources problems suffer from the free riding problem, however, and standard models predict inefficient outcomes. Experiments, however, suggest cooperation does exist and contribution rates towards public goods are 40-60 percent of the optimal level. Social preferences such as warm glow, altruism, inequity aversion, etc has also been used to explain the differences between outcomes and standard theory.

Coalitions have the potential to increase cooperation. However, higher benefits of cooperation or marginal propensity of consumption rate (MPCR) increases incentives for an individual to free ride in a public good game. Also the usual public goods model depict an inverse relation between MPCR and the coalition size, however this is in contrast to recent experiments and large size international Environmental Agreements (IEA).

Incorporating social preferences with coalition formation in a public good game, I explore the existence of large sized coalitions. Individuals are assumed to be inequity averse in this model. i.e. their payoffs are strictly increasing in the payoff of the least well off member of the society. This means that individual's utility function is a weighted average of the sum of their pecuniary payoffs and their aversion to inequity, the latter being a function of the minimum payoff received amongst all the individuals.

An individual's weights on monetary payoff and social preference is private information. Using backward induction in a two stage game of incomplete information, I establish conditions on social preferences under which individuals would join the coalition. Increase in MPCR leads to more people satisfying the cutoff and joining the coalition. Increase in MPCR also translates to higher likelihood of the coalition contributing to the public good.

2 Related Work

The existence of cooperation can be due to warm glow and altruism as shown inAndreoni (1995). Ostrom et al. (1994) report that punishment on excess use of common property resources leads to higher cooperation. Coalition formation is also a method well reported in literature to increase the membership. Barrett (1994) and Hoel and Schneider (1997) suggest that coalition incentivizes cooperation among coalition members and contribution rates increase as a result, hence solving the free riding problem to an extent. Also, Finus and Maus (2008) suggested that coalition can attract more members by lowering the required public good provisioning level. According to Fehr and Fischbacher (2002), local resources are managed well when users of common –pool resources organize and enforce their own rules, instead of following externally imposed norms.

Isaac and Walker (1988) provide evidence for existence of free riders,although less than the theoretical predictions. The authors demonstrate that marginal propensity of consumption rate (MPCR)is the primary determinant of contribution and there is no group size effect. High benefits of cooperation (MPCR) are associated with small coalition size as depicted by the usual public goods model (Komisar (1969)). Barrett (1994) shows that there is an inverse relationship between equilibrium coalition size and gains from cooperation. When there is more to be gained from cooperation, then the minimum viable coalition is smaller, whereas when gains are low, it takes more coalition members to abate. Burger and Kolstad (2009) confirm existence of large size coalition in an experimental setup which also, defies their theoretical model. Dannenberg, Lange and Sturm (2014) prove in an experimental setup that institutions that allow members to endogenously determine the terms of agreement may attract more members.

New theory models incorporate social behavioral components in the public good setup to explain cooperation. For instance, International Environmental Agreements (IEA) incur additional costs (Hoel and Schneider (1997)) to conform to social norms and convention as they play an important role in sustaining IEA. Rabin (1993) showed that by incorporating reciprocity, public goods game can be interpreted as coordination problem where we want to achieve full contribution. Falk and Fischbacher (2006); Charness and Rabin (2002) explain cooperation through equitable distribution and belief formation.

Kolstad and Ulph (2011) and Kolstad (2014) study how social preferences influences coalition formation and find that inclusion of social preferences lowers the threshold for contributing to the public good. In their model, for a given MPCR inclusion of social preferences reduces the size of the coalition. This is in contrast to what the above experimental results suggest. In this paper,I include social preferences in a public good framework and introduce heterogeneity through the weights placed by individuals on pecuniary payoff and social preference. By changing the MPCR, I want to show the existence of large sized coalitions.

Coalition formation in presence of social preferences can broaden the existing literature and answer the existence of large sized coalition as depicted in the aforementioned experiments. Heterogeneity of agents in terms of the different weights on social preferences can also help explain the likelihood by which an individual will join the coalition and vote to contribute for a public good.

3 Model

There are N individuals in the economy, all of whom are endowed with 1 unit. The model has a continuous public good with binary contributions, i.e. increasing in the contributions made by individuals and each individual has the following choice set $e_i = \{0, 1\}$, where e_i is the endowment individual has. $e_i = 0$ means the individual does not contribute for the public good and $e_i = 1$ is when he/she contributes to the public good.

Decisions are made in a two stage game, where players make a decision about joining the coalition in Stage I and in Stage II, coalition votes whether its member will contribute using majority rule. Coalition is a group of people in the society which takes collective decision. Coalition size is depicted by M and number of fringe members by F; $M + F = N$. Fringe members decide independently to contribute or not. The payoff function in the model is given as follows:

$$\begin{cases} \pi_i = \lambda_i(\gamma Q) + (1-\lambda_i)\gamma Q & \text{for } e_i = 1 \\ \pi_i = \lambda_i(1+\gamma Q) + (1-\lambda_i)\gamma Q & \text{for } e_i = 0 \end{cases} \tag{1}$$

where, λ_i is the weight on the monetary payoff and $1 - \lambda_i$ is the weight on the social preference. In a usual public goods game, we have $\lambda_i = 1$ and the utility function only comprises of the first term: the monetary or pecuniary payoff. In my model, I incorporate social preferences such that weights placed by an individual is private information in the game. Term adjacent to λ_i is the pecuniary or the monetary payoff the individuals receive.

Term adjacent to 1-λ_i depicts the social preference in the model. This is done using Rawlsian inequality which is the minimum payoff received by anyone in the population. By the definition, minimum payoff will be γQ, where again γ is the benefit of cooperation and Q is the number of contributors or the total contributions being made.

If the individual's choice contribution strategy is $e_i = 1$ then pecuniary payoff is γQ, where γ is the benefit of cooperation (MPCR) and Q is the sum of the total contributions being made. People who are free riding i.e. have the strategy $e_i = 0$, receive $1+\gamma Q$ as their pecuniary payoff. They have their endowment of 1 and the benefit from the contributions made by others. Since contributions are binary, Q can also be interpreted as the number of contributors to the public good.

We solve our model of incomplete information using backward induction. In Stage II the individuals know the size of the coalition. The coalition members decide using majority rule if the coalition contributes to the public good and fringe members decide independently if they would like to contribute. We solve for Stage II by comparing the expected payoff from contributing v/s expected payoff from not contributing.

4 Stage II: Contribute or not for the public good

Suppose that a coalition of size M has been formed in Stage 1 and for simplicity assume that M is odd, hence the majority is given by: $m' = \frac{M+1}{2}$. Within the coalition the decision to contribute is made by majority voting. Let F' fringe members be contributing for the public good, where $F' \subset F$.

Due to private information, individuals do not know the weight people place on pecuniary payoff and also on the social preference. For the expected payoff, we need the probability of contribution for every individual. Heterogeneity in preferences leads to a different probability for contributing to the public good. We order the probability of contributing to the public good as: p_1, p_2, p_3....$p_{m'}$.....p_M. Here p_1 is the probability of the person who is most likely to contribute to the public good. p_M is the probability of the person who is least likely to contribute to the public good and let it be represented by p. For technical reasons and without loss of generality, we substitute all the probabilities with p_M i.e. p. Substituting with the least probability i.e p will give us the least expected payoff from contributing.

p is the least probability of a coalition member saying yes for contribution towards the public good. It follows a binomial distribution and $0 \leq p \leq 1$. A coalition member will contribute for the public good if the payoff from contribution is at-least equal to the payoff from not contributing

Now we define the terms which we will be using in the analysis when individual 'i' votes yes to contribute to public good:

$$\theta^v = \left[\binom{M-1}{m'-1} (p)^{m'-1}(1-p)^{M-m'} + \binom{M-1}{m'} (p)^{m'}(1-p)^{M-m'-1} + ... \binom{M-1}{M-1} (p)^{M-1} \right]$$
$$= \left[\sum_{i=m'-1}^{M-1} \binom{M-1}{i} (p)^i (1-p) \right] \tag{2}$$

$$\eta^v = \left[\binom{M-1}{0} (1-p)^{M-1} + \binom{M-1}{1} (p)(1-p)^{M-2} + \cdots \binom{M-1}{m'-2} (p)^{m'-2}(1-p)^{M-m'+1} \right]$$
$$= \left[\sum_{i=0}^{m'-2} \binom{M-1}{i} (p)^i (1-p)^{M-(i+1)} \right] \tag{3}$$

θ^v reflects the term where at least m'-1 other players vote yes to contribute for the public good. Also, a yes by individual "i" will lead to at least the majority voting yes to contribute and hence the coalition will contribute for the public good.

η^v reflects the term where majority of the population does not vote to contribute for the public good. It lists all the cases from no one voting to contribute to the cases where majority is falling short by one vote. Although the individual "i" wants to contribute, majority is not matched and hence the coalition does not contribute.

Now we define the terms which we will be using in the analysis when individual "i" does not vote yes to contribute to public good:

$$\begin{aligned}\theta^{nv} &= \left[\binom{M-1}{m'}(p)^{m'}(1-p)^{M-m'-1} + \binom{M-1}{m'+1}(p)^{m'+1}(1-p)^{M-m'-2} + \ldots\ldots\binom{M-1}{M-1}(p)^{M-1}\right] \\ &= \left[\sum_{i=m'}^{M-1}\binom{M-1}{i}(p)^{i}(1-p)^{M-i-1}\right]\end{aligned} \tag{4}$$

$$\begin{aligned}\eta^{nv} &= \left[\binom{M-1}{0}(1-p)^{M-1} + \binom{M-1}{1}(p)(1-p)^{M-2} + \cdots\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\right] \\ &= \left[\sum_{i=0}^{m'-1}\binom{M-1}{i}(p)^{i}(1-p)^{M-(i+1)}\right]\end{aligned} \tag{5}$$

θ^{nv} reflects the term where at least the majority is voting to contribute. Since the individual "i" does not vote to contribute the majority is determined by at least m' voters. Although the individual does not want to contribute, the coalition contributes as the majority rule is satisfied. η^{nv} shows the terms where the majority voting is not satisfied. It shows all the cases where no one votes and the majority falling short by one.

Payoff when the individual "i" in a coalition of size M says yes to contribution:

$$\pi^v = [\theta^v]\,[\lambda_i(\gamma(M+F')) + \delta_i(\gamma(M+F'))] + [\eta^v]\,[\lambda_i(1+\gamma F') + \delta_i\gamma(F')]$$
$$= [\theta^v]\,[\gamma(M+F')] + [\eta^v]\,[\lambda_i + \gamma F'] \tag{6}$$

The term adjacent to θ^v is the payoff the individual receives when the coalition of size M and F' fringe member contribute to public good. The term adjacent to η^v is the payoff when the coalition is not contributing as the majority is not satisfied and only the F' fringe members contribute to the public good.

Now we compare this to the payoff the individual "*i*" receives when h/she does not vote to contribute:

$$\pi^{nv} = [\theta^{nv}]\,[\lambda_i(\gamma(M+F')) + \delta_i(\gamma(M+F'))] + [\eta^{nv}]\,[\lambda_i(1+\gamma F')_{+}\delta_i\gamma(F')]$$
$$= [\theta^{nv}]\,[\gamma(M+F')] + [\eta^{nv}]\,[\lambda_i + \gamma F'] \tag{7}$$

The term adjacent to θ^{nv} is the payoff individual receives when the coalition of size M and F' fringe members contribute. Again, we are depicting the fringe members who contribute by F' and we do not make any assumption on the number of fringe members contributing towards the public good. The term adjacent to η^{nv} is the payoff one receives when the coalition is not contributing as the majority rule is not satisfied and only the fringe members(F') contribute towards the public good.

An individual will compare π^v and π^{nv} to decide whether to contribute or not. After comparison we observe that $\pi^v \geq \pi^{nv}$ if:

$$\left[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\right][\gamma(M+F')-(\lambda_i+\gamma F')] \geq 0 \tag{8}$$

$$\lambda_i \leq \gamma M \tag{9}$$

We can interpret equation 8 as:

$\left[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\right][\gamma(M+F')]$ -$\left[\binom{M-1}{m'-1}(p)^{m'-1}(1-p)^{M-m'}\right][(\lambda_i+\gamma F')]$

The first term depicts the expected payoff when individual "i" votes to contribute. Since already m'-1 people are contributing, vote by "i" leads to majority and hence the coalition contributes and everyone in the coalition receives the payoff $\gamma(M+F')$. The second term depicts the expected payoff when the coalition of size M does not contribute. This is because individual "i" does not vote and hence the coalition only receives m'-1 votes and the majority is not satisfied. The resulting payoff is $\lambda_i + \gamma F'$, when the coalition is free riding. Thus here individual i is the pivotal voter and equation 8 shows the least expected marginal benefit a pivotal member receives from contributing.

The result is also intuitive since the pivotal member's vote decides whether a coalition will contribute or not. In the model if an agent is not pivotal, and the coalition contributes, his/her payoff is the same if he/she votes to contribute or not i.e $\gamma(M + F')$. Similarly if the coalition does not contribute, his/her payoff will be $\lambda_i + \gamma F'$ irrespective of his/her vote. Whereas if a pivotal member votes to contribute for public good, which in turn leads to a coalition contributing, the payoff of pivotal voter (or other members) will be $\gamma(M+F')$. If the pivotal voter does not vote to contribute and as a result the coalition also does not contribute, the resultant payoff will be $\lambda_i + \gamma F'$.The gain from voting for the pivotal voter is $\gamma M'$ and the gain from not voting is λ_i .

An individual will vote to contribute if the gain from voting is more than the gain from not voting to contribute which leads us to the threshold $\lambda_i \leq \gamma M$ in equation 9

Let $\lambda \hat{=} \gamma M$ If $\lambda_i \leq \hat{\lambda}$ then the individual votes yes to contribute . As we increase γ (benefit from cooperation), the cutoff increases and hence the probability of voting yes for contribution also increases. This leads to the first proposition of the paper :

Proposition I: If $\lambda_i \leq \hat{\lambda}$, where $\hat{\lambda} = \gamma M$,then the individual votes yes to contribute for the public good. If $\lambda_i \geq \hat{\lambda}$ then the individual does not vote yes to contribute for the public good. Increase in γ leads to higher likelihood of an individual contributing.

If the given cutoff is satisfied, the individual will vote yes to contribute for the public good. As we increase the benefit of cooperation (γ), the cutoff changes such that the probability of voting yes for contribution increases. The cutoff is independent of the action of fringe members. This can be because individual's payoffs is increasing in the minimum payoff received amongst individuals. The payoff will be highest when everyone is contributing and is independent of the actions of fringe members.

Now we analyze the cutoff rule or threshold for the fringe members. We will be checking for the behavior of one of the fringe members amongst F' members. A fringe member "*i* will contribute if the payoff from contribution is at-least greater than the payoff received from not contributing. By stage

II , everyone knows the size of the existing coalition.

The payoff a fringe member "i" receive when h/she decides to contribute:

$$\begin{aligned} \pi^c &= [\alpha]\,[\lambda_i(\gamma(M+F')) + \delta_i\gamma(M+F')] + [\beta]\,[\lambda_i(\gamma F')_{+}\delta_i\gamma(F')] \\ &= [\alpha]\,[\gamma(M+F')] + [\beta]\,[\gamma F'] \end{aligned} \tag{10}$$

In the above equations α depicts the case when coalition of size M is contributing for the public good as the majority rule is satisfied. The corresponding term is the payoff received when a coalition of size M and F' fringe members are contributing. β depicts when the coalition of size M is not contributing as majority rule is not satisfied. Thus only F' fringe members are contributing. The adjacent term is the payoff the fringe member receives when h/she is contributing with the other fringe members.

The payoff a fringe member receives when they decide not to contribute for the public good:

$$\begin{aligned} \pi^{nc} &= [\alpha]\,[\lambda_i(1+\gamma(M+F'-1)) + \delta_i\gamma(M+F'-1)] + [\beta]\,[\lambda_i(1+\gamma(F'-1)_{+}\delta_i\gamma(F'-1)] \\ &= [\alpha]\,[\gamma(M+F'-1)+\lambda_i] + [\beta]\,[\lambda_i+\gamma(F'-1)] \end{aligned} \tag{11}$$

α and β are defined as above, since the individual is not contributing, size of the fringe members is reduced to F'-1. The term adjacent to α is the payoff individual receives when h/she is free riding and coalition of size M and F'-1 fringe members are contributing. The term corresponding to β is the payoff individual receives when neither the coalition of size M and the individual "i" contributes. The individual free rides on the contribution of F'-1 fringe members.

An individual will compare π^c and π^{nc} to decide whether to contribute or not as a fringe member. After comparison we see that $\pi^c \geq \pi^{nc}$ if:

$$\lambda_i \leq \gamma \tag{12}$$

We can interpret equation 12 as the comparison between gain from contribution and the gain from begin a free rider. A fringe member's contribution does not influence the decision of coalition, thus their marginal gain from free riding will be λ_i . Similarly the marginal gain from contribution will be γ which is the MPCR or additional gain to a fringe member if he/she contributes. An individual contributes if gain from contribution is higher than the benefit from free riding i.e. $\lambda_i \leq \gamma$ which is the threshold in the above equation.

Let $\hat{\lambda} = \gamma$, if $\lambda_i \leq \hat{\lambda}$ is satisfied then the fringe member will contribute for the public good. This leads to our second proposition:

Proposition II: If $\lambda_i \leq \hat{\lambda}$, where $\hat{\lambda} = \gamma$, the fringe member will contribute for the public good. Increase in γ leads to higher likelihood of the fringe member contributing for the public good.

As γ increases, the cutoff will increase. This will lead to an increase in the likelihood that fringe member will contribute for the public good. The cutoff is independent of the coalition members. This might be because their payoff is an increasing function of the minimum payoff received by anyone. In order to increase the minimum payoff, which will be highest when everyone is contributing, the fringe members contribute irrespective of the coalition members.

5 Stage I: Decision to join the coalition

In the model, an individual will join coalition if the payoff from joining the coalition is at-least greater than the payoff from not joining the coalition. Notice that according to the above definition, member will join the coalition when they are indifferent between joining and not joining. M is any size of the coalition which will be formed and as we will see M will not affect the decision of joining the coalition. From stage II, we know the cases when a fringe member will contribute and also the cases when the coalition members will be contributing for the public good. Based on Stage II results, we can have three cases based on values of λ_i. Based on the cutoff $\lambda_i \leq \gamma M$ (the coalition members vote yes to contribute) and $\lambda_i \leq \gamma$(the fringe members will contribute), we have the following three cutoffs:

1)$\lambda_i \leq \gamma$ which also implies $\lambda_i \leq \gamma M$

2)$\gamma < \lambda_i \leq \gamma M$

3)$\lambda_i > \gamma M$

As we discussed in stage II, a coalition decides to contribute or not is based on majority rule, thus if there are people who have the cutoff $\lambda_i \leq \gamma M$ in majority, then the coalition as a whole will contribute.

λ_i is assumed to be uniformly distributed between 0 and 1. Hence the probability $\lambda_i \leq \gamma M$ is γM and the probability of $\lambda_i > \gamma M$ is $1 - \gamma M$. We assume for simplification that M is odd here. Thus majority rule means that at least $\frac{M+1}{2} = m'$ people should vote to contribute for the public good to be provided by a coalition of size M. The solutions to three cases will be discussed after describing the payoffs in all the cases. Again we do not make any restriction on Fringe members, using information from Stage II, we depict the fringe members who are contributing $(\lambda_i <)$ as F'.

Here we are solving for the cases when $\gamma M < 1$

Case I: $\lambda_i \leq \gamma$:

Here we solve for the case where an individual "i" satisfies the above cutoff.

The payoff when the individual "i" decides to join the coalition:

$$\pi_i = [\phi]\,[(\gamma(M + F')] + [\psi]\,[\lambda_i + \gamma F'] \tag{13}$$

where,

$$\phi$$

$$= \left[\binom{M-1}{m'-1}(\gamma M)^{m'-1}(1-\gamma M)^{M-m'} + \binom{M-1}{m'}(\gamma M)^{m'}(1-\gamma M)^{M-m'-1} + \cdots \binom{M-1}{M-1}(\gamma M)^{M-1}\right]$$

$$= \left[\sum_{i=m'-1}^{M-1}\binom{M-1}{i}(\gamma M)^i(1-\gamma M)^{M-i-1}\right] \tag{14}$$

ψ

$$= \left[\binom{M-1}{0}(1-\gamma M)^{M-1} + \binom{M-1}{1}(\gamma M)(1-\gamma M)^{M-2} + \cdots \binom{M-1}{m'-2}(\gamma M)^{m'-2}(1-\gamma M)^{M-m'+1}\right]$$

$$= \left[\sum_{i=0}^{m'-2} \binom{M-1}{i}(\gamma M)^{i}(1-\gamma M)^{M-(i+1)}\right] \quad (15)$$

ϕ reflects the term where at-least m'-1 want to contribute for the public good. When the individual "i" joins the coalition, there will be majority of the people who would want to contribute as h/she satisfies the cutoff($\lambda_i \leq \gamma M$). The adjacent term shows the payoff as a result of the coalition of any size M and F' fringe members contributing. F' are the fringe members who contribute and have the cutoff $\lambda_i \leq \gamma$.

ψ reflects the term where majority of the population does not want to contribute for the public good, irrespective of the player i's decision. As a result the coalition votes to not provide the public good. The adjacent term is the payoff when the coalition is not contributing and only the fringe members who meet the cutoff(F') contributes.

The payoff when the member decides to not join the coalition:

$$[\phi'] \left[(\gamma(M + F')\right] + [\psi'] \left[\gamma(F' + 1)\right] \tag{16}$$

where,

$$\phi'$$

$$= \left[\binom{M-1}{m'-1}(\gamma(M-1))^{m'-1}(1-\gamma(M-1))^{M-m'} + \binom{M-1}{m'}(\gamma(M-1))^{m'}(1-\gamma(M-1))^{M-m'-1} + \cdots \binom{M-1}{M-1}(\gamma(M-1))^{M-1}\right]$$

$$= \sum_{i=m'-1}^{M-1} \binom{M-1}{i}(\gamma(M-1))^{i}(1-\gamma(M-1))^{M-i-1} \tag{17}$$

$$\psi'$$

$$= \left[\binom{M-1}{0}(1-\gamma(M-1))^{M-1} + \binom{M-1}{1}(\gamma(M-1))(1-\gamma(M-1))^{M-2} + \cdots \binom{M-1}{m'-2}(\gamma(M-1))^{m'-2}(1-\gamma(M-1))^{M-m'+1}\right]$$

$$= \left[\sum_{i=0}^{m'-2} \binom{M-1}{i}(\gamma(M-1))^{i}(1-\gamma(M-1)^{M-(i+1)}\right] \tag{18}$$

If one person out of the M does not want to join the coalition, the coalition size reduces to M-1. Hence the probability of a coalition member contributing, is given by the cutoff $\lambda_i \leq \gamma(M-1)$ which is $\gamma(M-1)$. The probability of a coalition member not contributing is given by the cutoff: $\lambda_i > 1-\gamma(M-1)$ i.e. $1-\gamma(M-1)$.

The individual in this case, satisfies: $\lambda_i \leq \gamma$, thus he/she will contribute as a fringe member, and as a result the size of fringe member increases to F'+1. In 16 ϕ' depicts the cases when at-least majority of the $M-1$ members of the coalition want to contribute and the adjacent term is the payoff received when a coalition of size M-1 and F'+1 fringe members are contributing. ψ' reflects the term where majority of the M-1 members are not contributing and the adjacent terms reflects the payoff when only F'+1 fringe members are contributing.

An individual will compare 13 and 16 to decide whether to contribute for the public good or not.

Case II:$\gamma < \lambda_i \leq \gamma M$

Now the individual will not be contributing as a fringe member because the cutoff is not satisfied. The payoff when the member decides to join the coalition:

$$\pi_i = [\phi]\left[(\gamma(M+F')\right] + [\psi]\left[\lambda_i + \gamma F'\right] \tag{19}$$

ϕ and ψ are expressed by 14 and 15.

The payoff when the member decides to not join the coalition:

$$\pi_i = [\phi'] [\lambda_i + (\gamma(M - 1 + F')] + [\psi'] [\lambda_i + \gamma(F')] \tag{20}$$

ϕ' and ψ' are expressed by 17 and 18. In 20, the term adjacent to ϕ' the payoff received by the individual when coalition of size M-1 and F' fringe members are contributing. The term adjacent to ψ' depicts the payoff the individual receives when only F' fringe members contribute for the public good.

An individual will compare 19 and 20 to decide whether to contribute for the public good or not.

Case III: $\lambda_i > \gamma M$

In this case, individual will not be willing to contribute even if h/she joins the coalition. Although having people who satisfy $\lambda_i \leq \gamma M$ in majority (at-least m'), will make the coalition contribute for the public good. The payoff when the member decides to join the coalition:

$$\pi_i = [\phi''] [(\gamma(M + F')] + [\psi''] [\lambda_i + \gamma F'] \tag{21}$$

here:

$$\phi'' = \left[\binom{M-1}{m'}(\gamma M)^{m'}(1-\gamma M)^{M-m'-1} + \binom{M-1}{m'+1}(\gamma M)^{m'+1}(1-\gamma M)^{M-m'-2} + \right.$$
$$\left. \cdots \binom{M-1}{M-1}(\gamma M)^{M-1}\right]$$
$$= \sum_{i=m'}^{M-1}\binom{M-1}{i}(\gamma M)^{i}(1-\gamma M)^{M-i-1} \tag{22}$$

$$\psi'' = \left[\binom{M-1}{0}(1-\gamma(M-1))^{M-1} + \binom{M-1}{1}(\gamma(M-1))(1-\gamma(M-1))^{M-2} + \right.$$
$$\left. \cdots \binom{M-1}{m'-1}(\gamma(M-1))^{m'-1}(1-\gamma(M-1))^{M-m'}\right]$$
$$= \left[\sum_{i=0}^{m'-1}\binom{M-1}{i}(\gamma(M-1))^{i}(1-\gamma(M-1)^{M-(i+1)}\right] \tag{23}$$

ϕ'' reflects the term where majority of the coalition wants to contribute for the public good and the adjacent term shows the payoff when the coalition of size M and F' fringe members contribute for the public good. ψ'' is the term depicting the cases when majority is not in favor of contribution and the adjacent term is the payoff received when only the fringe members are contributing.

The payoff when the member decides to not join the coalition:

$$[\phi']\,[\lambda_i + (\gamma(M-1+F')] + [\psi']\,[\lambda_i + \gamma(F')] \tag{24}$$

ϕ' and ψ' are expressed by 17 and 18.

An individual will compare 21 and 24 to decide whether they want to contribute for the public good. I now discuss the solution for all the above 3 cases. In order to analyze the 3 cases, we need to study them separately when $\gamma M < 1$ and when $\gamma M \geq 1$.

Solution for: $\gamma M < 1$

In order to arrive at a solution, I use the expected value of λ_i. Since λ_i is uniformly distributed between 0 and 1, expected value of $\lambda_i = 1/2$. I take γ to be given as in the usual public goods game. Using the inequality $\lambda_i \leq \gamma M$ and $\lambda_i > \gamma M$, expected value of M is derived. Using this information we can analyze whether an individual would join a coalition or not based on the payoffs specified in the three cases above.

Taking $\lambda = 1/2$ we look at the 3 specific cases mentioned above. Since we have specified M is an odd number, we need M to be at least 3 in our analysis. Given M is at least 3, $\lambda = 1/2$ and $\gamma M < 1$, we can not have $\gamma > 0.5$, hence the first case is not possible under this scenario.

Result for case 2:$\gamma < \lambda_i \leq \gamma M$

Taking $\lambda = 1/2$ and $\gamma = 0.1$, possible values of M given $\lambda_i \leq \gamma M < 1$ are 5, 7 and 9. Comparing 19 and 20 we find that individuals will not join the coalition. The results hold true for $\gamma = 0.17$, where expected value of M is

3 and 5. The highest value of γ given $\lambda = 1/2$ and M at least 3, is $\gamma < 0.4$, for which also the results hold true. Taking $\gamma = 0.07$, we get that expected value of M lies between 7 and 13. For $\gamma = 0.05$, we can get expected value of M to be between 11 and 19. For both of these cases we again have the result that individual will not join the coalition. The result was analyzed for other values of γ and the individual will not join the coalition in case II.

Result for case 3:$\lambda_i > \gamma M$

Taking $\lambda = 1/2$ and given M should be at least 3 and $1 > \lambda_i > \gamma M$. We have that γ should be at most 0.16. Thus taking $\gamma = 0.16$ and $M = 3$, we arrive at the result where the individual will not join the coalition. When $\gamma = 0.09$, the possible values of M can be 3 and 5. For these values also we find that individual will not join the coalition. When $\gamma = 0.05$, the possible values of M lie between 3 to 9 . For $\gamma = 0.07$, we have M ranging from 3 to 7. In both the cases the individual will not join the coalition. The result holds true for other values of γ.

However one can derive the results for case 1 using $\lambda_i \leq 1/2$. The highest value of λ possible given $\lambda_i \leq \gamma$ and $\gamma M < 1$ is 0.3. With $\lambda = 0.3$ and $\gamma = 0.34$, expected value of M can be 3. Similarly, when $\lambda = 0.19$, M can be 3 and 5. In both the cases, the individual will join the coalition. The difference between the two equation is positive for lower values of λ as well. Case 2 and case 3, were also tested for $\lambda < 1/2$ and $\lambda > 1/2$. The results remain the same and the individuals will not join the coalition. This leads us to the third proposition.

Proposition III: For $\gamma M \leq 1$ and given values of γ, an individual will join the coalition when $\lambda_i \leq \gamma$ (for $\lambda < 1/2$) and not join the coalition when $\lambda_i > \gamma$.

The case I ($\lambda_i \leq \gamma$) results could only be computed when $\lambda < 1/2$ for the reasons specified above.

Solution for $\gamma M \geq 1$

When $\gamma M \geq 1$ we can also solve for $\gamma > 0.5$. The probability individual has $\lambda_i \leq \gamma M$ i.e. γM will be 1 and probability any individual has cutoff $\lambda_i > \gamma M$ which is $1 - \gamma M = 0$, since $\lambda_i < 1$. Comparing equation 13 and 16, one finds that $\phi = \phi' = 1$ and $\psi = \psi' = 0$. Plugging this into the 13 and 16 shows that the individual in the first case will be indifferent between joining the coalition and not joining the coalition. Based on our definition of joining the coalition, the individual joins the coalition if h/she is indifferent.

For case II, comparing 19 and 20, one finds that $\phi = \phi' = 1$ and $\psi = \psi' = 0$. Plugging these values in 19 and 20 shows that individual will not join the coalition as 19 < 20.

For case III, comparing 21 and 24, one finds that $\phi'' = \phi' = 1$ and $\psi'' = \psi' = 0$. Plugging these values in 21 and 24 shows that individual will not join the coalition as 21 < 24. This leads us to the final proposition in the paper.

Proposition IV: When $\gamma M \geq 1$, individual with $\lambda_i \leq \gamma$ will join the coalition and individuals with cutoff $\lambda_i > \gamma$ will not be joining the coalition.

From Proposition III and IV, we find that individuals with relatively lower weight on pecuniary payoff ($\lambda_i \leq \gamma$) will join the coalition and individuals with higher weight on pecuniary payoff will not join the coalition. The individuals who will be joining the coalition are the ones who satisfy $\lambda_i \leq \gamma$ or λ_i can also be interpreted as the proportion of people(M) who satisfy this cutoff (out of N). As γ increases, more individuals satisfy this cutoff, thus leading to more people joining the coalition. Higher γ also leads to an increase in the cutoff of the people who will vote yes to contribute for public good in Stage II ($\lambda_i \leq \gamma M$). Thus an increase in γ or higher benefits of cooperation can increase the size of a coalition and also increase the likelihood by which an existing coalition will contribute for the public good. In the public goods experiments, higher γ is depicted by $\gamma > 0.5$ and $\gamma M \geq 1$ should be used for predictions. Hence we are able to show that an increase in the benefits of cooperation i.e. MPCR leads to a bigger coalition size which is also contributing for the public goods.

The results also helps us to estimate the equilibrium number of individuals in the coalition.

Proposition V

In the equilibrium M is given by the individuals who satisfy $\lambda_i \leq \gamma$. Increase in γ leads to more people joining the coalition and a higher likelihood of a coalition contributing to the public good.

The results also establish a relation between coalition formation in public goods game along with voting rule. From our results we know that individuals with $\lambda_i \leq \gamma$ will join the coalition. Individuals who satisfy $\lambda_i \leq \gamma$ also satisfy $\lambda_i \leq \gamma M$. In other words, individuals who join the coalition will also satisfy the cutoff for contributing in Stage II. Individuals can predict from stage II that if there are people with cutoff : $\lambda_i \leq \gamma M$ in majority , coalition will contribute for the public good.

The cutoff for joining the coalition makes sure that everyone in the coalition will be contributing for public good. This can also incentivise the individuals to contribute for the public good as they know from Stage II that all the individuals in the coalition will contribute for the public good. The chances of there being a free rider is reduced and this leads to increased cooperation. This is also consistent with Gunnthorsdottir, Houser and McCabe (2007) who find that cooperators contribution decreases when the frequency of free riders increases. These results will hold true in any kind of voting rule. This leads us to the last proposition and a corollary from these results.

Proposition VI: An individual i who joins the coalition will always contribute for the public good. An increase in MPCR leads to more people joining the coalition and higher likelihood of individuals contributing for the public good.

An immediate corollary from the above proposition is the following:

Corollary

An individual who joins the coalition will also contribute for the public good in case of unanimous voting. As MPCR increases, more people join the coalition and everyone will vote to contribute. Thus the public good will be provided in case of unanimous voting as well.

6 Conclusion

In this paper, I explore the disparity between experimental results and thetheoretical findings regarding the relation between the size of the coalitionand benefits of cooperation. Incorporating social preferences in a public good game with coalition explains the existence of large sized IEA. Heterogeneity of social preferences predicts who will contribute to the public good. From Stage II, I derive separate cutoffs which if satisfied will lead to contribution by both the fringe members and coalition members. Using this information in stage I, I derive the cutoffs to be satisfied by individuals who will join the coalition. Individuals with relatively lower weight on pecuniary payoff satisfy the cutoff of joining the coalition and also contributing as a coalition member.

As MPCR increases, more individuals satisfy the cutoff. This results in coalition of larger size and higher likelihood that a coalition will contribute for the public good. Existence of people who satisfy these cutoff i.e. those who have lower weight on pecuniary payoff in majority can lead to the large sized coalitions contributing for the public good and henceforth increase cooperation. Our results also apply to unanimous voting rules . If a individual joins a coalition h/she will contribute for the public good. Increase in the benefits of cooperation leads to more individuals joining the coalition and all of them contributing for the public good. Future work will test my theoretical findings in the lab. As compared to the previous experiments, I will be incorporating social preferences to validate the positive relation between MPCR and size of the coalition.

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