Understanding Persistent Stagnation*

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- preliminary -

Abstract

The fear of persistent stagnation has never been greater in advanced economies as central banks continue to struggle with near-zero interest rates and low inflation. We theoretically explore issues of long-run stagnation in a representative agent framework. We analytically compare *expectations-driven* stagnation to a *secular stagnation* episode. This contrast provides novel policy implications. Our findings extended to a quantitative DSGE model in which we reassess Japan's experience from a prolonged liquidity trap.

Keywords: Expectations-driven trap, secular stagnation, monetary policy, zero lower bound, inflation dynamics.

JEL Classification: E31, E32, E52.

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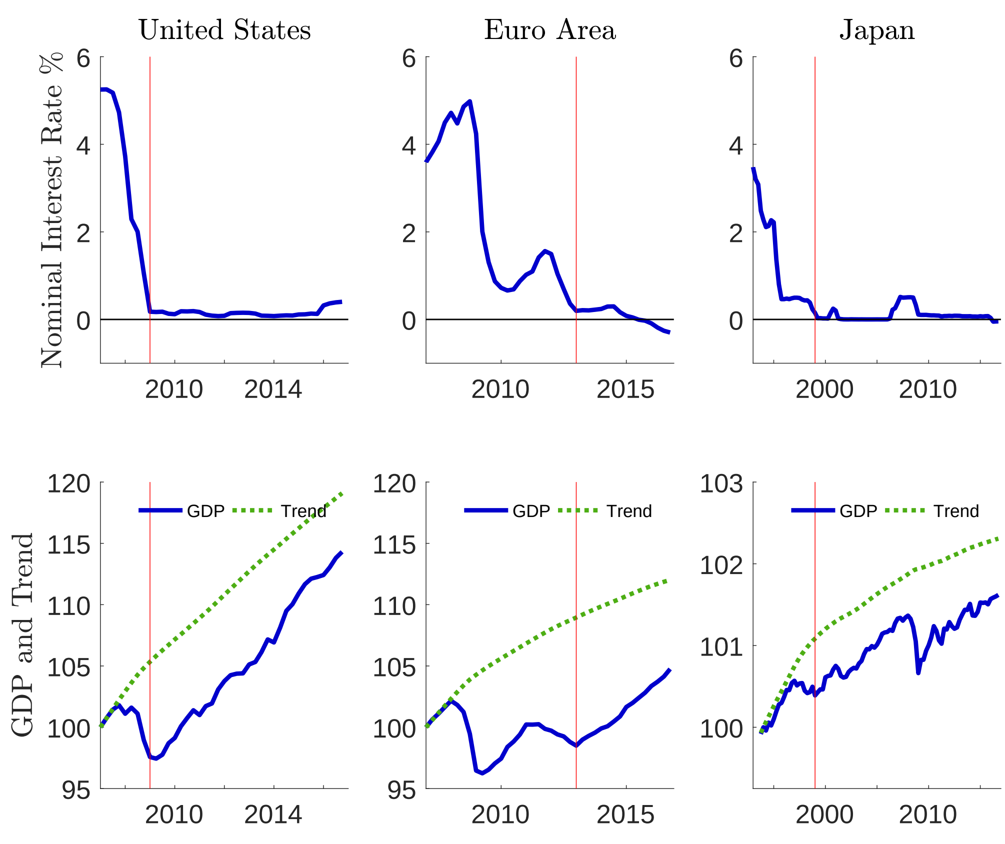
1 Introduction

Concerns of persistent stagnation began with the onset of Japan's prolonged period of zero nominal interest rates in mid-90s. Since the financial crisis of 2008-09, these concerns have spread to the rest of the advanced world. Figure 1 shows the trajectory of short term nominal interest rates and output in the United States, the Euro Area and Japan. In all cases, central banks lowered interest rates to near zero levels. Yet, output remains stagnant.

Figure 1: Interest Rate and Output Dynamics



Notes: The vertical red lines correspond to the quarter in which the nominal interest rate was below 25 basis points for the first time in the sample shown in each graph. GDP (solid blue line) is indexed equal to 100 in 2007Q1 for the U.S. and Euro Area and in 1994Q1 for Japan. GDP trend (green dotted line) is constructed using a backward looking 10-year moving average of GDP growth.

The macroeconomics literature has primarily sought to reconcile the case of persistent stagnation in two ways: through an *expectations-driven liquidity trap* as in Benhabib, Schmitt-Grohé and Uribe (2001, 2002) or Alvin Hansen's *secular stagnation* hypothesis, which entails a permanent decline in the natural interest rate (Summers, 2013). These distinct concepts about the source of permanent stagnation have only been analyzed in separate theoretical frameworks. In particular, *secular stagnation* has been studied in models with heterogeneous agents (Eggertsson, Mehrotra and Robbins, 2018). In this paper, we provide a DSGE framework that allows us to consider the ideas of *secular stagnation* and *expectations-driven* liquidity traps in a unified framework.

The textbook representative agent model fixes the steady-state natural rate to equal the inverse of the household's discount factor and cannot accommodate the *secular stagnation* hypothesis. We modify the representative agent's intertemporal Euler equation such that the steady state features a negative relation between output and real interest rate in the steady state, similar to a static IS-curve. This modification breaks the connection between the natural interest rate and the discount factor, thus allowing for the possibility of a permanently negative natural interest rate.

Our framework, thus, accommodates two different explanations for persistent stagnation. The *expectations-driven* liquidity trap, commonly known as the BSGU steady state, arises in the presence of a non-linearity in the monetary policy rule introduced by the zero lower bound constraint on short term nominal interest rates. Pessimistic inflationary expectations become self-fulfilling in this setting. Combined with long-run money non-neutrality, the BSGU steady state can account for permanent output stagnation, below-target inflation and zero nominal interest rates (Aruoba, Cuba-Borda and Schorfheide, 2017*b*; Schmitt-Grohé and Uribe, 2017).

We show that the existence of the BSGU steady state depends on the assumptions on long-run nominal rigidities. Appropriate bounds on downward nominal wage rigidity can preclude the existence of the BSGU steady state. We build a tractable model with downward nominal wage rigidity to demonstrate our key results analytically. In the model, the BSGU steady state does not exist when nominal wages are sufficiently rigid downwards. Conversely, the BSGU steady state exists only when nominal wages are sufficiently flexible downwards. A policy implication of this finding is that policies that increase the degree of downward nominal wage rigidity can preclude the expectations-driven traps, extending the set of tools identified by Benhabib et al. (2002) and Woodford (2003, Ch 2).[1] Furthermore, this novel finding extends the unconventional implications associated with increases in price or wage flexibility in the New Keynesian models (Eggertsson, 2011). Relaxing nominal rigidities opens the economy to possibility of expectations driven traps.

[1] Benhabib et al. (2002) propose monetary and fiscal policies that violate the households' transversality conditions along the deflationary equilibrium paths.

We label this result as the *curse of flexibility*.

On the other hand, the *secular stagnation* steady state emerges because of constraints on monetary policy's ability to replicate the permanently low natural interest rate. Due to the zero lower bound constraint on nominal interest rates, output remains permanently stagnant and the central bank cannot attain its inflation target. An increase in liquidity premia associated with government bonds, a reduction in the marginal impatience of the household or an increase in marginal (labor-dependent) tax on interest incomes can generate the decline of the natural rate in our framework. These explanations serve as analytical counterparts for fundamental forces such as aging, inequality, debt deleveraging (Eggertsson et al. 2018), international reserve accumulation, or a scarcity of safe assets (Caballero, Farhi and Gourinchas, 2015; Eggertsson, Mehrotra, Singh and Summers, 2016) in our representative agent framework.

We find that policy interventions to deal with unemployment lead to contrasting outcomes depending on the source of stagnation. Policy interventions such as increased government spending are effective at reducing unemployment under secular stagnation. Yet, the same intervention is contractionary when implemented around the BSGU steady state. Labor market reforms that increase wage flexibility are expansionary under BSGU trap but contractionary under secular stagnation (*paradox of flexibility*). Similarly, positive supply shocks that increase productive potential of the economy are contractionary under secular stagnation (*paradox of toil*) but increase employment under BSGU trap. Because of the opposite outcomes implied by policy interventions, our exercise suggests caution in implementing policies without identifying the nature of stagnation.

We present alternate micro-foundations to modify the Euler equation in an observationally equivalent way. Following Feenstra (1986) and Fisher (2015), a functional equivalence can be shown between using bonds in the utility and entering transactions costs of bonds in the budget constraint (bonds in advance may be modeled as a special case of bonds in the utility). Additionally, we show that a similar modification can also be brought about by Uzawa-Epstein preferences, or through increase in marginal (labor-dependent) tax rate on interest incomes.[2] These alternate micro-foundations essentially make government bonds matter. In an earlier literature, it has been shown that interest rate pegs can achieve price level determinacy if government bonds provide

[2] In this paper, we restrictive our quantitative analysis to the bonds-in-utility modification because of this recent popularity. In ongoing work, we contrast the dynamics associated with different micro-foundations away from ZLB.

transactions services (see Canzoneri and Diba 2005 for closed economy, and Calvo and Végh 1995; Lahiri and Végh 2003 for open economy settings). We emphasize that the modified Euler equation does not, by itself, give rise to price-level determinacy in our setting. This is crucial for the model to exhibit the two different stagnation steady states depending on bounds on nominal rigidities. Moreover, we show that a breakdown of Ricardian equivalence is also not sufficient for the existence of this secular stagnation steady state. The key element for that result is the modified Euler equation that features a negative steady state relation between output and real interest rate, as in the static IS curve.

In a quantitative small scale DSGE model, we find that either of the stagnation steady states can exist for reasonable parameters. Like in the analytical model, the BSGU steady state exists when the degree of nominal rigidity is weak enough to allow low levels of inflation in equilibrium. There is a large literature that has modeled liquidity trap as a result of a temporary decline in the natural rate. In those log-linearized models, *deflationary black holes* emerge as the duration of temporary liquidity trap is increased, with inflation and output tending to negative infinity (Eggertsson, 2011).[3] Instead, the modified Euler equation in our setting can give rise to a bounded secular stagnation steady state.

An important application of a bounded secular stagnation steady state in the representative agent framework is that it can be used to estimate the DSGE model for Japan under the assumption of secular stagnation. We show that it is particularly straightforward to log-linearize the model around this stagnation episode and construct impulse responses using the standard tools. For a country such as Japan, where the nominal interest rates have been close to zero for over two decades, standard DSGE models linearized around the targeted inflation steady state require agents to expect recovery in the medium to long-run (See Bianchi and Melosi (2017) and Gust, Herbst, López-Salido and Smith (2017)). Lubik and Schorfheide (2004), and more recently Bianchi and Nicolo (2017), provide an alternative method of conducting inference with the log-linearized model around the locally indeterminate BSGU steady state, where agents can have long-run deflationary expectations. We illustrate that, because of the local determinacy of the secular stagnation steady state, the modified Euler equation may also be a useful tool for researchers interested in estimating

[3] Eggertsson and Singh (2018) show that these properties with galactic deflation are an artifact of conducting inference using the log-linearized solution. In the non-linear model, the equilibrium may cease to exist as the expected duration of the liquidity trap is increased beyond few quarters.

models with long-run stagnation.

Variance decomposition of the estimated model finds a significant role for sunspot shocks under the BSGU hypothesis. In contrast, fundamental shocks drive the first-order dynamics under secular stagnation. Researchers interested in understanding historical drivers of inflation and output may use the methods presented in this paper to compare them under either of the stagnation hypotheses. A takeaway from our analysis is that it is possible to enrich the model in additional dimensions that can aid empirical identification of the stagnation episodes.

Our work complements the recent analyses of Michau (2018), Michaillat and Saez (2018) and Ono and Yamada (2018) who use the bonds-in-utility assumption to analyze a unique persistent stagnation scenario. We distinctly focus on understanding the differences between the two stagnation concepts analytically and quantitatively. The policy implication that a flattening of the aggregate supply curve precludes the existence of the BSGU steady state is related to the R&D policies advocated by Benigno and Fornaro (2017) in a New Keynesian model with endogenous growth. Our analysis shows that such policy implications apply even in the case of liquidity traps with exogenous productivity growth.

This paper is also related to the work by Mertens and Ravn (2014), who contrast the temporary expectations driven trap to a temporary fundamentals driven trap. Their model cannot accommodate the permanent fundamentals driven trap. Our paper is also complementary to Schmitt-Grohé and Uribe (2017), who analyze the case of permanent expectations driven traps. Recently, Mertens and Williams (2018) use the prices of derivatives, along with theoretical predictions from the standard New Keynesian model, to empirically identify whether the US economy corresponds to the targeted inflation steady state or the BSGU steady state. We leave it to future work to combine their methods with our framework to differentiate between the targeted inflation and the secular stagnation steady states.

The layout for the remainder of the paper is as follows. Section 2 illustrates key insights from introducing the modified Euler equation in an analytically tractable setting with downward nominal wage rigidity. We show that the bounds on deflation are sufficient to preclude the possibility of expectations trap even with a standard Euler equation. The modified Euler equation, by allowing a negative relation between output and real interest rate, opens up the possibility of a secular stagnation steady state. Section 3 describes a quantitative small-scale DSGE model commonly used in the literature. Section 4 shows the quantitative robustness of the steady state insights derived from the simple model and compares the dynamics around each steady state. In Section 5, we show that the model can be calibrated with reasonable parameters that does not allow the researcher to separately identify the BSGU stagnation trap from the the secular stagnation in the case of Japan. Section 6 concludes.

2 Key insights in a two equations setup

We begin with a simple model that analytically captures the key insights. Using this model, we characterize and formally define the different steady states: *targeted inflation* state, *secular stagnation* steady state, and the *expectations driven trap* (also referred to as the BSGU steady state).

Suppose the representative agent supplies labor $\bar{h} = 1$ inelastically and maximizes the following utility function involving a consumption good C_t and (real) stock of nominal bonds B_t:

$$\max_{\{C_t, B_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ln(C_t) + \delta_t V(\frac{B_t}{P_t}) \right]$$

where $\delta_t \geq 0$ regulates the marginal utility from holding the risk-free bonds supplied by the government. We assume that these bonds are in net zero supply and the felicity from holding these bonds is such that $V'(0) = 1$. The household earns wage income $W_t h_t$, interest income on past bond holdings, dividends Φ_t from firms' ownership and makes transfers T_t to the government. An interior solution to household optimization yields the Euler equation:

$$U'(C_t) = \beta R_t \mathbb{E}_t \left[\frac{U'(C_{t+1})}{\Pi_{t+1}} \right] + \delta_t$$

Consumption goods are produced by competitive firms with labor as the only input using the technology

$$Y_t = F(h_t) = h_t^{\alpha}, \quad \text{where } 0 < \alpha < 1$$

These firms set price of the final good P_t to equate marginal product of labor to the marginal cost.

$$F'(h_t) = \frac{W_t}{P_t}$$

We introduce a very stylized form of downward nominal wage rigidity (following Schmitt-Grohé and Uribe 2017):[4]

$$W_t \geq (\gamma + (1-\gamma)(1-u_t)^{\alpha})W_{t-1} \equiv \tilde{\gamma}(u_t)W_{t-1}$$

where $\gamma \in (0,1)$ and $u_t \equiv 1 - \frac{h_t}{\bar{h}}$ is involuntary unemployment. This downward rigidity implies that employment cannot exceed the total labor supply in the economy i.e. $h_t \leq 1$. We further assume that the following slackness condition holds:

$$(\bar{h} - h_t)(W_t - \tilde{\gamma}(u_t)W_{t-1}) = 0$$

We close the model by assuming a government that balances budget,

$$B_t + T_t = R_{t-1}B_{t-1}$$

and a monetary authority that sets nominal interest rate on the government bonds using the following Taylor rule

$$R_t = \max\{1, (1+r^*)\Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\alpha_\pi}\}$$

where $(1+r^*) \equiv \frac{1-\delta_t}{\beta}$ is the natural interest rate, Π^* is the inflation target set by the central bank, and $\alpha_\pi > 1$.[5] The zero lower bound (ZLB) constraint on the short term nominal interest rate introduces an additional nonlinearity in the policy rule. Finally, we assume that the resource constraints hold in the aggregate:

$$C_t = Y_t, \quad \text{and} \quad B_t = 0.$$

[4] In Appendix C, we provide analytical results for the particular downward nominal wage rigidity assumed by Eggertsson et al. (2018).

[5] The natural interest rate is defined as the real interest rate on one period government bonds that would prevail in the absence of nominal rigidities.

2.1 Equilibrium

Let $w_t \equiv \frac{W_t}{P_t}$ denote the real wage. The competitive equilibrium is given by the sequence of seven endogenous processes $\{C_t, Y_t, R_t, \Pi_t, h_t, w_t, u_t\}$ that satisfy the conditions (1) - (7) for a given exogenous sequence of process $\{\delta_t\}_{t=0}^{\infty}$ and the initial condition w_{-1}:

$$
\begin{aligned}
1 &= \beta E_t \left[\frac{C_t}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} \right] + \delta_t C_t && (1) \\
Y_t &= h_t^{\alpha}. && (2) \\
\alpha h_t^{\alpha-1} &= w_t && (3) \\
h_t \leq \bar{h}, \quad w_t &\geq (\gamma + (1-\gamma) h_t^{\alpha}) \frac{w_{t-1}}{\Pi_t}, \quad (\bar{h} - h_t) \left(w_t - (\gamma + (1-\gamma) h_t^{\alpha}) \frac{w_{t-1}}{\Pi_t} \right) = 0 && (4) \\
u_t &= 1 - \frac{h_t}{\bar{h}} && (5) \\
R_t &= \max\{1, (1 + r_t^*) \Pi^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\alpha_\pi} \} && (6) \\
Y_t &= C_t && (7)
\end{aligned}
$$

where the exogenous sequence of natural interest rate is given by $1 + r_t^* \equiv \frac{1-\delta_t}{\beta}$.

2.2 Non-stochastic Steady-state

In the steady state, we can simplify the system of equations to an aggregate demand block and an aggregate supply block.

Aggregate Demand AD curve is a relation between output and inflation and is derived by combining the Euler equation and the Taylor rule. Mathematically, the AD curve is given by

$$
Y_{AD} = \frac{1}{\delta} \begin{cases} 1 - \beta \Gamma^* \Pi^{\alpha_\pi - 1}, & \text{if R } > 1, \\ 1 - \frac{\beta}{\Pi}, & \text{if R } = 1 \end{cases} \tag{8}
$$

where $\Gamma^* = (1 + r^*) (\Pi^*)^{1-\alpha_\pi}$. Away from the ZLB, the AD curve is a downward sloping graph, and it becomes upward sloping when the nominal interest rate is constrained by the ZLB. The kink in the aggregate demand curve occurs at the inflation rate at which monetary policy is constrained by the ZLB.

$$
\Pi_{kink} = \left(\frac{1}{(1 + r^*) \Pi^*} \right)^{\frac{1}{\alpha_\pi}} \Pi^*
$$

The solid red line in in Figure 2 plots the aggregate demand curve with positive natural rate. When $R^* \equiv (1+r^*)\Pi^* > 1$, the kink in AD curve occurs at inflation rate below Π^*. For the natural interest rate to be positive, the premium on government bonds must be low enough i.e. $\delta < 1-\beta$.[6]

Aggregate Supply: Because of the assumptions of downward nominal wage rigidity and capacity constraints on production, the AS curve features a kink at full employment level of output and gross inflation rate equal to one. When inflation rate is less than one, the downward wage rigidity constraint becomes binding. As a result, inflation cannot fall to completely adjust any demand deficiency and firms layoff workers. The aggregate supply curve can be summarized by:

$$Y_{AS} \le 1, \quad \Pi \ge (\gamma + (1-\gamma)Y_{AS}), \quad (Y_{AS}-1)\left(1-(\gamma+(1-\gamma)Y_{AS})\frac{1}{\Pi}\right) = 0 \tag{9}$$

When $h = \bar{h} \equiv 1$, $\Pi \ge 1$. The AS curve is a vertical line at full employment. For $h < \bar{h}$, $\Pi = \gamma + (1-\gamma)h^{\alpha}$. That is, AS curve is linear and upward sloping with slope$=1-\gamma$ for $y < 1$. The kink in the AS curve occurs at the coordinate $Y = 1, \Pi = 1$. Because of this assumed linear aggregate supply curve under deflation, the degree of nominal rigidity γ also determines the lower bound on inflation. The solid blue line in Figure 2 plots the aggregate supply curve.

Figure 2

[6]For illustration in this section, we use the following parameters: $\beta = 0.77$, $\alpha = 0.5$, $\alpha_\pi = 2$, $\gamma = \{0.5, 0.9\}$,and $\delta = \{0.1, 0.3\}$.

With this two equation model, we can characterize and prove the existence of different steady states. Proposition 1 formalizes that the unintended steady state can be eliminated by assuming a flat enough Phillips curve i.e. $\gamma > \beta$, as shown in Figure 2.

Proposition 1 *(Disarming the Perils): Let* $\Pi^* = 1$*,* $0 < \delta < 1 - \beta$*, and* $\gamma > \beta$*.There exists a globally unique steady state. It features full employment, inflation at the policy target and positive short term nominal interest rate.*

Proof. Note that the downward sloping portion of aggregate demand always goes through $Y = 1$, $\Pi = 1$ with $\Pi^* = 1$. When $\delta < 1 - \beta$, $R^* > 1$, thus, the kink in the AD occurs at inflation rate below 1. AS is linear relationship while AD is strictly convex. At Π_{kink}, $Y_{AD} > 1 > Y_{AS}$. When $\Pi = \gamma$, $Y_{AS} = 0 < Y_{AD}$. The last inequality requires the assumption that $\gamma > \beta$. ∎

Any steady state, with locally determinate dynamics in its neighborhood, at which the central bank can meet its inflation target is *defined* as the *targeted inflation steady state*. The presence of full-employment steady state is contingent on the natural interest rate and the inflation target of the monetary authority. With a unity inflation target, it must be the case that the natural interest rate be non-negative, which is implied by the assumption of $\delta < 1 - \beta$. Figure 2 plots the AD-AS representation of the environment described in Proposition 1. A high enough nominal rigidity puts a sufficiently high lower bound on deflation, which prohibits the self-fulfilling deflationary expectations to manifest in the steady state. Technically, the set of equilibrium inflation rates is upper hemicontinuous in the lower bound on inflation rate γ.[7]

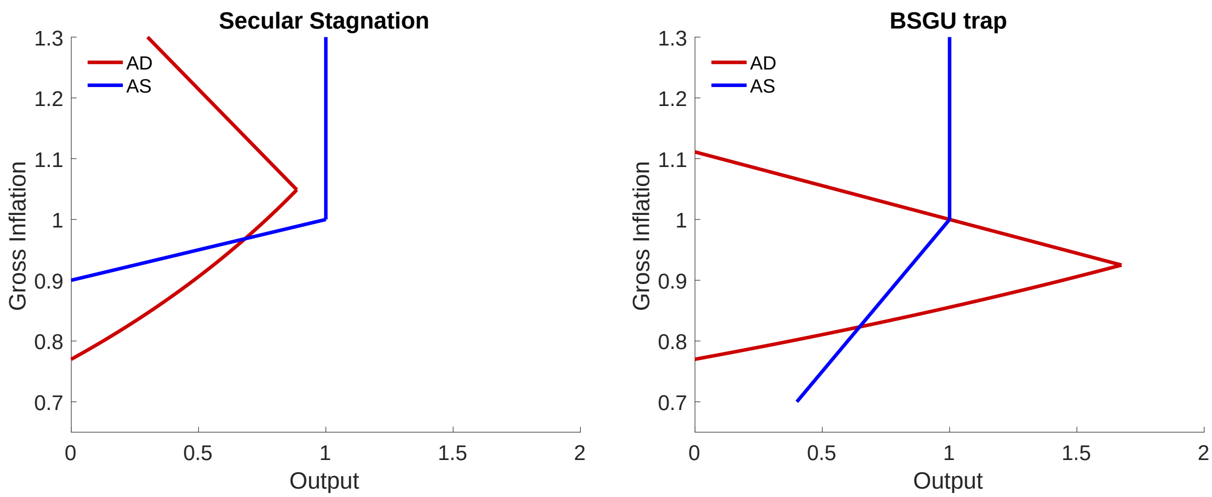
When the natural interest rate is negative, monetary policy can be constrained by the zero lower bound on short-term nominal interest rate and a modest inflation rate target. In that case, the nominal interest rate is permanently zero while there is unemployment and deflation in the economy. We characterize this possibility in Proposition 2.

Proposition 2 *(Secular Stagnation): Let* $\Pi^* = 1$*,* $\delta > 1-\beta > 0$*, and* $\gamma > \beta$*. There exists a unique steady state. It features unemployment and deflation, caused by a permanently negative natural interest rate. The equilibrium dynamics in this steady state's neighborhood are locally determinate.*

[7] The target inflation rate is the unique outcome when $\gamma > \beta$ and continues to be an equilibrium when $\delta = 0$. There is a kink at $\delta = 1 - \beta$, but the equilibrium inflation rate is continuous in $\gamma \leq \beta$.

Proof. When $\delta > 1 - \beta$, $R^* < 1$, thus, the kink in the AD occurs at inflation rate above 1. AS is linear relationship while AD is as strictly convex. At Π_{kink}, $Y_{AD} < 1 = Y_{AS}$. When $\Pi = \gamma$, $Y_{AS} = 0 < Y_{AD}$. The last inequality requires the assumption that $\gamma > \beta$. Thus, they must intersect at positive unemployment. We leave the proof of local determinacy to the online appendix. It involves log-linearizing the dynamic equilibrium around this secular stagnation steady state and showing that the Blanchard-Kahn determinacy conditions hold. ■

Figure 3



See left panel in Figure 3 for illustration of this unique steady state. The modified Euler equation allows for the possibility of a permanently negative natural interest rate, which is no longer fixed by the discount factor. Instead, the marginal utility of holding government bonds (δ) also determines the steady state natural interest rate. The stagnation steady state emerges when the premium on government bonds is sufficiently high driving the natural interest rate to negative levels. We formally *define* the *secular stagnation steady state* as the steady state with positive unemployment, zero nominal interest rate on short-term government bonds and exhibiting locally determinate equilibrium dynamics in its neighborhood.

Propositions 1 and 2 may serve as a counter-example to the folk wisdom that the imposition of a zero bound constraint on the monetary policy rule is sufficient to allow for the possibility of expectations driven traps. The primary ingredient for the absence of the expectations driven trap is a sufficiently high lower bound on deflation. Thus, the assumption of long-run money non-neutrality is not necessarily incompatible with the existence of such a locally indeterminate steady state. In

Proposition 3, the conditions conducive for the existence of such a steady state are shown in the case of our assumed nominal rigidity. Specifically, when there does not exist a very conservative lower bound on inflation, that is nominal wages are sufficiently downwardly flexible, the economy is open to the possibility of expectations driven traps (Schmitt-Grohé and Uribe, 2017).

Proposition 3 *(The Peril-ou-s nominal rigidity): Let* $\Pi^* = 1$*, and* $0 < \delta < 1 - \beta$*. If* $\gamma < \beta$ *there exists two steady states:*

1. *A unique full employment steady state, with inflation at the target rate.*
2. *A unique unemployment steady state with zero nominal interest rate and deflation. The local dynamics in a small neighborhood around the unemployment steady state are locally indeterminate.*

Without downward rigidity constraint on nominal wages (i.e. $\gamma = 0$*), there always exist two steady states: a unique deflationary steady state with zero nominal interest rates and a unique targeted inflation steady state.*

Proof. When $\delta < 1 - \beta$, $R^* > 1$, thus, the kink in the AD occurs at inflation rate below 1. Further, note that the downward sloping portion of aggregate demand always goes through $Y = 1$, $\Pi = 1$. This is the unique full employment steady state. At Π_{kink}, $Y_{AD} > 1 > Y_{AS}$. When $\Pi = \gamma$, $Y_{AS} = 0 > Y_{AD}$. Given that AS is strictly linear and AD is as strictly convex, this implies that there is a unique intersection at positive unemployment. At this second steady state, AS intersects AD from above and thus this is locally indeterminate. We leave the detailed proof following Blanchard and Kahn to the online appendix. ∎

See the right panel in figure 3 for illustration of two steady states with modified Euler equation. The co-existence of this unintended steady state requires that inflation can fall sufficiently low.

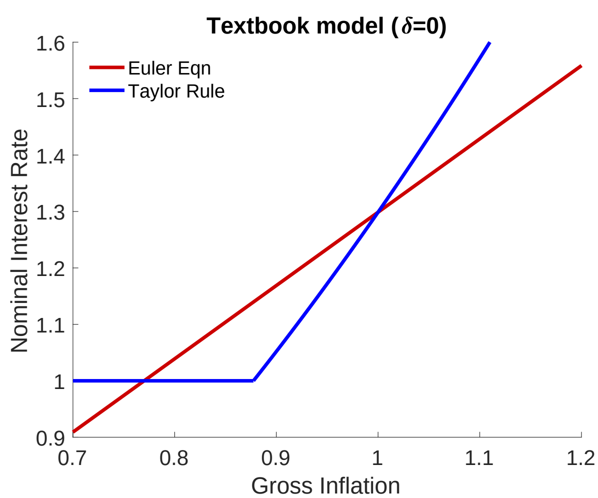
We define the *expectations driven* trap as the steady state with positive natural interest rate, unemployment, and deflation and in whose neighborhood the equilibrium dynamics are locally indeterminate. As a shorthand, henceforth, we refer to it as the *BSGU steady state.* In Section 2.3, we show that similar bounds on deflation are also enough to preclude expectations trap in a model with standard Euler equation. In Section 4, we show that putting a high enough lower bound on inflation excludes the BSGU steady state in the case of a quantitatively calibrated model as well.

2.3 Comparison with the textbook Euler Equation

We provide a brief comparison of results for the reader with the textbook Euler equation (Woodford, 2003). This serves to illustrate the use of two concepts in our framework - a) the modified Euler equation and b) bounds on deflation.

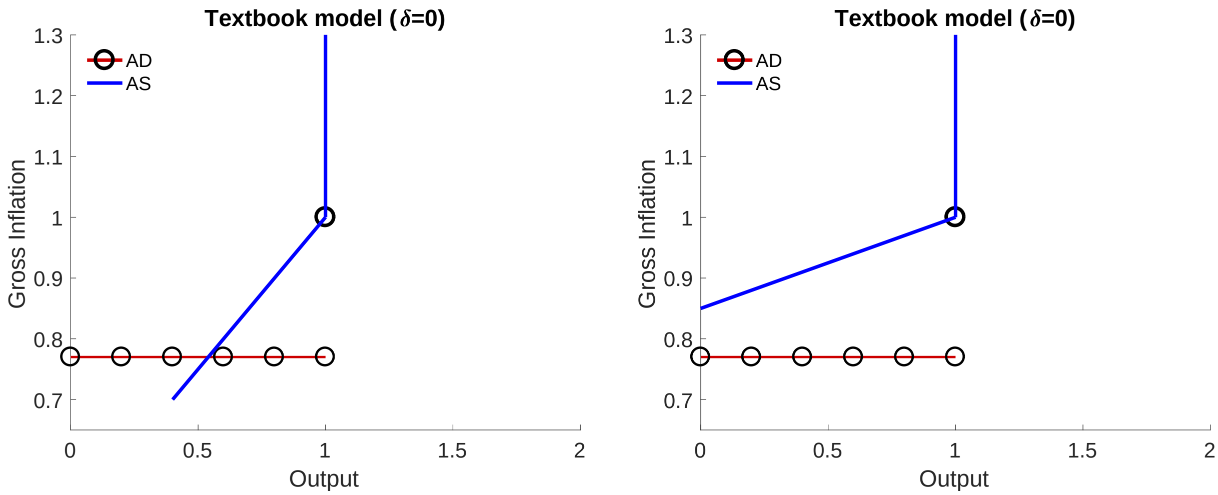
The textbook analysis usually assumes long-run money neutrality. Thus, there always exists a second steady state featuring deflation and zero nominal interest rate on the short-term government bonds. This point can be gleaned from the classic graphical representation of BSGU steady state in the (Π, i) space. Figure 4 plots the standard Euler equation along with the non-linear Taylor rule. There exist two intersections of the Taylor rule with the textbook Euler equation because of the zero lower bound constraint on the short term nominal interest rate. With the assumption of long-run neutrality, it suffices to look at the Euler equation and the Taylor rule to determine the equilibrium.

Figure 4



In an analogous AD-AS representation, we plot the textbook model in left panel of Figure 5. In the textbook model, the natural interest rate is always fixed at $\frac{1}{\beta} > 1$. As a result, the aggregate demand relationship is a horizontal line at $\Pi = \beta < 1$ when the nominal interest rate is constrained by the ZLB. And the aggregate demand is represented by the coordinate $(1, \Pi^*)$ in the (Y, Π) space when monetary policy is unconstrained. The aggregate supply curve in the textbook model is only used to pin down the equilibrium output (see for example Schmitt-Grohé and Uribe 2017).

Figure 5



However, the existence of this *unintended* deflationary steady state is contingent on the assumptions regarding the supply side of the economy. If the y-intercept of the aggregate supply curve is greater than β, then there does not exist a deflationary steady state, as shown in the right panel of Figure 5. Hence, even the textbook analysis is not complete without sketching out the assumptions on long-run nominal rigidities.

We wish to emphasize that while the modification to the Euler equation is not essential to eliminating the BSGU trap, it is essential for the model to generate a permanent liquidity trap with locally determinate steady state. The modified Euler equation, by allowing a negative relation between output and real interest rate, opens up the possibility of a secular stagnation steady state. This steady state cannot arise in the standard model because the AD curve is a flat line under liquidity trap, as shown in Figure 5. Furthermore, in the textbook models, β is usually chosen to lie in the range of 0.99 - 0.999 to match the relevant empirical real real interest rate. In order to eliminate the BSGU steady state, one needs to assume somewhat extreme form of downward wage rigidity. The model with $\delta > 0$ can allow researchers to match an empirically plausible steady state real interest rate, while imposing realistic restrictions on the extent of downward wage rigidity. Hence, the model with $\delta > 0$ has desirable quantitative features.[8]

[8] In Section A, we present the (Π, i) space representation of various steady states in our framework with the modified Euler equation, for completeness.

2.4 Comparative Statics: BSGU, SecStag and Targeted Inflation Steady State

We, now, show that the BSGU steady state and the secular stagnation steady state have different implications for policy. With the modified Euler equation, permanent changes in government spending can affect the steady state output. As a result, local to the BSGU steady state, there is a unique comparative static for permanent changes in such policies. The crucial assumption is that inflation expectations do not change drastically so that the economy moves to the full employment steady state.[9]

2.4.1 Government Spending multiplier

We introduce government spending financed by lump sum taxes in the model. The resource constraint in the economy is modified to

$$Y = C + G$$

For any given level of output, fiscal policy crowds out private consumption. Lower consumption implies higher marginal utility of consumption, which offsets the increased demand for liquidity. Because of inelastic labor supply, we do not have the supply side effect of increased labor supply. Thus, this permanent change in fiscal policy unambiguously increases the natural rate, as shown below:

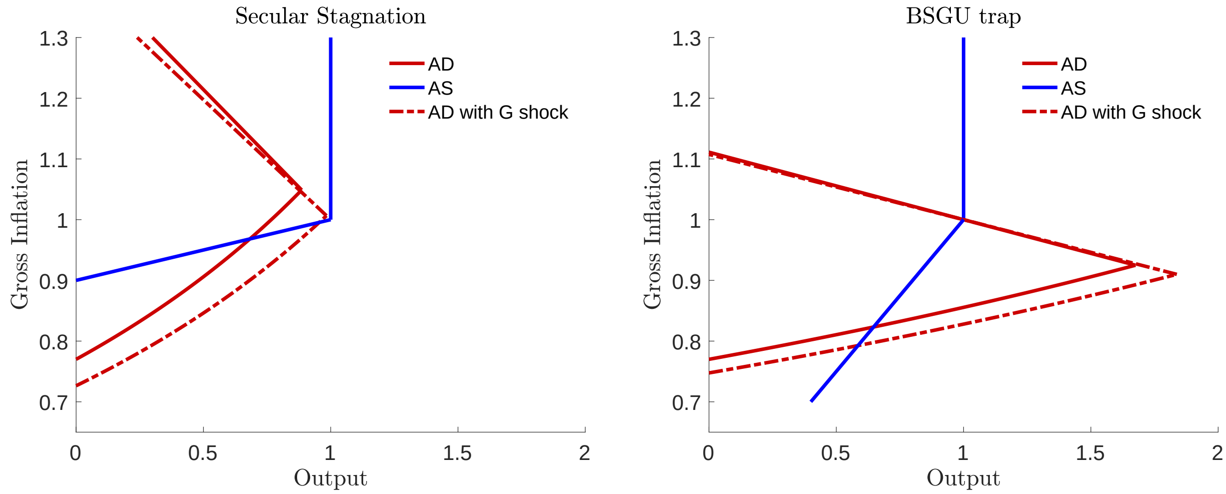
$$1 + r^* = \frac{1 - \delta(Y - G)}{\beta} = \frac{1 - \delta(1 - G)}{\beta} \implies \frac{dr^*}{dG} > 0$$

The subsequent rise in natural rate lowers the kink in the AD curve, since the position of the kink depends on the natural interest rate:

$$\Pi_{kink} = \left(\frac{1}{R^*}\right)^{\frac{1}{\alpha_\pi}} \Pi^*$$

[9] In subsequent comparative statics, there are no labor supply effects because we assume inelastic labor supply for analytical tractability. In Section 4, we show that these comparative statics can be generalized to a quantitative model with elastic labor supply.

Figure 6: Permanent increase in G



At the secular stagnation steady state, the upward sloping portion of the AD curve intersects the AS at a higher level of output since the AD curve is steeper than the AS curve locally. Thus, the policy experiment is expansionary when the economy suffers from fundamentals driven deficiency in aggregate demand (See the left panel in figure 6). Similarly, when the policy is implemented locally around the BSGU steady state, the policy brings about a permanent decline in steady state output because of the negative effect on inflation expectations (Mertens and Ravn, 2014).

2.4.2 Neofisherian Exit / Keynesian Quandary

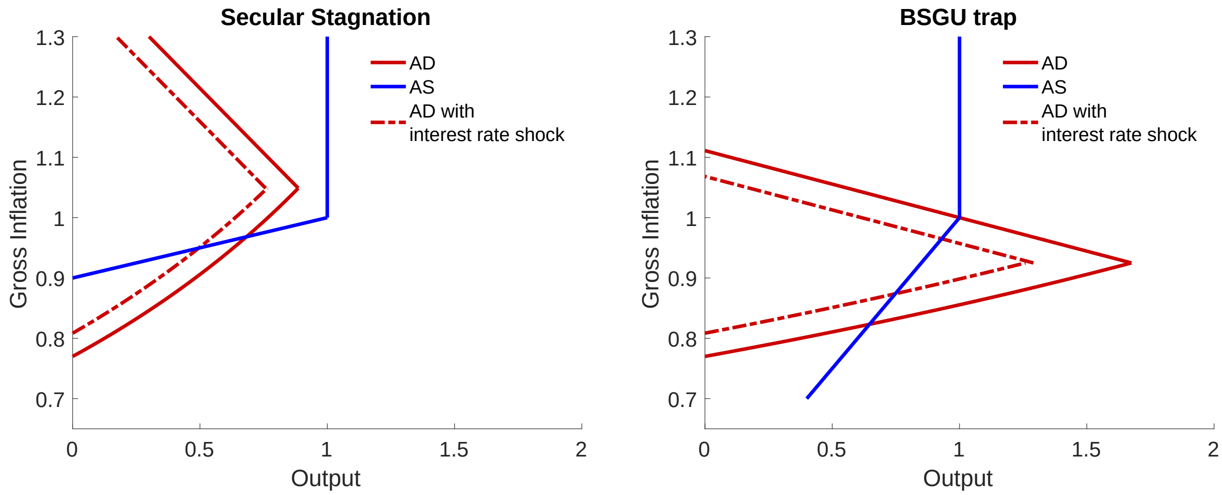
We now show the effects of a permanent increase in nominal interest rate. We model the policy as a permanent change in the intercept of the Taylor rule, a:

$$R^{new} = \max\{1 + a, a + R^* \left(\frac{\Pi}{\Pi^*}\right)^{\alpha_\pi}\} = a + R$$

where a is increased to a positive number from zero. This policy simultaneously increases the lower bound on nominal interest rate and thus does not have any effect on the placement of the kink in the aggregate demand curve. In the aggregate demand curve:

$$Y = \frac{1}{\delta}\left[1 - \frac{\beta(a + R)}{\Pi}\right]$$

Figure 7: Permanent increase in nominal interest rate

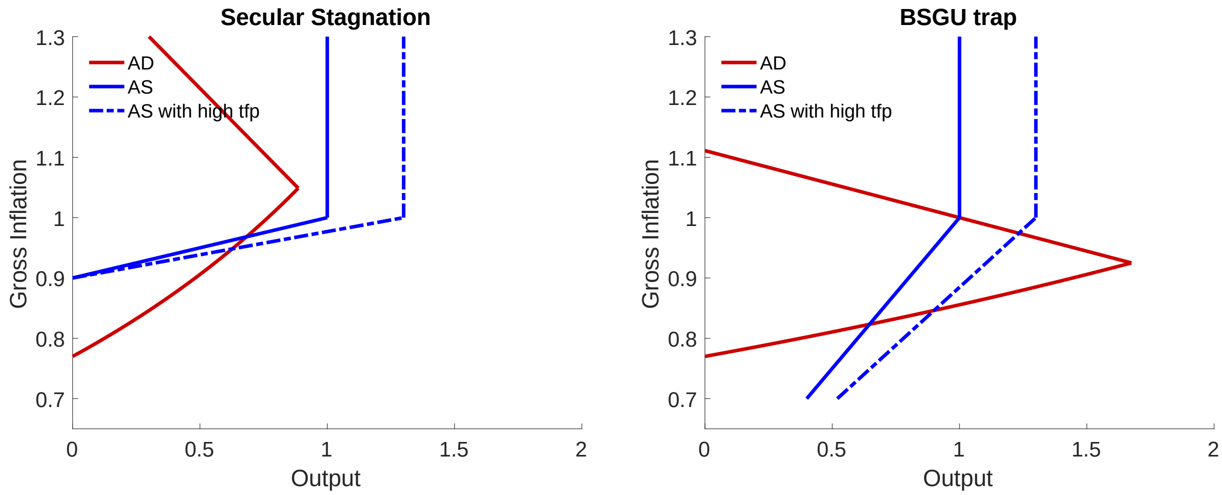


Given the inflation rate, an increase in a lowers output demanded. At the Sec Stag steady state, this induces deflationary pressures that increases the real interest rate gap and causes a further drop in output (See the left panel in figure 7). In contrast, an increase in nominal interest rate anchors agents' expectations to higher levels of inflation, thus obtaining the neo-Fisherian results (Schmitt-Grohé and Uribe, 2017).

2.4.3 Increasing Potential/Paradox of Toil

Eggertsson (2010) found that a temporary positive supply shock (cut in labor taxes, increase in TFP) can be contractionary under a temporary fundamentals driven liquidity trap, provided the supply shock is less persistent than the demand deficiency. This result has been labeled as the *Paradox of Toil*. Since the fundamentals driven trap is of permanent nature, the paradox of toil emerges in our setting regardless of the duration of the supply shock (See the left panel in figure 8). In contrast, a positive TFP shock is expansionary both at the targeted inflation steady state and the BSGU steady state (Mertens and Ravn, 2014).

Figure 8: Permanent increase in TFP



2.5 Discussion

2.5.1 Interpreting the modified Euler equation

The key ingredient in deriving the secular stagnation steady state in the representative agent setting is the modified Euler equation. The modification allows the natural interest rate to depend on the marginal utility of holding bonds δ, as well as the discount factor. A high demand for liquidity in the steady state can push the natural interest rate to a permanently negative level.

Now, we briefly sketch another alternative way to derive the wedge based on intertemporal discounting à la Uzawa-Epstein preferences that have been prominently used in the small open economy literature (Schmitt-Grohé and Uribe, 2003). The representative agent maximizes the following lifetime-utility function:

$$\max_{\{C_t,B_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \theta_t^t U(C_t)$$
$$\theta_0 = 1;$$
$$\theta_{t+1} = \beta(\tilde{C}_t)\theta_t \quad t \geq 0$$

where θ is the endogenous discount factor, and $\tilde{C}_t$ denotes the average per capita consumption, which the individual household takes as given.[10]

[10] This assumption, also used by Schmitt-Grohé and Uribe (2003), allows us to analytically relate the intertemporal discounting to the bonds-in-utility function. In ongoing work, we compare the dynamics implied by different assumptions in detail.

The household budget constraint is same as before and we assume that government bonds are in net zero supply. Let λ_t be the Lagrange multiplier, associated with the budget constraint. We can derive the following first order conditions:

$$\lambda_t = \beta(\tilde{C}_t)(1 + r_t)E_t\lambda_{t+1}; \quad \lambda_t = U'(C_t)$$

where $1 + r_t = R_t/\Pi_{t+1}$ is the real interest rate. These conditions imply the following Euler equation:

$$1 = \beta(\tilde{C}_t)\frac{R_t}{\Pi_{t+1}}\frac{U'(C_{t+1})}{U'(C_t)}$$

In equilibrium, individual and average per capita variables are identical. That is, $C_t = \tilde{C}_t$. The Euler equation, thus, simplifies to:

$$1 = \beta(C_t)\frac{R_t}{\Pi_{t+1}}\frac{U'(C_{t+1})}{U'(C_t)}$$

In the steady state:

$$1 = \beta(C)\frac{R}{\Pi}$$

If $\beta'(C) > 0$, the Euler equation exhibits a negative relationship between output and real interest rate, as in the static IS-curve. The converse assumption of increasing marginal impatience $\beta'(C) < 0$ is often made in the infinite horizon models to ensure a stable solution.[11] Because we do not allow for accumulation in our model, a stable steady state solution exists. In the case of $\beta(C) = \frac{\beta\frac{R}{\Pi}+\delta C}{\frac{R}{\Pi}}$ (with $\Pi > 0$), the modified Euler equation is exactly similar to the one derived with the bonds-in-utility function assumption:

$$1 = \beta\frac{R}{\Pi} + \delta C$$

The δ parameter can thus be interpreted as regulating the marginal impatience of the representative household.

In the previous section, we derived this modification in the Euler equation by adding bonds in the utility function. Fisher (2015) proves an observational equivalence of this assumption with risk-premia shocks assumed by Smets and Wouters (2007) in the budget constraint of the household.

[11] With capital accumulation, Das (2003) proves that assuming $\beta'(C) > 0$ does not necessarily preclude the existence of a stable solution.

Another interpretation of the shocks to δ is that these capture the *flight to liquidity* episode of the recent financial crisis (Krishnamurthy and Vissing-Jorgensen, 2012). Similar wedge in the Euler equation can be associated with the deterioration in liquidity properties of AAA rated corporate bonds in contrast to Treasury securities during the 2008-09 financial crisis (Del Negro, Eggertsson, Ferrero and Kiyotaki, 2017).

Yet another equivalent way to introduce the modification is to model exogenous changes in transaction costs in trading of financial assets where the liquid government bonds earn the transaction premia (Calvo and Végh 1995; Lahiri and Végh 2003). Finally, modeling labor income dependent taxes on earned interest income on government bonds can provide another meaningful micro-foundation.[12] For example, let the household pay a fraction $\tau(\tilde{C}_t)$ of the interest rate income in taxes. If $\tau'(C) < 0$, we can derive the same Euler equation as with bonds-in-utility/ Uzawa preferences:

$$1 = \beta \frac{(1 - \tau(C))R}{\Pi}$$

While these different interpretations follow naturally from extensive literatures, we also view this wedge in the Euler equation as a reduced form for fundamental factors such as aging, savings glut, reserve accumulation, inequality, debt deleveraging etc. microfounded in the secular stagnation literature (Eggertsson et al. 2016, 2018; Auclert and Rognlie 2018). Such a reduced form modification allows the researchers to employ existing tools to include secular stagnation as a complementary explanation in the estimated DSGE models, much like "β shock" is used in the temporary liquidity trap literature.[13]

2.5.2 Ricardian Equivalence and Price level determinacy

We presented alternate ways to micro-found the proposed modification to the Euler equation. Essentially these assumptions make government bonds matter, directly or indirectly. Thus, the Ricardian equivalence proposition (Barro, 1974) does not hold in our setting. However, we emphasize

[12] In the appendix, we show that depending on the income tax code (whether it is "progressive" or "regressive"), one can obtain a discounted, standard or amplified Euler equation analytically illustrating the key idea in Werning (2015)'s discussion of modification to the Euler equation under various fiscal transfer policies that may/may not dampen the forward guidance channel of monetary policy in McKay, Nakamura and Steinsson (2016)

[13] In the recent literature that augments DSGE models with endogenous growth, (mean zero) shocks to preference for bonds are added to get co-movement of investment and consumption as well to derive the *divine coincidence* benchmark (Garga and Singh, 2016).

that a mere breakdown of Ricardian equivalence is not enough to generate a secular stagnation steady state. Take for instance the textbook representative agent model. Instead of lump-sum non-distortionary taxation, assume linear distortionary labor taxes. The Euler equation remains unmodified and yet the Ricardian equivalence breaks down in such a setting. The modification to the Euler equation is crucial to allow for the existence of a locally determinate secular stagnation steady state.

Furthermore, it is a well known result in the literature that interest rate pegs can achieve price level determinacy if government bonds provide transactions services (see Canzoneri and Diba 2005 in closed economy and Calvo and Végh 1995; Lahiri and Végh 2003 in open economy settings). While government bonds matter and provide services to the representative consumer in our setting, we do not obtain price level determinacy just from the breakdown of Ricardian equivalence. This is in fact crucial to allow for the existence of BSGU steady state or the secular stagnation steady state, depending on the bounds on wage flexibility. For example, assume that the felicity from bonds in the utility takes a quadratic form: $\frac{\delta}{2}\left(\frac{B}{P}\right)^2$. Then the Euler equation is:

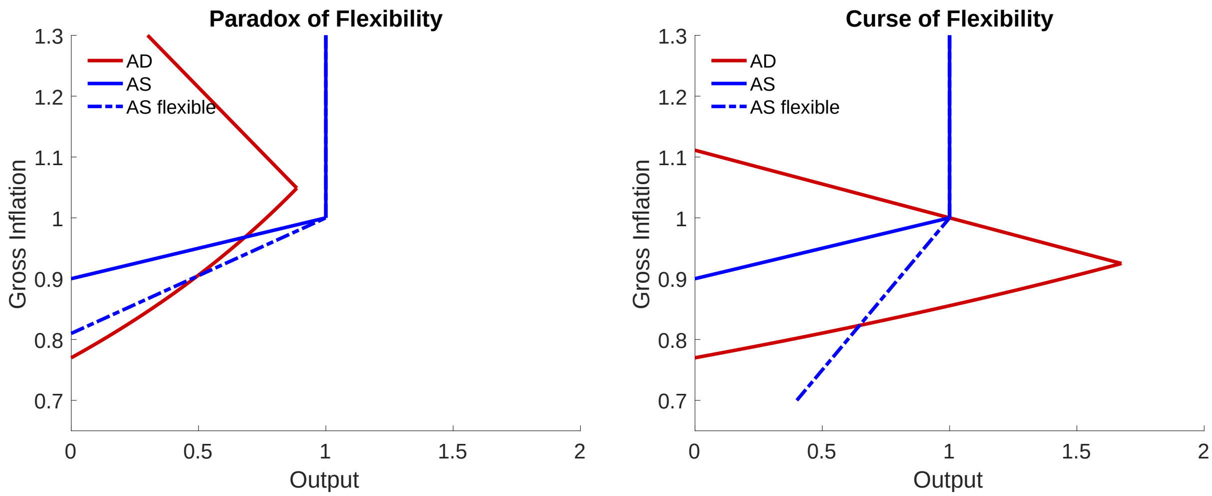
$$1 = \beta \frac{C_t}{C_{t+1}} \frac{R_t}{\Pi_{t+1}} + \delta C_t \frac{B_t}{P_t} \tag{10}$$

Given a path for nominal bonds $\{B_t\}$, with positive nominal bonds in steady state, the price level is uniquely determined even with a fixed nominal interest rate if there exists a steady state. Thus, this setting precludes the co-existence of a full employment and a BSGU steady state as shown in our analytical model. This example also clarifies that zero net supply of government bonds is not essential to the existence of secular stagnation steady state. For a given government policy, a linear felicity from real bonds would also modify the Euler equation so as to not feature the nominal price level in the Euler equation as in equation 10.

2.5.3 Perils of structural reforms: Curse of Flexibility

In models with temporary liquidity traps, a paradoxical result emerges with increasing flexibility of labor markets. A set of structural reforms that only increased wage flexibility increase deflationary pressures, worsening the liquidity trap. This is known as the *paradox of flexibility.* Similar insight extends to the case of secular stagnation, as shown in the left panel of Figure 9. If agents expect a long-lived liquidity trap driven by fundamentals, then an increase in nominal flexibility is deflationary.

Figure 9: Permanent increase in wage flexibility



Beyond the paradox, another reason to be cautious of undertaking supply side policy reforms emerges from our setting. These structural reforms that seek to increase downward wage flexibility also open the door to possibility of expectations driven traps (see the right panel in Figure 9). Conversely, structural reforms that flatten the Phillips curve by strengthening labor unions or legislating high enough minimum wages during recessions can preclude the possibility of BSGU trap. We label this as the *curse of flexibility*, because an economy at full employment may be subject to swings in expectations that lead to stagnation because of the downward flexibility in wages. In appendix D, we show that similar effect of flexibility can be derived in the overlapping generations model of Eggertsson et al. (2018).

In a recent work, Benigno and Fornaro (2017) construct a model with an expectations driven trap similar to the BSGU trap in terms of TFP growth and nominal interest rate. Their fiscal policy of precluding the stagnation trap imposes a lower bound on TFP growth rate through R&D subsidies. Thus, their suggested fiscal policy is analogous to a policy that puts a sufficiently high lower bound on nominal wage growth in our environment. Our analysis shows that policy reforms, that can preclude the BSGU steady state, are valid more broadly even in the textbook representative agent models with nominal rigidities.[14]

[14] Our analysis does not imply that imposing a lower bound on deflation is enough to preclude the BSGU steady state in more general settings. For example, in Benigno and Fornaro (2017) there is perfect downward nominal

3 Quantitative DSGE Model

We present a quantitative analysis based on a stylized small-scale New Keynesian model that has been widely studied in the literature (An and Schorfheide, 2007). Our model economy is composed of households, intermediate good producers, final good producers and a government. We briefly describe each of the decision problems next. A detailed derivation of equilibrium conditions is relegated to the appendix.

3.1 Households

$$\max_{C_t(k),H_t,B_t(k)} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t(k)/A_t)^{1-\tau} - 1}{1-\tau} - \chi_H \frac{H_t^{1+1/\eta}}{1+1/\eta} - \delta \frac{B_t}{A_t P_t} \right) \right],$$

subject to:

$$P_t C_t(k) + B_t(k) + T_t = W_t H_t + R_{t-1} B_{t-1}(k) + P_t D_t(k) + P_t SC_t,$$

The household derives utility from consumption C_t and from holding (real) stock of risk-free nominal bonds B_t, and disutility from hours worked H_t. The parameter χ_H scales the steady-state level of hours worked. The parameter δ regulates the marginal utility from holding bonds. The risk-free nominal bond pays a gross nominal interest rate R_t Each household supplies homogeneous labor services H_t in a competitive labor market taking the aggregate wage W_t as given. It collects interest payments on bond holdings, real profits D_t from intermediate good producers, pays lump sum taxes T_t, and receives payouts SC_t from trading a full set of state(k)-contingent securities.

rigidity, but endogenous growth opens up the possibility of a stagnation trap along the lines of BSGU steady state. Similarly, Heathcote and Perri (2018) model an economy with perfect downward nominal rigidity, precautionary savings and a liquid asset in net positive supply. Despite perfect downwardly rigid wages, a BSGU steady state exists in their model because of precautionary savings motive.

3.2 Final Good Firms

The perfectly competitive, representative, final good producing firm combines a continuum of intermediate goods indexed by $j \in [0,1]$ using the technology:

$$Y_t = \left(\int_0^1 Y_t(j)^{1-\nu_t} dj \right)^{\frac{1}{1-\nu_t}}.$$

Here $1/\nu_t > 1$ represents the elasticity of demand for each intermediate good. Profit maximization implies that the demand for intermediate goods is given by:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\nu_t} Y_t.$$

where the price of the final good P_t is given by :

$$P_t = \left(\int_0^1 P_t(j)^{\frac{\nu_t - 1}{\nu_t}} dj \right)^{\frac{\nu_t}{\nu_t - 1}}.$$

3.3 Intermediate Good Producers

Intermediate good j is produced by a monopolist who has access to the following production technology:

$$Y_t(j) = A_t H_t(j), \quad \text{with} \quad A_t = A_{t-1} z_t,$$

where A_t denotes the aggregate level of technology that is common to all firms, and z_t represents the stochastic (stationary) movements in TFP.

Intermediate good producers buy labor services $H_t(j)$ at a nominal price of W_t. Moreover, they face nominal rigidities in terms of price adjustment costs. These adjustment costs, expressed as a fraction of firms output, are defined by the function $\Phi_p(.)$:

$$\Phi_p \left(\frac{P_t(j)}{P_{t-1}(j)} \right) = \frac{\phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \Pi^* \right)^2 Y_t(j)$$

where Π^* is the inflation rate in the targeted steady state. Taking as given nominal wages, final good prices, the demand schedule for intermediate products and technological constraints, firm j chooses its the price $P_t(j)$ to maximize the present value of future profits:

$$\max_{\{P_{t+s}(j)\}} \quad \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s Q_{t+s|t} \left(\frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - \Phi_p \left(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} \right) Y_{t+s} - \frac{W_{t+s} Y_{t+s}(j)}{z_{t+s} P_{t+s}} \right) \quad ,$$

subject to

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\nu_t} Y_t.$$

Government Policies

The desired policy rate is set according to the following rule:

$$R_t^* = \left[r\Pi^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\psi_1} \left(\frac{Y_t}{Y_{t-1}} \right)^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R},$$

Here r is the steady state real interest rate, Π_t is the gross inflation rate defined as Π_t, and Π^* is the target inflation rate, which in equilibrium coincides with the steady state inflation rate. The actual policy rate relevant for agents decisions is subject to the zero lower bound constraint:

$$R_t \quad = \quad \max \{1, R_t^*\}$$

The government levies a lump-sum tax (subsidy) to finance any shortfalls in government revenues (or to rebate any surplus). The government's budget constraint is given by:

$$P_t G_t + R_{t-1} B_{t-1} \quad = \quad T_t + B_t,$$

where $G_t = \left(1 - \frac{1}{g_t}\right) Y_t$ is the government expenditure.

3.4 Resource constraint

We assume that the price adjustment costs are rebated back to the household in lump-sum fashion as part of the government transfers.[15] Hence, the market clearing resource constraint is given by:

$$C_t + G_t = Y_t$$

Finally, we assume nominal bonds are in net zero supply

$$B_t = 0$$

3.5 Shocks

There are three exogenous disturbances in the model: (i) exogenous changes to government expenditure g_t, (ii) exogenous changes to the growth rate of productivity z_t, and (iii) exogenous changes to the inverse demand elasticity for intermediate goods ν_t. We assume that these exogenous components obey the following auto-regressive processes:

$$\begin{aligned}
\ln g_t &= (1-\rho_g)\ln(g^*) + \rho_g \log g_{t-1} + \sigma_g \epsilon_{g,t}, && \epsilon_{g,t} \sim N(0,1) \\
\ln \nu_t &= (1-\rho_\nu)\ln(\nu^*) + \rho_\nu \log \nu_{t-1} + \sigma_\nu \epsilon_{\nu,t}, && \epsilon_{\nu,t} \sim N(0,1) \\
\ln z_t &= \rho_z \ln z_{t-1} + \sigma_z \epsilon_{z,t}, && \epsilon_{z,t} \sim N(0,1)
\end{aligned}$$

We allow the g_t and ν_t processes to have a non-zero mean, given by g^* and ν^*, that pin down the steady-state level of government consumption and the steady-state level of price markups, respectively.

3.6 Equilibrium Conditions and Competitive Equilibrium

Our model economy evolves around an stochastic balanced growth path given by the level of technology A_t. In order to solve the model we introduce the following stationary transformation: $\tilde{X}_t = X_t/A_t$. The derivations of the equilibrium conditions is relegated to the Appendix.

[15] An analogous interpretation would be to consider these costs as mental accounting costs for the firms or model these in the utility function of the representative agent.This assumption allows us to avoid unnatural results commonly associated with resource costs modeled in terms of output.

Definition 1 *A competitive equilibrium in terms the stationary variables is given by the sequence of quantities and prices* $\{\tilde{Y}_t, \tilde{C}_t, R_t, \Pi_{t+1}\}$ *which satisfy equations (11) - (14), given an exogenous sequence for processes* $\{g_t, \nu_t, z_t\}$*:*

$$1 = \beta \mathbb{E}_t \left[\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\tau} \frac{R_t}{z_{t+1}\Pi_{t+1}} \right] + \delta \tilde{C}_t^{\tau} \tag{11}$$

$$\zeta_t = \phi \nu_t \beta \mathbb{E}_t \left[\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\tau} \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t} \Pi_{t+1} \left(\Pi_{t+1} - \Pi^* \right) \right] \tag{12}$$

$$R_t = \max \left\{ 1, \left[r\Pi^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\psi_1} \left(\frac{\tilde{Y}_t}{\tilde{Y}_{t-1}} \right)^{\psi_2} \right]^{1-\rho_R} R_{t-1}^{\rho_R} \right\} \tag{13}$$

$$\tilde{C}_t = \frac{\tilde{Y}_t}{g_t}, \tag{14}$$

where $\zeta_t = (1-\nu_t) - \chi_H \tilde{C}_t^{\tau} \tilde{Y}_t^{1/\eta} + \nu_t \phi \left(\Pi_t - \Pi^* \right) \Pi_t - \frac{\phi}{2} (\Pi_t - \Pi^*)$.

As in our benchmark model in Section 2, equation (11) is the modified Euler equation in which the marginal utility of holding government bonds enters as an additional term.

4 Steady State of the Quantitative Model

4.1 Calibration

Because of the multiplicity of steady states, we calibrate parameters to match observed empirical moments in the data for Japan. Most of the calibrated parameter values are borrowed from Aruoba et al. (2017*b*), and presented in Table 1. The remaining parameters are set such that the economy is either in the secular stagnation steady state or the BSGU steady state.

The Frisch labor supply elasticity is fixed at 0.85, based on micro-level data based estimates of Kuroda and Yamamoto (2008). The (inverse) elasticity of demand for intermediate goods, ν, is set to a value of 0.1 to generate a steady state markup of 11% for the monopolistically competitive firms. Japan did not officially adopt an inflation target until 2013Q2. Consequently, we choose a zero inflation target $\Pi^* = 1$ for convenience. The labor disutility parameter χ_H is chosen so as to normalize the output in the targeted inflation steady state at one. Steady state government spending ratio is chosen from the consumption output ratio of 85.32% in the Japanese data.

Table 1: Parameters

Common Parameters				
η	ν	τ	Π^*	g^*
Inverse Frisch elasticity	Price s.s. markup	Intertemporal elasticity of substitution	Inflation target	Government Spending share
0.85	0.1	0.95	1	0.1468

Parameters Specific to Relevant Steady State				
		β	δ	ϕ
Steady State		Discount factor	Euler Equation Wedge	Adjustment cost parameter
Secular Stagnation		0.89	0.1305	2.45×10^3
BSGU		0.89	0.1253	7.57×10^2

Notes: The table shows the parameter values of the model for the baseline calibration.

The remaining parameters β, δ, ϕ are chosen to jointly match targets for natural interest rate, output gap and inflation in Japan. Since Japan has had a persistent stagnation, estimating output gap relative to a filter-based trend is problematic. This is because any such estimate of the trend is likely endogenous to the long-stagnation faced the economy. As such, Krugman (1998) and more recently Hausman and Wieland (2014) use indirect methods to quantify output gap in Japan. The most conservative estimate for output gap over 1995-2013 is -(minus) 4.5%, while the largest estimate in these studies is -(minus) 10%.

As a target for natural rate, we adopt two different targets, depending on the steady state. For the secular stagnation steady state, we chose the annual target of -(minus) 1%. This is based on two studies by Fujiwara et al. (2016) and Iiboshi et al. (2018) that separately estimate a series for the natural rate of interest in Japan based on the Laubach and Williams (2003) method. They find that the quarterly estimate was often -(minus) 0.5% since late 90s, and -(minus) 2% at the lowest level. In contrast, the BSGU steady state is calibrated to imply a (annualized) long-run real interest rate of 1%. For the sake of transparency, we fix $\beta = 0.89$, and find the remaining two parameters to target the respective steady state moments. The calibration implies an inflation rate in the empirically plausible range for either of the steady states. It is -1% in the secular stagnation and -3% under the BSGU steady state. Furthermore, the calibration implied slope of the log-linearized Phillips curve κ is 0.003 at the secular stagnation steady state, and 0.011 when calibrate the BSGU steady state. These values lie in the range of the conventional estimates found in the literature (see Aruoba, Bocola and Schorfheide 2017*a*).

Table 2: Model-Implied Values in Calibration of Steady State Parameters

	Natural Rate	Inflation	Output Gap
Secular Stagnation	-1	-1.1	-4.5
BSGU	1	-3	-4.5

Notes: The table shows the model-implied values in the calibration of the three steady state parameters (β, δ, ϕ).

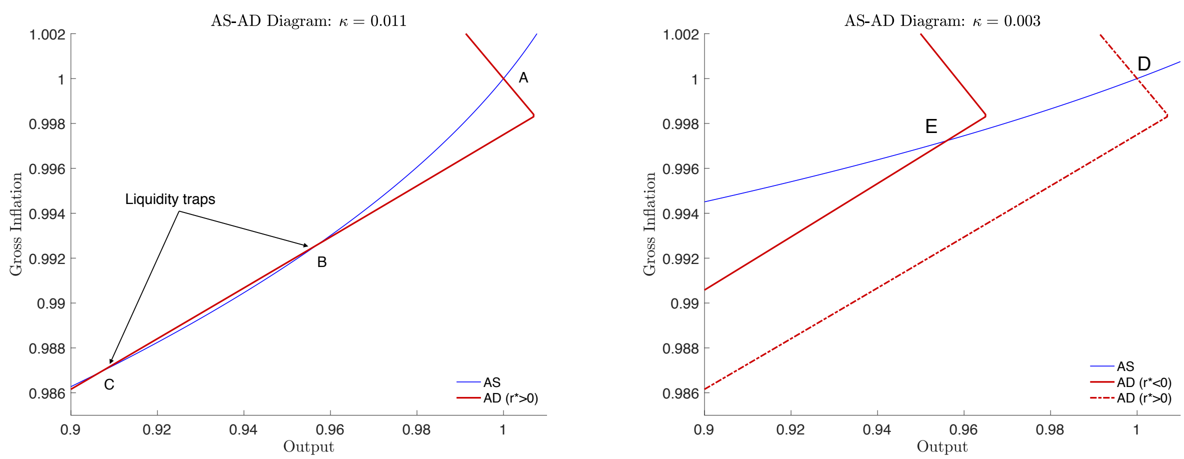
4.2 Steady state representation

In the last section, we show that the existence of a secular stagnation equilibrium does not depend on our choice of the downward nominal wage rigidity. In fact, a quantitative model with pricing frictions often used in quantitative work can accommodate the permanent stagnation steady state. The important ingredient is the modified Euler equation, which features a negative relationship between output and the short term real interest rate in steady state. We now show an analogous AS-AS representation of the quantitative steady state.

The left panel of figure 10 plots the aggregate demand and the aggregate supply curves for the case in which prices are relatively flexible. As in Section 2, we set $\delta < 1 - \beta$ which implies that the natural rate of interest is positive. The targeted-inflation steady state emerges when the aggregate supply curve intersect the downward sloping portion of the aggregate demand curve, and features target inflation equal Π^*, positive nominal interest rates and output normalized to one (depicted as *point A*).

Along with the targeted inflation steady state, there exist two additional steady states with zero interest rates. The steady state depicted at *point B* is locally indeterminate and satisfies the BSGU steady state criterion. At this intersection, the aggregate supply curve is steeper than the aggregate demand curve. Because intermediate good producers face convex costs of price adjustments, the aggregate supply curve is non-linear. The curvature of the aggregate supply flattens out as deflation increases the cost of adjusting prices. This opens up the possibility of an additional steady state, as depicted by *point C* where the aggregate supply schedule is flatter relative to the aggregate demand schedule. Although this steady state is locally determinate, features zero nominal interest rates and output permanently below full employment, it does not satisfy our definition of secular stagnation. This is because the natural rate remains positive at this steady state. For this reason, we do not pursue this steady state further in our analysis. We leave the investigation of this interaction of the AS curve and multiple stagnation steady states to future work.[16]

Figure 10: Steady State Equilibria of the Quantitative Model



In the right panel of Figure 10, we show the case of the aggregate supply curve with a relatively tight lower bound on inflation. When the natural rate is positive, $\delta < 1 - \beta$, the aggregate demand schedule intersects the aggregate supply curve at the targeted-inflation steady state (denoted by *point D*). In contrast, when the natural interest rate is negative, $\delta > 1 - \beta$, aggregate demand intersects aggregate supply at the secular stagnation steady state (denoted by *point E*). Consistent with the results in Section 2, Figure 10 shows that when the long-run aggregate supply curve does not accommodate high enough deflation, in this case because of the higher cost of price adjustment, the BSGU steady state does not exist. This highlights the importance of assumptions imposed on the supply side of the economy when analyzing alternative explanations for prolonged liquidity trap episodes.

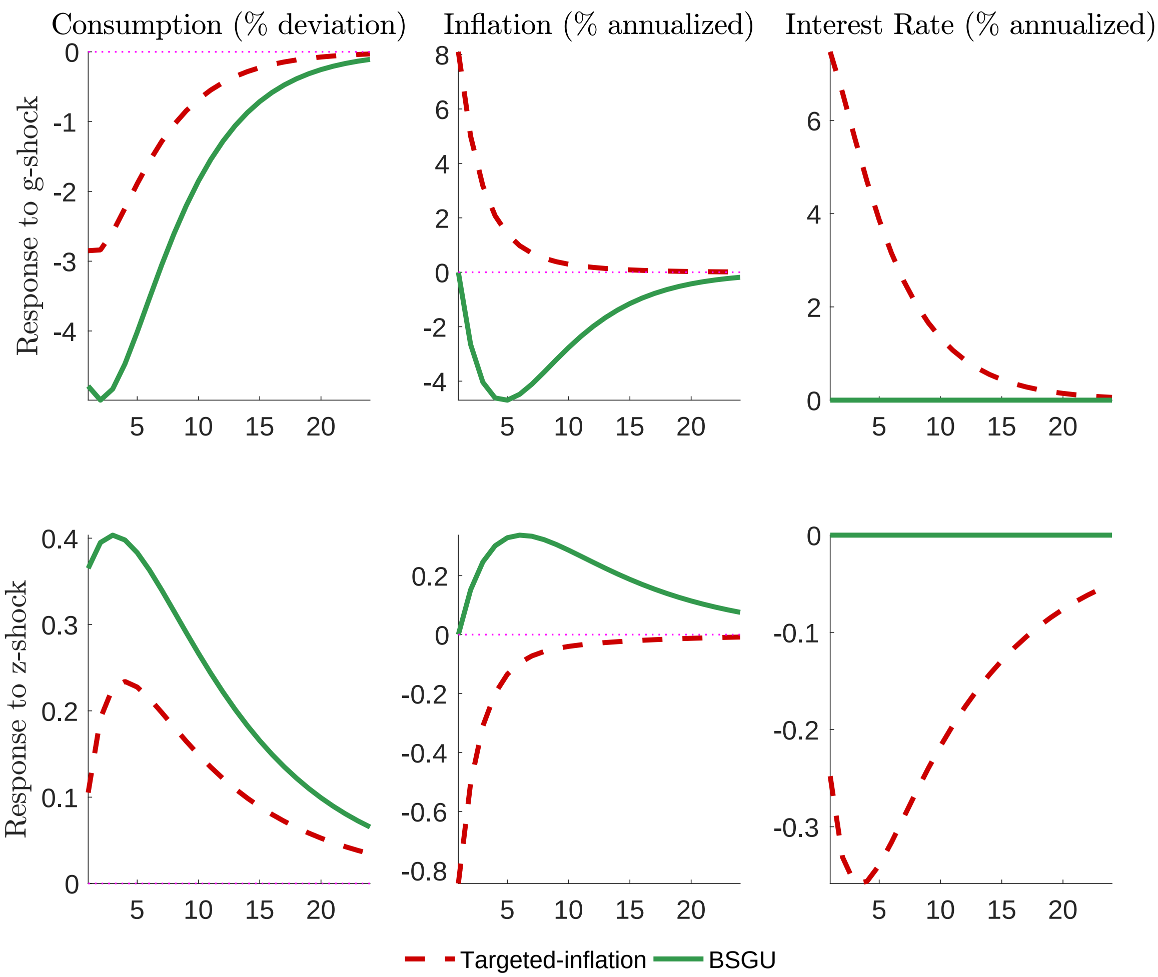
4.3 Impulse Response Functions

We now illustrate the difference in dynamics of BSGU and secular stagnation through impulse responses. Figure 11 contrasts impulse response function near the targeted-inflation steady state (red dashed line) to that near the BSGU steady state (solid green line) for consumption, inflation, and

[16] An interesting point to note is that multiple steady states with zero nominal interest rates may emerge due to the non-linearities induced by pricing frictions. Similar multiplicity can be obtained through changes in the curvature of the Euler equation, see for example Heathcote and Perri (2018) for a model with precautionary savings, perfectly downwardly rigid nominal wages and multiple zero interest rate steady states.

nominal interest rates after a one-time unanticipated shock to government expenditures and productivity. Near the targeted-inflation steady state, the increase in government expenditure crowds out consumption. The transmission channel is standard, higher demand creates upward pressure on prices. In response, the unconstrained portion of the policy rule calls for an endogenous increase in the nominal interest rate. Since the real rate rises in equilibrium, households reduce consumption expenditure. Similarly, a positive productivity shock lowers inflation, which is met with lower nominal and real interest rates and thus higher consumption.

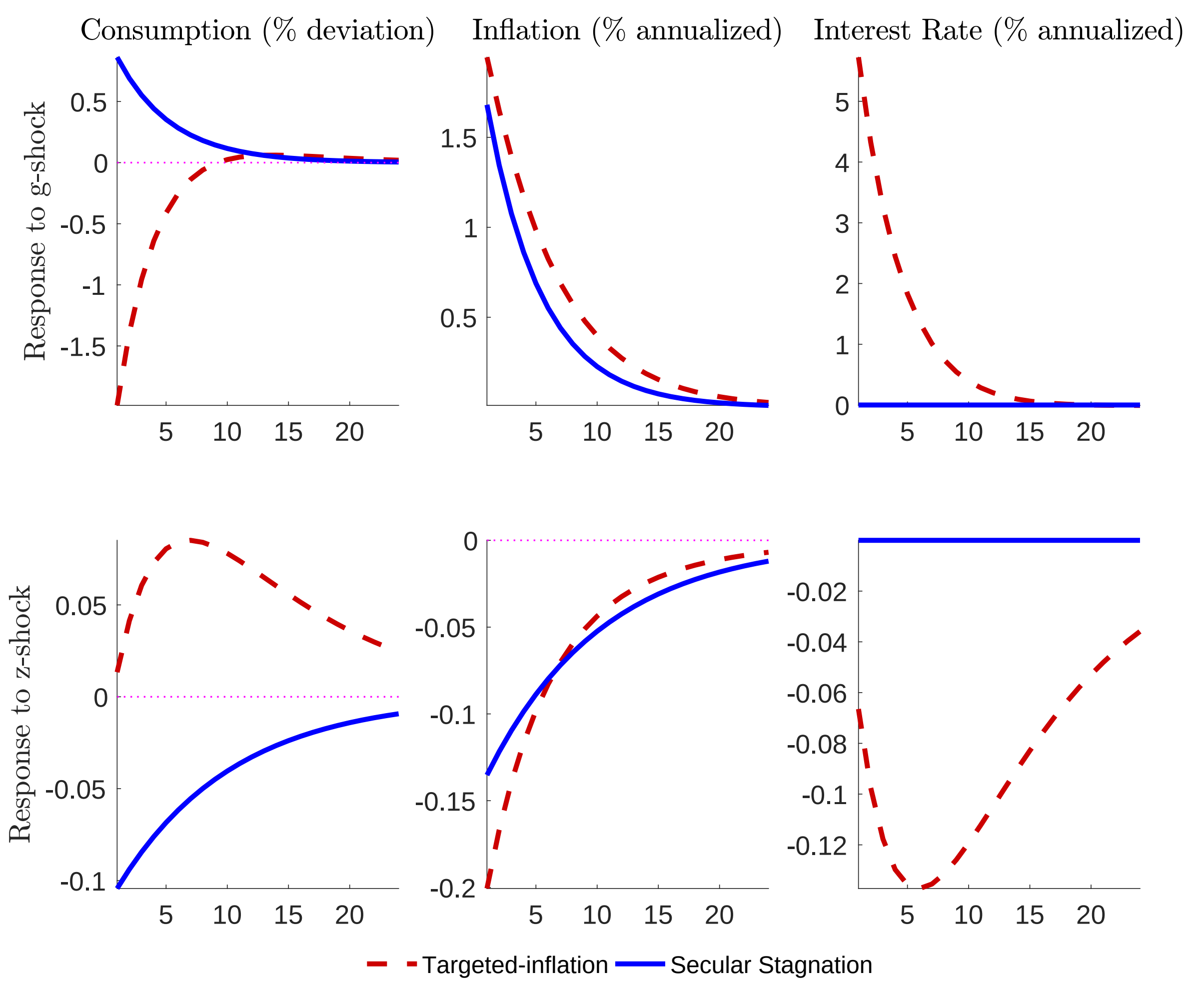
Figure 11: Targeted-Inflation vs BSGU



Notes: The dashed red lines correspond to the linear dynamics under near the targeted-inflation steady state. The solid greenlines are the corresponding dynamics near the secular stagnation steady state.

The dynamics in the neighborhood of BSGU steady state, are depicted by the solid green line in Figure 11. The responses of the economy to an increase in government expenditure are dramatically different. In the presence of deflationary expectations, higher government expenditure signals weaker demand which further reduces inflation expectations. Because the interest rate cannot fall below the zero lower bound, lower expected inflation raises the real rate. This exerts a contractionary pressure on household consumption. Similarly, higher productivity boosts inflation rather than lowering it as in the targeted-inflation steady state. The higher inflation lowers the real interest rate and boosts consumption. These results are similar to dynamic responses to documented in Mertens and Ravn (2014) and in the deflationary regime of Aruoba et al. (2017*b*).

Figure 12: Targeted-Inflation vs Secular Stagnation



Notes: The dashed red lines correspond to the linear dynamics under near the targeted-inflation steady state. The blue solid lines are the corresponding dynamics near the secular stagnation steady state.

Figure 12 compares the dynamics near the targeted-inflation steady state to the secular stagnation steady state. The impulse responses when the economy is near the targeted inflation steady state are shown in the dashed red line, and are qualitatively similar to the ones obtained in Figure 11. The quantitative differences in the two figures arise from the different calibrations in the cost of price adjustment. The solid blue line, depicts the dynamics under secular stagnation. Around the secular stagnation steady state, there is no crowding out of consumption in response to an increase in government expenditure. This is because there is no endogenous monetary policy reaction that counteracts the inflationary pressures of increased government spending. As a result, the government expenditure shock is expansionary. Similarly, a positive productivity shock puts a downward pressure on nominal wages and this further increases the real interest rate, forcing households to reduce their consumption expenditures (*paradox of toil*).

5 Reassessing Japan: Expectations Traps vs Secular Stagnation

We illustrate an important quantitative use of our framework. We estimate the model for Japan with zero nominal interest rates using standard rational expectations methods (Herbst and Schorfheide, 2016). The estimation is conducted by log-linearizing the model around the secular stagnation and the BSGU steady state. We find that either of the hypotheses is plausible to explain the long-run stagnation in Japan. Moreover, the estimated dynamics around either of the steady state feature different transmission mechanisms that we highlight.

Because we consider a period in which the ZLB was binding in Japan, we assume that agents expect to be in the liquidity trap permanently. While permanent shocks to expectations or fundamentals could move the economy back to the targeted inflation steady state, we do not allow for transitions between steady states. We leave a richer quantitative exercise for future work. Henceforth, ZLB permanently binds and we estimate the local dynamics around each steady state.

5.1 Data

We use data on output, consumption and inflation from Japan while imposing the zero nominal interest rates during the period 1995:Q4 to 2012:Q4. In Section 4.2, we calibrated the structural parameters $\tau, \beta, \delta, \phi, \nu$ to allow the model to be in secular stagnation or the BSGU steady state.

Because steady state features ZLB, we do not need to specify the parameters governing the monetary policy rule in equation (13). Instead, we peg the nominal interest rate to its theoretical lower bound of $R_t = 1$ throughout.

5.2 Solution and estimation

Following Lubik and Schorfheide (2004) the first order dynamics to the equilibrium conditions can be summarized by the following system of linear rational expectation equations (LRE):

$$\begin{aligned}
\Gamma_0(\theta)s_t &= \Gamma_1(\theta)s_{t-1} + \Psi(\theta)\boldsymbol{\varepsilon}_t + \Pi(\theta)\boldsymbol{\eta}_t, \\
s_t &= \left[\hat{Y}_t, \hat{\Pi}_t, \mathbb{E}_t\hat{Y}_{t+1}, \mathbb{E}_t\hat{\Pi}_{t+1}, \hat{g}_t, \hat{\nu}_t, \hat{z}_t\right]', \\
\boldsymbol{\varepsilon}_t &= \left[\epsilon_{g,t}, \epsilon_{\nu,t}, \epsilon_{z,t}\right]', \\
\boldsymbol{\eta}_t &= \left[\left(\hat{Y}_t - \mathbb{E}_{t-1}\hat{Y}_t\right), \left(\hat{\Pi}_t - \mathbb{E}_{t-1}\hat{\Pi}_t\right)\right]',
\end{aligned} \tag{15}$$

where the vector s_t contain endogenous variables including expectations, the vector $\boldsymbol{\varepsilon}_t$ contains the three structural shocks of the model, and the vector $\boldsymbol{\eta}_t$ corresponds to the one-period ahead forecast errors associated with the two expectational variables of the system. The matrices $\Gamma_0, \Gamma_1, \Psi, \Pi$ are functions of the structural parameters denoted by θ. We use standard Bayesian methods to estimate the following set of parameters:

$$\begin{aligned}
\theta^{SecStag} &= \{\rho_g, \rho_\nu, \rho_z, \sigma_g, \sigma_\nu, \sigma_z\} \quad \text{and} \\
\theta^{BSGU} &= \{\rho_g, \rho_\nu, \rho_z, \sigma_g, \sigma_\nu, \sigma_z, \sigma_\zeta, \sigma_{\zeta,\epsilon_g}, \sigma_{\zeta,\epsilon_\nu}, \sigma_{\zeta,\epsilon_z}\}
\end{aligned}$$

As discussed in Section 2, when the economy is at the secular stagnation steady state, equation 15 satisfies the Blanchard-Khan determinacy conditions and a unique solution can be constructed using standard methods. For the secular stagnation model we estimate the persistence and standard deviation of fundamental shocks g_t, ν_t, z_t. For the case in which the economy is near the BSGU steady state the LRE system exhibits self-fulfilling dynamics in which the expectational variables become linear functions of the structural shocks $\boldsymbol{\varepsilon}_t$ and the sunspot shock, which we denote by ζ_t. Following the method of Bianchi and Nicolo (2017), we modify equation 15 to construct a solution to the system of LRE in the presence of self-fulfilling expectations. To account for indeterminacy in the BSGU model we need to estimate the standard deviation of the sunspot shock σ_ζ and the covariances of the sunspot shock with the fundamental shocks: $\sigma_{\zeta,\epsilon_g}, \sigma_{\zeta,\epsilon_\nu}, \sigma_{\zeta,\epsilon_z}$

Table 3 presents the assumed priors and the estimated posteriors for θ approximated around the BSGU and the secular stagnation steady state. The priors for the common parameters across models are identical and we use fairly agnostic priors. We set priors over the correlations, instead of the covariances, of the fundamental shocks and the sunspot shock and use an uninformative prior in order to let the data pin down the preferred correlation structure to describe the dynamics under indeterminacy.

Table 3: DSGE Estimated Parameters

Parameter	Distribution	Para (1)	Para (2)	BSGU	Secular Stagnation
		Fundamental shocks parameters			
ρ_g	$\mathcal{B}$	0.6	0.2	0.8565	0.8369
				[0.7695, 0.9377]	[0.7584, 0.915]
ρ_ν	$\mathcal{B}$	0.6	0.2	0.1613	0.1425
				[0.0405, 0.27]	[0.0428, 0.2417]
ρ_z	$\mathcal{B}$	0.6	0.2	0.3295	0.5205
				[0.1419, 0.5018]	[0.2445, 0.8154]
σ_g	$\mathcal{IG}$	0.03	2	0.0080	0.0079
				[0.0068, 0.009]	[0.0067, 0.009]
σ_ν	$\mathcal{IG}$	0.03	2	0.0145	0.0129
				[0.0122, 0.0167]	[0.0109, 0.0151]
σ_z	$\mathcal{IG}$	0.03	2	0.0025	0.0046
				[0.0013, 0.0035]	[0.0022, 0.0069]
		Sunspot shock parameters			
σ_ζ	$\mathcal{IG}$	0.03	2	0.0027	-
				[0.0023, 0.0031]	-
$corr(\epsilon_z, \zeta)$	$\mathcal{U}$	-1	1	0.0904	-
				[-0.0608, 0.2595]	-
$corr(\epsilon_\nu, \zeta)$	$\mathcal{U}$	-1	1	0.9596	-
				[0.9326, 0.9881]	-
$corr(\epsilon_g, \zeta)$	$\mathcal{U}$	-1	1	0.0302	-
				[-0.0411, 0.1022]	
$\log(p(\mathcal{Y}_t))$				-298.2664	-292.0135

Notes: Para (1) and Para (2) list the means and the standard deviations for the Beta ($\mathcal{B}$) distributions; the upper and lower bound of the support for the Uniform distribution ($\mathcal{U}$); and s and v location parameters for the Inverse Gamma ($\mathcal{IG}$) distribution, where $p_{IG}(\sigma|v,s) \propto \sigma^{-v-1}\exp(-vs^2/2\sigma^2)$. Numbers in square brackets denote 90% credible intervals.

The estimated parameters governing the persistence and volatility of the fundamental shocks are similar across the BSGU and the secular stagnation models, with the exception of the parameters of the technology shock which have a higher persistence and volatility in the secular stagnation model. The standard deviation of the sunspot shock in the BSGU specification is statistically different from zero and with a magnitude similar to that of the technology shock. The estimated correlation between the fundamental and sunspot shocks varies substantially. The data favors a high correlation between the markup shock and the sunspot shock, while picking up a small correlation of the sunspot shock with the other two fundamental shocks. These results illustrate that the BSGU model relies on a different transmission mechanism than the secular stagnation model. In the next section we investigate in more detail the difference in the transmission mechanism of shocks across the two models.

5.3 Model comparison

The last row of Table 3 shows the overall fit of the two specifications using the marginal data density estimates, denoted with $\log(p(\mathcal{Y}_t))$. It represents the likelihood function integrated over the prior distribution of the parameters. Both models perform equally well in terms of fitting the Japanese data over 1995-2012, although secular stagnation hypothesis is somewhat favored. This illustrates that the secular stagnation hypothesis, where a persistent stagnation is driven by change in fundamentals is an equally plausible hypothesis when compared head to head with the expectations trap hypothesis.

To further explore the difference across models, Table 4 presents the conditional variance decomposition for output, consumption and inflation. Under secular stagnation, the model interprets the data only through fundamental shocks. Panel A shows that technology and the demand shocks explain most of the variation in consumption. In contrast, panel B and C show that consumption is mostly explained by technology shocks, while inflation is driven primarily by shocks to markup.

Under the BSGU hypothesis, in contrast, the first-order dynamics are less dependent on the fundamental shocks. Instead, sunspot shocks play a substantial role explaining between 50 to 60 percent of the forecast error variance of the output, consumption and inflation.

Because the dynamics under BSGU are indeterminate, in our estimation we chose a particular solution by allowing the correlations between fundamental and sunspot shocks to be estimated. In Table 5 we explore the role of the alternate correlations in accounting for the importance of sunspot shocks. We proceed by comparing the baseline estimated BSGU model with a model

Table 4: Variance Decomposition

A. Output

	Secular Stagnation			BSGU			
horizon	ϵ_z	ϵ_g	ϵ_v	ϵ_z	ϵ_g	ϵ_v	ζ
1	20.64	79.13	0.23	8.79	43.21	10.39	37.6
4	12.3	87.6	0.1	6.51	30.27	16.34	46.87
8	10.2	89.71	0.09	6.2	19.6	19.81	54.39
12	9.79	90.12	0.08	6.08	14.96	21.3	57.66
∞	9.67	90.25	0.08	5.18	16.31	21.55	56.96

B. Consumption

	Secular Stagnation			BSGU			
horizon	ϵ_z	ϵ_g	ϵ_v	ϵ_z	ϵ_g	ϵ_v	ζ
1	97.91	0.98	1.11	14.68	5.22	17.35	62.76
4	97.37	1.81	0.82	8.65	7.36	21.72	62.28
8	96.96	2.22	0.82	6.96	9.63	22.27	61.14
12	96.86	2.32	0.82	6.34	11.29	22.22	60.15
∞	96.83	2.35	0.81	5.15	16.76	21.43	56.66

C. Inflation

	Secular Stagnation			BSGU			
horizon	ϵ_z	ϵ_g	ϵ_v	ϵ_z	ϵ_g	ϵ_v	ζ
1	0.04	0.21	99.75	0.82	0.09	92.08	7.01
4	0.05	0.53	99.42	2.09	0.18	76.24	21.48
8	0.05	0.66	99.29	2.91	1.21	64.31	31.58
12	0.05	0.69	99.26	3.34	2.82	56.81	37.03
∞	0.05	0.7	99.25	4.32	12.85	33.2	49.63

where we impose zero correlation of sunspot shock with the fundamental shocks. Surprisingly, the sunspot shocks still account for roughly 40 percent of the variance of the three outcome variables. We then allow only one of the correlations between fundamentals and sunspot shocks to be set at its estimated value while the other correlations to set to zero. We find that it is the correlation of sunspot shocks with markups shocks that largely accounts for the additional importance of sunspot fluctuations under indeterminacy.

Table 5: BSGU: Fundamentals vs Sunspots

	Variance Decomposition ($h = \infty$)				
	Baseline	No correlation	$corr(\nu, \zeta)$	$corr(g, \zeta)$	$corr(z, \zeta)$
Output	56.96	43.74	59.92	43.84	43.42
Consumption	56.66	43.66	58.16	43.82	43.34
Inflation	49.63	44.48	51.78	44.63	44.16

Overall, we cannot tilt the balance in favor of either model. Importantly, our exercise illustrates that quantitative evaluation of secular stagnation models is feasible in an empirically realistic DSGE model. Aruoba et al. (2017*b*) find that expectations regarding the timing of exit from the liquidity trap play a key role in order to identify transitions across the targeted-inflation regime and the BSGU steady state. Incorporating these expectations in the context of the secular stagnation steady state is an important extension that we leave for future work.

6 Conclusion

In this paper, we propose a modification to the inter-temporal Euler equation of the representative household to contrast two sources of persistent stagnation. We show that such a modification can be micro-founded by any of the three deviations: a) bonds in the utility, b) Uzawa-Epstein preferences, and c) risk-premium shocks or shocks to transaction costs.

The analytical model is useful in many respects. An novel policy implication emerged from our analysis: Labor reforms that increase wage flexibility can make the economy vulnerable to expectations driven traps. Furthermore, the modified Euler equation allows researchers to estimate locally determinate models with permanent stagnation, and conduct policy counterfactuals.

We show that a quantitative model can accommodate either of the stagnation episodes for reasonable parameterizations. Thus suggesting caution in advocating policy reforms to escape stagnation, that work locally to one type of stagnation. Future research that develops techniques to empirically identify the steady state following Mertens and Williams (2018) is desirable.

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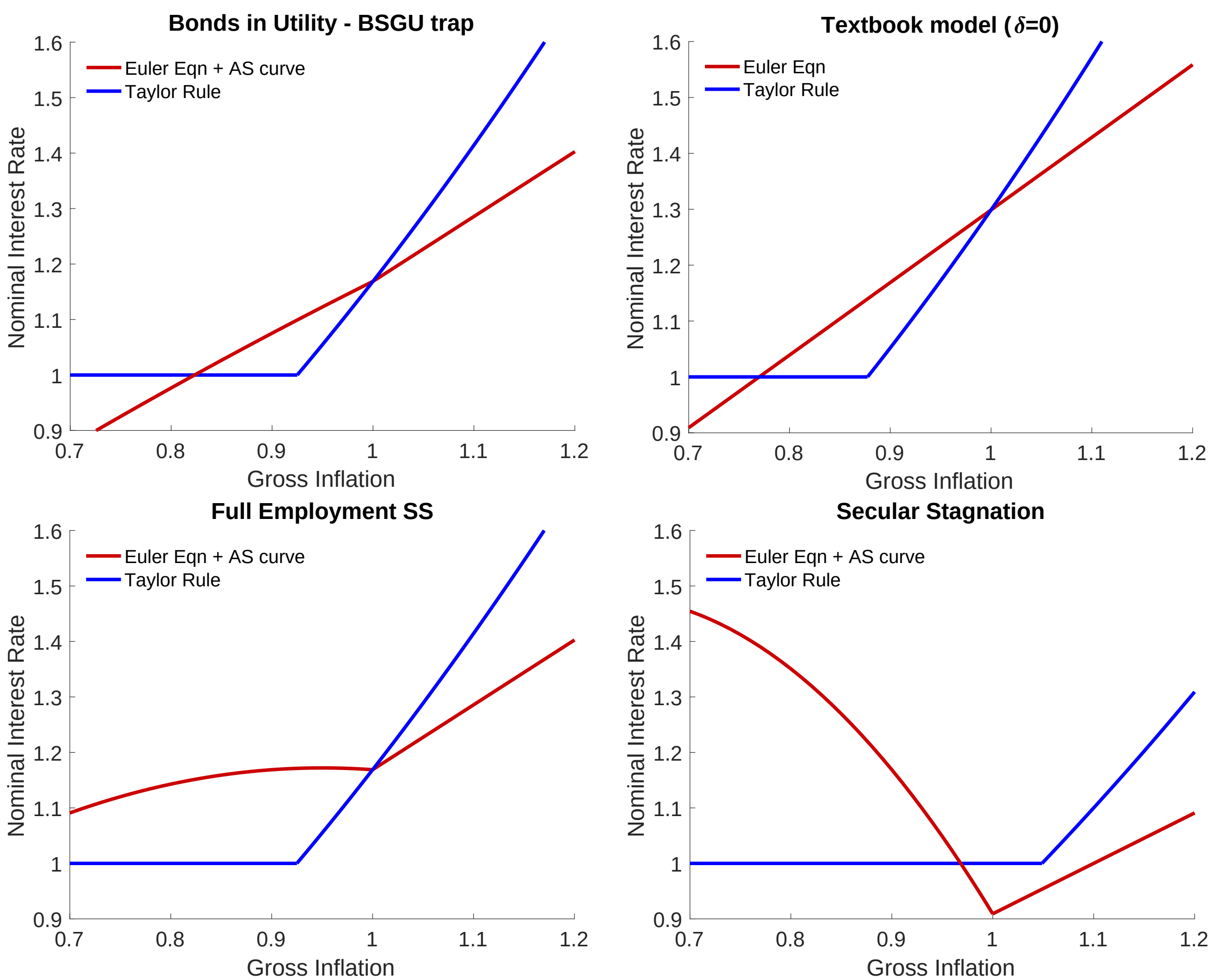
A i-Π graphs representation as in Benhabib et al. (2001)

in our model with $\delta > 0$, the Euler equation cannot be graphed without specifying the aggregate supply block. The top left panel in Figure 13 plots the (Π, i) space representation for the BSGU steady state result highlighted in Proposition 3. There is a kink in the combined Euler equation and AS curve when inflation rate is one. It is a straight line above inflation rate of one, and becomes concave for values of inflation below one. This curvature is inherited form the downward nominal wage rigidity. If this rigidity is strong enough, there does not exist a BSGU steady state (as in Proposition 1), illustrated in bottom left panel. Furthermore, when the constraints on monetary policy are binding such that it cannot replicate the natural rate, there exists a unique secular stagnation steady state as shown in the bottom right panel.

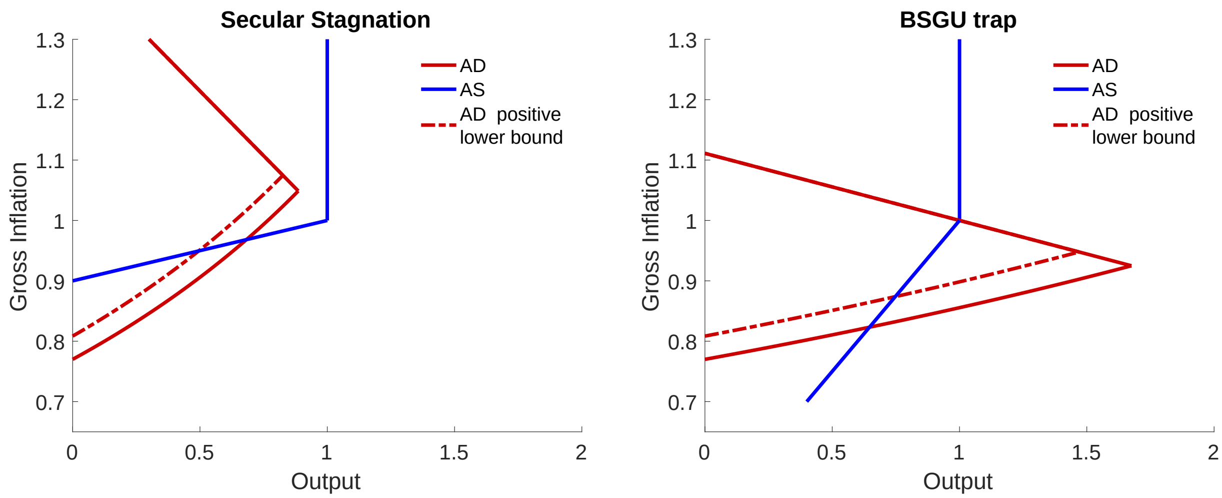
Figure 13



B Other comparative Statics

B.1 Changing Lower Bound

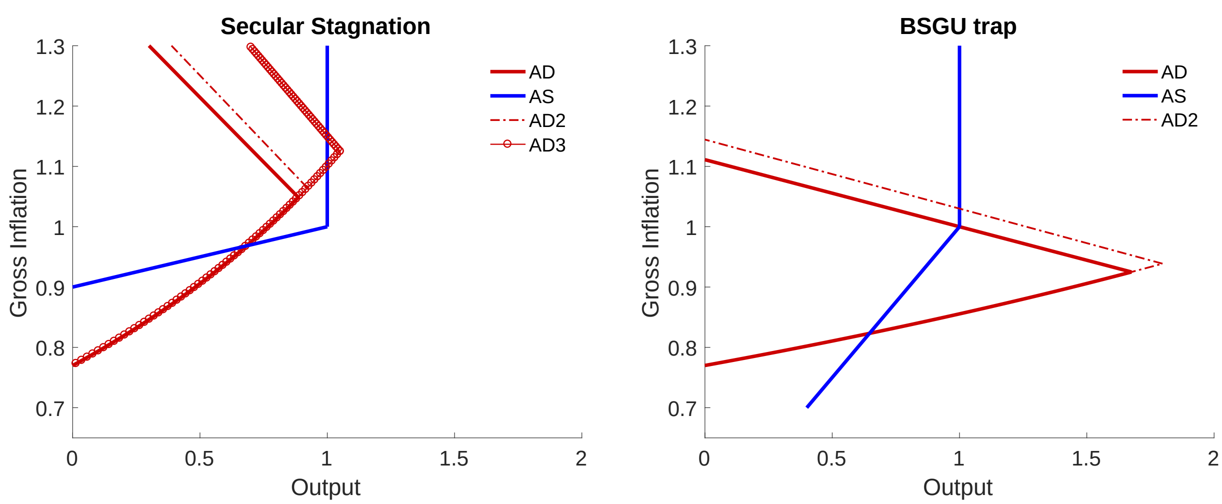
Figure 14: Permanent increase in lower bound



Increasing the lower bound from zero makes the BSGU steady state less contractionary and the reverse holds true for the Sec Stag steady state. The normal steady state is not impacted by the changing the lower bound.

B.2 Increasing Inflation Target / Timidity Trap

Figure 15: Higher inflation target



Increasing inflation target does not change the equilibrium allocation in panel a. In Panel b), increasing inflation target gives rise to the possibility of an equilibrium with full employment only if the inflation target is increased to a sufficiently high level. This formalizes the *timidity trap* argument made by Krugman.[17] In Panel c), there always exists the full employment equilibrium along with the unemployment equilibrium.

In the scenarios presented in panels b) and c), the economy can only move to full employment state if the central bank could credibly raise the inflation expectations to be consistent with the full employment equilibrium.

C Analytical results with EMR wage setting

$$\frac{\gamma}{\Pi} = 1 - (1-\gamma) y^{\frac{1-\alpha}{\alpha}}$$

Equilibria is given by:

$$1 = \frac{\beta R}{\Pi} + \delta C^{\sigma}$$

$$C = F(h)$$

$$F'(h) = w$$

$$\frac{\gamma}{\Pi} = 1 - (1-\gamma) C^{\frac{1-\alpha}{\alpha}} \quad \text{if } \Pi \leq 1$$

$$u = 1 - h$$

$$R = \max\{1, R^* \left(\frac{\Pi}{\Pi^*}\right)^{\alpha_\pi}\}$$

where $\alpha_\pi > 1$.

Aggregate Supply: At $Y = 1$, it becomes a vertical line. For $\Pi < 1$, it is strictly monotonic and upward sloping. At $\Pi = \gamma$, $Y = 0$.

The slope of the upward sloping portion of the aggregate demand is $\frac{\delta \Pi^2}{\beta} > 0$. The second derivative is strictly positive for $y \geq 0$ $(\frac{2\delta^2 \Pi^3}{\beta^2})$. As $y \to 0$, $\Pi_{AD} \to \beta$. When $R^* > 1$, $\Pi_{kink} < \Pi^* = 1$.

When $y = 0$, $\Pi_{AD} = \beta$ and $\Pi_{AS} = a$. When $y = 1$, $\Pi_{AD} = \frac{\beta}{1-\delta}$ and $\Pi_{AS} = 1$.

[17] See: Krugman, Paul R. "The Timidity Trap." New York Times March 20, 2014. And: Krugman, Paul R. "Timid Analysis (Wonkish)." New York Times March 21, 2014.

Proposition 4 *(Unique Steady State): Let $\Pi^* = 1$, $1 \geq \gamma > \frac{\beta}{1-\delta}$ and $\sigma = 1$. There exists a unique steady state. It is the full employment steady state.*

Proposition 5 *(Unique Steady State): Let $\Pi^* = 1$, $\delta > 1 - \beta$, $\gamma > \beta$ and $\sigma = 1$. There exists a unique steady state. It is the secular stagnation steady state.*

Proposition 6 *(Two Steady States): Let $\Pi^* = 1$, $\delta < 1 - \beta$, and $\sigma = 1$. If $a = \gamma < \beta$ there exists two steady states. There is a unique full employment steady state, and a BSGU steady state.*

With a more general downward wage rigidity assumption(as in SGU) , we could show that if there exists at least one equilibrium and none of the equilibria are full employment steady state, then at least one of the equilibria must be a secular stagnation steady state.

D BSGU steady state in Eggertsson et al. (2018)

In this section, we show that our result that the bounds on wage flexibility determine the existence of BSGU steady state extends to the OLG model of secular stagnation. We directly specify the bond-market equilibrium condition, and refer the reader to its detailed derivation in Eggertsson et al. (2018, Sec 3).

The bond market clearing condition:

$$1 + r_t = \frac{1+\beta}{\beta} \frac{D_t}{Y_t - D_{t-1}}$$

where D is the exogenous debt limit faced by the young borrowers. Rest of the equilibrium conditions are same as derived in Section 2. We list them here for completeness.[18]

Fisher equation:

$$1 + r_t = \frac{R_t}{\Pi_{t+1}}$$

Aggregate Supply block

$$\alpha h_t^{\alpha-1} = w_t$$

$$h_t \leq \bar{h}, \quad w_t \geq (\gamma + (1-\gamma) h_t^{\alpha}) \frac{w_{t-1}}{\Pi_t}, \quad (\bar{h} - h_t)\left(w_t - (\gamma + (1-\gamma) h_t^{\alpha}) \frac{w_{t-1}}{\Pi_t}\right) = 0$$

[18] It is straightforward to show existence of BSGU steady state even with the wage setting assumed by Eggertsson et al. (2018).

Monetary policy

$$R_t = \max\{1, (1 + r_t^*)\Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\alpha_\pi}\}$$

In the steady state, aggregate demand block is given by:

$$Y_{AD} = D + \begin{cases} \frac{1+\beta}{\beta}\frac{D}{\Gamma^*}\Pi^{1-\alpha_\pi}, & \text{if R } > 1, \\ \frac{1+\beta}{\beta}D\Pi, & \text{if R } = 1 \end{cases} \tag{D.1}$$

where $\Gamma^* = (1 + r^*) (\Pi^*)^{1-\alpha_\pi}$. The aggregate supply block in the steady state simplifies to:

$$Y_{AS} \leq 1, \quad \Pi \geq (\gamma + (1-\gamma)Y_{AS}), \quad (Y_{AS} - 1)\left(1 - (\gamma + (1-\gamma)Y_{AS})\frac{1}{\Pi}\right) = 0 \tag{D.2}$$

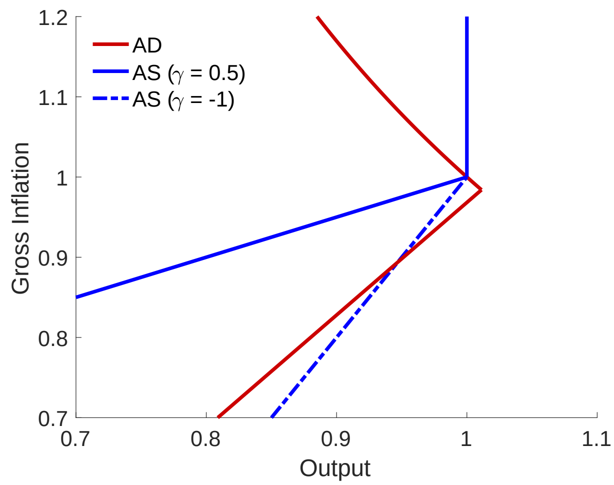


Figure 16: Changing the Lower Bound on Deflation in the EMR model

In Figure 16, we plot the aggregate demand the aggregate supply curves in the steady state, assuming that the natural rate is positive. We use the following parameters for illustration: $\beta = 0.77$, $D = 0.31$, $\Pi^* = 1$, $\alpha = 0.5$, $\gamma = 0.5$. With the exception of the debt limit D parameter, the remaining parameters are same as in our benchmark model. We plot the aggregate supply curve for two values of γ. When the lower bound on deflation is sufficiently low, the BSGU steady state co-exists along with the full employment steady state (see the dotted blue line).

E Quantitative Model with standard Euler equation

In Figure 17, we show the AS-AD diagram in the steady state of the quantitative model with the standard Euler equation. It shows the robustness of the corresponding result in the analytical model with a downward nominal wage rigidity plotted in Figure 2. Now the long-run aggregate supply is upward sloping even with positive inflation. Since the key equation is the aggregate demand equation and it remains unmodified in the quantitative model, the message from the analytical model goes through. Even in the textbook model, for there to exist an equilibrium with deflation and unemployment, aggregate supply curve needs to allow sufficient deflation. As in our discussion on figure 10 with the modified Euler equation, the specification of nominal rigidity is essential to the existence of the BSGU steady state.

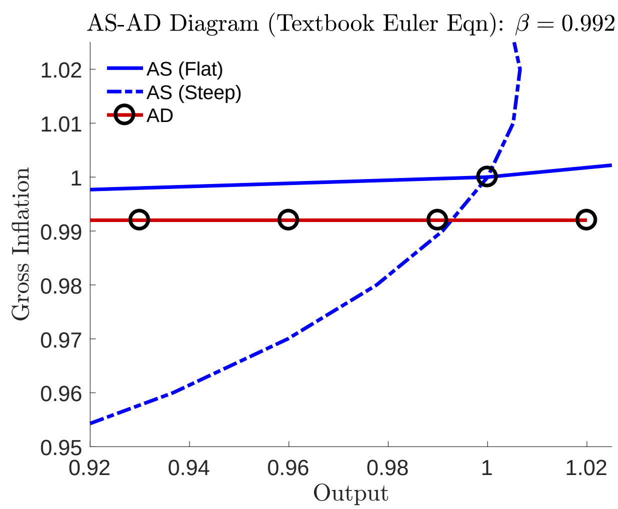
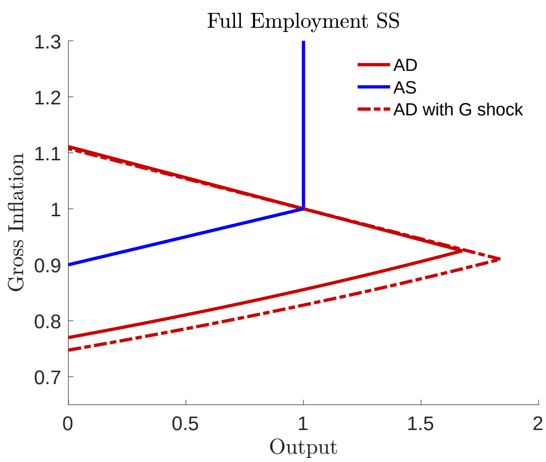


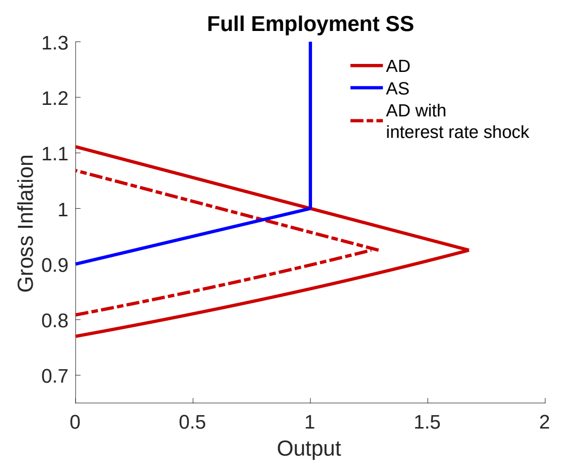
Figure 17: Steady State Equilibria

F Comparative Statics around the targeted inflation steady state

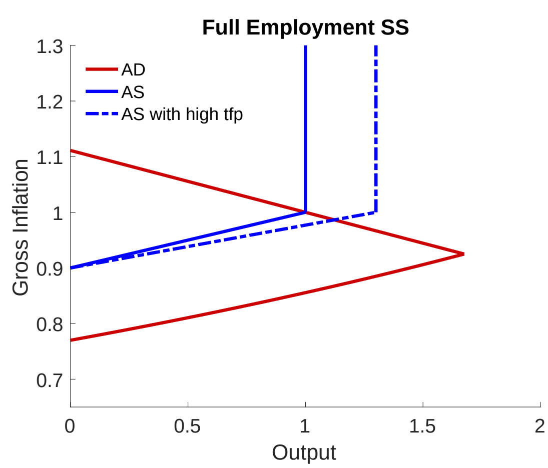
(a) Permanent increase in G



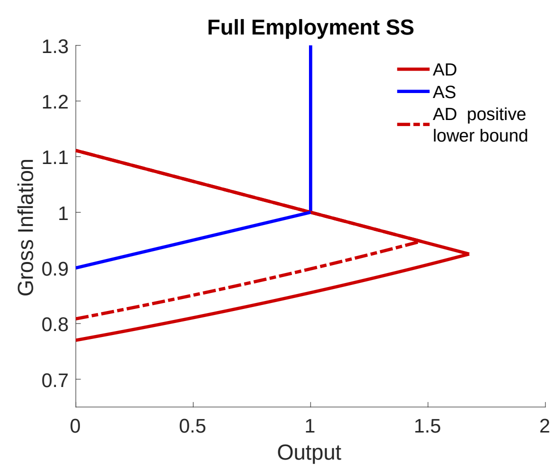
(b) Permanent increase in nominal interest rate



(c) Permanent increase in TFP



(d) Permanent increase in lower bound



(e) Higher inflation target