Bargaining for Assembly

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Abstract

A buyer wants to purchase multiple contiguous items from sellers holding an item each. We refer to such situations as assembly problems. We model contiguity of items through graphs where each node represents an item and an edge between two nodes denotes physical adjacency. The buyer wants to purchase a path of a desired length, called a feasible path. A seller is critical if he lies on every feasible path. We investigate subgame perfect equilibria of an infinite horizon alternate-offer bargaining game between the buyer and the sellers. We show that the buyer can extract full surplus within two periods if the valuations of the sellers are symmetric and there are no critical sellers. Further, we show that if the valuations of the sellers are asymmetric, there does not exist any equilibria where the buyer extracts full surplus. Our paper thus brings out the role of complementarity in location and competition in valuation in assembly problems.

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Keywords: Contiguity, Complementarity, Competition, Holdout, Land Acquisition, Bargaining, Assembly, Anticommons

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1 Introduction

A buyer wants to purchase multiple contiguous items from sellers owning an item each. We refer to such situations as assembly problems. Strategic delays or holdouts, as they are commonly known, are typical of such problems. By using such delays as threats, sellers can extract surplus from the buyer. We model contiguity of items through graphs where each node represents an item and an edge between two nodes denotes physical adjacency. The buyer wants to purchase a path of a desired length, called a feasible path. A seller is critical if he lies on every feasible path. Applied bargaining literature has studied holdout under certain bargaining protocols when the number of sellers is the same as the number of items required, i.e., when all sellers are critical in our sense. We investigate subgame perfect Nash equilibria (SPNE) of an infinite horizon alternate-offer bargaining game between the buyer and the sellers. We show that the buyer can extract full surplus within two periods if the valuations of the sellers are symmetric and there are no critical sellers. Further, we show that if the valuations of the sellers are asymmetric, there does not exist any equilibria where the buyer extracts full surplus. Our paper thus brings out the role of complementarity in location and competition in valuation in assembly problems.

Heller (2008) presents a lively account of how too much private ownership prevents assembly by creating a "gridlock". He has cited a number of examples including assembling individual patents for manufacturing a drug, assembling airwaves to establish a telecommunication network, mashup and remixing in the music industry and land acquisition for projects like airports, wind farms and large scale agriculture. In all these cases, ownership of a resource is fragmented. An individual owner, therefore, can potentially prevent assembly, which results in suboptimal use of the resource — a problem that Heller refers to as the "tragedy of the anticommons". This term is in sharp contrast to the well-known "tragedy of the commons" (Hardin, 1968) where absence of ownership results in overuse and eventual depletion of a resource.

The usual remedy for both the commons and the anticommons problems is to limit rights of use or rights of exclusion through social, legal or economic sanctions. Such sanctions, wherever applied, are strongly resisted on the grounds of violation of property rights. Coase (1960) conjectured that if property rights are well-defined and tradeable, competitive bargaining for such rights would lead to efficient outcomes in the absence of transaction costs. In the anticommons problem, complementarity of demand allows individual sellers to exert monopoly power to the extent that the buyer is not left with much incentive to implement her project. By allowing for different degrees of complementarity, we are able to show that when seller valuations are symmetric, the buyer is able to extract full surplus unless there is a critical seller. In this case, criticality is the only form of complementarity that can prevent the buyer from extracting full surplus. In contrast, when sellers have asymmetric valuations, efficient sellers are endowed with more bargaining power and the buyer is never able to extract full surplus. Thus, we are able to identify two sources of inefficiency in the assembly problem, viz., presence of critical sellers and asymmetry of valuations.

The bilateral trade model represents the most elementary form of a market for an indivisible item involving one buyer and one seller. Consider a simple model where a buyer and a seller bargain for the ownership of an indivisible item with complete information of valuations for the object for one period: the buyer makes the first offer which the seller may accept or reject. If we postulate that the seller decides to accept any offer that does not make him strictly worse-off, then this game has a unique subgame perfect Nash equilibrium outcome: the buyer offers the seller his exact valuation, the seller accepts, and thus the buyer extracts full surplus. Contrast this equilibrium outcome to that of the infinite horizon alternate offer bargaining model due to Rubinstein (1982): the buyer has to offer a strictly positive share of the surplus to the seller to avoid strategic delay or as a cost of holdout.

Roy Chowdhury and Sengupta (2012) have studied the problem of one buyer bargaining with multiple sellers holding an item each, where all items are complementary. In their model, the buyer begins bargaining by making simultaneous offers to all active sellers. A seller can accept or reject the offer she receives. On acceptance, the seller surrenders his plot in lieu of the cash offer and leaves the market. Sellers rejecting buyer's offer make counteroffers in the next period that the buyer can accept or reject. Bargaining continues till there is a consensus on trade. Roy Chowdhury and Sengupta (2012) show that with transparent protocols , buyer can extract higher surplus if he has an outside option. With less transparent protocols, however, holdout may be unavoidable even if the buyer has an outside option.

We model the case where there are possibly more sellers than the number of items required. In our model, sellers are located on nodes of a graph. Nodes are connected by an edge if the corresponding items are physically adjacent or complementary. A sequence of connected nodes is called a path. A path the same size as the number of items required is called a feasible path. Complementary nodes lie on the same path, but two disjoint paths are substitutable. A cycle is a sequence of connected nodes where the beginning and the terminal nodes are the same. Note that in this model, two paths are not completely substitutable if they share some nodes. A seller is critical if he lies in the intersection of all feasible paths. Sarkar (2017) has studied a similar problem with incomplete information. We find it a natural way to model different degrees of complementarity.

Using the same bargaining protocol as Roy Chowdhury and Sengupta (2012), we show the existence of subgame perfect Nash equilibria where the buyer can extract full surplus when there is no critical seller. Our results generalize the results by Roy Chowdhury and Sengupta (2012) by allowing for complementarity within feasible paths and substitutability among multiple paths. Further, while Roy Chowdhury and Sengupta (2012) require outside options to prevent holdout, we are able to avoid holdout by utilizing the competition between items that are substitutable. Such competition has the familiar flavour of Bertrand games well covered in the applied game theoretic literature.

The equilibria we characterize have the following features:

- If the underlying graph has a critical seller, the buyer can never extract full surplus. This covers the case studied by Roy Chowdhury and Sengupta (2012) with only one feasible path on which all sellers are critical.

- If the underlying graph contains a cycle of a minimal length, the buyer can extract full surplus in the first period itself regardless of whether she is making the first offer.

- In all other cases, the buyer can extract full surplus in the second period if she is making the first offer. She extracts full surplus in the first period if sellers are making the first offer.

In a much older paper, Asami (1985) modelled a land market with multiple buyers and multiple sellers as a cooperative game. In his model, each buyer is interested in buying k plots and each seller owns a plot located on a line. He finds that in a core allocation, competition forces agents to receive no surplus, while some agents, e.g. a critical seller or a lonely buyer are able to extract positive surplus. In contrast, our approach is non-cooperative and allows for general contiguity structures and valuations. However, it retains all the features of Asami (1985) pertaining to the single buyer problem.

We discuss the relevant literature in the next section. Subsequently, we lay down the preliminary structure of our model and present two important results from the literature. Then we present our main results for different cases of our model. All proofs are presented in the Appendix. The following section offers detailed discussion of the main results. The final section offers some concluding remarks.

2 Literature

The problem of holdout derives its significance from the fact that production may require several inputs that are complements owned by different agents. Each input owner has some bargaining power, which may lead to delay in negotiations and in some cases bargaining breakdowns.This problem has been studied in the land assembly context (Asami, 1985; O'Flaherty, 1994; Cai, 2000, 2003; Menezes and Pitchford, 2004; Miceli and Segerson, 2012; Roy Chowdhury and Sengupta, 2012; Göller and Hewer, 2015). Some other themes that have been studied in this context are secret offers (Noe and Wang, 2004; Krasteva and Yildirim, 2012a) and the choice of bargaining order over sellers (Krasteva and Yildirim, 2012b; Xiao, 2018).

Strategic exchange is usually modelled in economics using bargaining games, where agents on one side of the market propose prices (or, equivalently, shares of the surplus), and those on the other side accept or reject. The legitimate range of price offers, the sequencing of the prices offered and the possible length of the negotiation process are given by the bargaining protocol which is common knowledge (see Osborne and Rubinstein (1990) for a survey). For our exposition we follow the strategic bargaining literature in economics. The negotiation process we follow is a natural extension of Rubinstein (1982). We also assume complete information i.e. all relevant information pertaining to the game is common knowledge among players. So our model belongs to the class of models of strategic bargaining with complete information (e.g., Fernandez and Glazer (1991))

Closer to our setting, Menezes and Pitchford (2004) study a non-cooperative game of entry into an efficient bargaining process. They show that there is inefficient entry and relate it to the degree of complementarity in production. Cai (2000, 2003), shows how inefficiencies due to hold-out may arise by using a circular bargaining protocol, where the buyer follows a fixed order of bargaining with sellers and sellers he cannot agree with are pushed to the end of the queue. We do not study entry and we assume a simultaneous offers game. In this we are closest to Roy Chowdhury and Sengupta (2012). Also like most of the above papers (except Cai (2003)), we analyze the Cash Offers model, where payment is made immediately on agreement.

Roy Chowdhury and Sengupta (2012) study the same negotiation process as us, but they focus on the role of outside options of buyers and protocol trasperancy in creating or mitigating inefficiencies under very strict assumptions on complementarity. In our paper, the bargaining protocol is assumed to be transparent throughout and the buyer has no outside option. We introduce competition among sellers into the model and focus on its effect on holdout. With perfect seller competition, holdout should disappear. In our model, however, any particular seller may not be substitutable by any other. Buyer wants a cluster of inputs. One particular cluster of inputs may be substituted by another but any individual input within a particular cluster may not always be replaceable by a particular input in another cluster.

Sarkar (2017) obtained results in an incomplete information framework that are closely related to the ones presented here. He investigated the existence of direct mechanisms that are "successful" in the sense of Myerson and Satterthwaite (1983)[1] when agents have private and independent valuations and seller valuations are drawn from the same prior. He showed that:

- There does not exist a successful mechanism if the number of sellers is exactly equal to the number of items required, i.e., all sellers are critical.
- If there are multiple feasible paths, there are priors for which a successful mechanism exists.
- The set of priors for which a successful mechanism exists shrink with the number of critical sellers in the underlying graph.

Unfortunately, although a successful direct mechanism may exist for certain priors, it is not easy to interpret the form of such a mechanism[2]. Bargaining can be viewed as an indirect mechanism with a natural interpretation. Our paper suggests that bargaining with a generalized Rubinstein protocol may be used to implement efficiency under incomplete information.

We adopt the contiguity requirement on inputs following Sarkar (2017, 2018) and study the effect of competition on holdout using the bargaining protocol analyzed by Roy Chowdhury and Sengupta (2012).

There is a literature in contract theory in the broad lines of our contribution. Segal (1999) analyzes the problem of contracting with externalities. With public commitment, inefficiency arises because of externalities in agents' reservation utilities. Genicot and Ray (2006) analyse a game where a principal offers contracts to a set of agents whose outside

[1]A mechanism is "successful" in this sense if it is ex-post efficient, interim incentive compatible, interim individually rational and ex post budget balanced.

[2]See Krishna and Perry (2000) for the construction of a successful mechanism.

option depends on the number of agents not contracted. In this game, competition among agents is exploited to force agents to inferior contractual terms.

A natural follow-up of our exercise is to investigate the impact of formation of seller coalitions on equilibrium payoffs (see Ray (2007) for a survey of coalition formation). A complete analysis of this question is beyond the scope of this paper. In our concluding remarks, we provide an example to show that if the sellers are allowed to form coalitions, the buyer may not be able to extract full surplus even when sellers have identical valuations.

As a methodological note, notice that in the case where seller valuations are not equal, we have used a mixed strategy equilibrium proposed by Blume (2003). Kartik (2011) has shown that under mild assumptions, these equilibria are the only ones using undominated strategies.

3 Preliminaries

Sellers of items are located on nodes of a graph. Two adjacent sellers are connected by an edge. In an application like land acquisition, adjacency can be interpreted in the usual physical sense. In general, adjacency between a pair of nodes simply means that the corresponding items are complementary inputs in the production process the buyer uses. A path is a sequence of connected nodes. The buyer wants to purchase a path of desired length[3], say k. This implies that the buyer can combine any k mutually complementary items to produce output. We will denote a path by $\mathcal{P}$ and the corresponding sum of seller valuations by $\mathcal{S}$. An assembly problem with complete information is a tuple: $\langle \Gamma, k, v, \delta \rangle$. Here Γ is a graph of order n; positive integer k is the desired minimum length of the path buyer is interested in purchasing; the first component of $v \equiv (v_0, v_1, \ldots, v_n)$ denotes the valuation of the buyer for a path of length k or more, and other components denote the valuation of the sellers for their respective items; the real number $\delta \in [0, 1]$ denotes the common rate at which the $n + 1$ agents discount future payoffs. Note that efficiency

[3]This can be relaxed to include any special graph of a fixed size. Rights of passage directly motivates the desire to purchase a path in our case.

would require the buyer to purchase only paths of length k, unless some of the sellers have zero valuation. We assume that there exists a path $\mathcal{P} \in \Gamma$, such that it results in a positive surplus: $v_0 - \sum_{i \in \mathcal{P}} v_i > 0$. When Γ is a complete graph of order n, we will denote an assembly problem by $\langle n, k, v, \delta \rangle$. Note that in any complete graph of order $n > k$, any set of k nodes constitutes a feasible path.

A seller is critical if he lies on every feasible path. This implies that the corresponding item is complementary with respect to every feasible production plan. If there is only one feasible path in Γ, all sellers in that path are critical. But if there are multiple feasible paths, a seller must lie in their intersection in order to qualify as critical. If there are multiple feasible paths, the number of critical sellers cannot exceed $k - 1$: not all sellers on a single path can be critical. Paths that are of length less than k and do not have a intersection with any feasible path can be excluded from the analysis, because the buyer's valuation over such paths is zero.

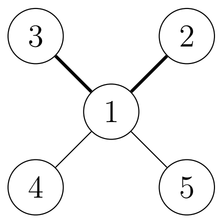

Figure 1: A feasible path in the star graph when $k = 3$; seller 1 is critical.

A graph with n nodes can have upto $\binom{n}{2}$ edges. Thus, the variety of possible graph structures can be large. We consider four major varieties among graphs with at least two feasible paths: (a) cycles of order $k + 1$, referred to as $\Gamma^{\triangle}$ (see Figure 2); (b) graphs with two disjoint paths, referred to as Γ^{D} (see Figure 3); (c) graphs with critical sellers, referred to as Γ^{*} (see Figure 4); (d) graphs where (i) there is no cycle of length $k + 1$, (ii) no two paths are disjoint and (iii) the intersection of all feasible paths is empty, referred to as Γ^{O} (see Figure 8). For convenience, we will refer variety (d) as *oddball.* Note that these varieties are mutually exclusive and any other graph in the class considered must have one or more of these graphs as subgraphs. Further, given our earlier interpretation of nodes and edges, these four varieties of graphs are naturally interpreted as follows. Suppose valuations of items are identical. In variety (a), every item on a feasible path can be completely substituted by another item on the graph. In variety (b), no individual item is completely substitutable, but a feasible path can be substituted completely by another feasible path. In variety (c), items belonging to critical sellers are not substitutable but those belonging to non-critical sellers are substitutable in a limited sense. In variety (d), items in the intersection of two or more feasible paths cannot be substituted with respect to these feasible paths, but they are substitutable with respect to other feasible paths. When seller valuations are not identical, substitution may be costly in economic terms over and above technological feasibility of substitution implied by the edges in the graph.

We apply a variant of the Rubinstein bargaining protocol due to Roy Chowdhury and Sengupta (2012). In each period, either the sellers propose individual offers of surplus shares to the buyer or the buyer proposes a vector of offers of surplus shares to n sellers. The sellers can individually accept or reject buyer's proposals. If a seller accepts buyer's proposal, the buyer immediately purchases his plot and the seller leaves the market. The sellers who have rejected buyer's offer propose individual shares to the buyer that the buyer may accept or reject. If the buyer accepts any of the seller offers, she immediately purchases his plot and the seller leaves the market. The buyer then resumes bargaining with rest of the active sellers. The game continues till the buyer is able to purchase k plots on a path.

Note that there can be offers of negative shares of surplus that force the agent to whom the offer is made to reject the offer in that period. Bilateral bargaining models, like that by Rubinstein (1982) do not include this feature: only non-negative offers are allowed in such bargaining games. Negative offers can be utilized by the agents, say, the buyer, to exclude other agents, say a particular seller, from the bargaining process at any particular stage and exploit more bargaining power in future.

Consider the two-person alternating offer bargaining game studied by Rubinstein (1982). Let players 0 and 1 be the buyer and seller of a plot with values v_0 and v_1 respectively. Player 0 moves first and proposes a sharing scheme for the surplus $v_0 - v_1$. Player 1 can either accept or reject the proposed share, say x. If he accepts, the game ends with both players sharing the surplus according to the scheme proposed by 0. The buyer has to pay the seller $v_1 + x(v_0 - v_1)$ in exchange of the plot. Consequently, the net payoff of the buyer and the seller are $(1 - x)(v_0 - v_1)$ and $x(v_0 - v_1)$. If he rejects then in the next stage 1 makes an proposal which 0 can accept or reject. Bargaining can go on infinitely till one of the two players accept the offer made by the other. The SPNE of this game, which is now a standard result, is presented below without a proof.

THEOREM **1** (**Rubinstein** (**1982**)) *Consider the model where the buyer bargains with one seller for one plot: $\langle n = 1, k = 1, v_0 > v_1, \delta \rangle$. There is a unique SPNE of the model described as follows:*

> *Agent i proposes a share $\frac{\delta}{1+\delta}$ of the surplus to j whenever she has to propose, and accept any share at least equal to $\frac{\delta}{1+\delta}$ whenever j has to propose.*

The game ends in the first period itself, with buyer proposing $\frac{\delta}{1+\delta}$ to the seller and the seller accepting it.

Roy Chowdhury and Sengupta (2012) study the pure strategy SPNE under Rubinstein bargaining protocol in a model where a buyer wants all plots held by n sellers. While they prove their result for the case when sellers' valuations are identical, their claim applies even when sellers valuations are not identical.

THEOREM **2** (**Roy Chowdhury and Sengupta** (**2012**)) *Consider the model $\langle n \geq 2, k = n, v_1 \leq \cdots \leq v_n, v_0 > \sum_{i=1}^{n} v_i, \delta \rangle$. The buyer's equilibrium payoff cannot be more than $\frac{1-\delta}{1+\delta}(v_0 - \sum_{i=1}^{n} v_i)$ for any $\delta > 0$.*

Note that all sellers are critical here. In terms of our model this situation pertains to the case with only one feasible path in a graph. Two types of equilibrium outcomes are identified: if $1 > \frac{n\delta}{1+\delta}$, the buyer offers $\frac{\delta}{1+\delta}$ to every seller in the first period and all of them accept. Otherwise, the buyer would offer zero in the first period, all but $r \geq 2$ sellers would accept, and in the second period these r sellers would demand P such that $1 - rP = \delta\left(1 - \frac{r\delta}{1+\delta}\right)$. Here, r is the highest positive integer such that $1 > \frac{r\delta}{1+\delta}$.

Two simple examples below illustrate the essential arguments of this paper.

Example 1 Consider the model $\langle n = 2, k = 1, v_0 > v_1 = v_2, \delta \rangle$. Suppose the buyer makes offers of zero surplus to seller 1 and negative surplus to seller 2. If seller 1 rejects the buyer's offer, he would compete with seller 2 in the next period and offer the entire surplus to the buyer. If sellers 1 and 2 are making offers in the first period, they cannot make equal positive claims: one of the sellers have the incentive to reduce her claim and increase payoff. On the other hand, if their claims are unequal, the seller with the lowest claim has the incentive to increase her claim slightly and increase his payoff. Consequently, none of the sellers 1 and 2 can extract any surplus. The game ends immediately with the buyer extracting full surplus. The equilibrium outcome is identical even when the sellers are proposing first.

The situation described in Example 1 is identical to the well-known Bertrand model of price competition between firms producing the same product at identical marginal costs. In this model, competition between the sellers drives prices down to the marginal cost. The buyer is able to extract full surplus. Note that in our model the competition is among feasible paths. Consequently, the richness of the underlying graph structure allows for results that are richer than simple Bertrand competition. However, the spirit of the argument applied for richer graph structures is in the nature of Bertrand competition.

The simple example below illustrates that the buyer may not be able to extract efficient surplus when seller valuations are not identical. This example is in the lines of Blume (2003) who characterizes a class of equilibria in the Bertrand model of price competition when firms have asymmetric marginal costs.

Example 2 Consider the land acquisition problem $\langle n, k, v, \delta \rangle$ such that $n = 2; k = 1, v_1 < v_2 < v_0$. We claim that the buyer cannot extract the efficient surplus in equilibrium. Consider the following strategies of the sellers: in any continuation game where the two sellers are making offers, seller 1 offers to sell at a price of v_2 and seller 2 mixes prices in $(v_2, v_2 + \gamma)$, $\gamma > 0$, with uniform probability constitutes an equilibrium. In any continuation game where the buyer is making an offer, seller 1 accepts a surplus of at least $\delta(v_2 - v_1)$ and seller 2 accepts any positive surplus. Given these strategies, following is a best response for the buyer: in any continuation game where the buyer is making an offer, she offers a surplus of $\delta(v_2 - v_1)$ to seller 1 and a negative surplus to seller 2. In any continuation game where the sellers are making an offer, she accepts any surplus offer that is less than or equal to $v_2 - v_1$. If the buyer proposes first, trade takes place in the first period itself with seller 1; seller 1 is able to extract a surplus of $\delta(v_2 - v_1)$. If the sellers propose first, trade takes place in the first period, where seller 1 is able to extract a surplus of $(v_2 - v_1)$. To check that this is an equilibrium, note that when making an offer, buyer cannot offer any higher surplus to seller 1 as it would be accepted. The buyer cannot offer positive surplus to seller 2, since he would accept it. Any lower surplus offer would be rejected by seller 1. The buyer cannot reject the offer of seller 1 either because that would reduce his share of surplus. Seller 1 cannot reduce his offer because it would be accepted. Any higher offer by seller 1 would be rejected, thus leading to a lower surplus for him. If $v_1 < v_0 < v_2$, only the trade with seller 1 is feasible. In this circumstance, we are back to the equilibrium outcome of the familiar bilateral bargaining model by Rubinstein (1982): in any continuation game where the buyer is making an offer, the buyer proposes a surplus of $\frac{\delta}{1+\delta}(v_0 - v_1)$ to seller 1 and a negative surplus to seller 2; seller 1 accepts any surplus at least equal to $\frac{\delta}{1+\delta}(v_0 - v_1)$. In any continuation game where the sellers are making an offer, seller 1 proposes a surplus of $\frac{\delta}{1+\delta}(v_0 - v_1)$ to and seller 2 proposes $v_0 - v_2$; the buyer accepts any surplus at least equal to $\frac{\delta}{1+\delta}(v_0 - v_1)$. Trade takes in the first period itself with seller 1.

4 Results

Here we consider assembly problems where the underlying graph has at least two different feasible paths. We distinguish between two cases: one, in which the seller valuations are equal, and two, where the seller valuations are unequal. To facilitate exposition, we present a set of examples after each result to illustrate the essential argument. Detailed proofs are presented in the appendix.

4.1 Equal seller valuations

In this subsection, we consider the case where all seller valuations are equal. The main result of this subsection is given below.

Theorem **3** *Consider an assembly problem $\langle \Gamma, k, v, \delta \rangle$ such that $v_1 = \cdots = v_n, v_0 > kv_1$. There exists a δ for which the buyer extracting full surplus in at most two periods is an equilibrium outcome if and only if Γ does not contain a critical seller.*

The formal proof of this result is given in Appendix A. Here we present four examples pertaining to the varieties of graph structures discussed in Section 3. In these examples, $k = 3$.

Example **3 (A 4-cycle)** Consider a cycle of length 4. Note that there are 4 feasible paths of length 3. Every pair of feasible paths has a non-empty intersection. But the intersection of all 4 feasible paths is empty. We argue that there exists an equilibrium where the buyer extracts full surplus.

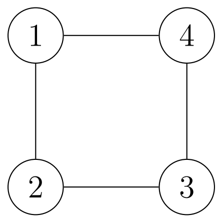

Figure 2: A cycle of length 4.

First note that bargaining continues if and only if there are at least two active sellers. Consider the following strategy of the buyer: She picks a feasible path. Whenever she is proposing, she offers sellers from the picked path their valuations (equivalently, zero surplus), and the remaining seller strictly less than his valuation (equivalently, negative surplus). Whenever the sellers are proposing, she accepts the required number of offers from the lowest seller claims, provided she can afford. Consequently, all active sellers claiming zero surplus whenever they are required to make an offer is a best response. To check this, note that no active seller can gain by deviating for one stage when all of them claim zero surplus. If active sellers make identical positive surplus claims, one of them can reduce his claim by a small amount and make a gain. If active sellers make unequal claims then a seller with lower claim can increase his claim by a small amount and make a gain. Now consider a stage where the buyer is making an offer. Active sellers who are made zero surplus offers would immediately accept: if any such seller rejects, he reaches a continuation game where the maximum he can gain is zero. Hence this is an equilibrium. Trade takes place in the first period itself with 3 sellers who are made zero surplus offers. Note that the equilibrium outcome does not change whether the buyer moves first, or the sellers.

Example 4 **(Two disjoint feasible paths)** Consider a graph with two disjoint paths of length 3 (see Figure 3 below). We will show that if the sellers move first, the buyer achieves full surplus in the first period itself. Consequently, if the buyer moves first and δ is large, there is an equilibrium where buyer extracts full surplus in the second period.

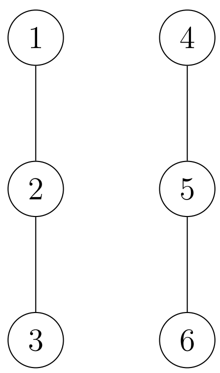

Figure 3: Graph with disjoint feasible paths

Consider the following strategy of the buyer: whenever the buyer is proposing, she makes negative offers to all sellers. Whenever the sellers are proposing, the buyer accepts the claims of sellers on a path with the lowest sum of claims provided her share of surplus is non-negative, and reject all other claims. In case the sum of claims on two feasible paths are same, she accepts claims from one of the paths chosen with equal probability. We claim that, given the above strategy, sellers in the two disjoint feasible paths claiming zero surplus whenever they are required to make an offer is a best response. No seller can gain by deviating for one stage when the sum of surplus claims on either path is zero. If the sum of surplus claims on both paths are equal and positive, a seller on either path can reduce his claim by a small amount and make a gain. If the sum of surplus claims on two paths are unequal, then any seller on on the path corresponding to the lower sum can increase his claim by a small amount and make a gain. Hence these are not equilibrium claims. To rule out other possible deviations, note that buyer can make zero surplus offers to sellers on both paths, and negative surplus offers to all other sellers; sellers on both paths would accept these offers. To ensure that this deviation in the first stage is not profitable for the buyer, we require $\delta > \frac{v_0-6v}{v_0-3v}$. The buyer can also make acceptable offers of surplus shares, $2\delta v$, to each seller on one path and negative offers to all other sellers, provided $v_0-3v-6\delta v > 0$. This is because, by rejecting a first period offer from the buyer, a seller on the chosen path competes with sellers on the other path; the highest surplus he can claim in a continuation game where he and the other sellers are making offers is $3v - v = 2v$.To ensure that this deviation in the first stage is not profitable for the buyer, we require $\delta > \frac{v_0-3v-6\delta v}{v_0-3v}$. Thus, provided $\delta > \max\{\frac{v_0-6v}{v_0-3v}, \frac{v_0-3v-6\delta v}{v_0-3v}\}$. Consequently, the buyer extracting full surplus in the second period is an equilibrium outcome in the strategies described above for large δ.

Example 5 **(Graph with critical sellers)** Consider the following line graph where there are two critical sellers. We argue that the buyer cannot claim full surplus in an equilibrium.

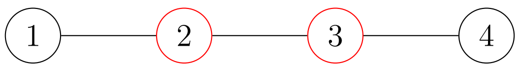

Figure 4: A line graph with two critical sellers marked red

Suppose the buyer makes offers first. Note that there is only one non-critical seller in both feasible paths. If she makes acceptable offers to both critical sellers in the first period itself, then she can pick a non-critical seller on any feasible path, offer him zero share of the surplus and make negative offers to all other sellers. If this non-critical seller rejects the zero offer, the most that he can claim in the next period is again zero, since there is at least one more competing non-critical seller on some other feasible path. Hence this seller must accept the offer in the first period. Thus the share of surplus the buyer can extract in this problem is at most the surplus she can extract in the problem $\langle n = k = 2, v, \delta \rangle$. By Theorems 1 and 2, we find that the buyer claims at most $\frac{1-\delta}{1+\delta}$ of full surplus in an equilibrium. Suppose sellers make the first offer. Note that by rejecting all seller offers, the buyer is able to induce a game where she makes the first offer. So either the sellers on a feasible path make offers that make the buyer indifferent between accepting and rejecting such offers, or they make offers that the buyer rejects in the first period. In either case, the buyer's equilibrium share of surplus cannot exceed δ times her equilibrium share of surplus in a continuation game where she has the first move.

EXAMPLE **6 (An oddball graph)** Consider the graph in Figure 8 below with $k = 3$. We will show that if the sellers move first, the buyer achieves full surplus in the first period itself. Consequently, if the buyer moves first and δ is large, there is an equilibrium where buyer extracts full surplus in the second period.

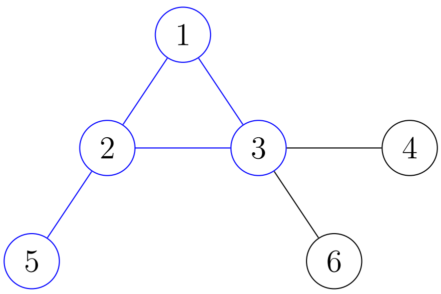

Figure 5: An oddball graph

Note that the graph has a subgraph, marked in blue, such that it contains a feasible path, and for each node x on this feasible path there exists another node outside this subgraph and a corresponding edge such that exclusion of x from the graph leaves a feasible path of length k. For instance, exclusion of node 2, would leave the graph with feasible paths $\{134\}$ and $\{136\}$. Further, for each node x on a feasible path, let $s(x)$ be the order of the smallest subgraph such that the union of this subgraph and the graph excluding x contains a feasible path. For example, in Figure 8, $s(1) = s(5) = 1$ and $s(2) = 2$.

Consider the following strategy of the buyer: In any continuation game where the buyer has the first move , the buyer makes negative offers to all sellers. In any continuation game where sellers have the first move, the buyer accepts the claims of sellers on a path with the lowest sum of claims provided her share of surplus is non-negative, and reject all other claims. In case the sum of claims on the two feasible paths are same, she accepts claims from one of the paths chosen with equal probability. We claim that given the above strategy, sellers claiming zero surplus at any subgame they are required to make an offer is a best response. To check this, note that no seller can gain by deviating for one stage when the sum of seller claims across paths is zero. This is because, for each node, there is always a feasible path in the graph that excludes it. If sums across feasible paths are positive, a seller on one of the paths can reduce his claim by a small amount and make a gain. If sums across paths are unequal, then a seller on a path with lower sum of claims can increase his claim by a small amount and make a gain. Hence these are not best responses.

To disallow possible deviations, note that buyer can make zero surplus offers to all sellers on the blue subgraph, and negative surplus offers to all other sellers; sellers on blue subgraph would accept these offers. To ensure that this deviation in the first stage is not profitable for the buyer, we require $\delta > \frac{v_0-4v}{v_0-3v}$. The buyer can also make acceptable offers of surplus shares to sellers on a path and negative offers to all other sellers. Seller corresponding to node x_i on the path picked accepts any surplus share at least equal to $\delta(s(x_i)-1)v$. This is because, by rejecting a first period offer from the buyer, a seller on the chosen path competes with sellers on the other path; the highest surplus he can claim in a continuation game where he and the other sellers are making offers is $(s(x_i)-1)v$.To ensure that this deviation in the first stage is not profitable for the buyer, we require $\delta > \frac{\sum_{i\in\mathcal{P}}(s(x_i)-1)v}{v_0-3v}$. Thus, provided $\delta > \max\left\{\frac{v_0-4v}{v_0-3v}, \frac{\sum_{i\in\mathcal{P}}(s(x_i)-1)v}{v_0-3v}\right\}$, the buyer extracting full surplus in the second period is an equilibrium outcome in the strategies described above.

4.2 Unequal seller valuations

In this subsection, we consider the case where seller valuations are not equal. In this case, the sum of seller valuations may differ over paths. The path corresponding to the least sum of seller valuations is efficient in the sense that it corresponds to highest potential surplus. It follows that if possible, the buyer would prefer to purchase the efficient path.

Let $\mathcal{P}_i$ denote the path corresponding to the i-th smallest sum of valuations on a path in Γ. We will refer to a set of assembly problems as *rich* if there does not exist two disjoint paths $\mathcal{P}_1$ and $\mathcal{P}_2$ such that $\mathcal{S}_1 = \mathcal{S}_2$. Suppose the richness condition is not satisfied. The buyer, if offering first, can offer negative surplus shares to all sellers who reject such offers. In the next period, sellers on $\mathcal{P}_1$ and $\mathcal{P}_2$ cannot claim any surplus: the buyer extracts full surplus in the second period. If the sellers are making offers first, sellers on these two paths cannot claim any surplus share.

Theorem 4 *Consider the rich class of assembly problems $\langle \Gamma, k, v, \delta \rangle$ such that $v_1 \leq \cdots \leq v_n$ with at least one strict inequality. There does not exist any equilibrium where the buyer extracts full surplus.*

We note that extracting full surplus implies trade taking place with only the k sellers on $\mathcal{P}_1$. There may exist equilibria where the buyer offers zero surplus to more than k sellers who accept. But this reduces the buyer's surplus strictly below $v_0 - \mathcal{S}_1$.

Also note that if the buyer extracts full surplus in an equilibrium, it cannot be that sellers accept over two different periods. Suppose the buyer makes zero offers to say $k-m$ sellers who accept in the first period, and the remaining m sellers make zero surplus claims in the next period. Then the buyer receives δ times the full surplus in period two. Since these m remaining sellers can earn at most zero in the second period, the buyer can offer them a small positive surplus in the first period itself and get more than δ times the full surplus. Similarly, suppose the sellers are proposing first, the buyer accepts m zero offers in the first period and makes zero offers to remaining $k-m$ sellers in the next period. At least one of these $k-m$ sellers can revise his first period downwards to receive a positive payoff in the first period itself. This observation leads to the following facts: (a) if the buyer extracts full surplus, all sellers realize zero surplus share in the same period; (b) we need to prove the claim of this theorem for the first two periods only.

For the formal proof of Theorem 4, see Appendix B. Here we present four examples pertaining to the varieties of graph structures discussed in Section 3. In all these examples, we have $k = 3$.

Example **7 (A 4-cycle)** Consider the cycle in Figure 2 where the numbers marking the nodes represent valuations. Suppose the buyer makes zero surplus offers to sellers on the efficient path $\{123\}$ in the first period and negative surplus offers to seller 4. At least one seller, say, seller 1, would reject this offer and claim a price of 4, the valuation of the seller outside this path, in the next period. The buyer must accept, provided the surplus on the path excluding seller 1, $v_0 - 7 > 0$. If this inequality does not hold, the buyer must offer $\frac{\delta}{1+\delta}$ times the efficient surplus, i.e., $v_0 - 6$ to this seller. So, suppose buyer makes negative offers to all sellers in the first period. Note that the sum of valuations on the four paths $\{123\}$, $\{234\}$, $\{341\}$ and $\{412\}$ are 6, 9, 8 and 7 respectively. Seller 1, being in the intersection of $\{123\}$ and $\{412\}$ can raise his price claim to 2: thus the sum of claims over the four paths become 7, 9, 9 and 8. Either the buyer accepts this claim, or she rejects and offers $\frac{\delta}{1+\delta}$ times the efficient surplus to this seller. Not all sellers would claim zero surplus when proposing first: for example, seller 1 can claim a price of 4, or if $v_0 - 7 < 0$, she can claim $\frac{1}{1+\delta}$ times the efficient surplus. If the buyer rejects all seller offers in the first period, then she is on a continuation game where she is proposing to all sellers. We have already argued that she cannot extract full surplus in such a continuation game.

Example **8 (Two disjoint feasible paths)** Consider Figure 3 where the numbers marking the nodes represent valuations. Suppose the buyer makes zero surplus offers to all sellers on $\{123\}$ in the first period and negative surplus offers to the remaining sellers. Seller 1 can reject this offer and claim a price equivalent to the sum of valuations on $\{456\}$, i.e., 15 in the next period which buyer must accept, provided the corresponding surplus is positive, i.e., $v_0 - 20 > 0$. If this inequality does not hold, the buyer must offer $\frac{\delta}{1+\delta}$ times the efficient surplus to this seller. So, suppose buyer makes negative offers to all sellers in the first period. If $v_0 > 15$, sellers on $\{123\}$ can make claims summing up to 15 in the second period. Either the buyer accepts this claim, or she rejects and offers $\frac{\delta}{1+\delta}$ times the efficient surplus, $v_0 - 6$ to sellers on $\{123\}$. Not all sellers would claim zero surplus when proposing first: as argued before, at least one seller can claim a price of 15, or, if $v_0 - 20 < 0$, she can claim $\frac{1}{1+\delta}$ times the efficient surplus $v_0 - 6$. If the buyer rejects all seller offers in the first period, then she is on a continuation game where she is proposing to all sellers. We have already argued that she cannot extract full surplus in such a continuation game.

Example **9 (Graph with critical sellers)** Consider the situation in Figure 8 where the numbers marking the nodes represent valuations. Suppose the buyer makes zero surplus offers in the first period. By Theorems 1 and 2, at least one critical seller , say, seller 2, rejects this offer and claims $\frac{1}{1+\delta}$ of the efficient surplus $v_0 - 6$ in the next period which buyer must accept. Consequently, the buyer cannot extract full surplus within the first two periods in an equilibrium. Suppose sellers make the first offers. As argued, critical seller 2 can claim strictly positive surplus. So full surplus extraction cannot take place in the first period. But if the buyer rejects all seller offers in the first period, then she is on a continuation game where she is proposing to all sellers. We have already argued that she cannot extract full surplus in such a continuation game.

Example **10 (An oddball graph)** Consider the assembly problem in Figure 8 where the numbers marking the nodes represent valuations. Suppose the buyer makes zero surplus offers to sellers on the efficient path $\{123\}$ in the first period. Seller 1 would reject this offer and claim a price of 4 in the next period which buyer must accept, provided $v_0 - 9 > 0$. If this inequality does not hold, the buyer must offer $\frac{\delta}{1+\delta}$ times the efficient surplus $v_0 - 6$ to this seller. So, suppose buyer makes negative offers to all sellers in the first period. Note that seller 1 lies in the intersection of multiple paths. He can raise his claim by at least 1. Either the buyer accepts this claim, or she rejects and offers $\frac{\delta}{1+\delta}$ times the efficient surplus $v_0 - 6$. Not all sellers would claim zero surplus when proposing first: as argued before, seller 1 can claim a price of 4, or if $v_0 - 9 < 0$, she can claim $\frac{1}{1+\delta}$ times the efficient surplus $v_0 - 6$. If the buyer rejects all seller offers in the first period, then she is on a continuation game where she is proposing to all sellers. We have already argued that she cannot extract full surplus in such a continuation game.

5 Discussion

Our first result claimed that if valuations of the sellers are identical and the underlying graph structure does not have a critical seller, there exist equilibria where the buyer extracts full surplus within two periods. Here we considered the simple advantages of position that certain sellers exact in a graph, and abstracted from advantages due to cost efficiency.

We considered four mutually exclusive and exhaustive categories of graphs, viz., (a) graphs containing cycles of order $k+1$, (b) graphs with two disjoint paths, (c) graphs with critical sellers, and (d) oddball graphs where (i) there is no cycle of length $k+1$, (ii) no two paths are disjoint and (iii) the intersection of all feasible paths is empty. These categories can be easily interpreted in terms of complementarity and substitutability as we have done in Section 3. Of particular interest is the $k+1$ cycle, where every item on a feasible path can be completely substituted by another item on the graph: only in this case, the buyer is able to extract full surplus in the first period, regardless of whether the buyer makes the first offer or the sellers. In other words, in this case, no seller has any positional advantage. Thus, it is comparable to the pure Bertrand competition visible in Example 1. At the other extreme is the graph with critical sellers: such critical sellers exhibit full positional advantage and prevent the buyer from extracting surplus beyond a point, regardless of whoever makes the first offer. Such sellers show full complementarity in an economic sense with respect to any feasible path on the graph.

The cases of graphs with disjoint feasible paths and oddball graphs lie between these two extremes. If the buyer picks a feasible path on any of these graphs, its nodes have limited substitutability. Note that our bargaining protocol only permits cash offers with full commitment. Consequently, once the buyer commits to a seller on a feasible path, she commits to all sellers in the feasible path. Thus the buyer has to cough up positive shares of the surplus if she is making the first offer. However, the buyer can avoid this commitment problem by making negative offers to all sellers and to push the outcome towards Bertrand competition in the second period. For a patient buyer, the loss of surplus by shifting the onus of bargaining to the sellers is not very significant.

The interpretation of these graphs in the context of anticommons applications like land acquisition is immediate. The notions of complements and substitutes also arise naturally in contexts like acquiring patent rights for drug manufacturing or obtaining rights for musical scores for a documentary.

Theorem 4 shows that full surplus extraction is not robust with respect to changes in the valuation structure. In fact, the buyer cannot extract full surplus whenever the valuation profile of the sellers shows a fairly general degree of asymmetry. The positional advantages that certain sellers hold become more pronounced when their valuations are asymmetric. In this sense, asymmetric valuations enable sellers in the efficient feasible path exercise monopoly power of a nature we had seen in Example 2. For the sake of completeness, we have characterized a set of equilibria for the variety of graphs studied here in Appendix B. However, there are multiple equilibria in such problems and at this stage we are unable to provide exact bounds on the surplus share the buyer can extract in various graph structures.

It may be noted that earlier inefficiency results in the literature, like Theorem 2, focussed on the extreme case where all sellers are critical and valuations are symmetric. Our generalized model, in contrast, shows that the inefficiency result pertains to the rather extreme case of graphs with critical sellers when valuations are symmetric. In our model, inefficiency is more pervasive when seller valuations are asymmetric.

An obvious extension of this exercise is to investigate the impact of coalition formation among sellers on the surplus shares. For instance, in a 3-cycle where 2 items are required and valuations are symmetric, any two sellers can form a coalition. Each member of the coalition would claim $\frac{1}{1+\delta}$ of the full surplus. Any seller whose claim is fulfilled, can offer to compensate the other. In this circumstance, at least one of the sellers gets positive surplus share, whereas the buyer loses some surplus share. A complete investigation of this question is, however, beyond the scope of current paper.

6 Concluding Remarks

In this paper, we modelled the assembly problem as a bargaining game between one buyer and multiple sellers located on the nodes of a graph. In our simple bargaining problem without transactions cost, the buyer, using competition between sellers, is able to implement an efficient project without significant delay when valuations are symmetric. Positional advantages, or equivalently complementarities, can be exercised only under extreme cases, when sellers are critical. The second result states that asymmetric seller valuations is a stronger force than complementarity: such asymmetry provides additional monopoly power to efficient sellers and prevent the buyer from efficient assembly. Thus, our results provide support to the Coase conjecture when sellers are not "monopolistic" in terms of positional or cost advantage.

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A Proof of Theorem 3

Consider a cycle of length $k+1$, say, $C(k+1)$. Note that there are $k+1$ feasible paths. Every pair of feasible paths has a non-empty intersection. But the intersection of all

$k+1$ feasible paths is empty. We characterize an equilibrium where the buyer extracts full surplus.

PROPOSITION **1** *Consider a assembly problem* $\langle \Gamma^{\triangle}, k, v, \delta \rangle$ *such that* $v_1 = \cdots = v_n, v_0 > kv_1$*. The buyer extracting full surplus is an equilibrium outcome.*

Proof: We will first prove the case of a graph which is a cycle of length $k+1$.

LEMMA **1** *Consider a assembly problem* $\langle C(k+1), k, v, \delta \rangle$ *such that* $v_1 = \cdots = v_{k+1}, v_0 > kv_1$*. The buyer extracting full surplus is an equilibrium outcome.*

Proof: Consider the following strategy of the buyer: She picks a feasible path. In any continuation game where $m < k$ plots have already been acquired and the buyer has the first move , the buyer offers $k-m$ sellers zero surplus and make negative offers to the remaining seller. In any continuation game where $m < k$ plots have already been acquired and sellers have the first move, the buyer accepts the lowest $k-m$ claims provided her share of the surplus is non-negative, and reject all other claims. In case more than $k-m$ sellers are making identical lowest offers, she accepts $k-m$ offers with equal probability.

We claim that given the above strategy, all active sellers claiming zero surplus at any subgame they are required to make an offer is a best response. Let x_i be the surplus claim of active seller i. No seller can gain by deviating for one stage when $x_i = x_j = 0, i \neq j$. Hence it is an equilibrium. If $x_i = x_j > 0, i \neq j$, either seller i or j can reduce his claim by a small amount and make a gain. If $x_i > x_j \geq 0, i \neq j$, then seller j can increase his claim by a small amount and make a gain. Hence these are not equilibrium claims.

At any subgame where the buyer is making an offer and m plots have already been acquired, the active seller who is made a negative offer rejects it. Simultaneously, $k-m$ would immediately accept corresponding zero offers, since if any of these sellers reject such offers, they reach a continuation game where the maximum he can gain by rejecting buyer's offers is zero. Hence this is an equilibrium. Trade takes place in the first period itself when $m = 0$, with k sellers who are made zero surplus offers. ■

Note that by Lemma 1 equilibrium outcome does not change whether the buyer moves first, or the sellers.

It follows immediately that such an equilibrium can be obtained for any graph containing a cycle of length $k+1$ as a subgraph. ■

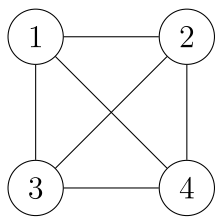

Figure 6: A complete graph of order 4; a cycle of order 4 is a subgraph.

Note that any complete graph of order $n > k$ contains a cycle of length $k+1$. This results in the following Corollary.

Corollary **1** *Consider a assembly problem $\langle n, k, v, \delta \rangle$ such that $v_1 = \cdots = v_n, v_0 > kv_1$. The buyer extracting full surplus is an equilibrium outcome.*

Remark **1** If $k = 2$, then the above result is also true for any graph containing a cycle of length more than $k+1$. But it is not true when $k > 2$. For instance, consider the cycle of length 5 when $k = 3$ (see Figure 7). Suppose the buyer wants to make offers that are acceptable to the sellers 1,2 and 3 in the first period itself. Sellers 1 and 2 will accept a zero surplus offer since if they reject, they have to compete with sellers 5 or 4. Seller 2, on the other hand, will not accept a surplus of less than δv, since if he rejects an offer, he has to compete with sellers 4 and 5 together. Therefore, the buyer has two ways to complete the transaction in the first period: either (i) she makes zero surplus offers to 4 sellers on the graph and makes a negative offer to the remaining seller, or (ii) she makes zero surplus offers to sellers 1 and 2, make a surplus offer of δv to seller 2, and negative offers to the remaining sellers. In this particular case, she would prefer (ii) over (i). Another alternative for the buyer is to make negative surplus offers to all sellers in the first period, thus letting sellers on two paths compete in the second period. This case is covered in Proposition 2 below.

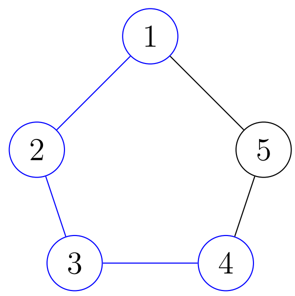

Figure 7: A cycle of length 5; Γ^{SO} in blue

Consider a graph Γ^D which contains two or more disjoint feasible paths. We will show that if the sellers move first, the buyer achieves full surplus in the first period itself. Consequently, if the buyer moves first and δ is large, there is an equilibrium where buyer extracts full surplus in the second period.

Proposition **2** *Consider a assembly problem $\langle \Gamma^D, k, v, \delta \rangle$ such that $v_1 = \cdots = v_n, v_0 > kv_1$. (a) If the sellers move first, the buyer achieves full surplus in the first period. (b) If the buyer moves first, for δ large enough, there is an equilibrium where buyer extracts full surplus in the second period.*

Proof: Consider the following strategy of the buyer: In any continuation game where the buyer has the first move , the buyer makes negative offers to all sellers. In any continuation game where sellers have the first move, the buyer accepts the claims of sellers on a path with the lowest sum of claims provided her share of surplus is non-negative, and reject all other claims. In case the sum of claims on the two feasible paths are same, she accepts claims from one of the paths chosen with equal probability.

We claim that, given the above strategy, sellers in the two disjoint feasible paths claiming zero surplus at any subgame they are required to make an offer is a best response. Let $\mathcal{P}_1$ and $\mathcal{P}_2$ be the two feasible paths in Γ^{D2}. Let x_i be the surplus claim of active seller i. No seller can gain by deviating for one stage when $\sum_{i \in \mathcal{P}_1} x_i = \sum_{i \in \mathcal{P}_2} x_i = 0$.

Hence it is an equilibrium. If $\sum_{i\in\mathcal{P}_1} x_i = \sum_{i\in\mathcal{P}_2} x_i > 0$, a seller on either path can reduce his claim by a small amount and make a gain. If $\sum_{i\in\mathcal{P}_1} x_i > \sum_{i\in\mathcal{P}_2} x_i$, then any seller on $\mathcal{P}_2$ can increase his claim by a small amount and make a gain. Hence these are not equilibrium claims.

Part (a) of the claim follows immediately. For part (b), note that buyer can make zero surplus offers to sellers on both paths, and negative surplus offers to all other sellers; sellers on both paths would accept these offers. To ensure that this deviation in the first stage is not profitable for the buyer, we require $\delta > \frac{v_0-2kv}{v_0-kv}$. The buyer can also make acceptable offers of surplus shares, $\delta(k-1)v$, to each seller on one path and negative offers to all other sellers, provided $v_0 - kv - \delta k(k-1)v > 0$. This is because, by rejecting a first period offer from the buyer, a seller on the chosen path competes with sellers on the other path; the highest surplus he can claim in a continuation game where he and the other sellers are making offers is $(k-1)v$.To ensure that this deviation in the first stage is not profitable for the buyer, we require $\delta > \frac{v_0-kv-\delta k(k-1)v}{v_0-kv}$. Thus, provided $\delta > \max\{\frac{v_0-2kv}{v_0-kv}, \frac{v_0-kv-\delta k(k-1)v}{v_0-kv}\}$, the buyer extracting full surplus in the second period is an equilibrium outcome in the strategies described above. ■

Suppose there are critical sellers on a graph. There cannot be more than $k-1$ critical sellers, otherwise there is only one feasible path and Theorem 2 applies. If there are c critical sellers on any graph, any feasible path of length k would have $k-c$ non-critical sellers. Since there are at least two feasible paths in this case, the number of non-critical sellers is at least $2(k-c)$. As the following result shows, critical sellers on the graph may extract positive surplus share. Since purchasing plots from critical sellers is obligatory, such sellers may support their claims by credible threats as observed in Theorems 1 and 2. At the same time, the buyer can exploit the competition among non-critical sellers and not allow them to extract any share of the surplus.

PROPOSITION **3** *Consider the assembly problem $\langle\Gamma^*, k, v, \delta\rangle$ such that Γ has $c < k$ critical sellers. If $1 \geq \frac{c\delta}{1+\delta}$, the buyer cannot claim more than $1 - \frac{c\delta}{1+\delta}$ times the efficient surplus in an equilibrium. Otherwise, there is an equilibrium where agreement takes place in the* *second period. In this equilibrium, buyer's share of the efficient surplus is* $\delta^2\left(1-\frac{r\delta}{1+\delta}\right)$*, where* r *is the highest positive integer such that* $1 > \frac{r\delta}{1+\delta}$*.*

Proof: Let $1 \geq \frac{c\delta}{1+\delta}$. Suppose the buyer makes offers first. Suppose there is only one non-critical seller in every feasible path, i.e., $k-c=1$. If she makes acceptable offers to all c critical sellers in the first period itself, then she can pick a non-critical seller on any feasible path, offer him zero share of the surplus and make negative offers to all other sellers. If this non-critical seller rejects the zero offer, the most that he can claim in the next period is again zero, since there is at least one more competing non-critical seller on some other feasible path. Hence this seller must accept the offer in the first period. Thus the share of surplus the buyer can extract in this problem is less than the surplus she can extract in the problem $\langle n \geq c, k=n, v, \delta\rangle$. By Theorems 1 and 2, we find that the buyer claims less than $1-\frac{c\delta}{1+\delta}$ times the efficient surplus in an equilibrium.

Suppose there are more than one non-critical seller in every feasible path, i.e., $k-c>1$. Consider the graph $\bar{\Gamma}$ derived from Γ^* in the following way: (a) $N(\bar{\Gamma}) = N(\Gamma^*) - C$, i.e., the set of nodes in $\bar{\Gamma}$ is the set of nodes in Γ^* without the critical sellers. (b) a pair of nodes a and b are adjacent in $\bar{\Gamma}$ if and only if they are adjacent in Γ^*, or there exists a path of length $c+2$ in Γ^* with a and b as the extreme nodes and c critical sellers in the middle. By construction, $\bar{\Gamma}$ does not have a critical seller when buyer wants to purchase $k-c$ contiguous items.

If she makes acceptable offers to all c critical sellers in the first period itself, then the remaining problem is identical to the problem $\langle\bar{\Gamma}, k-c, v, \delta\rangle$. Consequently, the share of surplus the buyer can extract in this problem in the second period is less than the surplus she can extract in the problem $\langle n \geq c, k=n, v, \delta\rangle$. By Theorems 1 and 2, we find that the buyer claims less than $\delta\left(1-\frac{c\delta}{1+\delta}\right)$ times the efficient surplus in an equilibrium.

Let $1 < \frac{c\delta}{1+\delta}$. Suppose r is the highest positive integer such that $1 > \frac{r\delta}{1+\delta}$.The buyer would pick a feasible path and offer zero to all sellers in the first period, all but $r \geq 2$ sellers would accept, and in the second period these r sellers would demand P share of the efficient surplus such that $1-rP = \delta\left(1-\frac{r\delta}{1+\delta}\right)$.

Suppose sellers make the first offer. Note that by rejecting all seller offers, the buyer is able to induce a continuation game where she makes the first offer. So either the sellers on a feasible path make offers that make the buyer indifferent between accepting and rejecting such offers, or they make offers that the buyer rejects in the first period. In either case, the buyer's equilibrium share of surplus cannot exceed δ times her equilibrium share of surplus in a continuation game where she has the first move. ■

Consider a graph Γ^O where (i) there is no cycle of length $k+1$, (ii) no two paths are disjoint and (iii) the intersection of all feasible paths is empty. This covers the case of Figure 7 with $k=3$ and the examples in Figure 8 below. We will show that if the sellers move first, the buyer achieves full surplus in the first period itself. Consequently, if the buyer moves first and δ is large, there is an equilibrium where buyer extracts full surplus in the second period.

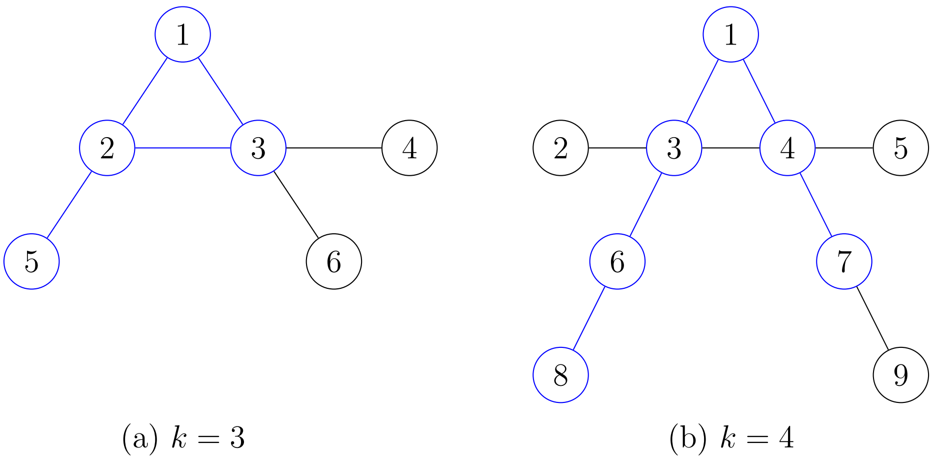

Figure 8: Two examples of Γ^O; Γ^{SO} in blue

We introduce some notation in the next two paragraphs that would be useful in proving the next result.

We note that each graph Γ^O has a subgraph Γ^{SO} such that (i) it contains a feasible path $\mathcal{P}$, (ii) for each node $x \in \mathcal{P}$ there exists a node $y \in \Gamma^O - \Gamma^{SO}$ and an edge $e(y,z), z \in \Gamma^{SO}$ such that $\Gamma^{SO} - x + z$ contains a feasible path of length k. For instance, in Figure 7, the path $\{1234\}$ qualifies as Γ^{SO}. Figure 8 shows two more examples. Observe that the order of any Γ^{SO} would vary from k to $n-1$. For any given Γ^O, let Γ^{SO*} be the smallest of all

$\Gamma^{SO} \subset \Gamma^O$ with order m^*.

Further, pick any feasible path $\mathcal{P}$ of length k on Γ^O. For each x on $\mathcal{P}$, let $s(x)$ be the order of the smallest subgraph ΓS of Γ^O such that $(\mathcal{P} - x) \cup \Gamma^S$ is a feasible path of length k. For example, in Figure 8 (a), $s(1) = s(5) = 1$ and $s(2) = 2$.

PROPOSITION 4 *Consider a assembly problem* $\langle \Gamma^O, k, v, \delta \rangle$ *such that* $v_1 = \cdots = v_n, v_0 > kv_1$. *(a) If the sellers move first, the buyer achieves full surplus in the first period. (b) If the buyer moves first, for* δ *large enough, there is an equilibrium where buyer extracts full surplus in the second period.*

Proof: Consider the following strategy of the buyer: In any continuation game where the buyer has the first move , the buyer makes negative offers to all sellers. In any continuation game where sellers have the first move, the buyer accepts the claims of sellers on a path with the lowest sum of claims provided her share of surplus is non-negative, and reject all other claims. In case the sum of claims on the two feasible paths are same, she accepts claims from one of the paths chosen with equal probability.

We claim that given the above strategy, sellers claiming zero surplus at any subgame they are required to make an offer is a best response. Let $\mathcal{P}_1, \ldots \mathcal{P}_m$ be the feasible paths in Γ^O. Let x_i be the surplus claim of active seller i. No seller can gain by deviating for one stage when $\sum_{i \in \mathcal{P}_1} x_i = \cdots = \sum_{i \in \mathcal{P}_m} x_i = 0$. This is because, for each x_i, there is always a feasible path in Γ^O that does not contain x_i. Hence it is an equilibrium. If $\sum_{i \in \mathcal{P}_1} x_i = \cdots = \sum_{i \in \mathcal{P}_m} x_i > 0$, a seller on either path can reduce his claim by a small amount and make a gain. If $\sum_{i \in \mathcal{P}_1} x_i > \sum_{i \in \mathcal{P}_2} x_i$, then any seller on $\mathcal{P}_2$ can increase his claim by a small amount and make a gain. Hence these are not equilibrium claims.

Part (a) of the claim follows immediately. For part (b), note that buyer can make zero surplus offers to all sellers on Γ^{SO*}, and negative surplus offers to all other sellers; sellers on Γ^{SO*} would accept these offers. To ensure that this deviation in the first stage is not profitable for the buyer, we require $\delta > \frac{v_0 - m^* v}{v_0 - kv}$. The buyer can also make acceptable offers of surplus shares to sellers on a path and negative offers to all other sellers. If $\mathcal{P}$ is the picked path and x_i is the node corresponding to seller i, he accepts any surplus share at least equal to $\delta(s(x_i)-1)v$. This is possible when $v_0 - kv - \delta\sum_{i\in\mathcal{P}}(s(x_i)-1)v > 0$. This is because, by rejecting a first period offer from the buyer, a seller on the chosen path competes with sellers on the other path; the highest surplus he can claim in a continuation game where he and the other sellers are making offers is $(s(x_i)-1)v$.To ensure that this deviation in the first stage is not profitable for the buyer, we require $\delta > \frac{v_0-kv-\delta\sum_{i\in\mathcal{P}}(s(x_i)-1)v}{v_0-kv}$. Thus, provided $\delta > \max\{\frac{v_0-m^*v}{v_0-kv}, \frac{v_0-kv-\delta\sum_{i\in\mathcal{P}}(s(x_i)-1)v}{v_0-kv}\}$, the buyer extracting full surplus in the second period is an equilibrium outcome in the strategies described above. ■

B Proof of Theorem 4

As noted in subsection 4.2, extracting full surplus implies trade taking place with only the k sellers on $\mathcal{P}_1$. Further, we also observed that, (a) if the buyer extracts full surplus, all sellers realize zero surplus share in the same period; (b) we need to prove the claim of this theorem for the first two periods only.

We now consider the four varieties of graphs alternately.

Consider a $k+1$ cycle $\langle C(k+1), k, v, \delta\rangle$ such that $v_1 \leq \cdots \leq v_{k+1}$ with at least one strict inequality. Suppose the buyer makes zero surplus offers in the first period. At least one seller, say i would reject this offer and claim $v_{k+1} - v_i$ in the next period which buyer must accept, provided $v_0 - \mathcal{S}_1 - (v_{k+1} - v_i) > 0$. If this inequality does not hold, the buyer must offer $\frac{\delta}{1+\delta}$ times the efficient surplus to this seller. So, suppose buyer makes negative offers to all sellers in the first period. Note that in a $k+1$- cycle, every pair of paths have a non-empty intersection. A seller in the intersection of $\mathcal{P}_1$ and $\mathcal{P}_2$ can raise his claim till the point the sum of claims on $\mathcal{P}_2$ and $\mathcal{P}_3$ are the same. Either the buyer accepts this claim, or she rejects and offers $\frac{\delta}{1+\delta}$ times the efficient surplus to this seller. Not all sellers would claim zero surplus when proposing first: as argued before, at least one seller can claim $v_{k+1} - v_i$, or if $v_0 - \mathcal{S}_1 - (v_{k+1} - v_i) < 0$, she can claim $\frac{1}{1+\delta}$ times the efficient surplus. If the buyer rejects all seller offers in the first period, then she is on a continuation game where she is proposing to all sellers. We have already argued that she cannot extract full surplus in such a continuation game.

Consider a assembly problem $\langle \Gamma^D, k, v, \delta \rangle$ such that $v_1 \leq \cdots \leq v_n$ with at least one strict inequality. Suppose the buyer makes zero surplus offers in the first period. At least one seller, say i would reject this offer and claim $\mathcal{S}_2 - v_i$ in the next period which buyer must accept, provided $v_0 - \mathcal{S}_2 - (\mathcal{S}_1 - v_i) > 0$. If this inequality does not hold, the buyer must offer $\frac{\delta}{1+\delta}$ times the efficient surplus to this seller. So, suppose buyer makes negative offers to all sellers in the first period. If $v_0 > \mathcal{S}_2$, sellers on $\mathcal{P}_1$ can make claims summing up to $\mathcal{S}_2$. Either the buyer accepts this claim, or she rejects and offers $\frac{\delta}{1+\delta}$ times the efficient surplus to this seller. Not all sellers would claim zero surplus when proposing first: as argued before, at least one seller can claim $\mathcal{S}_2 - v_i$, or if $v_0 - \mathcal{S}_2 - (\mathcal{S}_1 - v_i) < 0$, she can claim $\frac{1}{1+\delta}$ times the efficient surplus. If the buyer rejects all seller offers in the first period, then she is on a continuation game where she is proposing to all sellers. We have already argued that she cannot extract full surplus in such a continuation game.

Consider the $\langle \Gamma^*, k, v, \delta \rangle$ such that $v_1 \leq \cdots \leq v_n$ with at least one strict inequality. Suppose the buyer makes zero surplus offers in the first period. By Theorems 1 and 2, at least one critical seller rejects this offer and claims $\frac{1}{1+\delta}$ in the next period which buyer must accept. Consequently, the buyer cannot extract full surplus within the first two periods in an equilibrium. Suppose sellers make the first offers. Not all sellers claim zero surplus, because a critical seller can claim strictly positive surplus. So full surplus extraction cannot take place in the first period. But if the buyer rejects all seller offers in the first period, then she is on a continuation game where she is proposing to all sellers. We have already argued that she cannot extract full surplus in such a continuation game.

Consider the assembly problem $\langle \Gamma^O, k, v, \delta \rangle$ such that $v_1 \leq \cdots \leq v_n$ with at least one strict inequality. For each node x on a feasible path, let $\sigma(x)$ be the sum of valuations over the smallest subgraph such that its union with $\Gamma^O \backslash x$ contains a feasible path. Suppose the buyer makes zero surplus offers in the first period. At least one seller, say i would reject this offer and claim $\sigma(i) - v_i$ in the next period which buyer must accept, provided $v_0 - \mathcal{S}_1 - (\sigma(i) - v_i) > 0$. If this inequality does not hold, the buyer must offer $\frac{\delta}{1+\delta}$ times the efficient surplus to this seller. So, suppose buyer makes negative offers to all sellers in the first period. Note that in Γ^O, every pair of paths have a non-empty intersection. A seller in the intersection of $\mathcal{P}_1$ and $\mathcal{P}_2$ can raise his claim till the point the sum of claims on $\mathcal{P}_2$ and $\mathcal{P}_3$ are the same. Either the buyer accepts this claim, or she rejects and offers $\frac{\delta}{1+\delta}$ times the efficient surplus to this seller. Not all sellers would claim zero surplus when proposing first: as argued before, at least one seller can claim $\sigma(i) - v_i$, or if $v_0 - \mathcal{S}_1 - (\sigma(i) - v_i) < 0$, she can claim $\frac{1}{1+\delta}$ times the efficient surplus. If the buyer rejects all seller offers in the first period, then she is on a continuation game where she is proposing to all sellers. We have already argued that she cannot extract full surplus in such a continuation game.

C Equilibria with Unequal seller valuations

In this subsection, we characterize a selection of equilibria corresponding to the four types of graphs identified above.

Proposition 5 *Consider a assembly problem $\langle \Gamma^{\triangle}, k, v, \delta \rangle$ such that $v_1 \leq \cdots \leq v_n$ with at least one strict inequality. If $v_0 > kv_{k+1}$, there exists an equilibrium where if sellers are proposing first, the buyer gets a surplus of $v_0 - kv_{k+1}$; if the buyer is proposing first, she gets a surplus of $v_0 - (1-\delta)\sum_{i=1}^{k} v_i - \delta k v_{k+1}$.*

Proof: We will prove the case of a graph which is a cycle of length $k+1$. For any other graph containing a cycle, the buyer can implement the same surplus by giving negative offers to sellers not on the cycle.

Let $v_0 > kv_{k+1}$. Consider the following strategies: sellers $1, \ldots, k$, if proposing first, ask for $v_{k+1} - v_i$ amount of surplus and the seller $k+1$ asks for a price between $(v_{k+1}, v_{k+1} + \gamma)$, $\gamma > 0$, with uniform probability. The sellers $1, \ldots, k$ accept any surplus which is at least $\delta(v_{k+1} - v_i)$ when the buyer is making an offer. Seller $k+1$ accepts any positive surplus. If the sellers are proposing, the buyer accepts any price offer that is less than or equal to v_{k+1}. If the buyer is proposing, she offers each of the seller $1, \ldots, k$, $\delta(v_{k+1} - v_i)$ surplus and any negative surplus to seller $k+1$. Trade takes place in the first period itself. If the buyer is proposing first, she gets a surplus of $v_0 - (1-\delta)\sum_{i=1}^{k} v_i - \delta k v_{k+1}$, otherwise she gets a surplus of $v_0 - k v_{k+1}$. ■

Remark **2** If $v_0 \leq k v_{k+1}$, many different equilibrium outcomes are possible. If $v_0 < v_{k+1}$, then only the efficient path is feasible and we are back to the case studied by Roy Chowdhury and Sengupta (2012). Let us therefore suppose that $r < k$ is the highest positive integer such that $v_0 - r v_{k+1} > 0$. Then if the buyer is proposing, she offers $\delta(v_{k+1} - v_i)$ to sellers $1, \ldots, r$. She offers $\frac{\delta}{1+\delta}(v_0 - \sum_{i=1}^{k} v_i - \sum_{i=1}^{r}(v_{k+1} - v_i))$ to sellers $r+1, \ldots, k$ if $1 - \frac{(k-r)\delta}{1+\delta} > 0$. Otherwise, she offers zero to these $k - r$ sellers in the first period. If the sellers are proposing, the first $1, \ldots, r$ sellers demand $v_{k+1} - v_i$, while the rest of the sellers make offers that make the buyer indifferent between accepting their offers and making new offers in the next period. Note that in none of these equilibria, the buyer is able to extract the full efficient surplus.

We now turn to the case where the graph contains only disjoint feasible paths, e.g., Figure 3.

Proposition **6** *Consider a assembly problem $\langle \Gamma^D, k, v, \delta \rangle$ such that $v_1 \leq \cdots \leq v_n$ with at least one strict inequality. Let $\mathcal{P}_1$ and $\mathcal{P}_2$ be the two paths corresponding to the minimum and second smallest sum of valuations, $\mathcal{S}_1$ and $\mathcal{S}_2$. If $v_0 > \mathcal{S}_2$ and δ is large, there exists an equilibrium where sellers $i = 1, \ldots, k$ on $\mathcal{P}_1$ demand $\lambda_i(v_0 - \mathcal{S}_2)$, $\sum_{i=1}^{k} \lambda i = 1$ in the period they make offers. The buyer accepts these offers and gets a surplus of $\mathcal{S}_2 - \mathcal{S}_1$. If the buyer is making offers, she proposes negative surplus shares to all sellers, and extracts a surplus of $\mathcal{S}_2 - \mathcal{S}_1$ in the next period.*

Proof: Suppose the buyer makes offers to all k sellers on $\mathcal{P}_1$ in the first period and all but one seller accept. Such a seller can make at most $v_0 - S_2$ in the next period. Consequently, if the buyer wants to make all k sellers on $\mathcal{P}_1$ accept her offer in the first period, she has to offer them at least $\delta(v_0 - \mathcal{S}_2)$. Such offers are feasible when $v_0 - \mathcal{S}_1 - k\delta(v_0 - \mathcal{S}_2) > 0$. Suppose this inequality is not satisfied. Let $r < k$ be the positive integer such that $v_0 - \mathcal{S}_1 - r\delta(v_0 - \mathcal{S}_2) > 0$ but $v_0 - \mathcal{S}_1 - (r+1)\delta(v_0 - \mathcal{S}_2) \leq 0$. Then the buyer offers

$\delta(v_0 - \mathcal{S}_2$ to sellers $1, \ldots, r$ on $\mathcal{P}_1$. She offers $\frac{\delta}{1+\delta}(v_0 - \mathcal{S}_1 - r\delta(v_0 - \mathcal{S}_2)$ to the rest $k-r$ if $1 - \frac{(k-r)\delta}{1+\delta} > 0$. If $1 - \frac{(k-r)\delta}{1+\delta} < 0$, she offers zero surplus to these $k-r$ sellers. Some of these sellers accept zero offers, while others reject and make fresh offers in the next period to make the buyer indifferent between accepting their offers and making new offers in the next period. In none of these cases, buyers surplus exceeds $(v_0 - \mathcal{S}_1 - r\delta(v_0 - \mathcal{S}_2)$. Similarly, the buyer can make acceptable offers to sellers on $\mathcal{P}_2$ instead, offering them at least $\delta(V_0 - \mathcal{S}_1)$. Such offers are feasible when $v_0 - \mathcal{S}_2 - k\delta(v_0 - \mathcal{S}_1) > 0$. However, buyer gets more surplus by proposing acceptable offers to sellers on $\mathcal{P}_1$. If $\frac{1-\delta}{(k-1)\delta} < \frac{v_0 - \mathcal{S}_2}{v_0 - \mathcal{S}_1}$, then the buyer prefers to make negative offers to all sellers in the first period than making acceptable offers to sellers on $\mathcal{P}_1$. ■

Remark **3** Note that there is another equilibrium similar to Roy Chowdhury and Sengupta (2012) when $\mathcal{S}_1 < v_0 < \mathcal{S}_2$.

We now consider the case of graphs with critical sellers, e.g., Figure 4.

Proposition **7** *Consider the assembly problem $\langle \Gamma^*, k, v, \delta \rangle$ such that $v_1 \leq \cdots \leq v_n$ with at least one strict inequality. Suppose that Γ^* has $c < k$ critical sellers. If $1 \geq \frac{c\delta}{1+\delta}$, the buyer cannot claim more than $\left(1 - \frac{c\delta}{1+\delta}\right)$ times the efficient surplus in an equilibrium. Otherwise, there is an equilibrium where agreement takes place in the second period. In this equilibrium, buyer's share of the efficient surplus is at most $\delta^2 \left(1 - \frac{r\delta}{1+\delta}\right)$, where r is the highest positive integer such that $1 > \frac{r\delta}{1+\delta}$.*

Proof: Let $1 \geq \frac{c\delta}{1+\delta}$. Suppose the buyer makes offers first. If she makes acceptable offers to all c critical sellers in the first period itself, then the remaining problem is identical to the problem $\langle \bar{\Gamma}, k - c, v_{-C}, \delta \rangle$ where c is the number of critical sellers and v_{-C} is the profile of valuations excluding those of the critical sellers in Γ^*. By Theorems 1, 2 and B, buyer's share of the efficient surplus cannot exceed $1 - \frac{c\delta}{1+\delta}$.

Let $1 < \frac{c\delta}{1+\delta}$. Buyer cannot make acceptable offers to all critical sellers in the first period. Suppose r is the highest positive integer such that $1 > \frac{r\delta}{1+\delta}$.The buyer would pick a feasible path and offer zero to all critical sellers in the first period, all but $r \geq 2$ sellers would accept, and in the second period these r sellers would demand P share of $v_0 - \mathcal{S}_2$ such that $1 - rP = \delta\left(1 - \frac{r\delta}{1+\delta}\right)$.

Suppose sellers make the first offer. Note that by rejecting all seller offers, the buyer is able to induce a continuation game where she makes the first offer. So either the sellers on a feasible path make offers that make the buyer indifferent between accepting and rejecting such offers, or they make offers that the buyer rejects in the first period. In either case, the buyer's equilibrium share of surplus cannot exceed δ times her equilibrium share of surplus in a continuation game where she has the first move. ■

We now consider the case of graphs without $k+1$ cycles that do not have a critical seller and no two feasible paths are disjoint, e.g., Figure 8.

PROPOSITION 8 *Consider the assembly problem $\langle \Gamma^O, k, v, \delta \rangle$ such that $v_1 \leq \cdots \leq v_n$ with at least one strict inequality. There exists an equilibrium where the buyer does not achieve full surplus.*

Proof: For a seller corresponding to node $x \in \mathcal{P}_1$, let Γ^x be the smallest subgraph of Γ^O such that $(\mathcal{P}\backslash x) \cup \Gamma^x$ contains a feasible path. Let $\mathcal{S}(\Gamma^x)$ be the sum of valuations of sellers on Γ^x. If $\mathcal{S}(\Gamma^x) \geq v(x)$, where $v(x)$ is the valuation of the seller corresponding to x, and $\sum_{x \in \mathcal{P}_1} \delta\left(\mathcal{S}(\Gamma^x) - v(x)\right) \leq v_0 - \mathcal{S}_1$, then the buyer offers $\delta\left(\mathcal{S}(\Gamma^x) - v(x)\right)$ to corresponding sellers and they accept. If either of these conditions are violated for some $x \in \mathcal{P}_1$, but satisfied for $y \in \mathcal{P}_1$ then the buyer offers $\frac{\delta}{1+\delta}\left(v_0 - \mathcal{S}_1 - \sum_{y \in \mathcal{P}_1} \delta\left(\mathcal{S}(\Gamma^y) - v(y)\right)\right)$ to each x and $\delta\left(\mathcal{S}(\Gamma^y) - v(y)\right)$ to each y. Note that $\mathcal{P}_1$ and $\mathcal{P}_2$ have at least one node in common. Consequently, if the sellers are making offers, these sellers claim $\min\{v_0, \mathcal{S}_3\}$. The buyer therefore, chooses to make negative offers to all sellers in the first period, if $\delta(v_0 - \mathcal{S}_3)$ is greater than the surplus she can obtain by making positive surplus offers to sellers on $\mathcal{P}_1$. Also note that the buyer cannot make zero surplus offers to all sellers on a subgraph containing more than k sellers: a seller common to two feasible paths contained in the subgraph will reject such an offer. ■