WEALTH DISTRIBUTION AND SKILLS GENERATION UNDER PUBLIC AND PRIVATE EDUCATION SYSTEMS[1]

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Abstract

This paper aims to understand the differing impacts of wealth distribution on human capital accumulation and skilled-unskilled labour generation under three educational paradigms: private, public and a system of mixed education. When public and private educations co-exist, we find that an increase in the rate of income tax raises the steady-state human capital per unit of inheritance not only for the unskilled labours, but also for the skilled labours (tax-payers). Between the two separate education regimes (private and public), the per capita human capital required to join the skilled sector, is higher under private education than that under public education when the skilled population in the economy is sufficiently large. It is found that in an economy with an unequal wealth distribution, low unskilled wage, an improved quality of public education and a low weight assigned to the offspring's education in the preferences of the individuals, the majority of the population choose public education system.

Keywords: Public education; Private education; Human capital; Growth, Public choice

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1. Introduction

Both the developed and the developing countries aspire to achieve equity and efficiency while providing education to their children. The choice of education policy plays a crucial role in determining the national economic growth. Almost every economy delivers education in a system comprising public and private educations. There has been a long-standing debate on whether it is ethical to apply the free market principle to education or not. In Ben Porath (1967), Heckman (1976), Galor and Zeira (1993), Banerjee and Newman (1993), Alonso-Carrera and Freire-Seren (2004), Contreras (2008) education is financed privately. On the other hand, Beauchemin (2000), Blankenau and Simpson (2003), Tanaka (2003) deal with public education while Glomm and Ravikumar (1992), Boldrin (1993), Glomm (1997),Garcia-Penalosa and Walde (2000), Cardak (2004), Benos (2010), Ghate, Glomm and Stone III (2014), Roemer and Unveren (2017), study both public and private educations. Boldrin and Montes (2005) argue that market best level of education can be generated by education subsidy to young and pension to old. Provision of subsidy to education is even more justified in the presence of externalities in education (Bishnu, 2013) or if education is believed to create society of knowledgeable voters (Dee, 2004). Empirical studies on private and public educations include Marginson (2007), Coulson (2009). Investigating the effects of public vs. private education on growth and development in the presence of unequal wealth distribution raises a few questions. In the mixed education system, what is the impact of raising the tax rate for financing public education? In the benchmark cases of public and private education, which education system generates higher level of skill? In an economy with unequal wealth distribution, what are the factors that result in public or private education as a voting equilibrium outcome?

This present paper analyses the interaction between wealth distribution, skills generation and economic growth under private and public education systems. It demonstrates that in an economy with high income inequality, if the parents have low weights assigned to human capital of their offspring in their preference functions, then the majority of the population will choose public education system. And a low rate of interest, a low unskilled wage, an improved quality of public education will strengthen this majority voting outcome. Probabilistic voting? Markov perfect equilibrium?

This paper rests upon the assumption that the educational decisions as well as the decisions regarding the choice of the quality of schooling are made by the parents. Papers by Glomm and

Ravikumar (1992), Benabou (1992), Boldrin (1993) have a common assumption that each individual makes his own educational decisions. But, Glomm (1997) argues that in the developing countries where the average level of school years attended by the individuals is very low, the assumption of parental decision of schooling is appropriate to study the effects of formal schooling on long run economic growth.

The utility function in this model shows warm glow preferences where the parents care about both the quality of education of their offspring (as in Glomm and Ravikumar (1992)) and the bequest they leave to them (as in Galor and Zeira (1993)). This model is based on the assumption that the parent derives satisfaction from the human capital of the offspring that is a monotonic function of the quality of education. The amount of bequest a parent leaves to her child contributes to the utility of the parent and it adds to the accumulation of total wealth of a dynasty. So, there are two ways to account for intergenerational altruism in our model-one is allocating resources for the child's education and the second one is leaving a bequest.

Human capital accumulation function is assumed to depend on the quality of school that a child attends and has both private and public inputs to education. All the children go to schools of different qualities depending on their parents' income and ability to afford schooling quality. There exists a threshold level of skill which is required to work as a skilled labour and those, whose accumulated skill fall below this threshold level join the unskilled labour force. So, every parent incurs the education cost of the offspring and derives utility from the human capital that their children attain. Every child goes to school, but some may remain unskilled. In many developing countries, some children attend primary schools but drop out without completing and fail to become skilled enough to find jobs in the skilled sectors.

We start by analysing the case of a mixed education system where public and private educations co-exist in the economy, i.e., when the human capital accumulation function consists of both private and public inputs to education. Then we consider the two education systems separately. Under the public education system, all the children have access to equal quality of education because under this system, school quality is financed by the government and there is no option to purchase any educational inputs or improve education quality privately. And, under private education regime, the school quality is purchased privately. Under this system, the parents choose from a menu of private schools and purchase the best and affordable quality of education for their children. In case of co-existence of public and private education system, it is assumed

that human capital accumulation of each and every child requires investment from both government and own household. The human capital accumulation function is assumed to take Cobb-Douglas form. Therefore, in this case, public inputs and private inputs are neither perfect substitutes nor perfect complements.

We have found that when the public and the private educations co-exist, an increase in the rate of income tax raises the steady state human capital per unit of inheritance not only of the unskilled labours, but also of the skilled labours, who are the tax-payers.

The wealth dynamics of the present model shows that in the private education system, those who inherit a low bequest initially work as unskilled labours unless sufficient inheritance is received. Those who are sufficiently well inherited will work as skilled labours and enjoy a steady growth of income. In the long run, all the individuals join the skilled labour force and will face a steady growth of income. In the public education system, the human capital growth rate is identical for all the individuals of the society and is independent of individual bequest and parental human capital. In this system, either all the individuals work as skilled labours or all are unskilled labours. If all the individuals are employed in the skilled sector, the economy faces a continuous development but if all the individuals are in the unskilled sector, the economy faces stagnation. When the rate of interest and weights assigned to bequest in the parental utility function are sufficiently high, there will be throughout growth in both public and private education systems.

Policies prioritizing the private educations over public education have always been challenged by the public opinion. A Gallup/Phi Delta Kappa poll in the US asked the public in 2002, 2006, and 2007 if they would favour a plan in which a local school board would contract with private, profit-making corporations to run the entire operations of the public schools in their community. Not even one-third of the public supported it. Also, in a 1982 Roper Report poll, 63% of the country rejected the idea of having the government subcontract to private companies for the running of public schools.[3] So, it is important to study the public opinion on the choice of education. While discussing the choice of education system through voting, we find that even if the tax rate imposed is their preferred one, the skilled labours will always choose the private education system over public education system. An unskilled labour will vote for the public education system if her inheritance received is below a critical level. Finally, after assuming a Pareto distribution of wealth we arrive at the conclusion that an economy with an unequal wealth

[3]https://www.huffingtonpost.com/kathleen-weldon/public-funds-private-educ_b_6979498.html

distribution, a high value of the inequality parameter, a low rate of interest, a low unskilled wage, an improved quality of public education and the parents with low weights assigned to human capital of their offspring in their preference functions, will have the public education system chosen by the majority of its population.
The next section discusses the model in detail.

2. The Model

We consider a two period overlapping generations model in which every individual lives for two periods-as a child and as an adult. The population consists of N individuals. Each household consists of one parent and one child. It is assumed that the population does not grow. We rule out any possibility of child labour and assume that all the young individuals go to schools and all the adult individuals supply their labours inelastically to their occupations. In the first period, when an individual is a child, she receives the bequest left by her parent and devotes her time to accumulate skill through schooling. In the second period, when an individual is an adult, she chooses her occupation and starts a family. In this period, the adult individual chooses the optimal amount of consumption for her family, the optimum quality of schooling for her offspring and the optimum amount of bequest she would leave for her child.

All the adult individuals have identical preferences over consumption, bequest and the human capital of their offspring. The utility function representing these preferences is:

$$U_t = \alpha \ln X_t + \beta \ln b_t + (1-\alpha-\beta) \ln h_{t+1} \qquad (1)$$

where, X_t denotes the consumption when old and b_t is the bequest which is left for the next generation and h_{t+1} denotes the human capital of the offspring.

Human capital accumulation of the offspring is assumed to depend on the quality of school that a child attends and has both private and public inputs to education. We have not included the time devoted to education in the human capital accumulation function because we have assumed that all the children spend their fulltime to accumulate skill through schooling.

Human capital is accumulated according to the learning technology:

$$h_{t+1} = \theta q_t \qquad (2)$$

where, q_t is the quality of school and $\theta>0$ can be interpreted as the efficiency parameter of the education system.[4]

$q_t = e_t$ under private education system;

$= E_t$ under public education system and (3)

$= (e_t)^{\mu}(E_t)^{1-\mu}, 0 \le \mu \le 1$ when public and private educations co-exist.

The economic performances are viewed under three education systems: Private, Public and the Co-existence of Public and Private Educations. Under private education system $(\mu = 1)$, educational quality is privately financed. Parents choose from a menu of private schools and use family income to finance the affordable quality of education for the children. Under public education system $(\mu = 0)$, the quality of schooling is funded by the government who levies an income tax at a uniform rate on the income of the adult individuals who are working as skilled labours. So, under public education, all children have access to equal quality of education at zero price. Under the system where public and private educations co-exist, the human capital accumulation function comprises of both public and private inputs; the private input is financed by the parents and the public inputs are provided by the government, who taxes the skilled labours of the society.

In the beginning of the second period, an adult individual decides to join the skilled sector as opposed to the unskilled sector depending on the skill attained and the salary offered. It requires a threshold level of skill that satisfies the incentive of an individual to join the skilled labour force. Those who cannot accumulate that threshold skill level, work as unskilled labours. Also, we have assumed that the educational quality, which is measured by the education expenditure, is financed by the family income. If the family income is too low or if the inheritance a child receives is too low to purchase a good quality education, then she might not be able to

[4]Following Loury (1981), Lloyd Ellis (2000) we have assumed that human capital accumulation function is independent of parental human capital level but nevertheless it is dependent on parental human capital through education spending. We did avoid h in equation (2) due to mathematical simplification. Inclusion of h_t brings non-linearity in the dynamic system and makes it difficult to analyse the bequest dynamics.

accumulate skill, adequate enough to join the skilled sector. So, even though all the children go to school, not all become skilled labours.

It is empirically evident that a child's human capital is positively related to her parental level of human capital. In this model, the human capital accumulation function depends positively on the educational quality. Educational quality is again dependent on the educational expense which in turn is an increasing function of the parental income that is positively related to the level of skill for the skilled labours.

The production function of the unskilled sector is assumed to follow a linear technology given by $X_t^u = \underline{w}L_t$ where L_t is the number of unskilled workers at time t. Each unskilled worker employed in this sector earns a wage of $\underline{w}$, the average as well as marginal productivity of unskilled labours. Unlike Galor Zeira (1993), in the present model, there is constant returns to scale with respect to skilled sector. The production function of the skilled sector is given by $X_t^s = \delta H_t$, $where\ H_t = \sum h_t$ is the aggregate human capital of the economy, $\delta>0$ and h_t is the skill accumulated by an adult individual. So, each individual employed in skilled sector earns δh_t.

In the next section, competitive equilibrium of the model with co-existing public and private education systems is derived.

2.1. Economy with the Co-existence of Public and Private Education Systems:

Here, we analyse the case when public and private educations co-exist. Under this system, the human capital accumulation function is assumed to take the form of a Cobb-Douglas production function with the arguments being the private and the public inputs to education. Private inputs are purchased privately by the economic agents and public inputs are financed by the government using the tax revenues.

So, the human capital accumulation function is denoted by: $h_{t+1} = \theta e_t^{\mu} E_t^{1-\mu}$, where $0 < \mu < 1$, e_t and E_t denote private and public inputs to education respectively.

In our model, government imposes income tax at a uniform rate on the skilled labours' income. Income of the unskilled labours is exempted from taxation.

One rationalization for this assumption is that in most of the developing economies, the unskilled sector is mostly informal where collection of taxes is prohibitively expensive. Tax revenues are used to finance education and all children have access to the same quality of education at a zero price.[5]

So, the total tax revenue $= \tau_t \delta H_t$ and $E_t = \frac{\tau_t \delta H_t}{N}$

Hence, $h_{t+1} = \theta(e_t)^{\mu}\left(\frac{\tau_t \delta H_t}{N}\right)^{(1-\mu)}$ (4)

2.1.1. The Optimization Exercise of *an Unskilled Labour:*

An unskilled labour faces the budget constraint: $\underline{w} + (1+r)b_{t-1} = X_t + b_t + e_t$, where b_{t-1} is the inheritance received, r is the rate of interest which is assumed to be exogenously given, b_t is the bequest and e_t is the private input to education.

Each unskilled labour of the t^{th} period maximizes utility subject to the budget constraint and the human capital accumulation function with respect to the control variables X_t, b_t, and e_t.

An unskilled labour solves the following maximization exercise:

$$\max_{\{X_t, b_t, e_t\}} \alpha \ln X_t + \beta \ln b_t + (1-\alpha-\beta) \ln h_{t+1}$$

[5] Glomm (1997)

Subject to

$$X_t + b_t + e_t = \underline{w} + (1+r)b_{t-1} \text{ and } h_{t+1} = \theta(e_t)^{\mu}\left(\frac{\tau_t \delta H_t}{N}\right)^{(1-\mu)}$$

From the first order conditions we obtain the equilibrium values:

$$X^*_{t,u} = \frac{\alpha[\underline{w} + (1+r)b_{t-1}]}{[\mu + (\alpha+\beta)(1-\mu)]} \quad (5)$$

$$b^*_{t,u} = \frac{\beta[\underline{w} + (1+r)b_{t-1}]}{[\mu + (\alpha+\beta)(1-\mu)]} \text{ and} \quad (6)$$

$$e^*_{t,u} = \frac{\mu(1-\alpha-\beta)[\underline{w} + (1+r)b_{t-1}]}{[\mu + (\alpha+\beta)(1-\mu)]} \quad (7)$$

The equilibrium law of motion of human capital, for an unskilled labour is given by:

$$h^*_{t+1} = \theta\left[\left(e^*_{t,u}\right)^{\mu}\left(\frac{\tau_t \delta H^*_t}{N}\right)^{1-\mu}\right] \quad (8)$$

Therefore, $h^*_t = \theta\left[\left(e^*_{t-1}\right)^{\mu}\left(\frac{\tau_{t-1}\delta H^*_{t-1}}{N}\right)^{1-\mu}\right]$ (9)

(Subscript ‘u’, standing for unskilled agent is omitted for notational simplification)

We have, $\frac{h^*_{t+1}}{h^*_t} = \left(\frac{e^*_t}{e^*_{t-1}}\right)^{\mu}\left(\frac{H^*_t}{H^*_{t-1}}\right)^{1-\mu}$, (10)

when the tax rate is uniform over time.

From equation (9), $h^*_t = \theta\left[\left(\frac{e^*_{t-1}}{b_{t-1}}\right)^{\mu}\left(\frac{\tau_{t-1}\delta H^*_{t-1}}{N}\right)^{1-\mu}(b_{t-1})^{\mu}\right]$

$$= \theta\left\{\frac{\mu(1-\alpha-\beta)}{\beta}\right\}^{\mu}\left(\tau\delta h_{t-1}n^s\right)^{1-\mu}(b_{t-1})^{\mu} \quad (11)$$

So, the steady state value of human capital $\left(h^*\right)$ depends on b_{t-1}.

We can now easily obtain the steady state values of the choice variables for an unskilled labour.

At steady state, $b_t = b_{t-1} = b_u^*, b_u^*$ being the steady state value of b_t for an unskilled agent.

Substituting this in equation (6), we get the steady-state value of the bequest of the unskilled labour: $b_u^* = \dfrac{\beta \underline{w}}{\left[\{\mu + (1-\mu)(\alpha+\beta)\} - \beta(1+r)\right]}$ (12)

Similarly, at steady state, $h_t = h_{t-1} = h_u^*, h_u^*$ being the steady state value of h_t for an unskilled agent.

Substituting this and equation (12) in equation (11), we get the steady-state value of the human capital of the unskilled labour:

$$h_u^* = \frac{\theta^{\frac{1}{\mu}} \left(\tau \delta n^s\right)^{\frac{1-\mu}{\mu}} \mu(1-\alpha-\beta)\underline{w}}{\left[\{\mu + (1-\mu)(\alpha+\beta)\} - \beta(1+r)\right]} \quad (13)$$

From equation (13) we find that, as the proportion of skilled labours (n^s) in the total population increases, tax revenue increases, the quality of public inputs increases, the steady state human capital of unskilled worker increases. Steady-state value of e_t for an unskilled agent, e_u^* is obtained by putting the value of b_u^* from equation (12) in equation (7). This gives us:

$$e_u^* = \frac{\left[\mu \underline{w}(1-\alpha-\beta)\right]}{\left[\mu + (\alpha+\beta)(1-\mu) - \beta(1+r)\right]} \quad (14)$$

The bequest dynamics of unskilled sector is illustrated below in figure 1. From equation (6) we find that bequest line intersects the 45 degree line at M, the only equilibrium generated in the system. M is a stable equilibrium which corresponds to the steady state value of b_t,

$$b_u^* = \frac{\beta \underline{w}}{\left[\{\mu + (1-\mu)(\alpha+\beta)\} - \beta(1+r)\right]}$$

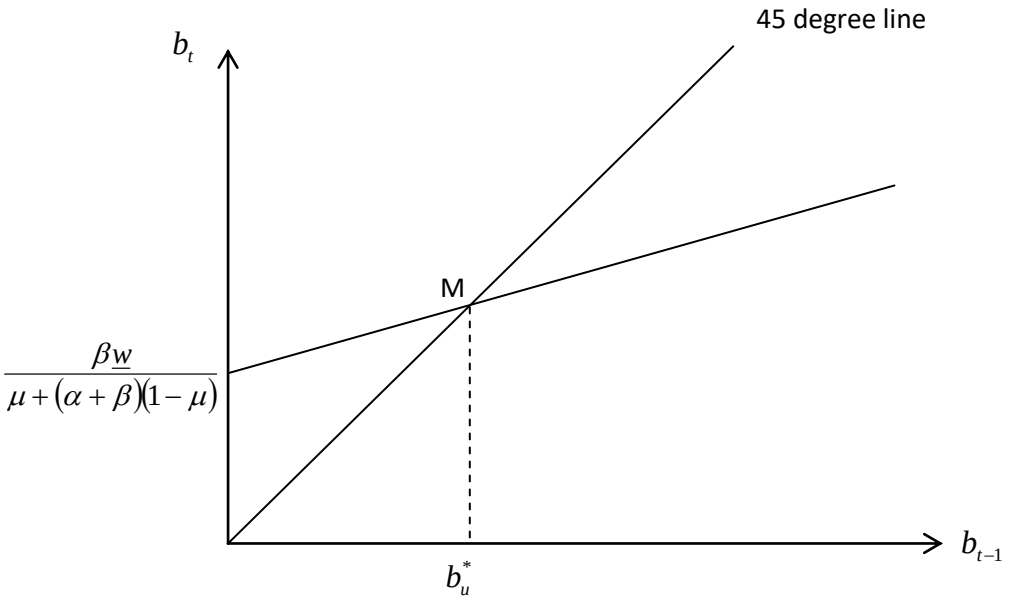

Figure 1: wealth dynamics of the unskilled sector under the education system with both public and private components

We assume $\mu + (1-\mu)(\alpha+\beta) > \beta(1+r)$. This assumption is necessary to ensure a positive value of the steady state value of bequest and also a stable equilibrium for the unskilled sector.

From equation (13), it is clear that when $\mu = 0$, $h_u^* = 0$ and when $\mu = 1$, $h_u^* = \frac{\theta(1-\alpha-\beta)\underline{w}}{1-\beta(1+r)}$.

(15)

So, when only private education is there in the economy, the steady state level of human capital becomes zero for the unskilled labours, but when the economy has only public education system, the steady state level of human capital has a constant value which depends on the unskilled wage and the rate of interest. We will study these situations in detail in sections 3 and 4.

2.1.2. The Optimization Exercise of a *Skilled Labour:*

A representative skilled labour, under this education system where public and private educations co-exist, faces the following maximization exercise:

$$\max_{\{X_t, b_t, e_t\}} \alpha \ln X_t + \beta \ln b_t + (1-\alpha-\beta) \ln h_{t+1}$$

Subject to

$$X_t + b_t + e_t = (1-\tau_t)\delta h_t + (1+r)b_{t-1} \text{ and } h_{t+1} = \theta(e_t)^{\mu}\left(\frac{\tau_t \delta H_t}{N}\right)^{(1-\mu)}$$

The first order conditions give us the equilibrium values:

$$X^*_{t,s} = \frac{\alpha[(1-\tau_t)\delta h_t + (1+r)b_{t-1}]}{[\mu + (\alpha+\beta)(1-\mu)]} \tag{16}$$

$$b^*_{t,s} = \frac{\beta[(1-\tau_t)\delta h_t + (1+r)b_{t-1}]}{[\mu + (\alpha+\beta)(1-\mu)]} \text{ and} \tag{17}$$

$$e^*_{t,s} = \frac{\mu(1-\alpha-\beta)[(1-\tau_t)\delta h_t + (1+r)b_{t-1}]}{[\mu + (\alpha+\beta)(1-\mu)]} \tag{18}$$

Now, for the skilled agents, the human capital of the parent, which determines their income, enters into the expressions of the optimum values of all the three choice variables. This creates computational difficulty for finding out the explicit expressions for the steady state values. So, instead, we study the dynamics in terms of $\left(\frac{b_t}{h_t}\right)$.

The equilibrium law of motion of human capital, for a skilled labour is given by:

$$h^*_{t+1} = \theta\left[\left(e^*_{t,s}\right)^{\mu}\left(\frac{\tau_t \delta H^*_t}{N}\right)^{1-\mu}\right] \tag{19}$$

Therefore, $h^*_t = \theta\left[\left(e^*_{t-1,s}\right)^{\mu}\left(\frac{\tau_{t-1}\delta H^*_{t-1}}{N}\right)^{1-\mu}\right]$ (20)

$$= \theta\left[\left(\frac{e^*_{t-1,s}}{b_{t-1,s}}\right)^{\mu}\left(\frac{\tau_{t-1}\delta H^*_{t-1}}{N}\right)^{1-\mu}\left(b_{t-1,s}\right)^{\mu}\right]$$

$$= \theta\left\{\frac{\mu(1-\alpha-\beta)}{\beta}\right\}^{\mu}\left(\tau\delta h_{t-1}n^s\right)^{1-\mu}\left(b_{t-1,s}\right)^{\mu} \tag{21}$$

Dividing equation (17) throughout by h_t, we get:

$$\frac{b_t}{h_t} = \frac{\beta\left[(1-\tau_t)\delta + (1+r)\frac{b_{t-1}}{h_t}\right]}{\left[\mu + (1-\mu)(\alpha+\beta)\right]}$$

Substituting the value of h_t from equation (21), we get:

$$\frac{b_t}{h_t} = \frac{\beta\left[(1-\tau)\delta + (1+r)\frac{1}{\theta}\left(\frac{\beta}{\mu(1-\alpha-\beta)}\right)^{\mu}(\tau\delta n^s)^{\mu-1}\left(\frac{b_{t-1}}{h_{t-1}}\right)^{1-\mu}\right]}{\left[\mu + (1-\mu)(\alpha+\beta)\right]} \quad (22)$$

Let, $\frac{b_t}{h_t} = Z_t$; so, $\frac{b_{t-1}}{h_{t-1}} = Z_{t-1}$ (23)

Therefore, $Z_t = \frac{\beta\left[(1-\tau)\delta + (1+r)\frac{1}{\theta}\left(\frac{\beta}{\mu(1-\alpha-\beta)}\right)^{\mu}(\tau\delta n^s)^{\mu-1}(Z_{t-1})^{1-\mu}\right]}{\left[\mu + (1-\mu)(\alpha+\beta)\right]}$ (24)

Now, $\frac{dZ_t}{dZ_{t-1}} = \frac{\beta}{\left[\mu + (1-\mu)(\alpha+\beta)\right]}(1+r)\left(\frac{1}{\theta}\right)\left(\frac{\beta}{\mu(1-\alpha-\beta)}\right)^{\mu}(\tau\delta n^s)^{\mu-1}(1-\mu)Z_{t-1}^{-\mu} > 0$ and

$$\frac{d^2Z_t}{dZ_{t-1}^2} = (-)\frac{\beta\mu(1+r)}{\left[\mu + (1-\mu)(\alpha+\beta)\right]}\left(\frac{1}{\theta}\right)\left(\frac{\beta}{\mu(1-\alpha-\beta)}\right)^{\mu}(\tau\delta n^s)^{\mu-1}(1-\mu)Z_{t-1}^{-\mu-1} < 0$$

So, $Z_t = f(Z_{t-1})$ is an upward rising concave function which is drawn below in figure 2. $Z_t = f(Z_{t-1})$ intersects the 45 degree line, generating a stable equilibrium at Z*, which is the steady state equilibrium value of Z_t. Any value of Z_{t-1}, different from Z*(higher or lower) will eventually converge to Z*.

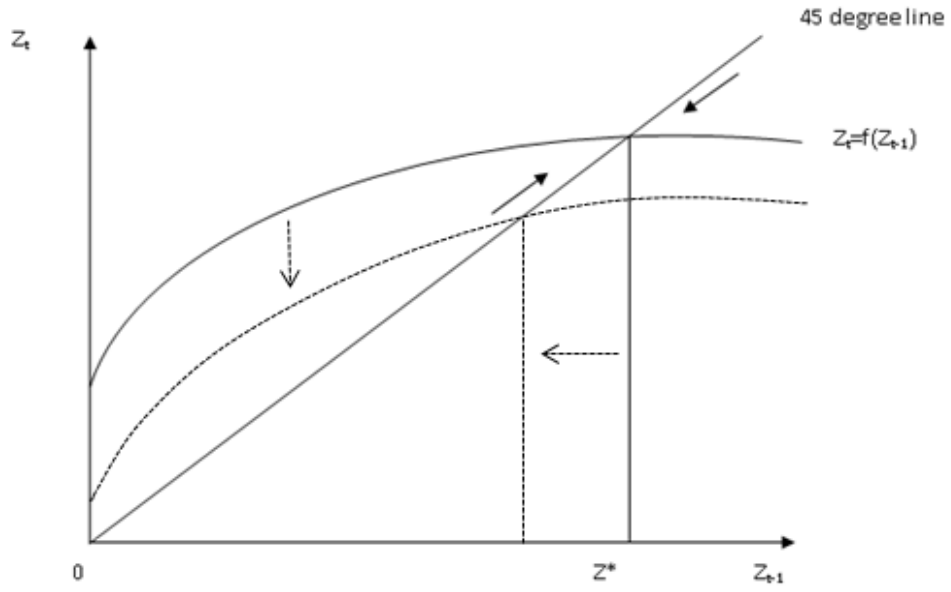

Figure 2: wealth dynamics of the skilled sector under the co-existence of public and private education system

From equation (24), we have $Z_t = \dfrac{\beta\left[(1-\tau)\delta + (1+r)\dfrac{1}{\theta}\left(\dfrac{\beta}{\mu(1-\alpha-\beta)}\right)^{\mu}(\tau\delta n^s)^{\mu-1}(Z_{t-1})^{1-\mu}\right]}{[\mu+(1-\mu)(\alpha+\beta)]}$

So, when $\mu = 0$, $Z_t = \dfrac{\beta\left[(1-\tau)\delta + (1+r)\dfrac{1}{\theta}(\tau\delta n^s)^{-1}(Z_{t-1})\right]}{(\alpha+\beta)}$

And when $\mu = 1$, $Z_t = \beta\left[(1-\tau)\delta + \dfrac{\beta(1+r)}{\theta\mu(1-\alpha-\beta)}\right]$ (25)

This implies that, unlike under complete private education the bequest to human capital ratio depends on its last period value under complete public education.

So we get a unique steady state value of $\dfrac{b_t}{h_t}$, $\left(\dfrac{b}{h}\right)^*_s$ for the skilled agents.

Equation (24) implies that

$$\frac{dZ_t}{d\tau}=\frac{\beta}{\left[\mu+(1-\mu)(\alpha+\beta)\right]}\left[-\delta+\frac{(1+r)}{\theta}\left(\frac{\beta}{\mu(1-\alpha-\beta)}\right)^{\mu}\left(\delta n^{s}\right)^{\mu-1}(\mu-1)\tau^{\mu-2}Z_{t-1}^{1-\mu}\right]<0$$

For an unskilled agent, we can obtain the value of $\left(\frac{b_u^*}{h_u^*}\right)$ by dividing equation (12) by equation (13).

This gives us $\left(\frac{b_u^*}{h_u^*}\right)=\frac{\beta}{\theta^{\frac{1}{\mu}}\left(\tau\delta n^{s}\right)^{\frac{1-\mu}{\mu}}\mu(1-\alpha-\beta)}$

And $\frac{d\left(\frac{b_u^*}{h_u^*}\right)}{d\tau}=\frac{b_u^*}{h_u^*}\left[\frac{-(1-\mu)}{\tau\mu}\right]<0$

So,. Since the unskilled agents have fixed incomes, which do not depend on their human capital, their bequests also are independent of level of human capital. On the other hand, skilled individuals' income and bequests depend on their human capital. Hence we get that in an education system where public and private educations co-exist, the ratio of wealth to human capital depends on its last period value only for the skilled agents but, for the unskilled agents this ratio is a constant.

Proposition 1: *With the imposition of income tax, the steady- state human capital per unit of inheritance increases not only for the unskilled labours, but also for the skilled labours.*

In other words, public education system, through tax imposition, reduces the role of inheritance in education system. This is shown in figure 2. When the tax rate is increased, as $\frac{dZ_t}{d\tau} < 0$, the $Z_t = f(Z_{t-1})$ line shifts to downward position lowering the steady state value of Z_t.

Next, we study the two benchmark cases ($(\mu = 0)$and $(\mu = 1)$) separately to get a clearer overview of the long run economic performances in our framework.

2.2. Private Education System (μ=1):

2.2.1. Competitive Equilibrium under Private Education system

The optimization problem of a representative individual employed in the unskilled sector in an economy under private education system is given by

$$\max_{\{X_t, b_t, e_t\}} \alpha \ln X_t + \beta \ln b_t + (1-\alpha-\beta) \ln h_{t+1}$$

subject to

$$\underline{w} + (1+r)b_{t-1} = X_t + b_t + e_t$$

and

$$h_{t+1} = \theta e_t$$

From the first order conditions we find the equilibrium values:

$$b_t = \beta [\underline{w} + (1+r)b_{t-1}], \quad (26)$$

$$X_t = \alpha [\underline{w} + (1+r)b_{t-1}], \quad (27)$$

$$e_t = (1-\alpha-\beta)\,[\,\underline{w} + (1+r)b_{t-1}] \qquad (28)$$

Similarly, a representative skilled labour, under private education system solves the following maximization problem

$$\max_{\{X_t, b_t, e_t\}} \alpha \ln X_t + \beta \ln b_t + (1-\alpha-\beta) \ln h_{t+1}$$

subject to $\delta h_t + (1+r)b_{t-1} = X_t + b_t + e_t$

$$h_{t+1} = \theta e_t$$

From the first order conditions of the maximization problem we find the optimum values:

$$X_t = \alpha\,[\,\delta h_t + (1+r)b_{t-1}] \qquad (29)$$

$$b_t = \beta\,[\,\delta h_t + (1+r)b_{t-1}] \qquad (30)$$

$$e_t = (1-\alpha-\beta)\,[\,\delta h_t + (1+r)b_{t-1}] \qquad (31)$$

From the 1st order conditions of both the optimization problems under private education system, we find that consumption of basic goods, investment on child's education and amount of bequest left for the next generation depend on own income and inheritance received by both skilled and unskilled adult individuals.

Hence, the equilibrium law of motion for human capital can be obtained as follows.

Under **private education system**, for unskilled labour, $h_{t+1} = \theta\,(1-\alpha-\beta)\,[\underline{w} + (1+r)b_{t-1}]$ (32)

and for skilled labour, $h_{t+1} = \theta\,(1-\alpha-\beta)\,[\delta h_t + (1+r)b_{t-1}]$ (33)

2.2.2. Bequest Dynamics under Private Education System

Now, we look at the *bequest decisions* under competitive equilibrium outcomes.

Now, from equation (32) we have, $h_t = \theta(1-\alpha-\beta)\left[\underline{w} + (1+r)b_{t-2}\right]$

Also, from equation (26) we have $b_{t-1} = \beta\left[\underline{w} + (1+r)b_{t-2}\right]$

Therefore, for an unskilled labour we have $\frac{h_t}{b_{t-1}} = \frac{\theta(1-\alpha-\beta)\left[\underline{w} + (1+r)b_{t-2}\right]}{\beta\left[\underline{w} + (1+r)b_{t-2}\right]} = \frac{\theta(1-\alpha-\beta)}{\beta}$

(34)

Similarly, using equation (33), for a skilled labour we have

$h_t = \theta(1-\alpha-\beta)\left[\delta h_{t-1} + (1+r)b_{t-2}\right]$ and from equation (30) we have $b_{t-1} = \beta\left[\delta h_{t-1} + (1+r)b_{t-2}\right]$

Therefore, again we have, $\frac{h_t}{b_{t-1}} = \frac{\theta(1-\alpha-\beta)}{\beta}$ (35)

Equations (34) and (35) imply that for both skilled and unskilled labours under private education system, human capital is directly proportional to the inheritance received.

We have assumed that all the children go to school, but not all can accumulate the skill which is required to join the skilled sector of the economy. Human capital accumulation function, in our model isdirectly dependent on the efficiency of the education system or the school quality, but the above two equations show that it also directly depends on the amount of inheritance received.

From equations (34) and (35)even though the ratio of h_t and b_{t-1}, are the same, the bequest equations will be different.

From equations (30) and (35), we have,

$$b_t = \beta\left[\frac{\delta.\theta.(1-\alpha-\beta)}{\beta} b_{t-1} + (1+r)b_{t-1}\right] = \left[\delta\theta(1-\alpha-\beta) + (1+r)\beta\right]b_{t-1} \quad (36)$$

So, we have the laws of motion for bequest equations under **private education system** given by:

For unskilled labours: $b_t = \beta\left[\underline{w} + (1+r)b_{t-1}\right]$

and for skilled labours : $b_t = \left[\delta\theta(1-\alpha-\beta) + (1+r)\beta\right]b_{t-1}$

Note that, for an unskilled labour, there is an intercept term in bequest function, but it is absent in the bequest function for a skilled labour. Also note that, slope of bequest function for skilled labour is always higher than that of an unskilled labour.

Now, an individual will choose to work as a skilled labour when the following incentive constraint is satisfied, i.e., when $\underline{w} + (1+r)b_{t-1} \leq \delta h_t + (1+r).b_{t-1}$

This gives us the threshold skill level, $h_t \geq \frac{\underline{w}}{\delta} = \bar{h}$

So, the condition for a young individual to join the skilled labour force is given by $h_{t+1} \geq \frac{\underline{w}}{\delta}$

Using equation (32) to substitute for h_{t+1} we have, $\theta(1-\alpha-\beta)[\underline{w} + (1+r)b_{t-1}] \geq \frac{\underline{w}}{\delta}$

The above inequality implies that, $b_{t-1} \geq \frac{\underline{w}[1-\delta\theta(1-\alpha-\beta)]}{\delta\theta(1-\alpha-\beta)(1+r)} = d$

This is the minimum level of parental inheritance required to make one's child a skilled labour on becoming an adult. This relation shows that higher the unskilled wage, higher is the inheritance needed to make one's child join the skilled sector and the more efficient the education system, lower is the inheritance needed to make one's child a skilled labour. Also, a lower preference of the parent towards schooling quality implies a higher inheritance needed to make her child a skilled labour.

We can now draw the diagram of the bequest dynamics for this education system.

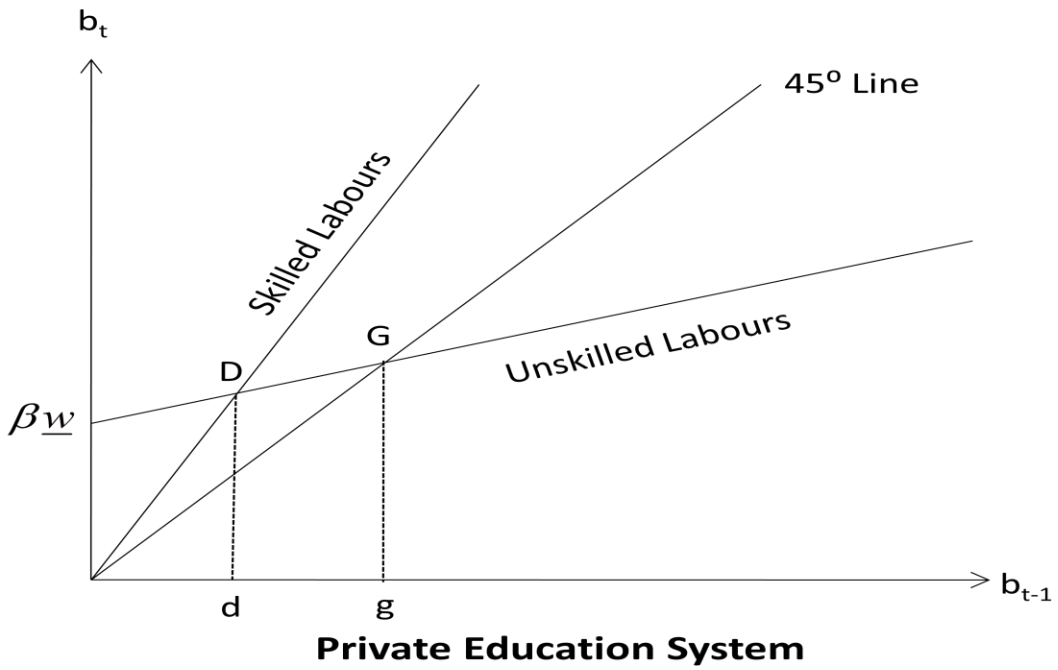

Figure 3*: Bequest lines for skilled labour and unskilled labour in the private education system*

To draw the above figure we assume $\left[\delta\theta(1-\alpha-\beta)+\beta(1+r)\right]>1$

Since the skilled labours' bequest line is a ray from the origin and lies above the 45 degree line, we do not get any equilibrium for the skilled labours implying that bequest of skilled individual will keep growing steadily across generations. If the above mentioned assumption is violated, skilled labours' bequest line will lie below 45 degree line and equilibrium bequest of skilled labour would be zero and this is much below than the equilibrium bequest of unskilled labour! To get rid of this absurd possibility we assume $\left[\delta\theta(1-\alpha-\beta)+\beta(1+r)\right]>1$

This diagram has two crucial points of intersection: D and G. D is the point where the skilled and unskilled labours' bequest lines intersect each other and G is the unskilled labours' equilibrium which is stable. 'g' is the equilibrium value of inheritance received for the unskilled labour, corresponding to G. The value of inheritance received corresponding to D is obtained by equating equations (26) and (30). This is the minimum level of inheritance required to become a skilled labour ('d').So, the value b_{t-1} that corresponds to D, is 'd'.

Now, the equilibrium bequest of unskilled labours-the value of 'g', is computed by equating b_t and b_{t-1} in equation (26). This gives us

$$g = \frac{\beta \underline{w}}{1-\beta(1+r)}$$

We assume this equilibrium bequest of unskilled labour is positive. For this, we require the condition $\beta(1+r)<1$. To see whether 'd' lies to the left or right of 'g', we need to consider the difference.

$$(g-d) = \frac{\underline{w}[\delta\theta(1-\alpha-\beta)+\beta(1+r)-1]}{[1-\beta(1+r)][\delta\theta(1-\alpha-\beta)(1+r)]}$$

Since, $[\delta\theta(1-\alpha-\beta)+\beta(1+r)]>1$ and $\beta(1+r)<1$ 'd' lies to the left of 'g'.

An individual will be on the skilled labours' bequest line when her parent is employed as a skilled labour. Since 'd' is the point beyond which the return from being a skilled labour is higher than that from being an unskilled labour, parents having bequest more than 'd' will join the skilled labour force. This implies that from the point 'd', all the individuals will be on the skilled labours' bequest line, enjoying a higher income. So, if the inheritance received (b_{t-1}) by any family is at and beyond the point, 'd' , then only an adult individual is willing to pay for a quality of schooling good enough for her child to accumulate the threshold skill level and join skilled sector. Only then, will their subsequent generations experience steady growth. Everyone with a lower amount of inheritance received will remain on the unskilled labours' bequest line, temporarily. From the dynamics, it is observed that, in the long-run, they will also join the skilled labour force and experience steady-growth[6]. But note that equilibrium G is never attained by anyone since beyond point 'd' everyone will be on skilled labours' bequest line.

This leads to our first proposition.

Proposition 2: *In the private education system, everyone in the economy will eventually work as skilled labour in the long run and face steady growth no matter what their inheritance level is.*

[6] If subsistence consumption is introduced in the utility function, private education system may generate low level equilibrium trap.

2.3. Public Education System (μ=0):

2.3.1. Decentralized Equilibrium under Public Education system

In the public education system, the incomes of the skilled labours are taxed but the unskilled labours are exempted from taxation.

The quality of schooling to which every child has access becomes: $E_t = \frac{\tau_t \delta H_t}{N}$,

where, τ_t is the tax rate imposed by the government at the t[th] period and $\tau_t.\delta.H_t$ is the total tax revenue collected at t[th] period. Please note that, if there is no skilled individual in the economy, tax revenue earned by the government is zero and consequently, schooling quality and human capital accumulation are zero under public education system.

So, under this system, an unskilled labour faces the budget constraint: $\underline{w} + (1+r)b_{t-1} = X_t + b_t$

Therefore, the optimization problem of an unskilled labour under public education system is given by:

$$\max_{\{X_t, b_t\}} \alpha \ln X_t + \beta \ln b_t + (1 - \alpha - \beta) \ln (h_{t+1})$$

subject to

$\underline{w} + (1+r)b_{t-1} = X_t + b_t$ and

$h_{t+1} = \theta E_t = \frac{\theta \tau_t \delta H_t}{N}$

From the first order conditions of the optimization problem we obtain the equilibrium values:

$$b_t = \frac{\beta [\underline{w} + (1+r)b_{t-1}]}{(\alpha + \beta)} \quad (37)$$

$$X_t = \frac{\alpha [\underline{w} + (1+r)b_{t-1}]}{(\alpha + \beta)} \quad (38)$$

The representative skilled labour faces the budget constraint: $(1-\tau_t)\delta h_t + (1+r)b_{t-1} = X_t + b_t$

Hence, the skilled labour solves the following problem:

$$\max_{\{X_t, b_t\}} \alpha \ln X_t + \beta \ln b_t + (1 - \alpha - \beta) \ln h_{t+1}$$

subject to $h_{t+1} = \theta\tau_t \, \delta \, H_t / N$

and $(1-\tau_t)\delta h_t + (1+r)b_{t-1} = X_t + b_t$

From the optimality conditions the following optimum choices are found out:

$$X_t = \frac{\alpha \, [\,(1-\tau_t)\delta . h_t + (1+r)b_{t-1}\,]}{(\alpha+\beta)} \qquad (39)$$

$$b_t = \frac{\beta \, [\,(1-\tau_t)\delta . h_t + (1+r)b_{t-1}\,]}{(\alpha+\beta)} \qquad (40)$$

Following these equilibrium choices, we can have the equilibrium law of motion for human capital. Under **public education system**, for both skilled and unskilled workers:

$$h_{t+1} = \theta\tau_t \, \frac{\delta . H_t}{N} \qquad (41)$$

2.3.2. Bequest Dynamics under Public Education System

The laws of motion of bequest under **public education system** are given by equations (37) and (40):

For unskilled workers, $b_t = \frac{\beta \, [\,\underline{w} + (1+r)b_{t-1}\,]}{(\alpha+\beta)}$

and for skilled workers, $b_t = \frac{\beta\,[\,(1-\tau_t)\delta h_t + (1+r)b_{t-1}\,]}{(\alpha+\beta)}$

Under this education system the human capital accumulation function is, $h_{t+1} = \theta E_t = \theta\tau_t \frac{\delta H_t}{N}$ so,

$h_t = \theta\tau_{t-1}\frac{\delta . H_{t-1}}{N}$

We have, $\frac{H_t}{h_t} = \frac{H_t N}{\theta\tau_{t-1}\delta H_{t-1}}$

And $\frac{h_{t+1}}{h_t} = \theta\tau_t \frac{\delta H_t}{N h_t} = \theta\tau_t \frac{\delta}{N}\frac{H_t N}{\theta\tau_{t-1}\delta H_{t-1}} = \frac{H_t}{H_{t-1}}$,

when uniform tax rate is imposed over time.

Again, $h_{t+1} = \theta\tau_t \delta H_t = \theta\tau_t \frac{X_t^{\,s}}{N} = \frac{\theta\tau_t \delta N^s h_t}{N} = \theta\tau_t \delta h_t \frac{N^s}{N}$,

Where, N^s is the number of skilled labours in the economy.

Then, $h_t = \theta\tau_{t-1}\delta h_{t-1}\frac{N^s}{N}$

Therefore, $\frac{h_t}{h_{t-1}} = \theta\tau_{t-1}\delta\frac{N^s}{N} = \frac{H_t}{H_{t-1}}$

So, when $\tau_t = \tau_{t-1}$, i.e., uniform tax rate is imposed across time, the human capital growth rate is identical for all individuals in the society and is independent of b_t and h_t.

Proposition 3: *In the public education system, human capital growth rate is identical for all the individuals of the society and is independent of individual bequest (b_t) and parental human capital (h_t).*

When N^s/N is independent of t then $h_t = \left(\theta\tau_{t-1}\delta\frac{N^s}{N}\right)^t h_0$ and

$$H_{t+1} = \left(\frac{N^s}{N} \theta \tau_t \delta \right)^{t+1} H_0$$

Hence, $g = \frac{N^s}{N} \theta \tau_t \delta$,where g is the growth rate of aggregate as well as individual human capital under public education regime.

Thus, under public education system the bequest dynamics of a skilled labour is $b_t = \frac{\beta}{(\alpha+\beta)} \left[(1-\tau_t)\delta \left(\theta \tau_t \delta \frac{N^s}{N} \right)^t h_0 + (1+r) b_{t-1} \right]$ and that of an unskilled labour is $b_t = \frac{\beta}{(\alpha+\beta)} \left[\underline{w} + (1+r) b_{t-1} \right]$.

As in the case of private education, here also we can compute the threshold level of skill required to join the skilled sector. Under public education system, the threshold level of skill required to become a skilled labour is: $h_t \geq \frac{\underline{w}}{(1-\tau_t)\delta} = \overline{h_1}$

So, the minimum level of skill, that the next generation needs to accumulate, is $h_{t+1} \geq \frac{\underline{w}}{(1-\tau_{t+1})\delta}$

(42)

Now, $h_{t+1} = \theta \tau_t n^s \delta h_t$, where $n^s = N^s / N$

Then, from equation (42): $\theta \tau_t n^s \delta h_t \geq \frac{\underline{w}}{(1-\tau_{t+1})\delta}$

So, under public education system, this relation does not depend on the inheritance received by the individuals. This implies that either all the individuals can accumulate the threshold skill level and end up being skilled labours or no one can accumulate that level of skill and every one remains unskilled.

We can now draw the diagram describing the bequest dynamics of public education system.

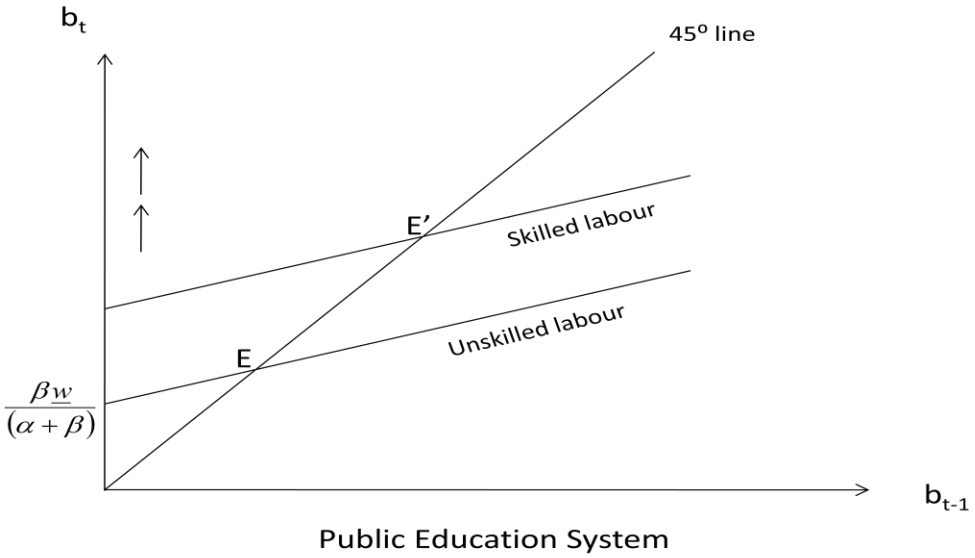

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Figure 4*: Bequest lines for skilled labour and unskilled labour in the public education system*

In this case, we have two stable equilibria, E', for the skilled labours and E, for the unskilled labours. But, under this system, the entire population will be either unskilled or skilled. If all the individuals can accumulate the threshold skill level that is required to become a skilled labour,(independent of b_{t-1}), then the economy will reach the skilled labours' equilibrium; otherwise, it will reach the bad equilibrium E. In the second case, the economy will face stagnation.

Since the bequest equation of the skilled labours, in the public education system, contains a time component in the intercept term, the skilled labours' bequest line will keep moving in the upward direction with time. So, as time increases, the economy under public education system with all its skilled individuals faces continuous development.

So, under public education system,

if $\frac{\beta}{(\alpha+\beta)}(1+r,)>1$ then there will be throughout growth of bequest and

if $\frac{\beta}{(\alpha+\beta)}(1+r)<1$, then there will be stagnation, but the skilled labours will face a continuous growth at each time point as their bequest line will keep moving upward as time increases.

For the first case, when the individuals have relatively higher elasticity of utility with respect to bequest compared to that with respect to consumption, public education may be recommended for any economy.

But, for the second case, if the entire population works in the unskilled sector under public education system, then private education is a better choice than public education. If the entire population works in the skilled sector, then the choice between private education and public education may be ambiguous.

Proposition 4: *In the public education system, either all the individuals work as skilled labours or all are unskilled labours. When the individuals do not derive much satisfaction from leaving a bequest for the next generation and the rate of interest is low* $\left(\frac{\beta}{(\alpha+\beta)}(1+r)>1\right)$ *then if all the individuals are employed in the skilled sector, the economy faces continuous development but if all the individuals work in the unskilled sector, the economy faces stagnation and there exist unique equilibrium bequest. When the individuals derive more satisfaction from leaving a bequest for the next generation and the rate of interest is sufficiently high* $\left(\frac{\beta}{(\alpha+\beta)}(1+r)<1\right)$ *then there does not exist any equilibrium and there will be throughout the growth of bequest, no matter all the labours work in the skilled sector or unskilled sector. However, the human capital accumulation remains halted in both the cases when all the labours work in the unskilled sector.*

So, following this analysis, we may say that, under private education system, initially, the individuals will work as skilled labours if their parental bequest is higher than the point 'd'. All the children of the unskilled labours having inheritance less than 'd' will work as unskilled labours until 'd' is reached. In the long run, they will enjoy the growth in bequest and they will also work as skilled labours. The children of the skilled labours with higher inheritance will join the skilled population and will experience steady growth in wealth. Thus, under private education system, the only non-trivial equilibrium is in the long run where everyone works as a skilled labour and they face steady growth. On the contrary, under public education system, two situations may emerge-either all the individuals work as skilled labours or all individuals work as unskilled labours. When all the individuals work as unskilled labours, the bequest may grow over

time or stagnate, but, the human capital accumulation remains halted. When all the individuals work as skilled labours, bequest and human capital both grow over time.

Under public education system, either all the individuals experience stagnation or all the individuals experience growth. Growth of the economy depends crucially on the term $\frac{\beta}{(\alpha+\beta)}(1+r)$

It is the slope of the bequest lines for both skilled and unskilled labours being greater than unity. This indicates that the preference towards bequest of the old individuals and the rate of interest earned on the inheritance received should be high enough to make the term greater than unity in order for the economy to achieve throughout growth. Otherwise, the economy will experience stagnation. So, in our model, there is a chance that the public education system ends up stuck at the unskilled labours' equilibrium. But if the economy can recover from the situation and make all the individuals skilled enough, it will be able to enjoy a significantly better and developed economic state with less inequality where all the individuals are educated, skilled and experiencing continuous growth.

Next, we find out the choice of education system by majority voting.

3. Choice of Education System (Private vs. Public)

In this section, we aim to find out the choice of education by the skilled and unskilled individuals of the economy by comparing their indirect utilities from the two systems to see who is better off under which system. First, we look at the private education system.

3.1. Indirect utilities under private education system:

Indirect utility is obtained by substituting the equilibrium values of the choice variables, X_t, b_t and e_t into the utility function given by equation (1).

Substituting X_t, b_t and e_t from equations (26), (27), (28) into the utility function given by (1) and using equation (2), we obtain the *unskilled labour's indirect utility under private education system* $(U_n^{\,p})$.

$$U_n^{\,p}$$

$$= \alpha \ln[\alpha\{\underline{w} + (1+r)b_{t-1}\}] + \beta \ln[\beta\{\underline{w} + (1+r)b_{t-1}\}] + (1-\alpha-\beta)\ln[\theta(1-\alpha-\beta)\{\underline{w} + (1+r)b_{t-1}\}]$$

$$= \varepsilon + \ln[\underline{w} + (1+r)b_{t-1}],$$

where, $\varepsilon = (\ln \alpha^{\alpha} + \ln \beta^{\beta} + \ln(1-\alpha-\beta)^{(1-\alpha-\beta)} + \ln \theta^{(1-\alpha-\beta)})$

Similarly, we substitute X_t, b_t and e_t from equations (29), (30) and (31) into the utility function and use equation (2) to obtain the *skilled labour's indirect utility under private education system* $(U_s^{\,p})$.

Thus,

$$U_s^{\,p} = \alpha \ln[\alpha\{\delta h_t + (1+r)b_{t-1}\}] + \beta \ln[\beta\{\delta h_t + (1+r)b_{t-1}\}] + (1-\alpha-\beta)\ln[\theta(1-\alpha-\beta)\{\delta h_t + (1+r)b_{t-1}\}]$$

$$= \varepsilon + \ln[\delta h_t + (1+r)b_{t-1}]$$

where ε is same as above.

3.2. Indirect utilities under public education system:

Under this education system, school quality is financed by tax revenues, for which the skilled labours are required to pay the taxes. We compute a tax rate that is preferred by a representative

skilled labour. This preferred tax rate is denoted by $\tilde{\tau}_t$,and is obtained by maximizing the indirect utility of the skilled labour under this system $\left(U_s^{\bar{p}}\right)$ with respect to the tax rate, τ_t .[7]

So, $\tilde{\tau}_t = \arg\max U_s^{\bar{p}}$

Now, $U_s^{\bar{p}}$ is obtained by substituting X_t and b_t from equations (39) and (40) in equation (1), where $h_{t+1} = \theta E_t = \frac{\theta \tau_t \delta H_t}{N}$

We get: $U_s^{\bar{p}}$

$$= \alpha \ln\left[\frac{\alpha\{(1-\tau_t)\delta h_t + (1+r)b_{t-1}\}}{(\alpha+\beta)}\right] + \beta \ln\left[\frac{\beta\{(1-\tau_t)\delta h_t + (1+r)b_{t-1}\}}{(\alpha+\beta)}\right] + (1-\alpha-\beta)\ln\left[\theta\tau_t \frac{\delta H_t}{N}\right]$$

(43)

So, $\tilde{\tau}_t = \arg\max U_s^{\bar{p}}$ implies that $\tilde{\tau}_t = \frac{(1-\alpha-\beta)\{\delta h_t + (1+r)b_{t-1}\}}{\delta h_t}$ (44)

Now, the skilled labour's indirect utility at the tax rate $\tilde{\tau}_t$ is obtained by substituting $\tilde{\tau}_t$ from equation (44) into equation (43) which gives:

$$U_s^{\bar{p}} = \varepsilon + \ln[\delta h_t + (1+r)b_{t-1}] + (1-\alpha-\beta)\ln(n^s)$$

The unskilled labours are not tax payers and for them, any tax rate is exogenously given. So, for any tax rate τ_t, their indirect utility is derived by substituting X_t and b_t from equations (37) and (38) into the utility function given by equation (1) where $h_{t+1} = \theta E_t = \frac{\theta \tau_t \delta H_t}{N}$

This gives us the indirect utility of the unskilled labour under public education system, $U_n^{\bar{p}}$

[7] This preferred tax rate can also be derived from the skilled labour's utility maximization problem considering τ_t as a choice variable, i.e., by solving the exercise: $\max_{\{X_t, b_t, \tau_t\}} \alpha \ln X_t + \beta \ln b_t + (1-\alpha-\beta)\ln(h_{t+1})$ subject to $h_{t+1} = \theta\tau_t \delta H_t / N$ and $(1-\tau_t)\delta h_t + (1+r)b_{t-1} = X_t + b_t$

$$= \alpha \ln\left[\frac{\alpha\{\underline{w}+(1+r)b_{t-1}\}}{(\alpha+\beta)}\right] + \beta \ln\left[\frac{\beta\{\underline{w}+(1+r)b_{t-1}\}}{(\alpha+\beta)}\right] + (1-\alpha-\beta)\ln\left[\frac{\theta\tau_t \delta H_t}{N}\right]$$

$$= \varepsilon - \ln(1-\alpha-\beta)^{(1-\alpha-\beta)} - \ln(\alpha+\beta)^{(\alpha+\beta)} + (\alpha+\beta)\ln[\underline{w}+(1+r)b_{t-1}] + (1-\alpha-\beta)\ln\tau_t + (1-\alpha-\beta)\ln(n^s) + (1-\alpha-\beta)\ln(\delta h_t)$$

Now, we compare the indirect utilities under the two education systems for both the skilled and the unskilled labours to find out the preferences of the individuals towards the education system.

The net indirect utility of a skilled labour defined by the indirect utility under public education system at her own preferred tax rate, $\tilde{\tau}_t$ over the indirect utility under private education system is denoted by Γ^s, where $\Gamma^s = U_s^{\bar{p}} - U_s^{p}$

$$= \varepsilon + \ln[\delta h_t + (1+r)b_{t-1}] + (1-\alpha-\beta)\ln(n^s) - \varepsilon - \ln[\delta h_t + (1+r)b_{t-1}]$$

$$= (1-\alpha-\beta)\ln(n^s)$$

Since n^s is the proportion of skilled labours in the total population, $\Gamma^s \leq 0$

So, $U_s^{\bar{p}} \leq U_s^{p}$, which implies that a skilled labour will always choose private education system, under which she does not have to pay any tax at all.

Proposition 5: *A skilled labour will always choose the private education system.*

Under public education system, skilled labours are taxed and this taxation reduces their real income. The tax revenues are used to finance the education of not their children only, but, all the children (including the children of the unskilled labours) of the economy. Instead of this, if the skilled labours were to pay for their children's education only, as in the case under private education system, then their children could have a better quality education.

Since the human capital of the offspring comes into the parents' preference, skilled labours' indirect utility declines after taxation. So, the skilled labours will always vote for the private education system.

The net indirect utility of an unskilled labour is denoted by Γ^n , which is the indirect utility of an unskilled labour under public education system over that under private education system.

$$\Gamma^n = U_n^{\bar{p}} - U_n^{p}$$

$$= \varepsilon - \ln(1-\alpha-\beta)^{(1-\alpha-\beta)} - \ln(\alpha+\beta)^{(\alpha+\beta)} + (\alpha+\beta)\ln[\underline{w} + (1+r)b_{t-1}] + (1-\alpha-\beta)\ln\tau_t + (1-\alpha-\beta)\ln(n^s) + (1-\alpha-\beta)\ln(\delta h_t) - \varepsilon - \ln[\underline{w} + (1+r)b_{t-1}]$$

$$= (1-\alpha-\beta)\ln\left[\frac{(n^s \tau_t \delta h_t)}{(1-\alpha-\beta)\{\underline{w} + (1+r)b_{t-1}\}}\right] - \ln(\alpha+\beta)^{(\alpha+\beta)}$$

$$= (1-\alpha-\beta)\ln\left(\frac{E_t}{e_t}\right) - \ln(\alpha+\beta)^{(\alpha+\beta)} = \ln\left[\frac{\left(\frac{E_t}{e_t}\right)^{(1-\alpha-\beta)}}{(\alpha+\beta)^{(\alpha+\beta)}}\right]$$

An unskilled labour will prefer the public education system when $\Gamma^n \geq 0$

i.e., if $\left(\frac{E_t}{e_t}\right)^{(1-\alpha-\beta)} \geq (\alpha+\beta)^{(\alpha+\beta)}$ which implies $E_t \geq (\alpha+\beta)^{\frac{(\alpha+\beta)}{(1-\alpha-\beta)}} e_t$

Substituting the value of e_t from equation (28), we get

$$\frac{E_t}{(\alpha+\beta)^{\frac{(\alpha+\beta)}{(1-\alpha-\beta)}}} \geq (1-\alpha-\beta)[\underline{w} + (1+r)b_{t-1}] \text{ which implies}$$

$$b_{t-1} \leq \frac{E_t - (1-\alpha-\beta)(\alpha+\beta)^{\left(\frac{\alpha+\beta}{1-\alpha-\beta}\right)}\underline{w}}{(1-\alpha-\beta)(\alpha+\beta)^{\left(\frac{\alpha+\beta}{1-\alpha-\beta}\right)}(1+r)} = \text{j}$$

j is that critical point of inheritance received below which the unskilled labours will choose public education system.

Proposition 6: *An unskilled labour will vote for the public education system if her inheritance received is below a critical level.*

We assume a Pareto distribution of wealth. Therefore, b_{t-1} follows a Pareto distribution with the density function given by $f(x)=\frac{\lambda m^{\lambda}}{x^{\lambda+1}}$; $x \geq m, \lambda > 0$, where λ is the Pareto inequality parameter.[8] Higher the value of λ, higher is the number of people having lower wealth levels.

The cumulative density function is given by $F(x)=\int_{m}^{x} \frac{\lambda m^{\lambda}}{X^{\lambda+1}} dX = 1-\left(\frac{m}{x}\right)^{\lambda}$

From this, we obtain the number of unskilled workers with the amount of inheritance less than j in an economy with N individuals and with inequality λ and that is given by

$$NF(j) = N\left[1-\left(\frac{m}{j}\right)^{\lambda}\right], \text{j} \geq \text{m}$$

$$= N\left[1-\left(\frac{m(1-\alpha-\beta)(\alpha+\beta)^{\frac{(\alpha+\beta)}{(1-\alpha-\beta)}}(1+r)}{E_t - \underline{w}(1-\alpha-\beta)(\alpha+\beta)^{\frac{(\alpha+\beta)}{(1-\alpha-\beta)}}}\right)^{\lambda}\right]$$

This is the number of unskilled labours who prefer public education system. The rest- the ones with higher amount of inheritance and all of the skilled labours will choose private education system.

The number of these individuals, preferring private education system, can be computed as below:

$$\left[\left(N-N^{s}\right)-NF(j)\right]+N^{s} = N\left[1-F(j)\right]$$

$$= N\left(\frac{m(1-\alpha-\beta)(\alpha+\beta)^{\frac{(\alpha+\beta)}{(1-\alpha-\beta)}}(1+r)}{E_t - \underline{w}(1-\alpha-\beta)(\alpha+\beta)^{\frac{(\alpha+\beta)}{(1-\alpha-\beta)}}}\right)^{\lambda}$$

[8] Following Roy Chowdhuri (2012)

Public education system will be the outcome of majority voting in this economy when

$$NF(j) \geq N(1-F(j))$$

i.e., when $F(j) \geq \frac{1}{2}$, which implies $\left[1-\left(\frac{m(1-\alpha-\beta)(\alpha+\beta)^{\frac{(\alpha+\beta)}{(1-\alpha-\beta)}}(1+r)}{E_t-\underline{w}(1-\alpha-\beta)(\alpha+\beta)^{\frac{(\alpha+\beta)}{(1-\alpha-\beta)}}}\right)^{\lambda}\right] \geq \frac{1}{2}$

so, $\frac{m(1-\alpha-\beta)(\alpha+\beta)^{\frac{(\alpha+\beta)}{(1-\alpha-\beta)}}(1+r)}{E_t-\underline{w}(1-\alpha-\beta)(\alpha+\beta)^{\frac{(\alpha+\beta)}{(1-\alpha-\beta)}}} \leq \left(\frac{1}{2}\right)^{\frac{1}{\lambda}}$ (45)

If the above inequality holds, then the majority of the economy votes for public education system.

λ is the Pareto inequality parameter of the economy with a Pareto distribution of wealth. So, a higher value of λ indicates a higher concentration of the less wealthy people, who will vote in favour of the public education system. From the above relation it is clear that as the inequality in the economy increases, the right hand side term in (45) goes up and it becomes more likely that the public education system will win in vote.

A higher value of the rate of interest (r), which is earned on the amount of inheritance received, indicates a higher amount of the total wealth an individual possesses. This tends to enable a person to afford private education for her child. So, a high value of r makes it difficult for the inequality (45) to hold by making the political equilibrium inclined towards private education, which most of the individuals in the society might prefer and can afford now.

An increase in the unskilled wage $(\underline{w})$ raises the term in the left hand side of the inequality (45) and therefore, makes it harder to hold. This is an obvious fact that a higher wage increases affordability, so, a raise in the wage of an unskilled labour raises her willingness to pay for her child's education and attracts her to the lucrative menu of private schools with various qualities.

An increase in E_t implies a better quality of public schooling for every child and it decreases the left hand side term of the inequality (45) making it easier to hold. Improvement of the quality of public schools makes it obvious for the parents to choose public education system. The parents'

preferences play an important role in determining this majority voting outcome. An increase in the term $(\alpha+\beta)$ lowers the left hand side of inequality (45) further. This shows that if the share of consumption or bequest or both in the parent's utility, increases, then it is more likely that the parents will go for public education. This is because of the fact that a high value of $(\alpha+\beta)$ implies a low value of $(1-\alpha-\beta)$, which is the share of human capital of the child in a parent's preference function.

We finally obtain our last proposition:

Proposition 7: *An economy with an unequal wealth distribution, a high value of the inequality parameter, a low rate of interest, a low unskilled wage, an improved quality of public education and the parents with low weights assigned to human capital of their offspring in their preference functions, will have the public education system chosen by the majority of its population.*

The result that a high value of the inequality parameter leading to choice of public education tallies with the result obtained by Glomm and Ravikumar (1992). Glomm and Ravikumar (1992) deal with homogenous labour force. But, our paper advances the literature by computing the political equilibrium of a society comprising both skilled and unskilled individuals and including some other factors such as unskilled wage, quality of public education and shares of consumption and bequest in the preferences of the individuals. We find that these factors may be held responsible besides a high value of the inequality parameter for public education as the choice of the majority.

Our voting outcome finds that even if the tax rate imposed is her preferred one, a skilled labour will always choose the private education system. An unskilled labour will vote for the public education system if her inheritance received is below a critical level. Even if the unskilled labours pay neither the income taxes nor allocate any resource for their children's education under public education, they do not unanimously choose it. A case study conducted in Nairobi finds that some poor parents, who believe that private schools offer better quality education than public schools, make great sacrifices to place their children in private schools even at significant financial cost[9]. This accounts for the unskilled labours' choice of education for their children and

[9]http://theconversation.com/why-poor-parents-in-nairobi-choose-private-over-free-primary-schools-91084

it implies that the quality of education is a crucial factor in this study and might outweigh the income factor.

4. Conclusion

The distribution of skills and educational attainment are related to the level of income inequality in a society and educational policies have a huge impact on the income distributions[10]. The present paper studies the impact of wealth distribution on the rate of growth of human capital under different educational systems and explains how it affects the choice of educational system. The results support the idea that suitable educational policies should be part of the effective strategies to address the issue of skill generation, income inequality and acceleration of growth. We start with a framework where public and private educations co-exist. Under this set up, we find the short run equilibrium as well as the long run dynamics for both types of individuals in the society- skilled and unskilled. Then we analyse the two benchmark cases (private and public) separately.

Under the mixed education system, we show that an increase in the rate of income tax raises the steady state human capital per unit of inheritance received not only for the unskilled labours, but also for the skilled labours.

The wealth dynamics of the present model shows that under private education system, the non-trivial equilibrium is that in the long run, everyone works as a skilled labour and faces steady growth. On the contrary, under public education system, two situations may emerge-either all the individuals work as skilled labours or all the individuals work as unskilled labours. When all the individuals work as unskilled labours, the bequest may grow over time or stagnate, but, the human capital accumulation remains halted. When all the individuals work as skilled labours, bequest and human capital both grow over time.

Our voting outcome finds that even if the tax rate imposed is her preferred one, a skilled labour will always choose the private education system and an unskilled labour will vote for the public education system if her inheritance received is below a critical level. Finally, after assuming a

[10]Checchi and Worfhorst (2014)

Pareto distribution of wealth we arrive at the conclusion that an economy with an unequal wealth distribution with a high value of the inequality parameter, a low rate of interest, a low unskilled wage, an improved quality of public education and the parents with low weights assigned to human capital of their offspring in their preference functions, will have the public education system chosen by the majority of its population.

Our study is subject to several limitations. We did not consider heterogeneous ability across individuals. Also, this paper does not consider other occupations such as entrepreneurship, self-employment etc. This paper considers labour demand function to be perfectly elastic, hence, does not consider any demand constraint. These issues may be considered in future research.

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