Competition, Collaboration and Organization Design*

Suraj Prasad[†] and Sebastian Tanase [‡]

August 24, 2018

Abstract

We look at the tradeoff between competition and collaboration, and its implications for organization design – in the sense of structure, culture (or shared preferences) and monetary incentives. In our setting, collaboration is essential but costly for agents with different preferences. We consider two structures: an *internally competitive* (or parallel) structure, where agents compete on quality for their projects to be selected by a principal, and an *internally noncompetitive* (or focussed) structure, where the principal mandates a project. As preferences diverge, internal competition leads to higher quality projects, until the need to compromise to facilitate collaboration undoes these gains. As a result, internal competition and a more coherent culture complement each other. We use our theory to understand recent organizational changes at Microsoft.

1 Introduction

In 2013, Microsoft embarked on a bold restructuring of the company, so that its divisions would "war no more" and instead "collaborate more closely".[1] As the New York Times reported, Microsoft's previous organization structure was satirized in a comic which "showed several isolated pyramids representing its divisions, each with a cartoon pistol aimed at the other."[2] In addition, Microsoft also appointed a new CEO, Satya Nadella, whose priority was to articulate the mission of the company more clearly and thus transform its culture to make it more coherent. In an email to the entire company, Nadella stressed that Microsoft is "a family of individuals united

---

*This paper is based on Sebastian Tanase's honors thesis. We thank Murali Agastya, Hongyi Li, Guillaume Roger, Jiemai Wu, and a seminar audience at the University of Woolongong for their comments.

†School of Economics, University of Sydney. suraj.prasad@sydney.edu.au

‡University of Sydney. sebastian@tanas.ee

1The company dissolved its eight product-based divisions and reorganized instead around functional themes, with a focus on it online or 'cloud' business. See "Microsoft Overhauls, the Apple Way", *New York Times*, July 11, 2013. Also see "Microsoft's Creative Destruction", *New York Times*, Feb 4, 2010.

2See footnote 1.

by a single, shared mission... [We will] collaborate across boundaries to bring the best of Microsoft." [3]

These organizational changes at Microsoft raise some interesting questions. Was the decision to suppress internal competition across Microsoft's warring divisions the right one? What role did the need for collaboration play in this decision? Was Microsoft correct to view internal competition and a coherent culture as substitutes? To answer these questions, we develop a model with i) a tradeoff between competition and collaboration; and where ii) this tradeoff is affected by organizational culture. This tradeoff is not unique to Microsoft. The retailing firm Sears Holdings, for instance, has an organizational model where each of its thirty odd business units compete for funds and resources.[4] This appears to have stifled collaboration across its units. As a former executive of Sears put it, "if you were in a different business unit, we were in two competing companies ...cooperation and collaboration were not there."

In our model, a principal (the business owner) hires two agents (each a manager of a separate division) to introduce a new product to the market. To do this, the principal first proposes a type of project — which we model as a point on a Hotelling line — and appoints one of the agents as its leader. The principal can make up to two proposals, each with a different leader. Taking Microsoft as an example, the project could be a software application which is optimized for off-site cloud servers (points on the left side of the line), or it could be an application that is optimized for an on-site physical server (points on the right side of the line). Following this, the project leader develops the project by exerting effort to improve its quality; project quality is only observable to this agent. The principal then observes a noisy signal of the relative quality of projects and selects one project to go to market. Finally, the non-leading agent has to collaborate on the selected project, after which it goes to market.

Our model has three key features. The first is the way we model collaboration. We assume that each agent has *access* to a critical resource in his division, which is essential for the project that the other agent develops. That is quality for one agent's project yields returns from a market only when *the other agent collaborates* by using his resource. The critical resource, for example, could be a team in the division that the agent is familiar with managing. Or the critical resource could reflect the tacit knowledge that an agent possesses. This is a stark, but simple, way to model gains from collaboration.[5]

Second, though collaboration is essential, it is costly because agents have different preferences over the types of projects that go to market. We model these preferences as each agent having an ideal point on the Hotelling line. The further these ideal

[3]"Exclusive: Satya Nadella reveals Microsoft's new mission statement, sees 'tough choices' ahead", *Geekwire*, June 25, 2015.

[4]"At Sears, Eddie Lampert's Warring Divisions Model Adds to the Troubles", *Bloomberg*, July 12, 2013.

[5]A further discussion of this assumption is given in Section 8.

points are, the more heterogeneous the organization (or the less coherent its culture). The intuition is that some projects will better line up with existing products that the manager in a division privately benefits or earns rents from. As an example, after-sales service may be easier, existing sales channels may already be established, or the project is otherwise more compatible with existing or planned products.[6] Returning to the Microsoft example, its "managers often grumble privately that one of the most dreaded circumstances of the company is having to 'take a dependency' on another group."[7] In our model, culture is endogenous but the degree of its coherence is limited — this could be thought of as the company's ability to articulate its mission clearly, so that it can attract people with more similar preferences.

Third, we consider two structures. In an *internally competitive* structure, the principal makes two proposals, and appoints a different leader for each one, so that agents develop projects in *parallel*. The principal then selects just one of the projects to go to market based on a noisy signal of the relative quality of projects. By contrast, in a *noncompetitive* (or *focussed*) structure, the principal makes only one proposal, and as such only one agent develops the project. Structure can thus be viewed as a switch which turns competition within the organization on or off.

The main tradeoff in our model is a simple one. Proposing projects that are close to each other (and thus closer to the other agent's ideal point) makes collaboration easy but reduces the agents' incentives to compete, leading to lower quality products. On the other hand, proposing projects that are further apart leads to more competition and higher quality products, but collaboration is difficult. The role of organization design — in the sense of structure, culture, and monetary incentives — is to manage this tradeoff.

In a noncompetitive structure, heterogeneity has no benefits but instead makes collaboration more difficult. So the principal's payoffs (weakly) decrease as agents' preferences diverge more. Under a competitive structure, however, the principal's payoffs are a little more nuanced. For low levels of heterogeneity, collaboration is easy and the principal can extract the benefits of competition (in the form of higher quality products) as preferences diverge. But beyond a threshold, collaboration becomes difficult. Because of this, agents have to compromise, undoing the gains in quality. Furthermore, monetary incentives have to be distorted upwards. The principal's utility thus decreases in heterogeneity after a point.

Comparing these structures gives us a rich set of patterns which depends largely on the principal's information. When the principal is poorly informed — that is when the principal observes only a very noisy signal of the relative quality of projects — then the noncompetitive structure is optimal regardless of preferences. With a

[6] The common link across these example is that the benefits accrue to the agent ex-post, after the product has gone to market.

[7] See "Microsoft Overhauls, the Apple Way", *New York Times*, July 11, 2013. Also see Iansiti and Serels (2013) for a case study on differences between Windows Azure and the Server and Tools Business (STB). As they point out, "a sense of rivalry developed between the two development teams," each of whom thought that "they were on a special mission."

poorly informed principal, the project selected boils down to luck, and thus agents have fewer incentives to compete with one another. In addition, incentives in the parallel structure are diluted since only one project goes to market. This result is fairly intuitive but uninteresting.

Where things start to get interesting is when the principal is better informed. Here we find that while internal competition works well for intermediate levels of heterogeneity, it is not optimal when preferences are very similar or when preferences diverge a lot. In a very homogenous organization, agents do not have much of an incentive to compete. In a very heterogeneous organization, by contrast, agents have an incentive to compete, but the level of compromise required to facilitate collaboration, undermines the gains from competition. Along with the diluted monetary incentives associated with parallel development of projects, this results in internal competition no longer being optimal, even though the potential benefits from competition are high.

We then turn to the problem of determining the optimal culture of the organization. When the degree of coherence is limited — say because the organization cannot articulate its mission clearly — the organization is necessarily heterogeneous. For this case, a noncompetitive structure is optimal. On the other hand, the organization is at least as homogeneous when more coherence is possible. In this case, internal competition is more likely to be optimal. Thus a coherent culture and internal competition complement each other. This is our main result, and the intuition underlying it is simple: with a more coherent culture, collaboration is secured with a lower level of compromise, so that competition across agents is unfettered. This result contrasts with Microsoft's view that a coherent culture and competition are substitutes. Indeed, given its new CEO's ability to clearly articulate the mission of the company, Microsoft may be better off by retaining its competitive structure.

In our model, diverging preferences on a Hotelling line provide an incentive for agents to compete. In this respect, our paper is related to some recent work in the political economy literature. Hirsch and Shotts (2015) model how a policy decision maker (principal) and their policy developers (agents) interact, given their heterogeneous policy preferences. They find, in contrast to our paper, that the principal's payoffs always increase in heterogeneity. Callander and Harstad (2015) conclude that a certain level of heterogeneity is required in government policy experiments to circumvent a free-riding problem and to induce other governments to also experiment with policy. Neither of these papers consider the problem of agents collaborating. One paper that does have elements of both competition and collaboration is Bonatti and Rantakari (2016). But they take both our key design variables of interest (preferences and the competitive structure) as fixed.

More generally there are several papers which show that heterogeneity in preferences has incentive benefits in an organizational context. We list a few here. Dewatripont and Tirole (2005) look at how heterogeneity in preferences affects incentives for agents to communicate with each other in a setting where communication is costly. Prendergast (2008) finds that organizations should optimally hire biased agents because specialized agents have stronger intrinsic incentives. In Landier, Sraer, and Thesmar (2009), diverging preferences across a decision maker and implementer forces a decision maker to use more information in the decision process, which leads to more effort from the implementer. On a different note, Prat (2002) considers the question of heterogeneity in the context of team theory. He finds that if agents' actions are complementary in the payoff function, then they should be homogeneous, whereas if their actions are substitutes then they should be heterogeneous.

There is a small literature in economics on culture in organizations. Some papers build on the ideas of Schein (1985), and treat culture in terms of shared beliefs or preferences. For example, Van den Steen (2010a) and Van den Steen (2010b) define culture in terms of shared beliefs, and find that a homogeneous culture where agents have the same prior beliefs, helps to eliminate agency problems within a firm. He also discusses how his models can be modified to account for shared preferences. Lazear (1995) considers an evolutionary model of preferences within the organization and how the principal can control the process. Our modelling of culture, based on the distribution of ideal points on a Hotelling line, follows Prasad and Tomaino (2017). The setting of that paper is different in that there is no need for collaboration and there are no monetary incentives. Culture is also sometimes thought of as shared knowledge. Crémer (1993) defines culture as the 'stock of knowledge shared by the members of an organization, but not to the general population from which they are drawn'. He finds that a culture which is very stable and 'integrated', improves communication within the organization, at the cost of ossifying the organization. Li (2017) looks at the diffusion of shared knowledge within an organization. A final interpretation of culture is as a way of selecting from multiple equilibria. Examples of papers in this vein are Kreps (1990), Carillo and Gromb (1999), and the surveys on corporate culture by Hermalin (2001) and Hermalin (2013).

We model internal competition as a tournament as in Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983). Our paper is also related to a literature on resource allocation in organizations. Stein (1997) shows the advantage of winner picking (where funds from projects are pooled and concentrated with one agent) when a principal has superior information over the value of projects. In Marino and Zabojnik (2004), a tournament across teams relaxes the budget for providing incentives and thus reduces free riding. In Friebel and Raith (2010), a centralized organization leads to competition over resources, but also makes it more costly to induce effort when information is elicited from an agent. Rantakari (2017) considers a setting where agents can communicate to the principal and shows that the optimality of a competitive structure over a more focussed one where one agent is favored, depends on whether information about quality is hard or soft.

We view collaboration as the use of a critical resource that an agent has access to.[8] But there are other ways to model collaboration. In Itoh (1991), an agent can choose the level of 'help' offered to another agent. Inducing help in his setting increases output but imposes more risk on an agent. When the marginal cost of help at zero is positive, inducing any help discontinuously shifts the risk borne by the agent, so that at the optimum there is no help or a sufficiently large amount of it. Conversely, the lack of collaboration can be modeled as 'sabotage' in tournaments where one agent exerts effort to reduce the other agent's output (Lazear (1989)).

2 Model

2.1 Actors

A risk neutral principal P hires two risk-neutral agents, A_1 and A_2, to introduce a product to the market.[9] Think of P as the business owner of the firm with a residual interest in the output of the firm. In this context, A_1 and A_2 are managers within the firm, each belonging to a separate division. The model has five key parts: production, organizational structure, preferences, contracts, and the timing. We discuss each of these below.

2.2 Production

Production has four phases: a proposal phase, a project development phase, a project selection phase and finally a collaboration phase.

Consider the proposal phase first. The firm bases its production on a project proposal, which is a point on the Hotelling line denoting the 'focus' or the type of a product. P chooses the location of the proposal. Taking Microsoft as an example, think of projects to the left end as optimizing for off-site cloud servers, whereas projects to the right end optimize for on-site physical servers. If this point is to the left of 0, we denote the project by x_1 and A_1 is made the project leader – we say that x_1 is A_1's project. Similarly, if this point is to the right of 0, we denote the project by x_2 and A_2 is made the project leader – we say that x_2 is A_2's project. P can make up to two proposals, each having a different project leader.

Following the proposal(s) made by the principal, the project leader — say A_i — exerts effort to improve the quality level q_i of his project. We call this the project development phase.[10] Effort to improve quality is costly for the leader, and as such the leader has a quadratic cost function of $\frac{1}{2}q_i^2$. The quality of the project is privately observable.

The next phase is the project selection phase. In this phase, if two projects are developed, P observes a noisy signal z_i of the relative quality of A_i's project with

[8] This notion of access also plays an important role in Rajan and Zingales (1998).

[9] For the sake of clarity, we will use female pronouns when referring to the principal, and male pronouns when referring to the agents.

[10] If there are two proposals, quality for each is chosen simultaneously.

respect to A_j's project, with:[11]

$$z_i = q_i - q_j - \varepsilon \text{ where } \varepsilon \sim N(0, \sigma^2) \tag{1}$$

where $\sigma^2 > 0$ is the variance of the noise term. After P observes the signal z_i, she selects the project to be taken to market.

The final phase is the collaboration phase. For quality from A_i's project to yield returns from the market, A_j must collaborate by providing a critical resource that he alone has access to. If $a_j = 1$, agent j collaborates on agent i's project and if $a_j = 0$ then agent j does not collaborate on i's project. Similarly $a_i = 1$ if A_i collaborates on A_j's project whereas $a_i = 0$ when A_i does not collaborate. If an agent does not collaborate on the other agent's project, it gets scrapped. If, on the other hand, the agent collaborates, then the return on the project from the market equals its quality.

Formally, a project with quality q, yields the return y which is given by:

$$y(q, a) = a\lambda q \tag{2}$$

where $\lambda > 0$ is a technological parameter.

2.3 Organizational Structure

We consider two organizational structures. If P makes just one proposal (say with A_1 as the project leader) we call the structure of the organization, *internally non-competitive* or *focussed*. On the other hand, if P makes two proposals (one with A_1 as the project leader and the other with A_2 as the project leader) we call the structure of the organization *internally competitive* or *parallel*.

2.4 Preferences

The principal has no preference over the type of project. She is solely interested in her profits. Thus her utility is given by:

$$U_P = y + B - w_1 - w_2 \tag{3}$$

where $B > 0$ is a parameter which captures other non-contractible benefits that P gets from the agents besides the project that goes to market.[12]

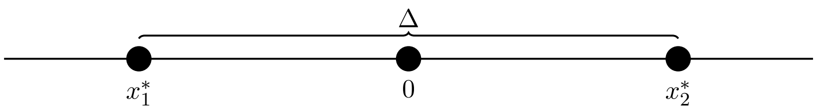
Agents, on the other hand, do have preferences over projects that go to market. We model these preferences as ideal points on the Hotelling line, symmetrically distributed around zero. A_1's ideal point is $x_1^* \leq 0$ and A_2's ideal point is $x_2^* \geq 0$. We define the culture of the firm, $\Delta \equiv x_2^* - x_1^*$, in terms of how close these ideal points are. We say that an organization has a more *coherent culture* when its preferences are

[11] For the case where only one project is developed, P does not get a signal.

[12] Think of B as the benefit the principal gets from the routine or perfunctory work that agents do apart from the project that they work on.

more homogeneous — that is when Δ is smaller. Figure 1 shows these ideal points for agents. As discussed in the introduction, the intuition for these preferences is that projects which align with existing products in a division, yield higher private benefits or rents for an agent.

Figure 1: Agents' Ideal Points.



The utilities of agents from a project proposal $\bar{x}$, that goes to market are given by:

$$U_i = \begin{cases} w_i + a_j(b - |x_i^* - \bar{x}|) - \frac{1}{2}q_i^2 & \text{(for a project leader)} \\ w_i + a_i(b - |x_i^* - \bar{x}|) & \text{(for a non-leading agent)} \end{cases} \tag{4}$$

where w_i is A_i's wage, and $b - |x_i^* - \bar{x}|$ is the agent's utility from a project with collaboration. In particular, $b \geq 0$ is the agent's private benefit from his ideal project, and $|x_i^* - \bar{x}|$ is the agent's loss from projects that are further away from his ideal one. As mentioned earlier, $\frac{1}{2}q_i^2$ is the project leader's cost of improving the quality of his project.

For the analysis in the paper, it will be easier to refer to the distances using the notation below:

$$\delta_{11} = |x_1^* - x_1| \qquad \delta_{12} = |x_1^* - x_2|$$
$$\delta_{21} = |x_2^* - x_1| \qquad \delta_{22} = |x_2^* - x_2|$$

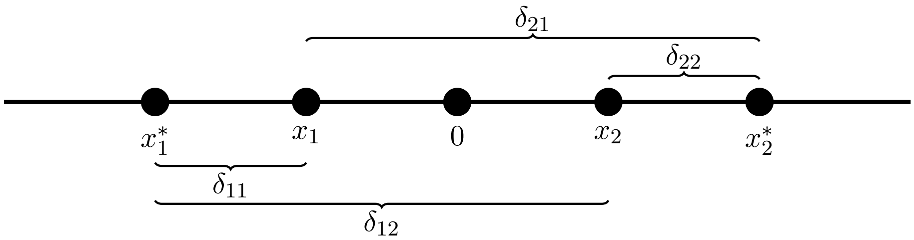
δ_{ij} can be read as the distance between agent i's ideal point, and agent j's project. When proposals lie in between agents' ideal points, then we can rewrite:

$$\delta_{ij} = \Delta - \delta_{jj} \tag{5}$$

where δ_{jj} is the extent of compromise that A_j makes to make the project more appealing to A_i. Figure 2 shows these distance costs for both agents.

In our framework, the culture of the organization (i.e. the distance between ideal points Δ) is endogenously chosen by P. In doing so, P faces a constraint $\Delta \geq k$; where $k > 0$ is a parameter which captures the limits to cultural alignment in the organization. k is our main parameter of interest. We interpret a low k as a setting where the company can articulate its mission clearly, so that it attracts employees with more similar preferences. In particular, within the context of our Microsoft example, a low k corresponds to the regime of its new CEO, Satya Nadella, who has the ability to communicate the mission of the company more clearly.

Figure 2: Preference Distance Costs.



2.5 Contracts

Next consider the contract that P can offer the agents. Each agent i is offered a take-it-or-leave-it linear contract which gives him a share s of the project return y, and a fixed wage w_0.[13] Our restriction to symmetric contracts is motivated by the fact that agents are identical at the time of hiring and that there are organizational costs associated with treating divisions differently with respect to incentives on a project.[14] Relaxing this assumption allows a principal to target quality and collaboration separately leading to an increase in utility under both structures. While this complicates our analysis, especially under the competitive structure, it does not change the main tradeoffs in our model. Later in our analysis, in Section 8, we discuss the implications of relaxing this assumption in more detail.

If an agent refuses the contract, all agents and the principal get their reservation utility of 0. We also impose a limited liability condition on the fixed wage w_0. Formally, A_i receives a wage given by:

$$w_i = w_0 + sy \quad (w_0 \geq \underline{w}) \tag{6}$$

2.6 Timeline

There are six distinct phases in the model. In the first phase which is the *hiring phase*, P offers take-it-or-leave-it contracts to two agents: A_1 and A_2. P chooses the ideal points for these agents (x_1^* for A_1 and x_2^* for A_2), the structure of the organization, and the contracts (w_0, s) for the agents in this phase. The next four phases are the four phases of production: the *proposal phase*, the *project development phase*, the *project selection phase* and the *collaboration phase*. Finally *payoffs* are realized. Figure 3 summarises the timing of the model.

We now impose some assumptions on the parameters of our model. Define h to be the value of the density function for ε at the point 0.[15]

[13]We assume that y is the only measure of the agents' performance that is verifiable and thus can be contracted on.

[14]Alternatively, there may be costs to the principal to commit to a proposal upfront.

[15]Given that ε is normally distributed with mean 0 and variance σ^2, $h \equiv \frac{1}{\sqrt{2\pi\sigma^2}}$.

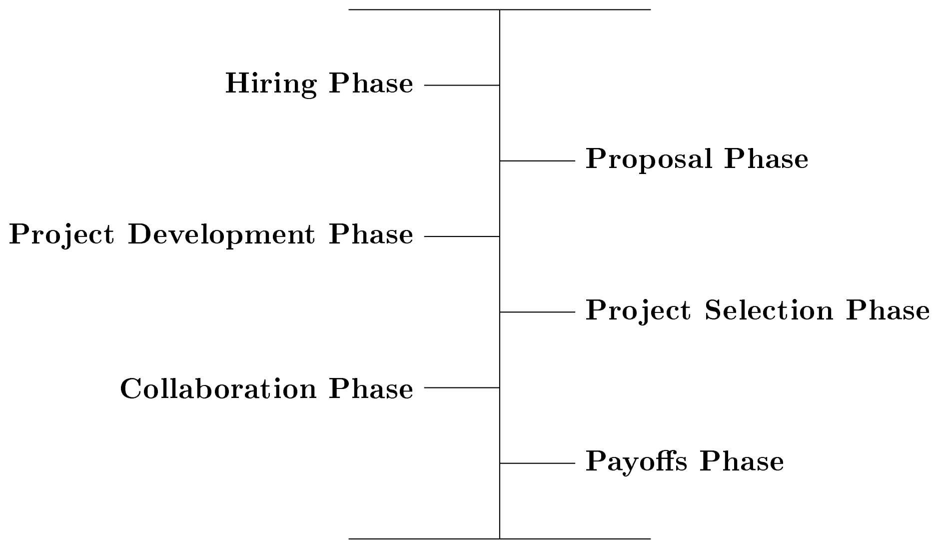
Assumption 1. *The parameters, λ, h, B and $\underline{w}$ satisfy the following conditions:*

i $\lambda h < 1$.

ii $B \geq 2\underline{w} \geq (\frac{\lambda}{2} + 2bh)^2$

The first condition is necessary for an equilibrium in pure strategies to exist. It is similar to a condition found in Nalebuff and Stiglitz (1983). The second condition ensures that the participation constraint holds for agents and for the principal. Abstracting away from participation constraints allows us to focus on the role that collaboration constraints play. In what follows, we also normalize $B = 2\underline{w}$ to minimize notation.

Figure 3: Generalised Timeline of the Model.



3 Efficient Benchmark

In this section we characterize the ideal points of agents, the organizational structure, the types of projects developed, agents' decisions on collaboration, and the levels of quality for a project, that maximize total surplus. Efficiency requires five things.

i. Collaboration — a non-leading agent must agree to collaborate on the leaders project ($a = 1$).

ii. Focussed structure — because quality is costly and only one product goes to market, only one project should be developed. This prevents the duplication of effort to improve quality. Assume that only A_1 develops a project.

iii. Proposals — a proposal must lie between ideal points. A proposal for A_1 to the left of x_1^* imposes an additional cost on both agents.

iv. Quality — to maximise total surplus, the First Order Condition requires $q_1 = \lambda$. Therefore, A_1 must fully internalise the benefit of quality to the principal.

v. Culture — the culture should be as coherent as possible to minimize the disutility from collaborating on a project. That is $\Delta = k$.

With the above criteria met, the total surplus (TS) becomes:

$$TS = \max\{B, B + \frac{1}{2}\lambda^2 + 2b - k\}$$

We now go on to solve for the equilibrium where agents collaborate. We do this in two steps. First, we take the culture of the organization Δ as given, and solve for the optimal level of incentives, quality, and compromise for agents. This gives us the principal's utility as a function of Δ. We do this separately for each structure. In the second step, we solve for the optimal structure and culture of the organization. The equilibrium concept we use is subgame perfect equilibrium.

4 Internally Noncompetitive Structure

In the noncompetitive organizational structure, the principal makes a project proposal to just one agent. We assume without any loss of generality that this agent is A_1. There is also no loss of generality in restricting the proposal to lie in between agents' ideal points; a proposal to the left of x_1^* does not affect incentives for quality but makes collaboration more difficult.

Consider the collaboration phase. A_2 will collaborate on A_1's project if his expected benefit from doing so exceeds his cost. The collaboration constraint for A_2 is thus given by:

$$s\lambda q_1 + b \geq \delta_{21} \tag{7}$$

Because the proposal lies between the agents' ideal points, we can use (5) and rewrite (7) as:

$$s\lambda q_1 + b \geq \Delta - \delta_{11} \tag{IC_{col}^N}$$

Next, consider the project selection phase. Because only one project is developed and because quality is non-negative, P lets the project go to market.

Going back one stage to the development phase, A_1's choice of quality solves the following problem:

$$\max_{q_1} U_1 = s\lambda q_1 - \frac{1}{2}q_1^2 + b - \delta_{11} \tag{8}$$

The first order necessary condition yields:[16]

$$q_1 = s\lambda \tag{IC_q^N}$$

So P's optimization problem is to maximize her utility subject to an incentive constraint for quality (IC_q^N), an incentive constraint for collaboration (IC_{col}^N) and two feasibility constraints for compromise δ_{11} (a non-negativity constraint and a maximal-compromise constraint).[17] P's problem is thus:

$$\max_{s,q_1,\delta_{11}} U_P^N = (1-2s)\lambda q_1$$

subject to

$$q_1 = s\lambda \tag{IC_q^N}$$

$$s\lambda q_1 + b \geq \Delta - \delta_{11} \tag{IC_{col}^N}$$

$$0 \leq \delta_{11} \leq \frac{\Delta}{2} \tag{$Feas$}$$

The optimal level of incentives s, quality q_1, and compromise δ_{11} for the non-competitive structure are given by:

$$s^N = \begin{cases} \frac{1}{4} & \text{if } \Delta < \Delta_{full}^N \\ \frac{\sqrt{\Delta-2b}}{\sqrt{2}\lambda} & \text{if } \Delta_{full}^N \leq \Delta \leq \Delta_{shut}^N \end{cases} \tag{9}$$

$$q_1^N = \begin{cases} \frac{\lambda}{4} & \text{if } \Delta < \Delta_{full}^N \\ \frac{\sqrt{\Delta-2b}}{\sqrt{2}} & \text{if } \Delta_{full}^N \leq \Delta \leq \Delta_{shut}^N \end{cases} \tag{10}$$

$$\delta_{11}^N \in \begin{cases} [\max\left\{0, \Delta - b - \frac{\lambda^2}{16}\right\}, \frac{\Delta}{2}] & \text{if } \Delta < \Delta_{full}^N \\ \{\frac{\Delta}{2}\} & \text{if } \Delta_{full}^N \leq \Delta \leq \Delta_{shut}^N \end{cases} \tag{11}$$

Figure 4 shows the optimal level of s, Figure 5 shows the optimal quality level and Figure 6 shows the optimal level of compromise δ_{11}, all as functions of the level of heterogeneity.

[16]The second order condition is -1, so that the first order condition is also sufficient for a maximum.

[17]The collaboration constraints along with Assumption 1 imply that the agent is willing to participate. The details are in the Appendix.

To get some intuition for the solution above, note that the optimal incentive level in the absence of the collaboration and feasibility constraints is $s = \frac{1}{4}$. The tradeoff that P faces for this case is the standard one in the literature; increasing s leads to higher quality projects, but because of limited liability, P is forced to give up a share of the surplus to the agents. For low levels of heterogeneity, P can find a feasible level of compromise that supports this level of incentives. But at $\Delta^N_{full} \equiv \frac{\lambda^2}{8} + 2b$, agents cannot compromise any further. Beyond this cutoff, incentives have to be distorted upwards till $s = \frac{1}{2}$ at which point ($\Delta^N_{shut} \equiv \frac{\lambda^2}{2} + 2b$) P shuts down the project.

Finally, P's utility from the noncompetitive structure as a function of Δ is:

$$U_P^{N*} = \begin{cases} \frac{\lambda^2}{8} & \text{if } 0 \leq \Delta < \Delta^N_{full} \\ \lambda\sqrt{\frac{1}{2}(\Delta - 2b)} - (\Delta - 2b) & \text{if } \Delta^N_{full} \leq \Delta \leq \Delta^N_{shut} \\ 0 & \text{if } \Delta > \Delta^N_{shut} \end{cases} \tag{12}$$

Notice that P's utility is weakly decreasing in the level of heterogeneity. This is because heterogeneity has no incentive benefits under the noncompetitive structure whereas it has costs because P has to distort incentives to ensure collaboration. Figure 7 above, shows P's utility under the noncompetitive structure as a function of the level of heterogeneity.

5 Internally Competitive Structure

The internally competitive organizational structure, in contrast to the noncompetitive structure, has both agents developing projects in parallel. There is no loss of generality in focussing on a symmetric equilibrium, where i) agents choose the same level of quality, ii) the tournament is symmetric with A_i's project being selected if and only if $z_i \geq 0$, and iii) where proposals are symmetric ($\delta_{ii} = \delta_{jj}$) and lie between agents ideal points so that (5) holds.[18]

Let us start at the collaboration phase. Consider the collaboration phase. A_j will collaborate on A_i's project if his expected benefit from doing so exceeds his cost. The collaboration constraint for A_j is thus given by:

$$s\lambda q_i + b \geq \delta_{ji} \tag{13}$$

And once again, we can use (5) to rewrite the collaboration constraint in (13) as:

$$s\lambda q_i + b \geq \Delta - \delta_{ii} \tag*{(IC^C_{col})}$$

[18]See Section 10.2.5 in the Appendix.

Figure 4: Monetary Incentives in the Noncompetitive Structure ($h = \frac{7}{10}$, $b = \frac{1}{7}$, $\lambda = 1$).

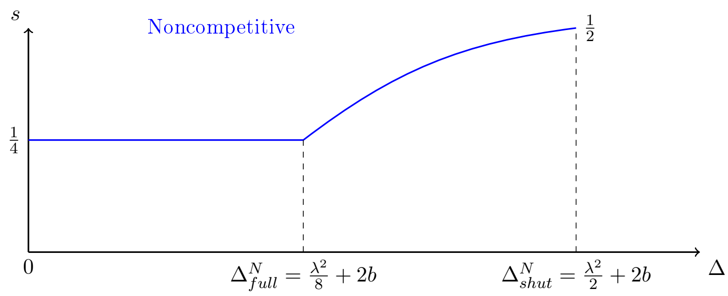


Figure 5: Quality in the Noncompetitive Structure ($h = \frac{7}{10}$, $b = \frac{1}{7}$, $\lambda = 1$).

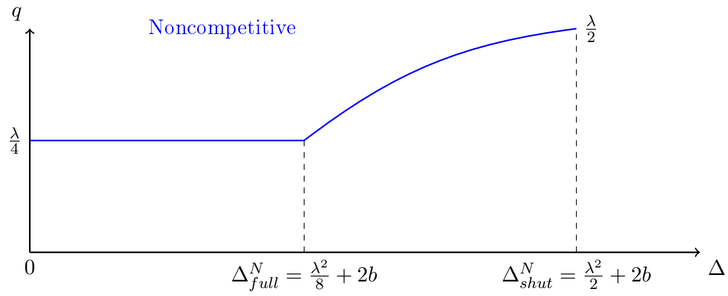


Figure 6: Compromise in the Noncompetitive Structure ($h = \frac{7}{10}$, $b = \frac{1}{7}$, $\lambda = 1$).

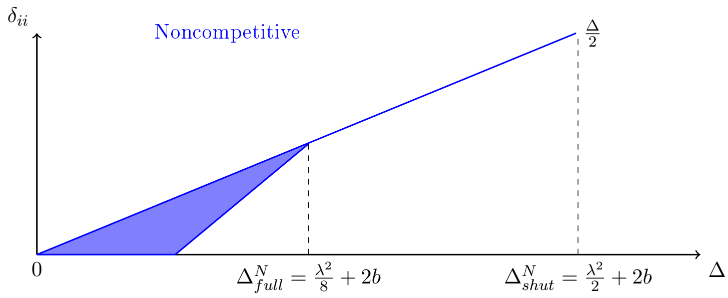
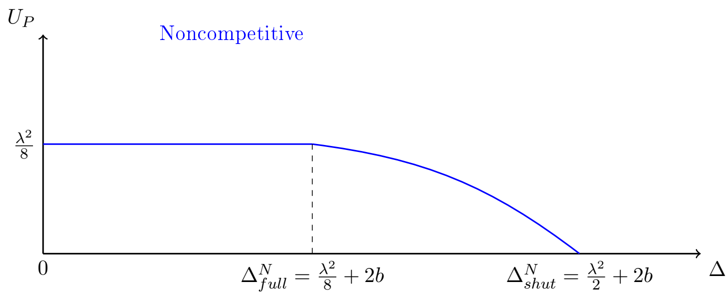


Figure 7: P's Utility in the Noncompetitive Structure ($h = \frac{7}{10}$, $b = \frac{1}{7}$, $\lambda = 1$).



Next, let us move to the project selection phase. As mentioned in the model setup, P receives a noisy signal z_i about the relative quality of projects. From the symmetry of the tournament, the probability that agent i's project gets selected is:

$$\begin{aligned} p &\equiv \Pr(\text{Agent } i \text{ wins}) \\ &= \Pr(z_i > 0) \\ &= \Pr(q_i - q_j > \varepsilon) \end{aligned}$$

Given this probability p, A_i's choice of quality solves the following problem:

$$\max_{q_i} EU_i = p\,(s\lambda q_i + b - \delta_{ii}) + (1-p)\,(s\lambda q_j + b - \delta_{ij}) - \frac{1}{2}q_i^2 \tag{14}$$

The First Order Condition for agent i in a symmetric equilibrium yields the following equation:[19]

$$\therefore q_i = \frac{1}{2}s\lambda + h(\delta_{ij} - \delta_{ii})$$

Using (5), the incentive for improving quality can be rewritten as:

$$q_i = \frac{1}{2}s\lambda + h(\Delta - 2\delta_{ii}) \tag{IC_q^C}$$

So P's optimization problem is to maximize her utility subject to an incentive constraint for quality (IC_q^C), an incentive constraint for collaboration (IC_{col}^C) and two feasibility constraints for compromise δ_{ii} (a non-negativity constraint and a maximal-compromise constraint).[20] P's problem is thus:[21]

$$\begin{aligned} &\max_{s,q,\delta_{ii}} U_P^C = (1-2s)\lambda q \\ &\text{subject to} \\ &\qquad q = \frac{1}{2}s\lambda + h(\Delta - 2\delta_{ii}) && (IC_q^C) \\ &\qquad s\lambda q + b \geq \Delta - \delta_{ii} && (IC_{col}^C) \\ &\qquad 0 \leq \delta_{ii} \leq \frac{\Delta}{2} && (Feas) \end{aligned}$$

[19] A derivation of this is left to the appendix. The Second Order Condition holds as long as $h\lambda < 1$, λ is sufficiently small and b is not too large relative to λ. See the appendix for further details.

[20] The collaboration constraints along with Assumption 1 imply that the agent is willing to participate. The details are in the Appendix.

[21] Given the symmetry in quality, we have dropped the subscript for q.

Comparing the competitive structure with the noncompetitive one, the only constraint that is different is the incentive constraint for quality. Agents now have an incentive to compete so that the project that they develop (which is closer to their ideal point) gets taken to market. This is reflected in the term $h(\Delta-2\delta_{ii})$. With this structure, there is now a tradeoff between competition and collaboration. Projects that are further apart – with a higher distance between ideal points Δ and a lower level of compromise δ_{ii} – induce more competition but make collaboration more difficult. And as Δ gets larger so that the culture is less coherent, this tradeoff becomes more severe. Also note that monetary incentives are diluted for this case. Because only one project gets selected to go to market, agents, under a parallel structure, have only half the incentives to invest in its quality in a symmetric equilibrium.

The optimal level of incentives, quality, and compromise are given by the expressions below:[22]

$$s^C = \begin{cases} \frac{1}{4} - \frac{h\Delta}{\lambda} & \text{if } \Delta < \Delta^C_{col} \\ \frac{-h\Delta+\sqrt{2(\Delta-b)+h^2\Delta^2}}{\lambda} & \text{if } \Delta^C_{col} \leq \Delta \leq \Delta^C_{int} \\ \frac{1-\sqrt{(1-\lambda h)(1+4h^2(2b-\Delta))}}{2\lambda h} & \text{if } \Delta^C_{int} < \Delta < \Delta^C_{full} \\ \frac{\sqrt{\Delta-2b}}{\lambda} & \text{if } \Delta^C_{full} \leq \Delta \leq \Delta^C_{shut} \end{cases} \tag{15}$$

$$q^C = \begin{cases} \frac{\lambda}{8} + \frac{h\Delta}{2} & \text{if } \Delta < \Delta^C_{col} \\ \frac{h\Delta+\sqrt{2(\Delta-b)+h^2\Delta^2}}{2} & \text{if } \Delta^C_{col} \leq \Delta \leq \Delta^C_{int} \\ \frac{\frac{1}{2}s^C\lambda+h(2b-\Delta)}{1-2hs^C\lambda} & \text{if } \Delta^C_{int} < \Delta < \Delta^C_{full} \\ \frac{\sqrt{\Delta-2b}}{2} & \text{if } \Delta^C_{full} \leq \Delta \leq \Delta^C_{shut} \end{cases} \tag{16}$$

$$\delta^C_{ii} = \begin{cases} 0 & \text{if } \Delta < \Delta^C_{col} \\ 0 & \text{if } \Delta^C_{col} \leq \Delta \leq \Delta^C_{int} \\ \Delta - s^C\lambda q^C - b & \text{if } \Delta^C_{int} < \Delta < \Delta^C_{full} \\ \frac{\Delta}{2} & \text{if } \Delta^C_{full} \leq \Delta \leq \Delta^C_{shut} \end{cases} \tag{17}$$

Figure 8 shows the optimal level of s, Figure 9 shows the optimal quality level and Figure 10 shows the optimal level of compromise δ_{ii}, all as functions of the level of heterogeneity.

To understand the solution above, it is useful to think of P's optimal incentive level in the absence of the collaboration constraint. Here, with competition, this

[22]This is assuming an interior solution holds with $s > 0$ when $\Delta < \Delta^C_{col}$. The necessary and sufficient condition that guarantees an interior s being optimal when the collaboration constraint does not bind, is $b < \frac{\lambda}{4h}$. When $b \geq \frac{\lambda}{4h}$, $s = 0$ when $\Delta \in [\frac{\lambda}{4h}, b]$ and P's utility for this range of Δ's is $\lambda h\Delta$.

optimal level is $s = \frac{1}{4} - \frac{h\Delta}{\lambda}$. The first part of the expression is the same as the noncompetitive case – P has to tradeoff more incentives for quality with giving up a larger share of the surplus. However, with the additional instrument of competition, she can afford to lower incentives and keep a larger share of the surplus. There is also no compromise at the optimum because it just reduces competition.

As heterogeneity increases, the collaboration constraint binds at $\Delta^C_{col} \geq b$. P can do two things at this point: she can increase monetary incentives or she can make agents compromise. At first, she only distorts monetary incentives upwards without making agents compromise so that she still gets the benefit of competition. But as heterogeneity gets larger, and collaboration gets more difficult, there is a cutoff (Δ^C_{int}) where agents are made to compromise. This goes on till agents cannot compromise any further (Δ^C_{full}). For levels of heterogeneity beyond this point, P raises incentives till they reach their maximum of $s = \frac{1}{2}$ at which point (Δ^C_{shut}) P shuts down the project.

Finally, P's utility from the competitive structure as a function of Δ is:

$$U_P^{C^*} = \begin{cases} \frac{\lambda^2}{16} + \frac{\lambda h \Delta}{2} + h^2\Delta^2 & \text{if } \Delta < \Delta^C_{col} \\ \frac{\lambda}{2}(h\Delta + \sqrt{h^2\Delta^2 + 2(\Delta - b)}) - 2(\Delta - b) & \text{if } \Delta^C_{col} \leq \Delta \leq \Delta^C_{int} \\ \left(1 - 2s^C\right)\lambda\left(\frac{\frac{1}{2}s^C\lambda + h(2b-\Delta)}{1-2hs\lambda}\right) & \text{if } \Delta^C_{int} < \Delta < \Delta^C_{full} \\ \frac{\lambda}{2}\sqrt{\Delta - 2b} - (\Delta - 2b) & \text{if } \Delta^C_{full} \leq \Delta \leq \Delta^C_{shut} \\ 0 & \text{if } \Delta > \Delta^C_{shut} \end{cases} \tag{18}$$

Figure 11 depicts P's utility.

The following lemma provides a simpler way to view P's utility as a function of Δ. The proofs of all the lemmas and propositions are in the Appendix.

Lemma 1. *There exists a $\Delta^{C^*} \in (\Delta^C_{col}, \Delta^C_{int})$ such that $U_P^{C^*}(\Delta)$ has a single peak at Δ^{C^*}. Δ^{C^*} is strictly increasing in b and in λ.*[23]

The lemma above says that P's utility initially increases in the level of heterogeneity, peaks at Δ^{C^*}, after which it starts to decrease. Thus a principal who faced no constraints on Δ, would choose Δ^{C^*} as the optimal level of heterogeneity under the competitive structure.

To understand this lemma, notice that for low levels of heterogeneity, pushing preferences apart leads to more competition which P benefits from. At some point, however, collaboration becomes difficult and the central tradeoff between competition and collaboration kicks in. This forces P at first to distort monetary incentives and then to induce more compromise which undoes the gains from competition.

[23]The comparative static of Δ^{C^*} with respect to the information parameter h is analytically less tractable, although numerical examples suggest that it is increasing.

Figure 8: Monetary Incentives in the Competitive Structure ($h = \frac{7}{10}$, $b = \frac{1}{7}$, $\lambda = 1$.)

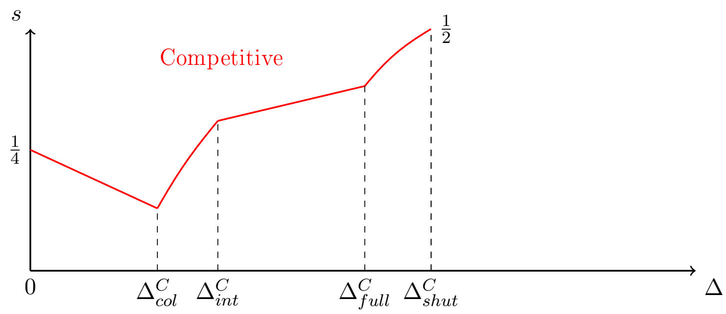


Figure 9: Quality in the Competitive Structure($h = \frac{7}{10}$, $b = \frac{1}{7}$, $\lambda = 1$.)

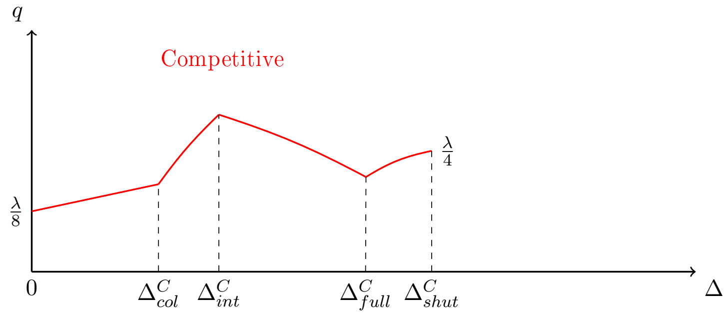


Figure 10: Compromise in the Competitive Structure ($h = \frac{7}{10}$, $b = \frac{1}{7}$, $\lambda = 1$.)

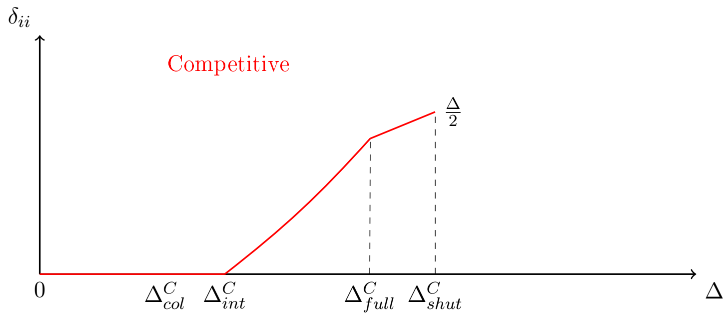
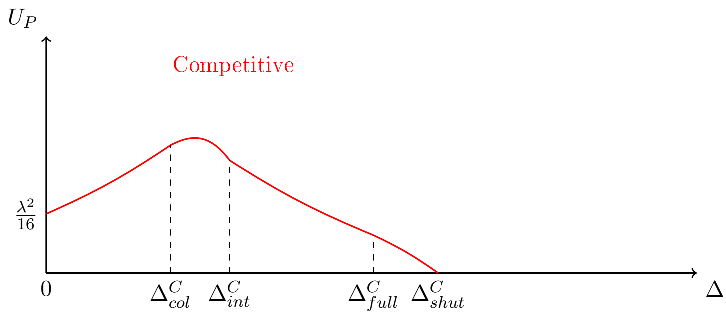


Figure 11: P's Utility in the Competitive Structure ($h = \frac{7}{10}$, $b = \frac{1}{7}$, $\lambda = 1$.)



This effect of compromise across heterogeneous agents undermining competition can be seen in Figure 9 where quality decreases in the region where compromise has an interior solution. To understand why this is the case, we can combine (IC_q^C) and (IC_{col}^C) for interior levels of compromise to get:

$$q = \frac{\frac{1}{2}s\lambda + h(2b - \Delta)}{1 - 2hs\lambda} \tag{19}$$

As Δ increases in this region with interior compromise, there are two effects. A direct effect which reduces quality in the equation above and an indirect effect which increases quality because incentives are rising. As can be seen from Figure 9, the direct effect dominates the indirect one, so that quality decreases in heterogeneity in this region.

One thing to notice is that P's maximum utility under the competitive structure involves no compromise. This relies on the linearity of the distance cost over the Hotelling line. Because P can choose the ideal points of agents, she can replicate project outcomes with compromise by hiring agents with more closely aligned preferences. This keeps incentives for quality the same but makes collaboration easier.

It is also useful to note that P's utility is strictly increasing in the parameters h, λ and b. This is stated in the lemma below.

Lemma 2. *The utility of the principal under the competitive structure, $U_P^{C^*}$, is strictly increasing in h, λ, and b.*

A higher h implies a more precise signal for the principal which leads to more competition across agents, thereby increasing P's utility. The private benefit b relaxes collaboration constraints. And finally, a more productive technology also increases P's utility.

6 Comparing Structures

We now turn to the comparison between both organizational structures. We assume that whenever P is indifferent between structures, she chooses the noncompetitive one.

The following proposition shows that when heterogeneity is very high, a noncompetitive structure is actually preferred to a competitive one:

Proposition 1. *The principal can continue operating a project profitably for higher levels of heterogeneity under the noncompetitive structure than under the competitive structure. That is, $\Delta_{shut}^N > \Delta_{shut}^C$.*

As the organization gets more heterogeneous, collaboration becomes more difficult until P is forced to shut a project down. This shut down condition for the noncompetitive structure is $\frac{\lambda^2}{2} + 2b$ whereas the condition under the competitive structure is $\frac{\lambda^2}{4} + 2b$. There are two parts to the shutdown condition. First as the agent's benefit from his ideal project b goes up, collaboration becomes easier under both structures moving the shut down level of heterogeneity out by $2b$ for both structures. Second, as the organization gets very heterogeneous, agents have to compromise fully (i.e. $\delta_{ii} = \frac{\Delta}{2}$) so that competition has no incentive benefits. Given that incentives are only half as effective in the competitive structure, the shut down level of heterogeneity under competition for this part, $\frac{\lambda^2}{4}$, is half that of the shutdown level under no competition, which is given by $\frac{\lambda^2}{2}$.

While the noncompetitive structure dominates the competitive one for a very homogenous organization and for a very heterogeneous organization, for intermediate levels of heterogeneity, a competitive structure can be better for the principal. This is demonstrated in the following proposition:

Proposition 2. *For a large enough private benefit b for the agents' ideal project and for a sufficiently large h so that the principal is well informed, there exists a level of heterogeneity for which P's utility under the competitive structure is larger than the maximum possible utility (over Δ) from the noncompetitive structure.*

Proposition 2 says that when the agent's private benefit from his ideal project b is large enough and when the signal about relative quality is not too noisy (so that h is high enough), then there is some Δ such that $U_P^{C^*}(\Delta) > \frac{\lambda^2}{8}$, which is the maximum possible utility under the noncompetitive structure. With a better informed principal, agents have stronger incentives to compete and increase quality whereas a larger private benefit b relaxes the collaboration constraint so that the principal can have a more heterogeneous organization to capture the benefits of competition.

Propositions 1 and 2 are contrary to what one might expect, given the existing literature. For example, in Hirsch and Shotts (2015), P's utility increases in the level of heterogeneity because agents with divergent preferences have a greater incentive to compete. These different results in our paper come from the fact that the model takes into account both collaborative and competitive effects.

The following lemma helps us to pin down the cases that can arise when we compare payoffs across structures.

Lemma 3. *i For $\Delta < \Delta^{C^*}$, $\frac{\partial U_P^{C^*}}{\partial \Delta}(\Delta) > \frac{\partial U_P^{N^*}}{\partial \Delta}(\Delta)$.*

ii For $\Delta > \Delta^{C^*}$, $\frac{\partial U_P^{C^*}}{\partial \Delta}(\Delta) < \frac{\partial U_P^{N^*}}{\partial \Delta}(\Delta)$.

The lemma tells us that P's utilities for both the structures can cross at most twice (once on the upward sloping part of $U_P^{C^*}$ and once on the downward sloping part of $U_P^{C^*}$). It is useful to define a cutoff level of heterogeneity $\tilde{k}$.

Definition 1. *The cutoff level of heterogeneity $\tilde{k}$ is defined as:*

$$\tilde{k} = \begin{cases} 0 & \text{if } U_P^{N*}(\Delta) \geq U_P^{C*}(\Delta) \text{ for all } \Delta \in [0, \Delta_{shut}^N] \\ \tilde{\Delta} & \text{if } U_P^{C*}(\Delta^{C*}) > U_P^{N*}(\Delta^{C*}) \text{ and where } U_P^{C*}(\tilde{\Delta}) = U_P^{N*}(\tilde{\Delta}) \text{ and } \tilde{\Delta} > \Delta^{C*}. \end{cases} \tag{20}$$

There are two main cases to consider when comparing structures.[24]

In the first case, where h tends to 0 and where the agents' benefit from their ideal project b is 0, the noncompetitive structure dominates the competitive structure for all levels of heterogeneity (see Figure 15). To understand why, notice that competition has no incentive benefits when the principal is uninformed. Furthermore monetary incentives under the competitive structure are only half as effective because only one of the two developed projects goes to market. As a result, P has to start distorting incentives upwards for a lower level of heterogeneity in the competitive structure (see Figure 12). Also quality is always higher in the noncompetitive structure for any Δ and less compromise is needed to secure collaboration (see Figure 13 and Figure 14).

The second case has positive and sufficiently large levels h and b. For this case, the noncompetitive structure is better for P for extreme values of Δ, low and high. For intermediate values of Δ, however, the competitive structure does better. U_P^{C*} and U_P^{N*} cross twice with $\tilde{k}$ being the larger of the two Δ's where the utilities for different structures cross. Figure 19 shows P's utility for this case. There are also clear patterns that emerge with the design variables: s, q and δ_{ii}. Monetary incentives under competition are lower for small levels of heterogeneity because P can use competition to induce quality instead of giving up a share of the surplus. But for higher levels of heterogeneity, compromise blunts competition, and requires the use of monetary incentives to get agents to collaborate. Because these monetary incentives are diluted in the competitive case, s is higher relative to the noncompetitive structure (see Figure 16). Quality, on the other hand, is larger under competition for intermediate levels of heterogeneity, where agents have some incentives to compete but where collaboration is not too difficult as well (see Figure 17). Finally, compromise is lower under the competitive structure (see Figure 18) where it is more costly to use.

7 Endogenous Culture

Finally, we solve for the optimal culture and structure of the organization. Let Δ^* be the optimal culture of the organization.

[24]There is a third case where $h > 0$ and $b = 0$. For all the numerical examples that we have looked at, the competitive structure never yields a higher utility for the principal relative to the noncompetitive one.

Figure 12: Comparing Structures: Monetary Incentives ($h \to 0$, $b = 0$, $\lambda = 1$).

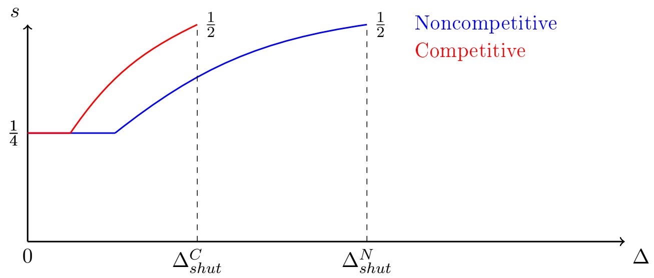


Figure 13: Comparing Structures: Quality ($h \to 0$, $b = 0$, $\lambda = 1$).

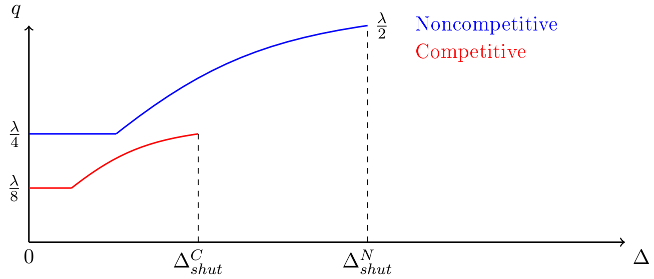


Figure 14: Comparing Structures: Compromise ($h \to 0$, $b = 0$, $\lambda = 1$).

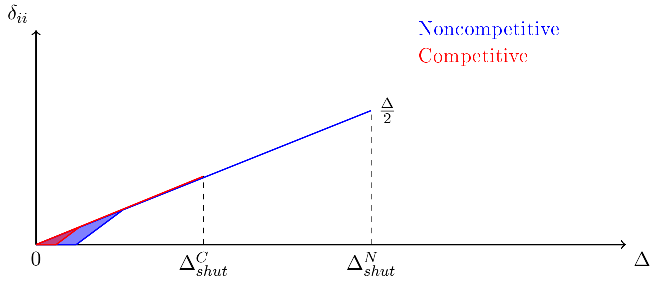


Figure 15: Comparing Structures: P's Utility ($h \to 0$, $b = 0$, $\lambda = 1$).

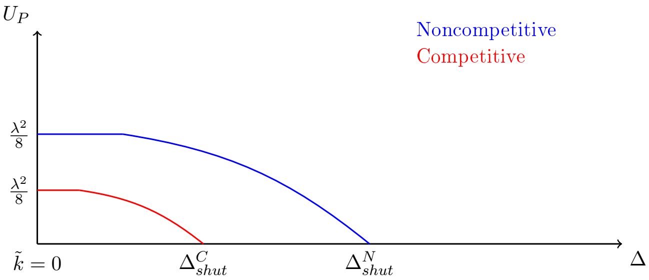


Figure 16: Comparing Structures: Monetary Incentives ($h = \frac{7}{10}$, $b = \frac{1}{7}$, $\lambda = 1$).

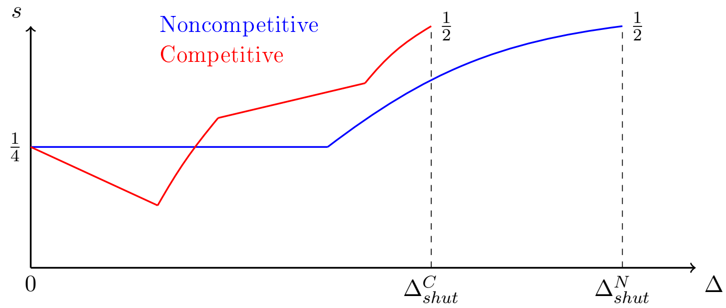


Figure 17: Comparing Structures: Quality ($h = \frac{7}{10}$, $b = \frac{1}{7}$, $\lambda = 1$).

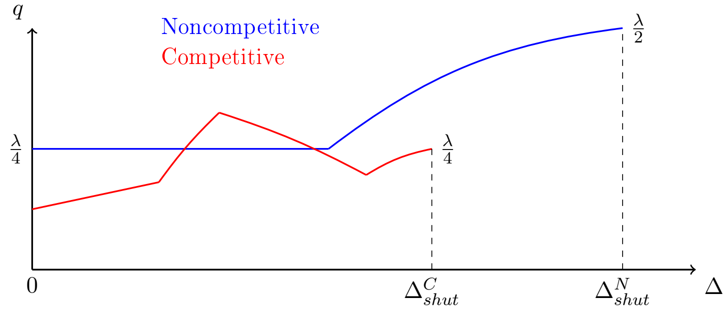


Figure 18: Comparing Structures: Compromise ($h = \frac{7}{10}$, $b = \frac{1}{7}$, $\lambda = 1$).

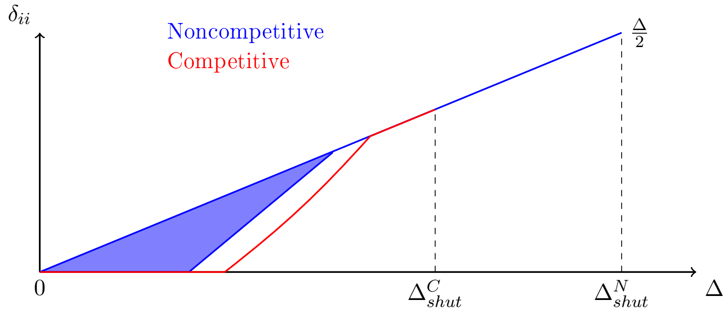
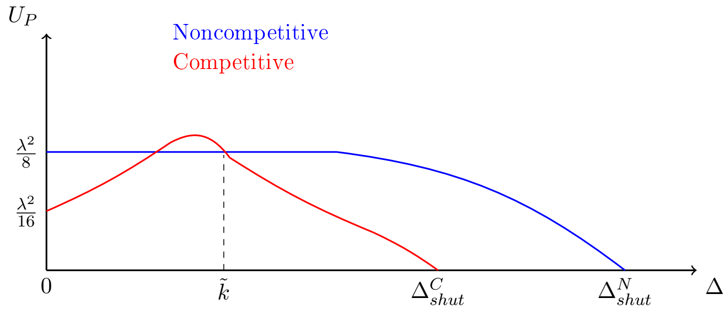


Figure 19: Comparing Structures: P's Utility ($h = \frac{7}{10}$, $b = \frac{1}{7}$, $\lambda = 1$).



Proposition 3. *The competitive structure is optimal if and only if $k < \tilde{k}$. Furthermore,*

$$\Delta^* \in \begin{cases} \{k\} & \text{if } k \geq \max\{\tilde{k}, \Delta^N_{full}\} \\ [k, \Delta^N_{full}] & \text{if } \tilde{k} \leq k \leq \Delta^N_{full} \\ \{k\} & \text{if } \Delta^{C^*} < k < \tilde{k} \\ \{\Delta^{C^*}\} & \text{if } k \leq \Delta^{C^*} \text{ and } \tilde{k} \neq 0 \end{cases} \tag{21}$$

Proposition 3 is the main result of our paper. It tells us that a coherent culture and internal competition complement one another. When k is large, the degree of coherence in the organization is limited. As such, the organization is necessarily heterogeneous and the noncompetitive structure is more likely to be optimal. When k is small, on the other hand, with the possibility of more coherence, the organization is not as heterogeneous. For this case, the competitive structure is more likely to be optimal. This is because collaboration can be achieved with less compromise leading to unrestrained competition.

The parameter k can be thought of as the ability of the organization to articulate its mission clearly so as to attract employees with similar preferences. Going back to our Microsoft example, a low k corresponds to the company under its new CEO, Satya Nadella. When Nadella took over, his top priorities were to, "Communicate clearly and regularly our sense of mission, world-view and business and innovation ambitions," and to "Drive cultural change from top to bottom, and get the right team in the right place".[25] These priorities have made a more coherent culture possible at Microsoft. But contrary to what the proposition recommends, Microsoft has moved away from its old structure of internal competition.

Internal competition and a coherent culture are complements when it comes to our main underlying parameter of interest: k. However, this complementarity may not hold if we vary the private benefit parameter b. This is because the optimal level of heterogeneity associated with the competitive structure Δ^{C^*} is increasing in b. There could be a range of parameters for which a switch to internal competition goes along with a more heterogeneous organization. Similarly, the complementarity may not hold with respect to varying the information parameter h.

8 Discussion of Assumptions

We have made a number of assumptions throughout the paper which deserve some further explanation.

Collaboration: One assumption that we have made is that collaboration by the non-leading agent is essential for a project to yield returns. This is a stark, but

[25] See Nadella, Shaw, and Nichols (2017).

tractable, way to capture gains from collaboration. The simplest way to think about these gains, is that divisions that make different products have different comparative advantages. Another interpretation within the context of Microsoft, is that while software is primarily developed for one platform, it is more valuable if it works well across multiple platforms. Relaxing the necessity of collaboration in production does not change the main tradeoff in our model. It only makes competition more valuable as preferences diverge more.

Quality Privately Observable: Another assumption made is that agents privately observe the quality of their own project. The principal only observes a noisy signal of the relative qualities when there are two projects. This assumption is necessary for the existence of a pure-strategy Nash equilibrium. If the principal has perfect information, the optimal choice of q by an agent is a mixed strategy. The intuition for why this is so is that, the competition for quality under perfect information in the competitive structure is essentially a Bertrand-style competition. However, unlike in a standard Bertrand-competition, producing quality when the marginal cost is higher than the marginal benefit is an optimal strategy, because an agent's utility experiences a discontinuous jump from the fact that a project closer to their ideal point is chosen.

Symmetric Monetary Incentives: Our restriction to symmetric monetary incentives is motivated by the fact that agents are identical at the time of hiring and that there may be costs to committing to a proposal upfront at the time of hiring. For example, different monetary incentives may signal certain productive attributes of a division which may be costly. Or there may be costs to treating divisions differently when its members are identical otherwise. We could relax this assumption and have different shares for the project leader (s') and for the non-leading agent (s) so that incentives can target quality and collaboration separately. The only thing that changes is the incentive constraints for quality. In the noncompetitive structure, the incentive constraint becomes:

$$q_1 = s'\lambda \tag{$IC_q^{N'}$}$$

and in the competitive structure, we have:

$$q_i = \frac{\frac{1}{2}s'\lambda + h(\Delta - 2\delta_{ii})}{1 - \lambda h(s' - s)} \tag{$IC_q^{C'}$}$$

This leads to an increase in the principal's utility for both structures. But it does not change the central mechanism whereby more homogeneity leads to less compromise and thus more intense competition. However, the analysis, especially for the competitive structure, is significantly more complicated and does not add any insights to our analysis.

Specialization: Agents in our framework can only develop projects on their side of the Hotelling line – that is we assume that agents are specialized when it comes to project development. Given agents may have a comparative advantage for certain types of projects, we think this is a reasonable assumption with the context of our Microsoft example. One way to relax this assumption is to modify our framework in two ways. First, assume that a leader with some small probability can 'hold back' the project at the last stage just before it goes to market. Second, suppose this holding back of a project in the last stage is very costly to the principal. Then, making the leader compromise too much by developing a project on the other side of the Hotelling line, relaxes the collaboration constraint, but will lead to the leader holding back the project at the very end with a small probability. For this modified framework, specialization arises optimally.

Participation: Throughout our analysis, we have abstracted from participation considerations by assuming a sufficiently large fixed wage $\underline{w}$ and a non-contractible benefit B that the principal gets from hiring the agent. This allows us to clearly identify the role that the collaboration constraint plays in our analysis. Lowering $\underline{w}$ makes the participation constraint bind for a certain level of heterogeneity whereas lowering the non-contractible benefit B affects the principal's participation constraint. An alternative assumption that we could have used is that the agent gains valuable human capital from working in the firm which more than offsets the costs of developing quality in a project.

Agents' private benefits b: In our model, agents get a private benefit $b \geq 0$ for their ideal project and these benefits get diluted for projects that are further away on the Hotelling line. There are other ways to model these private benefits.

First, we could assume that an agent gets a benefit $\hat{b}$ only when the project that he leads goes to market. With this specification, the incentive constraint under competition becomes:

$$q = \frac{1}{2}s\lambda + \hat{b}h + h(\Delta - 2\delta_{ii}) \qquad (\hat{IC}_q^C)$$

and the collaboration constraint for either structure is:

$$s\lambda q \geq \Delta - \delta_{ii} \qquad (\hat{IC}_{col})$$

Once again, all of our main results go through as long as $\hat{b}$ is not too large.[26]

Second, we could assume that the private benefit to the agent also depends on the quality of the project; for example suppose the benefit from the ideal project is $b + \tilde{b}q$. For this specification, the incentive constraints become:

[26] In particular, $\hat{b} \leq \frac{\lambda}{4h}$. For $\hat{b}$ above this threshold, the shut down level of Δ can be larger in the competitive case. However, for $\hat{b}$ above this threshold, optimal monetary incentives will be 0 for a range of Δ's.

$$q_1 = s\lambda + \tilde{b} \tag{$\tilde{IC_q^N}$}$$

and,

$$q = \frac{1}{2}(s\lambda + \tilde{b}) + h(\Delta - 2\delta_{ii}) \tag{$\tilde{IC_q^C}$}$$

and the collaboration constraint for either structure is:

$$(s\lambda + \tilde{b})q + b \geq \Delta - \delta_{ii} \tag{$\tilde{IC_{col}}$}$$

Once again, this does not change our main results. The level of heterogeneity where the principal shuts the project down is still lower under the competitive structure and for h and b sufficiently large, the competitive structure yields a higher utility than the noncompetitive one.

Incentives contingent on a product going to market: One option for the principal is to condition agents' pay on whether the project goes to market or not; call this contingent payment w_m. In terms of the model, this plays exactly the same role as the private benefit parameter b, except that the choice of this benefit is endogenous and costly for the principal. It turns out, however, that this contingent payment is zero at the optimum. To see why, notice that the purpose of this payment is to induce the non-leading agent to collaborate. When the collaboration constraint does not bind, this payment has no role to play. On the other hand, when the collaboration constraint does bind, the cost of this payment which is $2w_m$ offsets the benefit in terms of making collaboration easier. This can be seen from the principal's payoffs in (12) and (18) where the benefit from relaxing the collaboration constraint is less than or equal to $2w_m$.

9 Conclusion

In this paper, we construct a framework to understand the interplay between competition, collaboration and the culture of an organization. We model culture in a tractable way using a standard tool in the economics literature – the Hotelling line. We find that when collaboration is essential, a coherent culture and internal competition in the organization complement each other. The intuition is simple. A more coherent culture requires less compromise for agents to collaborate which leads to unfettered competition.

Going back to our motivating questions on the organizational changes at Microsoft, our theory suggests that Microsoft was right to switch to a noncompetitive structure given the conflicting interests across its divisions. The key here is the need for collaboration – without it, competition has larger benefits as preferences diverge. But as our analysis shows, Microsoft may be missing something by viewing a coherent culture and competition as substitutes; a coherent culture can harness the benefits of competition better when collaboration is essential. We believe that this insight can be applied across many organizational settings to manage the tradeoff between competition and collaboration.

More generally, as economists we have long understood the virtues of competition in providing incentives. Less well understood is the problem of how to use competition in environments that require collaboration. While our focus has mainly been on organizations, this tradeoff between competition and collaboration arises naturally in other contexts too. For example, in politics, parties may have different ideological preferences but still need to work together to get things done. We believe that our paper serves as a useful starting point to explore these themes.

10 Appendix

10.1 Derivation of Solution for the Noncompetitive Structure

10.1.1 Optimal Incentives, Quality, and Compromise

Consider P's optimization problem for the noncompetitive structure. Substituting out for q_1 we can rewrite P's problem as:

$$\max_{s,\delta_{11}} U_P^N = (1-2s)\lambda^2 s$$

$$\text{s.t } \max\{0, \Delta - b - s^2\lambda^2\} \leq \delta_{11} \tag{22}$$

$$\text{s.t } \delta_{11} \leq \frac{\Delta}{2} \tag{23}$$

Since δ_{11} does not affect P's utility directly, we can combine the constraints above as:

$$\Delta - b - s^2\lambda^2 \leq \frac{\Delta}{2} \tag{24}$$

We can then maximize P's utility with respect to s and set δ_{11} to satisfy (22) and (23).

The first order necessary conditions are:

$$\lambda^2(1-4s) = -\mu\lambda^2(2s)$$

$$\mu(\frac{\Delta}{2} - b - s^2\lambda^2) = 0$$

where μ is the non-negative multiplier.

There are two possible cases. First, suppose the constraint (24) above does not bind. Then from complementary slackness, $\mu = 0$, which yields $s = \frac{1}{4}$. This case holds as long as:

$$-(\frac{\lambda}{4})^2 < b - \frac{\Delta}{2} \tag{25}$$

The condition in (25) reduces to $\Delta < \frac{\lambda^2}{8} + 2b \equiv \Delta^N_{full}$. And $\delta_{11} \in [\max\{0, \Delta - b - \frac{\lambda^2}{16}\}, \frac{\Delta}{2}]$ so that (22) and (23) are satisfied.

Second, suppose (24) does bind. Then s is pinned down by:

$$s^2 = \frac{1}{2\lambda^2}(\Delta - 2b)$$

This yields $s = \frac{\sqrt{\Delta - 2b}}{\sqrt{2}\lambda}$. And $\delta_{11} = \frac{\Delta}{2}$ from (22) and (23).

Note that $\mu \geq 0$ if and only if $\frac{\sqrt{\Delta-2b}}{\sqrt{2}\lambda} \geq \frac{1}{4}$ which holds when $\Delta \geq \Delta^N_{full} + 2b$. Because the objective function is concave and the constraint function quasiconvex, these first order conditions are sufficient.

To compute the shut down condition, observe that P's optimal utility $\lambda\sqrt{\frac{1}{2}(\Delta - 2b)} - (\Delta - 2b) \geq 0$ if and only if $\Delta \leq \frac{\lambda^2}{2} + 2b$.

Finally, consider the participation constraints. A_2's participation follows from (7).

For A_1, because the proposal is on his side of the Hotelling line, (7) implies:

$$s\lambda q_1 + b \geq \delta_{11}$$

and since q_1 is bounded above by $\frac{\lambda}{2}$ and since Assumption 1 holds, it follows that:

$$\begin{aligned} &\underline{w} - \frac{1}{2}q_1^2 \geq 0 \\ &\therefore U_1 = \underline{w} + s\lambda q_1 - \frac{1}{2}q_1^2 + b - \delta_{11} \geq 0. \end{aligned}$$

10.2 Derivation of Solution for the Competitive Structure

10.2.1 Project Development Phase: First and Second Order Conditions

As mentioned in the body, the expected utility of A_i is given by:

$$EU_i = p\left(s\lambda q_i + b - \delta_{ii}\right) + (1-p)\left(s\lambda q_j + b - \delta_{ij}\right) - \frac{1}{2}q_i^2$$

The First Order Condition is therefore:[27]

$$\begin{aligned} 0 &= -q_i + s\lambda\left(q_i p'(Q) + p\right) - \delta_{ii}p'(Q) + \delta_{ij}p'(Q) - s\lambda q_j p'(Q) \\ \therefore q_i &= ps\lambda + s\lambda p'(Q)\left(q_i - q_j\right) + p'(Q)\left(\delta_{ij} - \delta_{ii}\right) \end{aligned}$$

where $Q \equiv q_i - q_j$. From the symmetry of quality and proposals, we have $q_i = q_j = q$ and $p = \frac{1}{2}$, and from (5) we have $\delta_{ij} = \delta_{ji} = \Delta - \delta_{ii}$. Thus the first order conditions can be written as:

$$\therefore q = \frac{1}{2}s\lambda + h\left(\Delta - 2\delta_{ii}\right)$$

For the q identified above to be a maximum, the Second Order Condition must be negative. Denoting the derivative of the probability density function as p'', the Second Order Condition becomes:

$$\text{SOC} = 2s\lambda p' + s\lambda p''\left(q_i - q_j\right) + p''\left(\delta_{12} - \delta_{11}\right) - 1$$

Consider three cases. When $Q = 0$, $q_i = q_j$, so that $p' = h$ and $p'' = 0$. Therefore, the SOC simplifies to:

$$2hs\lambda - 1 < 0$$

Since the maximum of s is $\frac{1}{2}$, we get a necessary condition that $\lambda h < 1$.

Next, suppose $Q > 0$, then $p'' < 0$ so that the second order condition holds when $\lambda h < 1$.

Finally suppose $Q < 0$. Remembering that p is normally distributed, we have:

$$\begin{aligned} p' &= \frac{1}{\sqrt{2\pi\sigma^2}} \times e^{\frac{-Q^2}{2\sigma^2}} \\ \implies p'' &= \frac{-Qp'}{\sigma^2} \end{aligned}$$

Thus we can rewrite the expression for the second order condition as:

$$\text{SOC} = 2s\lambda p' - 1 - \frac{p'Q}{\sigma^2} s\lambda Q - \frac{p'Q}{\sigma^2}(\delta_{12} - \delta_{11})$$

Since $(\delta_{ij} - \delta_{ii})$ is bounded above by Δ, which is in turn bounded above by $\frac{\lambda^2}{4} + 2b$ (as seen in Proposition 1), it follows that the second order condition holds when $\lambda h < 1$, λ is sufficiently small, and when b is not too large relative to λ.

10.2.2 Shut Down Condition

For $\Delta > \Delta^C_{shut} \equiv \frac{\lambda^2}{4} + 2b$ the constraints (IC^C_q) and (IC^C_{col}) cannot hold together. To see why, let $\delta_{ii} = \frac{\Delta}{2} - \alpha$ where $\alpha \geq 0$. Substituting δ_{ii} into (IC^C_q) gives us:

$$q = \frac{1}{2}s\lambda + 2h\alpha \tag{26}$$

Suppose $\Delta > \Delta^C_{shut} \equiv \frac{\lambda^2}{4} + 2b$. Then from (26) we have:

$$\begin{aligned} s\lambda q + b &= \frac{1}{2}s^2\lambda^2 + 2s\lambda h\alpha + b \\ &\leq \frac{\lambda^2}{8} + \lambda h\alpha + b \qquad (27) \\ &\leq \frac{\lambda^2}{8} + \alpha + b \qquad (28) \\ &< \frac{\Delta}{2} + \alpha \\ &= \Delta - \delta_{ii} \end{aligned}$$

where (27) follows from the fact that $s \leq \frac{1}{2}$ and where the (28) follows from the first part of Assumption 1. Thus the collaboration constraint does not hold for $\Delta > \Delta^C_{shut}$.

10.2.3 Optimal Incentives, Quality, and Compromise

To solve for the optimal level of incentives, quality and compromise under the competitive structure, we split the problem into two cases: a case where the collaboration constraint (IC_{col}^{C}) does not bind and a case where (IC_{col}^{C}) does bind.

Case 1: Non binding collaboration constraint

For the case where the collaboration constraint does not bind, P optimally sets $\delta_{ii} = 0$ (as compromise reduces quality). P's problem for this case, after substituting out for q is:

$$\max_{s} U_P^C = (1-2s)\lambda(\frac{1}{2}s\lambda + h\Delta)$$

The first order necessary condition yields:

$$s = \frac{1}{4} - \frac{h\Delta}{\lambda}$$

Since the objective function is concave in s, the first order necessary condition is sufficient. This case holds as long as:

$$s\lambda q + b > \Delta$$
$$\therefore \frac{1}{2}(\frac{\lambda^2}{16} - h^2\Delta^2) + b > \Delta$$

The cutoff Δ_{col}^{C} is determined implicitly by the equation:

$$\frac{1}{2}(\frac{\lambda^2}{16} - h^2\Delta_{col}^{C^2}) + b = \Delta_{col}^{C}$$

Case 2: Binding collaboration constraint

Next, consider the case where the collaboration constraint binds. Substituting out for δ_{ii} in the constraints and restricting our attention to levels of s where P's utility is non-negative, we can rewrite P's problem as:

$$
\begin{aligned}
\max_{s\in\left[\underline{s},\frac{1}{2}\right]} U_P^C =&(1-2s)\lambda q \\
\text{s.t. } q =& \frac{\frac{1}{2}s\lambda + h(2b-\Delta)}{1-2hs\lambda} \\
\text{s.t. } s\lambda q - \Delta \leq 0 & \quad (\mu_1) \\
\text{s.t. } \frac{1}{2}\Delta - s\lambda q \leq 0 & \quad (\mu_2) \\
\text{where } \underline{s} =& \begin{cases} 0 \text{ if } 2b \geq \Delta \\ \frac{2h}{\lambda}(\Delta - 2b) \text{ if } 2b < \Delta \end{cases}
\end{aligned}
$$

We first characterize the necessary first order conditions and then show that they are sufficient. The Kuhn-Tucker conditions are:

$$\lambda\left[(1-2s)\,q'\,(s) - 2q(s)\right] = \mu_1\lambda\left[sq'\,(s) + q\,(s)\right] - \mu_2\lambda\left[sq'\,(s) + q\,(s)\right] \tag{29}$$

where:

$$q'(s) = \frac{\lambda(1+4h^2(2b-\Delta))}{2(1-2hs\lambda)^2} \tag{30}$$

Note that the derivative in (30) is positive from part (i) of Assumption 1, as long as collaboration is feasible – that is the derivative is positive whenever $\Delta \leq \Delta_{shut}^C \equiv \frac{\lambda^2}{4} + 2b$.

Consider three subcases.

Subcase 1: $s\lambda q + b = \Delta$ (or $\delta_{ii} = 0$) $\implies \mu_1 \geq 0$, $\mu_2 = 0$. .

$$
\begin{aligned}
\therefore \frac{\Delta - b}{s\lambda} &= \frac{\frac{1}{2}s\lambda + h(2b-\Delta)}{1-2hs\lambda} \\
\implies s &= \frac{-h\Delta + \sqrt{2(\Delta-b) + h^2\Delta^2}}{\lambda} \\
\text{and } q &= \frac{h\Delta + \sqrt{2(\Delta-b) + h^2\Delta^2}}{2}
\end{aligned}
\tag{31}
$$

This case holds as long as:

$$
\begin{aligned}
&(1-2s)q'(s) - 2q(s) \geq 0 \\
\implies &\lambda(1-4s(1-\lambda hs)) \geq 4h(2b-\Delta)(1-\lambda h) \\
&\text{where } s \text{ satisfies (31)}
\end{aligned}
\tag{32}
$$

The cutoff Δ^C_{int} is determined from (32).

Subcase 2: $s\lambda q + b = \frac{1}{2}\Delta$ (or $\delta_{ii} = \frac{1}{2}\Delta$) $\implies$ $\mu_1 = 0$, $\mu_2 \geq 0$.

$$\begin{aligned}
\therefore \frac{\Delta - 2b}{2s\lambda} &= \frac{\frac{1}{2}s\lambda + h(2b - \Delta)}{1 - 2hs\lambda} \\
\implies s &= \frac{\sqrt{\Delta - 2b}}{\lambda} \\
\text{and } q &= \frac{\sqrt{\Delta - 2b}}{2}
\end{aligned} \tag{33}$$

This case holds as long as:

$$\begin{aligned}
&(1 - 2s)q'(s) - 2q(s) \leq 0 \\
\implies &\lambda(1 - 4s(1 - \lambda hs)) \leq 4h(2b - \Delta)(1 - \lambda h) \\
&\text{where } s \text{ satisfies (33)}
\end{aligned} \tag{34}$$

The cutoff Δ^C_{full} is determined from (34). It must be the case that $\Delta^C_{int} < \Delta^C_{full}$, otherwise there are multiple solutions, which cannot be the case as the objective function is strictly concave and the constraint set convex (as will be shown below).

Subcase 3: An interior δ_{ii} with $\mu_1 = 0$, $\mu_2 = 0$.

$$\begin{aligned}
\therefore 0 &= \lambda q_i(-2s) + (1 - 2s)q'(s) \\
\implies s &= \frac{1 - \sqrt{(1 - \lambda h)(1 + 4h^2(2b - \Delta))}}{2\lambda h} \\
\text{and } q &= \frac{\frac{1}{2}s\lambda + h\,(2b - \Delta)}{1 - 2hs\lambda}
\end{aligned} \tag{35}$$

where s satisfies (35)

This case holds as long as $\Delta^C_{int} \leq \Delta \leq \Delta^C_{full}$.

Since $q'(s)$ in (30) is positive, the function $s\lambda q(s)$ is strictly increasing in s. This implies that the constraint functions $s\lambda q - \Delta$ and $\frac{1}{2}\Delta - s\lambda q$ are quasiconvex. Also from Assumption 1, we have:

$$\begin{aligned}
\frac{\partial^2 (1 - 2s)\lambda q}{\partial s^2} &= \lambda((1 - 2s)q''(s) - 4q'(s)) \\
&= \frac{2\lambda q'(s)(\lambda h - 1)}{(1 - 2hs\lambda)} \\
&< 0
\end{aligned}$$

so that the objective function is strictly concave. Thus the first order conditions to this problem are both necessary and sufficient.

Finally, consider the participation constraint for the agents. For the non-leading agent, the collaboration constraint implies participation.

For the leading agent, notice that q_i is bounded above by $\frac{\lambda}{2} + 2bh$. From the second part of Assumption 1, it follows that $\underline{w} \geq \frac{1}{2}q_1^2$. Because the proposal for a leader is on his own side of the Hotelling line, the collaboration constraint therefore implies:

$$U_i = \underline{w} + s\lambda q_i - \frac{1}{2}q_i^2 + b - \delta_{ii} \geq 0.$$

.

10.2.4 Curvature of U_P^{C*}

We consider 4 different regions.

Region 1: $0 \leq \Delta \leq \Delta_{col}^C$.

U_P^{C*} is convex in Δ as the sum of convex functions is convex.

Region 2: $\Delta_{col}^C \leq \Delta \leq \Delta_{int}^C$.

For this region $U_P^{C*} = \frac{\lambda}{2}(h\Delta + \sqrt{h^2\Delta^2 + 2(\Delta - b)}) - 2(\Delta - b)$. Taking the second derivative with respect to Δ we get:

$$\begin{aligned} \frac{d^2 U_P^{C*}}{d\Delta^2} &= \frac{(h^2\Delta^2 + 2(\Delta - b))h^2 - (h^2\Delta + 1)^2}{(h^2\Delta^2 + 2(\Delta - b))^{\frac{3}{2}}} \\ &= \frac{-2bh^2 - 1}{(h^2\Delta^2 + 2(\Delta - b))^{\frac{3}{2}}} \\ &< 0 \end{aligned}$$

Thus U_P^{C*} is concave in Δ.

Region 3: $\Delta_{int}^C \leq \Delta \leq \Delta_{full}^C$.

Consider the optimization problem under a binding collaboration constraint (Case 2) on page 31. Substitute the quality constraint in this problem into the objective function. Since P's utility is linear in Δ and thus convex, P's utility U_P^{C*} is also convex in Δ.

Region 4: $\Delta_{full}^C \leq \Delta \leq \Delta_{shut}^C$.

U_P^{C*} is concave in Δ as a strictly increasing and concave transformation of a concave function is concave and as the sum of concave functions is concave.

10.2.5 Symmetry in Quality, Tournament, and Proposals

When quality is asymmetric in an equilibrium with collaboration, the principal (taking these strategies as given) selects the project with higher quality with probability 1. Thus this case reduces to the noncompetitive structure.

Since quality must be symmetric in an equilibrium with collaboration, the first order necessary conditions for each agent in the project development phase are:

For Agent i:

$$q = ps\lambda + p'(x)\left(\delta_{ij} - \delta_{ii}\right)$$

For Agent j:

$$q = (1-p)s\lambda + p'(x)\left(\delta_{ji} - \delta_{jj}\right)$$

Adding both the equations and multiplying both sides by $\frac{1}{2}$, we get the incentive constraint for quality:

$$q = \frac{s\lambda}{2} + \frac{h(x)}{2}((\delta_{ij} - \delta_{ii}) + (\delta_{ji} - \delta_{jj}))$$

where $h(x) \equiv p'(x)$. Suppose the tournament and the proposals are not symmetric at the optimum. Then set $\delta'_{ii} = \delta'_{jj} = \frac{\delta_{ii}+\delta_{jj}}{2}$, $\delta'_{ij} = \delta'_{ji} = \frac{\delta_{ij}+\delta_{ji}}{2}$ and $x = 0$. Since ε is normally distributed with mean 0, h is maximized at $x = 0$. This increases quality while collaboration still holds since $\delta'_{ij} = \delta'_{ji} \leq \max\{\delta_{ij}, \delta_{ji}\}$. It follows that there is no loss of generality in considering symmetric tournaments and symmetric proposals.

Finally, because the distance cost is linear, symmetric proposals that lie to either extreme of the ideal points, can be replaced by proposals corresponding to the ideal points to yield the same quality while making collaboration easier.

10.3 Proofs of Lemmas and Propositions

Proof of Lemma 1. From (18), P's optimal utility is strictly increasing in Δ when $\Delta \leq \Delta^C_{col}$. Also, using the envelope theorem $\frac{\partial U^C_P}{\partial \Delta} = -\dfrac{(1-2s^C)\lambda h}{(1-2hs^C\lambda)}$ when $\Delta^C_{int} \leq \Delta \leq \Delta^C_{full}$. Since $s^C < \frac{1}{2}$ in this region and since $\lambda h < 1$ from Assumption 1, it follows that P's utility is strictly decreasing in Δ for this range.

Next, consider the region Δ for $\Delta^C_{full} \leq \Delta \leq \Delta^C_{shut}$. Since P's utility is concave in Δ in this region and since the derivative of P's utility at Δ^C_{full} is negative, it follows that P's utility is decreasing in Δ for this region.

Finally, since $\frac{\partial U_P^C(\Delta_{col}^C)}{\partial \Delta} > 0$ and since $\frac{\partial U_P^C(\Delta_{int}^C)}{\partial \Delta} < 0$ and since P's optimal utility is twice continuously differentiable in Δ, it follows that there exists $\Delta^{C^*} \in (\Delta_{col}^C, \Delta_{int}^C)$ with $\frac{\partial U_P^C(\Delta^{C^*})}{\partial \Delta} = 0$. Since P's utility is concave in Δ in this interval, Δ^{C^*} is unique.

To see how Δ^{C^*} varies with the parameters note that Δ^{C^*} is implicitly defined by the equation:

$$\frac{\lambda h}{2} - 2 + \frac{\lambda}{2}\frac{h^2\Delta + 1}{\sqrt{h^2\Delta^2 + 2(\Delta - b)}} = 0$$

Since $U_P^{C^*}$ is strictly concave in this region, the sign of the partial derivative $\frac{\partial \Delta^{C^*}}{\partial b}$ has the same sign as:

$$\frac{(h^2\Delta + 1)b}{\sqrt{2(\Delta - b) + h^2\Delta^2}} > 0$$

Similarly, the sign of the partial derivative $\frac{\partial \Delta^{C^*}}{\partial \lambda}$ has the same sign as:

$$\frac{h}{2} + \frac{h^2\Delta + 1}{2\sqrt{h^2\Delta^2 + 2(\Delta - b)}} > 0$$

□

Proof of Lemma 2. Fix a specific value of h. Let $(s(h), \delta_{ii}(h))$ be the optimal incentive-compromise pair at h. Now consider $h' > h$. For the same incentive-compromise pair, P can induce a higher quality from (IC_q^C), and collaboration will still hold from (13). Since the profits of P are increasing in q for a given s ($\frac{\partial U_P^C}{\partial q} = (1 - 2s)\lambda$), an increase in h will increase P's profits. A similar argument holds for λ and b. □

Proof of Proposition 1. From Section 10.1.1 and Section 10.2.2 we have:

$$\Delta_{shut}^C = \frac{\lambda^2}{4} + 2b < \frac{\lambda^2}{2} + 2b = \Delta_{shut}^N.$$

□

Proof of Proposition 2. Let $bh \geq \frac{\lambda}{8}$. Since $\Delta_{col}^C \geq b$:

$$\begin{aligned} U_P^{C^*}(\Delta = b) &= \frac{\lambda^2}{16} + \frac{\lambda h b}{2} + h^2 b^2 \\ &> \frac{\lambda^2}{16} + \frac{\lambda h b}{2} \\ &\geq \frac{\lambda^2}{16} + \frac{\lambda^2}{16} \\ &\geq \max U_P^{N^*}(\Delta) \end{aligned}$$

$\square$

Proof of Lemma 3. i This part of the Lemma follows directly from Lemma 1 and the fact that $U_P^{N^*}$ is weakly decreasing in Δ.

ii Consider two possible cases.

First suppose $\Delta \leq \frac{\lambda^2}{8} + 2b \equiv \Delta_{full}^N$. Then from Lemma 1 and the fact that P's utility is constant for these Δ's under the noncompetitive structure, it must be the case that the slope of P's utility function is less under the competitive structure.

Second, suppose $\Delta > \frac{\lambda^2}{8} + 2b$. Notice that:

$$\frac{\partial U_P^{N^*}}{\partial \Delta}(\Delta) = -1 + \frac{\lambda}{2\sqrt{2}\sqrt{\Delta - 2b}} \tag{36}$$

Consider three subcases in the competitive structure. First, suppose full compromise is optimal. Then the derivative of P's utility with respect to Δ, under competition is:

$$\frac{\partial U_P^{C^*}}{\partial \Delta}(\Delta) = -1 + \frac{\lambda}{4\sqrt{\Delta - 2b}} \tag{37}$$

Thus the slope of P's utility with respect to Δ, is lower under the competitive structure for this subcase.

Second, suppose interior compromise is optimal. Since P's utility is convex in Δ and since the slope of P's utility with respect to Δ for this case is equal to (37) at the cutoff Δ_{full}^C, it must be the case that the slope of P's utility with respect to Δ, is less under the competitive structure for this subcase.

Third, suppose no compromise is optimal. Let $\Delta = \eta\frac{\lambda^2}{8}$ with $\eta > 1$ and let $b = \phi\frac{\lambda^2}{8}$ with $\phi \geq 0$. Then the slope of P's utility with respect to Δ is less under the competitive structure for this subcase if:

$$-1 + \frac{1}{\sqrt{\eta}} > -2 + \frac{\lambda h}{2} + \frac{1}{2}\left(\frac{h^2(2\phi+\eta)\lambda^2 + 8}{\sqrt{h^2(2\phi+\eta)^2\lambda^2 + 16(\eta+\phi)}}\right)$$

which can be rewritten as:

$$\frac{2 + \sqrt{\eta}(2 - \lambda h)}{2\sqrt{\eta}} > \frac{1}{2}\left(\frac{h^2(2\phi+\eta)\lambda^2 + 8}{\sqrt{h^2(2\phi+\eta)^2\lambda^2 + 16(\eta+\phi)}}\right)$$

Squaring both sides we get:

$$\frac{4+(2-\lambda h)^2\eta+4(2-\eta h)\sqrt{\eta}}{4\eta} > \frac{1}{4}\left(\frac{h^4\lambda^4(2\phi+\eta)^2+64+16h^2\lambda^2(2\phi+\eta)}{h^2\lambda^2(\phi+\eta)^2+16(\eta+\phi)}\right)$$

Simplifying this expression, we get:

$$\begin{aligned}64(\eta+\phi)+16(2-\lambda h)^2\eta(\eta+\phi)+64(2-\lambda h)(\eta+\phi)\sqrt{\eta}+4h^2\lambda^2(2\phi+\eta)^2 \\ +h^2\lambda^2(2-\lambda h)^2\eta(2\phi+\eta)^2+4h^2\lambda^2(2\phi+\eta)^2(2-\lambda h)\sqrt{\eta} \\ > 64\eta+h^4\lambda^4(2\phi+\eta)^2\eta+16h^2\lambda^2(2\phi+\eta)\eta\end{aligned} \tag{38}$$

Since $\lambda h < 1$ from Assumption 1 and since $\eta > 1$, it follows that: $64(\eta+\phi) \geq 64\eta$, $16(2-\lambda h)^2\eta^2 > 16h^2\lambda^2\eta^2$, $h^2\lambda^2(2-\lambda h)^2\eta(2\phi+\eta)^2 > h^4\lambda^4(2\phi+\eta)^2\eta$, and $16(2-\lambda h)^2\eta\phi+16h^2\lambda^2\eta\phi > 32h^2\lambda^2\eta\phi$.

Thus the left hand side of (38) is strictly larger than the right hand side of (38). □

Proof of Proposition 3. Consider four possible cases. First, let $k \geq \max\{\tilde{k}, \Delta^N_{full}\}$. For this region noncompetition yields a larger utility to P and since P's utility is strictly decreasing in Δ for $\Delta \geq k$, it follows that $\Delta^* = k$ is the optimal level of heterogeneity.

Second, let $\tilde{k} \leq k \leq \Delta^N_{full}$. For this region noncompetition yields a larger utility to P. Since P's utility is constant in the interval $[k, \Delta^N_{full}]$, any of the Δ's in the interval is optimal.

Third, let $\Delta^{C^*} < k < \tilde{k}$. For this region competition yields a larger utility to P. And since P's utility is strictly decreasing in the interval $[k, \tilde{k}]$, the optimal level of heterogeneity is k.

Finally, let $k \leq \Delta^{C^*}$. For this region competition yields a larger utility to P. And since P's utility is strictly increasing in the interval $[k, \Delta^{C^*}]$, the optimal level of heterogeneity is Δ^{C^*}.

□

References

Bonatti, A., and H. Rantakari (2016): "Politics of Compromise," *American Economic Review*, 106(2), 229–259.

Callander, S., and B. Harstad (2015): "Experimentation in Federal Systems," *The Quarterly Journal of Economics*, 130(2), 951–1002.

Carillo, J., and D. Gromb (1999): "On the Strength of Corporate Cultures.," *European Economic Review*, 43, 1021–1037.

Crémer, J. (1993): "Corporate Culture and Shared Knowledge.," *Industrial and Corporate Change*, 3, 351–386.

Dewatripont, M., and J. Tirole (2005): "Modes of Communication," *Journal of Political Economy*, 133(6), 1217–1238.

Friebel, G., and M. Raith (2010): "Resource Allocation and Organizational Form," *American Economic Journal: Microeconomics*, 2(2), 1–33.

Hermalin, B. (2001): *In C.L. Cooper, S. Cartwright, and P.C. Earley, eds., The International Handbook of Organizational Culture and Climate.*chap. Economics and Corporate Culture. John Wiley & Sons, New York.

——— (2013): *In R. Gibbons, and J. Roberts, eds., The Handbook of Organizational Economics.*chap. Leadership and Corporate Culture. Princeton University Press, Princeton.

Hirsch, A., and K. Shotts (2015): "Competitive Policy Development.," *American Economic Review*, 105(4), 1646–1664.

Iansiti, M., and A. Serels (2013): "Microsoft Server & Tools," *Harvard Business School Case 613-031*, pp. 1–21.

Itoh, H. (1991): "Incentives to Help in Multi-Agent Situations.," *Econometrica*, 59(3), 611–636.

Kreps, D. (1990): *In J.E. Alt and K.A. Shepsle, eds., Perspectives on Positive Political Economy.*chap. Corporate Culture and Economic Theory. Cambridge University Press, Cambridge.

Landier, A., D. Sraer, and D. Thesmar (2009): "Optimal Dissent in Organizations," *Review of Economic Studies*, 76, 761–794.

Lazear, E. (1989): "Pay Equality and Industrial Politics," *Journal of Political Economy*, 97(3), 561–580.

——— (1995): *In R. Siebert, ed., Trends in Business Organization: Do Participation and Cooperation Increase Competitiveness?*chap. Corporate Culture and Diffusion of Values. Mohr, Tubingen.

Lazear, E. P., and S. Rosen (1981): "Rank-order Tournaments as Optimum Labor Contracts.," *Journal of Political Economy*, 89(5), 841–864.

Li, H. (2017): "Developing Shared Knowledge.," *Journal of Law, Economics, & Organization*, 33(2), 332–376.

MARINO, A., AND J. ZABOJNIK (2004): "Internal Competition for Corporate Resources and Incentives in Teams," *The RAND Journal of Economics*, 35(4), 710–727.

NADELLA, S., G. SHAW, AND J. NICHOLS (2017): *Hit Refresh: The Quest to Rediscover Microsoft's Soul and Imagine a Better Future for Everyone*. Harper Collins, New York.

NALEBUFF, B., AND J. E. STIGLITZ (1983): "Prizes and Incentives: Towards a General Theory of Compensation and Competition.," *Bell Journal of Economics*, 14(1), 21–43.

PRASAD, S., AND M. TOMAINO (2017): "Resource Allocation and Culture in Organizations," *Working Paper*, pp. 1–38.

PRAT, A. (2002): "Should a Team Be Homogeneous?," *European Economic Review*, 46(7), 1187–1207.

PRENDERGAST, C. (2008): "Intrinsic Motivation and Incentives.," *American Economic Review Papers and Proceedings*, 98(2), 201–215.

RAJAN, R., AND L. ZINGALES (1998): "Power in a Theory of the Firm.," *Quarterly Journal of Economics*, 113(2), 387–432.

RANTAKARI, H. (2017): "Managerial Influence and Organizational Performance.," *Working Paper*, pp. 1–50.

SCHEIN, E. (1985): *Organizational culture and leadership.* Jossy-Bass, San Francisco.

STEIN, J. C. (1997): "Internal Capital Markets and the Competition for Corporate Resources.," *Journal of Finance*, 52(1), 111–133.

VAN DEN STEEN, E. J. (2010a): "Culture Clash: The Costs and Benefits of Homogeneity," *Management Science*, 56(10), 1718–1738.

——— (2010b): "On the Origin of Shared Beliefs (and Corporate Culture)," *RAND Journal of Economics*, 41(4), 617–648.