Can the optimal tariff be zero for a growing large country?

Takumi Naito*
Vanderbilt University and Waseda University

August 6, 2018

Abstract

Can the optimal tariff be zero for a growing large country? To pursue the possibility, we extend the Rivera-Batiz–Romer lab-equipment model of endogenous growth to include heterogeneous firms, asymmetric countries, and import tariffs. We find that each country's domestic revenue share is a sufficient statistic for its long-run growth rate, but it is not for its long-run welfare. A unilateral tariff reduction by either country always increases the balanced growth rate. A zero tariff is locally optimal for a country under a mild condition, which is automatically satisfied at a symmetric balanced growth path with the zero tariff.

JEL classification: F13; F43

Keywords: Optimal tariff; Large country; Lab-equipment model; Endogenous growth; Heterogeneous firms

*Takumi Naito. Department of Economics, Vanderbilt University, VU Station B #351819, 2301 Vanderbilt Place, Nashville, TN 37235-1819, USA. E-mail: tnaito@waseda.jp.

1 Introduction

It has been taken for granted among trade economists that the optimal tariff for a large country is positive.[1] Suppose that a large country increases a tariff on its imported good. On the one hand, this incurs distortions in consumption and production, which harm the country's welfare. On the other hand, the tariff-induced decrease in the country's import demand for the good drives down its world price (otherwise, the country would be a small country), and the resulting improvement in the country's terms of trade benefits its welfare. Since the former effect is negligible around a free trade equilibrium, increasing the tariff from zero necessarily increases the country's welfare. The optimal tariff, if exists, must balance out the negative distortionary effect and the positive terms of trade effect. The positivity of the optimal tariff for a large country is widely confirmed in general equilibrium trade models, from the two-good standard trade model (e.g., Kaldor, 1940)[2] to the Melitz (2003) model (e.g., Felbermayr et al., 2013; Demidova, 2017).[3]

A theoretical and practical argument against the implementation of the optimal tariff theory is that it results in the prisoner's dilemma. A tariff of country 1 harms country 2 through the latter's terms of trade deterioration, and vice versa. Due to the negative externalities, each country's welfare at a Nash equilibrium of a "tariff war" is typically lower than that at a free trade equilibrium, that is, the Nash equilibrium is Pareto inferior to the free trade equilibrium.[4] To avoid the prisoner's dilemma, the countries have to commit to reciprocity in tariff reductions, one of the founding principles of the GATT/WTO (e.g., Bagwell and Staiger, 1999). However, each country still has an incentive to gain by deviating unilaterally from the free trade regime, as the recent U.S.-China trade disputes suggest. Can there be a model in which the optimal tariff is zero for a large country? If so, then the model will provide stronger support for free trade without relying on reciprocity. The purpose of this paper is to create such a model.

We consider the role of economic growth in pursuing the possibility of a zero optimal tariff for a large country. While Rodriguez and Rodrik (2000) question the robustness of some major empirical studies reporting the positive relationship between trade liberalization and economic growth, more recent well-designed empirical papers such as Wacziarg and Welch (2008) and Estevadeordal and Taylor (2013) do find that the positive liberalization-growth relationship is robust, thereby overcoming Rodriguez and Rodrik's (2000) criticism. If it is true, then an increase in a tariff of a large country generates an additional welfare loss through slower growth, which might pull the country's optimal tariff down to zero. To characterize a large country's optimal tariff with the other country's tariff given, we have to allow for asymmetric countries. To this end, we extend the lab-equipment version of Rivera-Batiz and Romer (1991), the first and simplest two-country model of endogenous technological change, to include heterogeneous firms a la Baldwin and Robert-Nicoud (2008) (BRN hereafter), asymmetric countries a la Naito (2018), and import tariffs.[5]

The core of our model is a tradable differentiated intermediate good sector, where each potential entrant uses the knowledge good (produced one-to-one from the final good in an R&D sector) as the fixed input,

[1] A large country is defined as a country whose behavior affects the world prices of its traded goods.

[2] Horwell and Pearce (1970), Bond (1990), and Ogawa (2007) characterize the optimal tariff structure in multi-good settings. A general consensus is that there exists at least one good whose trade is taxed at the optimum.

[3] Felbermayr et al. (2013) assume CES preferences, whereas Demidova (2017) allows for variable markups.

[4] Johnson (1953) and Kennan and Riezman (1988) point out that, when a country is substantially larger than the other country, the former could have a higher welfare at the Nash equilibrium than at the free trade equilibrium.

[5] Rivera-Batiz and Romer (1991) consider two alternative specifications of R&D, namely the knowledge-driven specification (i.e., labor and public knowledge are used in R&D) and the lab-equipment specification (i.e., a composite final good is used in R&D). Recently, Naito (2017) extends BRN with the knowledge-driven specification to introduce asymmetric countries, whereas Naito (2018) formulates an asymmetric BRN model with the lab-equipment specification. Our model replaces iceberg trade costs in Naito (2018) with revenue-generating import tariffs, which make the analysis much more difficult due to the presence of tariff revenues.

and the final good as the variable input, following Rivera-Batiz and Romer (1991). An entrant pays two types of fixed costs: an entry cost to draw the unit final good requirement a (i.e., the inverse of productivity) from a Pareto distribution, and an overhead cost for each market if it is covered by the gross firm value at the realized a. As in most Melitz-type models, the equilibrium productivity distribution is determined by the zero cutoff profit and free entry conditions. The zero cutoff profit condition states that country j's fixed overhead cost in market k is equal to the gross firm value at the cutoff unit final good requirement a_{jk}, below which a firm makes a positive net firm value. The free entry condition requires that country j's fixed entry cost is equal to the sum of the expected net firm values over all markets. Country j's free entry condition binds the movement of its domestic and export cutoffs, whereas country j's zero cutoff profit condition for its export market $k(\neq j)$ is affected by country k's import tariff. Due to the technical difficulty of evaluating the future profits in general, we follow the literature by focusing on a balanced growth path (BGP), where all variables grow at constant (including zero) rates.

Before studying the full general equilibrium effects of a tariff change, we derive long-run growth and welfare formulas in line with Arkolakis et al. (2012). We find that country j's long-run growth rate of domestic varieties depends only and negatively on its revenue share of domestic varieties just like the ACR (Arkolakis–Costinot–Rodríguez-Clare) welfare formula.[6] However, country j's long-run welfare depends not only on its domestic revenue share, but also directly on its import tariff through its tariff revenue. Unlike Arkolakis et al. (2012) and the related studies considering iceberg trade costs as the only variable trade barriers, a country's domestic share, or sometimes called "autarkiness", is not a sufficient statistic for its welfare in our model with import tariffs. It is this property that leaves open the possibility that the optimal tariff for a large country is positive in asymmetric Melitz models such as Felbermayr et al. (2013) and Demidova (2017).

Based on the derived long-run growth and welfare formulas, we obtain the following main results. First, an increase in the import tariff of either country decreases the balanced growth rate. An increase in country 1's import tariff, with prices given, directly discourages country 2's exports, which keeps more of country 2's unproductive firms staying at their domestic market. This in turn encourages country 1's exports by productive firms and domestic selection of unproductive firms. In fact, for country 1's trade surplus to be cleared, its relative wage and hence its relative price of the final good go up, which indirectly affects countries' exports and domestic selection in the opposite directions of the aforementioned direct effects. It turns out that the direct effects are stronger than the indirect effects for the partner country 2, whereas the indirect effects dominate for the protecting country 1. Due to less exports and less domestic selection in both countries, both of them become more autarkic, and hence grow more slowly, on the new BGP than the old BGP. In the literature on endogenous growth and heterogeneous firms (e.g., BRN, 2008; Dinopoulos and Unel, 2011; Perla et al., 2015; Fukuda, 2016; Sampson, 2016; Ourens, 2016; Naito, 2017, 2018), only Naito (2017, 2018) allow for asymmetric countries, of which only Naito (2018) shows that unilateral trade liberalization in terms of an iceberg trade cost always raises long-run growth. We confirm the clean result of Naito (2018) even if complicated general equilibrium effects through tariff revenues are considered.

Second, a zero tariff is locally optimal for country j if its export revenue share relative to country $k(\neq j)$'s is smaller than an upper bound, which is larger than unity, at a BGP with the zero tariff. An increase in country j's import tariff creates a welfare trade-off between gains from tariff revenue and losses from autarkiness. The losses from autarkiness is relatively larger, the smaller the common subjective discount

[6] Naito (2018) shows that this is a consequence of the lab-equipment specification; we cannot obtain an ACR-like long-run growth formula at an asymmetric BGP of Naito (2017) with the knowledge-driven specification.

rate is, and/or the smaller country j's export revenue share relative to country k's is. If the above sufficient condition holds, then country j cannot gain by deviating unilaterally from its zero tariff. Moreover, the sufficient condition is automatically satisfied if the countries are similar, and/or the subjective discount rate approaches zero. Therefore, zero optimal tariffs for large countries occur quite naturally in our dynamic Melitz model. We also supplement the above local analytical result with numerical experiments in a wider domain of ad valorem tariff rates from 0 to 100%. We find that a country's optimal tariff is more likely to be zero, the more technologically advanced it is relative to the other country (in terms of the upper bound of a), simply because such country tends to have a relatively smaller export revenue share. This implies that, once economic growth is taken into account, a more technologically advanced and hence richer country (e.g., the United States) has a greater incentive to adopt free trade unilaterally.

The rest of this paper is organized as follows. Section 2 formulates the model. Section 3 characterizes a BGP, and derives long-run growth and welfare formulas. Section 4 studies the long-run effect of a tariff change. Section 5 examines if the optimal tariff can be zero for a growing large country. Section 6 concludes.

2 The model

We set up a Rivera-Batiz–Romer lab-equipment model with heterogeneous firms, asymmetric countries, and import tariffs.[7] In country $j(= 1, 2)$, there are a nontradable final good sector, a tradable intermediate good sector, and a nontradable R&D (i.e., knowledge good) sector. The final good is produced from a variety of differentiated intermediate goods and labor under constant returns to scale and perfect competition. Each intermediate good is produced using the knowledge good as the fixed input, and the final good as the variable input. The knowledge good is produced from the final good under constant returns to scale and perfect competition. We follow BRN's formulation of heterogeneous firms as closely as possible.

2.1 Households

The representative household in country j maximizes its overall utility $U_j = \int_0^\infty \ln C_{jt} \exp(-\rho t)dt$, subject to its budget constraint $\dot{W}_{jt} = r_{jt}W_{jt} + w_{jt}L_j + T_{jt} - E_{jt}; \dot{W}_{jt} \equiv dW_{jt}/dt, E_{jt} \equiv p^Y_{jt}C_{jt}$, with $\{r_{jt}, w_{jt}, T_{jt}, p^Y_{jt}\}_{t=0}^\infty$ and W_{j0} given, where $t(\in [0,\infty))$ is time (omitted whenever no confusion arises), C_j is consumption, ρ is the subjective discount rate, W_j is the asset, r_j is the interest rate, w_j is the wage rate, L_j is the supply of labor, T_j is the lump-sum transfer from country j's government, E_j is the consumption expenditure, and p^Y_j is the price of the final good. Parameters without country subscripts (e.g., ρ) are assumed to be the same across countries. It is straightforward to derive the Euler equation $\dot{E}_{jt}/E_{jt} = r_{jt} - \rho$.

2.2 Final good firms

The representative final good firm in country j maximizes its profit $\pi^Y_j = p^Y_j Y_j - \int_{\Theta_j} p_j(i)x_j(i)di - w_j L^Y_j$, subject to its production function $Y_j = A_j X_j^{\alpha_j}(L^Y_j)^{1-\alpha_j}, X_j = (\int_{\Theta_j} x_j(i)^{(\sigma-1)/\sigma}di)^{\sigma/(\sigma-1)}$, with p^Y_j, $\{p_j(i)\}_{i\in\Theta_j}$, and w_j given, where Y_j is the supply of the final good, Θ_j is the set of available varieties of intermediate goods, $p_j(i)$ is the demand price of variety i, $x_j(i)$ is the demand for variety i, L^Y_j is the demand for labor, A_j is an arbitrary constant, X_j is the index of the intermediate goods, $\alpha_j(\in (0, 1))$ is the Cobb-Douglas cost share of the intermediate goods, and $\sigma(> 1)$ is the elasticity of substitution across varieties.

[7] We omit non-tariff trade costs because adding them does not affect qualitative results.

The profit maximization problem is divided into three parts. First, minimizing $\int_{\Theta_j} p_j(i)x_j(i)di$, subject to $X_j = (\int_{\Theta_j} x_j(i)^{(\sigma-1)/\sigma}di)^{\sigma/(\sigma-1)}$, with $\{p_j(i)\}_{i\in\Theta_j}$ and X_j given, we obtain $\int_{\Theta_j} p_j(i)x_j(i)di = P_jX_j; P_j \equiv (\int_{\Theta_j} p_j(i)^{1-\sigma}di)^{1/(1-\sigma)}$, where P_j is the price index of the intermediate goods. Second, minimizing $P_jX_j + w_jL_j^Y$, subject to $Y_j = A_jX_j^{\alpha_j}(L_j^Y)^{1-\alpha_j}$, with P_j, w_j, and Y_j given, the minimized total cost is rewritten as $P_jX_j + w_jL_j^Y = c_j^Y(P_j, w_j)Y_j; c_j^Y(P_j, w_j) \equiv P_j^{\alpha_j}w_j^{1-\alpha_j}$, where A_j is set to $A_j \equiv \alpha_j^{-\alpha_j}(1-\alpha_j)^{-(1-\alpha_j)}$ to simplify the unit cost function $c_j^Y(P_j, w_j)$. Third, maximizing $\pi_j^Y = p_j^YY_j - c_j^Y(P_j, w_j)Y_j$, with p_j^Y, P_j, and w_j given, the first-order condition is given by $p_j^Y = c_j^Y(P_j, w_j)$, which is equivalent to the free entry condition $p_j^YY_j = \int_{\Theta_j} p_j(i)x_j(i)di + w_jL_j^Y$.

2.3 Intermediate good firms

The intermediate good firms are heterogeneous in the unit final good requirement a: the lower a is, the more productive a firm is. Each potential entrant in country j first pays $P_j^K\kappa_j^e$, where P_j^K is the price of the knowledge good, and κ_j^e is country j's one-time fixed entry cost in terms of the knowledge good. After that, a is randomly drawn from a cumulative distribution function $G_j(a); a \in [0, a_{j0}]$, and the corresponding probability density function $g_j(a)$, where a_{j0} is the upper bound of a in country j: the lower a_{j0} is, the more likely a firm is to be productive. If a is no more than its cutoff value a_{jk} (to be determined later), then a firm from country j incurs $P_j^K\kappa_{jk}$ to enter market $k(= 1, 2)$, where κ_{jk} is country j's one-time fixed overhead cost in market k in terms of the knowledge good; otherwise, the firm exits from market k without paying the fixed overhead cost. Free entry requires that the the fixed entry cost be equal to the sum of the expected net firm values over all markets.

An intermediate good firm indexed by a in country j maximizes its profit in market k, $\pi_{jk}(a) = p_{jk}^f(a)y_{jk}(a) - p_j^Y ay_{jk}(a)$, subject to the market-clearing condition $y_{jk}(a) = x_{jk}(a)$, the conditional demand function $x_{jk}(a) = p_{jk}(a)^{-\sigma}P_k^\sigma X_k = (\tau_{jk}p_{jk}^f(a))^{-\sigma}P_k^\sigma X_k$, with p_j^Y, P_k, and X_k given, where $p_{jk}^f(a)$ is the supply price of the firm's variety, $y_{jk}(a)$ is the supply of the firm's variety, $x_{jk}(a)$ is country k's demand for the firm's variety, $p_{jk}(a)$ is country k's demand price of the firm's variety, and $\tau_{jk}(\geq 1)$ is one plus country k's ad valorem tariff rate on imports from country j (with $\tau_{jj} = 1$), the only policy variable in this paper.[8] The profit-maximizing supply price is derived as $(p_{jk}^f(a) - p_j^Y a)/p_{jk}^f(a) = 1/\sigma \Leftrightarrow p_{jk}^f(a) = p_j^Y a/(1-1/\sigma)$. The corresponding revenue, gross profit, and gross firm value are given by $e_{jk}(a) \equiv p_{jk}^f(a)y_{jk}(a) = \tau_{jk}^{-\sigma}[p_j^Y a/(1-1/\sigma)]^{1-\sigma}P_k^\sigma X_k, \pi_{jk}(a) = e_{jk}(a)/\sigma = \tau_{jk}^{-\sigma}[p_j^Y a/(1-1/\sigma)]^{1-\sigma}P_k^\sigma X_k/\sigma$, and $v_{jkt}(a) \equiv \int_t^\infty \pi_{jks}(a)\exp(-\int_t^s(r_{ju}+\delta)du)ds$, respectively, where δ is the exogenous rate of a bad shock forcing a firm to exit (e.g., Melitz, 2003).

Country j's cutoff unit final good requirement in market k is determined by:

$$v_{jkt}(a_{jkt}) = P_{jt}^K\kappa_{jk}, j, k = 1, 2. \tag{1}$$

Eq. (1) is called the zero cutoff profit condition, meaning that the gross value of the cutoff firm just covers the fixed overhead cost. It is assumed that firms have to pay a larger fixed overhead cost for exports than domestic sales: $\kappa_{jk} > \kappa_{jj}, k \neq j$. Using Eq. (1) and $e_{jks}(a)/e_{jks}(a_{jkt}) = (a/a_{jkt})^{1-\sigma} = \pi_{jks}(a)/\pi_{jks}(a_{jkt})$, $v_{jkt}(a)$ is rewritten as $v_{jkt}(a) = (a/a_{jkt})^{1-\sigma}P_{jt}^K\kappa_{jk} \geq P_{jt}^K\kappa_{jk} \Leftrightarrow a \leq a_{jkt}$. This verifies that a firm with a in country j profitably enters market k if and only if $a \leq a_{jk}$. We assume that $a_{jk} < a_{jj}\forall j, k = 1, 2, k \neq j$, that is, only a fraction $G_j(a_{jk})/G_j(a_{jj})$ of country j's domestic surviving firms with $a \leq a_{jk}(< a_{jj})$ can also survive in their export market k.

[8] Applying Shephard's lemma to $\int_{\Theta_j} p_j(i)x_j(i)di = P_jX_j$ with $j = k$ gives $x_k(i) = (\partial P_k/\partial p_k(i))X_k = p_k(i)^{-\sigma}P_k^\sigma X_k$.

As explained in the first paragraph, country j's free entry condition is given by:

$$\textstyle\sum_k \int_0^{a_{jk}} (v_{jk}(a) - P_j^K \kappa_{jk}) g_j(a) da = P_j^K \kappa_j^e \Leftrightarrow \sum_k \kappa_{jk} H_{jk}(a_{jk}) = \kappa_j^e; \tag{2}$$

$$H_{jk}(a_{jk}) \equiv G_j(a_{jk}) h_{jk}(a_{jk}), h_{jk}(a_{jk}) \equiv (\overline{a}_{jk}(a_{jk})/a_{jk})^{1-\sigma} - 1,$$
$$\overline{a}_{jk}(a_{jk}) \equiv (\int_0^{a_{jk}} a^{1-\sigma} \mu_{jk}(a|a_{jk}) da)^{1/(1-\sigma)}, \mu_{jk}(a|a_{jk}) \equiv g_j(a)/G_j(a_{jk}),$$

where $H_{jk}(a_{jk})$ is country j's expected net firm value in market k relative to the fixed overhead cost $P_j^K \kappa_{jk}$, $h_{jk}(a_{jk})$ is the conditional version of $H_{jk}(a_{jk})$, $\overline{a}_{jk}(a_{jk})$ is the aggregate unit final good requirement of surviving firms, and $\mu_{jk}(a|a_{jk})$ is the probability density function conditional on survival, with $\int_0^{a_{jk}} \mu_{jk}(a|a_{jk}) da = 1$. In the same way as Melitz (2003, Appendix B), it can be verified that an increase in a_{jk} increases $H_{jk}(a_{jk})$ mainly by increasing the probability of survival $G_j(a_{jk})$. Then from Eq. (2), for $k \neq j$, a_{jj} and a_{jk} always move in the opposite directions. In other words, more domestic selection (i.e., a decrease in a_{jj}) implies more exports (i.e., an increase in a_{jk}), and vice versa.

Finally, let n_j^e be the number of entrants in country j. Then $n_{jk} \equiv n_j^e G_j(a_{jk})$ is the number of entrants in country j surviving in market k, or the number of varieties sold from country j to country k.

2.4 R&D firms

The representative R&D firm in country j maximizes its profit $\pi_j^K = P_j^K Q_j^K - p_j^Y D_j$, subject to the production function $Q_j^K = D_j$, with P_j^K and p_j^Y given, where Q_j^K is the supply of the knowledge good, and D_j is the demand for the final good from the R&D sector. The first-order condition is given by $P_j^K = p_j^Y$, which is equivalent to the free entry condition $P_j^K Q_j^K = p_j^Y D_j$.

2.5 Government

Country j's government budget constraint is given by $T_j = \sum_k (\tau_{kj} - 1) n_{kj} \int_0^{a_{kj}} p_{kj}^f(a) x_{kj}(a) \mu_{kj}(a|a_{kj}) da$. As usual, the government in country j collects its revenue only from its import tariff, and then transfers the revenue to country j's representative household.

2.6 Markets

The market-clearing conditions for the asset, labor, knowledge good, and final good are given by, respectively:

$$W_j = \textstyle\sum_k n_{jk} \int_0^{a_{jk}} v_{jk}(a) \mu_{jk}(a|a_{jk}) da, j = 1, 2,$$
$$L_j = L_j^Y, j = 1, 2,$$
$$Q_j^K = \overline{\kappa}_j (\dot{n}_{jj} + \delta n_{jj}); \overline{\kappa}_j \equiv (\textstyle\sum_k \kappa_{jk} G_j(a_{jk}) + \kappa_j^e)/G_j(a_{jj}), j = 1, 2,$$
$$Y_j = C_j + D_j + F_j; F_j \equiv \textstyle\sum_k n_{jk} \int_0^{a_{jk}} a y_{jk}(a) \mu_{jk}(a|a_{jk}) da, j = 1, 2,$$

where $\overline{\kappa}_j$ is an entrant's: "expected units of knowledge required to get a 'winner.' " (BRN, 2008, p. 25), or its expected total fixed costs in terms of the knowledge good conditional on domestic survival, and F_j is the demand for the final good from the intermediate good sector.

Country j's Walras' law and its market-clearing conditions imply that:[9]

$$\begin{aligned}&\textstyle\sum_k E_{jk} = \sum_k E_{kj};\, E_{jk} \equiv n_{jk} \displaystyle\int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk})da,\\ &E_{jk} = E_{kj}, k \neq j,\end{aligned}$$

where E_{jk} is country j's revenue of selling the intermediate goods to country k, or country k's expenditure for buying the intermediate goods from country j net of tariff. The first line shows country j's national budget constraint, saying that its total revenue of selling the intermediate goods to all destinations is equal to its total expenditure for buying the intermediate goods from all sources net of tariff. Subtracting country j's domestic revenue and expenditure from the first line, we obtain the second line, country j's zero balance of trade. Letting $\lambda_{jk} \equiv E_{jk}/\sum_l E_{jl}; \sum_k \lambda_{jk} = 1$, be the revenue share of varieties country j sells to country k, country j's zero balance of trade is rewritten as $\lambda_{jk}\sum_l E_{jl} = \lambda_{kj}\sum_l E_{kl}, k \neq j$.

In the next section, we characterize a BGP, where all variables grow at constant (including zero) rates.

3 Balanced growth path

3.1 Characterization

Let labor in country 2 be the numeraire: $w_2 \equiv 1$. Suppose that the world economy is on a BGP for $t \geq 0$. We first derive country j's growth rate of domestic varieties $\gamma_j^* \equiv (\dot{n}_{jj}/n_{jj})^*$ as (see Appendix A for derivation):

$$\gamma_j^* = [\alpha_j/(1-\alpha_j)](1/\sigma)\{1/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}w_j^* L_j/(p_j^{K*}\overline{\kappa}_j^*) - \rho - \delta, k \neq j,$$

where $p_j^K \equiv n_{jj}P_j^K$ is: "an 'intensive form' " (BRN, 2008, p. 25) of P_j^K normalizing the negative effect of variety growth on $P_j^K = p_j^Y = P_j^{\alpha_j} w_j^{1-\alpha_j}$, and a superscript asterisk represents a BGP. The above expression indicates that country j's growth rate depends on $p_j^{K*}/w_j^*, \overline{\kappa}_j^*$, and λ_{jk}^*. The first two correspond to: "the p_K-channel and the $\overline{\kappa}$-channel" (BRN, 2008, p. 27), respectively. The third one is included in the term $1/[1+(\tau_{kj}-1)\lambda_{jk}^*]$, which is equal to $\sum_l E_{jl}/(P_j X_j)$, the ratio of country j's total intermediate good revenue to its total intermediate good expenditure gross of tariff (see Appendix A for derivation). Since $\{a_{jk}^*\}$ are constant from Eq. (2), $\overline{\kappa}_j^*$ and $\lambda_{jk}^* = G_j(a_{jk}^*)\int_0^{a_{jk}^*} e_{jk}(a)\mu_{jk}(a|a_{jk}^*)da/\sum_l G_j(a_{jl}^*)\int_0^{a_{jl}^*} e_{jl}(a)\mu_{jl}(a|a_{jl}^*)da$ are constant.[10] Therefore, constancy of γ_j^* implies that p_j^{K*}/w_j^* is constant.

To see when p_j^{K*}/w_j^* is constant, we rewrite country j's intermediate good price index as:

$$\begin{aligned}&P_j = \{\textstyle\sum_k n_{kj}[\tau_{kj}p_k^Y \overline{a}_{kj}(a_{kj})/(1-1/\sigma)]^{1-\sigma}\}^{1/(1-\sigma)} = n_{jj}^{1/(1-\sigma)} p_j^Y \overline{m}_j/(1-1/\sigma);\\ &\overline{m}_j \equiv \{\textstyle\sum_k (n_{kk}/n_{jj})(G_k(a_{kj})/G_k(a_{kk}))[(\tau_{kj}p_k^Y/p_j^Y)\overline{a}_{kj}(a_{kj})]^{1-\sigma}\}^{1/(1-\sigma)},\end{aligned} \tag{3}$$

where $p_j^Y \overline{m}_j$ is: "a weighted average of firms' marginal selling costs in a particular market" (BRN, 2008, p. 24). Substituting Eq. (3) into $p_j^Y = P_j^{\alpha_j} w_j^{1-\alpha_j}$, and solving it for p_j^Y with $n_{jj}, \overline{m}_j$, and w_j given, we obtain $p_j^Y = [n_{jj}^{1/(1-\sigma)}\overline{m}_j/(1-1/\sigma)]^{\alpha_j/(1-\alpha_j)} w_j$, and hence $p_j^K = n_{jj}p_j^Y = n_{jj}^{1-[1/(\sigma-1)]\alpha_j/(1-\alpha_j)}[\overline{m}_j/(1-$

[9]Time differentiating country j's asset market-clearing condition, and using its no-arbitrage condition $\dot{v}_{jk}(a) = (r_j + \delta)v_{jk}(a) - \pi_{jk}(a)$, household budget constraint, free entry conditions for all sectors, and government budget constraint, we obtain country j's Walras' law: the sum of the values of excess demands for all markets is zero.

[10]The expression for λ_{jk}^* is immediately obtained by noting that $n_{jk} = n_{jj}G_j(a_{jk})/G_j(a_{jj})$.

$1/\sigma)]^{\alpha_j/(1-\alpha_j)}w_j$. For p_j^K/w_j to be constant in spite of variety growth, we must have $1-[1/(\sigma-1)]\alpha_j/(1-\alpha_j)=0$, or:

$$\alpha_j = 1-1/\sigma = (\sigma-1)/\sigma.$$

Then p_j^K is simplified to $p_j^K=[\overline{m}_j/(1-1/\sigma)]^{\sigma-1}w_j$, and thus the growth equation is finally rewritten as:

$$\begin{aligned}\gamma_j^* &= \{(1-1/\sigma)/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}w_j^*L_j/(p_j^{K*}\overline{\kappa}_j^*)-\rho-\delta\\ &= \{(1-1/\sigma)/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}L_j/\{[\overline{m}_j^*/(1-1/\sigma)]^{\sigma-1}\overline{\kappa}_j^*\}-\rho-\delta, k\neq j.\end{aligned}\tag{4}$$

Eqs. (3) and (4) imply that, leaving aside the cutoffs $\{a_{jk}^*\}$ for the moment, country j's growth rate γ_j^* is decreasing in its import trade cost $\tau_{kj}, k\neq j$, decreasing in the relative price of the final good of country k to country j p_k^Y/p_j^Y, and increasing in the relative number of domestic varieties of country k to country j n_{kk}/n_{jj}. The last one means that, whenever the relative number of domestic varieties of country 1 to country 2 $\chi\equiv n_{11}/n_{22}$ increases, country 2 grows faster whereas country 1 grows more slowly, thereby slowing down the increase in χ. On a BGP, χ^* is determined by the balanced growth condition:

$$\gamma_1^*=\gamma_2^*\equiv\gamma^*\Leftrightarrow\{1/[1+(\tau_{21}-1)\lambda_{12}^*]\}L_1/(\overline{m}_1^{*\sigma-1}\overline{\kappa}_1^*)=\{1/[1+(\tau_{12}-1)\lambda_{21}^*]\}L_2/(\overline{m}_2^{*\sigma-1}\overline{\kappa}_2^*),\tag{5}$$

where we call a common growth rate on a BGP γ^* the "balanced growth rate".

Since $\overline{m}_j^*$ is constant from Eq. (4), and χ^* is constant from Eq. (5), the relative price of the final goods $(p_1^Y/p_2^Y)^*$ is constant from Eq. (3). Using $p_j^K=n_{jj}p_j^Y=[\overline{m}_j/(1-1/\sigma)]^{\sigma-1}w_j$, this is given by:

$$(p_1^Y/p_2^Y)^*=(w_1^*/\chi^*)(\overline{m}_1^*/\overline{m}_2^*)^{\sigma-1}.\tag{6}$$

Eq. (6) implies that w_1^* is constant, and $p_j^{K*}=[\overline{m}_j^*/(1-1/\sigma)]^{\sigma-1}w_j^*$ implies that p_j^{K*} is constant. From Eqs. (3) and (6), $\overline{m}_1^*, \overline{m}_2^*$, and $(p_1^Y/p_2^Y)^*$ are solved as functions of $w_1^*, \chi^*, \{a_{jk}^*\}$, and $\{\tau_{jk}\}$.

To determine the cutoffs, we use Eqs. (1) and (2). Specifically, dividing Eq. (1) by itself with $j=k$ gives $v_{jk0}(a_{jk}^*)/v_{kk0}(a_{kk}^*)=P_{j0}^K\kappa_{jk}/(P_{k0}^K\kappa_{kk}), j\neq k$, which is rewritten as (see Appendix A for derivations):

$$a_{12}^*/a_{22}^*=v^{*-1}\tau_{12}^{-\sigma/(\sigma-1)}(\kappa_{12}/\kappa_{22})^{-1/(\sigma-1)},\tag{7}$$

$$a_{21}^*/a_{11}^*=v^*\tau_{21}^{-\sigma/(\sigma-1)}(\kappa_{21}/\kappa_{11})^{-1/(\sigma-1)}; v^*\equiv(p_1^Y/p_2^Y)^{*\sigma/(\sigma-1)}.\tag{8}$$

We call (7) and (8) the relative competitiveness conditions in markets 2 and 1, respectively: an increase in a_{jk}^*/a_{kk}^* means that country $j(\neq k)$ becomes relatively more competitive in market k because relatively more firms from the former enter the latter. This is true if country k liberalizes its imports (i.e., τ_{jk} decreases) and/or country j's final good becomes relatively cheaper (i.e., $(p_j^Y/p_k^Y)^*$ decreases). Remembering that $(p_1^Y/p_2^Y)^*$ is solved as a function of $w_1^*, \chi^*, \{a_{jk}^*\}$, and $\{\tau_{jk}\}$ from Eqs. (3) and (6), Eqs. (2), (7), and (8) are solved as:

$$a_{jk}^*=a_{jk}^*(w_1^*,\chi^*,\tau_{21},\tau_{12}), j,k=1,2.\tag{9}$$

Finally, country 1's (or 2's) zero balance of trade is rewritten as (see Appendix A for derivation):

$$\{\lambda_{12}^*/[1+(\tau_{21}-1)\lambda_{12}^*]\}w_1^* L_1 = \{\lambda_{21}^*/[1+(\tau_{12}-1)\lambda_{21}^*]\}L_2; \tag{10}$$
$$\lambda_{jk}^* \equiv (H_{jk}(a_{jk}^*)+G_j(a_{jk}^*))\kappa_{jk}/\textstyle\sum_l (H_{jl}(a_{jl}^*)+G_j(a_{jl}^*))\kappa_{jl}.$$

This determines the relative wage w_1^* in line with Krugman (1980). To sum up, the balanced growth condition (5), the cutoff functions (9), and the balanced trade condition (10), determine a BGP: $(\chi^*, \{a_{jk}^*\}, w_1^*)$.

Before studying the long-run effects of a tariff change, we derive long-run growth and welfare formulas, which will greatly simplify the following analysis.

3.2 Long-run growth and welfare formulas

Suppose that a is distributed as Pareto, which is popular in applications of the Melitz model:

$$G_j(a) \equiv (a/a_{j0})^\theta = a_{j0}^{-\theta} a^\theta; \theta > \sigma - 1,$$

where a_{j0} is a scale parameter representing the upper bound of a in country j, and θ is a shape parameter which is common across countries. Then we obtain:

$$\begin{aligned}
\overline{a}_{jk}(a_{jk})^{1-\sigma} &= [\beta/(\beta-1)]a_{jk}^{1-\sigma}; \beta \equiv \theta/(\sigma-1) > 1,\\
h_{jk}(a_{jk}) &= 1/(\beta-1), H_{jk}(a_{jk}) = G_j(a_{jk})/(\beta-1),\\
H_{jk}(a_{jk}) + G_j(a_{jk}) &= G_j(a_{jk})\beta/(\beta-1) = \beta H_{jk}(a_{jk}),\\
(H_{jk}' + g_{jk})a_{jk}/(H_{jk}+G_{jk}) &= g_{jk}a_{jk}/G_{jk} = H_{jk}'a_{jk}/H_{jk} = \theta \forall j,k; g_{jk} \equiv g_j(a_{jk}), G_{jk} \equiv G_j(a_{jk}).
\end{aligned}$$

Using Eq. (2), λ_{jk}^* and $\overline{\kappa}_j^*$ are simplified to, respectively:

$$\lambda_{jk}^* = H_{jk}(a_{jk}^*)\kappa_{jk}/\textstyle\sum_l H_{jl}(a_{jl}^*)\kappa_{jl} = H_{jk}(a_{jk}^*)\kappa_{jk}/\kappa_j^e \Rightarrow \widehat{\lambda}_{jk}^* = \theta\widehat{a}_{jk}^*; \widehat{\lambda}_{jk}^* \equiv d\ln\lambda_{jk}^* \equiv d\lambda_{jk}^*/\lambda_{jk}^*, \tag{11}$$
$$\overline{\kappa}_j^* = \beta\kappa_j^e/G_j(a_{jj}^*) \Rightarrow \widehat{\overline{\kappa}}_j^* = -\theta\widehat{a}_{jj}^*, \tag{12}$$

where a hat over a variable represents the logarithmic change, or rate of change, in the variable. Eqs. (11) and (12) imply that each of λ_{jk}^* and $\overline{\kappa}_j^*$ depends only on one cutoff: an increase in a_{jk}^* increases country j's probability of survival in market k, which increases the corresponding revenue share; an increase in a_{jj}^* makes it easier for country j's potential entrant to survive, thereby decreasing its: "expected units of knowledge required to get a 'winner.' " (BRN, 2008, p. 25). Using Eq. (11), the logarithmically differentiated form of Eq. (2) is given by:

$$0 = \textstyle\sum_k \lambda_{jk}^* \widehat{a}_{jk}^*. \tag{13}$$

Turning to Eq. (4), country j's growth rate is decreasing in $(p_j^{K*}/w_j^*)\overline{\kappa}_j^*$, the product of "the p_K-channel and the $\overline{\kappa}$-channel" (BRN, 2008, p. 27). Substituting Eq. (12) into this, and noting that $p_j^{K*} = n_{jj0}p_{j0}^Y =$

$n_{j0}^e G_j(a_{jj}^*)p_j^{Y*}$, where n_{j0}^e is predetermined, and $p_j^{Y*} \equiv p_{j0}^Y$ is evaluated at the initial period of a BGP, it is rewritten as $(p_j^{K*}/w_j^*)\overline{\kappa}_j^* = (n_{j0}^e G_j(a_{jj}^*)p_j^{Y*}/w_j^*)\beta\kappa_j^e/G_j(a_{jj}^*) = \beta\kappa_j^e n_{j0}^e p_j^{Y*}/w_j^*$. This means that country j's growth rate is decreasing in p_j^{Y*}/w_j^*, country j's price of the final good in terms of labor (i.e., the inverse of country j's real wage in terms of the final good). This is because the knowledge good is produced one-to-one from the final good. From $p_j^Y = P_j^{\alpha_j} w_j^{1-\alpha_j}$ and the zero cutoff profit condition (1) for domestic sales, country j's real wage in terms of the final good is expressed as (see Appendix B for derivation):

$$w_j^*/p_j^{Y*} = (p_j^{Y*}/P_j^*)^{\sigma-1} = [1 + (\tau_{kj} - 1)\lambda_{jk}^*]\beta\kappa_j^e n_{j0}^e/\{[a_{jj}^*/(1 - 1/\sigma)]^{\sigma-1}\kappa_{jj}\}, \tag{14}$$

where $P_j^* \equiv P_{j0}$ is evaluated at the initial period of a BGP. Using Eq. (14) and $(p_j^{K*}/w_j^*)\overline{\kappa}_j^* = \beta\kappa_j^e n_{j0}^e p_j^{Y*}/w_j^*$, Eq. (4) is simplified to:

$$\gamma_j^* = (1 - 1/\sigma)L_j/\{[a_{jj}^*/(1 - 1/\sigma)]^{\sigma-1}\kappa_{jj}\} - \rho - \delta. \tag{15}$$

In Eq. (15), compared with Eq. (4), $\overline{m}_j^*$ and $\overline{\kappa}_j^*$ are replaced by a_{jj}^* and κ_{jj}, respectively, and the term $1 + (\tau_{kj} - 1)\lambda_{jk}^*$ is eliminated. Eq. (15) implies that γ_j^* depends only on a_{jj}^*. Specifically, differentiating Eq. (15), and using Eqs. (5), (11), and (15), give:

$$d\gamma_j^* = -(\sigma - 1)(\rho + \delta + \gamma^*)\widehat{a}_{jj}^* = -[(\rho + \delta + \gamma^*)/\beta]\widehat{\lambda}_{jj}^*. \tag{16}$$

Eq. (16) is the ACR formula for long-run growth: country j grows more slowly if and only if domestic selection becomes weaker (i.e., a_{jj}^* increases), or equivalently, it becomes less open (i.e., λ_{jj}^* increases).

Country j's long-run welfare (expressed in flow terms) is given by (see Appendix B for derivation):[11]

$$\begin{aligned}
\rho U_j &= \ln E_j^* - \ln p_j^{Y*} + (1/\rho)\gamma^* = \ln L_j + \ln A_j - (\sigma - 1)\ln a_{jj}^* + \ln \eta_j^* + (1/\rho)\gamma^*; \\
A_j &\equiv \beta\kappa_j^e n_{j0}^e/\{[1/(1 - 1/\sigma)]^{\sigma-1}\kappa_{jj}\}, \\
\eta_j^* &\equiv (1 - 1/\sigma)\rho/(\rho + \delta + \gamma^*) + 1 + \sigma(\tau_{kj} - 1)\lambda_{jk}^* > 1, k \neq j.
\end{aligned} \tag{17}$$

In the far right-hand side of Eq. (17), the sum of the second and third terms corresponds to country j's real wage divided by $1 + (\tau_{kj} - 1)\lambda_{jk}^*$ as seen from Eq. (14). In the definition of η_j^*, the first, second, and third terms indicate country j's interest income, wage income, and tariff revenue, respectively.

Totally differentiating Eq. (17), and using Eqs. (11), (13), and (16), we obtain:

$$\begin{aligned}
\rho dU_j &= (\sigma/\eta_j^*)\lambda_{jk}^*\tau_{kj}\widehat{\tau}_{kj} - (\sigma - 1)[1 + (\sigma/\eta_j^*)(\tau_{kj} - 1)\beta(1 - \lambda_{jk}^*)]\widehat{a}_{jj}^* + \Gamma_j^* d\gamma^* \\
&= (\sigma/\eta_j^*)\lambda_{jk}^*\tau_{kj}\widehat{\tau}_{kj} - (1/\beta)[1 + (\sigma/\eta_j^*)(\tau_{kj} - 1)\beta(1 - \lambda_{jk}^*) + \Omega_j^*]\widehat{\lambda}_{jj}^*; \\
\Gamma_j^* &\equiv -[(1 - 1/\sigma)\rho/(\rho + \delta + \gamma^*)^2]/\eta_j^* + 1/\rho \\
&= [1/(\rho\eta_j^*)][(1 - 1/\sigma)\rho(\delta + \gamma^*)/(\rho + \delta + \gamma^*)^2 + 1 + \sigma(\tau_{kj} - 1)\lambda_{jk}^*] > 0, \\
\Omega_j^* &\equiv \Gamma_j^*(\rho + \delta + \gamma^*), k \neq j.
\end{aligned} \tag{18}$$

In Eq. (18), the ACR formula for long-run welfare, there are two terms in the far right-hand side. The first

[11] Supposing that the representative household receives a constant utility flow $\ln E_j^* - \ln p_j^{Y*} + (1/\rho)\gamma^* = \rho U_j$ discounted by a factor $\exp(-\rho t)$ over an infinite horizon, its present discounted value is $\int_0^\infty \rho U_j \exp(-\rho t)dt = \rho U_j(1/\rho) = U_j$.

term represents the direct effect of a change in country j's import tariff τ_{kj} on its long-run welfare through a change in its tariff revenue. The second term summarizes the effects of a change in country j's domestic revenue share λ_{jj}^{*} (or equivalently, its domestic cutoff a_{jj}^{*}), which commonly appears in the Armington, Krugman, Eaton–Kortum, and Melitz–Pareto models without import tariffs as shown by Arkolakis et al. (2012). Specifically, suppose that domestic selection becomes weaker in country j (i.e., a_{jj}^{*} increases). On the one hand, this decreases its long-run welfare by decreasing its real wage, and also its tariff revenue indirectly through a decrease in its revenue share of exported varieties. On the other hand, it decreases the balanced growth rate. This directly decreases the welfare by decreasing future consumption, but it indirectly increases the welfare by increasing the interest income from the asset. Since the direct growth effect is always stronger than the counteracting indirect growth effect as long as $Q_j^K = \overline{\kappa}_j^{*} n_{jj}(\gamma^{*}+\delta) \geq 0$, the decrease in the balanced growth rate necessarily decreases the welfare. Overall, weaker domestic selection (i.e., an increase in a_{jj}^{*}), or equivalently, more autarkiness (i.e., an increase in λ_{jj}^{*}), is bad for country j's long-run welfare.

Our results so far are summarized in the following proposition:

Proposition 1 *An increase in a country's domestic revenue share implies a decrease in the balanced growth rate, but it does not imply a decrease in its long-run welfare.*

As Eqs. (16) and (18) show, an increase in country j's domestic revenue share λ_{jj}^{*} necessarily decreases the balanced growth rate, and also partly decreases its long-run welfare. However, if the increase in λ_{jj}^{*} is caused by an increase in country j's import tariff τ_{kj}, which sounds quite natural, its long-run welfare partly increases through the increased tariff revenue. It is the last effect that usually causes a large country's optimal tariff to be positive. In the following sections, we solve for general equilibrium effects of a tariff change to see how much the optimal tariff is for a growing large country.

4 Long-run growth effect of a tariff change

From now on, we omit asterisks just for notational simplicity. The long-run growth effects of tariff changes are derived in six steps: (i) from Eqs. (3) and (6), we solve for $\widehat{p_1^Y/p_2^Y} = \widehat{p_1^Y/p_2^Y}(\widehat{w}_1, \widehat{\chi}, \{\widehat{a}_{jk}\}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; (ii) substituting the result from step (i) into the logarithmically differentiated forms of Eqs. (7) and (8), and combining them with Eq. (13), we solve for $\widehat{a}_{jk} = \widehat{a}_{jk}(\widehat{w}_1, \widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; (iii) substituting the result from step (ii) into Eq. (11), and substituting it into the logarithmically differentiated form of Eq. (10), we solve for $\widehat{w}_1 = \widehat{w}_1(\widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; (iv) substituting the result from step (iii) back into $\widehat{a}_{jj} = \widehat{a}_{jj}(\widehat{w}_1, \widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$, and substituting it into Eq. (16), we solve for $d\gamma_j = d\gamma_j(\widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$; (v) substituting the result from step (iv) into the differentiated form of Eq. (5), we solve for $\widehat{\chi} = \widehat{\chi}(\widehat{\tau}_{21}, \widehat{\tau}_{12})$; and (vi) substituting the result in step (v) back into $d\gamma_2 = d\gamma_2(\widehat{\chi}, \widehat{\tau}_{21}, \widehat{\tau}_{12})$, we solve for $d\gamma = d\gamma_2(\widehat{\tau}_{21}, \widehat{\tau}_{12})$. After long but clear calculations following these steps, we finally obtain (see Appendix C for derivation):

$$d\gamma = -\sigma(\rho+\delta+\gamma)[\lambda_{jk}\lambda_{kj}/(\lambda_{jk}+\lambda_{kj})](\widehat{\tau}_{kj}+\widehat{\tau}_{jk}), k \neq j. \tag{19}$$

Eq. (19) immediately implies that:

$$\partial\gamma/\partial \ln \tau_{kj} = \partial\gamma/\partial \ln \tau_{jk} = -\sigma(\rho+\delta+\gamma)\lambda_{jk}\lambda_{kj}/(\lambda_{jk}+\lambda_{kj}) < 0 \forall j, k, k \neq j.$$

Proposition 2 *An increase in the import tariff of either country by the same rate decreases the balanced growth rate by the same amount.*

Suppose that country 1 increases its import tariff τ_{21}. With p_1^Y/p_2^Y given, this makes country 2 relatively less competitive in market 1 (i.e., decreases a_{21} from Eq. (8)). Since country 2's expected profit from exports decreases, free entry requires that its expected profit from domestic sales increases, causing more unproductive firms to stay in their domestic market (i.e., a_{22} increases from Eq. (2)). Because of easier competition with country 2's domestic firms, more firms from country 1 start exporting (i.e., a_{12} increases from Eq. (7)). This drives more of country 1's unproductive firms out of their domestic market (i.e., decreases a_{11} from Eq. (2)). Country 1's increased import protection causes less exports and less domestic selection in country 2, whereas it causes more exports and more domestic selection in country 1, with p_1^Y/p_2^Y given.

In fact, the increase in τ_{21} affects p_1^Y/p_2^Y. With country 1 exporting more and importing less, it tends to run a trade surplus. For the surplus to be cleared, w_1 and hence p_1^Y/p_2^Y increase so that country 1 becomes relatively more costly in producing the intermediate goods (see Eqs. (6) and (10)).[12] This makes country 2 relatively more competitive in market 1, implying more exports and more domestic selection (see Eqs. (2) and (8)). Similarly, country 1 becomes relatively less competitive in market 2, causing less exports and less domestic selection (see Eqs. (2) and (7)). These indirect effects work in the opposite directions of the direct effects in the previous paragraph. It turns out that the direct effects outweigh the indirect effects for country 2, whereas the opposite is true for country 1. Since domestic selection becomes weaker in both countries, both countries grow more slowly, with χ given. Finally, even if χ adjusts to equalize countries' growth rates, the new balanced growth rate is lower than the old one.

Proposition 2 has both qualitative and quantitative implications. Qualitatively, even a unilateral tariff reduction always raises long-run growth. In the literature on endogenous growth and heterogeneous firms (e.g., BRN, 2008; Dinopoulos and Unel, 2011; Perla et al., 2015; Fukuda, 2016; Sampson, 2016; Ourens, 2016; Naito, 2017, 2018), only Perla et al. (2015), Sampson (2016), and Naito (2018) show that reductions in iceberg trade costs always raises long-run growth, of which only Naito (2018) deals with a unilateral trade cost reduction in an asymmetric-country setting. By considering revenue-generating import tariffs as more realistic policy variables for the first time in the literature, our result provides further support for the positive long-run growth effect of trade liberalization. Quantitatively, a 1% tariff reduction in either a larger or a smaller country has the same long-run growth effect. As trade theories tell us that a smaller country has a smaller terms of trade impact than a larger country, we might guess that a smaller country affects the balanced growth rate by less than a larger country. Our result demonstrates that this conjecture is not true.

Armed with Proposition 2, we characterize the optimal tariff of a large country in the next section.

5 Can the optimal tariff be zero for a growing large country?

Substituting Eqs. (16) and (19) into Eq. (18), the amount of change in country j's long-run welfare is expressed only in terms of the rates of changes in tariffs as:

$$\rho dU_j = \sigma\lambda_{jk}\{(\tau_{kj}/\eta_j)\widehat{\tau}_{kj} - [1 + (\sigma/\eta_j)(\tau_{kj} - 1)\beta(1 - \lambda_{jk}) + \Omega_j][\lambda_{kj}/(\lambda_{jk} + \lambda_{kj})](\widehat{\tau}_{kj} + \widehat{\tau}_{jk})\}, k \neq j. \quad (20)$$

Eq. (20) immediately implies that:

[12]In (10), it seems that an increase in w_1 directly increases country 1's trade surplus. However, the resulting increase in p_1^Y/p_2^Y indirectly decreases its surplus by decreasing its exports but increasing its imports. Since the sum of the indirect effects is stronger than the direct effect, country 1's trade surplus is decreasing in w_1. See Eq. (C.5) in Appendix C for details.

$$\rho \partial U_j/\partial \ln \tau_{jk} = -\sigma\lambda_{jk}[1+(\sigma/\eta_j)(\tau_{kj}-1)\beta(1-\lambda_{jk})+\Omega_j]\lambda_{kj}/(\lambda_{jk}+\lambda_{kj}) < 0,$$
$$\rho \partial U_j/\partial \ln \tau_{kj} = \sigma\lambda_{jk}\Psi_j; \Psi_j \equiv \tau_{kj}/\eta_j - [1+(\sigma/\eta_j)(\tau_{kj}-1)\beta(1-\lambda_{jk})+\Omega_j]\lambda_{kj}/(\lambda_{jk}+\lambda_{kj}), k \neq j. \quad (21)$$

An increase in either τ_{kj} or τ_{jk} decreases the balanced growth rate from Eq. (19). This implies from Eq. (16) that country j becomes less open, which is bad for its long-run welfare. Since the increase in the other country's tariff τ_{jk} does not provide the tariff revenue to country j, it necessarily decreases country j's long-run welfare. However, the increase in country j's own tariff τ_{kj} creates a trade-off between gains from tariff revenue and losses from autarkiness, as represented by the first and second terms, respectively, in the definition of Ψ_j in Eq. (21). If country j's optimal tariff is positive, then it must satisfy $\Psi_j = 0$. Alternatively, if $\Psi_j < 0$ at $\tau_{kj} = 1$, then the status quo of zero tariff is locally optimal (and globally optimal if $\rho\partial^2 U_j/\partial(\ln \tau_{kj})^2 < 0 \forall \tau_{kj} \geq 1$). The condition is rewritten as:

$$\Psi_j|_{\tau_{kj}=1} = 1/\eta_j - (1+\Omega_j)\lambda_{kj}/(\lambda_{jk}+\lambda_{kj}) < 0 \Leftrightarrow \lambda_{jk}/\lambda_{kj} < 1 - 1/\sigma + (\rho+\delta+\gamma)/\rho, k \neq j. \quad (22)$$

Proposition 3 *A zero tariff is locally optimal for country j if $\lambda_{jk}/\lambda_{kj} < 1 - 1/\sigma + (\rho+\delta+\gamma)/\rho, k \neq j$, at a BGP with $\tau_{kj} = 1$. In particular, it is true if the two countries are symmetric at the BGP.*

The sufficient condition for the zero optimal tariff (22) states that the export revenue share of country j relative to country k is smaller than the upper bound $1 - 1/\sigma + (\rho+\delta+\gamma)/\rho$, which is larger than unity. This means that the condition is automatically satisfied if $\lambda_{jk}/\lambda_{kj} = 1$, that is, the countries are symmetric. By continuity, Eq. (22) is true as long as the countries are similar. Moreover, the upper bound is monotonically decreasing in ρ.[13] As ρ becomes smaller and smaller, the permissible range of $\lambda_{jk}/\lambda_{kj}$ becomes larger and larger. In the limit, as ρ approaches zero from above, Eq. (22) is satisfied for all positive export revenue shares. Therefore, zero optimal tariffs for large countries are quite common in our model.

Proposition 3 just shows that a zero tariff is locally optimal for a large country, but it is still unclear if it is optimal for a wider and more relevant domain of tariffs. To consider this, we make some numerical experiments. We start from calculating a symmetric BGP with free trade as a benchmark. Key parameters are borrowed from other work: $\rho = 0.02$ from Acemoglu (2009); and $\sigma = 4, \theta = 4, \delta = 0.025$ from Balistreri et al. (2011).[14] The other parameters and initial conditions are set arbitrarily: $L_j = 1, \kappa_{jj} = 2, \kappa_{jk} = 4, \kappa_j^e = 2$, and $n_{20}^e = 1,000$ (implying that $n_{10}^e = 1,000$ at the benchmark BGP). Finally, country j's Pareto scale parameter a_{j0}, controlling its overall productivity in the intermediate good sector, is set to $a_{j0} = 2$ to produce a reasonable value of the balanced growth rate. The resulting values of key endogenous variables are calculated as follows: $p_1^Y/p_2^Y = 1, a_{jj} = 1.313, a_{jk} = 1.042, \chi = 1, w_1 = 1, \lambda_{jk} = 0.442$, and $\gamma = 0.02487$, or 2.487%. The balanced growth rate of two to three percent is realistic enough.

Next, to see how tariffs affect a country's long-run welfare, and how the relationship changes with technological asymmetry and time preference, in Fig. 1 we draw country 1's iso-welfare curves (expressed in flow terms) on the (τ_{21}, τ_{12}) plane for nine pairs of (a_{10}, ρ). We first look at the middle center panel corresponding to the benchmark case. τ_{kj} ranges from 1 to 2, meaning that country j's ad valorem tariff

[13] Eqs. (5), (9) (derived from Eqs. (2), (7), and (8), together with Eqs. (3) and (6)), and (10) imply that $\chi, \{a_{jk}\}$, and w_1 depend on σ and θ, but not on ρ and δ. Then from Eq. (15), $\rho + \delta + \gamma = (1-1/\sigma)L_j/\{[a_{jj}/(1-1/\sigma)]^{\sigma-1}\kappa_{jj}\}$ is independent of ρ and δ: an increase in ρ and/or δ decreases γ by the same amount so that both sides of this equation are unchanged.

[14] Assuming that $\delta = 0.025$ from Bernard et al. (2007), and $\sigma = 3.8$ from Bernard et al. (2003), Balistreri et al. (2011) estimate that θ ranges from 3.9 to 5.2. Felbermayr et al. (2013) also assume that $\sigma = 3.8$ and $\theta = 4$.

rate takes from 0 to 100%. The number attached to each iso-welfare curve indicates the value of country 1's long-run welfare (expressed in flow terms) ρU_1. All displayed iso-welfare curves are downward-sloping, and ρU_1 increases as we move down and to the left. This implies that, with τ_{12} given, reducing τ_{21} to $\tau_{21} = 1$ maximizes ρU_1. Therefore, a zero tariff is optimal for country 1 for this relevant domain of tariffs.

As we move up to the top center panel, where ρ increases to $\rho = 0.03$, iso-welfare curves become flatter, suggesting that the negative relationship between τ_{21} and ρU_1 becomes relatively weaker due to the decreased net growth effect on welfare Γ_1^*.[15] In contrast, as we move down to the bottom center panel, where $\rho = 0.01$, iso-welfare curves become steeper. Anyway, for all three panels in the center column, all iso-welfare curves are downward-sloping, and hence a zero tariff is optimal for country 1.

Again starting from the middle center panel, suppose that a_{10} decreases by 10% to $a_{10} = 1.8$. As country 1 becomes more technologically advanced than country 2, the former becomes richer and less open than the latter on a BGP with free trade: $w_1 = 1.697, \lambda_{12} = 0.330, \lambda_{21} = 0.561$. Downward-sloping iso-welfare curves become steeper because the increase in $\lambda_{21}/(\lambda_{12} + \lambda_{21})$ intensifies the negative second term in Ψ_j of Eq. (21), the losses from autarkiness. This is true for all other rows. Therefore, a zero tariff is still optimal for a more technologically advanced country.

As we move from the middle center panel to the middle right panel, where country 1 becomes less technologically advanced than country 2 (i.e., $a_{10} = 2.2 > 2 = a_{20}$), we have $w_1 = 0.620, \lambda_{12} = 0.550, \lambda_{21} = 0.341$, and iso-welfare curves become flatter in contrast to the previous paragraph. Moreover, the upper three iso-welfare curves turn from downward-sloping to upward-sloping to the right of them. This is because an increase in τ_{12} makes country 2 less open (i.e., decreases λ_{21}), which weakens country 1's losses from autarkiness. This implies that, with τ_{12} sufficiently large, country 1's government can increase its long-run welfare by either decreasing or increasing τ_{21}. For example, with $\tau_{12} = 1.2$ fixed, starting from $\tau_{21} = 1.5$ (where $5.8 < \rho U_1 < 6$), country 1 gains by setting $\tau_{21} = 1$ (where $\rho U_1 \approx 6$), and it gains more by setting $\tau_{21} = 2$ (where $6 < \rho U_1 < 6.2$).[16] For a less technologically advanced country, we cannot ensure that a zero tariff is always optimal. The three panels in the right column suggest that the tendency becomes stronger as ρ increases. However, they also indicate that, as long as the more technologically advanced country 2 sets a zero optimal tariff, it is still optimal for the less technologically advanced country 1 to choose a zero tariff.

Fig. 1 shows that, for a relevant domain of tariffs within 100%, a zero tariff is optimal for a large country if it is no less technologically advanced (and hence no poorer) than the other country; otherwise, a zero tariff might not be optimal for a sufficiently large tariff of the other country. Also, the more technologically advanced a country is relative to the other country, and/or the more patient countries are, the more likely its optimal tariff is to be zero. To sum up, the optimal tariff can be zero for a growing large country.

6 Concluding remarks

Our theory has important policy implications. If national leaders take economic growth seriously, as they almost always say they do, it makes little sense for their own countries to deviate from free trade. In the face of the recent U.S.-China trade disputes, a typical argument against them by trade economists is that they could end up with a prisoner's dilemma, and committing to the reciprocity principle of the GATT/WTO would be a solution. By incorporating the simplest and widely accepted endogenous technological change mechanism of the Rivera-Batiz–Romer lab-equipment type into an asymmetric Melitz model with import

[15] Another observation is that, for the same (τ_{21}, τ_{12}), the value of ρU_1 decreases despite that ρ increases. This implies that U_1 decreases by more than the increase in ρ.

[16] Country 1 could gain more if it could set its tariff beyond $\tau_{21} = 2$. The problem, then, is whether an optimal tariff exists.

tariffs, this paper provides a stronger argument that it is in each country's own interest to keep free trade even if it is large in an economic sense.

References

[1] Acemoglu, D., 2009. Introduction to Modern Economic Growth. Princeton University Press, Princeton.

[2] Arkolakis, C., Costinot, A., Rodríguez-Clare, A., 2012. New trade models, same old gains? American Economic Review 102, 94–130.

[3] Bagwell, K., Staiger, R. W., 1999. An economic theory of GATT. American Economic Review 89, 215–248.

[4] Baldwin, R. E., Robert-Nicoud, F., 2008. Trade and growth with heterogeneous firms. Journal of International Economics 74, 21–34.

[5] Balistreri, E. J., Hillberry, R. H., Rutherford, T. F., 2011. Structural estimation and solution of international trade models with heterogeneous firms. Journal of International Economics 83, 95–108.

[6] Bernard, A. B., Eaton, J., Jensen, J. B., Kortum, S., 2003. Plants and productivity in international trade. American Economic Review 93, 1268–1290.

[7] Bernard, A. B., Redding, S. J., Schott, P. K., 2007. Comparative advantage and heterogeneous firms. Review of Economic Studies 74, 31–66.

[8] Bond, E. W., 1990. The optimal tariff structure in higher dimensions. International Economic Review 31, 103–116.

[9] Demidova, S., 2017. Trade policies, firm heterogeneity, and variable markups. Journal of International Economics 108, 260–273.

[10] Dinopoulos, E., Unel, B., 2011. Quality heterogeneity and global economic growth. European Economic Review 55, 595–612.

[11] Estevadeordal, A., Taylor, A. M., 2013. Is the Washington Consensus dead? growth, openness, and the Great Liberalization, 1970s-2000s. Review of Economics and Statistics 95, 1669–1690.

[12] Felbermayr, G., Jung, B., Larch, M., 2013. Optimal tariffs, retaliation, and the welfare loss from tariff wars in the Melitz model. Journal of International Economics 89, 13–25.

[13] Fukuda, K., 2016. The effect of globalization in an endogenous growth model with firm heterogeneity, international spillover, and trade: a note. unpublished.

[14] Horwell, D. J., Pearce, I. F., 1970. A look at the structure of optimal tariff rates. International Economic Review 11, 147–161.

[15] Johnson, H. G., 1953. Optimum tariffs and retaliation. Review of Economic Studies 21, 142–153.

[16] Kaldor, N., 1940. A note on tariffs and the terms of trade. Economica 7, 377–380.

[17] Kennan, J., Riezman, R., 1988. Do big countries win tariff wars? International Economic Review 29, 81–85.

[18] Krugman, P., 1980. Scale economies, product differentiation, and the pattern of trade. American Economic Review 70, 950–959.

[19] Melitz, M. J., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. Econometrica 71, 1695–1725.

[20] Naito, T., 2017. Growth and welfare effects of unilateral trade liberalization with heterogeneous firms and asymmetric countries. Journal of International Economics 109, 167–173.

[21] Naito, T., 2018. A lab-equipment model of growth with heterogeneous firms and asymmetric countries. unpublished.

[22] Ogawa, Y., 2007. The structure of optimal tariff rates in a large country with market power. Economic Theory 33, 271–283.

[23] Ourens, G., 2016. Trade and growth with heterogeneous firms revisited. Journal of International Economics 100, 194–202.

[24] Perla, J., Tonetti, C., Waugh, M. E., 2015. Equilibrium technology diffusion, trade, and growth. unpublished.

[25] Rivera-Batiz, L. A., Romer, P. M., 1991. Economic integration and endogenous growth. Quarterly Journal of Economics 106, 531–555.

[26] Rodriguez, F., Rodrik, D., 2000. Trade policy and economic growth: a skeptic's guide to the cross-national evidence. NBER Macroeconomics Annual 15, 261–325.

[27] Sampson, T., 2016. Dynamic selection: an idea flows theory of entry, trade, and growth. Quarterly Journal of Economics 131, 315–380.

[28] Wacziarg, R., Welch, K. H., 2008. Trade liberalization and growth: new evidence. World Bank Economic Review 22, 187–231.

Appendix A. Derivations of key equations in section 3.1

Derivation of $\gamma_j^* = [\alpha_j/(1-\alpha_j)](1/\sigma)\{1/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}w_j^* L_j/(p_j^{K*}\overline{\kappa}_j^*) - \rho - \delta, k \neq j$

Using Eq. (2), the asset market-clearing condition $W_j = \sum_k n_{jk} \int_0^{a_{jk}} v_{jk}(a)\mu_{jk}(a|a_{jk})da$ is rewritten as:

$$\begin{aligned} W_j &= n_{jj}\textstyle\sum_k (G_j(a_{jk})/G_j(a_{jj})) \displaystyle\int_0^{a_{jk}} v_{jk}(a)\mu_{jk}(a|a_{jk})da \\ &= (n_{jj}/G_j(a_{jj}))P_j^K(\textstyle\sum_k \kappa_{jk} G_j(a_{jk}) + \kappa_j^e) = p_j^K \overline{\kappa}_j; p_j^K \equiv n_{jj}P_j^K. \end{aligned} \tag{A.1}$$

Time differentiating Eq. (A.1), and using Eq. (A.1), the no-arbitrage condition $\dot{v}_{jk}(a) = (r_j + \delta)v_{jk}(a) - \pi_{jk}(a)$ (derived by time differentiating $v_{jkt}(a) = \int_t^\infty \pi_{jks}(a)\exp(-\int_t^s (r_{ju}+\delta)du)ds$), $\pi_{jk}(a) = e_{jk}(a)/\sigma$, and $E_{jk} = n_{jk}\int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk})da$, we obtain:

$$\dot{W}_j = W_j(\gamma_j + r_j + \delta) - (1/\sigma)\textstyle\sum_k E_{jk}; \gamma_j \equiv \dot{n}_{jj}/n_{jj}.$$

Rewriting $P_j X_j = \sum_k n_{kj}\int_0^{a_{kj}} \tau_{kj} p_{kj}^f(a) x_{kj}(a)\mu_{kj}(a|a_{kj})da$ using $y_{kj}(a) = x_{kj}(a), e_{jk}(a) = p_{jk}^f(a)y_{jk}(a), E_{jk} = n_{jk}\int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk})da, E_{jk} = E_{kj}$, and $\lambda_{jk} = E_{jk}/\sum_l E_{jl}$, we obtain:

$$\textstyle\sum_l E_{jl} = \{1/[1+(\tau_{kj}-1)\lambda_{jk}]\}P_j X_j, k \neq j. \tag{A.2}$$

Applying Shephard's lemma to $c_j^Y(P_j, w_j)Y_j$, and using $p_j^Y = c_j^Y(P_j, w_j)$, the expenditures for the intermediate goods and labor are given by, respectively:

$$P_j X_j = \alpha_j p_j^Y Y_j, \tag{A.3}$$

$$w_j L_j^Y = (1-\alpha_j)p_j^Y Y_j. \tag{A.4}$$

Using Eqs. (A.2) and (A.3), the expression for $\dot{W}_j$ is rewritten as:

$$\dot{W}_j = W_j(\gamma_j + r_j + \delta) - \{(\alpha_j/\sigma)/[1+(\tau_{kj}-1)\lambda_{jk}]\}p_j^Y Y_j, k \neq j.$$

Multiplying $Y_j = C_j + D_j + F_j$ by p_j^Y, and using Eqs. (A.2), (A.3), $\pi_{jk}(a) = e_{jk}(a) - p_j^Y a y_{jk}(a) = e_{jk}(a)/\sigma, P_j^K Q_j^K = p_j^Y D_j, Q_j^K = \overline{\kappa}_j(\dot{n}_{jj} + \delta n_{jj}), F_j = \sum_k n_{jk}\int_0^{a_{jk}} a y_{jk}(a)\mu_{jk}(a|a_{jk})da, p_j^K = n_{jj}P_j^K$, and $E_{jk} = n_{jk}\int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk})da$, $p_j^Y Y_j$ is expressed as:

$$p_j^Y Y_j = \{1/\{1-(1-1/\sigma)\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}\}[E_j + p_j^K \overline{\kappa}_j(\gamma_j + \delta)], k \neq j. \tag{A.5}$$

Substituting Eq. (A.5) into the last expression for $\dot{W}_j$, and using Eq. (A.1), we obtain:

$$\begin{aligned} \dot{W}_j/W_j &= r_j + \{\{1-\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}/\{1-(1-1/\sigma)\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}\}(\gamma_j+\delta) \\ &\quad - \{\{(\alpha_j/\sigma)/[1+(\tau_{kj}-1)\lambda_{jk}]\}/\{1-(1-1/\sigma)\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}\}Z_j; \\ Z_j &\equiv E_j/W_j, k \neq j, \end{aligned}$$

where a transformed variable $Z_j = E_j/W_j$ is interpreted as country j's average propensity to consume out of asset. Substituting the above expression and the Euler equation $\dot{E}_j/E_j = r_j - \rho$ into $\dot{Z}_j/Z_j = \dot{E}_j/E_j - \dot{W}_j/W_j$ gives:

$$\begin{aligned}\dot{Z}_j/Z_j &= \{\{(\alpha_j/\sigma)/[1+(\tau_{kj}-1)\lambda_{jk}]\}/\{1-(1-1/\sigma)\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}\}Z_j - \rho \\ &\quad - \{\{1-\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}/\{1-(1-1/\sigma)\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}\}(\gamma_j+\delta), k \neq j. \qquad \text{(A.6)}\end{aligned}$$

Multiplying $L_j = L_j^Y$ by w_j, and using Eqs. (A.1), (A.4), and (A.5), we obtain:

$$\gamma_j = \{\{1-(1-1/\sigma)\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}]\}/(1-\alpha_j)\}w_jL_j/(p_j^K\overline{\kappa}_j) - Z_j - \delta, k \neq j. \qquad \text{(A.7)}$$

On a BGP, both $\dot{Z}_j/Z_j$ and γ_j are constant. Noting that $n_{jk} = n_{jj}G_j(a_{jk})/G_j(a_{jj})$, λ_{jk} is rewritten as $\lambda_{jk} = G_j(a_{jk})\int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk})da/\sum_l G_j(a_{jl})\int_0^{a_{jl}} e_{jl}(a)\mu_{jl}(a|a_{jl})da$, which depends only on country j's cutoffs $\{a_{jk}\}$. Since $\{a_{jk}\}$ are constant from Eq. (2), λ_{jk} is constant on a BGP. Then Eq. (A.6) implies that Z_j is constant on a BGP. From Eqs. (A.6), (A.7), and $\dot{Z}_j/Z_j = 0$, Z_j and γ_j are solved as:

$$Z_j^* = \rho + \{\{1-\alpha_j/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}/(1-\alpha_j)\}w_j^*L_j/(p_j^{K*}\overline{\kappa}_j^*), \qquad \text{(A.8)}$$
$$\gamma_j^* = [\alpha_j/(1-\alpha_j)](1/\sigma)\{1/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}w_j^*L_j/(p_j^{K*}\overline{\kappa}_j^*) - \rho - \delta, k \neq j. \qquad \text{(A.9)}$$

Derivations of Eqs. (7) and (8)

The right-hand side of $v_{jk0}(a_{jk}^*)/v_{kk0}(a_{kk}^*) = P_{j0}^K\kappa_{jk}/(P_{k0}^K\kappa_{kk}), j \neq k$, is simply rewritten as $(p_j^Y/p_k^Y)^*\kappa_{jk}/\kappa_{kk}$. In the left-hand side, $v_{jk0}(a)$ is given by:

$$v_{jk0}(a) = \pi_{jk0}(a)\Delta_{jk0}(a); \Delta_{jk0}(a) \equiv \int_0^\infty \exp(-\int_0^t (r_{js} + \delta - \dot{\pi}_{jks}(a)/\pi_{jks}(a))ds)dt.$$

Calculating $\Delta_{jk0}(a)$ requires calculating r_{js} and $\dot{\pi}_{jks}(a)/\pi_{jks}(a)$ on a BGP. For r_{js}, multiplying Eq. (A.8) by $W_j^* = p_j^{K*}\overline{\kappa}_j^*$ from Eq. (A.1), and using $\alpha_j = 1-1/\sigma$, we obtain:

$$\begin{aligned}E_j^* &= p_j^{K*}\overline{\kappa}_j^*\{\rho + \{1+(\sigma-1)(\tau_{kj}-1)\lambda_{jk}^*/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}w_j^*L_j/(p_j^{K*}\overline{\kappa}_j^*)\} \\ &= p_j^{K*}\overline{\kappa}_j^*\rho + \{1+(\sigma-1)(\tau_{kj}-1)\lambda_{jk}^*/[1+(\tau_{kj}-1)\lambda_{jk}^*]\}w_j^*L_j, k \neq j. \qquad \text{(A.10)}\end{aligned}$$

Since $\overline{\kappa}_j^*, \lambda_{jk}^*, w_j^*$, and p_j^{K*} are constant, E_j^* is constant from Eq. (A.10). This and the Euler equation imply that $r_j^* = \rho$.

For $\dot{\pi}_{jks}(a)/\pi_{jks}(a)$, noting that $P_jX_j = (\sigma-1)w_jL_j$ from Eqs. (A.3), (A.4), $L_j = L_j^Y$, and $\alpha_j = 1-1/\sigma$, $\pi_{jk}(a) = \tau_{jk}^{-\sigma}[p_j^Y a/(1-1/\sigma)]^{1-\sigma}P_k^\sigma X_k/\sigma$ is rewritten as:

$$\pi_{jkt}(a) = \tau_{jk}^{-\sigma}[a/(1-1/\sigma)]^{1-\sigma}(P_{kt}/p_{jt}^Y)^{\sigma-1}(1-1/\sigma)w_k^*L_k. \qquad \text{(A.11)}$$

Dividing Eq. (3) for $j = k$ by p_{jt}^Y gives $P_{kt}/p_{jt}^Y = n_{kkt}^{1/(1-\sigma)}(p_k^Y/p_j^Y)^*\overline{m}_k^*/(1-1/\sigma)$. Substituting this into Eq. (A.11), and noting that n_{kkt} grows at the rate γ^*, $\pi_{jkt}(a)$ grows at the rate $-\gamma^*$: $\dot{\pi}_{jkt}(a)/\pi_{jkt}(a) = -\gamma^*$.

Substituting the results into the definition of $\Delta_{jk0}(a)$, we obtain $\Delta_{jk0}(a) = 1/(\rho+\delta+\gamma^*)$, and hence:

$$v_{jk0}(a) = \pi_{jk0}(a)/(\rho + \delta + \gamma^*). \tag{A.12}$$

Dividing Eq. (A.12) by itself with $j = k$, and using Eq. (A.11), we obtain $v_{jk0}(a^*_{jk})/v_{kk0}(a^*_{kk}) = \pi_{jk0}(a^*_{jk})/\pi_{kk0}(a^*_{kk}) = \tau_{jk}^{-\sigma}[(p^Y_j/p^Y_k)^* a^*_{jk}/a^*_{kk}]^{1-\sigma}$. Therefore, $v_{jk0}(a^*_{jk})/v_{kk0}(a^*_{kk}) = P^K_{j0}\kappa_{jk}/(P^K_{k0}\kappa_{kk}), j \neq k$, is rewritten as:

$$\tau_{jk}^{-\sigma}[(p^Y_j/p^Y_k)^* a^*_{jk}/a^*_{kk}]^{1-\sigma} = (p^Y_j/p^Y_k)^*\kappa_{jk}/\kappa_{kk}, j \neq k.$$

Solving this for a^*_{12}/a^*_{22} and a^*_{21}/a^*_{11}, we obtain Eqs. (7) and (8), respectively.

Derivation of Eq. (10)

Substituting Eq. (A.2) and $P_jX_j = (\sigma - 1)w_jL_j$ into $\lambda_{jk}\sum_l E_{jl} = \lambda_{kj}\sum_l E_{kl}, k \neq j$, we obtain Eq. (10). For λ^*_{jk}, using Eqs. (1), (A.12), $\pi_{jk}(a) = e_{jk}(a)/\sigma$, and $e_{jk}(a)/e_{jk}(a^*_{jk}) = (a/a^*_{jk})^{1-\sigma}$, we obtain $\int_0^{a^*_{jk}} e_{jk}(a)\mu_{jk}(a|a^*_{jk})da = (h_{jk}(a^*_{jk}) + 1)\sigma(\rho + \delta + \gamma^*)P^K_{j0}\kappa_{jk}$. Substituting this into
$\lambda^*_{jk} = G_j(a^*_{jk})\int_0^{a^*_{jk}} e_{jk}(a)\mu_{jk}(a|a^*_{jk})da/\sum_l G_j(a^*_{jl})\int_0^{a^*_{jl}} e_{jl}(a)\mu_{jl}(a|a^*_{jl})da$ gives
$\lambda^*_{jk} = (H_{jk}(a^*_{jk}) + G_j(a^*_{jk}))\kappa_{jk}/\sum_l(H_{jl}(a^*_{jl}) + G_j(a^*_{jl}))\kappa_{jl}$.

Appendix B. Derivations of key equations in section 3.2

Derivation of Eq. (14)

From $p^Y_j = P_j^{\alpha_j} w_j^{1-\alpha_j}$ and $\alpha_j = 1 - 1/\sigma$, we obtain $w^*_j/p^{Y*}_j = (p^{Y*}_j/P^*_j)^{\sigma-1}$, where $P^*_j \equiv P_{j0}$ is evaluated at the initial period of a BGP.

Using (A.11), (A.12), and $p^{K*}_j = n^e_{j0}G_j(a^*_{jj})P^K_{j0}$, the zero cutoff profit condition (1) for domestic sales is rewritten as:

$$[a^*_{jj}/(1 - 1/\sigma)]^{1-\sigma}(P^*_j/p^{Y*}_j)^{\sigma-1}(1 - 1/\sigma)w^*_jL_j/(\rho + \delta + \gamma^*) = [p^{K*}_j/(n^e_{j0}G_j(a^*_{jj}))]\kappa_{jj}.$$

Using Eq. (5), Eq. (4) is rewritten as:

$$\begin{aligned}&: \rho + \delta + \gamma^* = \{(1 - 1/\sigma)/[1 + (\tau_{kj} - 1)\lambda^*_{jk}]\}w^*_jL_j/(p^{K*}_j\overline{\kappa}^*_j)\\ &\Leftrightarrow (1 - 1/\sigma)w^*_jL_j/(\rho + \delta + \gamma^*) = [1 + (\tau_{kj} - 1)\lambda^*_{jk}]p^{K*}_j\overline{\kappa}^*_j, k \neq j.\end{aligned} \tag{B.1}$$

Substituting Eq. (12) into Eq. (B.1), substituting it into the above zero cutoff profit condition, and solving it for $(p^{Y*}_j/P^*_j)^{\sigma-1} = w^*_j/p^{Y*}_j$, we obtain Eq. (14).

Derivation of Eq. (17)

Substituting $p^Y_{jt} = p^{K*}_j/n_{jjt} = p^{Y*}_j\exp(-\gamma^*t)$ into $U_j = \int_0^\infty(\ln E_{jt} - \ln p^Y_{jt})\exp(-\rho t)dt$, and applying integration by parts, we obtain:

$$\rho U_j = \ln E^*_j - \ln p^{Y*}_j + (1/\rho)\gamma^*.$$

Substituting Eq. (B.1) into Eq. (A.10), E_j^* is rewritten as:

$$E_j^* = \{w_j^* L_j/[1 + (\tau_{kj} - 1)\lambda_{jk}^*]\}\eta_j^*; \eta_j^* \equiv (1 - 1/\sigma)\rho/(\rho + \delta + \gamma^*) + 1 + \sigma(\tau_{kj} - 1)\lambda_{jk}^*, k \neq j.$$

Dividing this by p_j^{Y*}, and using Eq. (14), we obtain:

$$E_j^*/p_j^{Y*} = L_j\{(w_j^*/p_j^{Y*})/[1 + (\tau_{kj} - 1)\lambda_{jk}^*]\}\eta_j^* = L_j(A_j/a_{jj}^{*\sigma-1})\eta_j^*;$$
$$A_j \equiv \beta\kappa_j^e n_{j0}^e/\{[1/(1 - 1/\sigma)]^{\sigma-1}\kappa_{jj}\}, k \neq j.$$

Substituting this into $\rho U_j = \ln E_j^* - \ln p_j^{Y*} + (1/\rho)\gamma^*$, we obtain Eq. (17).

Appendix C. Derivation of Eq. (19)

Step (i): first of all, we logarithmically differentiate $\overline{m}_j$ in Eq. (3) to obtain:

$$\widehat{\overline{m}}_j = [1/(1 - \sigma)]\textstyle\sum_k \zeta_{kj} d\ln\{(n_{kk}/n_{jj})(G_k(a_{kj})/G_k(a_{kk}))[(\tau_{kj}p_k^Y/p_j^Y)\overline{a}_{kj}(a_{kj})]^{1-\sigma}\};$$
$$\zeta_{kj} \equiv \frac{(n_{kk}/n_{jj})(G_k(a_{kj})/G_k(a_{kk}))[(\tau_{kj}p_k^Y/p_j^Y)\overline{a}_{kj}(a_{kj})]^{1-\sigma}}{\sum_l (n_{ll}/n_{jj})(G_l(a_{lj})/G_l(a_{ll}))[(\tau_{lj}p_l^Y/p_j^Y)\overline{a}_{lj}(a_{lj})]^{1-\sigma}} = \frac{\tau_{kj}E_{kj}}{\sum_l \tau_{lj}E_{lj}}; \textstyle\sum_k \zeta_{kj} = 1,$$

where ζ_{kj} is rewritten using $e_{jk}(a) = \tau_{jk}^{-\sigma}[p_j^Y a/(1-1/\sigma)]^{1-\sigma}P_k^\sigma X_k$ and $E_{jk} = n_{jk}\int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk})da$ as the expenditure share of varieties country j buys from country k. Using Eq. (A.2), $E_{jk} = n_{jk}\int_0^{a_{jk}} e_{jk}(a)\mu_{jk}(a|a_{jk})da, E_{jk} = E_{kj}$, and $\lambda_{jk} = E_{jk}/\sum_l E_{jl}$, ζ_{kj} is related to λ_{jk} as:

$$\zeta_{kj} = \tau_{kj}\lambda_{jk}/[1 + (\tau_{kj} - 1)\lambda_{jk}] \geq \lambda_{jk}, k \neq j,$$

where the inequality follows from $\tau_{kj} - [1 + (\tau_{kj} - 1)\lambda_{jk}] = (\tau_{kj} - 1)(1 - \lambda_{jk}) \geq 0$. Due to the import tariff, country j's expenditure share of imported varieties is no less than its revenue share of exported varieties.

Using Eq. (13) to calculate $d\ln\{(n_{kk}/n_{jj})(G_k(a_{kj})/G_k(a_{kk}))[(\tau_{kj}p_k^Y/p_j^Y)\overline{a}_{kj}(a_{kj})]^{1-\sigma}\}$, we obtain:

$$\widehat{\overline{m}}_j = (1 - \zeta_{kj})\widehat{a}_{jj} + \zeta_{kj}\{[1/(1 - \sigma)]d\ln(n_{kk}/n_{jj}) + \{[\beta - (1 - \lambda_{kj})]/\lambda_{kj}\}\widehat{a}_{kk} + \widehat{\tau}_{kj} + \widehat{p}_k^Y - \widehat{p}_j^Y\}, k \neq j.$$

Then the difference between $\widehat{\overline{m}}_1$ and $\widehat{\overline{m}}_2$ is calculated as:

$$\widehat{\overline{m}}_1 - \widehat{\overline{m}}_2 = \xi_1\widehat{a}_{11} - \xi_2\widehat{a}_{22} - (\zeta_{21} + \zeta_{12})[\widehat{p}_1^Y - \widehat{p}_2^Y - \widehat{\chi}/(\sigma - 1)] + \zeta_{21}\widehat{\tau}_{21} - \zeta_{12}\widehat{\tau}_{12};$$
$$\xi_j \equiv 1 - \zeta_{kj} - \zeta_{jk}[\beta - (1 - \lambda_{jk})]/\lambda_{jk}, k \neq j.$$

Substituting this into the logarithmically differentiated form of Eq. (6), $\widehat{p}_1^Y - \widehat{p}_2^Y$ is solved as:

$$\widehat{p}_1^Y - \widehat{p}_2^Y = (1/\Delta)[\widehat{w}_1 - (1 - \zeta_{21} - \zeta_{12})\widehat{\chi}] + [(\sigma - 1)/\Delta](\zeta_{21}\widehat{\tau}_{21} - \zeta_{12}\widehat{\tau}_{12} + \xi_1\widehat{a}_{11} - \xi_2\widehat{a}_{22}); \quad \text{(C.1)}$$
$$\Delta \equiv 1 + (\sigma - 1)(\zeta_{21} + \zeta_{12}) > 1.$$

We assume that country j's import expenditure share is smaller than its domestic expenditure share:

$$\zeta_{kj} < 1/2 \forall j,k,k \neq j \Rightarrow \lambda_{jk} < 1/2. \tag{C.2}$$

Under Eq. (C.2), we have $1-\zeta_{21}-\zeta_{12}>0$ and $1-\lambda_{12}-\lambda_{21}>0$.

Step (ii): logarithmically differentiating Eqs. (7) and (8) gives:

$$\begin{aligned}
\widehat{a}_{12}-\widehat{a}_{22} &= -\widehat{v}-[\sigma/(\sigma-1)]\widehat{\tau}_{12}, \\
\widehat{a}_{21}-\widehat{a}_{11} &= \widehat{v}-[\sigma/(\sigma-1)]\widehat{\tau}_{21}.
\end{aligned}$$

Combining them with Eq. (13), we obtain:

$$\begin{aligned}
(1-\lambda_{12})\widehat{a}_{11}+\lambda_{12}\widehat{a}_{22} &= \lambda_{12}\{\widehat{v}+[\sigma/(\sigma-1)]\widehat{\tau}_{12}\}, \\
\lambda_{21}\widehat{a}_{11}+(1-\lambda_{21})\widehat{a}_{22} &= \lambda_{21}\{-\widehat{v}+[\sigma/(\sigma-1)]\widehat{\tau}_{21}\}.
\end{aligned}$$

Substituting Eq. (C.1) into $\widehat{v}=[\sigma/(\sigma-1)](\widehat{p}_1^Y-\widehat{p}_2^Y)$, and substituting it into the above expressions, they are rewritten as:

$$\begin{aligned}
\widetilde{\lambda}_{11}\widehat{a}_{11}+\widetilde{\lambda}_{12}\widehat{a}_{22} &= \lambda_{12}\{\widehat{V}+[\sigma/(\sigma-1)-(\sigma/\Delta)\zeta_{12}]\widehat{\tau}_{12}+(\sigma/\Delta)\zeta_{21}\widehat{\tau}_{21}\}, \\
\widetilde{\lambda}_{21}\widehat{a}_{11}+\widetilde{\lambda}_{22}\widehat{a}_{22} &= \lambda_{21}\{-\widehat{V}+[\sigma/(\sigma-1)-(\sigma/\Delta)\zeta_{21}]\widehat{\tau}_{21}+(\sigma/\Delta)\zeta_{12}\widehat{\tau}_{12}\}; \\
\widehat{V} &\equiv [\sigma/(\sigma-1)](1/\Delta)[\widehat{w}_1-(1-\zeta_{21}-\zeta_{12})\widehat{\chi}], \\
\widetilde{\lambda}_{jj} &\equiv 1-\lambda_{jk}-\lambda_{jk}(\sigma/\Delta)\xi_j, \widetilde{\lambda}_{jk}\equiv\lambda_{jk}+\lambda_{jk}(\sigma/\Delta)\xi_k, k\neq j.
\end{aligned}$$

Solving them for $\widehat{a}_{11}$ and $\widehat{a}_{22}$, we obtain:

$$\widehat{a}_{11}=(\lambda_{12}/|\widetilde{\lambda}|)\{\widehat{V}+\{[\sigma/(\sigma-1)]\widetilde{\lambda}_{22}-(\sigma/\Delta)\zeta_{12}\}\widehat{\tau}_{12}-\{\sigma/(\sigma-1)-(\sigma/\Delta)\zeta_{21}-[\sigma/(\sigma-1)]\widetilde{\lambda}_{22}\}\widehat{\tau}_{21}\}, \tag{C.3}$$

$$\widehat{a}_{22}=(\lambda_{21}/|\widetilde{\lambda}|)\{-\widehat{V}+\{[\sigma/(\sigma-1)]\widetilde{\lambda}_{11}-(\sigma/\Delta)\zeta_{21}\}\widehat{\tau}_{21}-\{\sigma/(\sigma-1)-(\sigma/\Delta)\zeta_{12}-[\sigma/(\sigma-1)]\widetilde{\lambda}_{11}\}\widehat{\tau}_{12}\}; \tag{C.4}$$

$$|\widetilde{\lambda}|\equiv\widetilde{\lambda}_{11}\widetilde{\lambda}_{22}-\widetilde{\lambda}_{12}\widetilde{\lambda}_{21}=(1/\Delta)\{(1-\zeta_{21}-\zeta_{12})(1-\lambda_{12}-\lambda_{21})+\sigma[\beta(\zeta_{21}+\zeta_{12})-\lambda_{12}-\lambda_{21}]\}>0.$$

Finally, $\widehat{a}_{12}$ and $\widehat{a}_{21}$ are obtained by substituting Eqs. (C.3) and (C.4) back into Eq. (13).

Step (iii): logarithmically differentiating Eq. (10), and using Eqs. (11) and (13), give:

$$-\theta\{1/[1+(\tau_{21}-1)\lambda_{12}]\}[(1-\lambda_{12})/\lambda_{12}]\widehat{a}_{11}-\zeta_{21}\widehat{\tau}_{21}+\widehat{w}_1=-\theta\{1/[1+(\tau_{12}-1)\lambda_{21}]\}[(1-\lambda_{21})/\lambda_{21}]\widehat{a}_{22}-\zeta_{12}\widehat{\tau}_{12}.$$

Substituting Eqs. (C.3) and (C.4) into the above expression, and noting that $\zeta_{kj}=\tau_{kj}\lambda_{jk}/[1+(\tau_{kj}-1)\lambda_{jk}]\Leftrightarrow 1-\zeta_{kj}=(1-\lambda_{jk})/[1+(\tau_{kj}-1)\lambda_{jk}], k\neq j$, we obtain:

$$0 = -\widetilde{B}\widehat{w}_1 + \widetilde{C}\widehat{\chi} + F_{21}\widehat{\tau}_{21} - F_{12}\widehat{\tau}_{12} \Leftrightarrow \widehat{w}_1 = (1/\widetilde{B})(\widetilde{C}\widehat{\chi} + F_{21}\widehat{\tau}_{21} - F_{12}\widehat{\tau}_{12}); \tag{C.5}$$

$$\widetilde{B} \equiv \beta(\sigma/\Delta)(2 - \zeta_{21} - \zeta_{12}) - |\widetilde{\lambda}|$$

$$= (1/\Delta)\{(1 - \zeta_{21} - \zeta_{12})[2\beta\sigma - (1 - \lambda_{12} - \lambda_{21})] + \sigma(\lambda_{12} + \lambda_{21})\} > 0,$$

$$\widetilde{C} \equiv \beta(\sigma/\Delta)(1 - \zeta_{21} - \zeta_{12})(2 - \zeta_{21} - \zeta_{12}) > 0,$$

$$F_{jk} \equiv \theta\{(1 - \zeta_{kj})\{[\sigma/(\sigma - 1)]\widetilde{\lambda}_{kk} - (\sigma/\Delta)\zeta_{jk}\}$$

$$+ (1 - \zeta_{jk})\{\sigma/(\sigma - 1) - (\sigma/\Delta)\zeta_{jk} - [\sigma/(\sigma - 1)]\widetilde{\lambda}_{jj}\}\} - |\widetilde{\lambda}|\zeta_{jk}, k \neq j.$$

Step (iv): substituting Eq. (C.5) into $\widehat{V} = [\sigma/(\sigma - 1)](1/\Delta)[\widehat{w}_1 - (1 - \zeta_{21} - \zeta_{12})\widehat{\chi}]$ gives:

$$\widehat{V} = [1/(\sigma - 1)][(\sigma/\Delta)/\widetilde{B}][|\widetilde{\lambda}|(1 - \zeta_{21} - \zeta_{12})\widehat{\chi} + F_{21}\widehat{\tau}_{21} - F_{12}\widehat{\tau}_{12}].$$

Substituting this back into Eqs. (C.3) and (C.4), noting that $\widetilde{\lambda}_{11} + \widetilde{\lambda}_{22} - 1 = |\widetilde{\lambda}|$ and $\Delta - \sigma(\zeta_{21} + \zeta_{12}) = 1 - \zeta_{21} - \zeta_{12}$, and substituting the results into Eq. (16), we obtain:

$$d\gamma_1 = -(\rho + \delta + \gamma)[(\sigma/\Delta)/\widetilde{B}]\lambda_{12}[(1 - \zeta_{21} - \zeta_{12})\widehat{\chi} + J_1\widehat{\tau}_{12} + I_1\widehat{\tau}_{21}], \tag{C.6}$$

$$d\gamma_2 = -(\rho + \delta + \gamma)[(\sigma/\Delta)/\widetilde{B}]\lambda_{21}[-(1 - \zeta_{21} - \zeta_{12})\widehat{\chi} + J_2\widehat{\tau}_{21} + I_2\widehat{\tau}_{12}]; \tag{C.7}$$

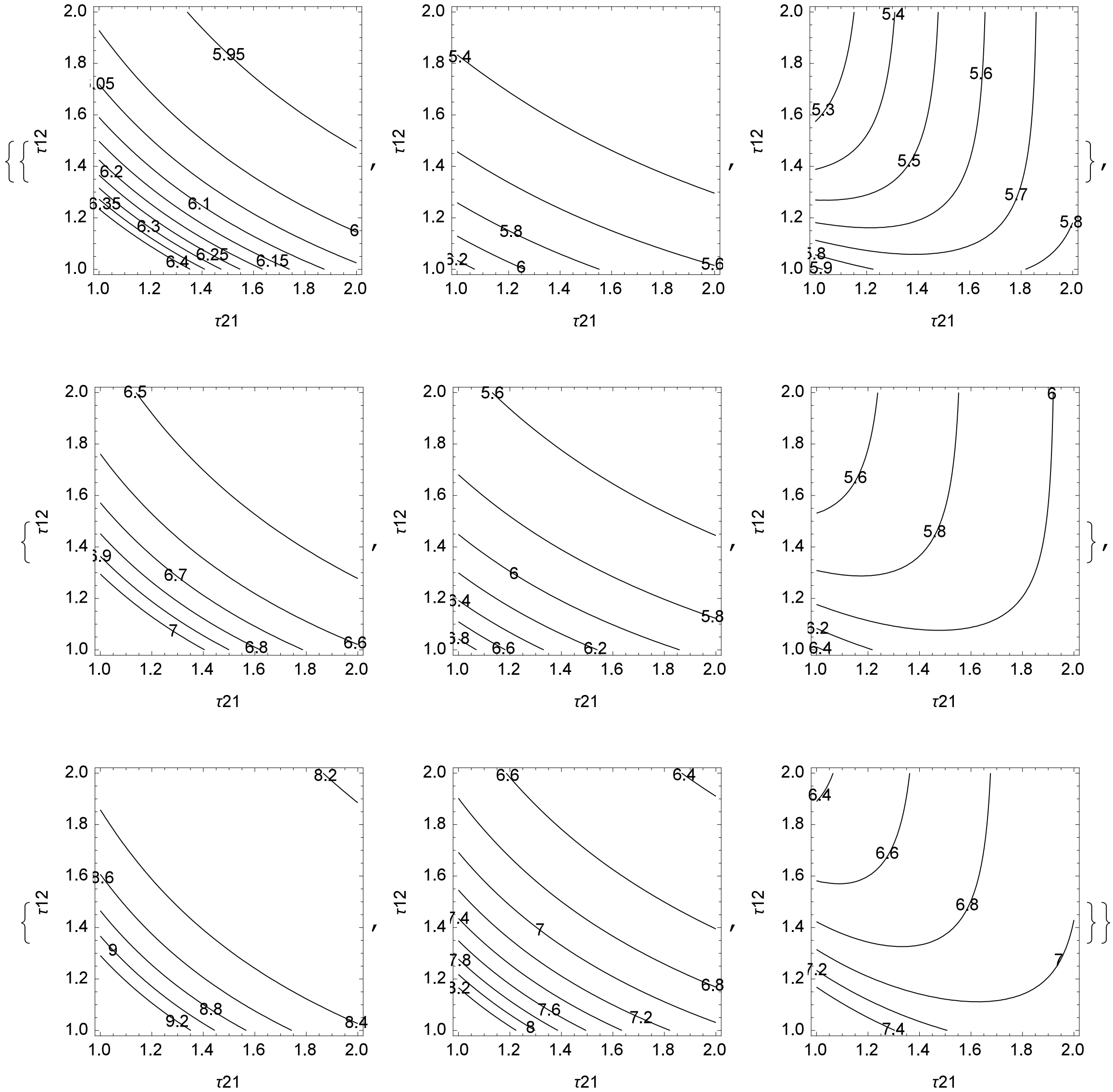
$$J_j \equiv (1 - \zeta_{kj} - \zeta_{jk})[\beta\sigma - (1 - \lambda_{kj})] + \sigma\lambda_{kj} > 0,$$

$$I_j \equiv J_j + 1 - \zeta_{kj} - \zeta_{jk} > J_j, k \neq j.$$

Step (v): substituting Eqs. (C.6) and (C.7) into the differentiated form of Eq. (5), $\widehat{\chi}$ is solved as:

$$\widehat{\chi} = [1/(1 - \zeta_{21} - \zeta_{12})][1/(\lambda_{12} + \lambda_{21})][(\lambda_{21}I_2 - \lambda_{12}J_1)\widehat{\tau}_{12} - (\lambda_{12}I_1 - \lambda_{21}J_2)\widehat{\tau}_{21}]. \tag{C.8}$$

Step (vi): substituting Eq. (C.8) back into Eq. (C.7), and noting that $I_j + J_k = \Delta\widetilde{B}\forall j, k, k \neq j$, we obtain Eq. (19), which does not depend on (C.2).



top left: $a_{10} = 1.8, \rho = 0.03$	top center: $a_{10} = 2.0, \rho = 0.03$	top right: $a_{10} = 2.2, \rho = 0.03$
mid left: $a_{10} = 1.8, \rho = 0.02$	mid center: $a_{10} = 2.0, \rho = 0.02$	mid right: $a_{10} = 2.2, \rho = 0.02$
bot left: $a_{10} = 1.8, \rho = 0.01$	bot center: $a_{10} = 2.0, \rho = 0.01$	bot right: $a_{10} = 2.2, \rho = 0.01$

Fig. 1. Country 1's iso-welfare curves on the (τ_{21}, τ_{12}) plane.