# Household Sorting in an Ancient Setting*

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April 1, 2019

Evolving

**Abstract**

We use archaeological data from two different ancient settlements of different historical eras on a Greek island to construct novel measures of wealth. Using these, we show that the wealthy tended to live closer to the center of the settlements. We build a monocentric city model with heterogeneous households, luxury goods and endogenous labor choices that is consistent with the rich living closer to the center. This result holds when transportation costs within the model are predominately time costs, as they mostly were in ancient history.

## 1 Introduction

What causes different households to choose to live in different parts of a city is one of the key questions in urban economics. We use archaeological data from within two ancient agglomerations to estimate how households sorted

*Gupta's research is supported by UK Economic and Social Research Council (ESRC) grant number ES/R006032/1. Halket's research is supported by the ESRC through the Centre for Microdata Methods and Practice (CeMMAP grant number ES/I034021/1).

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in early "urban" settlements. The data include precise locations for finds of several different qualities of a consumption good. We extend the canonical Muth-Mills model with heterogeneous households to include luxury goods in a tractable way and provide some conditions under which household location sorting in the model can be inferred from our data.

The nature of household sorting can have dramatic implications a wide variety of urban public policies on, e.g., transportation, public housing, public amenity provisions and the environment. Yet, inferring the way various amenities and technologies shape household location decisions even within variations of the static monocentric city model is challenging; "small" changes to the model environment can impart large qualitative changes on the model's outcomes.

If inferences are based on modern cities, these challenges are exacerbated by the pre-existing, "sticky" built and settled environment. Today's households make their choices conditional on the existing transportation network and character of the housing stock in various locations, which themselves are partially or wholly a product of the technologies and preferences of the past. Indeed, the possibility for differing equilibria based on legacy conditions are a feature of studies like Brueckner et al. [1999], Brueckner and Rosenthal [2009], Lee and Lin [2017].

Our work complements several strands of literature. It adds to the growing use of archaeological data to test economic theory. For instance Maurer et al. [2017] uses data from a similar period to ours to document trade and development patterns across settlements. Our paper is the first to our knowledge to look within settlements. We are able to do this by using detailed archaeological data collected at a fine spatial resolution and dated using cutting edge archaeological techniques. Both parametric and non-parametric estimates offer fairly clear pictures that the concentrations of wealth were highest closest to the settlements' centers.

We also add to the evidence on sorting in early cities cited in LeRoy and Sonstelie [1983] (who focus on census data from 19th century, North American cities). In their paper, the authors build a model with transportation choices. When the rich choose different transportation modes than the poor (e.g. the rich use an automobile and the poor walk), then the rich may choose to live in the suburbs. In addition to the alternative setting, our model adds to their work by providing various differing sorting conditions even when the transportation modes for all households are the same and also by explicitly including luxury goods in the model.

# 2 Data and Empirics

## 2.1 The Island and Data

The data are publicly available data[1] collected from the Greek island of Antikythera. Bevan and Conolly [2012] provide the following description of the island: *"Antikythera is a small island (ca. 20.8 sq.km) in the Mediterranean Sea. Despite being comparatively remote from larger land masses in Mediterranean terms, it lies along important routes of maritime interaction between the Peloponnese and Crete, and between the eastern and central Mediterranean. This geographical position has contributed to its very episodic history of human exploitation stretching back some 7,000 years, but with periods of substantial settlement followed by others of near complete abandonment. Highlights of this long-term history include evidence visits by Neolithic hunters from the Cyclades, Bronze Age farms with cultural links to Crete during the period of the Minoan palaces, a fortified settlement of Hellenistic pirates, a clutch of Late Roman communities, some glimpses of Middle Byzantine settlement and a recolonisation by west Cretan families in the late 18th century AD."*

Between 2005-07, the Antikythera Survey Project (ASP), co-directed by Andrew Bevan, James Conolly and Aris Tsaravopoulos (Greek Archaeological Service) conducted an intensive pedestrian survey of the island. The uniqueness of this exercise lay in the coverage of an entire island in a uniform manner with intensive survey methods.[2]The data offer a unique level

[1] see Bevan and Conolly [2014]

[2] Quoting from the description in Bevan and Conolly [2012] "...the entire island was fieldwalked in parallel lines 15-m apart. For certain interesting or problematic surface artefact scatters (particularly those of prehistoric date) this stage-one survey was followed

of detail in both the individual finds and their precise spatial locations.

We focus on two particular sets of finds: pottery and lithics. In the data, each piece of pottery is given a classification by Bevan and Conolly [2014] according to the quality of its fabrication: “Fine”, “Medium” or “Coarse”. In addition, as mentioned above, for each piece of pottery Bevan and Conolly [2014] assign a probability to it belonging to a particular chronological phase, using methods in Bevan et al. [2013]. [3]

Our study focuses on two major historical periods in the history of Antikythera: the Minoan period and the Hellenistic period. The Minoan period covers the time period between 2700-1200 BC when Antikythera was influenced by the Cretan civilization, thus ranging from the Early Bronze 2 phase to the Third Palace or Mycenaean phase in the Bevan et al. [2013] classification. The Hellenistic period covers 500BC-200AD, thus ranging from the Classical phase to the Early Roman phase in the Bevan et al. [2013] classification. The precise start and end dates of each phase are usually unknown, so our definition of the two phases necessarily must allow for some “lead” and “lag”.

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by more detailed stage-two collections on a 10×10-m grid. In terms of digital recording, this project was unusual for the detail of its treatment of the location, dating and other attributes of its artefacts. First, all artefacts and standing structures were entered individually in a database (with information on shape, size, decoration, fabric, date, location, etc.), rather than in aggregate, and these records were all the result of sustained laboratory study rather than decisions in the field. Second, the project sought to standardise the recording of the spatial location of all material culture, regardless of the survey method by which it was observed, such that all finds and observations had an effective spatial precision of ±10 m. Third and finally, it was the first substantial fieldwork project, to our knowledge, to adopt a probabilistic approach to assigning dates to individual collected artefacts.”

[3] The phases are: Middle to Late Neolithic (pre-4500 BC), Final Neolithic to Early Bronze 1 (ca. 4500-2700 BC), Early Bronze 2 (ca. 2700-2200 BC), Cretan late Prepalatial (ca. 2200-1950 BC), First Palace or Cretan Protopalatial (ca. 1950-1750 BC), Second Palace or Cretan Neopalatial (ca. 1750-1450 BC), Third Palace or Mycenaean (ca. 1450-1200 BC), Post Palatial to Protogeometric phases (1200-900 BC), Geometric phase (900-600 BC), Archaic phase (600-500 BC), Classical phase (500-325 BC), Hellenistic phase (325-0 AD), Early Roman phase (0-200 AD), Middle Roman phase (200-350 AD), Late Roman phase (350-650 AD), Early Byzantine phase (650-900 AD), Middle Byzantine phase (900-1200 AD), Early Venetian phase (1200-1400 AD), Middle Venetian phase (1400-1600 AD), Late Venetian phase (1600-1800 AD), Recent phase (1800-present), any other chronological phase.

We choose these two distinct time periods for our study because of the vastly different characteristics of settlement on Antikythera during them. Antikythera is well-known in the archaeological literature for exhibiting a high degree of historical variance in its settlement. Bevan et al. [2006] describe this phenomenon as one of "rollercoaster demographics". For a more complete history, see Bevan et al. [2006].

For the purposes of our study we highlight several elements of the island's history. The Minoan period is dominated by mostly by "cultivators" living in the fertile central part of the island and may have colonized the island from their larger neighbor, Crete. After the Minoan period, archaeologists have yet to find "good evidence... for much activity;" (Bevan et al. [2006]) in other words, it may have been abandoned (a situation comparable to its current lightly inhabited state).

During the Hellenistic period it was resettled but in a different part of the island. The island was, as Bevan et al. [2006] note, "dominated by a fortified town at a strategic position on its northern coast, overlooking a natural protected harbor. Documentary evidence suggests its role in piracy. Our survey indicates the presence of one or two other Hellenistic scatters on the island" which may have been "in some manner, part of the logistical and economic agenda of the fortified town itself."

Thus our choice of the two time periods is motivated precisely by archaeological and historical observations: these two periods correspond to distinct and prosperous phases in Antikythera's history. The discontinuity in settlement also makes the task of distinguishing between historical phases much simpler, in the words of Bevan et al. [2006] leading to a landscape that is "a less complicated palimpsest than in most other Mediterranean locations." Our interest lies in estimating wealth and population gradients relative to a "central" location, in the sense of being the center of economic activity. This center changed between the two time periods we focus on. Precisely, Figure 1 shows the island in its entirety, together with the location of the fertile center of the island, where most economic activity took place during the Minoan era and the port of Kastro in the northern part of Antikythera, which was the economic hub during the Hellenistic heyday of the island.

## 2.2 Measuring relative wealth

In this section we detail how we measure wealth gradients using the ASP data set. Our method covers the island of Antikythera with a fine grid of cells, and then measures pottery counts and wealth ratios for each of these cells. The exercise is conducted separately for both the Minoan and Hellenistic eras of settlement. How do we measure pottery counts and wealth? We first note that given data on both quantity of pottery as well as quality (fine, medium, coarse), we already have a natural separation of wealth and relative prices: raw pottery counts may be taken to be proxies for wealth and ratios of pottery counts by quality can measure relative wealth. Lithics provide an additional source of information, but because these are very basic implements used primarily in agrarian settings they are used to calculate gradients only for the much more ancient Minoan era.

More precisely, suppose that in a given cell $\mathcal{C}$ we observe $p_f, p_m$ and $p_c$ pieces of fine, medium and coarse pottery respectively (each individual piece denoted with $i$ subscript), as well as $\ell$ lithics. Denoting by $\pi_{M,i,f}$ and $\pi_{H,i,f}$ the probabilities of the $i$th piece of fine pottery belonging to the Minoan and Hellenistic periods, respectively, (and with similar probability notations for other pottery qualities) the wealth measures in cell $\mathcal{C}$ are

$$P_{\mathcal{C}}^{M} = \sum_{i=1}^{p_f} \pi_{M,i,f} p_{i,f} + \sum_{i=1}^{p_m} \pi_{M,i,m} p_{i,m} + \sum_{i=1}^{p_c} \pi_{M,i,c} p_{i,c} + \ell, \tag{1}$$

$$P_{\mathcal{C}}^{H} = \sum_{i=1}^{p_f} \pi_{H,i,f} p_{i,f} + \sum_{i=1}^{p_m} \pi_{H,i,m} p_{i,m} + \sum_{i=1}^{p_c} \pi_{H,i,c} p_{i,c}. \tag{2}$$

for the Minoan and Hellenistic periods respectively, while corresponding wealth ratios are

$$W_{\mathcal{C},q_1 q_2}^{M} = \frac{\sum_{i=1}^{p_{q_1}} \pi_{M,i,q_1} p_{i,q_1}}{\sum_{i=1}^{p_{q_2}} \pi_{M,i,q_2} p_{i,q_2}},\ W_{\mathcal{C},q_1 q_2}^{H} = \frac{\sum_{i=1}^{p_{q_1}} \pi_{H,i,q_1} p_{i,q_1}}{\sum_{i=1}^{p_{q_2}} \pi_{H,i,q_2} p_{i,q_2}}, \tag{3}$$

with $q_1 q_2 = fm, fc, mc$ for the Hellenistic period and $q_1 q_2 = fm, fc, mc, f\ell, m\ell, c\ell$ for the Minoan period. Note that since lithics are assigned to the Minoan era, we would have, for a given cell $\mathcal{C}$,

$$W^M_{\mathcal{C},q_1\ell} = \frac{\sum_{i=1}^{p_{q_1}} \pi_{M,i,q_1} p_{i,q_1}}{\ell}, q_1 = f, m, c. \tag{4}$$

In Figure 2, we plot the distribution of lithics over the island of Antikythera. We see that the lithics are primarily concentrated around the central, more agrarian parts of the island. This reflects the fact that lithics find greater use in more rural settings, in particular as agricultural implements and basic tools, and lends further support to the thesis that lithics were used in the Minoan era as the economic activity of a port, particularly one engaged in piracy related activities, is less likely to involve the use of such items. The next three figures similarly illustrate the distribution of pottery over the island, separately for the Minoan and Hellenistic eras. In each figure, the left panel shows pottery locations from the Minoan era while the right panel does so for the Hellenistic era. Each pottery location corresponds to a piece that belongs to the relevant era with nonzero probability.

Figure 3 plots the distribution of coarse pottery over the island. There are 5,497 pottery pieces of coarse quality in the Minoan era, and seem mostly concentrated around the fertile center of the island, while the 38 coarse pottery pieces that correspond to the Hellenistic era are almost entirely concentrated around the port of Kastro. Thus, not only do we evidence of the vastly changed economic structure of the island but also suggestions that it was more prosperous in the Hellenistic age. The comparison between pottery quantities of medium quality is somewhat closer: 906 pieces in the Minoan era versus 1069 pieces in the Hellenistic era. We plot these finds in Figure 4, noticing a similar pattern to the one observed for coarse pottery, with one exception. Examination of the right panel reveals the presence of some medium grade pottery in the fertile center of the island also in the Hellenistic era, although the vast majority of finds are still concentrated around Kastro. Nevertheless, this does suggest the presence of some prosperous farmsteads in the “hinterland” of the island even while the bulk of economic activity takes place around the port. Indeed, in Figure 5, we plot the finds of fine pottery and find the presence of some fine pieces as well in the fertile regions of the island during the Hellenistic phase. The overall distributions between eras still reflects the stark contrasts observed earlier, and the fact that there are 1262 pieces of fine Hellenestic pottery as opposed to just 226 Minoan pieces supports our earlier conjecture that, along with structural economic changes, the island underwent a fundamental change in prosperity between these eras.

The eyeballing exercise for the previous paragraph can be improved by constructing and plotting the cell-wise wealth measures defined in equations (1) and (2). We do this in Figure 6, where once again the left panels correspond to the Minoan era and the right panels to the Hellenistic era. Green dots correspond to low wealth, with the shading getting lighter as wealth increases and becoming shades of yellow through the middle ranges of wealth, and then turning shades of orange and finally red for the highest values of wealth. We observe that in the Minoan era wealth appears to be concentrated in the fertile center of the island. In the Hellenistic era we see, as anticipated, that wealth is mostly concentrated near the port. On the other hand, there is evidence of many "rural" areas with small populations in this era, possibly corresponding to a few prosperous farmsteads. This is evidenced by a prevalence of green dots in the fertile center of the island in the right panel of Figure 6, indicating sparsely populated cells.

To further illustrate the findings above, we obtain a smooth estimate of population and income over the island by plotting kernel density estimates of wealth distributions. The results are displayed in 3D in Figures 7. The upper and lower panels correspond to the Minoan and Hellenistic eras, respectively. The color scheme runs low-medium-high as green-yellow-red as described in the previous paragraph, but is now buttressed further with vertical heights measuring population and wealth densities. The figures are plotted in a northeasterly perspective from an elevated southwestern viewpoint. We observe the concentrations of wealth in the areas we saw previously in both eras. As the earlier figures suggested, the wealth distribution in the Minoan era is substantially less skewed that the Hellenistic distribution, although the peak certainly lies in the fertile center of the island.

## 2.3 Population and wealth gradients relative to center of economic activity

Our analysis in the previous section clearly indicates the presence of population and wealth gradients relative to the fertile center in the Minoan era and the port of Kastro in the Hellenistic era. In this section we estimate these gradients and discuss our finding in relation to the figures we have already discussed. We present both linear as well as nonparametric gradient estimates, the latter being computed using splines with the GAM package in R. As we will see below, nonparametric gradients allows us to capture nonlinearities in the gradients that reflect economic features of the island's population and wealth distributions. Solid lines correspond to gradients while asymptotic 95% confidence intervals (i.e. based on a standard normal critical value of 1.96) are traced out with dashed lines in each figure. Distances from the fertile center or port are standardized to lie in the interval $[0, 1]$ by dividing by the largest observed distance.

### 2.3.1 Absolute wealth gradients

Estimated wealth gradients are displayed in Figure 8, with linear fits in the upper panel and spline based nonparametric fits in the lower panel. In each figure, red lines correspond to the Minoan era and green lines to the Hellenistic era. Looking first at the linear fits, as expected, gradients for both eras are downward sloping. They also exhibit similar slopes. Nonlinearity in the gradients is captured by the nonparametric fits, which show humps in the gradients in both eras. This hump is more pronounced in the Minoan era, which is unsurprising as Figure 6 exhibited a less skewed wealth distribution relative center of economic activity in this era. On the other hand, Figure 6 also showed less wealthy areas far away from the port in the Hellenistic era, and these account for the small hump we see in the green gradient. The slight upwards bend observed in both gradients for the greatest distance can be ascribed to the presence of isolated communities in the coastal areas of the island, as seen be the presence of small quantities of pottery in some coastal areas in Figures 3-5, and consequently in Figure 6. Note though that confidence bands become rather wide at the extremities of distance, so this upwards bend could as much reflect the imprecision of these estimates due to sparse data.

### 2.3.2 Relative wealth gradients

Relative wealth gradients are displayed in Figures 10-12. Figure 10 plots the ratios defined in (3) for the Hellenistic era as a function of normalized distance from the port, with linear fits in the upper panel and nonparametric fits based on splines in the lower panel. Fine to coarse (FC, green) and medium to coarse (MC, blue) ratios are both decreasing in distance but the fine to medium (FM, red) ratio appears to be increasing in the linear fit. However this anomaly is corrected by capturing nonlinearity in the nonparametric fit in the lower panel, which shows downward sloping gradients in all three ratios. In Figure 11 we plot similar fits for the Minoan era, as functions of distance from the fertile center. Here we find that MC appears to increase with a linear fit, but a nonparametric fit in fact reveals a hump in the gradient, which ultimately slopes downward. Another notable difference is the very gentle gradient of FC, while the gradients of FM and MC are much more pronounced. For the Minoan era, further information on relative gradients can be obtained by plotting the ratios of various pottery qualities relative to lithics, as defined in (4). We do this in two separate figures due to the vast differences in scale. Figure 12 plots gradients for fine pottery to lithics and medium pottery to lithics ratios, while Figure 13 does so for the coarse pottery to lithics ratio. All three gradients are seen to be sharply downward sloping, regardless of linear or nonparametric fitting.

# 3 A monocentric city model with household heterogeneity and luxury goods

## 3.1 Introduction

We build a monocentric city model with household heterogeneity, a set of consumption goods, land and leisure. Even though the model uses preferences with meaningful wealth and substitution effects from leisure, we are able to obtain some sorting results for the competitive equilibria. We show that when commuting costs are dominated by time costs, households sort such that high ability (high income) households live close to the city center whereas when commuting costs are mostly in goods, households sort in opposite fashion. In the former case, luxury goods consumption is higher in the city center. In the latter case, luxury goods consumption will be higher in the periphery as long as substitution effects between land and non-durable goods are not too strong.

### 3.2 Setup

Households have preferences over a vector of $I$ non-durable consumption goods, land and leisure ($c, a, l$, respectively) denoted by $u(c, a, l)$. Households are endowed with one unit of time which they may use for work, commuting or leisure. Households are heterogeneous in the productivity of their work time, denoted by $z \sim F_z$, where $F_z$ is the distribution of population abilities with support $Z$ and density $f_z$. A unit of work time is converted into $z$ units of any in a set of non-durable goods $i \in I$. In equilibrium this will mean that the relative price of each good $i$ is the same. We normalize this price to 1.

All households live in a monocentric city and "commute" into the center of the city.[4] Commuting from home a distance $r$ costs $t_1(r)$ in time and $t_0(r)$ in the numeraire good. The supply of land at a distance $r$ in the economy is given by the density $f_r : \Re_+ \to \Re_+$.

We assume an initial endowment for land, $\theta : Z \to \Re_+$ for each household. In competitive equilibrium it must be the case, for each household, that

$$\sum_i c_i + ap(r) + lz \leq \Lambda + z(1 - t_1(r)) - t_0(r) \tag{5}$$

[4]We leave unspecified whether this is to consume location based amenities, buy goods or work.

where $p(r)$ is the price of land,

$$\Lambda = \int p(r) f_r(r)\, dr$$

is the total value of land in the city and (5) is the household's budget constraint.[5]

## 3.3 Preferences

We assume that households have a constant elasticity of substitution preferences over land, leisure and a composite $g : I \to R$ of the non-durable goods:

$$u(c, a, l) = \frac{\left(\omega_1 g(c)^{\frac{\varepsilon-1}{\varepsilon}} + \omega_2 a^{\frac{\varepsilon-1}{\varepsilon}} + \omega_3 l^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\gamma\varepsilon}{\varepsilon-1}}}{\gamma}.$$

with $\varepsilon, \gamma < 1$.We assume the composite $g$ is:

$$g(c) = \prod_{i \in I} (c_i - \beta_i)^{\alpha_i} + \sum_{i \in I} \beta_i \tag{6}$$

where $\beta_i \geq 0$ is a preference parameter and $\sum_{i \in I} \alpha_i = 1$ with $\alpha_i > 0$.

## 3.4 First order conditions

### 3.4.1 Non-durable consumption choices

The first order conditions for the household imply that

$$\frac{(c_i - \beta_i)}{\alpha_i} = \frac{(c_j - \beta_j)}{\alpha_j} = v(x)$$

where $v$ is the indirect sub-utility function for preferences $g$ given to spending on non-durable goods $x$. Using the budget constraint for this sub-problem ($x = \sum_{i \in I} c_i$) we get

$$v(x) = x$$

[5] We are thus assuming for convenience that land ownership is spread equally among the population.

and

$$c_i = \alpha_i(x - \sum_{j \in I} \beta_j) + \beta_i \tag{7}$$

If the goods can be ordered such that $(\beta_i - \alpha_i \sum_{j \in I} \beta_j)$ is decreasing in $i$, than higher $i$ goods are more "luxuriousness." (I.e. the elasticity of good $i$'s consumption share of total good spending $x$ with respect to $x$ is increasing in $i$.)

### 3.4.2 Consumption expenditures, land, leisure and location

The first order conditions for the rest of the household's problem, using the fact that $g(c) = x$ where $x$ is the amount the household will spend on non-durables, are:

$$\begin{aligned}
\lambda_c &= u_c(x, a, l) && (8)\\
\lambda_c p_r(r) &= u_a(x, a, l) && (9)\\
\lambda_L(r, z) + \lambda_c z &= u_l(x, a, l) && (10)\\
a(r, z)\frac{dp_r}{dr} &= -z\frac{\partial t_1(r)}{\partial r} - \frac{\partial t_0(r)}{\partial r} && (11)
\end{aligned}$$

Equation 11 becomes the Alonso-Muth condition by examining the slope of the bid-rent curves $\Psi(r, z; \bar{u})$. In equilibrium, $\frac{dp_r}{dr}(r) = \Psi_r(r^*(z), z; u^*)$.

Strict sorting occurs if everywhere:

$$\frac{\partial^2 \Psi}{\partial r \partial z}(r, z) \gtrless 0$$

Subbing in we get that land demand is:

$$a(r, z) = \frac{\Lambda + z(1 - t_1(r)) - t_0(r)}{\underbrace{\omega_1\left(\frac{1}{\omega_1}\right)^{1-\varepsilon} + \omega_2\left(\frac{p_r(r)}{\omega_2}\right)^{1-\varepsilon} + \omega_3\left(\frac{z + \lambda_L(r, z)}{\omega_3}\right)^{1-\varepsilon}}_{\equiv P(r,z)}}\left(\frac{p_r(r)}{\omega_2}\right)^{-\varepsilon} \tag{12}$$

so

$$\frac{\partial^2\Psi(r,z)}{\partial r\partial z} = \frac{\frac{dt_1(r)}{dr}}{a(r,z)}\left[\frac{z}{a(r,z)}\frac{\partial a(r,z)}{\partial z} - 1\right] + \frac{\frac{dt_0(r)}{dr}}{(a(r,z))^2}\frac{\partial a(r,z)}{\partial z}. \tag{13}$$

Note that

$$\frac{\partial P(r,z)}{\partial z} \geq 0,$$

strictly so if $1 - t_1(r) > l(r,z)$. Differentiating 12:

$$\begin{aligned}
\frac{\partial a(r,z)}{\partial z} &= \frac{1-t_1(r)}{P(r,z)}\left(\frac{p_r(r)}{\omega_2}\right)^{-\varepsilon} - \frac{\Lambda + z(1-t_1(r)) - t_0(r)}{(P(r,z))^2}\left(\frac{p_r(r)}{\omega_2}\right)^{-\varepsilon}\frac{\partial P(r,z)}{\partial z} \\
&= a(r,z)\left(\frac{1-t_1(r)}{\Lambda + z(1-t_1(r)) - t_0(r)} - \frac{\partial P(r,z)}{\partial z}\frac{1}{P(r,z)}\right).
\end{aligned}$$

*Case* 1. Commuting costs are only in time: $t_0 \approx 0$.

The cross-derivative of the bid-rent curve then becomes:

$$\frac{\partial^2\Psi(r,z)}{\partial r\partial z} = \frac{\frac{dt_1(r)}{dr}}{a(r,z)}\left[\frac{z(1-t_1(r))}{\Lambda + z(1-t_1(r))} - \frac{\partial P(r,z)}{\partial z}\frac{z}{P(r,z)} - 1\right] < 0$$

where the inequality follows because $\frac{z(1-t_1(r))}{\Lambda+z(1-t_1(r))} < 1$. So the highest types live strictly closer to the city center.

Non-durable expenditures $x(r,z)$ are similarly

$$x(r,z) = \frac{\Lambda + z(1-t_1(r))}{P(r,z)}\left(\frac{1}{\omega_1}\right)^{-\varepsilon}. \tag{14}$$

The first-order condition for expenditures can be rewritten as:

$$\omega_1(x(r,z))^{-\frac{1}{\varepsilon}}P^{\frac{\gamma\varepsilon-\varepsilon+1}{(\varepsilon-1)(1-\gamma)\varepsilon}} = \lambda_c.$$

If $\gamma\varepsilon - \varepsilon + 1 < 0$ (i.e. if $\varepsilon > \frac{1}{1-\gamma}$) then $\frac{\partial P^{\frac{\gamma\varepsilon-\varepsilon+1}{(\varepsilon-1)(1-\gamma)\varepsilon}}}{\partial z} > 0$,which in turn implies that $\frac{\partial x(r,z)}{\partial z} > 0$. Note that a conventional estimate of risk aversion and elasticity of substitution have $\gamma \approx -1$ and $\varepsilon \approx 0.9$, which would satisfy the inequality. Under the same parameter conditions, $\frac{\partial P^{\frac{\gamma\varepsilon-\varepsilon+1}{(\varepsilon-1)(1-\gamma)\varepsilon}}}{\partial r} < 0$ and then the first-order condition similarly implies that $\frac{\partial x(r,z)}{\partial r} < 0$. Given negative assortative matching on location, the total derivative $\frac{dx(r,z)}{dz} = \frac{\partial x(r,z)}{\partial z} + \frac{\partial x(r,z)}{\partial r}\frac{\partial r(z)}{\partial z} > 0$ follows. Therefore, higher types live closer to the center and spend more on non-durables.

*Case* 2. Commuting costs are only in goods: $t_1 \approx 0$.

The cross-derivative of the bid-rent curve then becomes:

$$\frac{\partial^2 \Psi(r,z)}{\partial r \partial z} = \frac{\frac{dt_0(r)}{dr}}{a(r,z)}\left[\frac{1}{\Lambda + z - t_0(r)} - \frac{\partial P(r,z)}{\partial z}\frac{z}{P(r,z)}\right]$$

Non-durable expenditures $x(r,z)$ are similarly

$$x(r,z) = \frac{\Lambda + z - t_0(r)}{P(r,z)}\left(\frac{1}{\omega_1}\right)^{-\varepsilon}. \tag{15}$$

The first-order condition for expenditures can still be rewritten as:

$$\omega_1 (x(r,z))^{-\frac{1}{\varepsilon}} P^{\frac{\gamma\varepsilon-\varepsilon+1}{(\varepsilon-1)(1-\gamma)\varepsilon}} = \lambda_c.$$

As in the case above, if $\gamma\varepsilon - \varepsilon + 1 < 0$ (i.e. if $\varepsilon > \frac{1}{1-\gamma}$) then $\frac{\partial P^{\frac{\gamma\varepsilon-\varepsilon+1}{(\varepsilon-1)(1-\gamma)\varepsilon}}}{\partial z} > 0$ and $\frac{\partial P^{\frac{\gamma\varepsilon-\varepsilon+1}{(\varepsilon-1)(1-\gamma)\varepsilon}}}{\partial r} < 0$ which in turn imply that $\frac{\partial x(r,z)}{\partial z} > 0$ and $\frac{\partial x(r,z)}{\partial r} < 0$. It also implies $\frac{\partial a(r,z)}{\partial z} > 0$ and thus $\frac{\partial^2 \Psi(r,z)}{\partial r \partial z} > 0$. Which means households positively sort. However such sorting is not sufficient to determine which types of households spend more on luxuries. Higher land prices close to the center encourage households living there to spend more of their income on the non-durables. On the other hand the households living further away are more productive and have higher incomes. Which effect is stronger will depend on parameterizations.

## 4 Conclusion

Modern cities are shaped by amalgam of forces, some present and some from historical sources. Modern transportation networks often are partially molded by historic networks (in part to reduce frictions to rights of way). Modern public goods often have explicit links to the location preferences of past generations (the Louvre and the Frick Museums were formally residences of their patrons).

Ancient settlements, especially those that were built without meaningful antecedents, offer a different laboratory to test urban economics models. Here we integrate archaeological data from two past settlements into a simple monocentric city model. We show how to infer the spatial distribution of wealth from the data and then how the simple model, calibrated with modern preferences but ancient transportation costs, matches the data.

## 5 Figures

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Figure 1: Antikythera

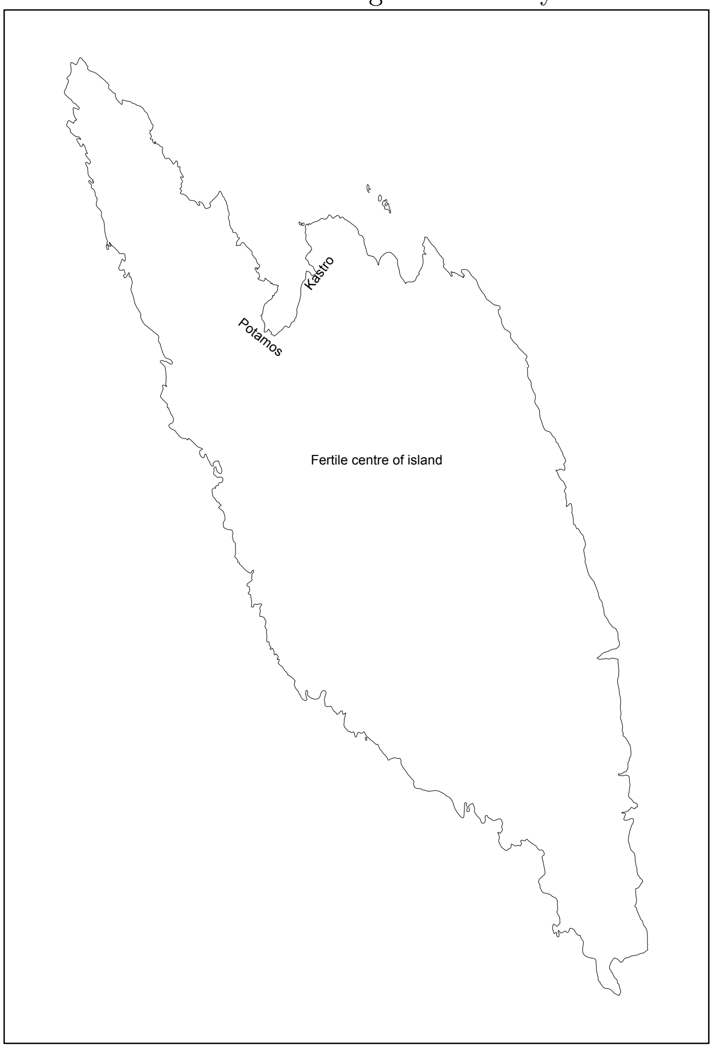


Figure 2: Lithics

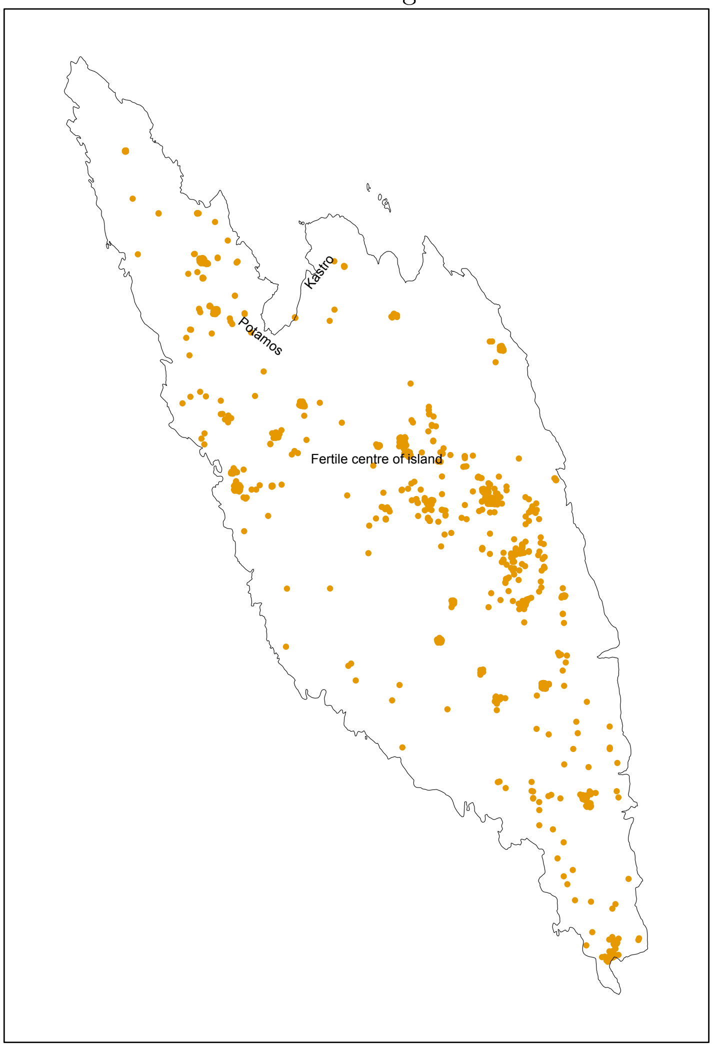


Figure 3: Coarse pottery finds

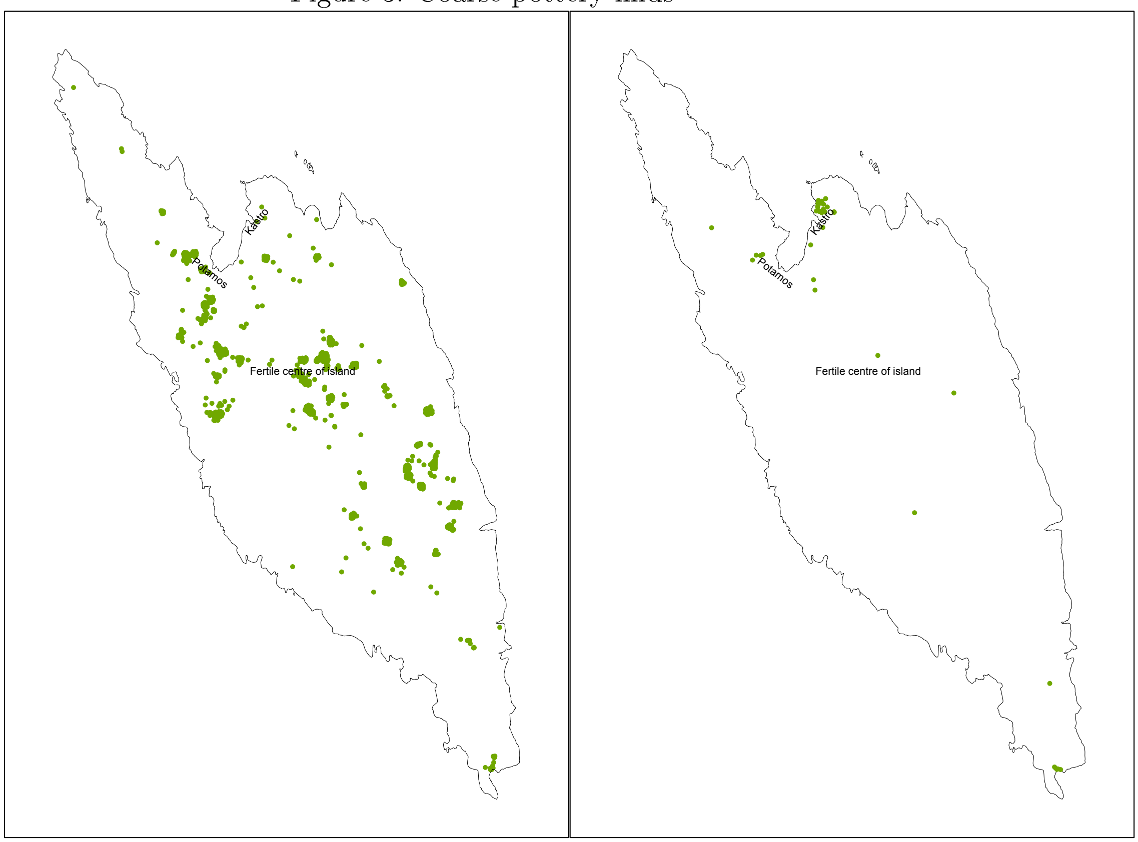


Figure 4: Medium pottery finds

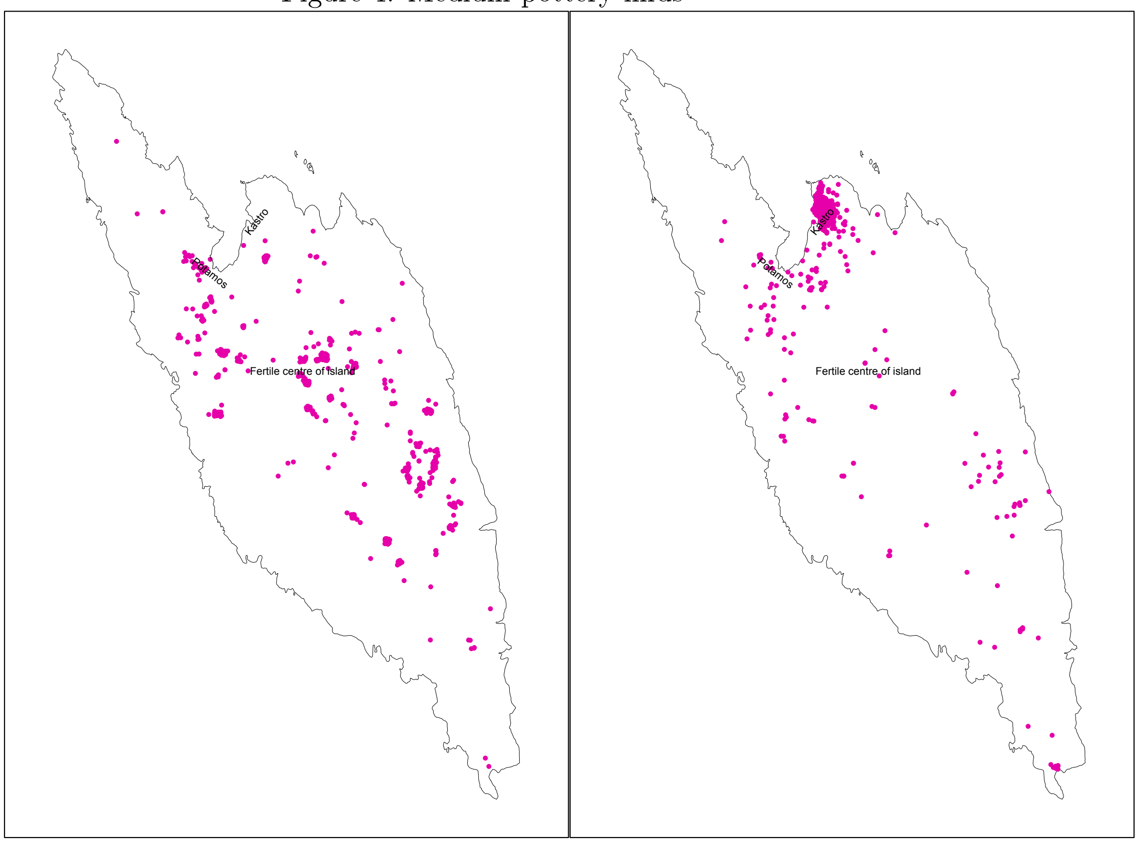


Figure 5: Fine pottery finds

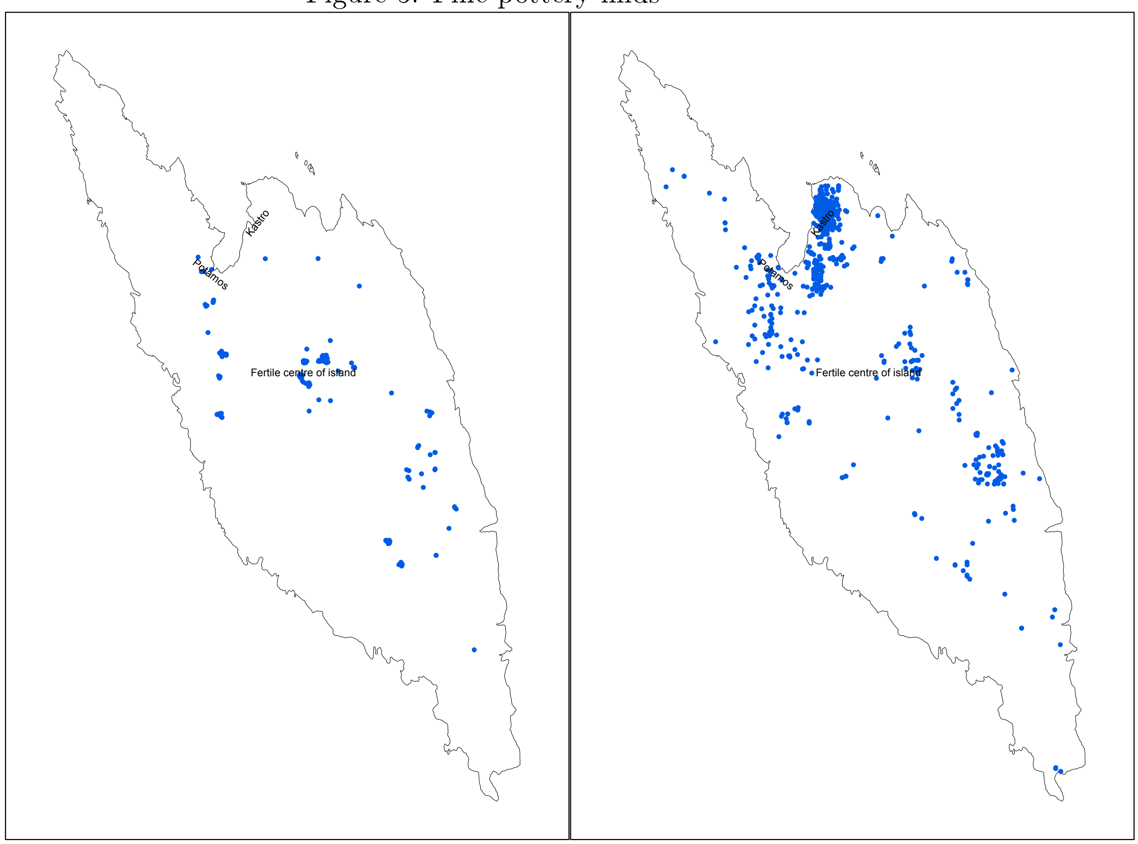


Figure 6: Wealth scores

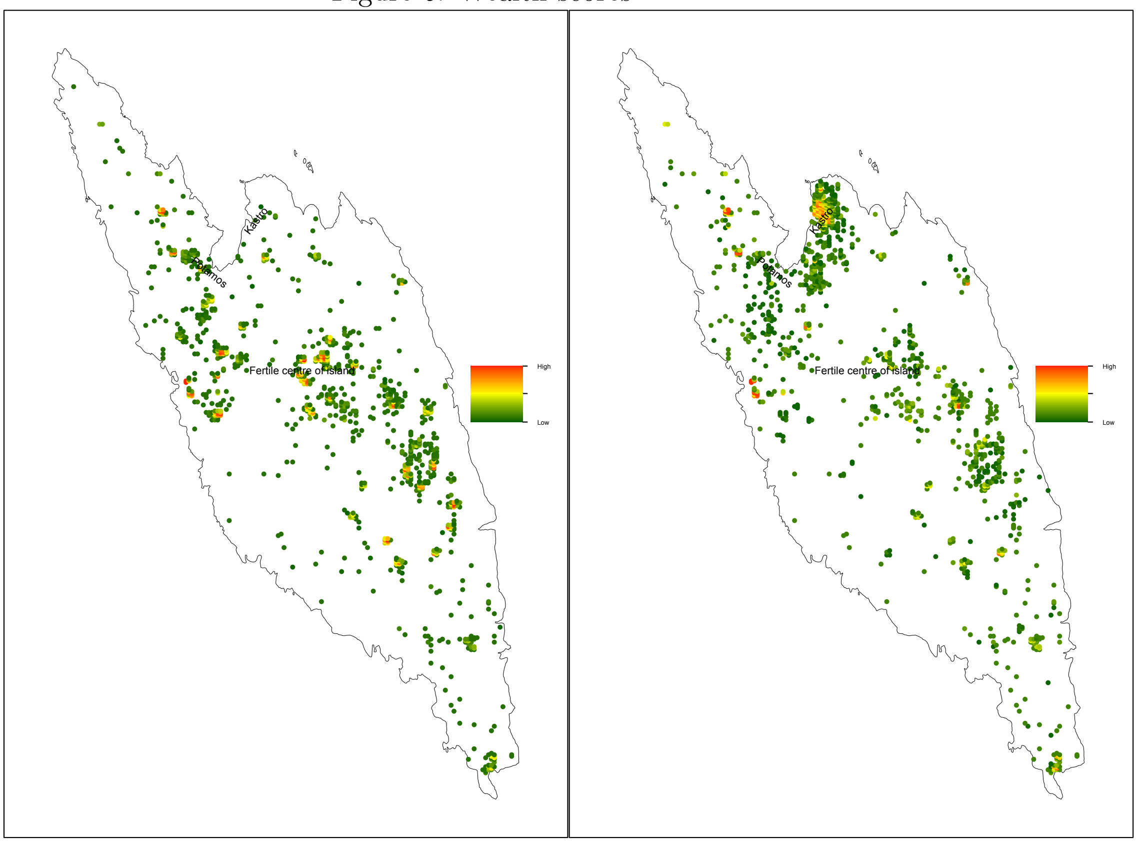
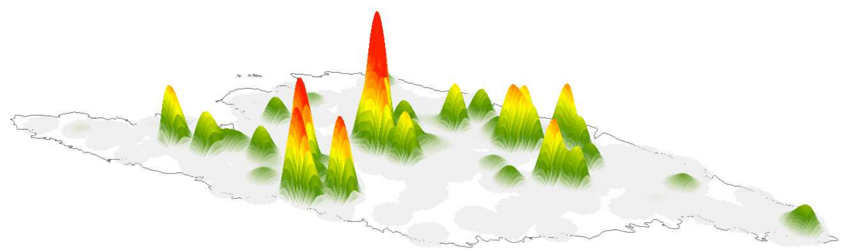


Figure 7: Wealth kernel densities



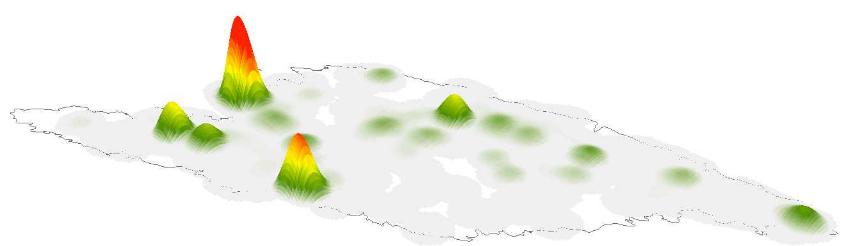


Figure 8: Linear and nonparametric wealth gradients

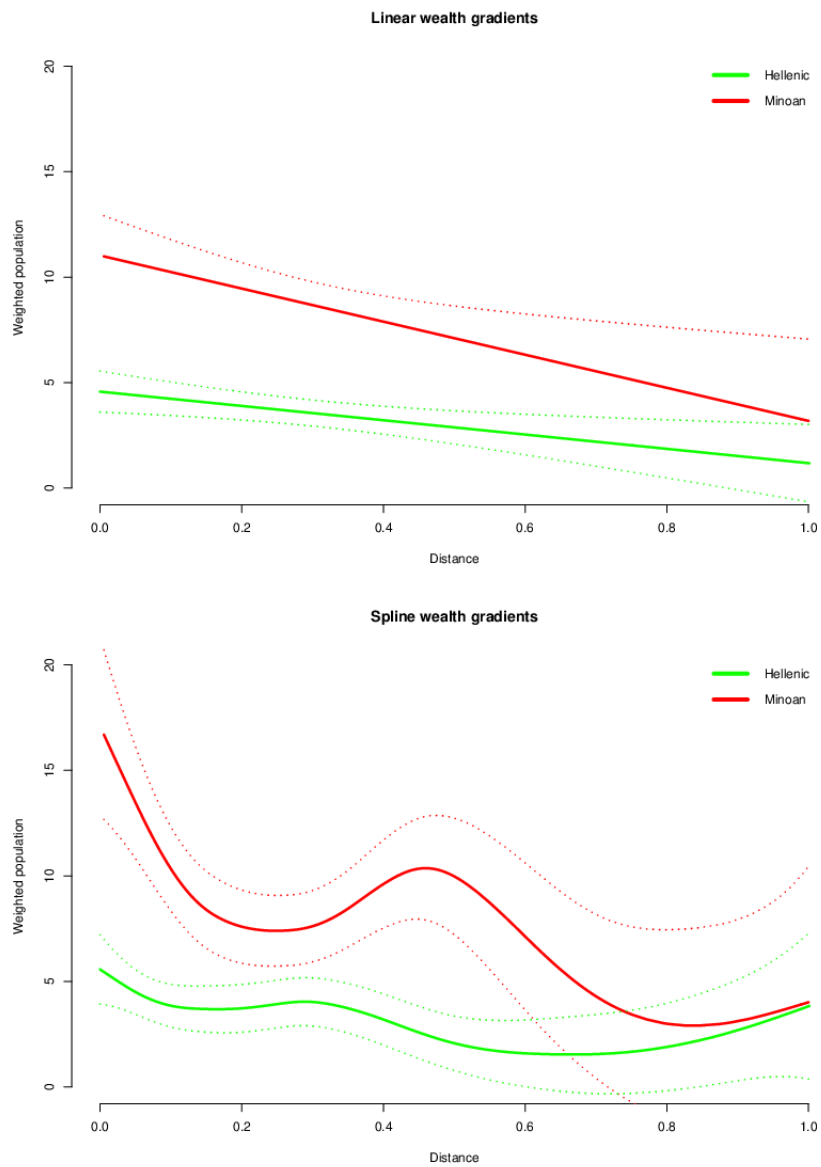
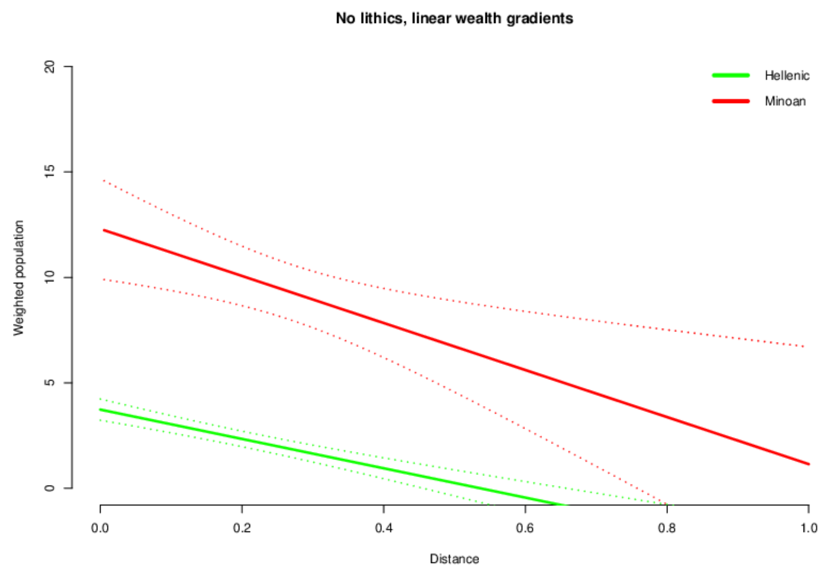


Figure 9: Linear and nonparametric population gradients: excluding lithics



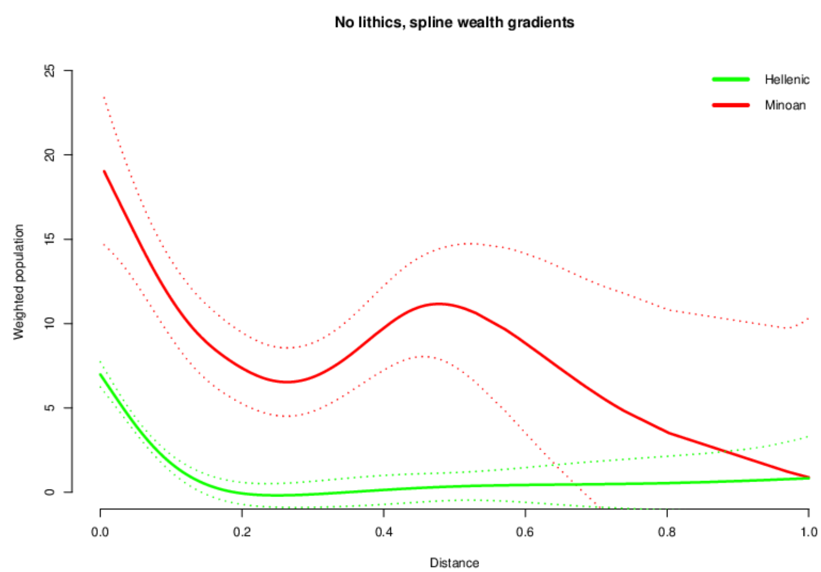



Figure 10: Relative wealth gradients: Hellenistic era

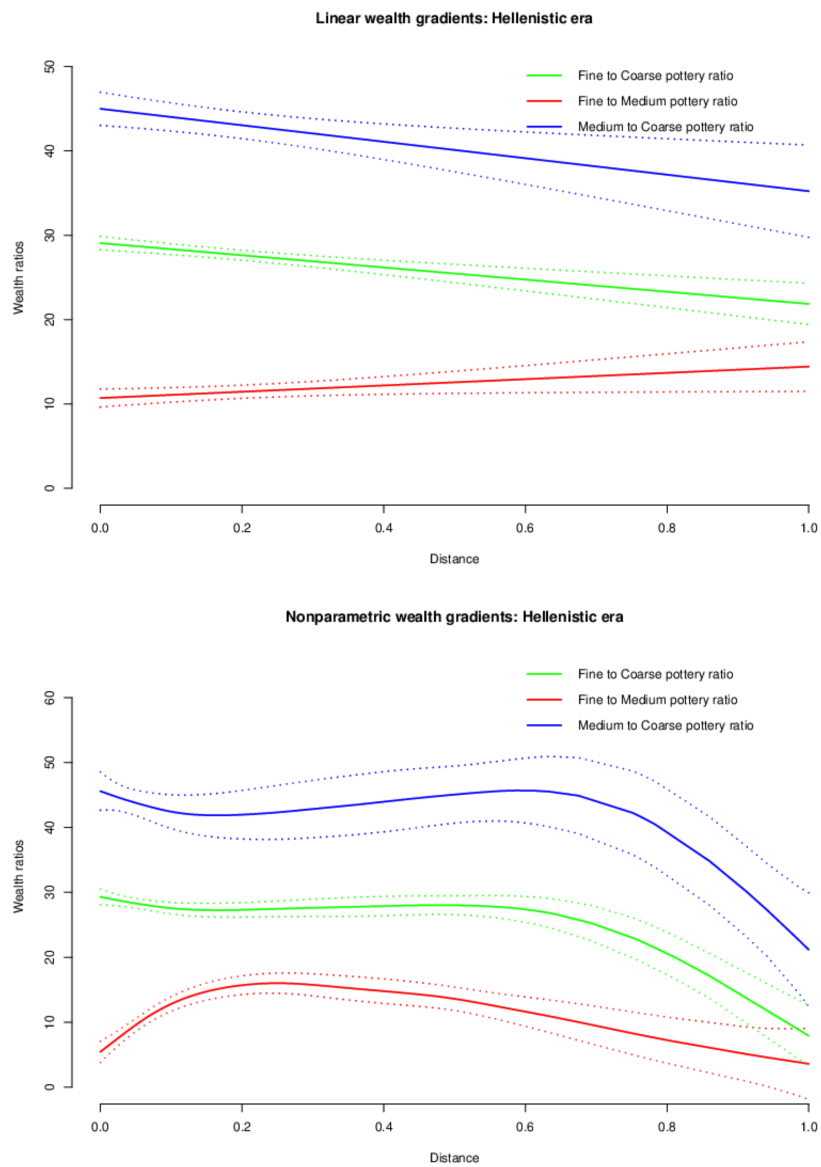


Figure 11: Relative wealth gradients: Minoan era, no lithics

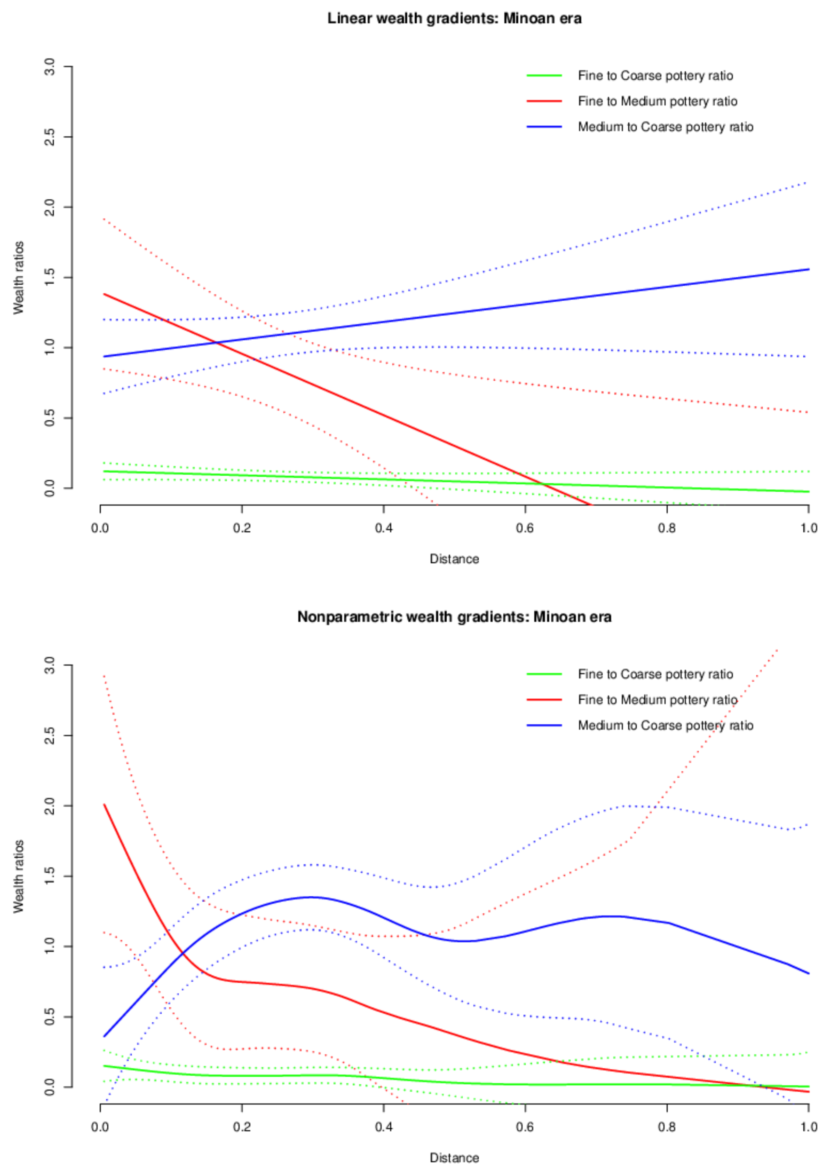


Figure 12: Relative wealth gradients: Minoan era, fine and medium pottery to lithics ratio

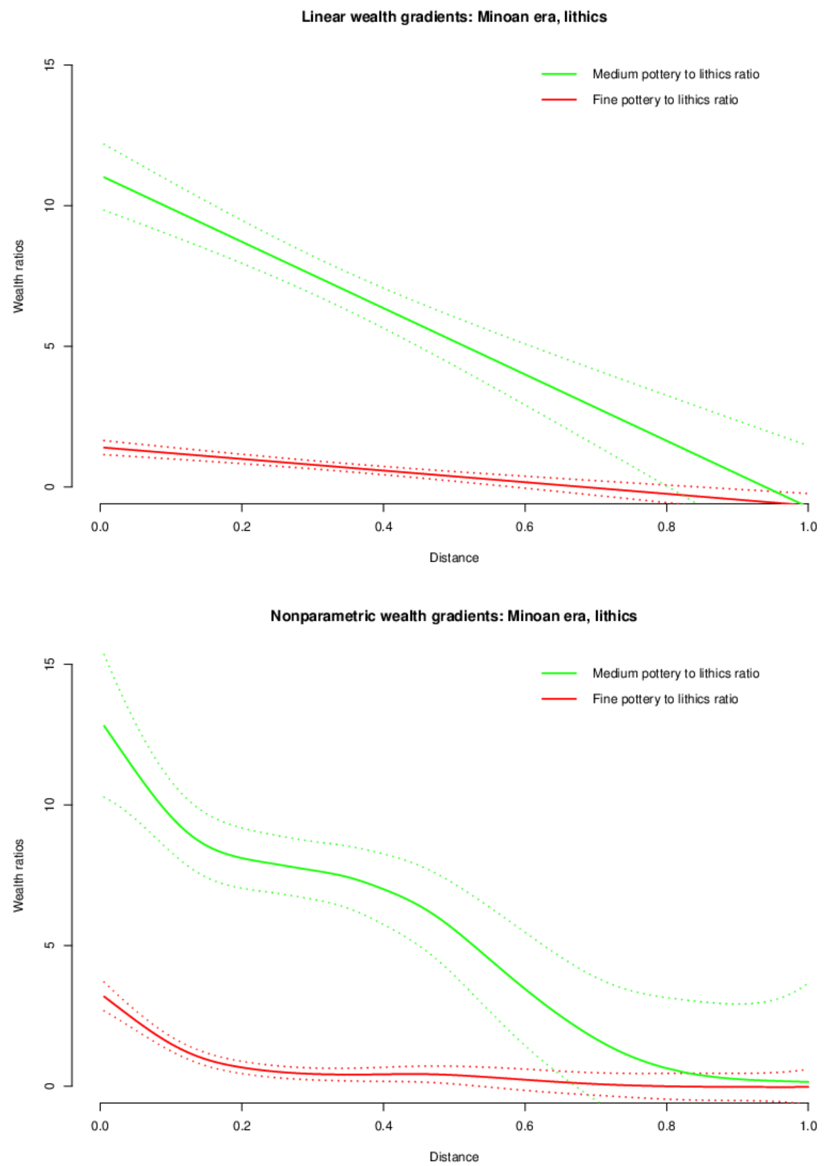
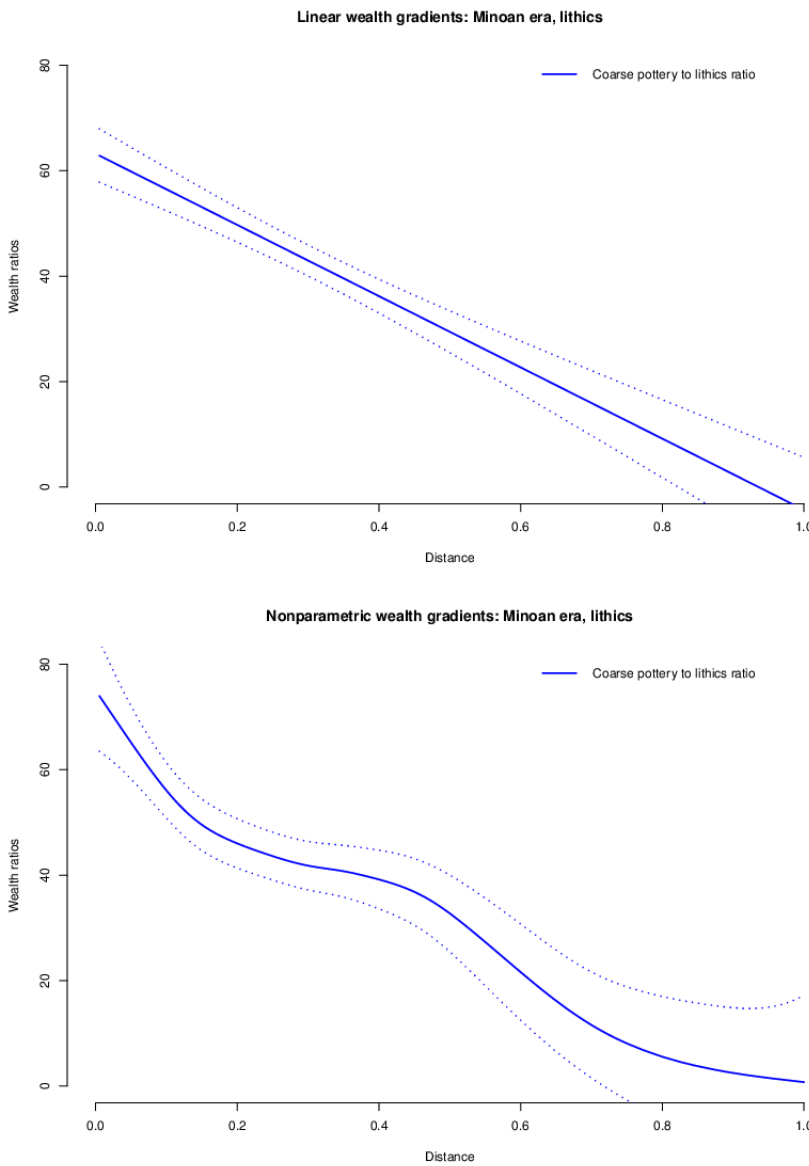


Figure 13: Relative wealth gradients: Minoan era, coarse pottery to lithics ratio



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