Reversal of Bertrand-Cournot Ranking for Optimal Privatization Level

Arindam Paul* and Parikshit De†

Abstract

We consider a vertically related differentiated product mixed duopoly market where a public and private firm compete in the downstream market. A foreign monopolist input supplier supplies the input of production in the upstream market. The public firm is partially privatized. The welfare maximising (public) regulator chooses the optimal privatization level for the public firm . We show that both under Cournot and Bertrand competition the Public firm becomes partially privatized. Moreover under Bertrand competition the privatization level is always larger than that under Cournot competition.

1 Introduction

Consider a simple vertically related mixed duopoly market where a public and private firm compete in the downstream market (final imperfectly substitute output market). The production of final commodity requires an input that is imported from the upstream market controlled by a foreign supplier who is a monopolist. Quite naturally, the monopolist foreign firm can discriminate input price that it charges. The public firm in the downstream market is partially privatized. The privatization level of the public firm can be optimally chosen by a regulator with the objective of

*Economic Research Unit, Indian Statistical Institute, Kolkata, India. Email: arish309@gmail.com

†Department of Economic Sciences , Indian Institute of Science Education and Research, Bhopal, India. Email: parikshitde@iiserb.ac.in

the welfare maximization.

We consider two type of downstream competition, namely, Cournot and Bertrand competition. In case of former, the firms compete in quantities while for latter they compete ins price. Under this setup, firstly, we find that under both Cournot and Bertrand the public firm becomes partially privatize. Secondly, our Bertrand-Cournot comparison reveals that the optimum level privatization is higher under Bertrand than that under Cournot competition. These findings are contrary to the existing results in the literature. Assuming Cournot competition Fujiwara [7] shows that in equilibrium the public firm becomes partially privatized. Assuming Bertrand competition Ohnishi [16] shows that in equilibrium the public firm becomes fully public. Recently Mitra.et.al [13] reestablished the validity of this existing results for very general demand functions.

A common feature of all these above papers is that the marginal cost of production is fixed and identical across the all firms. Allowing monopolist input supplier in the vertical structure, here we endogenise the marginal cost of each firm and obtain unconventional results in this context. Intuitively, in presence of competitive input supplier, in downstream market privatization level affects the welfare only via the output of the firms. In case of Cournot competition we have partial privatization due to the following reasons: Firstly, under no privatization the increase in effective product diversitywith privatization level increases the welfare. Secondly, at full privatization the benefit and cost of the society both increases due to decrease in the privatization level but former grow at less rapid rate than the latter. Hence we have partial privatization. However, for all privatization level, the benefit and cost of the society both increases due to decrease in the privatization level under Bertrand competition but the former grows at lesser rate than the latter. Therefore, we have no privatization. When we have monopolist input supplier in the vertical structure, the explanation for Cournot similar to that of Fujiwara [7] since we would not have price discrimination (since not profitable) in the input market. However, the story gets altered for the Bertrand competition. In case of Bertrand competition we not only have price discrimination but also due the price discrimination now the benefit and cost of the society both decreases due to increase in the privatization level from **Cournot (optimal) privatization level** but former decreases at less rapid rate than the latter. Therefore privatization under Bertrand is higher.

1.1 Related literature

Industrial economists are often interested in comparing different market structures which are primarily based on their market outcomes and then try to determine the best market structure considering either the society's welfare or the firm's profit and sometimes considering both. In this context, the "Cournot-Bertrand comparison" is one such important comparison that has often been analyzed in the literature of industrial economics. The first study with differentiated products was made by Singh and Vives [18]. They conclude that under Cournot duopoly each firm in the industry produces less, charges more and earns higher profit than under Bertrand duopoly. Further, they argued that the latter is efficient than the former in terms of welfare ranking. We refer to these rankings as the standard rankings. Subsequent studies in this literature have mainly concentrated in determining the circumstances where these standard rankings are either partially reversed or fully reversed. One such contribution by Hackner [9] shows that the standard rankings are dependent on the duopoly assumption and they get reversed under sufficient quality differences with increasing number of firms. However they do not consider the welfare rankings between Cournot and Bertrand. Hsu and Wang [10] conclude that the standard rankings hold in case of welfare with any number of firms. Amir and Jin [2], have extended the "Cournot-Bertrand comparison" by including the following market indicators:- mark-up output ratio, average output, average price and Herfindahl index. Except for Singh and Vives [18], the aforementioned studies deals with oligopoly market with linear demand. On the other hand, Vives [20] and Okuguchi [17] have worked with oligopoly markets assuming general non-linear demand functions. Subsequent studies by Mukherjee [15] and Cellini.et.al [6] for free entry; Symeonidis [19] and [11] for endogenous Research & Development expenditure; López and Naylor [12] for the wage bargaining provided evidence on partial reversal of the standard rankings. Arya.et.al [4] and Alipranti.et.al [1] have shown the complete reversal of the standard rankings with a vertically related producer along with Ghosh and Mitra [8] who get the same with mixed market.

Literature on privatization can be classified into two broad categories. First category include the papers in which the privatization is a discrete variable (See Anderson et al. [3], Barcena-Ruiz and Garzon [5].). Second category include the papers in which the privatization is a continuous variable (See Matsumura [14], Fujiwara [7], Ohnishi [16].).

1.2 Demand Side

The utility of the representative consumer is quasilinear in the competitive sector's output and is give by

$$\mathcal{U}(q_1, q_2, y) = U(q_1, q_2) + y$$

where q_i be the output produce by Firm i, $i = 1, 2$. The sub-utility that depends on the imperfectly competitive sector output is quadratic and is given by

$$U(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + q_2^2 + 2sq_1q_2\right)$$

where $a > 0$ represent the test parameter and $s \in (0, 1)$ represent the degree of product substitution. Therefore the representative consumer's problem is given by

$$\max_{q_1, q_2, y} \mathcal{U}(q_1, q_2, y) = U(q_1, q_2) + y$$
$$\textit{Sub to} \;\; p_1q_1 + p_2q_2 + y \leq I$$

where p_i be the price of commodity i and I be the income of the consumer. Given the quasilinear specification of the utility function the consumer's problem can be reduce to

$$\max_{q_1, q_2} U(q_1, q_2) - p_1q_1 - p_2q_2.$$

Therefore from the first order condition of the consumer's optimization we have the inverse demand function that Firm i faces

$$p_i(q_1, q_2) = \frac{\partial U}{\partial q_i}(q_1, q_2) = a - q_i - sq_j \quad \forall i, j = 1, 2 \ \& \ i \neq j. \tag{1}$$

Given $s \in (0, 1)$ the inverse demand function is invertible and we can solve for q_i to obtained the direct demand function that Firm i faces

$$D_i(p_i, p_j) = \frac{a}{1+s} - \frac{p_i}{1-s^2} + \frac{sp_j}{1-s^2} \quad \forall i, j = 1, 2 \ \& \ i \neq j. \tag{2}$$

1.3 Supply Side

Suppose that the production of the final commodity of each firm of the imperfect competitive sector uses same technology and production process required a key input on the one-to-one basis. Moreover this key input is imported from a foreign firm (Firm U). Firm U is a monopolist in the input market. There is no any other cost of production for final commodity of the imperfect competitive sector. Therefore the profit of the Firm i is given by

$$\pi_i = (p_i - z_i)q_i \tag{3}$$

where the z_i be the input price charged by Firm U to Firm i for purchase of it's input. Therefore the profit of the Firm U is given by

$$\pi_U = \sum_{k=1}^{2} z_k q_k. \tag{4}$$

The consumer surplus is given by

$$CS = U(q_1, q_2) - p_1 q_1 - p_2 q_2 \tag{5}$$

and the welfare of the society is given by

$$W = CS + \pi_1 + \pi_2. \tag{6}$$

The Firm 1 is a public firm which is partially privatize and maximizes weighted average of it's profit and welfare where the weight attached to it's profit is the privatization ratio. Therefore if $\theta \in [0, 1]$ represent the level of privatization then the Firm 1's objective function is

$$V_1 = \theta\pi_1 + (1 - \theta)W \tag{7}$$

and Firm 2 is a private firm which maximizes it's profit.

2 Game Structure and Main Results

The sequence of events are given by the following three stage game

- **Stage-I** Regulator or Planer select optimal privatization ratio θ.
- **Stage-II** Firm U chooses (z_1, z_2) by maximizing it's profit.
- **Stage-III** Firm 1 and Firm 2 compete in the market. While competing in the market we consider two specific mode of competition- Cournot competition and Bertrand competition. In case of former firms compete with quantity while in case of latter firms compete with price.

We use backward induction method to solve this three stage game separately for Cournot and Bertrand competition. Therefore first given any θ and given any pair of (z_1, z_2) we solve the Stage-III outcome of each mode of competition separately then using these market outcomes and given any θ we solve for optimal pair of (z_1, z_2). Finally using optimal Stage-II and Stage-III outcomes we solve for optimal privatization θ.

Proposition 1 Given this market structure we have the following results:

(i) The Publicly-Regulated-Firm is partially privatized in the context of both Bertrand and Cournot.

(ii) Under Bertrand the optimal level of privatization is higher than that of Cournot.

We use Lemma 1 to establish the Proposition 1.

Lemma 1 Followings are true for the Stage-II input price of foreign input seller:

(i) Assuming downstream Cournot competition, the foreign monopolist input supplier does not discriminates the input price and optimally charges $z_1^{QQ}(\theta) = z_2^{QQ}(\theta) = a/2$ in the Stage-II independent of privatization ratio set by the regulator in Stage-I.

(ii) Assuming downstream Bertrand competition, given any privatization ratio ($\theta \in [0, 1]$) set by the regulator, the foreign monopolist input supplier discriminates the input price in the Stage-II. If $(z_1^{PP}(\theta), z_2^{PP}(\theta))$ be the optimal choice of Stage-II input prices by the monopolist then we have following (a) $z_1^{PP}(\theta) < z_2^{PP}(\theta)$.

Given the Lemma 1(i) we have partial privatization under downstream Cournot competition following the analysis of Fujiwara [7]. However, given the Lemma 1(ii) the analysis of the Ohnishi [16] for Bertrand competition can not be simply extended in presence of monopoly input supplier. In case of price competition, at the **optimal level of privatization under quantity competition**, input supplier is charging relatively higher input price to the private firm. The over all output availability now would be lesser leading to reduced welfare. Hence welfare under Bertrand competition will increase at the optimal level of privatization under Cournot (See fig 1).

3 Simulation exercise and other market outcomes

We have the following simulation results[1]:

- For all $s \in (0, s_1]$ output of Firm 1 is larger under Cournot than that of Bertrand where $s_1 \geq 0.9$.

[1]Table of simulation is available at request

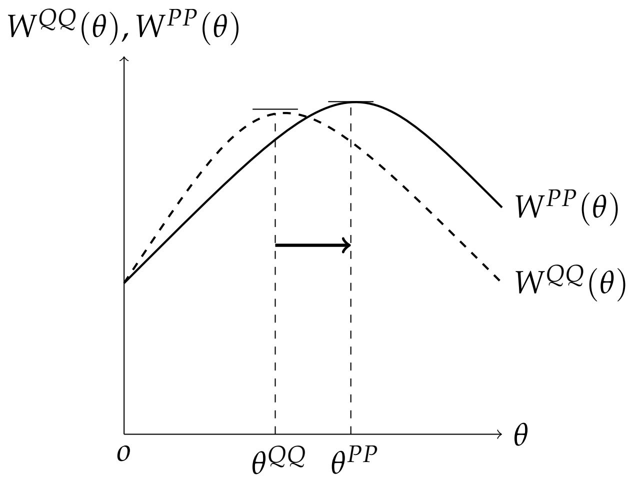

Figure 1: Graphical explanation of Proposition 1.

- For all $s \in (0, s_2]$ output of Firm 2 is larger under Cournot than that of Bertrand where $0.5 \leq s_2 \leq 0.75$.
- Firm 1 charges higher price under Cournot than that of under Bertrand.
- For all $s \in (0, s_3]$ Firm 2 charges higher price under Bertrand than that of Cournot where $0.5 \leq s_3 \leq 0.75$.
- Profit of Firm 2, Consumer Surplus and Welfare are higher under Bertrand than what we have under Cournot.

4 Appendix

Proof of the Lemma 1: We complete the proof in two steps.

Proof of the Lemma 1(i): Assuming downstream Cournot competition, in Stage-III given any θ and any pair of (z_1, z_2), Firm 1 maximizes

$$V_1^{QQ}(q_1, q_2; z_1, z_2, \theta) = \theta \pi_1^{QQ}(q_1, q_2; z_1) + (1 - \theta) W^{QQ}(q_1, q_2; z_1, z_2)$$

by choosing q_1 given q_2 where $\pi_1^{QQ}(q_1, q_2; z_1) = (p_1(q_1, q_2) - z_1)q_1$ and $W^{QQ}(q_1, q_2; z_1, z_2) = (a -$

$z_1)q_1 + (a - z_2)q_2 - \frac{1}{2}\left(q_1^2 + q_2^2 + 2sq_1q_2\right)$ and Firm 2 will maximizes $\pi_2^{QQ}(q_1, q_2; z_2) = (p_2(q_1, q_2) - z_2)q_2$ by choosing q_2 given q_1. If $(q_1^{QQ}(z_1, z_2; \theta), q_2^{QQ}(z_1, z_2; \theta))$ be the optimal Stage-III choice vector then it satisfy the condition $\partial V_1^{QQ}(q_1^{QQ}(z_1, z_2; \theta), q_2^{QQ}(z_1, z_2; \theta); z_1, z_2, \theta)/\partial q_1 = 0$ and the condition $\partial \pi_2^{QQ}(q_1^{QQ}(z_1, z_2; \theta), q_2^{QQ}(z_1, z_2; \theta); z_1, z_2, \theta)/\partial q_2 = 0$ simultaneously. Therefore in stage-III Firm 1 produces

$$q_1^{QQ}(z_1, z_2, \theta) = \frac{2(a - z_1) - s(a - z_2)}{2(1 + \theta) - s^2}$$

and Firm 2 produces

$$q_2^{QQ}(z_1, z_2, \theta) = \frac{(1 + \theta)(a - z_2) - s(a - z_1)}{2(1 + \theta) - s^2}.$$

Hence in Stage-II Firm-U choose the vector of input price that maximizes his own profit given the Stage-III choice of Firm 1 and Firm 2. Formally the problem of Firm U is

$$\max_{(z_1, z_2)} \Pi_U^{QQ}(z_1, z_2, \theta) = z_1 q_1^{QQ}(z_1, z_2; \theta) + z_2 q_2^{QQ}(z_1, z_2; \theta).$$

Therefore, $(z_1^{QQ}(\theta), z_2^{QQ}(\theta))$ will satisfy the first order conditions, $\partial \Pi_U^{QQ}(z_1^{QQ}(\theta), z_2^{QQ}(\theta), \theta)/\partial z_1 = 0$ and $\partial \Pi_U^{QQ}(z_1^{QQ}(\theta), z_2^{QQ}(\theta), \theta)/\partial z_2 = 0$. Solving we get $z_1^{QQ}(\theta) = z_2^{QQ}(\theta) = a/2$.

Proof of the Lemma 1(ii): Assuming downstream Bertrand competition, in Stage-III given any θ and any pair of (z_1, z_2), Firm 1 maximizes

$$V_1^{PP}(p_1, p_2; z_1, z_2, \theta) = \theta \pi_1^{PP}(p_1, p_2; z_1) + (1 - \theta) W^{PP}(p_1, p_2; z_1, z_2)$$

by choosing p_1 given p_2 where $\pi_1^{PP}(p_1, p_2; z_1) = (p_1 - z_1)D_1(p_1, p_2)$ and $W^{PP}(p_1, p_2; z_1, z_2) = W^{QQ}(D_1(p_1, p_2), D_2(p_1, p_2); z_1, z_2)$ and Firm 2 will maximizes $\pi_2^{PP}(p_1, p_2; z_2) = (p_2 - z_2)D_2(p_1, p_2)$ by choosing p_2 given p_1. If $(p_1^{PP}(z_1, z_2; \theta), p_2^{PP}(z_1, z_2; \theta))$ be the optimal Stage-III choice vector then it satisfy the condition $\partial V_1^{PP}(p_1^{PP}(z_1, z_2; \theta), p_2^{PP}(z_1, z_2; \theta); z_1, z_2, \theta)/\partial p_1 = 0$ and the condition $\partial \pi_2^{PP}(p_1^{PP}(z_1, z_2; \theta), p_2^{PP}(z_1, z_2; \theta); z_1, z_2, \theta)/\partial p_2 = 0$ simultaneously. Therefore in stage-III Firm 1 charges

$$p_1^{PP}(z_1, z_2, \theta) = z_1 + \frac{(2\theta - s^2)(a - z_1) - (2\theta - 1)(a - z_2)}{2(1 + \theta) - s^2}$$

and Firm 2 charges

$$p_2^{PP}(z_1, z_2, \theta) = \frac{(1+\theta(1-s^2))(a-z_2) - s(a-z_1)}{2(1+\theta) - s^2}.$$

Hence in stage-III Firm 1 produces

$$q_1^{PP}(z_1, z_2, \theta) = \frac{(2-s^2)(a-z_1) - s[(2-\theta) - s^2(1-\theta)](a-z_2)}{(1-s^2)[2(1+\theta) - s^2]}$$

and Firm 2 produces

$$q_2^{PP}(z_1, z_2, \theta) = \frac{(1+\theta(1-s^2))(a-z_2) - s(a-z_1)}{(1-s^2)[2(1+\theta) - s^2]}.$$

Hence in Stage-II Firm-U choose the vector of input price that maximizes his own profit given the Stage-III choice of Firm 1 and Firm 2. Formally the problem of Firm U is

$$\max_{(z_1, z_2)} \Pi_U^{PP}(z_1, z_2, \theta) = z_1 q_1^{PP}(z_1, z_2; \theta) + z_2 q_2^{PP}(z_1, z_2; \theta).$$

Therefore, $(z_1^{PP}(\theta), z_2^{PP}(\theta))$ will satisfy the first order conditions, $\partial\Pi_U^{PP}(z_1^{PP}(\theta), z_2^{PP}(\theta), \theta)/\partial z_1 = 0$ and $\partial\Pi_U^{PP}(z_1^{PP}(\theta), z_2^{PP}(\theta), \theta)/\partial z_2 = 0$. Solving we get

$$z_1^{PP}(\theta) = \frac{[4(1+\theta) - (1-\theta^2)s - (1+\theta)s^2 + \theta(1+\theta)s^3]a}{8(1+\theta) - (5-2\theta+\theta^2)s^2 + (1-\theta)^2 s^4} \tag{8}$$

and

$$z_2^{PP}(\theta) = \frac{[4(1+\theta) + 2(1-\theta)s - (4-3\theta+\theta^2)s^2 + (1-\theta)s^3 + (1-\theta)^2 s^4]a}{8(1+\theta) - (5-2\theta+\theta^2)s^2 + (1-\theta)^2 s^4}. \tag{9}$$

Given $8(1+\theta) - (5-2\theta+\theta^2)s^2 + (1-\theta)^2 s^4 > 0, 4(1+\theta) - (1-\theta^2)s - (1+\theta)s^2 + \theta(1+\theta)s^3 > 0$ and $4(1+\theta) + 2(1-\theta)s - (4-3\theta+\theta^2)s^2 + (1-\theta)s^3 + (1-\theta)^2 s^4 > 0$ for all $(s, \theta) \in [0,1] \times (0,1)$

therefore we have $z_1^{PP}(\theta) > 0$ and $z_2^{PP}(\theta) > 0$. Using equation (8) and (9) we get,

$$z_d^{PP}(\theta) := z_1^{PP}(\theta) - z_2^{PP}(\theta) = -\frac{(1-\theta)[(3+\theta) - (3-\theta)s - (1+\theta)s^2 + s^3]a}{8(1+\theta) - (5-2\theta+\theta^2)s^2 + (1-\theta)^2 s^4}. \tag{10}$$

Given for all $(s,\theta) \in [0,1] \times (0,1)$ we have $(3+\theta) - (3-\theta)s - (1+\theta)s^2 + s^3 > 0$ and $8(1+\theta) - (5-2\theta+\theta^2)s^2 + (1-\theta)^2 s^4 > 0$ therefore we have $z_1^{PP}(\theta) < z_2^{PP}(\theta)$ for all $\theta \in [0,1)$ but for $\theta = 1$ we have $z_1^{PP}(\theta) = z_2^{PP}(\theta)$. Hence the result.

Proof Of Proposition 1 Assuming the downstream Cournot competition in the Stage-I the regulator will maximize the welfare

$$W^{QQ}(q_1^{QQ}(z_1^{QQ}(\theta), z_2^{QQ}(\theta), \theta), q_1^{QQ}(z_1^{QQ}(\theta), z_2^{QQ}(\theta), \theta), z_1^{QQ}(\theta), z_2^{QQ}(\theta))$$

by choosing $\theta \in [0,1]$. It can be shown that the unique optimal level of privatization is $\theta^{QQ} = s(1-s)/(4-3s)$. However, assuming the downstream Cournot competition in the Stage-I the regulator will maximize the welfare

$$W^{QQ}(q_1^{PP}(z_1^{PP}(\theta), z_2^{PP}(\theta), \theta), q_1^{PP}(z_1^{PP}(\theta), z_2^{PP}(\theta), \theta), z_1^{PP}(\theta), z_2^{PP}(\theta))$$

by choosing $\theta \in [0,1]$. It can be shown that the unique optimal level of privatization θ^{PP} is obtained by solving the equation

$$f(\theta) = \left[\begin{array}{c} -s^4(1-s)(s+1)^4\theta^4 + 2s^2(1-s)(2s^4+2s^3-s^2+4s+2)(s+1)^2\theta^3 \\ +6s^2(1-s)(1+s)(10+4s+2s^3+s^4+s^5)\theta^2 \\ +(-4s^9+22s^7-2s^6-6s^5+30s^4-36s^3+44s^2+32s-128)\theta \\ +s(s+2)(s+1)(s^6-4s^5+2s^4+10s^3-19s^2+2s+16) \end{array}\right] = 0. \tag{11}$$

Finally given $f(\theta^{QQ}) > 0$ for all $s \in (0,1)$ we can conclude that $\theta^{QQ} < \theta^{PP}$.

References

[1] Alipranti, M., Milliou, C., & Petrakis, E. (2014). Price vs. quantity competition in a vertically related market. Economics Letters, 124, 122-126.

[2] R. Amir and J. Y. Jin. Cournot and bertrand equilibria compared: substitutability, complementarity and concavity. International Journal of Industrial Organization, 19 :303–317, 2001.

[3] Anderson, S. P., de Palma, A., Thisse, J.-F., 1997. Privatization and efficiency in a differentiated industry. European Economic Review 41, 1635-1654.

[4] Arya, A., Mittendorf, B., & Sappington, D. E. (2008). Outsourcing, vertical integration, and price vs. quantity competition. International Journal of Industrial Organization, 26, 1-16.

[5] Bárcena-Ruiz, J. C., & Garzón, M. B. (2018). Privatisation and vertical integration under a mixed duopoly. Economic Systems, 42(3), 514-522.

[6] Cellini, R., Lambertini, L., & Ottaviano, G. I. (2004). Welfare in a differentiated oligopoly with free entry: a cautionary note. Research in Economics, 58, 125-133.

[7] Fujiwara, K., 2007. Partial privatization in a differentiated mixed oligopoly. Journal of Economics 92, 51-65.

[8] Ghosh, A., Mitra, M., 2010. Comparing Bertrand and Cournot in mixed markets. Economics Letters 109, 72-74.

[9] Häckner, J., 2000. A note on price and quantity competition in differentiated oligopolies. Journal of Economic Theory 93, 233-239.

[10] Hsu, J., & Wang, X. H. (2005). On welfare under Cournot and Bertrand competition in differentiated oligopolies. Review of Industrial Organization, 27, 185-191.

[11] Lin, P., & Saggi, K. (2002). Product differentiation, process R & D, and the nature of market competition. European Economic Review, 46, 201-211.

[12] López, M. C., & Naylor, R. A. (2004). The Cournot–Bertrand profit differential: a reversal result in a differentiated duopoly with wage bargaining. European Economic Review, 48(3), 681-696.

[13] Mitra, M., Pal, R., Paul, A., & Sharada, P. M. (2017). Equilibrium co-existence of public and private firms and the plausibility of price competition.

[14] Matsumura, T., 1998. Partial privatization in mixed duopoly. Journal of Public Economics 70, 473-483.

[15] Mukherjee, A. (2005). Price and quantity competition under free entry. Research in Economics, 59, 335-344.

[16] Ohnishi, K., 2010. Partial privatization in price-setting mixed duopoly. Economics Bulletin 30, 309-314.

[17] Okuguchi, K. (1987). Equilibrium prices in the Bertrand and Cournot oligopolies. Journal of Economic Theory, 42, 128-139.

[18] Singh, N. and Vives, X., 1984. Price and quantity competition in a differentiated duopoly. RAND Journal of Economics 15, 546-554.

[19] Symeonidis, G. (2003). Comparing Cournot and Bertrand equilibria in a differentiated duopoly with product R & D. International Journal of Industrial Organization, 21, 39-55.

[20] Vives, X. (1985). On the efficiency of Bertrand and Cournot equilibria with product differentation. Journal of Economic Theory, 36, 166-175.