Sovereign Debt: A Quantitative Comparative Investigation of the Partial Default Mechanism*

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Abstract

We quantitatively explore the implications of the partial default mechanism for the dynamics of sovereign debt and default in a small open economy. The model features endogenous partial default and preemptive recovery on the defaulted amount with direct utility cost of default, instead of the exclusion from international financial markets. The model is calibrated to Argentina and compared to the traditional full default models. We show that with our partial default framework, (1) the model with endowment not only matches the mean spread on debt and the debt-to-output ratio, like the traditional models, but also matches both the default frequency and the default rate; (2) the model with production, with investment as another margin to smooth consumption, improves the fit with data for the volatilities of consumption and spread on debt as well as the debt service-to-output ratio (without significant change in other moments), partially offsetting the weak performance of the endowment model; and (3) furthermore, the non-exclusion from international markets provides a more realistic pattern of the impulse responses of various macro variables to economic shocks, which gives a better understanding of the propagation mechanism of partial default.

Keywords: Sovereign debt, partial default, small open economy model, business cycle
JEL Classification: E32; F34; F41; F44; F47; H63

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1 Introduction

Sovereign defaults are common, especially in emerging and developing countries. The standard theory of sovereign default, which investigates the incentives and consequences of default, assumes that countries always default on all of their debt and are excluded from the international capital market after default for some period of time. However, the empirical regularities show that countries always default on only part of their debt and continue to borrow while having debt arrears. Besides being not an accurate assumption, the standard full default models of both the short-term and long-term debt have their limitations in terms of simultaneously predicting some critical features of the debt dynamics, like the debt-to-output ratio and the default frequency, although they can predict that default happens in recessions and that the country spreads are counter-cyclical. In this paper, we solve the partial default models of a small open economy to quantitatively investigate the responses of the borrowing, default, and pricing of sovereign debt to economic shocks. The simulations show that the partial default models can match the country spreads on external debt and default-related statistics without adverse effect on their ability to match the major business cycle moments. Moreover, the partial default models can also match the rate of (partial) default, which the traditional Eaton and Gersovitz (1981) style (full) default models are not designed to and cannot match.

The partial default models of this paper, with endowment and with production, both have three key features: Firstly, the rate of (partial) default is endogenously-determined. Secondly, there is a preemptive recovery rate associated with default,[1] which adds to the future debt obligations of the country. Thirdly, there is no exclusion from the international capital market after default. These three key features allow us to improve the predictions of the debt and the default-related statistics, simultaneously matching the mean spread on debt, the debt-to-output ratio, the default frequency, as well as other macroeconomic business cycle facts. Moreover, these key features allow us to match the partial default rate, and specifically, the non-exclusion from the international capital market enables us to generate meaningful impulse responses of various macroeconomic variables to economic shocks that lead to default, which the traditional full default models are not able to do.

The models, both with endowment and with production, consist of two types of agents. The first type includes the households in a small open economy. The government, acting on the behalf of the households in the small open economy, issues one-period bonds and maximizes the life-time utility of the representative household. The government is allowed to partially default on its outstanding debt with an exogenous preemptive recovery rate.

[1] See Asonuma and Trebesch (2016).

The default amount is endogenously-determined, which allows us to endogenize the partial default rate.

According to the contemporary empirical studies,[2] sovereign countries are often able to borrow soon after default. The small open economy is not excluded from the international capital market after default. While the non-exclusion shuts down one channel of penalizing defaulting countries, the default leads to a direct loss of utility for the representative household. The disutility endogenously depends on the amount of default and is a reduced form for various losses such as reputation, trade loss, and other costs resulting from default. The default risk premium is embedded in pricing of the new-issuance.

The new-issuance and the preemptive recovery payment of default are indistinguishable when rolled over into the next period and they compose the new total debt obligation. Due to the preemptive recovery feature, the bond pricing in the partial default models with one-period debt acquires features similar to that for the long-term debt models in the Eaton and Gersovitz's (1981) framework. The reason is that, repayment, and here the price of debt in each period depends not only on payment in the next period but also the next period's price of bond due to the rollover of any defaulted amount.

The second type agents are many international investors. They can invest into either the sovereign debt subject to partial default or an international risk-free asset. They are risk-neutral and, therefore, care only about the expected returns. Thus, the expected gross return on sovereign debt is equal to the gross return on the risk-free asset.

While the models are solved using the well-established techniques of value function iteration combined with the interpolation methods to approximate the value function, the bond price function, and other policy functions, we need to use an irregular discrete state space for debt. The need for using an irregularly spaced grid arises from the fact that our bond pricing function has potentially a large discontinuity near the zero-debt level, especially for the bad states (of the economy's productivity). Thus, to mitigate the adverse effects of this discontinuity on the process of interpolation, we put a point close to the zero-debt point on the bond grid. Moreover, we also put additional grid points for the low values of debt to more accurately estimate the bond price in that region. The expectation of the value and the bond price functions are computed using the Gauss-Hermite points and weights. We use a one-loop method,[3] combined with precomputation (Maliar, Maliar, and Judd, 2011), to speed up computations and solve for the equilibrium using a very tight convergence criterion on the bond price function, the value function, and all the policy functions.

For the quantitative analysis, the benchmark models, both with endowment and with

[2]See Subsection 2.1 Empirical Evidence.

[3]See Arellano, Bai, and Mihalache (2018) and Hatchondo, Martinez, and Sapriza (2010).

production, are calibrated to match the moments of the economic data for Argentina. Following the literature on sovereign default, we examine the ability of our models to match the moments associated with the debt dynamics; in addition, we also track the performance of the models in terms of the stylized business cycle facts. The model with endowment simultaneously matches the mean spread on debt, the debt-to-output ratio, and the default frequency, as well as other macroeconomic cyclical facts. The simulations of the model with endowment also match the default rate, which the traditional Eaton and Gersovitz (1981) style (full) default models, by construction, cannot. The impulse responses of various macroeconomic and debt variables to economic shocks are also used to investigate the propagation mechanism of the shocks in the presence of partial default. We find that when responding to a significant adverse shock, the country defaults and the international markets demand a spread on the debt issued by the country. Despite the increased cost due to the spreads, the country continues to issue new debt. It uses the proceeds from this new-issuance and from running a net trade surplus to service the debt not defaulted upon. For an extreme shock, the default is full and there is no need to service any debt and, therefore, both the new-issuance and the trade surplus fall to zero in the initial period and the country endogenously enters into autarky in both the financial and the good markets.

In the calibration process of the partial default model with endowment, the volatilities of consumption (trade balance) and interest spreads are over-predicted. To improve the model's fit to these moments we add production to the model. In the partial default model with production, the adjustment in investment provides another margin for a country to smooth consumption, which brings the simulations of the standard deviation of consumption (relative to that of the output) more in line with the data. By leading to less reliance on international borrowing, it also helps reduce somewhat the high standard deviation of spreads. The overall fit of the calibrated model with production to the target moments, the mean spread on debt, the debt-to-output ratio, the default frequency, the default rate, and the standard deviation of investment relative to that of the output, is reasonable. Relative to the model with endowment, the model with production does slightly worse in terms of the default frequency and default rate, but better in terms of the debt-service-to-output ratio. Specifically, the model can simultaneously match the mean spread on debt and the debt-to-output ratio. It also closely matches the relative volatility of investment. The other business cycle statistics such as the correlation of output with consumption, trade balance, and interest spreads are relatively unaffected by including production in the model.

The paper is organized as follows: Section 2 discusses the partial default-related empirical evidence and literature. Section 3 presents the models with endowment and with production. Section 4 calibrates the models and discusses the computational algorithm and

Section 5 analyzes the quantitative implications of the models *vis-à-vis* the empirical facts and investigates the impulse responses. Section 6 concludes.

2 Empirical Evidence and Literature

This section begins with presenting the evidence on the partial nature of sovereign default. Thereafter, it provides the estimates for the partial default rate and identifies the empirical characteristics of the partial default rate. These empirical facts are either relevant to the specifications of our model or for the comparison with the results from the model simulation. Following the literature, a more detailed analysis is provided for the data for Argentina. The literature on sovereign default, in general, and on partial default, in particular, that relates to the paper is also discussed.

2.1 Empirical Evidence

We focus on the three empirical facts: (1) Sovereign default is always partial and the existence of positive debt arrears is frequent; (2) Countries are often able to borrow soon after default; and (3) The partial default rate is counter-cyclical and it can be used as the proxy for the *realized* default risk.

1. Sovereign default is always partial and the existence of positive debt arrears is frequent: The partial default rate is defined as the ratio of the (end-of-period cumulative) debt arrears on external debt to the (beginning-of-period) debt service obligations, which consist of the previous period debt arrears and the amounts of principal and interest due:

$$Partial\ Default\ Rate = \frac{Debt\ Arrears}{Debt\ Service\ Obligations} = \frac{Debt\ Arrears}{Debt\ Arrears + Actural\ Debt\ Service}$$

The expression on the right hand side of the second equality sign follows from the fact that the debt service obligations can also be written as the (end-of-period cumulative) debt arrears and actual debt service payments (*e.g.*, also see Arellano, Mateos-Planas, and Rios-Rull, 2013):[4] Figure 1 plots the time series of the partial default rate for Argentina over the period 1970–2013. The frequency of positive arrears and the partial default rate (conditional on positive arrears) are 0.7045 and 0.4876, respectively (as shown in Table 1). Debt arrears exist in both the good and the bad times. Table 2 reports the key statistics of the partial

[4] Partial default mechanism is commonly-used in the sovereign debt literature. The value of partial default rate is different from debt forgiveness ("haircut") or debt renegotiation. According to Uribe and Schmitt-Grohé (2017), "haircut" is defined as the resulting loses inflicted to creditors from default on a fraction of the outstanding debt.

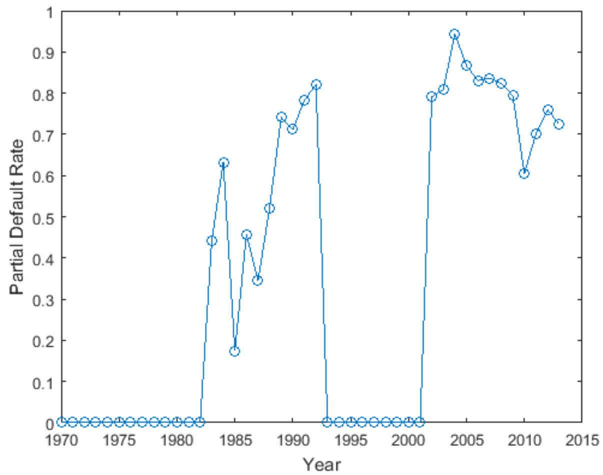

Figure 1: Partial Default Rate – Argentina

default rate (1970–2013) of other emerging countries defined by the IMF analysts in the Americas: partial default is common, and the partial default rate varies across countries. These findings motivate one of the main assumptions in this paper that sovereign default can be partial. Moreover, we allow it to be endogenously determined by the choice of the borrowing sovereign. The partial default rate conditional on the positive arrears for Argentina is one of our empirical targets for the calibration.

2. Countries are often able to borrow soon after default: Holding positive debt arrears does not cause an exclusion from the international capital market. Accordingly, this paper assumes that there is no exclusion from the international capital market after default. Arellano, Mateos-Planas, and Rios-Rull (2013) showed that countries continue to borrow during the periods with positive debt arrears: the new loans-to-output ratio conditional on positive debt arrears of the cross-country data (1970–2010) is 1.7 percent. This value is 2.3 percent for Argentina. Hatchondo, Martinez, and Sapriza (2009) suggested that according to the related empirical studies, if the qualities of policies and institutions are used as controls, the market access is not significantly influenced by the previous default decisions. The standard assumption of exclusion was also dropped in the long-term bonds analysis in Hatchondo and Martinez (2009). The authors stated that in the past three decades, the sovereign debt market has become more competitive and the increasing competition diminishes creditors' ability to cooperate to exclude default countries from capital markets.[5] Continuing to access to the international capital market is also a *puzzle* mentioned in Tomz and Wright (2010): foreign investments are always observed in practice even in countries that cannot commit on their debt obligations.

[5] Also see Wright (2005).

Table 1: External Sovereign Debt – Argentina (1970–2013)

Variables	Unconditional		Conditional on Positive Arrears		Conditional on Above Output Trend		Conditional on Below Output Trend	
	$\bar{x}$	σ_x	$\bar{x}$	σ_x	$\bar{x}$	σ_x	$\bar{x}$	σ_x
Positive Arrears Frequency	0.7045		1.0000		0.7500		0.6500	
Partial Default Rate	0.3435	0.3721	0.4876	0.3548	0.2707	0.3467	0.4310	0.3913
Debt Arrears/GDP	0.0387	0.0593	0.0550	0.0642	0.0214	0.0306	0.0595	0.0774
Debt Service/GDP	0.0252	0.0116	0.0270	0.0120	0.0249	0.0106	0.0256	0.0129
External Debt/GDP	0.2570	0.1906	0.3182	0.1917	0.2158	0.0978	0.3064	0.2568

Table 2: Key Statistics of the Partial Default Rate for Emerging Countries in Americas (1970–2013)

Country	*Partial Default Rate*				
	Mean	Std. Dev.	Positive Arrears Frequency	Mean Conditional on Positive Arrears	*External Sovereign Debt-to-GDP Ratio*
Argentina	0.3435	0.3721	0.7045	0.4876	0.2570
Brazil	0.1348	0.2049	1.0000	0.1348	0.1524
Colombia	0.0161	0.0347	0.5455	0.0295	0.2026
Ecuador	0.1605	0.2868	0.7955	0.2018	0.3783
Mexico	0.0015	0.0085	0.2045	0.0074	0.2127
Peru	0.2613	0.3934	0.7045	0.3709	0.3423
Venezuela, RB	0.0635	0.1258	0.7727	0.0821	0.2439

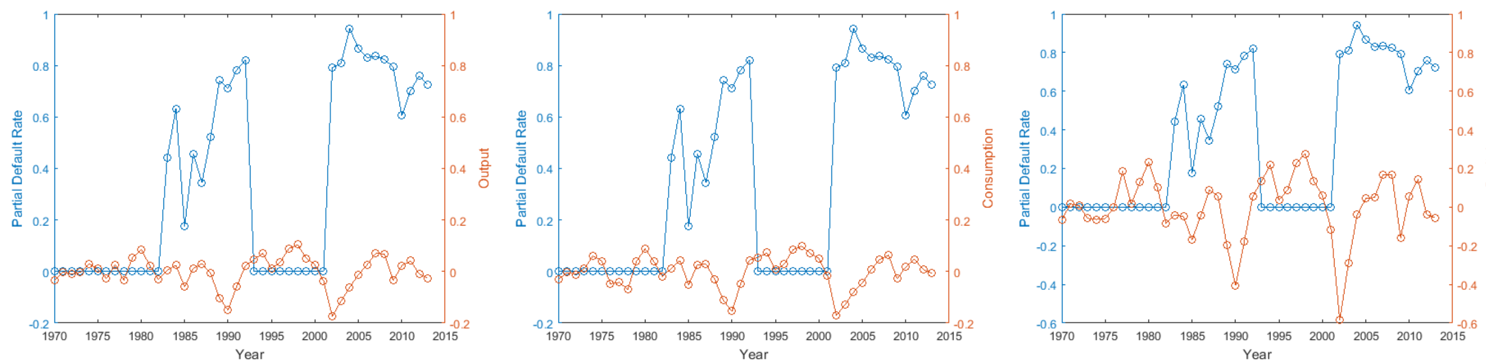

Figure 2: Partial Default Rate and Cyclical Component of Output, Consumption, and Capital

Table 3: External Sovereign Debt and Business Cycles – Argentina

Variables $corr\,(x,\,y)$	Partial Default Rate	External Debt/GDP	GDP	Consumption	Gross Capital Formation
Partial Default Rate	1.0000***				
External Debt/GDP	0.4866***	1.0000***			
GDP	-0.3810**	-0.5749***	1.0000***		
Consumption	-0.3900***	-0.5731***	0.9539***	1.0000***	
Gross Capital Formation	-0.3740**	-0.5550***	0.9388***	0.8263***	1.0000***

Notes: Intra-temporal correlations

* 10% significant level, ** 5% significant level, and *** 1% significant level

3. The partial default rate is counter-cyclical and it can be used as the proxy for the realized default risk: Figure 2 plots the partial default rate against the cyclical components of the real output, final consumption, and gross capital formation of Argentina, showing the counter-cyclical property of the partial default rate. The correlations of the partial default rate with the output, consumption, and gross capital formation are -0.3810, -0.3900, and -0.3740, respectively (as shown in Table 3).[6]

2.2 Literature Review

The partial default mechanism and the determinants of the default costs in this paper are related to the studies which have explored the partial default of debt and its corresponding penalties. The representative studies include Alfaro and Kanczuk (2009), Aguiar, Amador, Farhi, and Gopinath (2014), Arellano, Mateos-Planas, and Rios-Rull (2013), Dubey, Geanakoplos, and Shubik (2005), Easton and Rockerbie (1999), and Walsh (2016).[7] Easton and Rockerbie (1999) stressed the importance of incorporating default as a matter of degree in models of lending to less-developed countries. Alfaro and Kanczuk (2009) discussed the arguments in favor and against the short-term and long-term debt in the model with partial default via taxation and assumed that the cost of default is a higher future interest rate but not the exclusion from the capital market. Arellano, Mateos-Planas, and Rios-Rull (2013) showed that sovereign defaults are far from the binary events and built a partial default model with endowment, thinking of the partial default as a more expensive way to borrow. They also assumed non-exclusion from the international capital market. Aguiar, Amador, Farhi, and Gopinath (2014) studied partial sovereign default-by-inflation in the context of a monetary union. In a standard general equilibrium model, which allows partial default, Dubey, Geanakoplos, and Shubik (2005) note that because of the consideration of reputation, or because of the collateral guarantees, there exists at least partial payment. Their penalty of disutility is linear and separable in default as in our case, allowing the marginal rate of substitution between goods to depend on the level of default. Walsh (2016) analyzes the portfolio choice problems of a small open economy under the partial default mechanism. The author notes that partial default is a simple and tractable way to introduce the endogenous default and haircuts: partial default is important, because the degree of default, not just

[6] The correlations of the partial default rate with the lagged values of business cycle variables are also negative. The largest negative correlation coefficients are the coefficients of the lagged output, consumption, gross capital formation with the partial default rate, which indicates economy deteriorates first, then the partial default rate spikes up as the default risk is *realized*. These inter-temporal correlation coefficients are -0.4821, -0.5002, and -0.4524, respectively.

[7] Other studies concerning the default costs are Hatchondo and Martinez (2009), Hatchondo, Martinez, and Sapriza (2009), and Tomz and Wright (2010), which have been discussed in "Empirical Evidence".

whether it occurs, is of central economic significance. Following Dubey, Geanakoplos, and Shubik (2005), Walsh (2016) assumes a proportional and linear cost of default. In addition, the author assumes that the marginal default cost declines as the borrower's wealth goes up. As the result, there exists a minimum level of the consumption for a given level of the liquid net wealth.

Most existing studies related to the sovereign debt and default treat default as a full and binary event. The representative studies include Aguiar and Gopinath (2006), Arellano (2008), and Yue (2010), as well as the studies emphasizing the role of debt maturity: Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), and Chatterjee and Eyigungor (2012). These studies assumed the default causes a multiple-period exclusion from the international capital market as well as output loss. On the other hand, the empirical facts show that most emerging countries usually do not default on all of their debt and that financial autarky is counterfactual. Even when there is continued default on newly-maturing debt, there still exists the ongoing relationships between creditors and debtors. Moreover, while the theoretical models of full default imply a very tight relationship between the spread and the default frequency/probability, their numerical simulations fail to simultaneously match both dimensions of the data. This mismatch sometimes leads to spurious conclusions regarding the relation between the default frequency and the average country spreads and introduces a negative bias in the spread-default frequency differential.

The literature on sovereign default uses models with both endowment and production. Aguiar and Amador (2014) provided the benchmark full default models of sovereign debt in both an endowment economy and a production economy. Using the model with production, they explored the effect of "sovereign debt overhang": a country with a large external sovereign debt position has greater temptation to default and therefore cannot be trusted to leave large investment un-expropriated. Aguiar, Amador, and Gopinath (2009) explored the joint dynamics of the sovereign debt, investment, and expropriation risk of a small open economy model. They showed that the combination of the political economic risk and the risk of losing office generates the perpetual cycles in both the debt and the investment.[8] Maliar, Maliar, and Sebastian (2008) studied the effect of sovereign risk on the capital flows from rich to poor countries in a two-country model. Tomz and Wright (2010) studied the joint dynamics of the debt and foreign direct investment using a small open economy model. In their model, all of the financial capital (debt) is used for production. Compared with these studies, this paper assumes that the debt finance is used for the purpose of consumption smoothing and investment, and that the debt can be defaulted partially.

[8]The authors further showed that the long-run expected tax on capital varies with the state of the economy and that investment is distorted more in recessions, generating the persistent effects from i.i.d. shocks.

3 The Models

This section describes the two versions of the partial default models of a small open economy with infinite time periods, $t = 1, 2, \ldots$. The first one is an endowment model whereas the second one introduces production.

3.1 The Model with Endowment

3.1.1 The Small Open Economy

We treat a country as a small open economy which consists of households and the government. The benevolent government acts as a planner and maximizes the utility of the representative household.

The exogenous output/endowment of the economy is given by

$$Y = \theta \bar{Y}, \tag{1}$$

where $\bar{Y} > 0$ is a parameter and θ is the output shock which follows an exogenous $AR(1)$ process:

$$\ln \theta = \rho \ln \theta_{-1} + \varepsilon, \qquad \varepsilon \sim N(0, \sigma_\varepsilon^2). \tag{2}$$

ρ is the auto-correlation coefficient and ε is the error term that has a normal distribution with 0 mean and variance σ_ε^2.

The Government

The government borrows in the international market and lacks commitment. Therefore, it can default on its debt obligation. Let A be the debt obligation of the government at the beginning of a period and let D be the amount on which it chooses to default. Then, we have

$$0 \leq D \leq A \tag{3}$$

and the partial default rate for the period is D/A.

The government chooses the new debt level A' which is the payable in the next period. This debt consists of two components: (1) new borrowing by issuing of fresh debt B' at the price q; and (2) a preemptive recovery payment on the amount of partial default, D, on the current debt, A. Let $\tilde{R}$ $(0 < \tilde{R} < 1)$ be the exogenous preemptive recovery rate. Then the preemptive recovery payment is $\tilde{R}D$ and we have

$$A' = B' + \tilde{R}D. \tag{4}$$

Thus, after being rolled into the next period debt obligation, A', the new-issuance B' and the preemptive recovery payment $\tilde{R}D$ are indistinguishable. The default does not lead to the exclusion of the country from the international capital market but does result in a direct utility loss $G(D)$ to the household.

The bond price, q, depends on the gross amount of borrowing today (i.e., A') and today's realized output shock: $q(A'; \theta)$. The dependence on A' arises from the fact that higher borrowing increases the likelihood of default, which in turn reduces the bond price ($q_{A'} < 0$). q depends on today's output shock, θ, as it predicts tomorrow's output due to the $AR(1)$ nature of the endowment process and, hence, the ability to pay. In particular, higher θ implies higher bond prices when it is positively autocorrelated.

There is an exogenous borrowing constraint applied to limit the total debt:[9]

$$A' \leq \bar{A} \tag{5}$$

The Representative Household

The social planner solves the optimization problem of the representative household, which captures all economic decisions of the small open economy in the model. The household's life-time utility is the expected sum of their per-period utility

$$U(C) - G(D)$$

which depends on consumption, C, and default amount, D, and is discounted with a factor $\beta \in (0,\, 1)$.

The budget constraint is:

$$C = \theta\bar{Y} - (A - D) + q(A'; \theta)\left(A' - \tilde{R}D\right), \tag{6}$$

which states that the household's consumption is the residual of the endowment after the partial repayment $(A - D)$ on the outstanding debt (as it is subject to the partial default), augmented by the new-issuance, $B' = A' - \tilde{R}D$.

The state of the small open economy is characterized by $\{A; \theta\}$ and the household's decisions for tomorrow's debt, consumption, and default are functions of these state variables: $A'(A; \theta)$, $C(A; \theta)$, and $D(A; \theta)$. The new-issuance $B'(A; \theta)$ can be calculated from (4). In

[9] Usually, the upper bond is set as a fixed share of output to rule out the Ponzi schemes.

addition, the general equilibrium bond price function depends on $\{A'; \theta\}$, giving us $q(A', \theta)$ as noted before.

3.1.2 International Investors

There are many identical risk-neutral foreign investors, who are willing to absorb the risk from the random output shocks affecting the domestic economy. Foreign investors have two investment options: the sovereign debt with partial default, and the international risk-free assets with the constant interest rate r_f.

The absence of arbitrage for investors requires:

$$q(A'; \theta) = \frac{1}{1+r_f} E\left[\left(1 - \frac{D'}{A'}\right) + q(A''; \theta')\frac{\tilde{R}D'}{A'})|\theta\right]. \tag{7}$$

Note that whereas the left-hand side of the equation is the bond price, its right-hand side represents the present value of the expected future returns from investing in the sovereign debt discounted at the rate $(1 + r_f)$, the international risk-free (gross) interest rate. More specifically, on the right-hand side, the first term in the square bracket, $(1 - D'/A')$, is the net-of-default repayment on one unit of debt in the next period and the second term is the market value of the preemptive recovery on the amount defaulted for one unit of debt. When these two terms are discounted by the international interest rate, no-arbitrage requires their sum to be equal to the bond price.

The bond price function (7) is similar in structure to that for the long-term debt.[10] Note that, for the long-term debt, the current price q depends not only on the expected payments tomorrow, but also on the price of the (residual-maturity) bond, q', in the next period. A similar structure arises here due to the rolling over of the preemptive recovery default amount on the debt to the next period.

3.1.3 The Optimization Problem

The recursive formulation of the household's optimization problem is given by

$$V(A; \theta) = \max_{A', C, D} \{U(C) - G(D) + \beta E[V(A'; \theta')|\theta]\} \tag{8}$$

subject to (3) , (5), and (6), with B' given by (4).

Definition. A *recursive competitive equilibrium* for the small open economy is a process for

[10] See Equation (6) in Chatterjee and Eyigungor (2012).

the bond price $q(A'; \theta)$ and a set of decision rules: $A'(A; \theta)$, $C(A; \theta)$, $D(A; \theta)$, and $B'(A; \theta)$, such that:

(1) Given q, the decision rules for A', C, D, and B' solve the recursive problem (8), and

(2) The bond price q satisfies (7).

3.1.4 Solving the Model

In the following paragraphs, the optimization problem is solved and the equilibrium is analyzed. The trade-offs between the consumption and the borrowing and between the consumption and the default, conditional on the initial debt and realization of output, are discussed.

The *Bellman equation* of the model is:

$$V(A; \theta) = \max_{A', C, D} \{U(C) - G(D) + \beta E[V(A'; \theta')|\theta]\}$$

$$+\lambda_1 \left(\theta\bar{Y} - (A - D) + q(A'; \theta)(A' - \tilde{R}D) - C\right) + \lambda_2 \left(\bar{A} - A'\right) + \lambda_3 (D - 0) + \lambda_4 (A - D)$$

where λ_1, λ_2, λ_3, and λ_4 are the multipliers for (6), (5), and (3), respectively.

The *Kuhn-Tucker conditions* for the inequality constraints are:

$$\lambda_2 \left(\bar{A} - A'\right) = 0 \tag{9}$$

$$\lambda_3 (D - 0) = 0 \tag{10}$$

$$\lambda_4 (A - D) = 0 \tag{11}$$

The *Euler equations* for the total debt obligation A' and default D are:

$$U_C \left[q + q_{A'} \left(A' - \tilde{R}D\right)\right] - \lambda_2 = \beta E\left[(U_{C'} - \lambda_4')\,|\theta\right] \tag{12}$$

$$U_C \left[1 - q\tilde{R}\right] = G_D - \lambda_3 + \lambda_4 \tag{13}$$

For the purpose of understanding the intuition behind the Euler equations, it is useful to consider their simplified version, when the solutions for A' and D are interior. In that case, (12 – 13) simplify to:

$$U_C \left[q + q_{A'} \left(A' - \tilde{R}D\right)\right] = \beta E\left[U_{C'}|\theta\right] \tag{14}$$

$$U_C \left[1 - q\tilde{R}\right] = G_D \tag{15}$$

Equation (14) equates the marginal benefit of one unit of additional borrowing (which is on the left-hand side) with the marginal cost (on the right-hand side). The marginal cost is the discounted expected present value of the marginal utility of an additional unit of consumption in the next period, which is foregone due to borrowing. The marginal benefit is the marginal utility of the gross receipts from the marginal debt issuance, which is $q\left(A';\,\theta\right)$; partially offset by the reduced receipts, $q_{A'}\left(A' - \tilde{R}D\right)$, on infra-marginal debt.

Equation (15) describes the static trade-off between consumption and default. $\left(1 - q\tilde{R}\right)$ on the left-hand side is the net increase in consumption due to default on one unit of debt. Thus, in the equilibrium with the interior solution for default, the left-hand side is the marginal benefit, whereas the right-hand side is the marginal cost/disutility of default.

The full model consists of the new-issuance (4), the budget constraint (6), the bond price function (7), Kuhn-Tucker Conditions $(9-11)$, and Euler equations (12) and (13), which can be solved for the endogenous variables: A', B', C, q, D, λ_2, λ_3, and λ_4. After including the $AR\left(1\right)$ process (2) of the output shock θ, the system with endowment consists of 9 equations in 9 variables.

3.2 The Model with Production

3.2.1 The Small Open Economy

In this subsection, we consider the small open economy with production. The output is given by

$$Y = \theta K^{\alpha}, \tag{16}$$

where α is the share of the capital, K, which depreciates at the rate $\delta\,(0 < \delta < 1)$. Labor is supplied inelastically so we suppress it for brevity. θ now is denoted as the productivity shock and follows an exogenous $AR\left(1\right)$ process:

$$\ln\theta = \rho\ln\theta_{-1} + \varepsilon, \qquad \varepsilon \sim N(0, \sigma_{\varepsilon}^2) \tag{17}$$

where ρ is the auto-correlation coefficient and ε is the error term that follows a normal distribution with 0 mean and variance σ_{ε}^2.[11]

[11] Instead of using the Markov chain with a very large number of points on productivity, to accurately capture the mean and standard deviation of the spread generated by the model, we obtain similar accuracy with a *AR(1)* process with more modest sized productivity and coupled with the off-grid interpolation and the approximation of expectation over the productivity (θ) with Gauss-Hermite quadrature. See Section 4 for more details.

The Government

The description of the government remains unchanged. It begins the period with the debt obligation A and chooses the new debt level A' which is the payable in the next period. The equations (3 – 5) continue to apply to the production model. As before, the default does not lead to the exclusion of the country from the international capital market, but does result in a direct utility loss $G(D)$ to the household.

Besides depending on the gross amount of borrowing today, A', and today's realized output shock, θ, the bond price, q, now also depends on the capital accumulated today, K': $q(K', A'; \theta)$. q depends on K' because a higher stock of future capital implies higher output, thereby giving more confidence to the international investors by affecting the likelihood of default and, hence, the cost of borrowing ($q_{K'} > 0$).

The Representative Household

As in the model with endowment, the social planner solves the optimization problem of the representative household, where the budget constraint for this production model is:

$$C + K' - (1-\delta)K = \theta K^{\alpha} - (A - D) + q(K', A'; \theta)\left(A' - \tilde{R}D\right) - \frac{\phi}{2}\left(\frac{K'}{K} - 1\right)^2 K, \quad (18)$$

which states that the household's consumption and investment ($I = K' - (1-\delta)K$) is the residual of the output after the partial repayment ($A - D$) on the outstanding debt, augmented by the new-issuance, $B' = A' - \tilde{R}D$, and a new term capturing the capital adjustment costs. We assume a standard quadratic capital adjustment cost function to calibrate the volatility of investment later in Subsection 4.1.

The state of the small open economy with production is characterized by $\{K, A; \theta\}$, and the household's decisions for tomorrow's capital and debt, consumption, and default are functions of these state variables: $K'(K, A; \theta)$, $A'(K, A; \theta)$, $C(K, A; \theta)$, and $D(K, A; \theta)$. The new-issuance $B'(K, A; \theta)$ is determined by (4).

3.2.2 International Investors

As in the model with endowment, the absence of arbitrage for investors requires:

$$q(K', A'; \theta) = \frac{1}{1+r_f} E\left[\left(1 - \frac{D'}{A'}\right) + q(K'', A''; \theta')\tilde{R}\frac{D'}{A'}|\theta\right] \quad (19)$$

and the bond price function (19) is again similar in the structure to that for the long-term debt.

3.2.3 The Optimization Problem

The recursive problem for the production model is:

$$V\left(K,\, A;\, \theta\right) = \max_{K',\, A',\, C,\, D} \left\{U\left(C\right) - G\left(D\right) + \beta E\left[V\left(K',\, A';\, \theta\right)|\theta\right]\right\} \tag{20}$$

subject to (3), (5), and (18), with B' given by (4).

Definition. A *recursive competitive equilibrium* for the small open economy with production is a process for the bond price $q\left(K',\, A';\, \theta\right)$ and a set of decision rules: $K'\left(K,\, A;\, \theta\right)$, $A'\left(K,\, A;\, \theta\right)$, $C\left(K,\, A;\, \theta\right)$, $D\left(K,\, A;\, \theta\right)$, and $B'\left(K,\, A;\, \theta\right)$ such that:

(1) Given q, the decision rules of K', A', C, D, B' solve the recursive problem (20), and

(2) The bond price q satisfies (19).

3.2.4 Solving the Model

We follow the outline for the endowment model here and, in the following paragraphs, solve the optimization problem and analyze the equilibrium. Also, the trade-offs between the consumption and investment, the consumption and borrowing, and the consumption and default, conditional on the capital, debt, and realization of productivity are discussed.

The *Bellman equation* in this case is:

$$V\left(K,\, A;\, \theta\right) = \max_{K',\, A',\, C,\, D} \left\{U\left(C\right) - G\left(D\right) + \beta E\left[V\left(K',\, A';\, \theta\right)|\theta\right]\right\}$$

$$+\lambda_1 \left(\theta K^{\alpha} - \left(A - D\right) + q\left(K',\, A';\, \theta\right)\left(A' - \tilde{R}D\right) - \frac{\phi}{2}\left(\frac{K'}{K} - 1\right)^2 K - K' + \left(1-\delta\right)K - C\right)$$

$$+\lambda_2\left(\bar{A} - A'\right) + \lambda_3\left(D - 0\right) + \lambda_4\left(A - D\right)$$

where λ_1, λ_2, λ_3, and λ_4 are the multipliers of (18), (5), and (3).

The *Kuhn-Tucker conditions* for the inequality constraints remain unchanged as in $(9-11)$. So, they are skipped.

The *Euler equations* for capital K', debt A', and default D are:

$$U_C\left[1 + \phi\left(\frac{K'}{K} - 1\right) - q_{K'}\left(A' - \tilde{R}D\right)\right] = \beta E\left\{U_{C'}\left[\alpha\theta' K'^{\alpha-1} + \left(1-\delta\right) + \frac{\phi}{2}\left(\frac{K''}{K'} - 1\right)\left(\frac{K''}{K'} + 1\right)\right]|\theta\right\} \tag{21}$$

$$U_C\left[q + q_{A'}\left(A' - \tilde{R}D\right)\right] - \lambda_2 = \beta E\left[\left(U_{C'} - \lambda_4'\right)|\theta\right] \tag{22}$$

$$U_C\left[1-q\tilde{R}\right]=G_D-\lambda_3+\lambda_4 \tag{23}$$

where $q_{K'}$ and $q_{A'}$ represent the marginal effects of the capital and the debt on the bond price, respectively.

We begin by noting that equations $(22-23)$ are unchanged from the endowment model and so does their interpretation. There is now a new Euler equation for K' in (21). The left-hand side of (21) is the marginal cost of investing in capital. It consists of three components: the cost of purchasing capital, the adjustment cost associated with changing the capital stock, and finally saving on borrowing due to increase in bond price ($q_{K'}>0$) due to reduced risk of default. The right-hand side of (21), the marginal benefit, which also consists of three components: the marginal product of capital, the undepreciated stock left, and finally the last term capturing the reduction in future adjustment costs.

The full model with production consists of the new-issuance (4), the budget constraint (18), the bond price function (19), the Kuhn-Tucker conditions $(9-11)$, and the Euler equations $(21-23)$, which can be solved for the endogenous variables: A', B', K', q, C, D, λ_2, λ_3, and λ_4. After including the $AR(1)$ process (17) of the productivity shock θ, the system consists of 10 equations in 10 variables.

4 Calibration and Numerical Solution of the Model

The benchmark models with endowment and with production outlined in the previous section are too complex to be solved analytically. Therefore, one needs to turn to solving them numerically. This section deals with this task. The first step for finding the numerical solution is to choose the appropriate analytical forms for various functions and to assign suitable values to various parameters. This first step, called calibration, is the object of the next subsection. The following subsection provides the details of the computational algorithm used to solve the suitably-parameterized and calibrated models.

4.1 Functional Forms and the Choice of Parameters

We consider a period to be a year, thus, making the model have an annual frequency. The functional forms of the preferences for both the model with endowment and the model with production are:

$$U(C)=\frac{C^{1-\sigma}}{1-\sigma} \tag{24}$$

Table 4: Parameters Selected Directly

	Value	Source or Basis
Risk aversion, σ	2	Literature survey
Auto-correlation, ρ	.86759	CE (2012): .948503 (Quarterly)
S.D. of auto-correlation error, σ_ε	.0413	CE (2012): .027092 (Quarterly)
Output scalar, $\bar{Y}$	10	Arellano (2008)
Risk-free interest rate, r_f	.0406	CE (2012): .01 (Quarterly)
Output adj. parameter, γ	0	-
Borrowing limit, $\bar{A}$	8.1	Prevent Ponzi schemes without binding
Capital share, α	.33	RBC literature survey
Capital depreciation rate, δ	.08	RBC literature survey

$$G(D) = \begin{cases} \tilde{\kappa}_1 D & D > 0 \\ 0 & D = 0 \end{cases} \tag{25}$$

$$\tilde{\kappa}_1 = \kappa_1 \theta^{\gamma} \tag{26}$$

$$\tilde{R} = \bar{R}\theta^{\gamma_2} \tag{27}$$

The utility function is the standard form in the macroeconomic literature, being CRRA in consumption. The flexible specification for $G(D)$ allows the disutility function to vary depending on both the default and the exogenous shocks. With $\kappa_1 > 0$ and non-zero γ, the utility cost rises monotonically with the default. The marginal disutility of default depends on the exogenous process of productivity. The preemptive recovery rate, $\tilde{R}$, also varies with realization of the productivity process when γ_2 is non-zero.

Given these functional forms and their parametric assumptions, the numerical specification of the model with endowment requires giving values to 11 parameters. They are: two preference parameters β and σ, two parameters of the $AR(1)$ process ρ and σ_ε and the steady state output/endowment $\bar{Y}$, two disutility parameters κ_1 and γ, two parameters describing the preemptive recovery $\bar{R}$ and γ_2, the risk free interest rate r_f, and the borrowing limit $\bar{A}$. For the model with production, besides the above 11 parameters, numerical values are also needed for the three production-related parameters: α, δ, and ϕ.

The parameters directly-selected are listed in Table 4. The coefficient of the relative risk aversion, σ, is set to 2, which is a standard value used in the studies of macroeconomic fluctuations. The parameters of the $AR(1)$ process, ρ and σ_ε, are estimated from the stochastic process for the productivity in Chatterjee and Eyigungor (2012). We annualize the simulated quarterly process of Chatterjee and Eyigungor (2012) and de-trended it using the HP Filter with the annual smoothing parameter of 100. The estimated values of ρ and σ_ε are

0.86759 and 0.0413, respectively. The risk free rate r_f is set to 0.0406 by annualizing the risk free interest rate in Chatterjee and Eyigungor (2012).[12] At the current stage, we do not investigate the role of the marginal disutility parameter γ, leaving it equal 0. The borrowing upper bound, $\bar{A}$, is set to be 81 percent of the normalized output/endowment, which is a reasonable value for the indebtedness of a small open emerging economy.[13] The steady state value of the output/endowment is normalized to 10, as in Arellano (2008).

The remaining 4 parameters for the endowment model, the discount factor β, the disutility parameter κ_1, the preemptive recovery rate $\bar{R}$, and the output adjustment parameter γ_2 are calibrated to match the following moments: the mean spread 0.0815, the debt-to-output ratio 0.25, the average partial default rate conditional on positive arrears 0.488, the default frequency 0.125. Table 5 summaries the value of parameters and the targets for the model with endowment.

For the model with production, we set α and δ to be 0.33 and 0.08, respectively, which are widely-accepted values in the real business cycle literature.[14] The remaining 5 parameters for the production model, the discount factor β, the disutility parameter κ_1, the preemptive recovery rate $\bar{R}$, the output adjust parameter γ_2, and the capital adjustment cost parameter ϕ are calibrated to match the following moments: the mean spread 0.0815, the debt-to-output ratio 0.25, the average partial default rate conditional on positive arrears 0.488, the default frequency 0.125, and the relative standard deviation of investment 3.79.[15] Table 6 summaries those values for the model with production.[16]

[12] Based on the data from FRED, the constant maturity rate of 1-year U.S. Treasury Bill is 0.0317 for the period 1994–2013 and is 0.0572 for the period 1970–2013.

[13] In Table 2, the maximum annual average external debt-to-GDP ratio among the emerging countries in the Americas is 0.38.

[14] Aguiar, Amador, and Gopinath (2009) set the annual capital share $\alpha = \frac{1}{3}$ for the numerical analysis of Argentina. In Alburqueque (2003), the annual capital share was set to 0.4, which coincides with the estimation of the capital share for the U.S. economy and several developing countries. Usually, the capital share lies between 0.3 and 0.4, and its value is higher for developing countries. More closed a country is, lower its capital share is.

The capital depreciation-to-GDP share for Argentina in 1990s, which was estimated by Coremberg (2004) with the Hedonic Valuation method, had an average value of 0.104. In Berlamann and Wesselhoft (2014), the time-varying capital depreciation schemes used to estimate the capital stock of 103 countries were borrowed from the U.S. economy over the period 1950–2011, from 0.055 to 0.078 for various non-residential assets. The non-residential assets depreciation rate for the U.S. economy during 1961–2001 in Kamps (2006) is 0.085. In Barro and Sala-i-Martin (2003), the depreciation rate for the overall of structures and equipments is around 0.050 per year.

[15] See Uribe and Schmitt-Grohé (2017), Table 1.3.

[16] According to Chatterjee and Eyigungor (2012), in Argentina, around 30 percent of debt defaulted in 2001 was repaid, and eventually only 70 percent of the debt was the truly unsecured portion. In Yue (2010), the debt recovery rate was 0.27. The value used in Arellano, Mateos-Planas, and Rios-Rull (2013), and Bi (2008) is 0.3. We set the parameter $\bar{R}$ controlling the preemptive debt recovery rate to 0.2, which along with the output adjustment as in Equation (27) leads to the preemptive recovery rate in a reasonable range.

Table 5: The Model with Endowment: Parameters Selected by Matching Data Moments

	Value	Statistics
Discount factor, β	.85547	CE (2012): mean spread, .0815 (Annualized)
Disutility cost, κ_1	.011055	CE (2012): debt-to-output ratio, .25 (Annualized)
Preemptive recovery, $\bar{R}$	.2	CE (2012): default frequency, .125 (Annualized)
Output adj. parameter 2, γ_2	-1.2	Data: partial default rate (cond.) .488

Table 6: The Model with Production: Parameters Selected by Matching Data Moments

	Value	Statistics
Discount factor, β	.855	CE (2012): mean spread, .0815 (Annualized)
Disutility cost, κ_1	.01338	CE (2012): debt-to-output ratio, .25 (Annualized)
Preemptive recovery, $\bar{R}$	.2	CE (2012): default frequency, .125 (Annualized)
Output adj. parameter 2, γ_2	-2.4	Data: partial default rate (cond.) .488
Capital adj. cost, ϕ	5	Uribe and Schmitt-Grohé (2017): $\sigma(i)/\sigma(y)$, 3.79

4.2 Details of the Computational Algorithm

Since the policy functions describing the models' dynamics have kinks, we solve the model using value function iteration on a fine discrete grid, but allowing the off-grid interpolation of the policy functions and the value function.

While all our state variables, K, A, and θ, take values over a continuum, we need to discretize their respective ranges for the purposes of solving the model using the value function iteration. For A, the grid is set to consist of 292 points in both the endowment and production models. For the production model, the K grid has 161 points. Finally, for θ, for both our models, we use a 17-state grid that covers values over the range $[-4\sigma_\varepsilon, +4\sigma_\varepsilon]$ as generated by Tauchen's (1986) algorithm.

In both models, the bond price functions have a potentially large discontinuity at a zero-debt level ($A' = 0$) when the productivity level (θ) is sufficiently low. The reason is that for zero-debt the bond price is the risk-free price ($1/(1 + r_f)$), whereas when the productivity is below a threshold (which would depend on the past history) the country may want to default for any positive level of debt, A', thus causing the bond price to suddenly transition from the risk-free price to a much lower price for any small positive value of A'.

This discontinuity poses a problem for value function iteration method, when it is combined with off-grid interpolation: the bond price for a low value of borrowing (A') may be affected by this discontinuity due to interpolation. To solve this problem, we add a point on the bond grid very close to zero ($A' = 10^{-3}$). Moreover, as the interpolation methods other than the linear interpolation use values not just from adjacent points, but other nearby points as well, we add additional grid points for the low values of debt in order to more accurately estimate the bond price in that region (resulting in an *unevenly-distributed* discrete state space).

As θ follows an $AR(1)$ process and, hence, takes values on a continuum, to compute the expectations in the value function and bond price function (in (8) and (7), or (20) and (19)) we use the Gauss-Hermite quadrature with 11 nodes for a highly accurate approximation.

The computational algorithm iterates over two unknown functions: the value function (given the bond price function) and the bond price function, with various policy functions as by-products. We use what in the literature is called *one-loop algorithm*, which decreases computation time significantly. Further gains in computation time are achieved by using precomputation (Maliar, Maliar, and Judd, 2011) and generating functional approximation of the expectation of the value function and the bond price function. The usual nested-loop approach solves for value function in the inner loop until convergence for the current bond price function and then updates the bond price function, which is iterated over in the outer loop. The one-loop algorithm instead starts with the current bond price function and iterates over the value function. Then it uses the policy functions associated with the current iteration of the value function to update the bond price function in the same loop. Hence, the name one-loop algorithm. We continue the iteration to convergence in not just the bond price function, but also in the value function and the policy functions for the state variables, using a very tight convergence criterion. Table 7 summarizes the grid specification and the convergence criteria applied in this paper.

5 Results

This section analyzes the properties of the models in terms of their ability to match the moments of data that have been emphasized in the literature. We also evaluate the models' performance *vis-à-vis* some additional moments that our models can match given the partial nature of default, which the standard full default models in the literature cannot do due to the nature of the assumptions they make. More significantly, as our models do not have the exclusion from international market pursuant to default, we can generate meaningful impulses responses to shocks that are large enough to cause default. These impulse responses provide further insight into the mechanism that operates in the models. We discuss the results of the endowment model in Subsection 5.1 and those of the model with production in Subsection 5.2.

Table 7: Grid Specification and Convergence Criteria

	Baseline Model (Endowment)	Baseline Model (Production)	Chatterjee and Eyigungor (2012)	Arellano (2008)
Grid specification				
# of grid points for A	292	292	350	200
minimum A	0	0	0	-1.5
maximum A	8.1	8.1		3.3
# of grid points for K	N/A	161	N/A	N/A
minimum K	N/A	8	N/A	N/A
maximum K	N/A	24	N/A	N/A
# of grid points for θ	17	17	200	21
minimum θ	-4σ_ε	-4σ_ε		-3σ_ε
maximum θ	4σ_ε	4σ_ε		3σ_ε
# of GH Nodes for θ	11	11	N/A	N/A
Convergence criterion	1e-5 for 3 consecutive iterations (bond price, value, and policy functions)	1e-5 for 3 consecutive iterations (bond price, value, and policy functions)	1e-5 for 2 consecutive iterations (bond price)	1e-7 (bond price)

5.1 The Endowment Economy

We begin our discussion of the endowment economy with the analysis of the properties of the policy functions, which is followed by looking at the simulated paths of important macro variables, with the discussion of the impulse responses at the end.

5.1.1 Policy Functions

Before turning to the policy functions proper, we make some remarks about the value function shown in Figure 3, which is a function of the current productivity (θ) and the current level of indebtedness (A). As expected, the value function is increasing in θ as a higher value of productivity implies higher welfare. On the other hand, intuitively, greater indebtedness translates into lower welfare. In terms of the curvature, the value is concave in the level of productivity, a property it inherits from the utility function due to the diminishing marginal utility of consumption.

However, as the world risk-free rate is (constant and) exogenous to the economy, the trade-off *vis-à-vis* borrowing does not show similar diminishing returns (at least no as sharp) and, therefore, value function is very linear in A. This has the important implications for solving the model, which our computation approach takes into account. In particular, it makes the policy function for borrowing very sensitive to the bond price. Therefore, we not

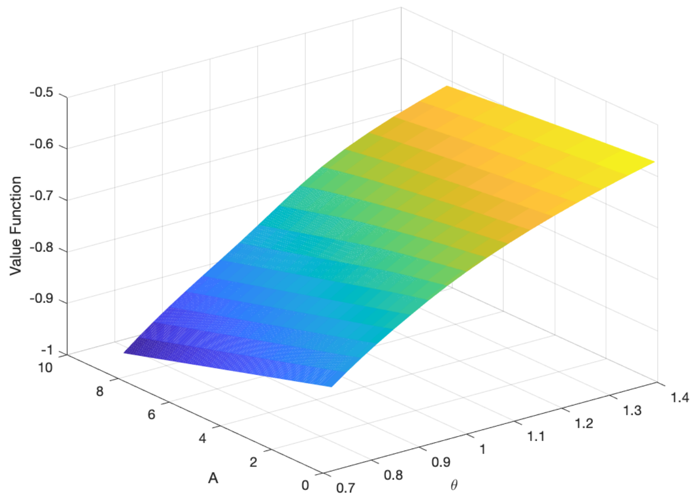

Figure 3: Value Function (Endowment)

only look for the convergence in bond price and value functions, but, in fact, in all policy functions (including the bond policy function) as well, as mentioned in the previous section.

Figure 4 plots the policy functions for bond price, default, debt obligation, new-issuance, consumption, and net export. Recall, while all other policy functions are functions, like the value function, of the current productivity (θ) and the current level of indebtedness (A), the bond price function is, instead, determined by the current productivity (θ) and the newly chosen level of indebtedness (A').

We begin with the bond price function in Figure 4 which shows that when times are good (high θ), there is no default risk and bonds issued by the sovereign command the risk-free price. However, as the current productivity level falls to the middle levels, future default becomes more probable (as θ follows an $AR(1)$ process). Therefore, the bond price starts taking a hit and the level of borrowing starts to matter for pricing of bonds; higher borrowing implies a greater decrease in bond price for a current level of productivity. For the low levels of productivity, the bond price collapses completely to zero or very low levels.

A similar argument applies to the variation in the default decision with productivity and indebtedness in Figure 4. Good times generate no default; normal times make default decisions dependent on current indebtedness (A) and the default behaves with obvious partial nature; and there is full default (or nearly full default, closing in on the upper bound of debt) in really bad times. In fact, it is useful to explore the congruence in the default decision and bond prices (high default today goes with low bond price today) in Figure 4 a bit further. Note that this happens despite the fact that whereas the bond price is a function of A', the default decision depends on A. The reason is that due to the $AR(1)$ process of productivity, the current θ determines both the current incentive to default as well as, by determining the future productivity, the future likelihood of default and, hence, the current bond price.

The policy functions for the gross borrowing (A') and the new-issuance (B') in the middle row of Figure 4 are fairly intuitive as well. The gross borrowing and new-issuance increase with the current debt obligation (A). There is no new debt issued when times are really bad (low θ) which are also the times when there is full default. In those times, the only implicit borrowing is the preemptive amount of default that is rolled over (recall, $A' = B' + \tilde{R}D$). The debt obligation at the middle levels productivity comprises the positive new-issuance and the preemptive recovery payment of default. Thus, the small open economy continues to borrow when it has debt arrears. For the high levels of θ, the debt obligation is only the new-issuance because of no default. A' and B' are also generally increasing in θ mainly driven by the impatience of the agents. However, when current borrowing is very low, there is a very slight non-monotonicity in both A' and B' in θ; when times are really good and there is no debt to begin with, even with being impatient, the country does not find it optimal to keep increasing the borrowing with θ.

The last row of Figure 4 shows the policy functions for consumption and net export. For the low levels of θ, the net export is zero or nearly zero and the economy is in autarky in trade. Thus, the consumption equals endowment and is not a function of A. Note that as these are also times when the new-issuance is zero and there is full default, there is also financial autarky (no capital inflows or outflows), although the country keeps accumulated arrears in the form of preemptive recovery payments. For the middle/normal levels of θ, there is a trade deficit for low levels of indebtedness and trade surplus occurs when the level of indebtedness is high. This is also true for the trade balance given high shocks. In effect, the country services debt by exporting when it is already indebted and imports to consume more (due to the impatience) when it does not have as much debt currently. The behavior of net export is non-monotonic in θ and is driven by a similar logic. For a given A, as θ increases, first the country pays by exporting more. However, as θ increases beyond a certain value, the country is rich enough that international markets are willing to lend more (or give higher prices for bonds) and it is able to afford higher consumption. These two forces together cause the net export to fall and ultimately turn into the net import. Interestingly, while the consumption is strictly decreasing in A as expected for high θ, it is constant for middle values of θ when the default is partial. This is driven by the Euler equation (15) for the case when default is interior/partial and the linearity of $G(D)$ in D: together they imply consumption is independent of the current level of indebtedness.

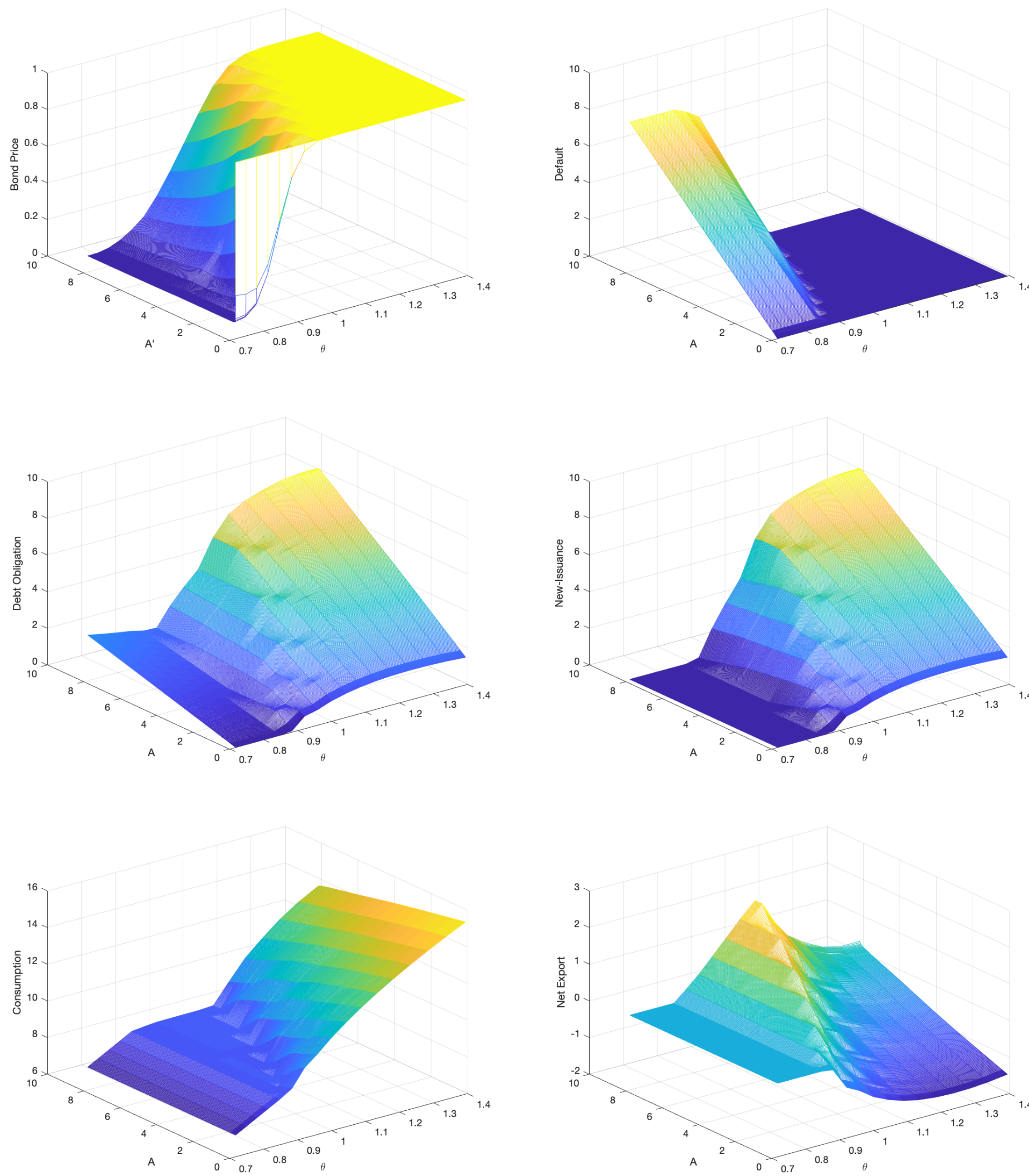

Figure 4: Policy Functions (Endowment)

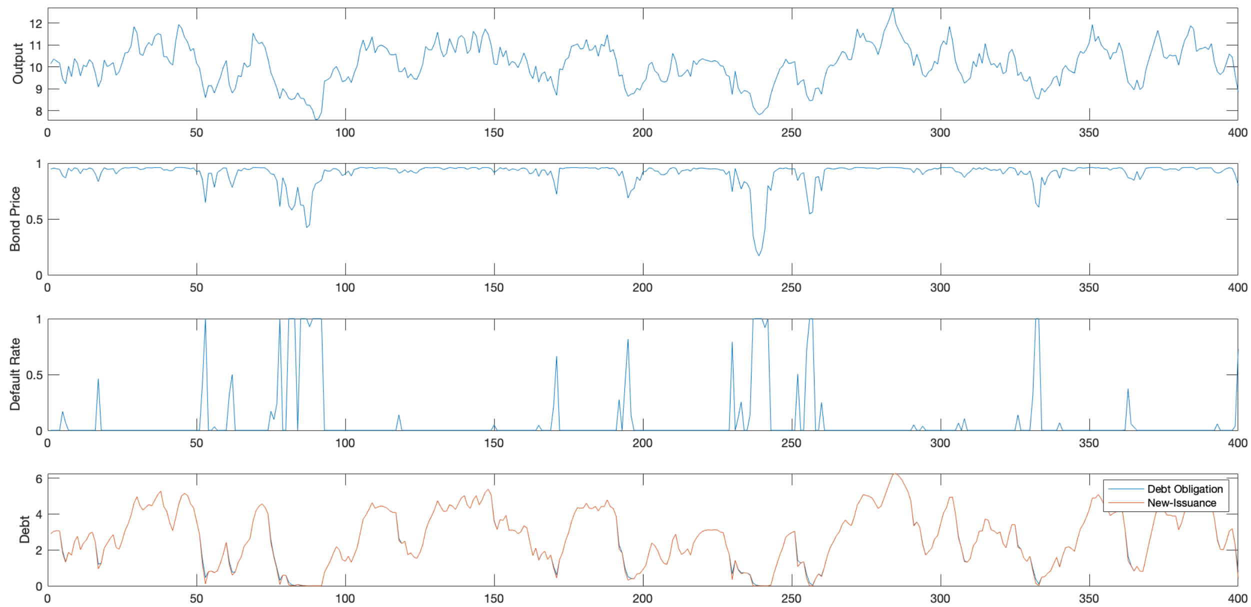

Figure 5: Simulated Results – 4 Century Sample (Endowment)

5.1.2 Findings

Figure 5 plots the time series of the simulated output/endowment, bond price, default rate, and debt (total and new-issuance) for a randomly-selected 4 centuries. When the low output results in a default episode with positive default rate, it is accompanied by a decreasing bond price, i.e., increasing country spread. The crash in bond price endogenously forces the country into financial autarky. Note that the default is frequently partial, consistent with the empirical facts. So, is the fact that, even during periods of default, the country continues to issue new debt.

Table 8 reports how the partial default model with endowment does in terms of matching the data moments that have been focus of interest in the literature *vis-à-vis* other models of sovereign debt and default. The second column lists the data for Argentina to which the model was calibrated. All data values, except the partial default rate, are from Chatterjee and Eyigungor (2012) and Arellano (2008). Column 3 to Column 5 report, for purposes of comparison, the corresponding simulated results of Arellano (2008) baseline model, and Chatterjee and Eyigungor (2012) short-term debt model and long-term debt model, respectively.

Arellano's (2008) baseline model of the short-term debt approximately matches the probability of default,[17] the volatility of the trade balance, and the debt service-to-GDP ratio,

[17] See Footnote 21 in Arellano (2008).

Table 8: Main Findings (Endowment)

	CE Data (Arellano Data)	Arellano Baseline	CE-ST	CE-LT	Partial Default Baseline
Mean Spread	.0815	.0358	**.0815**	**.0815**	**.0830**
S.D. of Spread	.0443	.0636	**.0443**	**.0443**	.2982
Debt-to-Output	1	.0595	**.7**	**.7**	**.994**
$\sigma(c)/\sigma(y)$	1.09	1.1	1.59	1.11	1.79
$\sigma(nx/y)/\sigma(y)$	.17	**.26**	1.06	.2	1.11
$corr(c, y)$	.98	.97	.73	.99	.9
$corr(nx/y, y)$	-.88 (-.64)	-.25	-.16	-.44	-.57
$corr(r - r_f, y)$	-.79 (-.88)	-.29	-.55	-.65	-.32
Debt Service-to-Output	.053	**.056**	.699	.055	.242
Default Frequency	.125 (.03)	**.03**	.073	.068	**.126**
Default Rate (Cond.)	.488	1	1	1	**.502**

which are shown in bold. However, the simulated results cannot predict well the mean spread and the high debt-to-output ratio. Chatterjee and Eyigungor (2012) change the data moments targeted to the mean spread, the standard deviation of spread, and the debt-to-output ratio for their models of the short-term and long-term debt. Although they perfectly match the mean spread and the standard deviation of the spread, their simulated results cannot fully explain the debt-to-output ratio. Both the short-term and long-term debt models fail to generate the empirical default frequency.

The last column of Table 8 reports the simulated results of our partial default model with endowment. The model can simultaneously match the mean spread, the debt-to-output ratio, and the default frequency. Given the default is endogenously-determined, the simulated results closely predict the partial default rate, which the full default models are not able to predict.

Table 8 also reports some other key statistics of the cyclical components of macroeconomic variables. The partial default model with endowment matches well the positive correlation between consumption and output, the counter-cyclical trade balance and country spreads. Compared with the full default model of the short-term debt in Chatterjee and Eyigungor (2012), the partial nature of default in our (short-term debt) model introduces the features in the bond price schedule similar to those present in the bond price schedule of the long-term debt model. This allows us to generate a better prediction for debt service without targeting.

The partial default model with endowment does over-predict the volatilities of consumption (trade balance) and interest spreads. To improve the model's fit to these moments we add production to the model. In a model with production, adjustment in investment pro-

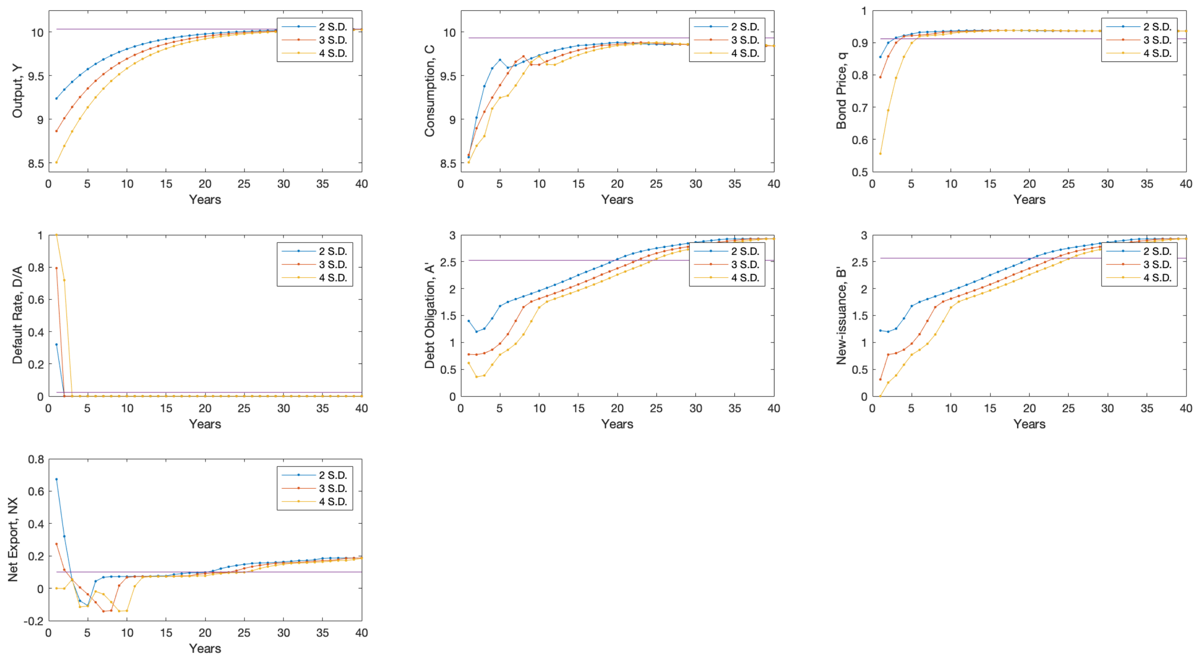

Figure 6: One-Period Shock, with Initial Debt at the 50th Percentile of the Distribution of the Simulated Data

vides another margin for a country to smooth consumption, which should bring the currently high value of the relative standard deviation of consumption (and net exports) more in line with the data. By leading to less reliance on international borrowing and net exports, it may also help with reducing the high standard deviation of spreads.

5.1.3 Impulse Response Functions

As noted earlier, our model allows us to investigate the impulse response functions of various macroeconomic variables, as in our partial default framework, we do not assume the exclusion from the international capital market after default. We use these impulse responses to understand the operational mechanism in the model.

Given an initial debt level of $A = 2.5263$ at the 50th percentile (median) of the distribution of the simulated data sample, Figure 6 plots the impulse response functions of the output, consumption, and bond price in the first row, the impulse response functions of the debt obligation, default rate, and new-issuance in the second row, and the net export in the third row, under one-time two to four standard deviations of the exogenous shock at the beginning of the year 1, respectively.

The responses of a two standard deviations (2 S.D.) shock are shown in blue. The output goes down to 9.24, decreasing by approximately 7.6 percent. The consumption decreases sharply by 14.4 percent to 8.56 and is, therefore, more volatile than output. On the debt

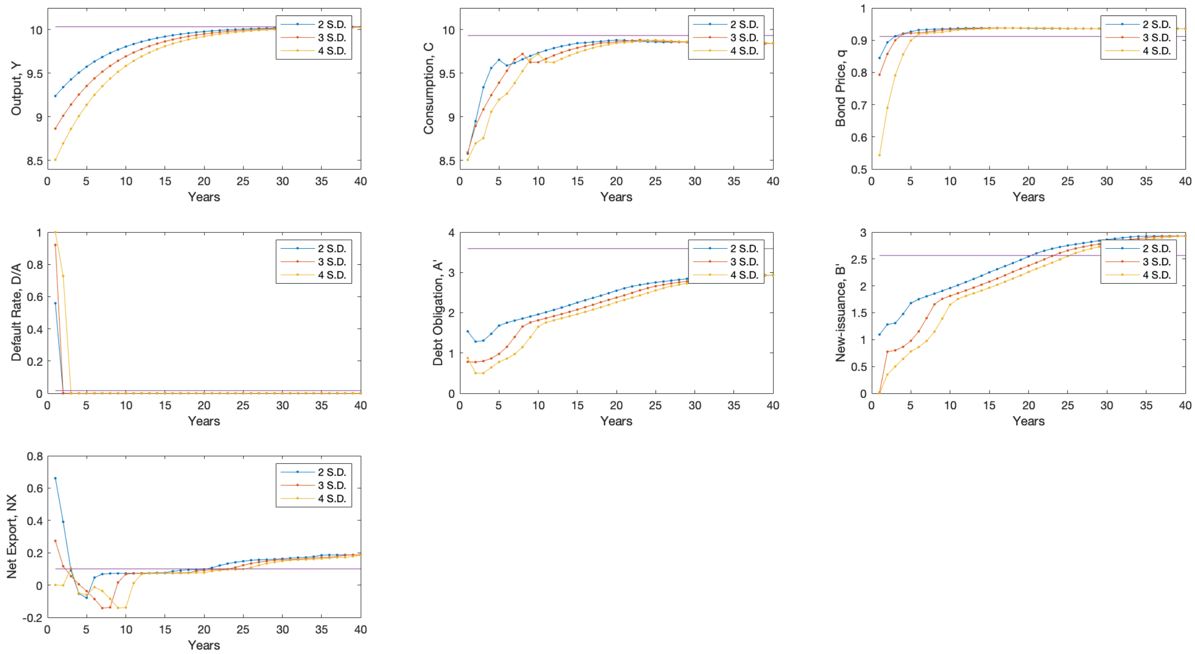

Figure 7: One-Period Shock, with Initial Debt at the 75th Percentile of the Distribution of the Simulated Data

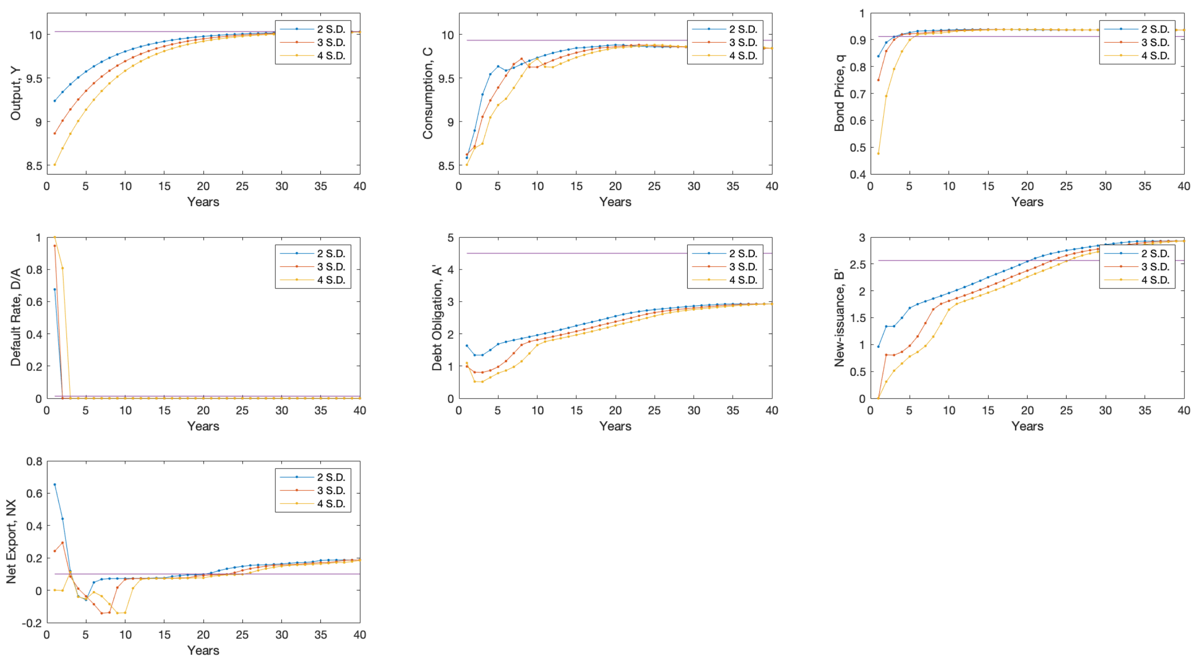

Figure 8: One-Period Shock, with Initial Debt at the 90th Percentile of the Distribution of the Simulated Data

market, there is one-period partial default with the default rate equal to 0.32. The bond price goes down for one period to 0.86, opening up a spread of about 1,100 basis points relative to the risk-free world interest rate. Yet, the country issues new debt amounting to about 1.22 because it allows the country to service part of the debt (not defaulted on). The jump in trade surplus to approximately 0.67 is another way the country services its debt. The economy starts to recover at the year 2. It takes about 40 years to regain a new steady state, where there is less consumption, more trade surplus, no default, and higher debt stock with the bond price at the risk-free level.

The responses of a three standard deviations (3 S.D.) shock are shown in red in Figure 6. In this case, the output goes down further to 8.9, decreasing by approximately 11 percent. The consumption decreases with the similar magnitude (14.1 percent) as in the 2 S.D. case, to 8.59 and is still more volatile than output. On the debt market, there is one-period partial default but with a higher default rate of 0.79 approximately. In this case, therefore, the bond price goes down for one period to 0.79, with a higher spread of about 1,700 basis points to reflect an even higher default risk. There is still issuance of new debt, but it is more modest at about 0.31. Again, the proceeds allow the country to service the debt not defaulted on. There is also an expansion of the trade surplus, albeit by a smaller amount, to approximately 0.27, which further assists in servicing the non-defaulted debt. Once again, the economy starts to recover at the year 2 and it takes about 40 years to regain a new steady state, where there are less consumption, more trade surplus, no default, and higher debt stock with the bond price at the risk-free level.

The responses of a four standard deviations (4 S.D.) shock are shown in yellow in Figure 6. This time output falls by about 15 percent to about 8.5. The consumption decreases to 8.5 as well, still matching the fall in output, if not exceeding. There is now multi-period default with full default in the year 1 and a default rate of about 0.72 in the year 2. These default trends are reflected in the bond price which tanks for one period to 0.56 implying a 4,000+ point spread, which remains as high as 1,700 basis points in the year 3. Given such high spread, in contrast to 2 S.D. and 3 S.D. cases, the country no longer finds it optimal to issue any new debt. Not only is there no new debt issuance, there is no trade surplus either, like the 2 S.D. and 3 S.D. cases. These outcomes are consistent with its decision of full default, as it obviates the need to issue new debt or run trade surplus to service debt not defaulted on. In fact, full default and, hence, no repayment coupled with no new issue of debt implies that the country endogenously enters financial autarky. Moreover, with the net export at zero, there is no goods trade either; the country consumes what it produces. However, international trade and lending recover soon after the default. The economy starts to recover at the year 2 and it takes about 40 years to regain a new steady state, where there

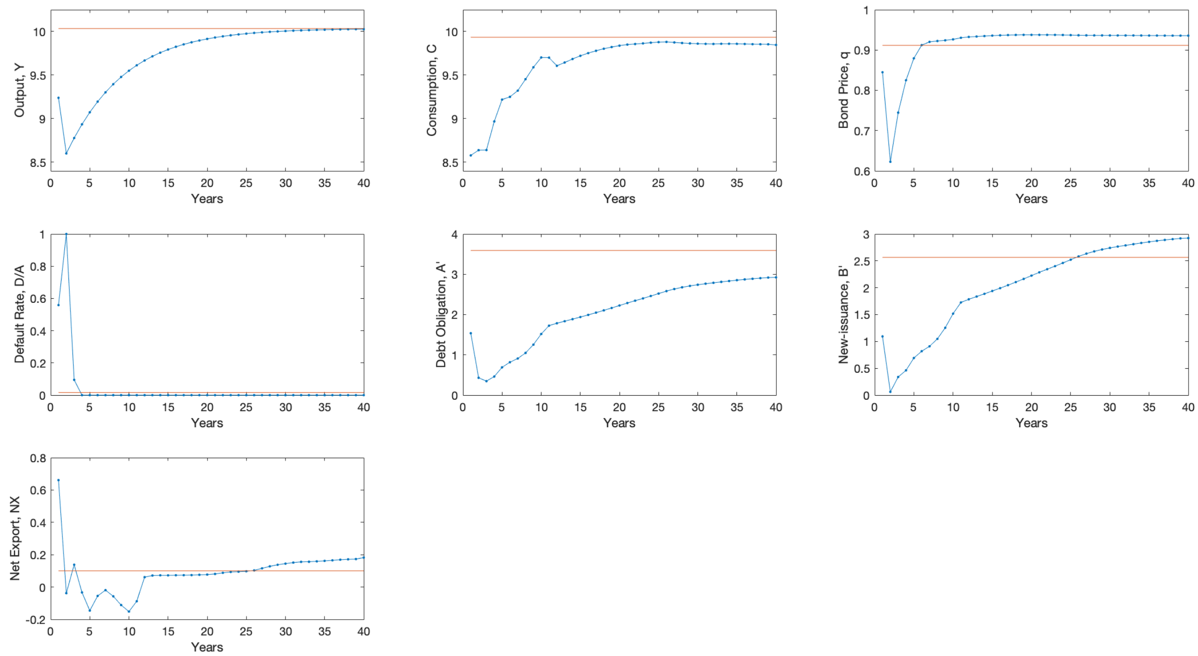

Figure 9: Two-Period 2 S.D. Shocks, with Initial Debt at the 75th Percentile of the Distribution of the Simulated Data

are less consumption, more trade surplus, no default, and higher debt stock with the risk free bond price.

Figure 7 and Figure 8 tell a similar story given the initial debt level at the 75th percentile (3rd quartile; $A' = 3.5910$) and at the 90th percentile ($A' = 4.4974$) of the distribution of the simulated data sample, respectively.

In Figure 9, we give a one-time, two-period two standard deviations shock to the economy with initial debt at 75th percentile (3rd quartile) of the distribution of the simulated data sample. The second period shock is not anticipated in the first period. The response in the year 1 is thus the same as in Figure 7 for 2 S.D. The explanation for those responses is same as for Figure 6 above. A second unanticipated shock, however, changes the dynamics in interesting and intuitive ways relative to a one-period 4 S.D. shock (in Figure 7). The consumption now recovers more slowly, remaining at its trough for 3 years. The default now continues for 3 years (instead of 2 years) with full default in the middle year, the year 3. This is also mirrored in the delayed recovery of bond price, which also reaches its trough in the year 2 (of 0.62), instead of the year 1, although it falls less (0.62 vs. 0.56). The paths of A' are, however, very similar in the two cases, falling until year 3. The initial increase in the trade surplus of year 1 of 0.67 gives way to zero or mildly negative values for a longer period up to year 11 (in comparison to up to the year 6). The economy starts to recover at the end of the year 2. It takes about 40 years to regain a new steady state, where there

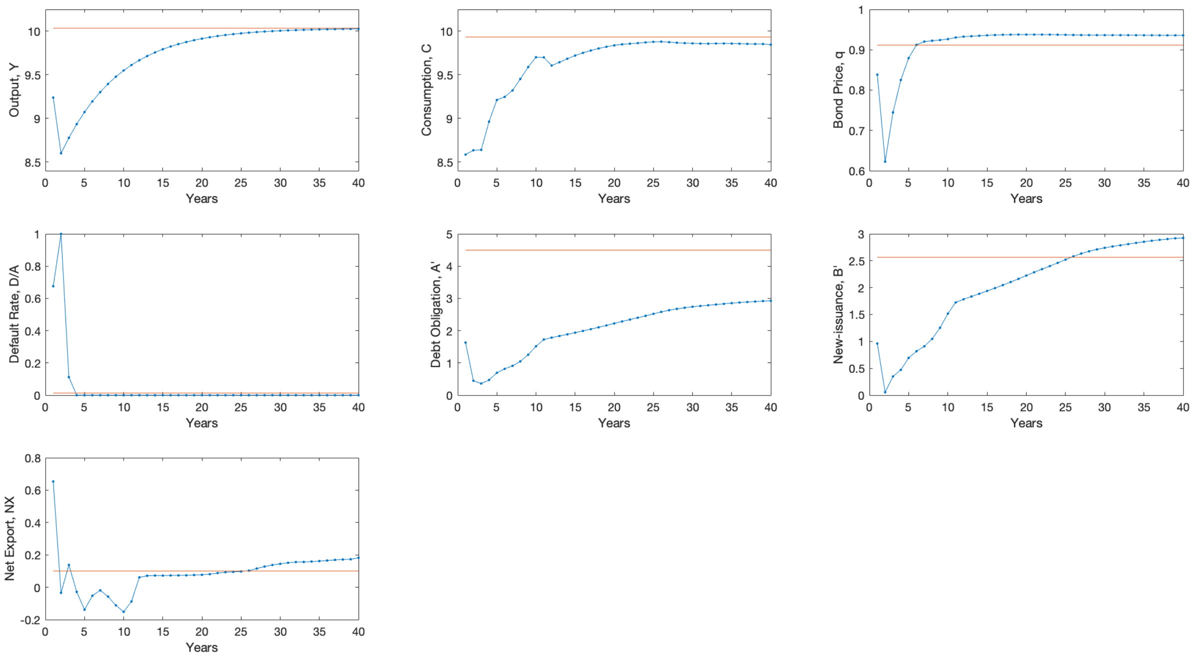

Figure 10: Two-Period 2 S.D. Shocks, with Initial Debt at the 90th Percentile of the Distribution of the Simulated Data

is less consumption, more trade surplus, no default, and lower debt stock with the risk free bond price.

Figure 10 tells a similar story given the initial debt level at the 90th percentile of the distribution of the simulated data.

5.2 The Production Economy

We now turn to the model with production. Recall, the motivation for including the production is the fact that the endowment model over-predicts the volatilities of consumption (trade balance) and interest spreads. The intuition for why the production may be helpful along this dimension is as follows: Production requires labor (inelastically-supplied in the model) and capital. Therefore, the country not only consumes and borrows from the international market, but also makes the decision about investment in capital. The adjustment in this investment in capital now provides an additional margin for the country to smooth consumption in response to shocks. It is, therefore, anticipated that it will bring down the high value of the relative standard deviation of consumption (to output) to be more in line with the data. By reducing the reliance on the international borrowing, it may also help reduce the high standard deviation of spreads (as driven by Euler equation (21)).

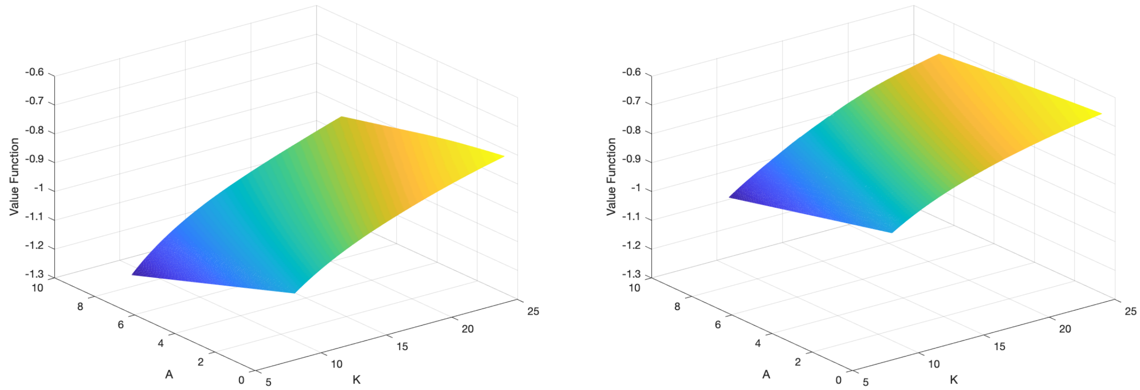

Figure 11: Value Functions (Production – θ, 1st and 3rd Quartiles of Shocks)

5.2.1 Policy Functions

Figure 11 plots the value functions at the current level of capital (K) and indebtedness (A), for low and high current productivity (θ), respectively. As expected, the value function is increasing in K as higher value of capital implies higher welfare. On the other hand, greater indebtedness translates into lower welfare. In terms of the curvature, the value is concave in the capital. However, similar to the value function in the model with endowment, the value function of the model with production is very linear in A. Comparing the value functions at the low (the 1st quartile of the shocks) productivity and high (the 3rd quartile of the shocks) productivity, we show that the value function is increasing with respect to θ as a higher value of productivity implies higher welfare.

Figure 12 and Figure 13 plot the policy functions for bond price, default, debt obligation, new-issuance, consumption, and capital at the low productivity and high productivity, respectively. We begin with the bond price function of the low productivity shock in Figure 12, which shows that when the capital stock is high there is no default risk and bonds issued by the sovereign command the risk-free price. However, as the capital level (K') falls, future default becomes more probable. Therefore, the bond price starts taking a hit ($q_{K'} > 0$) and the level of borrowing starts to matter for pricing of bonds; higher borrowing implies a greater decrease in the bond price for a given level of capital ($q_{A'} < 0$). For the low levels of K', the bond price collapses completely or gets close to zero. A similar argument applies to the variation in the default decision with K and indebtedness A in Figure 12. High K and low A generate no default. Normal levels of K and A make the default behave with an obvious partial nature; and there is full default (or nearly full default, closing in onto the upper bound of debt) for really low K levels.

The policy functions for the gross borrowing (A') and the new-issuance (B') in the middle

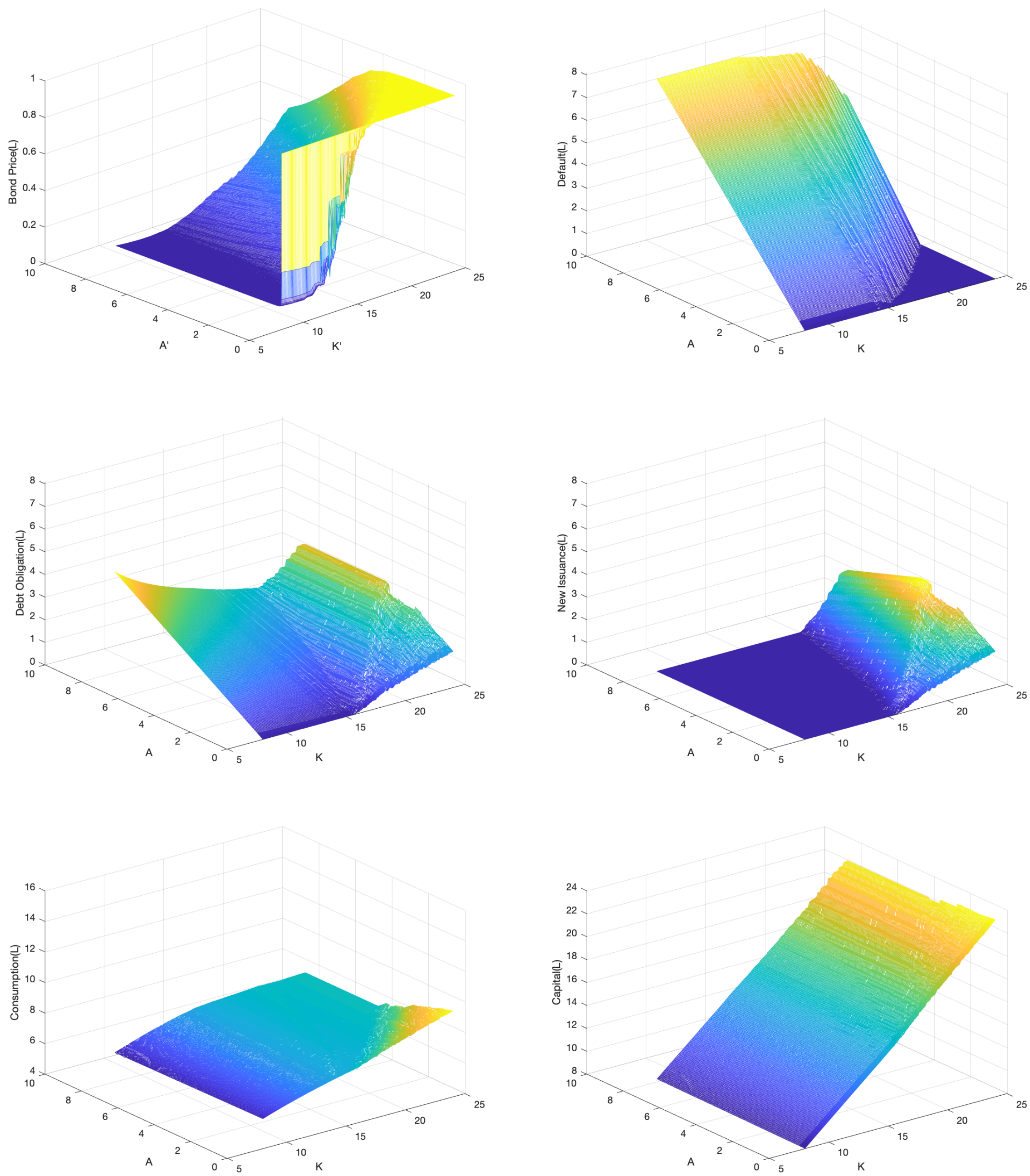

Figure 12: Policy Functions (Production – Low θ, 1st Quartile of Shocks)

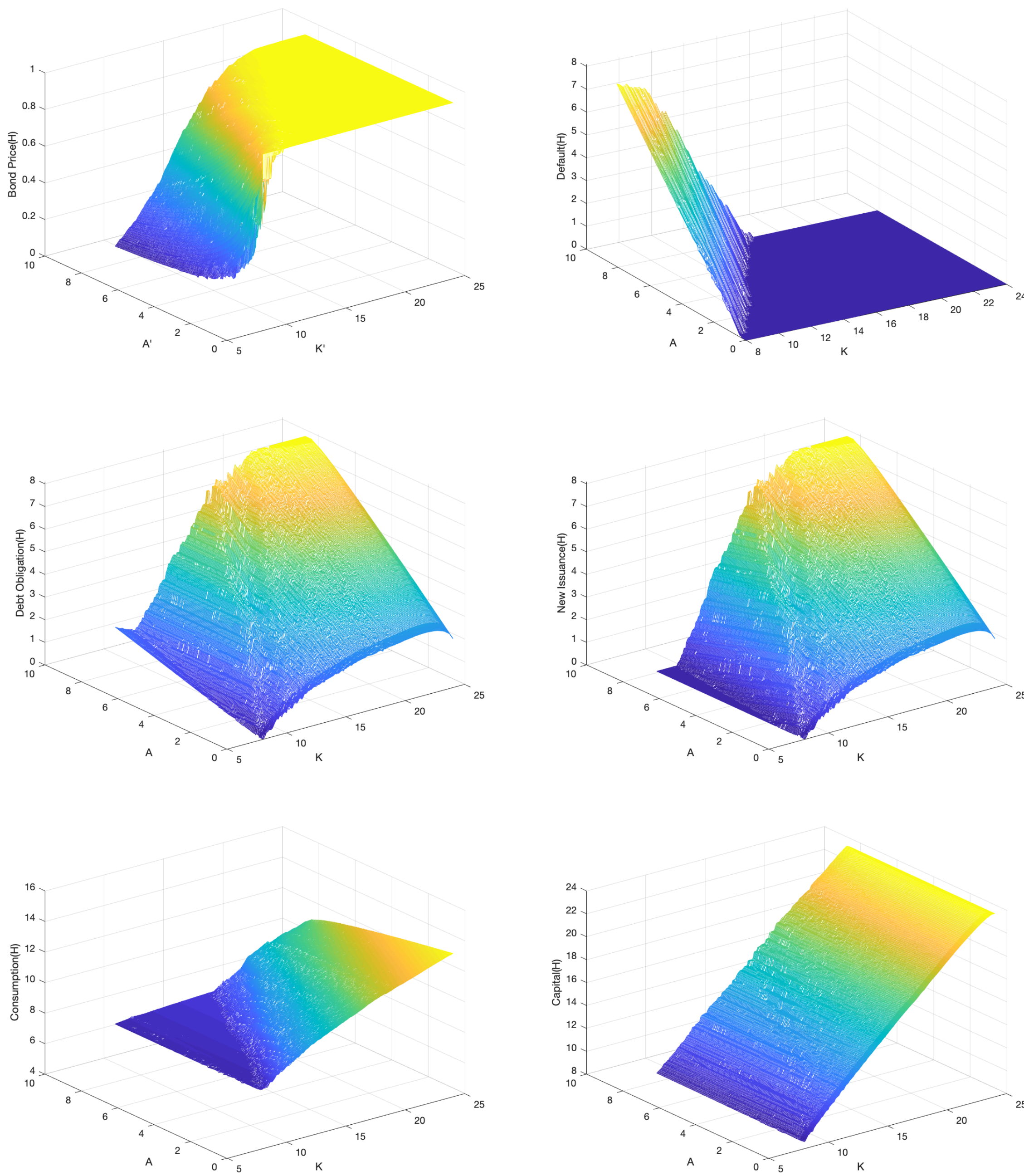

Figure 13: Policy Functions (Production – High θ, 3rd Quartile of Shocks)

row of Figure 12 are fairly intuitive. There is no new debt issued when K is low, which are also the times when there is full default. In those times, the only implicit borrowing is the preemptive amount of default that is rolled over (recall, $A' = B' + \tilde{R}D$). The debt obligation at the middle levels of K, or high levels of K combined with high levels of A, composes of the positive new-issuance and the preemptive recovery payment of default. Thus, the small open economy continues to borrow when it has debt arrears. For the high levels of K combined with low levels of A, the debt obligation consists of the new-issuance above because of no default.

The last row of Figure 12 shows the policy functions for consumption (C) and capital (K'). The capital decision is relatively independent with respect to the current debt level (A), while it is increasing with respect to the current capital stock (K). The consumption at low K equals output minus the sum of domestic investment and net export, and is not a function of A, as these are also times when the new-issuance is zero and there is full default. There is also financial autarky (no capital inflows or outflows), although the country keeps accumulated arrears in the form of preemptive recovery payments.

Figure 13 tells a similar story given the productivity shock at its 75th percentile (the 3rd quartile), where there is a higher bond price, less default, higher debt stock (new-issuance), and more consumption, given the similar levels of current capital and indebtedness (K, A) ((K', A') for bond price).

5.2.2 Findings

The last column of Table 9 reports that how the partial default model with production does in terms of matching the data moments that have been focus of interest in the literature *vis-à-vis* other models of sovereign debt and default, as well as the partial default model with endowment. We calibrate five parameters of the model (β, κ_1, γ_2, $\bar{R}$, and ϕ) to simultaneously match five targets: the mean spread on debt, the debt-to-output ratio, the default frequency, the default rate, and $\sigma(i)/\sigma(y)$ (as shown in Table 6). The overall fit of the calibrated model with production to these target moments is reasonable. Relative to the model with endowment, the model with production does slightly worse in terms of the default frequency and default rate, but better in terms of the debt service-to-output ratio. Specifically, the model can simultaneously match the mean spread and the debt-to-output ratio. It also closely matches the relative volatility of investment.

Table 9 also reports other key statistics of the cyclical components of macroeconomic variables. Recall, our main motivation for including production is to reduce the volatility of consumption relative to that of the output. Along this dimension, the model does quite well by reducing $\sigma(c)/\sigma(y)$ from 1.79 to 1.43. It was also conjectured that the standard deviation

Table 9: Main Findings (Production)

	CE Data (Arellano Data)	Arellano Baseline	CE-ST	CE-LT	Partial Default Baseline Endowment	Partial Default Baseline Production
Mean Spread	.0815	.0358	**.0815**	**.0815**	**.0830**	**.0815**
S.D. of Spread	.0443	.0636	**.0443**	**.0443**	.2982	.2629
Debt-to-Output	1	.0595	**.7**	**.7**	**.994**	**.975**
$\sigma(c)/\sigma(y)$	1.09	1.1	1.59	1.11	1.79	1.43
$\sigma(nx/y)/\sigma(y)$	.17	**.26**	1.06	.2	1.11	1.15
$corr(c, y)$	.98	.97	.73	.99	.90	.85
$corr(nx/y, y)$	-.88 (-.64)	-.25	-.16	-.44	-.57	-.48
$corr(r - r_f, y)$	-.79 (-.88)	-.29	-.55	-.65	-.32	-.23
Debt Service-to-Output	.053	**.056**	.699	.055	.242	.103
Default Frequency	.125 (.03)	**.03**	.073	.068	**.126**	**.199**
Default Rate (Cond.)	.488	1	1	1	**.502**	**.583**
$corr(i, y)$	0.71	N/A	N/A	N/A	N/A	0.84
$\sigma(i)/\sigma(y)$	3.79	N/A	N/A	N/A	N/A	**4.51**

of the spread may also go down. While that does happen, the improvement is much more modest: from 0.2982 to 0.2629. The other business cycle statistics such as the correlation of output with consumption, trade balance and interest spreads are relatively unaffected by including production in the model.

6 Conclusion

The standard theory of sovereign default, which investigates the default incentives and consequences, usually assumes that countries always default on all of their debt and they are excluded from the international capital market after default. In this paper, we solve the partial default models of a small open economy to quantitatively investigate the responses of the borrowing, default, and pricing of sovereign debt to economic shocks.

The partial default models with both endowment and production in this paper are built with three key features. Firstly, the default is endogenously-determined, which allows us to endogenize the partial default rate. Secondly, the preemptive recovery payment on the default enables the bond pricing in the partial default models of the short-term debt to acquire the features similar to those for the long-term debt model in the Eaton and Gersovitz's (1981) framework. Thirdly, there is no exclusion from the international capital market after default, which allows us to investigate the impulse responses of various macroeconomic and debt variables to economic shocks.

The benchmark models with endowment and with production are both calibrated to match the economic data moments of Argentina. The model with endowment simultaneously matches the mean spread on debt, the debt-to-output ratio, the default frequency, as well as other macroeconomic cyclical facts. The simulation results also match the partial default rate, which is the key proxy of the realized default risk. The results explain why countries default in bad times, capturing the empirical facts that countries always default partially and will still be in the international capital market and be able to borrow soon after default. The results also match the fact that the consumption is more volatile and net trade turns to surplus with default in emerging countries.

In the calibration process of the partial default model with endowment, the volatilities of consumption (trade balance) and interest spreads are over-predicted. To improve the model's fit to these moments we add production to the model. In the partial default model with production, the adjustment in investment provides another margin for a country to smooth consumption which brings the simulations of the relative standard deviation of consumption more in line with the data. By leading to less reliance on international borrowing, it also helps reduce the high standard deviation of spreads. The overall fit of the calibrated model with production to the target moments, the mean spread, the debt-to-output ratio, the default frequency, the default rate, and the relative standard deviation of investment to that of output, is reasonable. Relative to the model with endowment, the model with production does slightly worse in terms of the default frequency and default rate, but better in terms of the debt service-to-output ratio. Specifically, the model can simultaneously match the mean spread and the debt-to-output ratio. It also closely matches the relative volatility of investment. The other business cycle statistics such as the correlation of output with consumption, trade balance, and interest spreads are relatively unaffected by including production in the model.

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