Intra-Household Conflict and Female Labour Force Participation

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Abstract

The low level of participation of women in the labour force has emerged as an impediment to economic development in India over the past few decades. Our study provides a theoretical framework to understand how social structure influences women's labour market choices. We consider household decision making in a two person household with a wife and a husband, and analyze decision making in a non-patriarchal and patriarchal social structure. The patriarchal regime is characterized by men having control over women's labour supply decisions and the non-patriarchal regime is characterized as being gender neutral. We find that the patriarchal social structure generates inefficiencies as women are prevented from joining the labour force even if they potentially earn more than their spouses. Adding more structure to the framework and introducing a sector which allows couples to purchase household help from the market, we see that the inefficiency of the patriarchal system persists. Further, the model helps explain the U-shaped labour supply curve of women with respect to women's education level. Hence, the study highlights the need for policies that increase the bargaining power of women in patriarchal societies and target high levels of education for women as a solution.

KEYWORDS: Social Structure, Nash Bargaining, Female Labour Force Participation

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1 Introduction

The past few decades has seen the emergence of India as the fastest growing major economy in the world (World Bank, 2019). However, the growth story is mired in several contradictions. One of the critically debated contradiction is the low level of Female Labour Force Participation (FLFP). The low level of FLFP is puzzling, since it has accompanied rapid fertility transition and broad improvements in women's education attainment, which would tend to be supportive of an increase in participation of women in the labour market (Afridi et al., 2017; Fletcher et al., 2017; Kapsos et al., 2014; Klasen and Pieters, 2015). According to the National Sample Survey (NSS) 68^{th} round National Employment and Unemployment survey, 2011-12, the overall FLFP by usual primary status in the age group 25 and above is 26.32 percent. If we further split the FLFP by urban and rural, we see that the urban FLFP is 19.58 percent and the rural FLFP is 29.22 percent. Comparing India's FLFP with the rest of the world, we see that, according to ILO(2013), India ranks 121 out of 131 countries across the world and one of the lowest in South Asia (Andres et al., 2017).

The low FLFP is a cause of concern for several reasons. The first is that India currently has a large working age population with few dependents. Given that women make up nearly half the working age population, having so few women participating in the labour force has enormous economic implication. Esteve-Volart (2004) in their study show that, for India, a 10 per cent increase in female-to-male ratio of managers would increase per capita total output by 2 per cent and a 10 percent increase in female-to-male ratio of total workers would increase per capita total output by 8 percent. The second is that women's participation in the labour force also has implication on the extent to which they can benefit from economic growth. This is because employment and earnings are important determinants of women's bargaining power in the family(Anderson and Eswaran, 2009). The third is that there are positive spillovers from women's earned income on child indicators. There is fair evidence that children enjoy better educational outcomes when mothers earn a wage income(Afridi et al., 2012; Luke and Munshi, 2011). All these factors are extremely critical, because, despite the high rate of economic growth, India's per capita income and Human Development Indicators are very low when compared to the rest of the world and a higher level of FLFP could potentially improve these indicators.

The existing literature tries to explain the phenomenon through socio-economic factors that influence both the demand and supply of women's labour supply. On the demand side, it is argued that economic growth has not translated into higher job creation in general and in particular for women (Fletcher et al., 2017; Kannan and Raveendran, 2012; Klasen and Pieters, 2015; Naidu, 2016). This means that even though women want to work, they do not find suitable jobs.

On the supply side, the predominant argument is that women face social stigma when they engage in paid work. Fletcher et al. (2017) argue that Indian households require women to prioritize household work and may even explicitly constrain work by married women. Further evidence on social strictures on women's participation in paid work is provided by Ghai (2018), who finds that Indian States which have stronger social strictures on women are less likely to have women engaging in paid work, especially at higher levels of education.

The supply side explanation to the low FLFP is the starting point of our study. We provide a theoretical framework to understand the economic rationale behind these strictures on women's participation in the labour force. The basis of the model is the conflict of preference within a two individual household, where the couple pools resources and takes decisions collectively. When it comes to labour supply, economic efficiency would require the higher earning spouse to work full time, irrespective of gender. This provides the households with the highest income and welfare. However, the low FLFP observed in India seems to reflect economic inefficiency. We hence investigate further for a possible explanation.

The answer might lie in the nature of social norms in India. In patriarchal societies, it is within the control of men to prevent their wives from entering the labour force (Derné, 1994). If women are unable to join the labour force early in their working lives, they are forever deprived of the credentials and networks that help an individual remain in the labour force. Thus, not participating in the labour force early in their working lives precludes future access to labour markets. The lack of access to paid work means that they loose bargaining power within marriage (Anderson and Eswaran, 2009). In patriarchal societies, the labour supply of married women is determined solely to profit their husbands and hence generates a dead-weight loss. The husband has the option of letting the wife work or follow a gender based division of work, where women don't work to prioritize care giving. The first option of letting the wife work has the benefit that the household income and hence welfare is higher. The second option of preventing the wife from participating in the labour force comes at the cost of a lower family income and welfare. Despite the loss in overall household income, the husband might still exercise this option because he can potentially gain from it. This is because the loss in the wife's bargaining power due to her not having access to labour markets, even if she breaks away from marriage, means that husband can corner a larger share of the smaller household income.

The model goes further to explain another set of recent empirical findings that link the education and income level of the husband and wife, and FLFP. Klasen and Pieters (2015) provides empirical support for a U-shaped relation between FLFP and the education level of the wife in urban India. For rural India, Afridi et al. (2017) find a decline in FLFP with respect to female education. Our model helps explain the response of FLFP to husband's wage or household income in a cross section as well as the response of FLFP to women's education levels.

The rest of the chapter is organized into 5 sections. The second section that follows describes the general framework of household decision making. The third section explains the supply of labour by households when the spouses have no conflict of preference. The fourth section describes the household's labour supply when there is conflict of preferences between spouses. The fifth section introduces a market for household help and we show that this can generate a U-shaped relation between the labour supply of the wife, and her own education and husband's income. The sixth and the last section summarizes the key findings of this chapter.

2 Framework of Household Decision Making

In this section we introduce the general framework of household decision making. We analyze the behavior of a representative household which comprises of a husband and a wife, each endowed with a unit of time. They can allocate time between market work ($l_i, i = w, h$) and household work($t_i, i = w, h$). Further, the wife and husband earn market wages αw and w per unit of time spent on market work, respectively. Here, α denotes the relative wage of wife with respect to the husband's wage w. In marriage, the couple pools resources when making decisions for the household.
Each spouse cares about two household goods, a wife specific and husband specific private good, denoted by x_w and x_h respectively. They also care about a household public good that is produced using time allocated to household work. Cobb Douglas utility functions that capture these preferences are given by:

$$u_h = (x_h)^{\sigma}(x_w)^{1-\sigma}T^{\beta} \tag{1}$$
$$u_w = (x_h)^{1-\sigma}(x_w)^{\sigma}T^{\beta} \tag{2}$$

Here, $\sigma = \left\{\frac{1}{2}, 1\right\}$ and $\beta \in (0, 1)$ are parameters that represent the importance of private consumption and household time to each spouse. Further, the household time allocation T is $t_i, i = w, h$, when they are unmarried and $t_w + t_h$ when married. We now analyze the work choices of the spouses in the household assuming identical preferences.

3 Household Decision Making without Conflict of Preferences

In this section we assume that wife and husband have identical preferences with $\sigma = \frac{1}{2}$. Hence,

$$u_h = (x_h)^{\frac{1}{2}}(x_w)^{\frac{1}{2}}T^{\beta} \tag{3}$$
$$u_w = (x_h)^{\frac{1}{2}}(x_w)^{\frac{1}{2}}T^{\beta} \tag{4}$$

This gives us a model of household decision making were the spouses have no conflict of preferences and hence optimize their common utility functions subject to their budget constraint. Further, we assume that household time is not marketed and has no close substitute. The household's optimization problem is given by:

$$\max_{x_w, x_h, t_w, t_h} (x_h)^{\frac{1}{2}}(x_w)^{\frac{1}{2}}T^{\beta}$$

subject to :

$$x_w + x_h = (1 - t_w)\alpha w + (1 - t_h)w$$
$$T = t_w + t_h$$

We set up the Lagrangian for the household's optimization problem which is as follows:

$$\max_{x_w, x_h, t_w, t_h} L = (x_h)^{\frac{1}{2}}(x_w)^{\frac{1}{2}}T^{\beta} + \lambda((1 - t_w)\alpha w + (1 - t_h)w - x_h - x_w)$$

The first derivatives of the Lagrangian are as follows:

$$\begin{aligned}
\frac{\partial L}{\partial x_w} &= \frac{1}{2}(x_h)^{\frac{1}{2}}(x_w)^{-\frac{1}{2}}T^{\beta} - \lambda && (5)\\
\frac{\partial L}{\partial x_h} &= \frac{1}{2}(x_h)^{\frac{-1}{2}}(x_w)^{\frac{1}{2}}T^{\beta} - \lambda && (6)\\
\frac{\partial L}{\partial t_w} &= \beta(x_h)^{\frac{1}{2}}(x_w)^{\frac{1}{2}}T^{\beta-1} - \lambda\alpha w && (7)\\
\frac{\partial L}{\partial t_h} &= \beta(x_h)^{\frac{1}{2}}(x_w)^{\frac{1}{2}}T^{\beta-1} - \lambda w && (8)\\
\frac{\partial L}{\partial \lambda} &= (1-t_h)w + (1-t_w)\alpha w - x_w - x_h && (9)
\end{aligned}$$

Setting the first derivatives to zero, we see that the allocation of time depends on who has the lower cost of household work. We analyze this as follows:

3.1 The wife's wage is less than that of the husband or $\alpha < 1$

In this case the wife makes the first contributions to household work and the husband contributes any residual household time. We start by assuming that the husband works in the market full time and only the wife contributes towards household work (or $t_h = 0$ and $T = t_w$). The constrained optimization problem is solved by setting equations 5, 6, 7 and 9 to zero and we find the following:

$$x_w = x_h = \frac{\alpha w t_w}{2\beta} \tag{10}$$

using this in the budget constraint we find that:

$$t_w = \frac{(1+\alpha)\beta}{\alpha(1+\beta)}$$

We know that the wife's time allocation cannot exceed 1 and hence we see that:

$$t_w = \begin{cases} 1 & if\ \alpha \leq \beta \\ \frac{(1+\alpha)\beta}{\alpha(1+\beta)} & if\ \alpha > \beta \end{cases} \tag{11}$$

We now derive the wife's and husband's private consumption for the interior and boundary solution of t_w.

3.1.1 The wife's household time allocation has an interior solution, $\alpha > \beta$

In this case we have, $t_h = 0$ and $t_w = \frac{(1+\alpha)\beta}{\alpha(1+\beta)}$. Hence, $T = \frac{(1+\alpha)\beta}{\alpha(1+\beta)}$. The private consumption levels are given by:

$$x_w = x_h = \frac{(1+\alpha)w}{2(1+\beta)}$$

3.1.2 The wife's household time allocation has a boundary solution, $\alpha \leq \beta$

Since $t_w = 1$, we cannot set $\frac{\partial L}{\partial t_w}$ to zero. Hence we set $\frac{\partial L}{\partial t_h}$ to zero and check the conditions under which the husband will start contributing to household work. Hence, now $T = 1 + t_h$, or the total time allocated to household production is full time allocation of the wife and time contributions of the husband. Now, we go back to the first order conditions and set 5, 6,8 and 9 to zero and we find the following:

$$x_w = x_h = \frac{(1 + t_h)w}{2\beta} \tag{12}$$

using this in the budget constraint we find that:

$$t_h = \frac{\beta - 1}{\beta + 1}$$

Since, $\beta < 1$, we are again at a boundary solution of $t_h = 0$ and $T = 1$ for husband's time. We cannot set $\frac{\partial L}{\partial t_h}$ to zero since at this point $\frac{\partial L}{\partial t_h} < 0$. Hence setting 5, 6, and 9 to zero we find:

$$x_w = x_h = \frac{w}{2}$$

3.2 The wife's wage is greater than that of the husband or $\alpha \geq 1$

In this case the husband makes the first contributions to household work and any residual demand is contributed by the wife.We start by assuming that the wife works in the market full time and only the husband contributes towards household work (or $t_w = 0$ and $t = t_h$). The constrained optimization problem is solved by setting equations 5, 6, 8 and 9 to zero and we find the following:

$$x_w = x_h = \frac{wt_h}{2\beta} \tag{13}$$

using this in the budget constraint we find that:

$$t_h = \frac{(1 + \alpha)\beta}{(1 + \beta)}$$

However, we know that the husband's time endowment is 1 and hence:

$$t_h = \begin{cases} 1 & if \ \alpha \geq \frac{1}{\beta} \\ \frac{(1+\alpha)\beta}{(1+\beta)} & if \ \alpha < \frac{1}{\beta} \end{cases} \tag{14}$$

We now derive the wife's and husband's private consumption for the interior and boundary solution of t_h.

3.2.1 The husband's household time allocation has an interior solution, $\alpha < \frac{1}{\beta}$

In this case we have, $t_h = \frac{(1+\alpha)\beta}{(1+\beta)}$ and $t_w = 0$. Hence, $T = \frac{(1+\alpha)\beta}{(1+\beta)}$. The private consumption levels are given by:

$$x_w = x_h = \frac{(1 + \alpha)w}{2(1 + \beta)}$$

3.2.2 The husband's household time allocation has a boundary solution, $\alpha \geq \frac{1}{\beta}$

Since $t_h = 1$ we cannot set $\frac{\partial L}{\partial t_h}$ to zero. We set $\frac{\partial L}{\partial t_w}$ to zero to check the conditions under which the wife will start contributing to household work. Hence, $T = 1 + t_w$, or the total time allocated to household production is full time work of the husband and time contributions of the wife. Now, we go back to the first order conditions and set equations 5, 6, 7 and 9 to zero and we find the following:

$$x_w = x_h = \frac{(1+t_w)\alpha w}{2\beta} \tag{15}$$

$$t_w = \frac{\beta - 1}{\beta + 1}$$

However, since $\beta < 1$, t_w cannot fall below 0. Hence, $t_w = 0$ and $T = 1$. Here we again have a boundary solution with respect to t_w and cannot set $\frac{\partial L}{\partial t_w}$ to zero. Setting equations 5, 6 and 9 to zero we find the following:

$$x_w = x_h = \frac{\alpha w}{2}$$

We now summarize the wife's and husband's market work decision, $l_i = 1 - t_i$, *where* $i = w, h$, corresponding to different levels of the wife's relative wage α.

$$l_h = \begin{cases} 1 & if\ \alpha \in (0,1) \\ \frac{1-\alpha\beta}{1+\beta} & if\ \alpha \in \left[1, \frac{1}{\beta}\right) \\ 0 & if\ \alpha \in \left[\frac{1}{\beta}, \infty\right) \end{cases} \tag{16}$$

$$l_w = \begin{cases} 0 & if\ \alpha \in (0,\beta] \\ \frac{1-\frac{\beta}{\alpha}}{\beta+1} & if\ \alpha \in (\beta,1) \\ 1 & if\ \alpha \in [1,\infty) \end{cases} \tag{17}$$

Each spouse's labour supply, under collective decision making without conflict of preferences, is shown in figures 1 and 2.

The following proposition summarizes the implications of household decision making on the labour supply of the spouses when there is no conflict of preferences between them.

Proposition 1: *When there is no conflict of preferences between the wife and husband, labour supply is determined by considerations of efficiency. The spouse who earns more will always work full time irrespective of gender. The other spouse will either do full time domestic work, or split his/her time between market and domestic work.*

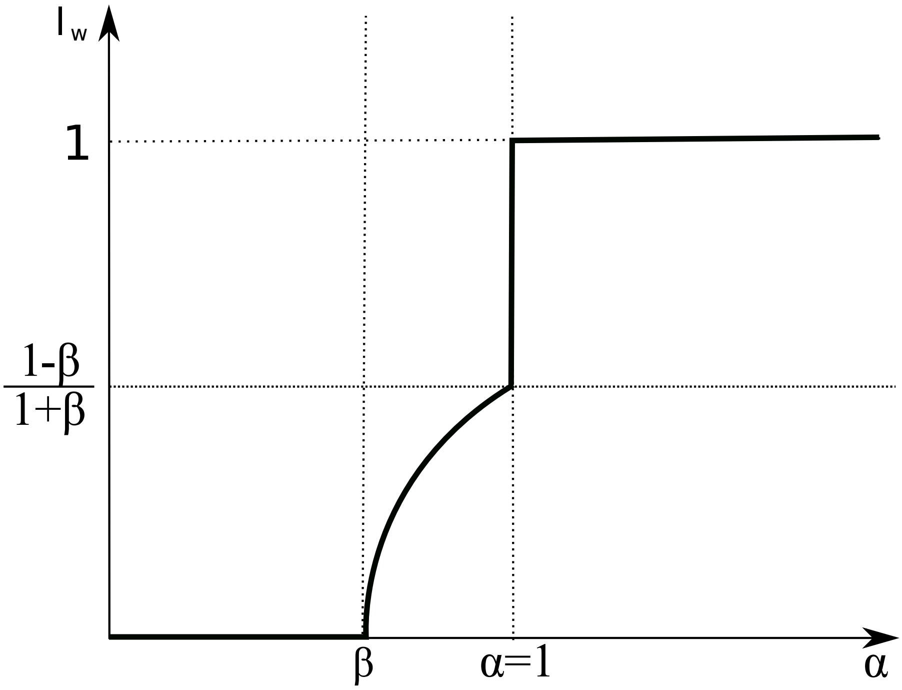

Figure 1: The wife's supply of market labour (l_w) as a function of her relative wage α under collective decision making without conflict of preferences.

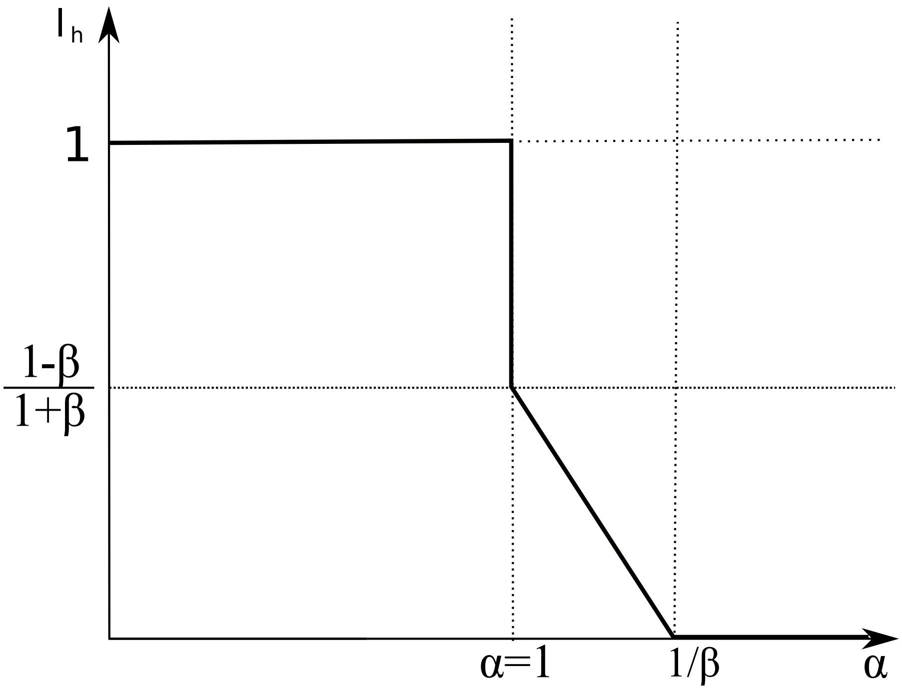

Figure 2: The husband's supply of market labour (l_h) as a function of the wife's relative wage (α) under collective decision making without conflict of preferences.

4 Household Decision Making with Conflict of Preferences

We introduce the idea of conflict of preferences by assuming that the husband doesn't care about the wife's private consumption and vice versa or we assume $\sigma = 1$. The utility functions describing their preferences are now as follows:

$$u_h = x_h \cdot t^\beta$$
$$u_w = x_w \cdot t^\beta$$

We also define two regimes of household decisions making. A non-patriarchal regime and a patriarchal regime.
In the non-patriarchal regime, the couple pools their time endowment and jointly decides private consumption of each spouse and their time allocations to household work. The household decision making is formulated as a bargaining problem which is solved using the Nash Bargaining solution. This is as follows:

$$\max_{x_w, x_h, T} N_N = \left[x_h \cdot T^\beta - R_h\right]\left[x_w \cdot T^\beta - R_w\right]$$

s.t.

$$\begin{aligned} x_w + x_h &= (1 - t_h)w + (1 - t_w)\alpha w \\ T &= t_w + t_h \end{aligned}$$

Here, R_h and R_w are the wife's and husband's threat points. The threat point in marriage is their indirect utility outside marriage.
In the patriarchal regime, the husband has the choice of preventing the wife from participating in the labour-force. If the wife is prevented from joining the labour force, time allocation is gender specific with the wife doing household work full time and the husband engaging in the labour-force while supplying any residual household time. Once the decision of letting the wife join the labour force has been made, the household decision making problem is again formulated as a bargaining problem where the couple decides their private consumption and the husband's time allocation. The bargaining problem when the wife is allowed to work is the same as in non-patriarchal regime. However, if the wife is prevented from joining the labour force, the household decision making problem is again posed as bargaining problem and solved using the Nash Bargaining solution. In this regime, the wife looses all her bargaining power since she can never enter the labour force even as a single person household. Thus, her threat point goes to zero. The household's objective function is hence as follows:

$$\max_{x_w, x_h, T} N_P = \left[x_h \cdot T^\beta - R_h\right]\left[x_w \cdot T^\beta\right]$$

s.t.

$$\begin{aligned} x_w + x_h &= (1 - t_h)w \\ T &= 1 + t_h \end{aligned}$$

Here, R_h is the husband's threat point and the wife's threat point $R_w = 0$. We now proceed to analyze the couple's labour market decisions in each of these regimes.

4.1 The Non-Patriarchal Regime

We start by identifying the couple's threat points which in marriage are their indirect utilities when single. The couple's indirect utilities when single are obtained from the following decision making problems:

$$\max_{x_h,t_h} x_h \cdot t_h^\beta \quad s.t. \;\; x_h = (1-t_h)w$$

$$\max_{x_w,t_w} x_w \cdot t_w^\beta \quad s.t. \;\; x_w = (1-t_w)\alpha w$$

Setting up the Lagrangian for the husband's decision making problem, we have:

$$L = x_h \cdot t_h^\beta + \lambda[(1-t_h)w - x_h]$$

The first order conditions (FOCs) are as follows:

$$\begin{aligned}
\frac{\partial L}{\partial x_h} &= t_h^\beta - \lambda = 0 \\
\frac{\partial L}{\partial t_h} &= \beta x_h t_h^{1-\beta} - \lambda w = 0 \\
\frac{\partial L}{\partial \lambda} &= (1-t_h)w - x_h = 0
\end{aligned}$$

Solving the FOCs simultaneously we find:

$$\begin{aligned}
x_h &= \frac{w}{1+\beta} \\
t_h &= \frac{\beta}{1+\beta}
\end{aligned}$$

Since the wife and husband are identical in preferences structure and differ only by wages, the wife's choices are as follows: Solving the FOCs simultaneously we find:

$$\begin{aligned}
x_w &= \frac{\alpha w}{1+\beta} \\
t_w &= \frac{\beta}{1+\beta}
\end{aligned}$$

The indirect utility functions of the wife (V_0^w) and husband (V_0^h) are as follows:

$$\begin{aligned}
V_0^h &= \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \\
V_0^w &= \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}
\end{aligned}$$

For the Nash Bargaining solution we have $R_h = V_0^h$ and $R_w = V_0^w$. The household's optimization problem is hence given as follows:

$$\max_{x_w,x_h,T} N_N = \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}\right]\left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}\right]$$

s.t.

$$\begin{aligned} x_w + x_h &= (1-t_h)w + (1-t_w)\alpha w \\ T &= t_w + t_h \end{aligned}$$

Setting up the Lagrangian we have:

$$\begin{aligned} \max_{x_w,x_h,t_h,t_w} L_N = &\left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}\right]\left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}\right] \\ &+\lambda_\alpha\left[(1-t_h)w + (1-t_w)\alpha w - x_h - x_w\right] \end{aligned}$$

The first derivatives of the Lagrangian are as follows:

$$\frac{\partial L_N}{\partial x_w} = T^\beta\left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}\right] - \lambda \tag{18}$$

$$\frac{\partial L_N}{\partial x_h} = T^\beta\left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}\right] - \lambda \tag{19}$$

$$\frac{\partial L_N}{\partial t_h} = \beta x_w T^{\beta-1}\left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}\right] + \beta x_h T^{\beta-1}\left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}\right] - \lambda w \tag{20}$$

$$\frac{\partial L_N}{\partial t_w} = \beta x_w T^{\beta-1}\left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}\right] + \beta x_h T^{\beta-1}\left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}\right] - \lambda \alpha w \tag{21}$$

$$\frac{\partial L_N}{\partial \lambda} = (1-t_h)w + (1-t_w)\alpha w - x_w - x_h \tag{22}$$

The value of α determines who has a lower opportunity cost of supplying household time and hence, is a critical to decision making.

4.1.1 The wife's wage is less than that of the husband or $\alpha < 1$

Revisiting the first derivatives, if $\alpha < 1$, then the wife's opportunity cost of household work is lower and hence, she will take the lead in allocating time towards household work. The Husband will only supply any residual household time if the wife exhausts her time endowment. We start by assuming that $T = t_w$ and $t_h = 0$ and the first derivatives of the Lagrangian are now as follows:

$$\frac{\partial L_N}{\partial x_w} = t_w^\beta\left[x_h \cdot t_w^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}\right] - \lambda \tag{23}$$

$$\frac{\partial L_N}{\partial x_h} = t_w^\beta\left[x_w \cdot t_w^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}\right] - \lambda \tag{24}$$

$$\frac{\partial L_N}{\partial t_h} = \beta x_w t_w^{\beta-1}\left[x_h \cdot t_w^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}\right] + \beta x_h t_w^{\beta-1}\left[x_w \cdot t_w^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}\right] - \lambda w \tag{25}$$

$$\frac{\partial L_N}{\partial t_w} = \beta x_w t_w^{\beta-1}\left[x_h \cdot t_w^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}\right] + \beta x_h t_w^{\beta-1}\left[x_w \cdot y_w^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}\right] - \lambda \alpha w \tag{26}$$

$$\frac{\partial L_N}{\partial \lambda} = w + (1-t_w)\alpha w - x_w - x_h \tag{27}$$

We now set the derivatives of the Lagrangian with respect to x_h, x_w, t_w and λ to zero. Equating equations 23 and 24-

$$t_w^\beta \left[x_w \cdot t_w^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] = t_w^\beta \left[x_h \cdot t_w^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] \tag{28}$$

or

$$t_w^\beta (x_h - x_w) = (1-\alpha) \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \tag{29}$$

Setting equations 23, 24 and 26 to zero and solving simultaneously we get:

$$x_h + x_w = \frac{\alpha w t_w}{\beta} \tag{30}$$

Using this in the budget constraint we find that:

$$t_w = \frac{(1+\alpha)\beta}{\alpha(1+\beta)} \tag{31}$$

However, the wife's time endowment is capped at 1. Hence we have:

$$t_w = \begin{cases} 1 & if\ \alpha \leq \beta \\ \frac{(1+\alpha)\beta}{\alpha(1+\beta)} & if\ \alpha > \beta \end{cases} \tag{32}$$

We solve for the couple's choice of private goods and their indirect utilities for both the interior and boundary solution.

CASE 1: The wife's household time allocation has an interior solution, $\alpha > \beta$

Here we have $t_h = 0$, $t_w = \frac{(1+\alpha)\beta}{\alpha(1+\beta)}$ and $T = \frac{(1+\alpha)\beta}{\alpha(1+\beta)}$. Using these in equations 29 and 30 we get:

$$x_h + x_w = \frac{(1+\alpha)w}{1+\beta} \tag{33}$$

$$x_h - x_w = \frac{(1-\alpha)\alpha^\beta w}{(1+\beta)(1+\alpha)^\beta} \tag{34}$$

Solving the above two equations simultaneously we get:

$$x_h = \frac{\left[(1+\alpha)^{1+\beta} + (1-\alpha)\alpha^\beta\right] w}{2(1+\beta)(1+\alpha)^\beta}$$

$$x_w = \frac{\left[(1+\alpha)^{1+\beta} - (1-\alpha)\alpha^\beta\right] w}{2(1+\beta)(1+\alpha)^\beta}$$

The indirect utility functions are hence:

$$V_h = \frac{\left[(1+\alpha)^{1+\beta} + (1-\alpha)\alpha^\beta\right] \beta^\beta w}{2\alpha^\beta (1+\beta)^{1+\beta}}$$

$$V_w = \frac{\left[(1+\alpha)^{1+\beta} - (1-\alpha)\alpha^\beta\right] \beta^\beta w}{2\alpha^\beta (1+\beta)^{1+\beta}}$$

CASE 2: The wife's household time allocation has a boundary solution, $\alpha \leq \beta$

In this case $t_w = 1$ and given that it is a boundary condition we cannot set $\frac{\partial L_N}{\partial t_w}$. We now set $\frac{\partial L_N}{\partial t_h}$ to zero so that we can check if the husband contributes to household work. We proceed by assuming that the wife works at home full time and the residual time demand is supplied by the husband. Now, $T = 1 + t_h$, The first order conditions that are to be solved simultaneously are:

$$\frac{\partial L_N}{\partial x_w} = T^\beta \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] - \lambda \tag{35}$$

$$\frac{\partial L_N}{\partial x_h} = T^\beta \left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] - \lambda \tag{36}$$

$$\frac{\partial L_N}{\partial t_h} = \beta x_w T^{\beta-1} \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] + \beta x_h T^{\beta-1} \left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] - \lambda w \tag{37}$$

$$\frac{\partial L_N}{\partial \lambda} = (1 - t_h)w - x_w - x_h \tag{38}$$

Setting the first order conditions to zero and equating 35 and 36-

$$T^\beta \left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}} \right] = T^\beta \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] \tag{39}$$

or

$$T^\beta (x_h - x_w) = (1 - \alpha) \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \tag{40}$$

Equating equations 35, 36 and 37 to zero and solving simultaneously we get:

$$x_h + x_w = \frac{Tw}{\beta} \tag{41}$$

Using this in the budget constraint we find that:

$$t_h = \frac{\beta - 1}{\beta + 1}$$

However, since $\beta < 1$, we have $t_h = 0$ and $T = 1$. We again have a boundary solution with respect to husband's household time allocation and hence cannot set $\frac{\partial L_N}{\partial t_h}$ to zero since it can be verified to be negative. Setting the first derivatives of the Lagrangian with respect to x_h, x_w and λ to zero we get:

$$x_h + x_w = w \tag{42}$$

Using $T = 1$ in 40 and solving simultaneously with 42 we have:

$$x_h = \frac{\left[(1+\beta)^{1+\beta} + (1-\alpha)\beta^\beta \right] w}{2(1+\beta)^{1+\beta}}$$

$$x_w = \frac{\left[(1+\beta)^{1+\beta} - (1-\alpha)\beta^\beta \right] w}{2(1+\beta)^{1+\beta}}$$

The indirect utility functions are hence:

$$V_h = x_h = \frac{\left[(1+\beta)^{1+\beta} + (1-\alpha)\beta^\beta \right] w}{2(1+\beta)^{1+\beta}}$$

$$V_w = x_w = \frac{\left[(1+\beta)^{1+\beta} - (1-\alpha)\beta^\beta \right] w}{2(1+\beta)^{1+\beta}}$$

4.1.2 The wife's wage is greater than that of the husband or $\alpha \geq 1$

In this case, the wife's opportunity cost of household work is higher than that of the husband. Hence, time allocations to household work are made by the husband first and any residual requirement is supplied by the wife. We start by assuming that $T = t_h$ and $t_w = 0$ and the first derivatives of the Lagrangian are now as follows:

$$\frac{\partial L_N}{\partial x_w} = t_h^{\beta}\left[x_h \cdot t_h^{\beta} - \frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}}\right] - \lambda \tag{43}$$

$$\frac{\partial L_N}{\partial x_h} = t_h^{\beta}\left[x_w \cdot t_h^{\beta} - \frac{\beta^{\beta} \alpha w}{(1+\beta)^{1+\beta}}\right] - \lambda \tag{44}$$

$$\frac{\partial L_N}{\partial t_h} = \beta x_w t_h^{\beta-1}\left[x_h \cdot t_h^{\beta} - \frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}}\right] + \beta x_h t_h^{\beta-1}\left[x_w \cdot t_h^{\beta} - \frac{\beta^{\beta} \alpha w}{(1+\beta)^{1+\beta}}\right] - \lambda w \tag{45}$$

$$\frac{\partial L_N}{\partial t_w} = \beta x_w t_h^{\beta-1}\left[x_h \cdot t_h^{\beta} - \frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}}\right] + \beta x_h t_h^{\beta-1}\left[x_w \cdot y_w^{\beta} - \frac{\beta^{\beta} \alpha w}{(1+\beta)^{1+\beta}}\right] - \lambda \alpha w \tag{46}$$

$$\frac{\partial L_N}{\partial \lambda} = w + (1 - t_h)\alpha w - x_w - x_h \tag{47}$$

Setting the first derivatives with respect to x_h, x_w, t_h and λ to zero and equating 43 and 44 we have :

$$t_h^{\beta}\left[x_w \cdot t_h^{\beta} - \frac{\beta^{\beta} \alpha w}{(1+\beta)^{1+\beta}}\right] = t_h^{\beta}\left[x_h \cdot t_h^{\beta} - \frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}}\right] \tag{48}$$

or

$$t_h^{\beta}(x_h - x_w) = (1-\alpha)\frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}} \tag{49}$$

Setting equations 43, 44 and 45 to zero and solving simultaneously we have:

$$x_h + x_w = \frac{t_h w}{\beta} \tag{50}$$

Using this in the budget constraint we find that:

$$t_h = \frac{(1+\alpha)\beta}{1+\beta} \tag{51}$$

However, we know that the husband's time allocation for household time cannot exceed his time endowment of 1 unit. Hence:

$$t_h = \begin{cases} \frac{(1+\alpha)\beta}{1+\beta} & if\ \alpha < \frac{1}{\beta} \\ 1 & if\ \alpha \geq \frac{1}{\beta} \end{cases} \tag{52}$$

We solve for the couple's choice of private goods and their indirect utilities for both the interior and boundary solution.

CASE 1: The husband's household time allocation has an interior solution, $\alpha < \frac{1}{\beta}$

Here, we have $t_w = 0$, $t_h = \frac{(1+\alpha)\beta}{1+\beta}$ and $T = \frac{(1+\alpha)\beta}{1+\beta}$. Replacing these in equations 49 and 50 , we get:

$$\begin{aligned} x_h - x_w &= \frac{(1-\alpha)w}{(1+\beta)(1+\alpha)^\beta} \\ x_h + x_w &= \frac{(1+\alpha)w}{1+\beta} \end{aligned}$$

Solving simultaneously for x_h ans x_w we get :

$$x_h = \frac{\left[(1+\alpha)^{1+\beta} + 1 - \alpha\right] w}{2(1+\beta)(1+\alpha)^\beta}$$
$$x_w = \frac{\left[(1+\alpha)^{1+\beta} - 1 + \alpha\right] w}{2(1+\beta)(1+\alpha)^\beta}$$

The indirect utility functions are hence:

$$V_h = \frac{\left[(1+\alpha)^{1+\beta} + 1 - \alpha\right] \beta^\beta w}{2(1+\beta)^{1+\beta}}$$
$$V_w = \frac{\left[(1+\alpha)^{1+\beta} - 1 + \alpha\right] \beta^\beta w}{2(1+\beta)^{1+\beta}}$$

CASE 1: The husband's household time allocation has a boundary solution, $\alpha \geq \frac{1}{\beta}$

In this case, we have $t_h = 1$. Given that t_h is at the boundary, we cannot set $\frac{\partial L_N}{\partial t_h}$ to zero. However, we need to check if the wife too contributes to household work. For this we repeat the optimization exercise setting $t_h = 1$ and $T = 1 + t_w$:

$$\max_{x_w, x_h, t_w} L_N = \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}\right]\left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}\right] + \lambda\left[(1-t_w)\alpha w - x_w - x_h\right]$$

FOCs:

$$\frac{\partial L_N}{\partial x_w} = T^\beta \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}\right] - \lambda \tag{53}$$
$$\frac{\partial L_N}{\partial x_h} = T^\beta \left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}\right] - \lambda \tag{54}$$
$$\frac{\partial L_N}{\partial t_w} = \beta x_w T^{\beta-1}\left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}\right] + \beta x_h T^{\beta-1}\left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}\right] - \lambda \alpha w \tag{55}$$
$$\frac{\partial L_N}{\partial \lambda} = (1-t_w)\alpha w - x_w - x_h \tag{56}$$

Setting the first order conditions to zero and equating 53 and 54-

$$T^\beta \left[x_w \cdot T^\beta - \frac{\beta^\beta \alpha w}{(1+\beta)^{1+\beta}}\right] = T^\beta \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}}\right] \tag{57}$$

or

$$T^{\beta}(x_h - x_w) = (1-\alpha)\frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}} \tag{58}$$

Using 57 while equating 53 and 55:

$$x_h + x_w = \frac{T\alpha w}{\beta} \tag{59}$$

Using these in the budget constraint we find that:

$$t_w = \frac{\beta - 1}{\beta + 1}$$

However, since $\beta < 1$, we must be hitting a boundary condition with respect to t_w. Checking $\frac{\partial L_N}{\partial t_w}$, we see that $\frac{\partial L_N}{\partial t_w} < 0$. Hence, $t_w = 0$ and $T = 1$. Hence, setting the derivatives of the Lagrangian with respect to x_h, x_w and λ to zero we find:

$$\begin{aligned} x_h - x_w &= (1-\alpha)\frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}} \\ x_h + x_w &= \alpha w \end{aligned}$$

Solving simultaneously for x_h and x_w:

$$\begin{aligned} x_h &= \frac{\left[\alpha(1+\beta)^{1+\beta} + (1-\alpha)\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}} \\ x_w &= \frac{\left[\alpha(1+\beta)^{1+\beta} - (1-\alpha)\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}} \end{aligned}$$

The indirect utility functions are hence:

$$\begin{aligned} V_h &= \frac{\left[\alpha(1+\beta)^{1+\beta} + (1-\alpha)\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}} \\ V_w &= \frac{\left[\alpha(1+\beta)^{1+\beta} - (1-\alpha)\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}} \end{aligned}$$

From the analysis of household decision making in the non-patriarchal regime, we learn the following about the couple's labour supply ($l_i = 1 - t_i \ where \ i = w, h$) and welfare:

$$l_h = \begin{cases} 1 & if \ \alpha \in (0,1) \\ \frac{1-\alpha\beta}{1+\beta} & if \ \alpha \in \left[1, \frac{1}{\beta}\right) \\ 0 & if \ \alpha \in \left[\frac{1}{\beta}, \infty\right) \end{cases}$$

$$l_w = \begin{cases} 0 & if \ \alpha \in (0,\beta] \\ \frac{1-\frac{\beta}{\alpha}}{\beta+1} & if \ \alpha \in (\beta, 1) \\ 1 & if \ \alpha \in [1,\infty) \end{cases}$$

$$V_h = \begin{cases} \frac{\left[(1+\beta)^{1+\beta}+(1-\alpha)\beta^{\beta}\right]w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in (0,\beta] \\ \frac{\left[(1+\alpha)^{1+\beta}+(1-\alpha)\alpha^{\beta}\right]\beta^{\beta}w}{2\alpha^{\beta}(1+\beta)^{1+\beta}} & if\ \alpha \in (\beta,1) \\ \frac{\left[(1+\alpha)^{1+\beta}+1-\alpha\right]\beta^{\beta}w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[1,\frac{1}{\beta}\right) \\ \frac{\left[\alpha(1+\beta)^{1+\beta}+(1-\alpha)\beta^{\beta}\right]w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[\frac{1}{\beta},\infty\right) \end{cases}$$

$$V_w = \begin{cases} \frac{\left[(1+\beta)^{1+\beta}-(1-\alpha)\beta^{\beta}\right]w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in (0,\beta] \\ \frac{\left[(1+\alpha)^{1+\beta}-(1-\alpha)\alpha^{\beta}\right]\beta^{\beta}w}{2\alpha^{\beta}(1+\beta)^{1+\beta}} & if\ \alpha \in (\beta,1) \\ \frac{\left[(1+\alpha)^{1+\beta}-1+\alpha\right]\beta^{\beta}w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[1,\frac{1}{\beta}\right) \\ \frac{\left[\alpha(1+\beta)^{1+\beta}-(1-\alpha)\beta^{\beta}\right]w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[\frac{1}{\beta},\infty\right) \end{cases}$$

Each spouse's labour supply under collective decision making is shown in figures 3 and 4. We note here that the labour supply curves of the spouses, despite the conflict of preferences, is identical to when there is no conflict of preference.

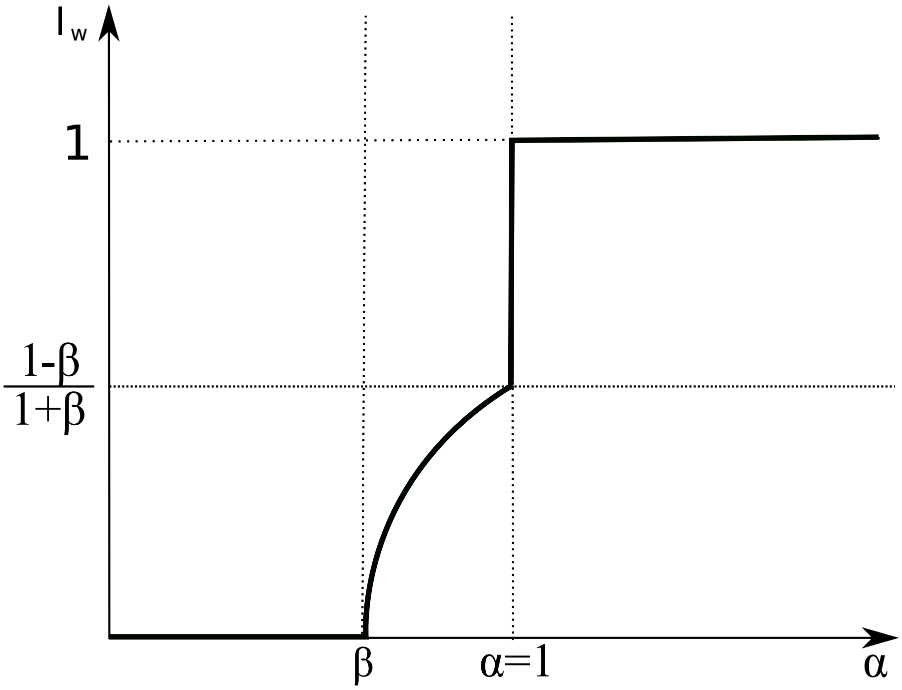

Figure 3: The wife's market labour supply (l_w) as a function of her relative wage α under the collective regime.

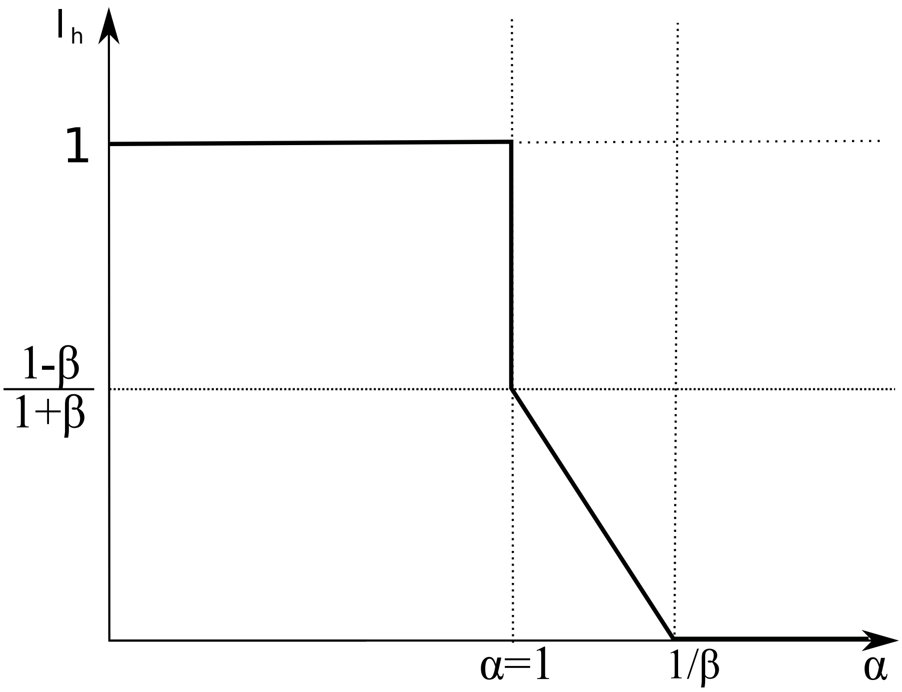

Figure 4: The husband's market labour supply (l_h) as function of the wife's relative wage α under the collective regime.

4.2 The Patriarchal Regime

In this section we explore if the husband gains from segregation of work by gender. In this regime the husband has the option of forcing the wife to stay home full time and not participate in the labour force. If he chooses this option, he engages in the labour force and supplies any residual household time. In this case the husband's threat point remains the same as in the non-patriarchal regime. However, the wife's threat point shifts to 0 if the husband chooses to prevent her from working as she has no outside option given that she is not part of the labour force. The husband chooses between the two regimes depending on which one makes him better off. Further, we also check if the welfare of the wife falls below her reservation utility when single.
The problem which involves bargaining over the household's resources is as follows, if the husband chooses to prevent the wife from working:

$$\max_{x_w, x_h, t_h} N_P = \left[x_h \cdot (T)^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] \left[x_w \cdot (T)^\beta \right]$$

s.t.

$$\begin{aligned} x_w + x_h &= (1 - t_h) w \\ T &= t_h + 1 \end{aligned}$$

Setting up the Lagrangian:

$$\max_{x_w, x_h} L_P = \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] \left[x_w \cdot T^\beta \right] + \lambda \left\{ (1 - t_h) w - x_w - x_h \right\}$$

The first derivatives of the Lagrangian are as follows:

$$\frac{\partial L_P}{\partial x_w} = T^\beta \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] - \lambda \tag{60}$$

$$\frac{\partial L_P}{\partial x_h} = T^\beta \left[x_w \cdot T^\beta \right] - \lambda \tag{61}$$

$$\frac{\partial L_P}{\partial t_h} = \beta x_h T^{\beta-1} \left[x_w \cdot T^\beta \right] + \beta x_w T^{\beta-1} \left[x_h \cdot T^\beta - \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \right] - \lambda w \tag{62}$$

$$\frac{\partial L_P}{\partial \lambda} = (1 - t_h) w - x_w - x_h \tag{63}$$

Setting the first derivatives to 0 and equating 60 and 61 we get:

$$T^\beta (x_h - x_w) = \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \tag{64}$$

Setting equations 60, 61 and 62 to zero we get:

$$x_h + x_w = \frac{Tw}{\beta}$$

Using this in the budget constraint we get:

$$t_h = \frac{\beta - 1}{\beta + 1} \tag{65}$$

Since $\beta < 1$, it must be that we cannot set $\frac{\partial L_P}{\partial t_h}$ to zero. Checking at the boundary we find that at $t_h = 0$, $\frac{\partial L_P}{\partial t_h} < 0$. Hence we have $t_h = 0$ and $T = 1$. Setting derivatives with respect to x_h, x_w and λ to zero we get:

$$\begin{aligned} x_h - x_w &= \frac{\beta^\beta w}{(1+\beta)^{1+\beta}} \\ x_h + x_w &= w \end{aligned}$$

Solving the above equations simultaneously for x_h and x_w we get:

$$\begin{aligned} x_h &= \frac{\beta^\beta + (1+\beta)^{1+\beta}}{2} \frac{w}{(1+\beta)^{1+\beta}} \\ x_w &= \frac{-\beta^\beta + (1+\beta)^{1+\beta}}{2} \frac{w}{(1+\beta)^{1+\beta}} \end{aligned}$$

The couple's indirect utility functions are given by:

$$\begin{aligned} V_h &= \frac{\beta^\beta + (1+\beta)^{1+\beta}}{2} \frac{w}{(1+\beta)^{1+\beta}} \\ V_w &= \frac{-\beta^\beta + (1+\beta)^{1+\beta}}{2} \frac{w}{(1+\beta)^{1+\beta}} \end{aligned}$$

4.2.1 The husband's choice between the patriarchal and non-patriarchal regimes

Whether or not the husband chooses the patriarchal solution over the non-patriarchal solution solely depends on what gives him a higher welfare. We compare his welfare in the two regimes for different values of his wife's relative wage α.

CASE 1: $\alpha \in (0, \beta]$

The husband's indirect utility under the patriarchal solution, and denoted by subscript Ph:

$$V_{Ph} = \frac{[\beta^\beta + (1+\beta)^{1+\beta}]w}{2(1+\beta)^{1+\beta}}$$

The husband's indirect utility under the non-patriarchal solution, and denoted by subscript Nh:

$$V_{Nh} = \frac{\left[(1+\beta)^{1+\beta} + (1-\alpha)\beta^\beta\right] w}{2(1+\beta)^{1+\beta}}$$

Comparing the indirect utilities of the husband, we find:

$$V_{Ph} - V_{Nh} = \frac{\alpha \beta^\beta w}{(1+\beta)^{1+\beta}} > 0 \tag{66}$$

We hence see that $V_{Ph} > V_{Nh}$ for all α in this range.

CASE 2: $\alpha \in (\beta, 1)$

The husband's indirect utility under the patriarchal solution, denoted by subscript Ph:

$$V_{Ph} = \frac{[\beta^\beta + (1+\beta)^{1+\beta}]w}{2(1+\beta)^{1+\beta}}$$

The husband's indirect utility under the non-patriarchal solution, denoted by subscript Nh:

$$V_{Nh} = \frac{\left[(1+\alpha)^{1+\beta} + (1-\alpha)\alpha^{\beta}\right]\beta^{\beta} w}{2\alpha^{\beta}(1+\beta)^{1+\beta}}$$

Comparing the indirect utilities of the husband, we find:

$$V_{Ph} - V_{Nh} = \left[\frac{(1+\beta)^{1+\beta}}{\beta^{\beta}} + \alpha - \frac{(1+\alpha)^{1+\beta}}{\alpha^{\beta}}\right]\frac{\beta^{\beta} w}{2(1+\beta)^{1+\beta}} \tag{67}$$

Using numerical methods, we show that $V_{Ph} > V_{Nh}$ for all α in this range. Refer to Appendix A.

CASE 3: $\alpha \in \left[1, \frac{1}{\beta}\right)$

The husband's indirect utility under the patriarchal solution, denoted by subscript Ph:

$$V_{Ph} = \frac{[\beta^{\beta} + (1+\beta)^{1+\beta}]w}{2(1+\beta)^{1+\beta}}$$

The husband's indirect utility under the non-patriarchal solution, denoted by subscript Nh:

$$V_{Nh} = \frac{\left[(1+\alpha)^{1+\beta} + 1 - \alpha\right]\beta^{\beta} w}{2(1+\beta)^{1+\beta}}$$

Comparing the indirect utilities of the husband, we find:

$$V_{Ph} - V_{Nh} = \left[\frac{(1+\beta)^{1+\beta}}{\beta^{\beta}} + \alpha - (1+\alpha)^{\beta}\right]\frac{\beta^{\beta} w}{2(1+\beta)^{1+\beta}} \tag{68}$$

Using numerical methods, we can show that $V_{Ph} > V_{Nh}$ if and only if $\alpha \leq \overline{\alpha}$ in this range. We see that $\overline{\alpha} > 1$. Further, it is less than $\frac{1}{\beta}$ only for $\beta \in (0, 0.68)$. Refer to Appendix A for the detailed analysis.

CASE 4: $\alpha \in \left[\frac{1}{\beta}, \infty\right)$

The husband's indirect utility under the patriarchal solution, denoted by subscript Ph:

$$V_{Ph} = \frac{[\beta^{\beta} + (1+\beta)^{1+\beta}]w}{2(1+\beta)^{1+\beta}}$$

The husband's indirect utility under the non-patriarchal solution, denoted by subscript Nh:

$$V_{Nh} = \frac{\left[\alpha(1+\beta)^{1+\beta} + (1-\alpha)\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}}$$

Comparing the indirect utilities of the husband, we find:

$$V_{Ph} - V_{Nh} = \left\{(1+\beta)^{1+\beta} - \alpha\left[(1+\beta)^{1+\beta} - \beta^{\beta}\right]\right\}\frac{w}{2(1+\beta)^{1+\beta}} \tag{69}$$

$V_{Ph} > V_{Nh}$ if only if:

$$\alpha \leq \frac{(1+\beta)^{1+\beta}}{(1+\beta)^{1+\beta} - \beta^{\beta}} \equiv \overline{\alpha}$$

Plotting $\overline{\alpha}$ as a function of β, we find that $\overline{\alpha} \geq \frac{1}{\beta}$ only for $\beta \in (0.68, 1)$.
We now summarize the wife's and husband's labour supply ($l_i = 1 - t_i,\ i = w, h$) under the patriarchal regime. If $\overline{\alpha} \in \left(1, \frac{1}{\beta}\right)$:

$$l_h = \begin{cases} 1 & if\ \alpha \in (0,1) \\ 1 & if\ \alpha \in [1, \overline{\alpha}] \\ \frac{1-\alpha\beta}{1+\beta} & if\ \alpha \in \left(\overline{\alpha}, \frac{1}{\beta}\right) \\ 0 & if\ \alpha \in \left[\frac{1}{\beta}, \infty\right) \end{cases}$$

$$l_w = \begin{cases} 0 & if\ \alpha \in (0,\beta] \\ 0 & if\ \alpha \in (\beta, 1) \\ 0 & if\ \alpha \in [1, \overline{\alpha}] \\ 1 & if\ \alpha \in (\overline{\alpha}, \infty) \end{cases}$$

Figures 5 and 6 provide a graphical description of this case.

If $\overline{\alpha} \in \left[\frac{1}{\beta}, \infty\right)$:

$$l_h = \begin{cases} 1 & if\ \alpha \in (0,1) \\ 1 & if\ \alpha \in [1, \overline{\alpha}] \\ 1 & if\ \alpha \in \left(\overline{\alpha}, \frac{1}{\beta}\right) \\ 0 & if\ \alpha \in \left[\frac{1}{\beta}, \infty\right) \end{cases}$$

$$l_w = \begin{cases} 0 & if\ \alpha \in (0,\beta] \\ 0 & if\ \alpha \in (\beta, 1) \\ 0 & if\ \alpha \in [1, \overline{\alpha}] \\ 1 & if\ \alpha \in (\overline{\alpha}, \infty) \end{cases}$$

Figures 7 and 8 provide a graphical description of this case.

4.2.2 Welfare of the wife in the patriarchal regime

The patriarchal solution in marriage does not always provide a woman with her utility when she is single. The wife's indirect utility under the patriarchal solution, denoted by subscript (Pw) is:

$$V_{Pw} = \frac{[-\beta^{\beta} + (1+\beta)^{1+\beta}]w}{2(1+\beta)^{1+\beta}}$$

Her utility as single and denoted by V_{Sw} is:

$$V_{Sw} = \frac{\beta^{\beta}\alpha w}{(1+\beta)^{1+\beta}}$$

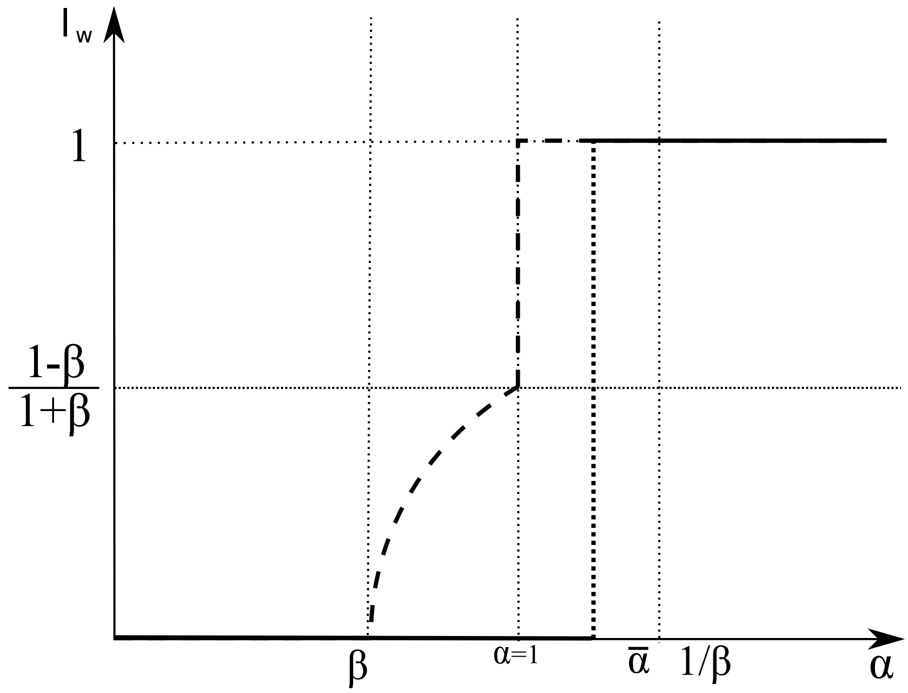

Figure 5: Wife's market labour supply as a function of her relative wage α for $\overline{\alpha} \in \left(1, \frac{1}{\beta}\right)$. The Dashed line indicates the labour supply in the non-patriarchal regime.

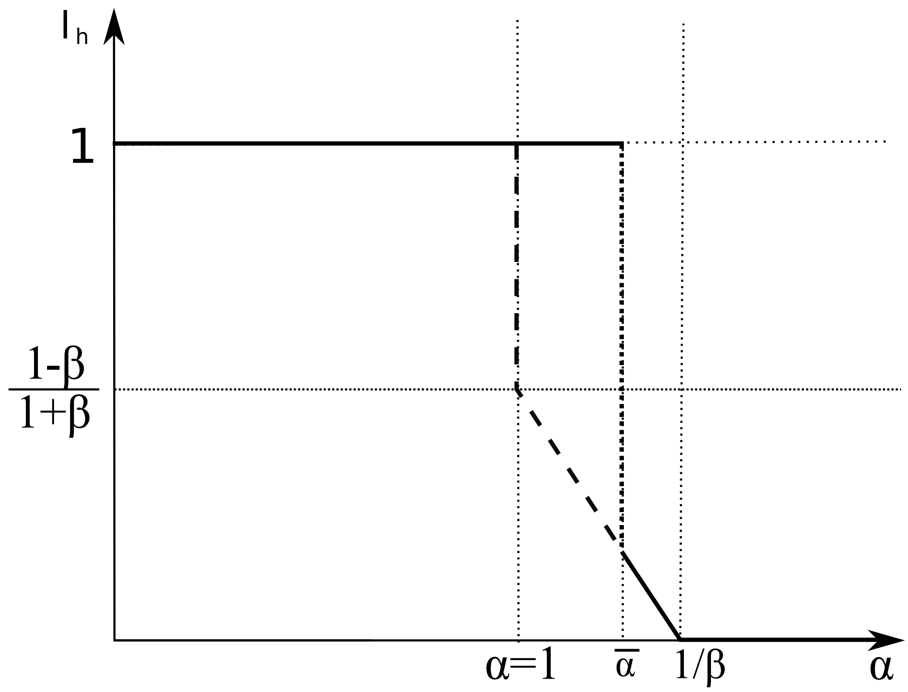

Figure 6: Husband's market labour supply as a function of the wife's relative wage α for $\overline{\alpha} \in \left(1, \frac{1}{\beta}\right)$. The Dashed line indicates the labour supply in the non-patriarchal regime.

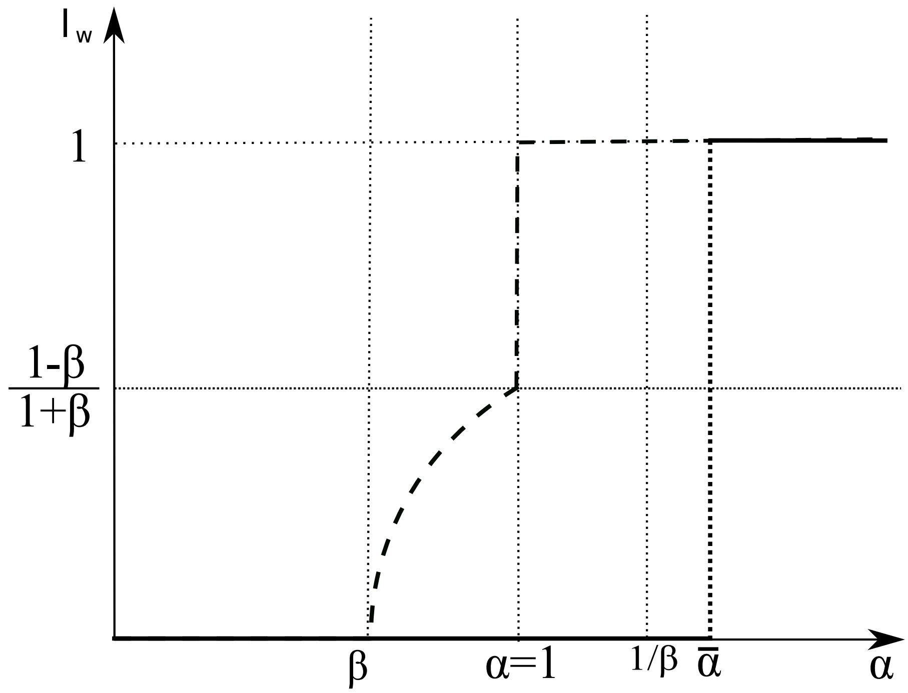

Figure 7: Wife's market labour supply (l_w) as a function of her relative wage α for $\overline{\alpha} \in \left[\frac{1}{\beta}, \infty\right)$. The Dashed line indicates the labour supply in the non-patriarchal regime.

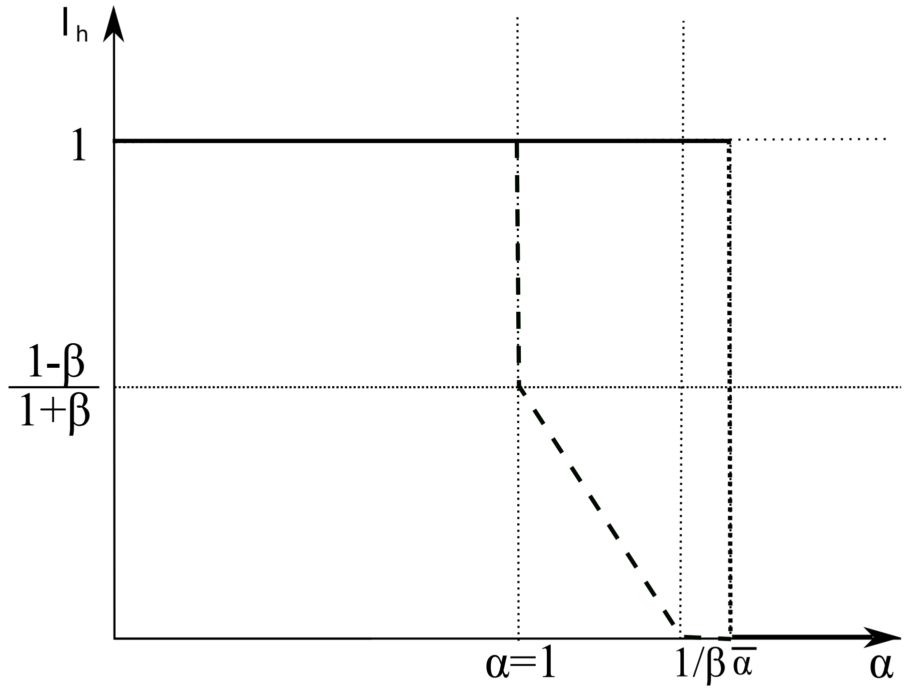

Figure 8: Husband's market labour supply (l_h) as a function of the wife's relative wage α for $\overline{\alpha} \in \left[\frac{1}{\beta}, \infty\right)$. The Dashed line indicates the labour supply in the non-patriarchal regime.

Comparing the two we find that:

$$V_{Pw} - V_{Sw} = \left[\frac{(1+\beta)^{1+\beta}}{2\beta^{\beta}} - \frac{1}{2} - \alpha \right] \frac{\beta^{\beta} w}{(1+\beta)^{1+\beta}}$$

$V_{Pw} - V_{Sw} \geq 0$ if and only if:

$$\alpha \leq \alpha^{D} = \frac{1}{2} \left[\frac{(1+\beta)^{1+\beta}}{\beta^{\beta}} - 1 \right]$$

Plotting α_D as a function of β shows us that α_D is increasing in β. Refer to Appendix A for the numeric plot.

We now summarize the findings of household decision making when there is a conflict of preference between the wife and husband in the following three propositions:

Proposition 2: *When household decisions are made in the non-patriarchal regime, labour supply is determined by considerations of economic efficiency. The spouse who earns more will always work full time irrespective of gender. The other spouse will either do full time domestic work, or split his/her time between market and domestic work.*

Proposition 3: *When household decisions are made in the patriarchal regime, labour supply is determined by considerations of the husband's welfare. The husband faces the dilemma of letting the wife work and enjoying a higher household income, or preventing the wife from entering the labour force to gain bargaining power in the household at the cost of a lower household income. As a result, in the patriarchal regime, under certain conditions, women will be working at home full time even though it would be efficient for them to join the labour force.*

Proposition 4: *When household decisions are made in the patriarchal regime, the wife faces a loss of bargaining power, because she cannot be part of the labour force. As a result of this, she will often not even receive the welfare that she enjoyed when she was single. Over this range of parameters, she will rationally choose to remain single provided society allows that option.*

5 Market Purchased Time in Household Decision Making with Conflict of Preferences

We now extend the model by allowing household time (t_b) to be purchased from the market. This market bought household time is assumed to be a perfect substitute for a couple's own time contributions to household work. Whether or not the household purchases time from the market depends on whether purchasing household help is cheaper than the opportunity cost of not working for the spouses.
We assume that marriage markets are characterized by exogenous matching based on education and is such that a matched couple has identical education level. Men in the

economy earn wages which are proportional to their education level. The wage function at any point of time is given by:

$$w = \pi e$$

Here, π is a positive constant.

Women, however, face discrimination in the job markets and only earn a fraction α^d of the husband's wage even though they have the same education level. However, we assume that there is a household help sector in the economy which allows households to purchase household time at a price $\overline{w}$. This price also acts as a support wage for women. Hence, the relative wage α of wife is α_d when both spouses have education levels that earn them incomes higher than the household help sector price. However, as wage level of women falls below $\overline{w}$ the wage discrimination declines and at very low education levels of the couple, turns in favour of women. Figure 9 shows the relation the husband's wage and the relative wage α of the wife. We now analyze the household's labour market choices in

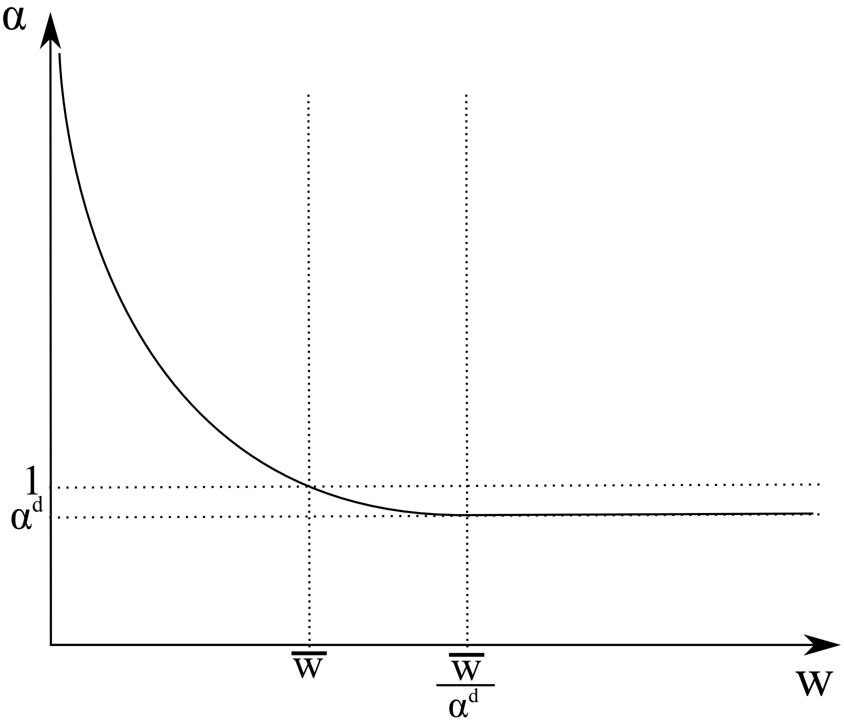

Figure 9: Response of wife's relative wage α to husband's wage w.

each of the two household regimes with objective of mapping the labour supply of women with respect to their husband's wage rate and the wife's education level.

5.1 The Non-Patriarchal Regime

We analyze the household's labour market choices when decision making is in the non-patriarchal regime. We have three cases. The first is when the wife's and husband's wage rate is greater than the price of purchasing household help. The second is when the husband's wage rate is greater but the wife's wage is less than the price of purchasing household help. The third is when both spouses earn less than the price of purchasing household help.

5.1.1 The price of purchasing household time is less than both the wife's and husband's market wages, $\overline{w} < w$ and $\overline{w} < \alpha_d w$.

In this case, the couple will choose to work full time in the market and purchase all the required household time from the market. The household time that is purchased from the market is denoted by t_b. Since the wife and husband earn more than the price of purchasing household time, relative wage of the wife is α_d. Since the household decision making problem is solved using the Nash Bargaining Solution, we first find each spouse's threat-points which comes from the following optimization exercises:

$$\max_{x_h, t_b} x_h \cdot t_b^{\beta} \quad s.t. \;\; x_h + \overline{w} t_b = w \tag{70}$$

$$\max_{x_w, t_b} x_w \cdot t_b^{\beta} \quad s.t. \;\; x_w + \overline{w} t_b = \alpha_d w \tag{71}$$

Solving this we find the following: For the man:

$$\begin{aligned} x_h &= \frac{w}{1+\beta} \\ t_b &= \frac{\beta w}{(1+\beta)\overline{w}} \\ V_0^h &= \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} \end{aligned}$$

For the woman:

$$\begin{aligned} x_w &= \frac{\alpha_d w}{1+\beta} \\ t_b &= \frac{\beta \alpha_d w}{(1+\beta)\overline{w}} \\ V_0^w &= \frac{(\alpha_d)^{1+\beta} \left(\frac{\beta w}{\overline{w}}\right)^{\beta} w}{(1+\beta)^{1+\beta}} \end{aligned}$$

The indirect utility of the husband and wife, when single, gives us their threat points. This allows us to define the household's decision making problem which is as follows:

$$\max_{x_w, x_h, t_b} N = \left[x_h \cdot t_b^{\beta} - \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} \right] \left[x_w \cdot t_b^{\beta} - \frac{(\frac{\beta w}{\overline{w}})^{\beta} \alpha_d^{1+\beta} w}{(1+\beta)^{1+\beta}} \right]$$

s.t.

$$x_w + x_h + \overline{w} t_b = (1+\alpha_d) w$$

Setting up the Lagrangian we have:

$$\begin{aligned} \max_{x_w, x_h, t_b} L = & \left[x_h \cdot t_b^{\beta} - \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} \right] \left[x_w \cdot t_b^{\beta} - \frac{(\frac{\beta w}{\overline{w}})^{\beta} \alpha_d^{1+\beta} w}{(1+\beta)^{1+\beta}} \right] \\ & + \lambda \left[(1+\alpha_d) w - x_h - x_w - \overline{w} t_b \right] \end{aligned}$$

First derivatives:

$$\frac{\partial L_N}{\partial x_w} = t_b^\beta \left[x_h \cdot t_b^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}}\right] - \lambda \tag{72}$$

$$\frac{\partial L_N}{\partial x_h} = t_b^\beta \left[x_w \cdot t_b^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta \alpha_d^{1+\beta} w}{(1+\beta)^{1+\beta}}\right] - \lambda \tag{73}$$

$$\frac{\partial L_N}{\partial t_b} = \beta x_w t_b^{\beta-1} \left[x_h \cdot t_b^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}}\right] + \beta x_h t_b^{\beta-1} \left[x_w \cdot t_b^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta \alpha_d^{1+\beta} w}{(1+\beta)^{1+\beta}}\right] - \lambda \overline{w} \tag{74}$$

$$\frac{\partial L_N}{\partial \lambda} = (1+\alpha_d)w - x_h - x_w - \overline{w} t_b \tag{75}$$

We set all the first derivatives of the Lagrangian to zero and find the following:

$$t_b^\beta (x_h - x_w) = (1 - \alpha_d^{1+\beta}) \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}} \tag{76}$$

and,

$$x_h + x_w = \frac{t_b \overline{w}}{\beta} \tag{77}$$

Using 77 in the budget constraint:

$$t_b = \frac{\beta w (1+\alpha_d)}{\overline{w}(1+\beta)}$$

Using optimal t_b in 76 and 77, and solving simultaneously for x_h and x_w :

$$x_h = \frac{1}{2}\left[(1+\alpha_d)^{1+\beta} + (1 - \alpha_d^{1+\beta})\right] \frac{w}{(1+\alpha_d)^\beta (1+\beta)}$$
$$x_w = \frac{1}{2}\left[(1+\alpha_d)^{1+\beta} - (1 - \alpha_d^{1+\beta})\right] \frac{w}{(1+\alpha_d)^\beta (1+\beta)}$$

We can now calculate the indirect utility functions of the wife and husband:

$$V_h = \frac{1}{2}\left[(1+\alpha_d)^{1+\beta} + (1 - \alpha_d^{1+\beta})\right] \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}}$$
$$V_w = \frac{1}{2}\left[(1+\alpha_d)^{1+\beta} - (1 - \alpha_d^{1+\beta})\right] \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}}$$

5.1.2 The price of purchasing household time is more than the wife's market wages and less than the husband's wages, $\alpha_d w \leq \overline{w}$ and $\overline{w} < w$.

Since the wife's wage is supported by the household help sector price, she is indifferent between purchasing household help or supplying it herself. We assume that she chooses to work full time in the market and purchase all the needed household help from the market. The household decision making problem is identical to the previous case with the only difference that the relative wage $\alpha = \frac{\overline{w}}{w} > \alpha_d$. Hence we have:

$$
\begin{aligned}
t_b &= \frac{\beta w(1+\alpha)}{\overline{w}(1+\beta)} \\
x_h &= \frac{1}{2}\left[(1+\alpha)^{1+\beta} + (1-\alpha^{1+\beta})\right]\frac{w}{(1+\alpha)^{\beta}(1+\beta)} \\
x_w &= \frac{1}{2}\left[(1+\alpha)^{1+\beta} - (1-\alpha^{1+\beta})\right]\frac{w}{(1+\alpha)^{\beta}(1+\beta)} \\
V_h &= \frac{1}{2}\left[(1+\alpha)^{1+\beta} + (1-\alpha^{1+\beta})\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} \\
V_w &= \frac{1}{2}\left[(1+\alpha)^{1+\beta} - (1-\alpha^{1+\beta})\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}
\end{aligned}
$$

5.1.3 The price of purchasing household time is more than wage rate of both spouses, $\alpha_d w < \overline{w}$ and $w \leq \overline{w}$.

Here, $\alpha = \frac{\overline{w}}{w} > 1$. As the husband's wage falls, α keeps rising and the analysis is identical to the case when $\alpha > 1$ and there is no household help sector.

Considering that $w = \frac{\overline{w}}{\alpha}$ we see that:

$$
l_h = \begin{cases} \frac{1-\alpha\beta}{1+\beta} & if\ \alpha \in \left[1, \frac{1}{\beta}\right) \quad or\ w \in (\beta\overline{w}, \overline{w}] \\ 0 & if\ \alpha \in \left[\frac{1}{\beta}, \infty\right) \quad or\ w \in (0, \beta\overline{w}] \end{cases}
$$

$$
l_w = 1 \qquad if\ \alpha \in [1, \infty)\ or\ w \in (0, \overline{w}]
$$

$$
V_h = \begin{cases} \frac{\left[(1+\alpha)^{1+\beta}+1-\alpha\right]\beta^{\beta} w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[1, \frac{1}{\beta}\right) \quad or\ w \in (\beta\overline{w}, \overline{w}] \\ \frac{\left[\alpha(1+\beta)^{1+\beta}+(1-\alpha)\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[\frac{1}{\beta}, \infty\right) \quad or\ w \in (0, \beta\overline{w}] \end{cases}
$$

$$
V_w = \begin{cases} \frac{\left[(1+\alpha)^{1+\beta}-1+\alpha\right]\beta^{\beta} w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[1, \frac{1}{\beta}\right) \quad or\ w \in (\beta\overline{w}, \overline{w}] \\ \frac{\left[\alpha(1+\beta)^{1+\beta}-(1-\alpha)\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}} & if\ \alpha \in \left[\frac{1}{\beta}, \infty\right) \quad or\ w \in (0, \beta\overline{w}] \end{cases}
$$

Summarizing the results of the non-patriarchal regime and replacing $\alpha = \frac{\overline{w}}{w}$ we have the following:

$$
l_h = \begin{cases} 0 & if\ w \in (0, \beta\overline{w}] \\ \frac{1-\frac{\overline{w}\beta}{w}}{1+\beta} & if\ w \in (\beta\overline{w}, \overline{w}] \\ 1 & if\ w \in (\overline{w}, \infty) \end{cases}
$$

$$l_w = 1 \qquad \forall\ w \in (0, \infty)$$

$$V_h = \begin{cases} \frac{\left[\frac{\overline{w}}{w}(1+\beta)^{1+\beta} + (1-\frac{\overline{w}}{w})\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}} & if\ w \in (0, \beta\overline{w}] \\ \frac{\left[(1+\frac{\overline{w}}{w})^{1+\beta} + 1 - \frac{\overline{w}}{w}\right]\beta^{\beta} w}{2(1+\beta)^{1+\beta}} & if\ w \in (\beta\overline{w}, \overline{w}] \\ \frac{1}{2}\left[(1+\frac{\overline{w}}{w})^{1+\beta} + 1 - \left(\frac{\overline{w}}{w}\right)^{1+\beta}\right] \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ w \in \left(\overline{w}, \frac{\overline{w}}{\alpha_d}\right] \\ \frac{1}{2}\left[(1+\alpha_d)^{1+\beta} + 1 - \alpha_d^{1+\beta}\right] \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ w \in \left(\frac{\overline{w}}{\alpha_d}, \infty\right) \end{cases}$$

$$V_w = \begin{cases} \frac{\left[\frac{\overline{w}}{w}(1+\beta)^{1+\beta} - 1 + \frac{\overline{w}}{w}\beta^{\beta}\right] w}{2(1+\beta)^{1+\beta}} & if\ w \in (0, \beta\overline{w}] \\ \frac{\left[(1+\frac{\overline{w}}{w})^{1+\beta} - 1 + \frac{\overline{w}}{w}\right]\beta^{\beta} w}{2(1+\beta)^{1+\beta}} & if\ w \in (\beta\overline{w}, \overline{w}] \\ \frac{1}{2}\left[(1+\frac{\overline{w}}{w})^{1+\beta} - 1 + \left(\frac{\overline{w}}{w}\right)^{1+\beta}\right] \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ w \in \left(\overline{w}, \frac{\overline{w}}{\alpha_d}\right] \\ \frac{1}{2}\left[(1+\alpha_d)^{1+\beta} - 1 + \alpha_d^{1+\beta}\right] \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} & if\ w \in \left(\frac{\overline{w}}{\alpha_d}, \infty\right) \end{cases}$$

5.2 The Patriarchal Regime

In the patriarchal regime, the husband has the option of making the wife supply household help full time and engaging himself in the labour market. Any residual demand for household time is met by the husband or purchased from the market depending on its affordability. We analyze this regime of household decision making for two cases, the first when the husband's wage is greater than the price of purchasing household time and the second when it is less.

5.2.1 The price of purchasing household time is less than the husband's wages, $\overline{w} < w$.

Here, the relative wage of the wife is $\alpha \in [\alpha_d, 1)$. In this case, we assume that the husband works full time and the wife stays at home full time. Any additional household time over and above the wife's contribution is purchased from the market. The husband's threat point remains the same as in the corresponding case in the non-patriarchal regime. However, when the husband prevents the wife from entering the labour force, her threat point falls to zero. Hence, the couple's threat points are as follows:

$$\begin{aligned} V_0^h &= \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} \\ V_0^w &= 0 \end{aligned}$$

The household decision making problem is hence given as follows:

$$\max_{x_w, x_h, t_b} \left[x_h \cdot (T)^{\beta} - \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} \right] \left[x_w \cdot (T)^{\beta} \right]$$

s.t.

$$\begin{aligned} x_w + x_h + \overline{w}t_b &= w \\ T &= 1 + t_b \end{aligned}$$

Setting up the Lagrangian we have:

$$\max_{x_w, x_h, t_b} L_P = \left[x_h \cdot (1+t_b)^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}}\right]\left[x_w \cdot (1+t_b)^\beta\right] + \lambda\left[w - x_w - x_h - \overline{w}t_b\right]$$

First derivatives of the Lagrangian are as follows:

$$\frac{\partial L_P}{\partial x_w} = (1+t_b)^\beta \left[x_h \cdot (1+t_b)^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}}\right] - \lambda \tag{78}$$

$$\frac{\partial L_P}{\partial x_h} = (1+t_b)^\beta \left[x_w \cdot (1+t_b)^\beta\right] - \lambda \tag{79}$$

$$\begin{aligned} \frac{\partial L_P}{\partial t_b} &= \beta x_w (1+t_b)^{\beta-1} \left[x_h \cdot (1+t_b)^\beta - \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}}\right] \\ &\quad + \beta x_h (1+t_b)^{\beta-1}\left[x_w \cdot (1+t_b)^\beta\right] - \lambda \overline{w} \end{aligned} \tag{80}$$

$$\frac{\partial L_P}{\partial \lambda} = w - x_h - x_w - \overline{w}t_b \tag{81}$$

We set all the first derivatives of the Lagrangian to zero and find the following:

$$(1+t_b)^\beta (x_h - x_w) = \frac{(\frac{\beta w}{\overline{w}})^\beta w}{(1+\beta)^{1+\beta}} \tag{82}$$

and,

$$x_h + x_w = \frac{(1+t_b)\overline{w}}{\beta} \tag{83}$$

Using 83 in the budget constraint:

$$t_b = \frac{\frac{\beta w}{\overline{w}} - 1}{1+\beta}$$

We can see here that the household time bought can potentially have a boundary solution i.e.

$$t_b = \begin{cases} 0 & if\ w \leq \frac{\overline{w}}{\beta} \\ \frac{\frac{\beta w}{\overline{w}} - 1}{1+\beta} & if\ w > \frac{\overline{w}}{\beta} \end{cases} \tag{84}$$

If t_b hits a boundary solution, it must mean that we cannot set $\frac{\partial L}{\partial t_b} = 0$ given that $\frac{\partial L}{\partial t_b} < 0$ at $t_b = 0$, which is easily verified. We hence solve for the unknowns in both cases.

Purchased household time has a boundary solution, $w \leq \frac{\overline{w}}{\beta}$

Here we have $t_h = 0$, $t_w = 1$, $t_b = 0$ and $T = 1$. We set the derivatives of the Lagrangian with respect to x_h, x_w and λ to zero. This gives us:

$$x_h + x_w = w \tag{85}$$

and

$$x_h - x_w = \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} \tag{86}$$

Solving the linear equations, we get:

$$V_h = x_h = \frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{\left(\frac{\beta w}{\overline{w}}\right)^{\beta}} + 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$
$$V_w = x_w = \frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{\left(\frac{\beta w}{\overline{w}}\right)^{\beta}} - 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

Purchased household time has an interior solution, $w > \frac{\overline{w}}{\beta}$

Here we have $t_h = 0$, $t_w = 1$, $t_b = \frac{\frac{\beta w}{\overline{w}} - 1}{1+\beta}$ and $T = \frac{\beta\left(\frac{w}{\overline{w}}+1\right)}{1+\beta}$. Using this in 82 and 83:

$$x_h + x_w = \frac{\overline{w} + w}{(1+\beta)} \tag{87}$$

and

$$x_h - x_w = \frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}} \tag{88}$$

Solving the linear equations, we get:

$$x_h = \frac{1}{2}\left[\left(1 + \frac{\overline{w}}{w}\right)^{1+\beta} + 1\right]\frac{(\delta)^{\beta} w}{(1+\delta)^{\beta}(1+\beta)}$$
$$x_w = \frac{1}{2}\left[\left(1 + \frac{\overline{w}}{w}\right)^{1+\beta} - 1\right]\frac{(\delta)^{\beta} w}{(1+\delta)^{\beta}(1+\beta)}$$

The indirect utility functions are given by:

$$V_h = \frac{1}{2}\left[\left(1 + \frac{\overline{w}}{w}\right)^{1+\beta} + 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$
$$V_w = \frac{1}{2}\left[\left(1 + \frac{\overline{w}}{w}\right)^{1+\beta} - 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

The husband has the option of preventing his wife's participation in the labour force, but he chooses this option over the non-patriarchal solution only if it increases his welfare. Hence, we compare the husband's welfare under the patriarchal solution with his welfare under the non-patriarchal solution to figure out which one he will choose. The woman's utility under dictatorship may or may not exceed her reservation utility as single. Despite this, she is married because of the exogenous nature of match making in the marriage market which doesn't give her an option of being single. The welfare of the husband for different ranges of w for both regimes is as follows:

$$\boxed{w \in \left(\overline{w}, \frac{\overline{w}}{\beta}\right]}$$

The indirect utility of the husband under patriarchal solution, denoted by the subscript Ph is:

$$V_{Ph} = \frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{(\frac{\beta w}{\overline{w}})^{\beta}} + 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

The indirect utility to the husband under the non-patriarchal solution, denoted by the subscript Nh is:

$$V_{Nh} = \frac{1}{2}\left[(1+\alpha)^{1+\beta} + 1 - (\alpha)^{1+\beta}\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

Comparing the two, we find that:

$$V_{Ph} - V_{Nh} = \frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{(\frac{\beta w}{\overline{w}})^{\beta}} - (1+\alpha)^{1+\beta} + (\alpha)^{1+\beta}\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

$V_{Ph} - V_{Nh} \geq 0$ iff:

$$\frac{(1+\beta)^{1+\beta}}{(\frac{\beta w}{\overline{w}})^{\beta}} - (1+\alpha)^{1+\beta} + (\alpha)^{1+\beta} \geq 0$$

or,

$$w \leq \overline{\delta}\overline{w} \quad here\ \overline{\delta} = \frac{1}{\beta}\left[\frac{(1+\beta)^{1+\beta}}{(1+\alpha)^{1+\alpha} - \alpha^{1+\alpha}}\right]^{\frac{1}{\beta}}$$

$$\boxed{w \in \left(\frac{\overline{w}}{\beta}, \infty\right)}$$

The indirect utility of the husband under patriarchal solution, denoted by the subscript Ph is:

$$V_{Ph} = \frac{1}{2}\left[\left(1 + \frac{\overline{w}}{w}\right)^{1+\beta} + 1\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

The indirect utility to the husband under the non-patriarchal solution, denoted by the subscript Nh is:

$$V_{Nh} = \frac{1}{2}\left[(1+\alpha^{e})^{1+\beta} + (1 - (\alpha^{e})^{1+\beta})\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

Comparing the two, we find that:

$$V_{Ph} - V_{Nh} = \frac{1}{2}\left[\left(1 + \frac{\overline{w}}{w}\right)^{1+\beta} - (1+\alpha)^{1+\beta} + (\alpha)^{1+\beta}\right]\frac{(\frac{\beta w}{\overline{w}})^{\beta} w}{(1+\beta)^{1+\beta}}$$

$V_{Ph} - V_{Nh} \geq 0$ iff:

$$\left(1 + \frac{\overline{w}}{w}\right)^{1+\beta} - (1+\alpha)^{1+\beta} + (\alpha)^{1+\beta} \geq 0$$

or,

$$w \leq \overline{\delta}\overline{w} \qquad here\ \overline{\delta} = \frac{1}{[(1+\alpha)^{1+\alpha} - \alpha^{1+\alpha}]^{\frac{1}{1+\beta}} - 1}$$

We see that $\overline{\delta}$, in both the cases, is an increasing function of wage, when $w \in \left(\overline{w}, \frac{\overline{w}}{\alpha_d}\right]$. $\overline{\delta}$ reaches a maximum value $\overline{\delta}_d$ when $\alpha = \alpha_d$. Hence the binding constraint for the patriarchal solution to be better than the non-patriarchal solution for the husband is $w \leq \overline{\delta}_d\overline{w}$.

The analysis of the husband's welfare under patriarchy tells us that, as the husband's wage rises, purchasing household time becomes progressively cheaper. When $w > \overline{\delta}_d\overline{w}$, the husband is better-off following the non-patriarchal solution and letting the wife participate in the labour market.

5.2.2 The price of purchasing household time is more than the husband's wages, $w \leq \overline{w}$

In this range as the husband's wage keeps falling, α keeps rising. This corresponds to the patriarchal regime when $\alpha > 1$ with no market for household help. The couple's labour supply is as follows if $\overline{\alpha} \in \left(1, \frac{1}{\beta}\right)$:

$$l_h = \begin{cases} 0 & if\ w \in (0, \beta\overline{w}] \\ \frac{1 - \frac{\beta\overline{w}}{w}}{1+\beta} & if\ w \in \left(\beta\overline{w}, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 1 & if\ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \overline{w}\right] \end{cases}$$

$$l_w = \begin{cases} 1 & if\ w \in \left(0, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 0 & if\ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \overline{w}\right] \end{cases}$$

If $\overline{\alpha} \in \left[\frac{1}{\beta}, \infty\right)$:

$$l_h = \begin{cases} 0 & if\ w \in \left(0, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 1 & if\ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \overline{w}\right] \end{cases}$$

$$l_w = \begin{cases} 1 & if\ w \in \left(0, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 0 & if\ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \overline{w}\right] \end{cases}$$

Summarizing the couple's labour supply for household decision making in a patriarchal regime, where household help can be purchased from the market, is as follows if $\overline{\alpha} \in \left(1, \frac{1}{\beta}\right)$:

$$l_h = \begin{cases} 0 & if \ w \in (0, \beta\overline{w}] \\ \frac{1-\frac{\beta\overline{w}}{w}}{1+\beta} & if \ w \in \left(\beta\overline{w}, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 1 & if \ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \infty\right) \end{cases}$$

$$l_w = \begin{cases} 1 & if \ w \in \left(0, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 0 & if \ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \overline{\delta}_d\overline{w}\right] \\ 1 & if \ w \in \left(\overline{\delta}_d\overline{w}, \infty\right) \end{cases}$$

If $\overline{\alpha} \in \left[\frac{1}{\beta}, \infty\right)$:

$$l_h = \begin{cases} 0 & if \ w \in \left(0, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 1 & if \ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \infty\right) \end{cases}$$

$$l_w = \begin{cases} 1 & if \ w \in \left(0, \frac{\overline{w}}{\overline{\alpha}}\right) \\ 0 & if \ w \in \left[\frac{\overline{w}}{\overline{\alpha}}, \overline{\delta}_d\overline{w}\right] \\ 1 & if \ w \in \left(\overline{\delta}_d\overline{w}, \infty\right) \end{cases}$$

Since the wage rate is a function of education i.e. $w = \pi e$, we can map the couple's labour supply with respect to their common education level as well. This is as follows if $\overline{\alpha} \in \left(1, \frac{1}{\beta}\right)$:

$$l_h = \begin{cases} 0 & if \ e \in (0, \frac{\beta\overline{w}}{\pi}] \\ \frac{1-\frac{\beta\overline{w}}{w}}{1+\beta} & if \ e \in \left(\frac{\beta\overline{w}}{\pi}, \frac{\overline{w}}{\pi\overline{\alpha}}\right) \\ 1 & if \ e \in \left[\frac{\overline{w}}{\pi\overline{\alpha}}, \infty\right) \end{cases}$$

$$l_w = \begin{cases} 1 & if \ e \in \left(0, \frac{\overline{w}}{\pi\overline{\alpha}}\right) \\ 0 & if \ e \in \left[\frac{\overline{w}}{\pi\overline{\alpha}}, \frac{\overline{\delta}_d\overline{w}}{\pi}\right] \\ 1 & if \ e \in \left(\frac{\overline{\delta}_d\overline{w}}{\pi}, \infty\right) \end{cases}$$

If $\overline{\alpha} \in \left[\frac{1}{\beta}, \infty\right)$:

$$l_h = \begin{cases} 0 & if\ w \in \left(0, \frac{\overline{w}}{\pi\overline{\alpha}}\right) \\ 1 & if\ w \in \left[\frac{\overline{w}}{\pi\overline{\alpha}}, \infty\right) \end{cases}$$

$$l_w = \begin{cases} 1 & if\ e \in \left(0, \frac{\overline{w}}{\pi\overline{\alpha}}\right) \\ 0 & if\ e \in \left[\frac{\overline{w}}{\pi\overline{\alpha}}, \frac{\overline{\delta}_d\overline{w}}{\pi}\right] \\ 1 & if\ e \in \left(\frac{\overline{\delta}_d\overline{w}}{\pi}, \infty\right) \end{cases}$$

We now focus on the labour supply of the wife as a function of the husband's wage rate and her education level. Figures 10 and 11 plot the wife's labour supply as a function of husband's wage and education level respectively. The following proposition summarizes the findings of this section:

Proposition 5: *The labour force participation of married women follows a U-shaped pattern with respect to her education. Participation remains low for intermediate levels of education, and is high for low and high levels of education.*

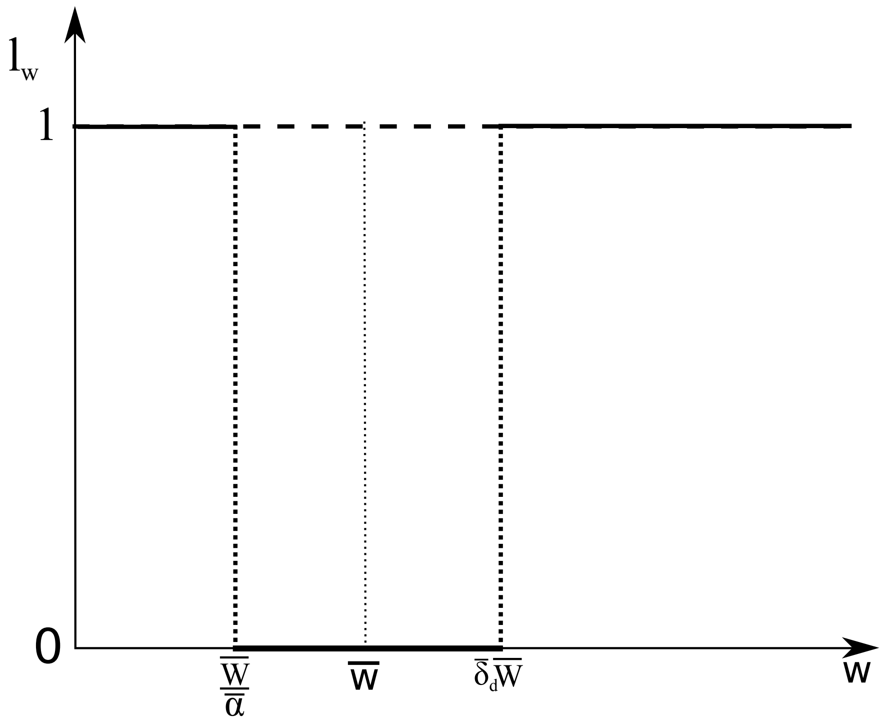

Figure 10: Wife's labour supply (l_w) as a function of her husband's wage rate w in the patriarchal regime. The Dashed line indicates the labour supply in the non-patriarchal regime.

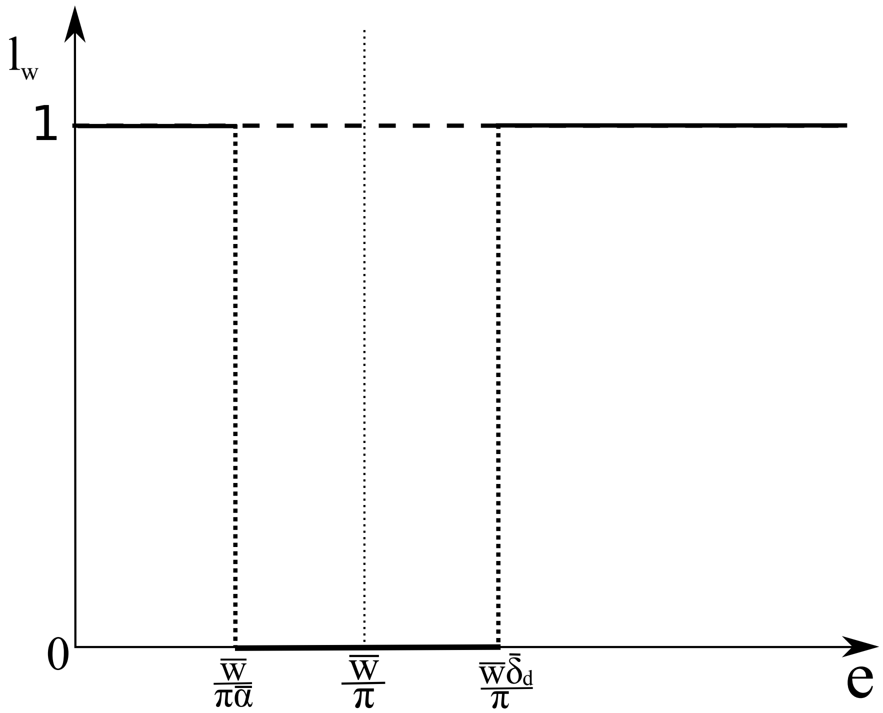

Figure 11: Wife's labour supply (l_w) as a function of her education level e in the patriarchal regime. The Dashed line indicates the labour supply in the non-patriarchal regime.

6 Conclusions

We have two key findings in this chapter. The first is that patriarchal households, where men have the choice between a gender neutral and a gender specific division of work, will choose women's participation in the labour force to maximize the welfare of men and this generates a dead-weight loss in the households. The labour supply that results from this household structure may lead to women not participating in the labour force even if they have a higher earning potential than their husbands. The second is that when we introduce a market for household help, we see that women's labour supply, as a function of her own education and husband's wage, follows a U-shaped, which is observed in the empirical literature. This paper hence provides a theoretical framework that helps explain the puzzling empirical findings on female labour force participation.

Our study suggests that the patriarchal structure of households can have extremely adverse welfare implications for married women and leads to loss of efficiency in economy due to dead-weight losses in households. Thus it is imperative to focus on policies that raise women's bargaining power within the family. Policies that put money in the hands of women will improve efficiency in household decision making. This is because money in the hands of women will help increase their bargaining power in the household, which in turn will reduce the gain to the husband from preventing the wife from working. Hence, this study lends further support to the argument that programs which have conditional cash transfers as incentives should be designed to transfer the money to women. This has the dual benefit of providing incentives to adopt the program as well as reducing dead-weight losses by raising women's say in the household. Further, the U-shaped labour supply curve of women with respect to education tells us that higher education of women need not always lead to higher labour force engagement in patriarchal societies. Unless women are empowered to break out of the patriarchal household set-up, education will not have an unambiguous positive effect on their welfare.

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Appendices

A Numerical Analysis

A.1 Numerical Analysis for Welfare Comparisons between the Patriarchal and Non-Patriarchal Solutions

The husband will choose to prevent his wife from joining the labour market if exercising this option improves his welfare.

A.1.1 CASE: $\alpha \in (\beta, 1)$

$$V_{Ph} - V_{Nh} = \left[\frac{(1+\beta)^{1+\beta}}{\beta^\beta} + \alpha - \frac{(1+\alpha)^{1+\beta}}{\alpha^\beta}\right] \frac{\beta^\beta w}{2(1+\beta)^{1+\beta}} \tag{89}$$

$V_{Ph} \geq V_{Nh}$ iff:

$$\frac{(1+\beta)^{1+\beta}}{\beta^\beta} + \alpha - \frac{(1+\alpha)^{1+\beta}}{\alpha^\beta} \geq 0 \tag{90}$$

We plot the $V_{Ph} - V_{Nh}$ for different values of β in figures 12, 13, 14 and 15. It is seen that $V_{Ph} - V_{Nh}$ never falls below 0 and is lower bounded by β.

A.1.2 CASE: $\alpha \in \left[1, \frac{1}{\beta}\right)$

$$V_{Ph} - V_{Nh} = \left[\frac{(1+\beta)^{1+\beta}}{\beta^\beta} + \alpha - (1+\alpha)^\beta\right] \frac{\beta^\beta w}{2(1+\beta)^{1+\beta}} \tag{91}$$

$$V_{Ph} - V_{Nh} \geq 0$$

Iff,

$$\frac{(1+\beta)^{1+\beta}}{\beta^\beta} + \alpha - (1+\alpha)^\beta \geq 0$$

We plot $\frac{(1+\beta)^{1+\beta}}{\beta^\beta} + \alpha - (1+\alpha)^\beta$ for different values of $\beta \in (0, 1)$. The numerical plots are given in figures 16, 17, 18 and 19. We observe for the range $\alpha \in \left[1, \frac{1}{\beta}\right)$, there exists an $\overline{\alpha}$ such that when $\alpha \leq \overline{\alpha}$, $V_{Ph} \geq V_{Nh}$. However,$\alpha \in \left[1, \frac{1}{\beta}\right)$ only if $\beta \in (0, 0.68)$. Further,we observe that $\overline{\alpha}$ is a decreasing function of β.

A.1.3 CASE: $\alpha \in \left[\frac{1}{\beta}, 1\right)$

$$V_{Ph} - V_{Nh} \geq 0$$

Iff,

$$\alpha \leq \overline{\alpha} = \frac{(1+\beta)^{1+\beta}}{(1+\beta)^{1+\beta} - \beta^\beta}$$

The plot of $\overline{\alpha}$ as a function of β is given in figure 20. We identify numerically that $\overline{\alpha}$ exceed $\frac{1}{\beta}$ only for $\beta \in (0.68, 1)$. Further,we observe that $\overline{\alpha}$ is a decreasing function of β.

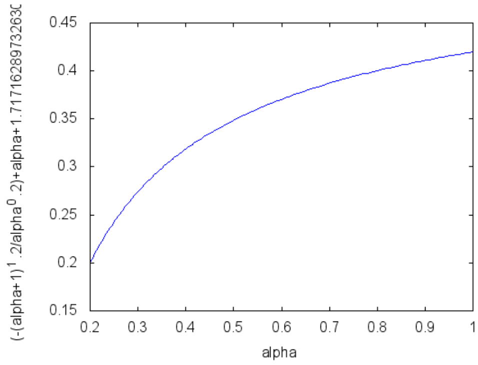

Figure 12: $V_{Ph} - V_{Nh}$ for $\beta = 0.2$ for $\alpha \in (\beta, 1)$

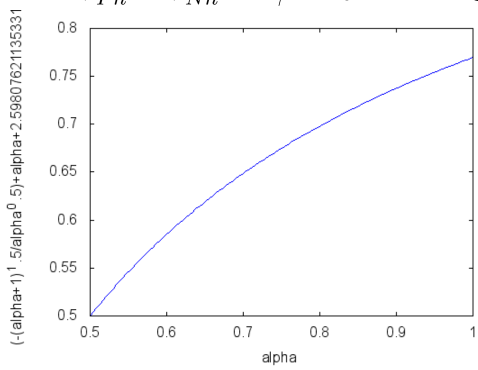

Figure 13: $V_{Ph} - V_{Nh}$ for $\beta = 0.5$ for $\alpha \in (\beta, 1)$

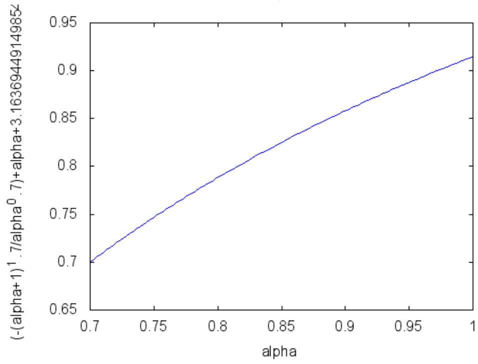

Figure 14: $V_{Ph} - V_{Nh}$ for $\beta = 0.7$ for $\alpha \in (\beta, 1)$

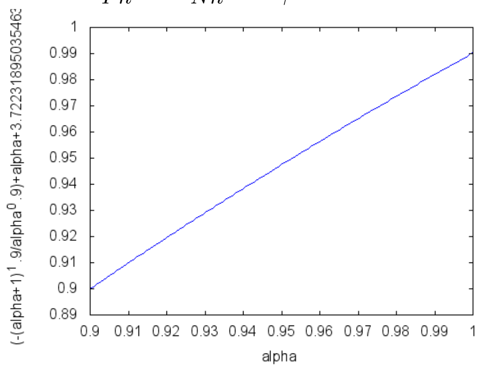

Figure 15: $V_{Ph} - V_{Nh}$ for $\beta = 0.9$ for $\alpha \in (\beta, 1)$

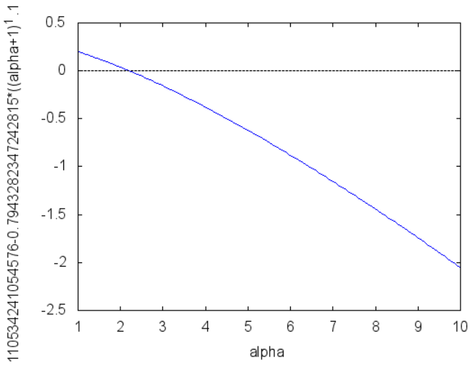

Figure 16: $V_{Ph} - V_{Nh}$ for $\beta = 0.1$, with numerically identified $\overline{\alpha} = 2.2$.

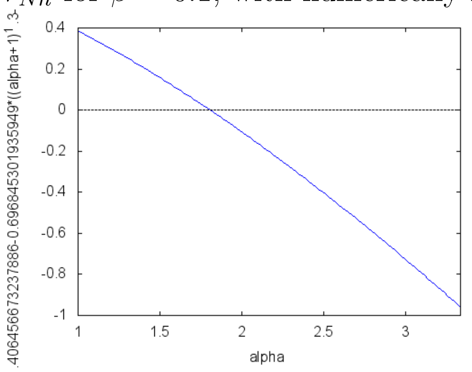

Figure 17: $V_{Ph} - V_{Nh}$ for $\beta = 0.3$ with numerically identified $\overline{\alpha} = 1.8$.

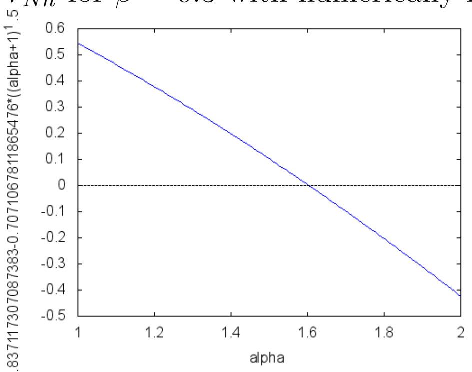

Figure 18: $V_{Ph} - V_{Nh}$ for $\beta = 0.5$ with numerically identified $\overline{\alpha} = 1.475$.

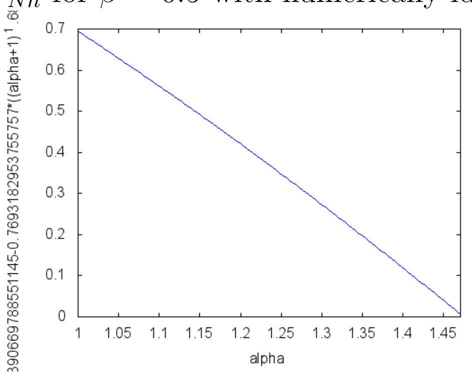

Figure 19: $V_{Ph} - V_{Nh}$ for $\beta = 0.68$ with numerically identified $\overline{\alpha}$ exceeding $\frac{1}{\beta}$ and hence dictatorship holds in the entire range.

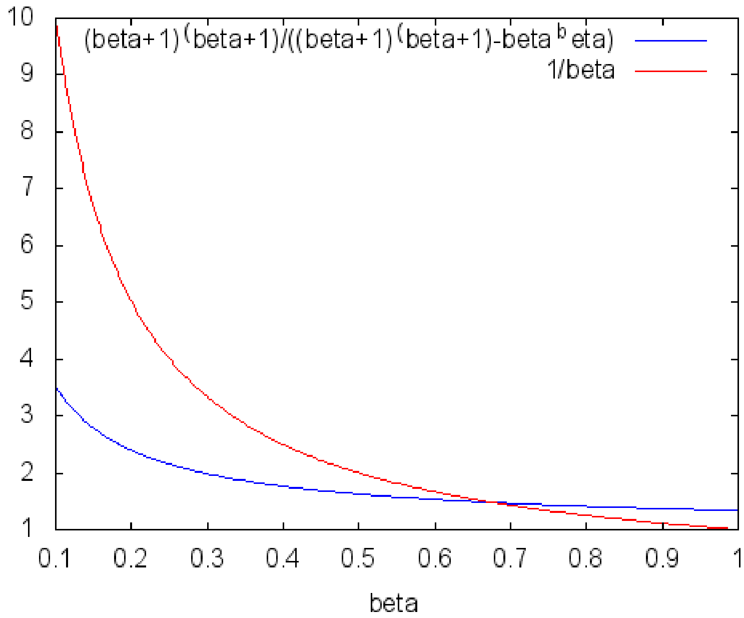

Figure 20: $\overline{\alpha}$ and β for $\beta \in (0, 1)$.

A.2 Welfare of the Wife under the Patriarchal solution

$V_{Pw} - V_{Sw} \geq 0$ if and only if:

$$\alpha \leq \alpha^D = \frac{1}{2}\left[\frac{(1+\beta)^{1+\beta}}{\beta^\beta} - 1\right]$$

The plot of α^D as a function of β shown in figure 21.

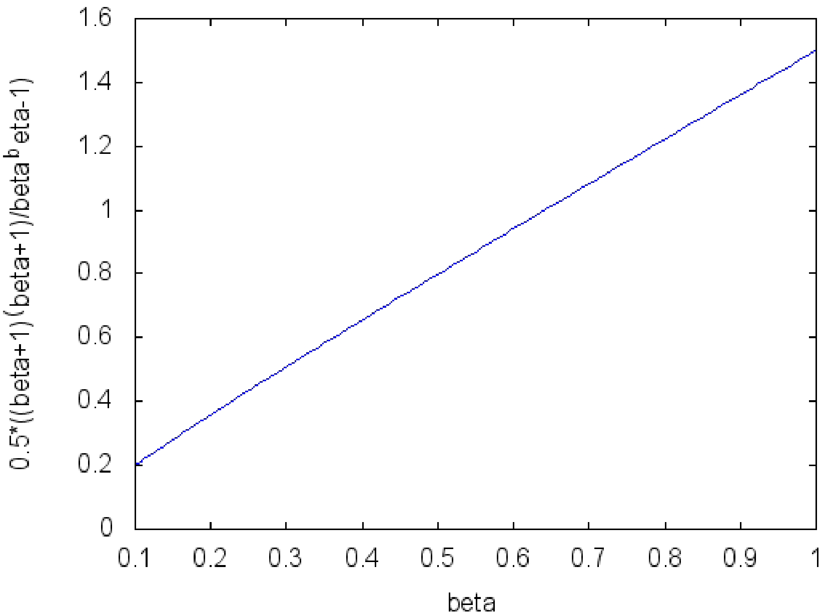

Figure 21: α^D as a function of β for $\beta \in (0,1)$.