**First price sealed bid auction with jealousy under incomplete information**

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**Abstract**

The present paper discusses the first-price sealed-bid auction with jealousy under incomplete information when the participating bidders are using the same bidding function. First, we will find the symmetric Bayesian Nash equilibrium when there are two bidders then we will generalize this model to the numbers of the participating bidders. In this part, we will show that the individual bidder will bid more than her expected value for the object due to the presence of the emotion of jealousy. It is noted that when this auction mechanism is highly competitive, the symmetric Bayesian Nash equilibrium will converge to the winner's curse. Finally, we will discuss the Bayesian Nash equilibrium with linear bidding function for two bidders as an application of this work. In this way, we will explain the effect of the emotion of jealousy on the decision-making behaviour of the individual bidder in a first-price sealed-bid auction mechanism under incomplete information.

**Keywords: Jealousy, winner’s curse, symmetric bidding function.**

**JEL: C720, D910.**

**Introduction**

An auction mechanism is a process in which an object is sold through the bidding process to the participating bidders. In first-price sealed bid auction, each bidder submits a sealed bid to the seller and the bidder with the highest bid wins the auction and gets good after paying his bid. When we come to the sale price of an object which is offered through an auction, then with symmetric bidders when the valuations of the participating bidders are conditional on the value of the object, the sale price converges to the real value in a highly competitive sealed-tender (Wilson, Robert 1977). It is also explained that when every participating bidder has the private information, the winning bid to converge in probability to the true value of the object at auction even though no every bidder is unaware of his true value (Milgrom, Paul R. 1979). Talking about the optimal strategy by the participating bidders, when the valuations approximations of the participating bidders', are not stochastically independent, honesty is a Nash equilibrium under imperfect information (Myerson, Roger B. 1981). In an auction, bidders bid aggressively with an increase in the number of the participating bidders. This tendency of bidding high is just because of the emotions and some factors regarding the object which is offered through an auction mechanism under incomplete information. Participating bidders bid more than their true valuations for the object because they are influenced by these emotions and the factors. This tendency of bidding is called the winner’s curse. With the optimal linear contract, the bidding competition is strictly positive for all the finite numbers of the potential bidders but vanishes when the number of participating bidders are infinite (McAfee, R. Preston, and John McMillan 1986). In winner’s curse, the bidding strategy of the winning bidder conveys bad news to her, because it means she has overestimated the value of the object (McAfee, R. Preston, and John McMillan 1987).

Humans are emotional. Jealousy is one of the important emotion. People get jealous when they expect something for themselves but their rivals get that. A bidder can’t neglect the emotion of jealousy when she is going to an auction. In this paper, we will see the effect of this emotion on the Nash equilibrium through the change in the bidding behaviour of the participating bidders. It is assumed that the participating bidders are using the same bidding function.

This paper is divided into two parts. The first part will find the symmetric Bayesian Nash equilibrium with two bidders then we will generalise the model to the number of the participating bidders when the non-linear bidding function for every bidder is same and also explain the symmetric Bayesian Nash equilibrium when the valuation of every bidder is uniformly distributed on the interval [0,1]. We will also discuss the existence of the problem of the winner’s curse in a highly competitive first price auction. At the end of the first part, we will find the symmetric Bayesian Nash equilibrium with two bidders when the bidding function is linear and the valuation of every bidder is uniformly distributed on the interval [0,1] as an application of our work. In the second part, we will discuss the results.

1. **Models**
2. **Two bidders case:**

**Model:**

Consider a risk-neutral seller who desires to sell an indivisible object to one of the two risk – neutral bidders: bidder 1 and bidder 2. The valuations of bidder and bidder 2 for the object are and respectively. These valuations are mutually independent and identically distributed. is the cumulative distribution function of the bidder valuation for the object with density functions , where . The density functions are common knowledge, known to both the participating bidders. The seller doesn’t know exact valuations of the participating bidders for the object but he knows the distribution from which each valuation is drawn. Bidder bids when her valuation is . A bidder will bid higher amount when her valuation is high. So, without loss of generality we will focus on increasing bidding functions. The bidding function of bidder can be written as , mapping each of her value into a non – negative bid. We assume that both the bidders will use the same bidding function, . Similar to the earlier chapters we assume that bidder suffers a loss of utility of the magnitude in terms of jealousy if she loses the auction. Therefore, we wish to find out the symmetric Bayesian Nash equilibrium with jealousy in the first price auction with incomplete information. The next result states and proves the optimal bidding function below:

**Result 1:**

*The symmetric Bayesian Nash equilibrium with jealousy in the first price sealed bid auction under incomplete information with two bidders will be,*

**Proof:** Let bidder bids and bidder 2 bids , where is the bidding function. Then, bidder wins the auction if . Let the inverse of exists. Then, we can write that the bidder 1 wins the auction if and only if , where is the inverse of B. So, the probability that the bid will be the winning bid will be . Thus the expected payoff to bidder 1 can be written as:

So, bidder ’s expected payoff will be maximum when .

Let the Nash equilibrium is i.e. each bidder’s bid is equal to the best response taken by herself based on her own valuation. Then for bidder 1 we can write . As the inverse of exists. Then, . So, we can write,

Multiplied by both sides, we get,

To solve this differential equation, we take integration to both sides of this equation. So,

Where is an arbitrary constant.

This equation holds for every , then we can write as,

Divided both sides by , then we get,

Use, . Where is an arbitrary constant.

Now, when

And, because with 0 valuation, bidder bids nothing.

So,

Thus we can write as

Similarly, the bidding function for bidder 2 will be

Therefore,

will be the only symmetric Bayesian Nash equilibrium.

And, the best responses are

So, player will bid when her bid is less than or equal to the valuation of bidder , and will bid , if is greater than the valuation of bidder . As both the bidders are symmetric, bidder will bid in the same manner.

Next as an illustrative example we will discuss the symmetric Bayesian Nash equilibrium with two bidders when each bidder’s valuations are uniformly distributed.

**Example: Find the symmetric Nash equilibrium when each bidder’s valuation is uniformly distributed on . Then , and .**

**Result 1.1:**

*The symmetric Bayesian Nash equilibrium with jealousy in the first price sealed bid auction under incomplete information with two bidders when each bidder’s valuation is uniformly distributed on will be,*

**Solution:**

We have that

is the only symmetric Bayesian Nash equilibrium.

By solving we get,

Use, then

And, the best responses are

So, player will bid when her bid is less than or equal to the valuation of bidder , and will bid , if is greater than the valuation of bidder . As both the bidders are symmetric, bidder will bid in the same manner.

Therefore, the Bayesian Nash equilibrium is .

Now, we are generalizing our model for the number of participating bidders in which we will discuss the symmetric Bayesian Nash equilibrium.

1. **Generalization of the model to number of bidders.**

**Model:**

Consider a risk-neutral seller desires to sell an indivisible object to one of the risk – neutral bidders.The valuations of each bidder for the object is , where These valuations are mutually independent and identically distributed. is the cumulative distribution function of the bidder valuation for the object with density functions, where The density functions are common knowledge, known to all the participating bidders. The seller doesn’t know exact valuations of the participating bidders for the object but he knows the distribution from which each valuation is drawn. Bidder bids when her valuation is . A bidder will bid higher amount when her valuation is high. So, we are focusing on increasing bidding functions. The bidding function of bidder can be written as , mapping each of her value into a non – negative bid. We assume that all the bidders are ex-ante symmetric, then, they will follow the same bidding function, . The variable of jealousy is defined by . The loser gets the loss of in terms of jealousy. Therefore, we wish to find out symmetric Bayesian Nash equilibrium with jealousy under incomplete information.

**Result 2:**

*The symmetric Bayesian Nash equilibrium with jealousy in the first price sealed bid auction under incomplete information with number of participating bidders will be,*

**Proof:** Let bidder bids and all other bidders bid , where is the bidding function. Then, bidder wins the auction if . Let the inverse of exists. Then, we can write that the bidder wins the auction if and only if , where is the inverse of . So, the probability that the bid will be the winning bid will be . Thus the expected payoff to bidder can be written as:

So, bidder ’s expected payoff will be maximum when .

Let the Nash equilibrium is i.e. each bidder’s bid is equal to the best response taken by herself based on her own valuation. As the inverse of exists. Then, . So, we can write,

Multiplied by both sides, we get,

To solve this differential equation, we take integration to both sides of this equation. So,

Where is an arbitrary constant.

This equation holds for every , then we can write as,

Divided both sides by , then we get,

Use, . Where is an arbitrary constant.

Now, when

And, because with 0 valuation, bidder bids nothing.

So,

Thus we can write as

Therefore,

will be the only symmetric Bayesian Nash equilibrium.

And, the best responses are

So, player will bid when her bid is less than or equal to the maximum valuations among remaining of the participating bidders, and will bid, if is greater than the maximum valuations among remaining of the participating bidders. As both the bidders are symmetric, bidder will bid in the same manner.

One can solve a special case of the above model with bidders where each bidder’s valuation is uniformly distributed. The proof will be similar to the previous case. We state the result below:

**Result 2.1:**

*The symmetric Bayesian Nash equilibrium with jealousy in the first price sealed bid auction under incomplete information with number of participating bidders when each bidder’s valuation is uniformly distributed on will be,*

Finally, we check whether the highly competitive auction under incomplete information converges to the winner’s curse problem and we characterize that in our case with jealousy.

1. **Existence of the problem of winner’s curse in highly competitive auction:**

The problem of winner’s curse will arise in the highly competitive auction.

**Result 3:**

*The symmetric Bayesian Nash equilibrium with jealousy in a highly competitive first price sealed bid auction under incomplete information will be,*

*Therefore, in a highly competitive auction, every bidder will bid .*

**Proof:** From the bidder’s case, we have symmetric Bayesian Nash equilibrium

In a highly competitive auction, . Then,

Therefore, in a highly competitive auction, every bidder will bid .

Therefore, the Bayesian Nash equilibrium in a highly competitive auction is .

Finally as an application we find the Bayesian Nash equilibrium with two bidders when the valuation of the bidders are uniformly distributed and the bidding function is linear of the form .

1. **Application**

**Model:**

Consider, there are two bidders have their valuations , which is independently drawn from the uniform distribution on . Each potential bidder is allowed to submit a sealed bid, , the highest bidder gets the good after paying her bid. Let the bidding function is . If the bidder wins the auction she gets the expected payoff . If she loses the auction she gets the payoff . The expected payoff function for bidder 1 for each is

**Result 4:**

*The Bayesian Nash equilibrium with jealousy in the first price sealed bid auction under incomplete information with two bidders when the valuation of the bidders, are uniformly distributed and the bidding function for every bidder is linear will be,*

**Proof:** Consider that bidders s highest bid is , when . So, bidder 1 will not bid more than . As, is uniformly distributed on and if and only if . So, we can write expected payoff function to bidder 1 as

As, is uniformly distributed on we put .

So, bidder ’s expected payoff will be maximum when . So,

Similarly for bidder ,

Thus the Bayesian Nash equilibrium of this game is,

and the best responses are

So, player will bid when her bid is less than or equal to the bid of bidder , and will bid , if is greater than the bid of bidder . As both the bidders are symmetric, bidder will bid in the same manner.

Therefore, the Bayesian Nash equilibrium is .

1. **Conclusion:**

In the first-price sealed bid auction with jealousy under incomplete information, the symmetric Bayesian Nash equilibrium is . When we solve this model with two bidders when each bidder’s valuation is uniformly distributed, we get the Bayesian Nash equilibrium . In this case, we can see that both the bidders are rational on their valuation. They are bidding expected value of their valuations. But they are not rational on the emotion of jealousy. They are paying the full amount of the jealousy which they can pay to win the auction. As a result, they are bidding more than their expected values because they are influenced by the emotion of jealousy. So, the emotion of jealousy is affecting the bidding strategies of both the bidders and changing the regular Nash equilibria in first-price sealed bid auction under incomplete information with two bidders.

When the model is generalized to number of participating bidders, the symmetric Bayesian Nash equilibrium is When we solve this model with bidders when each bidder’s valuation is uniformly distributed, we get the Bayesian Nash equilibrium . In this case, we can see that similar to the previous result all the bidders are rational on their valuation. They are bidding expected value of their valuations. But they are not rational on the emotion of jealousy. They are paying the full amount of the jealousy which they can pay to win the auction. As a result, they are bidding more than their expected values because they are influenced by the emotion of jealousy. So, the emotion of jealousy is affecting the bidding strategies of all the bidders and changing the regular Nash equilibria in first-price sealed bid auction under incomplete information with number of participating bidders.

In a highly competitive first-price sealed bid auction, every bidder will bid . Therefore, the Bayesian Nash equilibrium is . We can see that when the auction is highly competitive all the bidders will bid more than their true valuation. So, the emotion of jealousy is affecting the bidding strategies of all the bidders and changing the regular Nash equilibria in highly competitive first-price sealed bid auction under incomplete information. When the auction is highly competitive, the regular case explains that the seller’s revenue will be free of the variable of jealousy. The emotion of jealousy is an important emotion. This emotion can’t be neglected by the bidder when she will be going for an auction. Thus due to the emotion of jealousy among the participating bidders, seller’s revenue will be more than the revenue generated in the regular case. It will increase the seller’s revenue with the amount , where subscript is being used for the highest bidder. Thus, the emotion of jealousy is a reason for the existence of the problem of winner's curse and the higher seller’s revenue in a highly competitive auction.

Finally, when the valuation of the bidders, are uniformly distributed and the bidding function for every bidder is linear the Bayesian Nash equilibrium is . In this case, we can see that both the bidders are taking the jealousy factor into account, but not completely influenced by this emotion. So, they will bid the expected value. Thus, they are bidding more than their expected values because they are partially influenced by the emotion of jealousy. They are also rational on the emotion of jealousy as they are on their valuation. So, the emotion of jealousy is affecting the bidding strategies of both the bidders and changing the regular Nash equilibria in first-price sealed bid auction under incomplete information with linear bidding function.

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