Online Platform Quality, Discount and Advertising: A Theoretical Analysis

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Abstract

This paper discusses the optimal strategies of two-sided monopoly platform to attract and keep members of two distinct sides to the platform when buyers' side is affected by discount offered by platform. The study also introduces service quality provided by platform to buyers. Monopoly platform uses informative advertising technique to transmit information about discount details to buyers. The effect of change in discount on pricing structure, service quality and level of advertising has been evaluated by using a simple model setup of two-sided market structure. It is observed that platform gets to finance additional expense caused by increase in discount by raising price charged on buyers' side. Platform charges a lesser per transaction fee on sellers for attracting them to platform so that they could serve the expanded market on buyer side who get attracted due to increase in discount. Quality of Service (QoS) and level of advertisement are increasing in discount, conditional upon satisfaction of parametric restriction. Results are also derived for social optimum equilibrium and have been compared with optimal monopoly equilibrium. Monopoly platform sets lower advertising level compared to welfare perspective which is in stark contrast to existing literature. QoS chosen by monopoly platform is smaller compared to social optimum.

Keywords: Two sided markets, discount, quality of service, advertisement, social optimum

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I. Introduction:

In the last few years the study of two sided markets has gained renewed interest in economics and industrial organization literature. The significant increase in use of internet worldwide in the past years has marked the evolution of online platform in digital marketplace. Another catalyst that has contributed to rapid expansion of two-sided market, is the wireless handheld devices such as smartphones, tablets etc. Such technology has made possible to attain a much wider reach of internet, increasing the number of internet users. These two factors has paced up the growth of two-sided market in online marketplace. In 2017, retail e-commerce sales is estimated at \$2.304 trillion across the globe, a 24.8% jump from the previous years. M-commerce sales worldwide is estimated at \$1.357 trillion in 2017 and this amount of sale accounts for 58.9% of digital market. By 2021, m-commerce market is expected to be reached to 72.9% of overall e-commerce sales (McNair, 2018). Two sided markets can be defined as a market structure where intermediary provides a common platform to different groups of agents for making transaction with each other by charging appropriate price to each side (Rochet and Tirole, 2004). The one of the defining characteristics of two sided markets is the presence of indirect network externality or intergroup network externality which is associated with the value created by the participation of other group users. Rochet & Tirole (2003) and Caillaud & Jullien (2003) assumed the existence of indirect externality in both sides and both directions of the platform. Under two-sidedness, cross group externality (or, indirect externality) has an important implications as more members on a side helps to attract members of other side and vice-versa. Platform should internalize these externalities in deciding optimal pricing settings. Credit and debit card, computer operating systems, television networks, media markets, shopping malls, video game console, online trading platforms are examples of such market structures. In case of video game, consumers and software developers constitute two groups that engage in trade with each other. Videogame platforms require gamers to induce software developers to design games to their platforms and games to attract gamers to buy and use their games console (Rochet and Tirole, 2006). Although these two sided industries have different business standards but have adopted similar pricing strategies to get and maintain two sides on board.

The analysis in this paper is related to the growing literatures that focuses on the complexity of pricing structures in two sided markets. Papers by Rochet and Tirole (2003,2004,2006), Armstrong

(2006), Caillaud and Jullien (2003) provide valuable insights about workings of two sided markets and conclude that standard supply-demand analysis of one sided markets would not be sufficient to capture the pricing strategies of such markets. The literature of two sided markets is precisely concerned with optimal pricing strategy by taking into account externality that arises through interactions between demands of distinct group of users. Caillaud and Jullien, (2003) proposed a model setup of imperfect competition with indirect network externality and also suggested "divide and conquer" strategy where one side of market is subsidized and profits are earned from other side by charging a higher price to that side. Rochet and Tirole (2003) advocated that the monopoly platform's total price is determined by a variant of monopolistic Lerner condition and optimal price structure is governed by relative magnitude of price elasticities of demand of both sides. According to the literature, in two sided markets with network externalities, distribution of total price between two distinct groups determines volume of transaction i.e., platform is not only concerned about total price but also optimal division of total price between two groups (Rochet and Tirole, 2004). Rochet and Tirole (2006) incorporated usage and membership externality and derived optimal pricing under two sidedness. Armstrong (2006) introduced the concept of "two-part tariffs" similar to Rochet and Tirole (2006). Two-part tariff comprises of a fixed fee and a marginal price for each member of other side who participates in platform. Cross-group externality, levy of fee as lump-sum basis or per transaction basis and decision of members to single home or multi-home are influential in determining prices offered to both sides (Armstrong, 2006). Evans (2003) performed empirical analysis and advocated that differential pricing strategies are used to get multiple sides on board and price continues to play vital role in maintaining customers on the platform. The need to get two sides on platform for a successful transaction gives rise to "chicken and egg" problem: agents of a side will be willing to join the platform if they see many members of other side participating at the same platform (Caillaud and Jullien, 2003). Hagiu (2006) solved the famous "chicken and egg" problem by developing a model where producers enter to platform before consumers. Hagiu (2009) showed that product variety is a key determinant of optimal pricing settings. Two key insights of this paper has made important contribution on literature of two-sided markets. Firstly, consumers' preference for variety makes products less substitutable and thus gives suppliers opportunity to extract higher share of the joint surplus generated from transaction between two sides. Monopoly platform derives relatively more profits from the side that possesses more market power over other side. Secondly, stronger consumers' preference for variety or more market power of producers over consumers makes competing platform's price cutting strategy less effective, resulting in smaller consumer price cut in equilibrium. Many researchers have studied two-sided market structure empirically in recent years and contributed to existing literatures.

Our paper has contributed some valuable insights to the literature of two sided markets for better understanding of functioning of such markets. Many issues having important implications on price settings of two sided markets have been covered by existing literature. This paper introduces some economic and strategic factors responsible for determining optimal access to platform connecting two sides. Providing members lucrative offers has become a crucial strategy adopted by platform to attract members and this has a serious impact on pricing structure. This area has largely remained unexplored in two sided market structure. Narasimhan (1984) had taken couponing as a device of price discrimination since it allows to charge a lower price for a particular segment of consumers. Bester and Petrakis (1996) studied couponing as a price discrimination device in competitive market structure in single sided market. According to them, couponing to buyers increases competition between sellers in equilibrium, resulting in lower price and profits. In this paper we assume an amount of discount is being offered to buyers when they involve in transaction with sellers in monopoly platform and analyze that discount not only affects price charged on buyer but also price paid by sellers to platform, although discount adds utility to buyers' side only. We assume that platform uses advertising device to make sure that buyers receive information about discount. So, in our model advertising is informative in nature. Another variable has been introduced in our model is the quality of service provided by platform since it has major implication in attracting customers. Results from empirical analysis examines that service quality has strong effect on consumers' behavioural intensions (Zeithaml *et al.*, 1996). A study showed ten determinants of service quality by using a conceptual service quality model and demonstrated that delivering service quality is an important strategy to survive in competitive world (Parasuraman *et al.*, 1985). Many researches exhibited the strong role of service quality in determining market share and return on investment and also suggested to close the 'quality gap' between what consumers expect and what they get. In this study we attempt to find how pricing structure, service quality and advertising levels are affected by discount offered to buyers' side.

We also study provision of service quality and level of advertisement under social optimum structure and compare the results obtained in two regimes. A noticeable result is that monopoly platform sets a lower advertising level which counter the common theory that states that monopoly chooses a highly inefficient level of advertising. Indirect network externality and non-accountability of utilities produce divergence in results in two cases. The analysis of this study intends to fill the gap in the literature and can have major implications in describing pricing structures in two sided markets.

The rest of the paper is organized as follows. Section 2 outlines theoretical model set up and key concepts under monopoly market equilibrium followed by the comparative static analysis. Section 3 presents analysis of social optimum. In section 4, comparison of results of two regimes is discussed. Section 5 gives concluding remarks. Appendix contains proofs and computations.

2. Theoretical Framework:

2.1 The Model Setup

The theoretical model evaluates the profitability of two sided market when it provides a certain amount of cash incentive (or, discount) to its buyer. We have laid out a very simple model of two sided markets and it is similar to the one developed by Rochet and Tirole (2003). But we revise the model set up to account for the possibility of discount offerings given to buyers' side by platform. The analysis of two sided market under monopoly structure in section 2.1 will be used to compare the results under welfare analysis derived in section 3.

The model setup consists of three groups of agents. The potential value from transaction is generated by the interaction between two distinct groups of the market whom we will denote as Buyers' side (indexed by B) and sellers' (indexed by S). Such interactions between these two end-users are mediated by a platform. Indirect network externality is present in such market structure as members of a side are concerned about the number of members of other side participating the platform. To deliver services, the platform charges a fixed membership fee A_i ($\forall$ i $\in$ {B, S}) and a usage fee a_i ($\forall$ i $\in$ {B, S}) per transaction to end-users of each side. Intragroup heterogeneity can be observed in each side in terms of the benefit Θ_B derived per transaction by buyers and profit

π_S obtained per transaction by sellers. Buyer side of the platform derives satisfaction from the quality of service provided by platform while making a transaction in that platform and this satisfaction adds to the utility of buyer. We assume quality of service (QoS) to be endogenous to the system and this is indexed by 's'. Furthermore, the monopoly platform offers a kind of cash-incentive (or, discount) per transaction to buyers in an effort to build a larger buyer base and it is indexed by *d* where $0<d<1$. The discount also provides value to buyers and this is being added to their utility functions when buyers engage transaction with sellers.

The platform uses advertising message to provide information about discount offerings to buyers. So, advertising in our model is informative in nature and endogenous to the system. We assume that a fraction λ of buyers receives advertising signal sent by platform proposing the amount of discount "d". It is assumed that buyers who have been exposed to advertising signal, participate in platform and get involved in transaction in that platform. The structure of advertising cost is left quite general. The expenditure of advertising to attain a reach of λ fraction of buyers is assumed to be lump-sum and quadratic in nature. The marginal cost of advertising is increasing in the number of buyers reached. This assumption is in line with the standard advertising literature developed by Grossman and Shapiro (1984). The cost of ensuring that a fraction λ of buyers receives information about discount given by platform becomes a convex function of λ. We assume there is no other way to make transaction between these two groups, they can only transact through the platform. Every user who joins the platform, ends up making a transaction in the platform.

A. Buyers

In this subsection, buyers' participation decision has been analyzed. Buyers obtain additional utility Θ_B by entering into transaction with each additional seller where Θ_B is uniformly distributed over the continuum $[0, \bar{\Theta}]$. Net surplus for a representative buyer from making a transaction in platform that is supported by N_S sellers is,

$$U_B = [\Theta_B \, s\,(1+d) - a_B]\, N_S - A_B \tag{1}$$

There exists indirect network externality since buyers are interested in purchasing variety of products, so the net surplus from transaction is increasing in the number of sellers supported by platform. It has been assumed that each buyer enters into transaction with each seller for once. So, the total number of transaction buyers will make, is equivalent to total number of sellers participated in the same platform.

It is assumed that only those buyers who extract non-negative net surpluses from transaction, are willing to join the platform. So, the expected number of buyers connecting to the platform can be derived as,

$$N_B = \Pr(U_B \geq 0)$$

$$= \Pr\left((\Theta_B s(1+d) - a_B) - \frac{A_B}{N_S} \geq 0\right) \qquad \text{(Substituting from (1))}$$

Rochet and Tirole defined "per transaction price" for buyers as (Rochet and Tirole, 2004);

$$p_B = a_B + \frac{A_B}{N_S} \qquad (2a)$$

Therefore, the expected number of buyer willing to join the platform is,

$$N_B = \Pr\left[\Theta_B s\,(1+d) \geq p_B\right] \qquad \text{(using (2a))}$$

$$= \Pr\left[\Theta_B \geq \frac{p_B}{s(1+d)}\right]$$

Let, $\Theta^* = \frac{p_B}{s(1+d)}$; where Θ^* to be the marginal consumer who is indifferent between joining and not joining the platform. Therefore,

$$N_B = \Pr(\Theta_B \geq \Theta^*);$$

$$= (\bar{\Theta} - \Theta^*)$$

Then the spectrum of buyers who derive non-negative net surpluses and are willing to participate in platform can be obtained as,

$$N_B = \left(\bar{\Theta} - \frac{p_B}{s(1+d)}\right)$$

The spectrum of buyers in the domain $\Theta^* \leq \Theta_B \leq \bar{\Theta}$ will be willing to participate in platform and enter in transaction with sellers; but buyers in the interval $0 \leq \Theta_B \leq \Theta^*$ will not be prepared to join the platform. So, the market segment for buyers is not fully covered as buyers in the range $0 \leq \Theta_B \leq \Theta^*$ will neither join platform nor involve in transaction with sellers. Among the buyers willing to participate in platform, only a fraction λ receives advertising sent by platform. According to our assumption, buyers who are exposed to advertisement, only join the platform. Therefore, the expected number of buyers who will ultimately join the platform can be derived as,

$$\lambda N_B = \lambda \int_{\theta^*}^{\bar{\theta}} d\theta = \lambda \left(\bar{\theta} - \frac{P_B}{s(1+d)}\right) \tag{3}$$

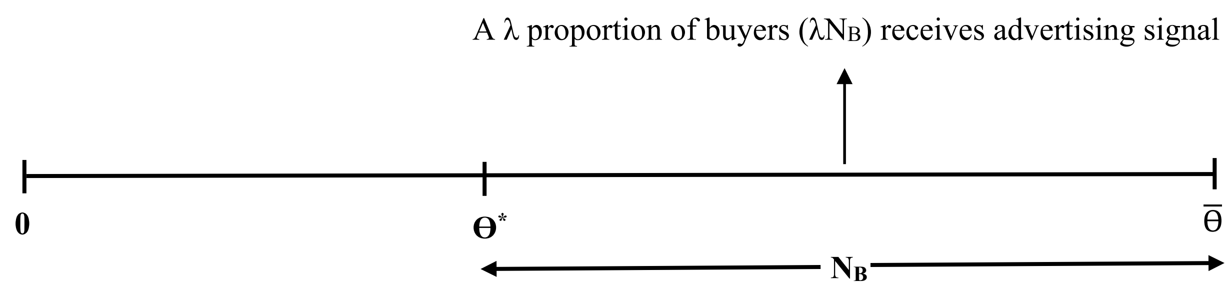


Figure 1: Consumer Spectrum under Advertising in Platform

B. Sellers

Participation decision of sellers has been discussed in this subsection. Each seller derives additional utility (profit) of π_S from entering into transaction with each buyer who has participated in the platform where π_S is uniformly distributed over the continuum $[0, \bar{\pi}]$. Net profit of a representative seller from making transaction in platform which is adopted by (λN_B) buyers is,

$$\Pi_S = (\pi_S - a_S)\,\lambda\, N_B - A_S \tag{4}$$

The fraction of buyers (λN_B) receives advertising massage from platform and decides to connect to the platform. So, a seller enters into transaction with (λN_B) buyers. Here also, the presence of indirect network externality is noticed since seller's profit from transaction is increasing with the adoption of more buyers.

The number of sellers who like to join the platform can be defined as,

$N_S = \Pr(\Pi_S \geq 0)$

$= \Pr((\pi_S - a_S) - \frac{A_S}{\lambda N_B} \geq 0)$

Following Rochet and Tirole (2004), we can define 'per transaction price' for sellers as;

$$p_S = a_S + \frac{A_S}{\lambda N_B} \tag{2b}$$

So, expected number of sellers who will join the platform is,

$N_S = \Pr(\pi_S \geq P_S)$;

Let, $\pi^* = P_S$ where π^* to be the marginal seller who is indifferent from joining and not joining the platform, then expected number of sellers joining the platform can be obtained as,

$N_S = [\bar{\pi} - \pi^*]$

$$= [\bar{\pi} - P_S] \tag{5}$$

The market for sellers is also partially covered as sellers in the range $0 \leq \pi_S \leq \pi^*$ will not be interested in participating in platform as those sellers derive negative surpluses from making transaction in platform.

The expected number of buyers and sellers (derived in the expressions (3) and (5) respectively) faced by platform depends negatively on 'per transaction price' (P_B for buyers and P_S for sellers).

2.1.1 Market Equilibrium in Two Sided Market under Monopoly Platform:

We consider that both distinct groups are served by a monopoly platform. This section analyses the optimal pricing structure of a monopoly platform.

The profit of the monopoly platform is given by,

$$\Pi_P = A_B\lambda N_B + A_S N_S + (a_B + a_S - d)\lambda N_B N_S - \frac{\alpha}{2}s^2 - \frac{\beta\lambda^2}{2} \tag{6}$$

The first two terms of (6) represents the revenue earned by platform from both sides through fixed membership fees. The third term constitutes the net revenue earned by platform through per transaction usage fee where d is the amount of cash incentive (or, discount) per transaction provided by platform to buyers and $\lambda N_B N_S$ is the total volume of transaction. The fourth term represents lump-sum quadratic quality cost borne by platform for providing service to buyers' side. The last term exhibits the cost of advertisement incurred by platform.

Incorporating the expression of 'per transaction price' from (2a) and (2b), equation (6) can be rewritten as,

$$\Pi_P = (P_B + P_S - d)\lambda N_B N_S - \frac{\alpha}{2}s^2 - \frac{\beta\lambda^2}{2} \tag{7}$$

Setting the values of N_B and N_S from equations (3) and (5) in equation (7), we have,

$$\Pi_P = \lambda\,(P_B + P_S - d)\,(\bar{\theta} - \frac{P_B}{s(1+d)})\,(\bar{\pi} - P_S) - \frac{\alpha}{2}s^2 - \frac{\beta\lambda^2}{2} \tag{8}$$

The platform chooses the profit-maximizing level of prices, quality and level of advertisement. Maximizing platform profit given by expression (8) w.r.t. P_B and P_S will yield profit maximizing amount of per transaction prices charged on buyers and sellers respectively by platform. We derive First Order Conditions assuming the existence of interior solution.

$$\frac{d\Pi_P}{dP_B} = \lambda\,(\bar{\pi} - P_S)\,[(\bar{\theta} - \frac{P_B}{s(1+d)}) - \frac{(P_B+P_S-d)}{s(1+d)}] = 0 \tag{9}$$

$$\frac{d\Pi_P}{dP_S} = \lambda\,(\bar{\theta} - \frac{P_B}{s(1+d)})\,[(\bar{\pi} - P_S) - (P_B+P_S-d)] = 0 \tag{10}$$

We have checked Second Order Sufficient Condition for profit maximization problem and all conditions have been satisfied. We derive the profit maximizing prices as,

$$P_B = \frac{[2\bar{\theta}s(1+d) + d - \bar{\pi}]}{3} \tag{11}$$

$$P_S = \frac{[2\bar{\pi} + d - \bar{\theta}s(1+d)]}{3} \tag{12}$$

The expected number of users faced by platform in each side is given by,

λ N_B =$\lambda \frac{[\bar{\theta}s(1+d)+\bar{\pi}-d]}{3s(1+d)}$ (13)

$N_S = \frac{[\bar{\theta}s(1+d)+\bar{\pi}-d]}{3}$ (14)

Using (11), (12), (13) and (14), the platform's profit can be derived as,

$\Pi_P = \lambda \frac{[\bar{\theta}s(1+d)+\bar{\pi}-d]^3}{27s(1+d)} - \frac{\alpha}{2} s^2 - \frac{\beta\lambda^2}{2}$ (From (7)) (15)

Next, Π_P is maximized w.r.t. s (QoS) to derive optimum amount of s to be provided by monopoly platform. First Order Condition with respect to'ˈs' can be obtained as,

$\frac{d\Pi_P}{ds} = \lambda \frac{(\bar{\theta}s(1+d)+\bar{\pi}-d)^2(2\bar{\theta}s(1+d)-\bar{\pi}+d)}{27s^2(1+d)} - \alpha s = 0$ (16)

Equation (16) can be simplified as, $\lambda\, \Phi(s) = \alpha s$

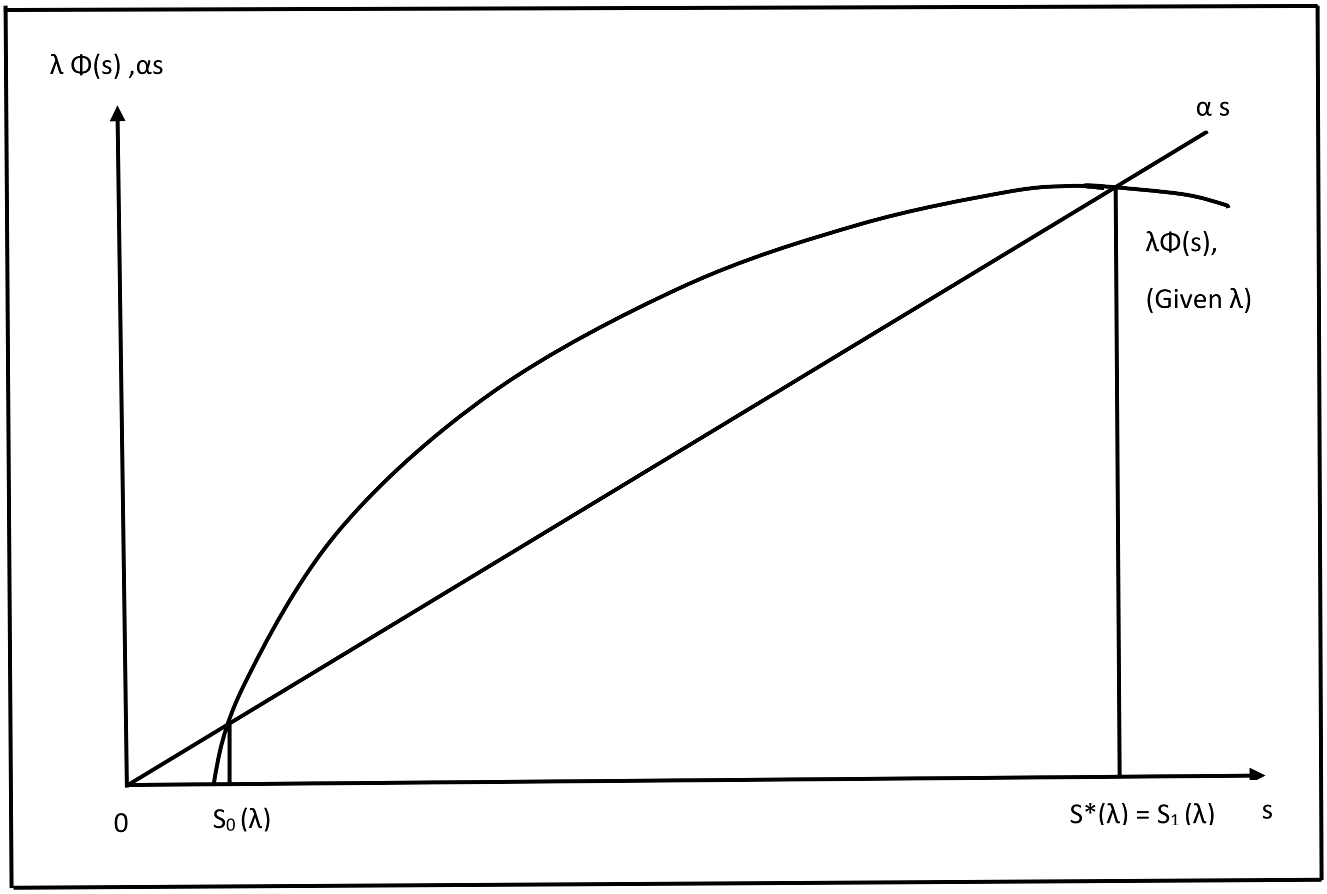
Where, $\Phi(s) = \frac{(\bar{\theta}s(1+d)+\bar{\pi}-d)^2(2\bar{\theta}s(1+d)-\bar{\pi}+d)}{27s^2(1+d)}$ (16a)

We have, $\Phi'(s) > 0$ and $\Phi''(s) < 0$ (See **Appendix**)

So, $\lambda\, \Phi(s)$ can be represented by concave upward sloping curve for any given value of λ and αs is represented by a linear curve passing through the origin. The equilibrium value of s can be found at the intersection of these two curves for any given value of λ. Figure 1 depicts the scenario diagrammatically. We can see from the figure that $\lambda\, \Phi(s)$ and αs curves intersect each other at $s_0(\lambda)$ and $s_1(\lambda)$ but the second order sufficient condition for profit maximization is satisfied at $s^*(\lambda) = s_1(\lambda)$, where $\alpha > \lambda\, \Phi'(s) = \lambda[\frac{2}{27(1+d)}\{\frac{(\bar{\pi}-d)^3}{s^3} + (1+d)^3\, \bar{\theta}^3\}]$

Hence, from (16) we get a condition where the profit maximizing level of quality of service provided by monopoly platform is a function of λ. We can also determine the range within which the unique profit maximizing value of s will lie. From the condition $P_B>0$, we have $s > \frac{(\bar{\pi}-d)}{2\bar{\theta}(1+d)}$ and from $P_S > 0$, we get $s < \frac{(2\bar{\pi}+d)}{\bar{\theta}(1+d)}$; so, the unique value of s* will fall within the range $\frac{(\bar{\pi}-d)}{2\bar{\theta}(1+d)} < s < \frac{(2\bar{\pi}+d)}{\bar{\theta}(1+d)}$.

Figure 2: Determination of Service Quality under Monopoly Platform



Implementing implicit function theorem in the expression (16), we derive the effect of change in advertisement on QoS. So, we have,

$$\frac{ds}{d\lambda} = -\frac{\frac{\delta^2 \Pi_P}{\delta\lambda\delta s}}{\frac{\delta^2 \Pi_P}{\delta s^2}}$$

The denominator of the above expression is in itself negative in sign because of the Second Order Condition of profit maximization with respect to service quality. We must get the sign of numerator to find out the direction of change in association between service quality and advertisement provided by platform.

$$\frac{\delta^2 \Pi_P}{\delta\lambda\delta s} = \frac{(\bar{\theta}s(1+d)+\bar{\pi}-d)^2(2\bar{\theta}s(1+d)-\bar{\pi}+d)}{27s^2(1+d)} > 0$$

Hence, it establishes a positive association between QoS and level of advertisement *i.e.,* $\frac{\delta s}{\delta\lambda} > 0$.

This result can also be explained graphically from Figure 2. The equilibrium value of s is found for a given value of λ. As λ increases, the concave curve shifts to upward direction, so level of quality rises. As the monopoly platform focuses more on buyers targeted through advertising signal, then it tends to provide improved QoS to retain those targeted customers. Better will be the service then more buyers will get attracted creating bandwagon effect and this will raise the profitability opportunity of monopoly platform.

Setting the profit maximizing level of service quality $s^*(\lambda)$ in the equation (15), Profit of the monopoly platform becomes,

$$\Pi_P = \lambda \frac{[\bar{\theta}s^*(\lambda)(1+d)+\bar{\pi}-d]^3}{27s^*(\lambda)(1+d)} - \frac{\alpha}{2}s^*(\lambda)^2 - \frac{\beta\lambda^2}{2} \tag{17}$$

Finally, the profit maximizing level of advertisement is determined by optimizing equation (17).

First Order Condition of profit maximization with respect to λ is,

$$\frac{d\Pi_P}{d\lambda} = \frac{[\bar{\theta}s^*(\lambda)(1+d)+\bar{\pi}-d]^3}{27s^*(\lambda)(1+d)} - \beta\lambda$$

$$+ \lambda \frac{(\bar{\theta}s^*(\lambda)(1+d)+\bar{\pi}-d)^2(2\bar{\theta}s^*(\lambda)(1+d)-\bar{\pi}+d)}{27s^{*2}(\lambda)(1+d)} \frac{\delta s}{\delta\lambda} - \alpha s^*(\lambda)\frac{\delta s}{\delta\lambda} = 0$$

Or, $\frac{[\bar{\theta}s^*(\lambda)(1+d)+\bar{\pi}-d]^3}{27s^*(\lambda)(1+d)} - \beta\lambda + \alpha s^*(\lambda)\frac{\delta s}{\delta\lambda} - \alpha s^*(\lambda)\frac{\delta s}{\delta\lambda} = 0$ (using 16)

$$\text{Or, } \lambda = \frac{[\bar{\theta}s^*(\lambda)(1+d)+\bar{\pi}-d]^3}{27\beta s^*(\lambda)(1+d)} \tag{18}$$

The second order sufficient condition (S.O.C) for profit maximization w.r.t level of advertisement (λ) requires $\left[\frac{(\bar{\theta}s^*(\lambda)(1+d)+\bar{\pi}-d)^2(2\bar{\theta}s^*(\lambda)(1+d)-\bar{\pi}+d)s^{*'}(\lambda)}{27s^{*2}(\lambda)(1+d)} - \beta\right] < 0.$ From (18) we get a condition which can be solved for equilibrium level of advertisement. By replacing the equilibrium value of advertisement in condition (16), we can get unique equilibrium value of QoS as s^*.

The profit of platform can be written as by setting the expression of λ from (18) in (17), we have,

$$\Pi_P = \frac{[\bar{\theta}s^*(\lambda)(1+d)+\bar{\pi}-d]^6}{27^2\beta s^*(\lambda)^2(1+d)^2} - \frac{\alpha}{2} s^*(\lambda)^2 - \frac{\beta}{2}\frac{[\bar{\theta}s^*(\lambda)(1+d)+\bar{\pi}-d]^6}{27^2\beta^2 s^*(\lambda)^2(1+d)^2}$$

$$= \frac{[\bar{\theta}s^*(\lambda)(1+d)+\bar{\pi}-d]^6}{27^2\beta s^*(\lambda)^2(1+d)^2}\left(1-\frac{1}{2}\right) - \frac{\alpha}{2} s^*(\lambda)^2$$

$$= \frac{1}{2}\frac{[\bar{\theta}s^*(\lambda)(1+d)+\bar{\pi}-d]^6}{27^2\beta s^*(\lambda)^2(1+d)^2} - \frac{\alpha}{2} s^*(\lambda)^2 \qquad (19)$$

The price charged on buyers' side and sellers' side by platform are derived as function of λ and the expressions are respectively,

$$P_B^* = \frac{[2\bar{\theta}(1+d)s^*(\lambda)+d-\bar{\pi}]}{3} \text{ and } P_S^* = \frac{[2\bar{\pi}+d-\bar{\theta}(1+d)s^*(\lambda)]}{3} \qquad (20)$$

The total number of transaction on a monopoly platform providing 's' QoS and 'd' amount of cash incentive can be obtained as ,

$$V = \lambda N_B^* N_S^* = \frac{[\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d]^5}{243\beta(1+d)^2 {s^*}^2(\lambda)} \qquad (21)$$

Next subsection will analyze and discuss the findings and results of the model.

2.2 Analysis and Results under Monopoly Platform

From profit maximizing outcomes derived in last subsection, we get following propositions. Propositions have added some of the valuable insights in our model. We can employ the comparative static exercise to see the responsiveness of equilibrium due to change in per transaction discount (d).

Lemma 1: More number of sellers will join the platform for an increase in amount of discount if $\bar{\pi} > 2+3d$.

(Proof is given in the Appendix)

Providing discount to buyers is used as a strategy to attract more buyers to platform. To serve the complete buyer spectrum, there should be sufficient sellers in the platform. Only a fall in per transaction fee charged to sellers can attract more number of sellers. This strategy is being persuaded to enlarge the seller spectrum which is ensured by the parametric restriction $\overline{\pi} > 2+3d$.

Given Lemma 1, Proposition 1 and 2 shows how market equilibrium values will respond for a change in amount of discount.

Proposition 1 shows the change in level of advertisement and service quality of monopoly platform in response to an increase in discount provided by monopoly platform, given Lemma 1.

Proposition 1: For $\overline{\theta} \geq \max\,[\overline{\theta}^{*}, \overline{\theta}']$

- **(i)** *An increase in discount will raise the level of advertisement of monopoly platform.*
- **(ii)** *An increase in discount increases service quality of monopoly platform.*

(Proof is given in the Appendix)

Level of advertising can be employed as a strategy by platform to channel the information about the available discount offerings to buyers' side, given that platform has sufficient number of sellers. Platform tries to expand its reach to buyers as much as possible by using the extent of advertising signals so that buyers could get information about discount and get attracted to participate to the platform. Platform will be induced to spend more on advertisement in response to an increase in discount when expanded spectrum of buyers join the platform and engage in transaction with larger number of sellers.

Providing better and improved service quality to buyers can be taken as a strategy by platform for attracting and retaining buyers to the platform given the existence of vast number of sellers in platform ensured by Lemma 1. The intuition behind the positive association between discount and service quality may be that the platform needs to upgrade its QoS to retain those buyers who get enticed by the lucrative offerings made by the platform when many number of buyers enter to platform due to increase in the amount of discount.

Next we analyze the effect of change in discount on per transaction fee charged on sellers' side and buyers' side, volume of transaction and profit of platform given Lemma 1.

Proposition 2: For $\overline{\theta} \geq \max\,[\overline{\theta}^{*}, \overline{\theta}']$

- **(i)** *An increase in discount will unambiguously lower per transaction fee charged on sellers.*
- **(ii)** *An increase in discount provided by platform will unambiguously raise per transaction fee charged on buyers.*
- **(iii)** *An increase in discount unambiguously raises the volume of transaction.*
- **(iv)** *An increase in discount will unambiguously raise profit of monopoly platform.*

(Proofs are given in the Appendix)

Providing discount to buyers is taken as a strategy to attract more buyers to platform. This expanded spectrum of buyer side requires a large number of seller to make successful transaction. To serve the expanded buyer spectrum, there should exist sufficient number of sellers in the platform. So, monopoly platform will charge a lower price to sellers in response to an increase in discount to induce more sellers to the platform so that they could serve the expanded market on buyer's side who get attracted through discount policy. This will produce a negative association between discount level and per transaction fee imposed on sellers.

Given that platform has sufficient number of sellers, as discount provided to buyer increases, platform will spend more on advertisement and charge a higher price to buyers. So, the platform gets to finance the additional expense generated due to increase in discount by increasing per transaction fees to buyers when more number of buyers join the platform.

Given a sufficient number of sellers, an increase in discount will attract large number buyers to the platform which gets ensured by $\overline{\theta} > \overline{\theta}'$. These expanded spectrum of buyers will interact with vast number of sellers and engage in transaction, resulting in higher volume of transaction. So, increase in discount translates into a larger number of transaction between buyers and sellers.

Having a broad spectrum of sellers ensured by Lemma 1 will incentivize platform to increase amount of discount given to buyer side. Providing a greater amount of discount will attract more buyers to the platform. These buyers will get involved in transaction with larger spectrum of sellers and that will increase the volume of transaction (Proved in Proposition 2(iii)) and thus the profitability of platform. An increase in discount will affect profit of the platform positively if there exists expanded spectrum of buyers which will be ensured by the condition $\overline{\Theta} > \overline{\Theta}^*$.

3. Welfare Analysis:

In this section, we study the welfare implications of providing discount to buyers. Here, we take the price settings similar to the structures derived in the monopoly case under section 2.1.1. We only derive QoS and level of advertising under social welfare analysis and investigate the variation of the results obtained in sections 2 and 3.

In the last section, the network platform enjoys monopoly power in deciding QoS, s and level of advertisement, λ. Monopoly platform maximizes its own profit without considering the well-being of other agents. It only engages in those activities that will maximize its profit. However, the objective of social planner is completely different from that of monopolist. He will choose that level of advertisement and QoS which will ensure the welfare of society as a whole. Maximizing social welfare will raise economic wellbeing of all groups of agents.

In our model, total social welfare generated through platform services comprises of net buyers' surplus, net sellers' surplus and monopolist profit function, conditional upon buyers' and sellers' participation in the platform. So, the social welfare function can be described as,

$$W = \lambda \int_{\pi^*}^{\overline{\pi}} d\pi \int_{\Theta^*}^{\overline{\Theta}} (\Theta_B s\,(1+\,d) - P_B) d\Theta + \,\lambda \int_{\Theta^*}^{\overline{\Theta}} d\Theta \int_{\pi^*}^{\overline{\pi}} (\pi_S - P_S) d\pi$$

$$+\quad \lambda\,(P_B\,+P_S\,-\mathrm{d}) \int_{\pi^*}^{\overline{\pi}} d\pi \int_{\Theta^*}^{\overline{\Theta}} d\Theta - \frac{\alpha}{2} s^2 - \frac{\beta\lambda^2}{2} \tag{22}$$

The first two terms of equation (22) represent buyers' net surplus and sellers' net profit respectively. All the subsequent terms depict the profit of monopoly platform. By simplifying the expression (22) and setting values of P_B, P_S similar to monopoly price structure (derived in (11) and (12)) we obtain social welfare as a function of QoS (s) and level of advertisement (λ) (See **Appendix**).

$$W(s, \lambda) = \frac{2\lambda}{27} \frac{[\overline{\Theta}s(1+\mathrm{d}) + \overline{\pi} - d]^3}{s(1+d)} - \frac{\alpha}{2} s^2 - \frac{\beta\lambda^2}{2} \tag{23}$$

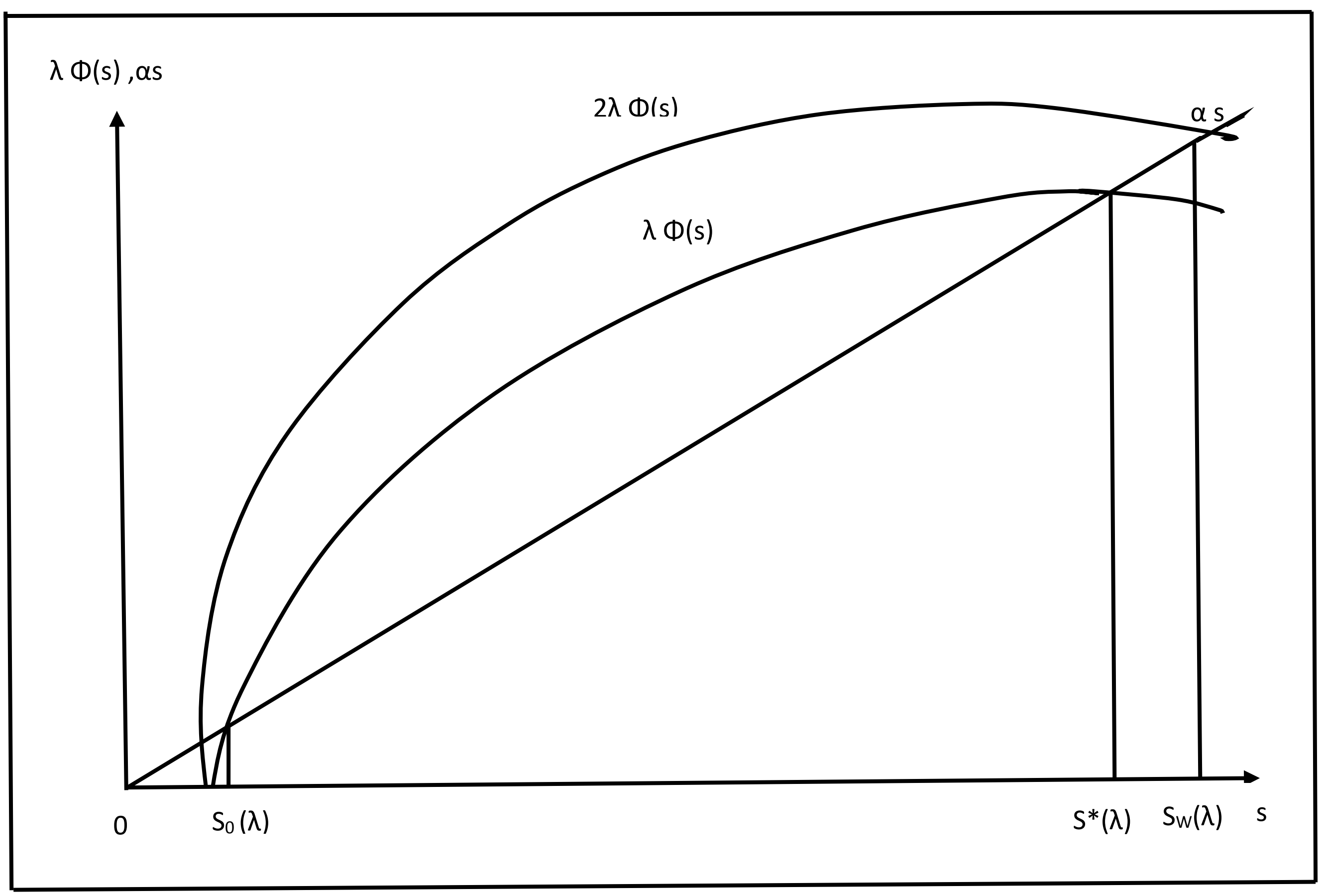
QoS has been determined by maximizing social welfare function expressed by (23). First Order Condition for welfare maximization w.r.t. s yields,

$$\frac{dW}{ds} = \frac{2\lambda}{27}\frac{(\bar{\theta}s(1+d)+\bar{\pi}-d)^2(2\bar{\theta}s(1+d)-\bar{\pi}+d)}{s^2(1+d)} - \alpha\, s = 0 \tag{24}$$

Similar to the analysis in monopoly case, equation (24) can be simplified as, $2\lambda\, \Phi(s) = \alpha\, s$

$2\lambda\, \Phi(s)$ can be represented by concave upward sloping curve for a given value of λ and it will lie above the curve $\lambda\, \Phi(s)$. Socially optimum level of quality can be found at the intersection of upward sloping curve and the linear curve, αs. Welfare maximizing level of service quality will be found at $s_w(\lambda)$ where second order sufficient condition (i.e., $2\lambda\, \Phi'(s) < \alpha$) is satisfied. Socially optimum service quality, $s_w(\lambda)$ will be situated at a higher level compared to the monopoly service level s^*. The situation can be described graphically in Figure 3.

Figure 3: Determination of Socially Optimum Service Quality (s_w)



We apply the implicit function theorem on (24) to derive the effect of change of advertisement on service quality. Hence, we have,

$$\frac{\delta^2 W}{\delta s^2}\frac{ds}{d\lambda} + \frac{\delta^2 W}{\delta\lambda\delta s} = 0$$

$$\text{Or, } \frac{ds}{d\lambda} = -\frac{\frac{\delta^2 W}{\delta\lambda\delta s}}{\frac{\delta^2 W}{\delta s^2}} \tag{25}$$

The denominator of (25) is in itself negative in sign because of the S.O.C of welfare maximization with respect to service quality. We must derive the sign of numerator to find out the change of direction of service quality due to change in advertisement provided by platform.

$$\frac{\delta^2 W}{\delta\lambda\delta s} = \frac{2}{27}\frac{(\overline{\theta}s(1+d)+\overline{\pi}-d)^2(2\overline{\theta}s(1+d)-\overline{\pi}+d)}{s^2(1+d)} > 0$$

Hence, positive association between QoS and level of advertisement has been established. This result is similar to the outcome that is established in monopoly case.

Social welfare function in (23) can be rewritten as,

$$W = \frac{2\lambda}{27}\frac{[\overline{\theta}s_w(\lambda)(1+d)+\overline{\pi}-d]^3}{s_w(\lambda)(1+d)} - \frac{\alpha}{2}s_w(\lambda)^2 - \frac{\beta\lambda^2}{2}$$

Finally we will find socially optimum level of advertisement by optimizing social welfare function w.r.t. level of advertisement, λ. FOC will yield the following result.

$$\frac{dW}{d\lambda} = \frac{2[\overline{\theta}s_w(\lambda)(1+d)+\overline{\pi}-d]^3}{27s_w(\lambda)(1+d)} - \beta\lambda + 2\lambda\frac{(\overline{\theta}s_w(\lambda)(1+d)+\overline{\pi}-d)^2(2\overline{\theta}s_w(\lambda)(1+d)-\overline{\pi}+d)}{27s_w(\lambda)^2(1+d)}\frac{\delta s}{\delta\lambda} - \alpha s_w(\lambda)\frac{\delta s}{\delta\lambda} = 0$$

$$\text{Or, } \frac{2[\overline{\theta}s_w(\lambda)(1+d)+\overline{\pi}-d]^3}{27s_w(\lambda)(1+d)} - \beta\lambda + \alpha\, s_w(\lambda)\frac{\delta s}{\delta\lambda} - \alpha s_w(\lambda)\frac{\delta s}{\delta\lambda} = 0 \qquad \text{(using (24))}$$

$$\text{Or, } \lambda_w = \frac{2[\overline{\theta}s_w(\lambda)(1+d)+\overline{\pi}-d]^3}{27\beta s_w(\lambda)(1+d)} \tag{26}$$

By solving (26), we obtain the value for welfare maximizing level of advertisement (λ_w) provided by platform. Inserting λ_w in (24), we can derive socially optimum level of service quality.

4. Comparison between Socially Optimum Outcome and Monopolistic Market Outcome under Two-sided Market:

Results derived in above two sections 2 and 3 show that the socially optimum structure does differ from the optimal market outcome in the monopoly case. In this section, we will study the divergence of results and attempt to give a plausible intuition of this divergence between two cases derived under section 2 and 3. The main intuition of this divergence falls upon the fact that monopoly firm is a profit-maximizing entity and will choose a strategy that will maximize its own profit whereas central planner will take wellbeing of each agent participating in platform into account. To analyze the scenario, we compare quality of service provided to buyers and level of advertising messages sent by platform under two regimes.

Proposition 3: *Monopoly platform provides lower amount of quality of service and level of advertisement compared to social optimum.*

We can find that welfare maximizing service quality (s_w) lies above the profit maximizing service quality (s^*). Thus, the socially optimum outcome requires platform to set service quality at a higher level since $s_w > s^*$. If platform sets a higher service quality, buyers will be interested to make transaction and retained to that platform. By seeing larger availability of buyers', sellers will get attracted and this will not only add to the profitability prospective of platform but also increases net surpluses of buyers and sellers through indirect externality component. Thus, giving better service by platform is welfare enhancing. This result is identical to the existing literature. Another noticeable result obtained in our model unambiguously shows that monopoly platform sends lower amount of advertisement to its buyers compared to that of social welfare perspective since $\lambda_w > \lambda^*$. A common feature of monopoly market structure is that a market sets a highly inefficient level of advertising. However in our study, we notice a counterintuitive result. The result is attributable to the fact that advertising makes buyers informed about discount offerings given by platform and this will attract more buyers. Because of the presence of indirect network externality, sellers also get attracted, resulting in enhancement of profit of platform. This will also beneficial from perspective of buyers' and sellers' as receiving more advertisement about discount offerings sent by platform will add to their utilities. So, indirect network externality plays a major role in deciding this divergence of outcomes under welfare prospective and monopoly case. To avoid such divergence, monopoly needs to take indirect network externality into account by considering welfare perspective of both groups of agents.

5. Concluding Remarks:

This paper is motivated by the prevailing condition in two sided markets where markets offer discount to its buyers to build a larger customer base. The study analyses the effect of discount offered to buyers on pricing structure, service quality and extent of advertisement of monopoly platform with indirect network effects by developing the structure of two sided markets similar to Rochet and Tirole (2003). It can be observed from profit maximization outcome that discount has considerable impact on optimal pricing structure of both sides, although discount is provided to only buyers' side. It is because of the presence of indirect network externality in two sided market. Discount is offered to buyers for attracting them to platform. A large number of sellers is needed to serve the expanded market on buyer side. This is why sellers are charged lower per transaction price by platform when platform decides to increase the amount of discount. It is found that price charged on buyers' side rises with discount offered to them and it is profitable for platform to offer discount as profit of platform is increasing in discount when a large number of buyers join to platform in response to an increase in discount. It is further observed that quality of service and advertising level provided by platform increase with the amount of discount, conditional upon the parametric restriction. When platform increases its discount amount, it seeks to transmit the information of discount to a wider section of buyers so that more buyers could avail the benefit and this can be ensured by increasing the extent of advertisement. Further, it will strive to deliver a better service to buyers' in an effort to retain existing customers and to entice non-users.

Results obtained in monopoly case carry over to social optimum. The finding of social optimum structure under two side markets counter the common feature of monopoly market which states that market uses highly inefficient level of advertising. Our result tells that monopoly platform in two sided market sets a lower level of advertising compared to the socially optimum level of advertising. More will be the advertising level, more buyers will get information about discount and more buyers will get to avail this benefit. Then larger section of buyers will be attracted. By seeing wider availability of buyers, more sellers will be interested in making transaction in this platform. So, setting a higher advertising not only adds to platform's profit but also benefits buyers' side and sellers' side. Similar finding can be obtained in case of QoS where platform chooses a lower level of quality compared to social optimum. The intuition behind the divergence in result of both regimes can be rest upon the role of indirect network externality in two sided market structure and non-accountability of utilities of both side in determining monopoly optimum outcome. One extension of our model would be to analyze how these results shape up under network competition.

Appendix

Derivation of Slope and Curvature of Φ(s) function

From the expression (16a), we define Φ (s) as,

$$\Phi\,(s) = \frac{(\overline{\theta}s(1+d)+\overline{\pi}-d)^2(2\overline{\theta}s(1+d)-\overline{\pi}+d)}{27s^2(1+d)}$$

$$= \frac{1}{27(1+d)}\left(\overline{\theta}(1+d)+\frac{\overline{\pi}-d}{s}\right)^2(2\overline{\theta}s(1+d)-\overline{\pi}+d)$$

Differentiating the above expression w.r.t. s, we obtain,

$$\Phi'(s) = \frac{1}{27(1+d)}[2\left(\overline{\theta}(1+d)+\frac{\overline{\pi}-d}{s}\right)(-\frac{\overline{\pi}-d}{s^2})\,(2\overline{\theta}s(1+d)-\overline{\pi}+d)$$

$$+\left(\overline{\theta}(1+d)+\frac{\overline{\pi}-d}{s}\right)^2\{2\overline{\theta}\,(1+d)\}]$$

$$= \frac{2\left(\overline{\theta}(1+d)+\frac{\overline{\pi}-d}{s}\right)}{27\,(1+d)}[-\frac{2\overline{\theta}(1+d)(\overline{\pi}-d)}{s}+\frac{(\overline{\pi}-d)^2}{s^2}+\overline{\theta}^2(1+d)^2+\frac{\overline{\theta}(1+d)(\overline{\pi}-d)}{s}]$$

$$=\frac{2\left(\overline{\theta}(1+d)+\frac{\overline{\pi}-d}{s}\right)}{27\,(1+d)}\left[\frac{(\overline{\pi}-d)^2}{s^2}+\overline{\theta}^2(1+d)^2-\frac{\overline{\theta}(1+d)(\overline{\pi}-d)}{s}\right]$$

$$=\frac{2\left(\overline{\theta}(1+d)+\frac{\overline{\pi}-d}{s}\right)}{27\,(1+d)}\left[\frac{\overline{\pi}-d}{s}\left\{\frac{\overline{\pi}-d}{s}-\overline{\theta}(1+d)\right\}+\overline{\theta}^2(1+d)^2\right]$$

$$=\frac{2}{27(1+d)}\left[\frac{\overline{\pi}-d}{s}\left\{\frac{(\overline{\pi}-d)^2}{s^2}-\overline{\theta}^2(1+d)^2\right\}+\overline{\theta}^2(1+d)^2\left(\overline{\theta}(1+d)+\frac{\overline{\pi}-d}{s}\right)\right]$$

$$=\frac{2}{27(1+d)}\left[\frac{(\overline{\pi}-d)^3}{s^3}+\overline{\theta}^3(1+d)^3\right]\;>0 \tag{A1}$$

Differentiating (A1) w.r.t s, we obtain the curvature of Φ (s). Therefore,

$$\Phi''(s)=\frac{2}{27(1+d)}\left[-\frac{3(\overline{\pi}-d)^3}{s^4}\right]<0$$

So, Φ (s) is an upward sloping concave curve.

Proof of Lemma 1

From (20), we define, $P_S=\frac{[2\overline{\pi}+d-\overline{\theta}(1+d)s^*(\lambda)]}{3}$

We can derive the effect of change in discount on price charged on seller, P_S.

$$\frac{dP_S}{dd}=\frac{\delta P_S}{\delta d}+\frac{\delta P_S}{\delta s}\frac{ds}{d\lambda}\frac{d\lambda}{dd}$$

$$=\left[\frac{\{1-\overline{\theta}s^*(\lambda)\}}{3}-\frac{\overline{\theta}(1+d)}{3}\frac{ds}{d\lambda}\frac{d\lambda}{dd}\right]$$

From Proposition 1(i), $\frac{d\lambda}{dd}>0$. Therefore, $\frac{dP_S}{dd}<0$ if $s^*(\lambda)>\frac{1}{\overline{\theta}}$. Now, the range within which the profit maximizing value of QoS will lie is $\left[\frac{(\overline{\pi}-d)}{2\overline{\theta}(1+d)},\frac{(2\overline{\pi}+d)}{\overline{\theta}(1+d)}\right]$. The association between P_S and d is negative if $\left[\frac{(\overline{\pi}-d)}{2\overline{\theta}(1+d)}-\frac{1}{\overline{\theta}}\right]>0$. So, more number of sellers will get attracted for an increase in d by reducing the price charged to sellers if $\overline{\pi}>2+3d$.

Proof of Proposition 1(i)

Second order sufficient condition for profit maximization w.r.t QoS (s) is as follows,

$$\alpha > \lambda\, \Phi'(s) = \lambda\, [\frac{2}{27(1+d)}\{\frac{(\bar{\pi}-d)^3}{s^3} + (1+d)^3\, \bar{\theta}^3\}]$$

$$\text{Or, } [\frac{2}{27(1+d)}\{\frac{(\bar{\pi}-d)^3}{s^3} + (1+d)^3\, \bar{\theta}^3\}] < \frac{\alpha}{\lambda}$$

$$\text{Or, } [\frac{(\bar{\pi}-d)^3}{s^3} + (1+d)^3\, \bar{\theta}^3] < \frac{27\alpha(1+d)}{2\lambda}$$

$$\text{Or, } \frac{(\bar{\pi}-d)^3}{s^3} < [\frac{27\alpha(1+d)}{2\lambda} - (1+d)^3\, \bar{\theta}^3]$$

$$\text{Or, } \frac{1}{s^3} < \frac{1}{(\bar{\pi}-d)^3}[\frac{27\alpha(1+d)}{2\lambda} - (1+d)^3\, \bar{\theta}^3]$$

$$\text{Or, } s^3 > \frac{(\bar{\pi}-d)^3}{[\frac{27\alpha(1+d)}{2\lambda} - (1+d)^3\, \bar{\theta}^3]}$$

$$\text{Or, } s > \frac{(\bar{\pi}-d)}{[\frac{27\alpha(1+d)}{2\lambda} - (1+d)^3\, \bar{\theta}^3]^{1/3}}$$

Therefore, satisfaction of S.O.C requires equilibrium level of quality (s^*) to be greater than A where A = $\frac{(\bar{\pi}-d)}{[\frac{27\alpha(1+d)}{2\lambda} - (1+d)^3\, \bar{\theta}^3]^{1/3}}$.

Now, from F.O.C for profit maximization w.r.t λ, we have

$$\frac{d\Pi_P}{d\lambda} = \frac{[\bar{\theta}s^*(\lambda)(1+d)+\bar{\pi}-d]^3}{27s^*(\lambda)(1+d)} - \beta\lambda = 0$$

To find the effect of change in d on λ, we use implicit function theorem. Therefore, we get,

$$\frac{\delta^2\Pi_P}{\delta\lambda^2}\frac{d\lambda}{dd} + \frac{\delta^2\Pi_P}{\delta d\delta\lambda} = 0$$

$$\text{Or, } \frac{d\lambda}{dd} = -\frac{\frac{\delta^2\Pi_P}{\delta d\delta\lambda}}{\frac{\delta^2\Pi_P}{\delta\lambda^2}}$$

The denominator of above expression is in itself negative in sign because of the S.O.C of profit maximization with respect to level of advertisement. We need to derive the sign of numerator to find out the change of direction of advertisement due to change in discount provided by platform.

$$\frac{\delta^2 \Pi_P}{\delta d \delta\lambda} = [\frac{3(\overline{\Theta} s^*(\lambda)(1+\mathrm{d})+\overline{\pi}-d)^2\{\overline{\Theta} s^*(\lambda)-1\}}{27 s^*(\lambda)(1+d)} - \frac{[\overline{\Theta} s^*(\lambda)(1+\mathrm{d})+\overline{\pi}-d]^3}{27 s^*(\lambda)(1+d)^2}]$$

$$= \frac{(\overline{\Theta} s^*(\lambda)(1+\mathrm{d})+\overline{\pi}-d)^2}{27 s^*(\lambda)(1+d)} [3\{\overline{\Theta} s^*(\lambda) - 1\} - \frac{(\overline{\Theta} s^*(\lambda)(1+\mathrm{d})+\overline{\pi}-d)}{(1+d)}]$$

$$= \frac{(\overline{\Theta} s^*(\lambda)(1+\mathrm{d})+\overline{\pi}-d)^2}{27 s^*(\lambda)(1+d)^2} [2\overline{\Theta} s^*(\lambda)(1+\mathrm{d}) - \overline{\pi} - 2d - 3]$$

Given Lemma 1, B will fall within the interval $[\frac{(\overline{\pi}-d)}{2\overline{\Theta}(1+d)}, \frac{(2\overline{\pi}+d)}{\overline{\Theta}(1+\mathrm{d})}]$ where $B=\frac{(\overline{\pi}+2d+3)}{2\overline{\Theta}(1+d)}$. For a positive association between extent of advertisement and amount of discount, profit maximizing value of service quality ($s^*(\lambda)$) should exceed B. From S.O.C, we have $s^* > A$. To satisfy two conditions, we need to have $s^*>A>B$. Therefore,

$$\frac{(\overline{\pi}-d)}{[\frac{27\alpha(1+d)}{2\lambda} - (1+d)^3\, \overline{\Theta}^3]^{1/3}} > \frac{(\overline{\pi}+2d+3)}{2\overline{\Theta}(1+d)}$$

Or, $\frac{(\overline{\pi}-d)^3}{[\frac{27\alpha(1+d)}{2\lambda} - (1+d)^3\, \overline{\Theta}^3]} > \frac{(\overline{\pi}+2d+3)^3}{8\overline{\Theta}^3 (1+d)^3}$

Or, $[(\overline{\pi} + 2d + 3)^3 \frac{27\alpha(1+d)}{2\lambda} - (\overline{\pi} + 2d + 3)^3(1 + d)^3\, \overline{\Theta}^3] < 8\overline{\Theta}^3 (1 + d)^3(\overline{\pi} - d)^3$

Or, $\overline{\Theta}^3 (1 + d)^3[(\overline{\pi} + 2d + 3)^3 + (2\overline{\pi} - 2d)^3] > (\overline{\pi} + 2d + 3)^3 \frac{27\alpha(1+d)}{2\lambda}$

Or, $9 (\overline{\pi} + 1) [(\overline{\pi} - d)^2 + 3(\mathrm{d} + 1)^2]\, \overline{\Theta}^3 (1 + d)^3 > (\overline{\pi} + 2d + 3)^3 \frac{27\alpha(1+d)}{2\lambda}$

Or, $\overline{\Theta}^3 > \frac{3\alpha(\overline{\pi}+2d+3)^3}{2\lambda(\overline{\pi}+1)\, [(\overline{\pi}-d)^2 + 3(\mathrm{d} + 1)^2]\, (1+d)^2}$

Or, $\overline{\Theta} > [\frac{3\alpha(\overline{\pi}+2d+3)^3}{2\lambda(\overline{\pi}+1)[(\overline{\pi}-d)^2 + 3(\mathrm{d} + 1)^2]\, (1+d)^2}]^{1/3}$

Or, $\overline{\Theta} > \overline{\Theta}^*$; where $\overline{\Theta}^* = [\frac{3\alpha(\overline{\pi}+2d+3)^3}{2\lambda(\overline{\pi}+1)[(\overline{\pi}-d)^2 + 3(\mathrm{d} + 1)^2]\, (1+d)^2}]^{1/3}$

$\therefore \frac{d\lambda}{dd} > 0$ when $\overline{\Theta} > \overline{\Theta}^*$ (Hence proved)

Proof of Proposition 1(ii)

The total effect of discount on QoS is sum of direct effect and indirect effect of discount on service quality.

$$\frac{ds}{dd} = \left(\frac{ds}{dd}\right)_\lambda + \frac{ds}{d\lambda}\,\frac{d\lambda}{dd}$$

From (16), we get,

$$\frac{d\Pi_P}{ds} = \lambda\frac{(\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d)^2(2\bar{\theta}(1+d)s^*(\lambda)-\bar{\pi}+d)}{27s^{*2}(\lambda)(1+d)} - \alpha\, s^*(\lambda) = 0$$

Or, $\frac{(\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d)^5(2\bar{\theta}(1+d)s^*(\lambda)-\bar{\pi}+d)}{27^2\beta s^{*3}(\lambda)(1+d)^2} - \alpha\, s^*(\lambda) = 0$ (Setting the value of λ from (17))

To find the effect of change in d on s, we use implicit function theorem. Therefore, we get,

$$\frac{\delta^2\Pi_P}{\delta s^2}\left(\frac{ds}{dd}\right)_\lambda + \frac{\delta^2\Pi_P}{\delta d\delta s} = 0$$

Or, $\left(\frac{ds}{dd}\right)_\lambda = -\frac{\frac{\delta^2\Pi_P}{\delta d\delta s}}{\frac{\delta^2\Pi_P}{\delta s^2}}$

The denominator of above expression is in itself negative in sign because of the S.O.C of profit maximization with respect to service quality. We need to derive the sign of numerator to find out the change of direction of service quality due to change in discount provided by platform.

$$\frac{\delta^2\Pi_P}{\delta d\delta s} = \frac{1}{27^2\beta s^{3*}(\lambda)}\Big[\frac{5(\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d)^4(2\bar{\theta}(1+d)s^*(\lambda)-\bar{\pi}+d)(\bar{\theta}s^*(\lambda)-1)}{(1+d)^2}$$

$$+\frac{(\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d)^5(2\bar{\theta}s^*(\lambda)+1)}{(1+d)^2} - \frac{2(\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d)^5(2\bar{\theta}(1+d)s^*(\lambda)-\bar{\pi}+d)}{(1+d)^3}\Big]$$

$$= \frac{1}{27^2\beta s^{*3}(\lambda)}\Big[\frac{(\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d)^5(2\bar{\theta}s^*(\lambda)+1)}{(1+d)^2}$$

$$+\Big\{\frac{(\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d)^4(2\bar{\theta}(1+d)s^*(\lambda)-\bar{\pi}+d)}{(1+d)^2}\left(5(\bar{\theta}s^*(\lambda)-1) - \frac{2(\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d)}{(1+d)}\right)\Big\}\Big]$$

The first term in bracketed portion is positive. So, we will concentrate on second term to find out the association between s and d. Therefore,

$$\frac{(\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d)^4(2\bar{\theta}(1+d)s^*(\lambda)-\bar{\pi}+d)}{(1+d)^3}\left[3\bar{\theta}(1+d)s^*(\lambda)-5-2\bar{\pi}-3d\right]$$

$$=\frac{(\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d)^4 3\bar{\theta}(2\bar{\theta}(1+d)s^*(\lambda)-\bar{\pi}+d)}{(1+d)^2}\left[s^*(\lambda)-\frac{(5+2\bar{\pi}+3d)}{3\bar{\theta}(1+d)}\right] \qquad (A2)$$

From Lemma 1, C will fall within the interval $[\frac{(\bar{\pi}-d)}{2\bar{\theta}(1+d)}, \frac{(2\bar{\pi}+d)}{\bar{\theta}(1+d)}]$ where C=$\frac{(2\bar{\pi}+3d+5)}{3\bar{\theta}(1+d)}$. Service quality and amount of discount will be positively related if profit maximizing value of service quality ($s^*(\lambda)$) exceeds C. From S.O.C, we have $s^* > A$. So, we need to have $s^*>A>C$. Therefore,

$$\frac{(\bar{\pi}-d)}{[\frac{27\alpha(1+d)}{2\lambda}-(1+d)^3\bar{\theta}^3]^{1/3}} > \frac{(2\bar{\pi}+3d+5)}{3\bar{\theta}(1+d)}$$

Or, $\frac{(\bar{\pi}-d)^3}{[\frac{27\alpha(1+d)}{2\lambda}-(1+d)^3\bar{\theta}^3]} > \frac{(2\bar{\pi}+3d+5)^3}{27\bar{\theta}^3(1+d)^3}$

Or, $[(2\bar{\pi}+3d+5)^3\frac{27\alpha(1+d)}{2\lambda} - (2\bar{\pi}+3d+5)^3(1+d)^3\bar{\theta}^3] < 27\bar{\theta}^3(1+d)^3(\bar{\pi}-d)^3$

Or, $\bar{\theta}^3(1+d)^3[(2\bar{\pi}+3d+5)^3+(3\bar{\pi}-3d)^3] > (2\bar{\pi}+3d+5)^3\frac{27\alpha(1+d)}{2\lambda}$

Or, $\bar{\theta}^3 > \frac{27\alpha(2\bar{\pi}+3d+5)^3}{2\lambda(1+d)^2[(2\bar{\pi}+3d+5)^3+(3\bar{\pi}-3d)^3]}$

Or, $\bar{\theta} > [\frac{27\alpha(2\bar{\pi}+3d+5)^3}{2\lambda(1+d)^2[(2\bar{\pi}+3d+5)^3+(3\bar{\pi}-3d)^3]}]^{1/3}$

Or, $\bar{\theta} > \bar{\theta}'$; where $\bar{\theta}' = [\frac{27\alpha(2\bar{\pi}+3d+5)^3}{2\lambda(1+d)^2[(2\bar{\pi}+3d+5)^3+(3\bar{\pi}-3d)^3]}]^{1/3}$

If buyers' spectrum expands beyond a critical value ($\bar{\theta}'$) i.e., when $\bar{\theta} > \bar{\theta}'$, then $(\frac{ds}{dd})_\lambda > 0$

From Proposition 1(i), it is clear that indirect effect of discount on service quality is also positive.

$\therefore \frac{ds}{dd} > 0$ when $\bar{\theta} \geq \max[\bar{\theta}^*, \bar{\theta}']$

Proof of Proposition 2(i)

Fromexpression (20), we define, $P_S = \frac{[2\overline{\pi}+d-\overline{\theta}(1+d)s^*(\lambda)]}{3}$

We can derive the effect of change in discount on price charged on seller, P_S.

$$\frac{dP_S}{dd} = \frac{\delta P_S}{\delta d} + \frac{\delta P_S}{\delta s}\frac{ds}{d\lambda}\frac{d\lambda}{dd}$$

$$= [\frac{\{1-\overline{\theta}s^*(\lambda)\}}{3} - \frac{\overline{\theta}(1+d)}{3}\frac{ds}{d\lambda}\frac{d\lambda}{dd}]$$

From Proposition 1(i), we know that $\frac{d\lambda}{dd}>0$ when $\overline{\theta} > \overline{\theta}^*$. So, second term is negative. From S.O.C, we have $s^* > A$. For the first term of the bracketed portion to be negative, we need $s^*>A>\frac{1}{\overline{\theta}}$

.

It is already proven in the previous Appendix portion that A>B (A3)

Now, we derive that B $(=\frac{(\overline{\pi}+2d+3)}{2\overline{\theta}(1+d)}) > \frac{1}{\overline{\theta}}$ (A4)

From Condition (A3) and (A4), we obtain $A>\frac{1}{\overline{\theta}}$

$\therefore \frac{dP_S}{dd}< 0$ when $\overline{\theta} > \overline{\theta}^*$ (Hence proved)

Proof of Proposition 2(ii)

From (20), we have, $P_B* = \frac{[2\overline{\theta}(1+d)s^*(\lambda)+d-\overline{\pi}]}{3}$

Applying Envelope Theorem, we derive the effect of change in discount on P_B.

$$\frac{dP_B}{dd} = \frac{\delta P_B}{\delta d} + \frac{\delta P_B}{\delta s}\frac{ds}{d\lambda}\frac{d\lambda}{dd}$$

$$=\frac{1}{3}[(2\overline{\theta}s^*(\lambda)+1) + 2\overline{\theta}(1+d)\frac{ds}{d\lambda}\frac{d\lambda}{dd}] > 0 \text{ when } \overline{\theta} > \overline{\theta}^*$$

Proof of Proposition 2(iii)

Volume of transaction is termed as V. We have,

$$V = \lambda N_B^* N_S^* = \frac{[\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d]^5}{243\beta(1+d)^2 {s^*}^2(\lambda)}$$

$$\frac{dV}{dd} = \frac{\delta V}{\delta d} + \frac{\delta V}{\delta s}\frac{\delta s}{\delta \lambda}\frac{\delta \lambda}{\delta d}$$

$$= [\frac{5\{\bar{\theta}s^*(\lambda)-1\}[\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d]^4}{243\beta(1+d)^2 {s^*}^2(\lambda)} - \frac{2[\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d]^5}{243\beta(1+d)^3 {s^*}^2(\lambda)}]$$

$$+ [\frac{5\bar{\theta}(1+d)[\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d]^4}{243\beta(1+d)^2 {s^*}^2(\lambda)} - \frac{2[\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d]^5}{243\beta(1+d)^3 {s^*}^3(\lambda)}] \frac{\delta s}{\delta \lambda}\frac{\delta \lambda}{\delta d}$$

$$= \frac{[\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d]^4}{243\beta(1+d)^2 {s^*}^2(\lambda)} [5\{\bar{\theta}s^*(\lambda) - 1\} - \frac{2[\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d]}{(1+d)}]$$

$$+ \frac{[\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d]^4}{243\beta(1+d)^2 {s^*}^2(\lambda)} [5\bar{\theta}(1+d) - \frac{2[\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d]}{s^*(\lambda)}] \frac{\delta s}{\delta \lambda}\frac{\delta \lambda}{\delta d}$$

$$= \frac{[\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d]^4}{243\beta(1+d)^3 {s^*}^2(\lambda)} [3\bar{\theta}(1+d)s^*(\lambda) - 2\bar{\pi} - 3d - 5]$$

$$+ \frac{[\bar{\theta}(1+d)s^*(\lambda)+\bar{\pi}-d]^4}{243\beta(1+d)^2 {s^*}^3(\lambda)} [3\bar{\theta}(1+d)s^*(\lambda) - 2\bar{\pi} + 2d] \frac{\delta s}{\delta \lambda}\frac{\delta \lambda}{\delta d} \qquad (A5)$$

From Lemma 1, C and D will fall within the interval $[\frac{(\bar{\pi}-d)}{2\bar{\theta}(1+d)}, \frac{(2\bar{\pi}+d)}{\bar{\theta}(1+d)}]$ where $C=\frac{(2\bar{\pi}+3d+5)}{3\bar{\theta}(1+d)}$ and $D=\frac{(2\bar{\pi}-2d)}{3\bar{\theta}(1+d)}$. Volume of transaction and amount of discount will be positively related if profit maximizing value of service quality ($s^*(\lambda)$) exceeds C and D. From S.O.C, we have $s^* > A$. So, we need to have $s^*>A>C$.

We already prove that A>C when $\bar{\theta} > \bar{\theta}'$; where $\bar{\theta}' = [\frac{27\alpha(2\bar{\pi}+3d+5)^3}{2\lambda(1+d)^2[(2\bar{\pi}+3d+5)^3+(3\bar{\pi}-3d)^3]}]^{1/3}$

If buyers' spectrum expands beyond a critical value ($\bar{\theta}'$) then first term of (A5) is positive and this satisfies S.O.C of profit maximization w.r.t s. Now second term of the expression of (A5) is positive if $s^*>A>D$.

It is already proven previously that A>C (A6)

Now, we derive that C ($=\frac{(2\bar{\pi}+3d+5)}{3\bar{\Theta}(1+d)}$) > D ($=\frac{(2\bar{\pi}-3d)}{3\bar{\Theta}(1+d)}$) (A7)

From Condition (A6) and (A7), we obtain A>D

Therefore, $\frac{dV}{dd}>0$ if $\bar{\Theta} \geq \bar{\Theta}'$ (Hence proved)

Proof of Proposition 2(iv)

$$\Pi_P = \frac{1}{2}\,\frac{[\bar{\Theta}s^*(\lambda)(1+\mathrm{d})+\bar{\pi}-d]^6}{27^2\beta s^*(\lambda)^2(1+d)^2} - \frac{\alpha}{2}\,s^*(\lambda)^2$$

Differentiating the profit function w.r.t. d, we get,

$$\frac{d\Pi_P}{dd} = \frac{\delta\Pi_P}{\delta d} + \frac{\delta\Pi_P}{\delta\lambda}\frac{d\lambda}{dd}$$

$$= \frac{1}{2}\,\frac{1}{27^2\beta s^*(\lambda)^2}\left[\frac{6[\bar{\Theta}s^*(\lambda)(1+\mathrm{d})+\bar{\pi}-d]^5\{\bar{\Theta}s^*(\lambda)-1\}}{(1+d)^2} - \frac{2[\bar{\Theta}s^*(\lambda)(1+\mathrm{d})+\bar{\pi}-d]^6}{(1+d)^3}\right] \text{ (Since, } \frac{\delta\Pi_P}{\delta\lambda}=0\text{)}$$

$$= \frac{[\bar{\Theta}s^*(\lambda)(1+\mathrm{d})+\bar{\pi}-d]^5}{27^2\beta s^*(\lambda)^2(1+d)^3}\left[3\{\bar{\Theta}s^*(\lambda)-1\}(1+d) - \bar{\Theta}s^*(\lambda)(1+\mathrm{d}) - \bar{\pi} + \mathrm{d}\right]$$

$$= \frac{[\bar{\Theta}s^*(\lambda)(1+\mathrm{d})+\bar{\pi}-d]^5}{27^2\beta s^*(\lambda)^2(1+d)^3}\left[2\,\bar{\Theta}s^*(\lambda)(1+\mathrm{d}) - 3 - 2d - \bar{\pi}\right]$$

$$= \frac{2\,\bar{\Theta}\,[\bar{\Theta}s^*(\lambda)(1+\mathrm{d})+\bar{\pi}-d]^5}{27^2\beta s^*(\lambda)^2(1+d)^2}\left[s^*(\lambda) - \frac{3+2d+\bar{\pi}}{2\,\bar{\Theta}(1+d)}\right]$$

$\therefore \frac{d\Pi_P}{dd}>0$ when $\bar{\Theta} > \bar{\Theta}^*$ (Hence proved)

Derivation of Equation (23)

From (22), Social welfare function can be written as,

$$W = \lambda\int_{\pi^*}^{\bar{\pi}} d\pi \int_{\Theta^*}^{\bar{\Theta}} (\Theta_B s\,(1+d) - P_B)\,d\Theta + \lambda\int_{\Theta^*}^{\bar{\Theta}} d\Theta \int_{\pi^*}^{\bar{\pi}} (\pi_S - P_S)\,d\pi$$

$$+ \quad \lambda\,(P_B + P_S - \mathrm{d})\int_{\pi^*}^{\bar{\pi}} d\pi \int_{\Theta^*}^{\bar{\Theta}} d\Theta - \frac{\alpha}{2}s^2 - \frac{\beta\lambda^2}{2}$$

$$= \lambda \int_{\pi^*}^{\bar{\pi}} d\pi \int_{\Theta^*}^{\bar{\Theta}} \Theta_B s\,(1+\ d) d\Theta + \ \lambda \int_{\Theta^*}^{\bar{\Theta}} d\Theta \int_{\pi^*}^{\bar{\pi}} \pi_S\, d\pi - \lambda\, \mathrm{d} \int_{\pi^*}^{\bar{\pi}} d\pi \int_{\Theta^*}^{\bar{\Theta}} d\Theta$$

$$- \ \frac{\alpha}{2} s^2 - \frac{\beta\lambda^2}{2}$$

$$= \lambda\, \mathrm{s}\,(1+\mathrm{d})\,\left(\frac{\bar{\Theta}^2}{2} - \ \frac{\Theta^{*2}}{2}\right) \int_{\pi^*}^{\bar{\pi}} d\pi + \lambda \frac{1}{2} (\bar{\pi}^2 - \pi^{*2}) \ \int_{\Theta^*}^{\bar{\Theta}} d\Theta - \lambda\, \mathrm{d}\, (\bar{\pi} - \pi^*)(\bar{\Theta} - \Theta^*)$$

$$- \ \frac{\alpha}{2} s^2 - \frac{\beta\lambda^2}{2}$$

$$= \lambda\, \mathrm{s}\,(1+\mathrm{d})\,\left(\frac{\bar{\Theta}^2}{2} - \ \frac{\Theta^{*2}}{2}\right) (\bar{\pi} - \pi^*) + \lambda \frac{1}{2} (\bar{\pi}^2 - \pi^{*2})\ (\bar{\Theta} - \Theta^*)$$

$$- \lambda\, \mathrm{d}\, (\bar{\pi} - \pi^*)(\bar{\Theta} - \Theta^*) \ - \ \frac{\alpha}{2} s^2 - \frac{\beta\lambda^2}{2}$$

$$= \frac{\lambda}{2} (\bar{\pi} - \pi^*)(\bar{\Theta} - \Theta^*)\ [\mathrm{s}\,(1+\mathrm{d})\,(\bar{\Theta} + \Theta^*) + (\bar{\pi} + \pi^*) - 2\mathrm{d}] - \ \frac{\alpha}{2} s^2 - \frac{\beta\lambda^2}{2} \quad \text{(A8)}$$

Putting, $\Theta^* = \frac{P_B}{s(1+d)}$ and $\pi^* = P_S$ in (A8), we have,

$$W\,(P_B, P_S, s, \lambda) = \frac{\lambda}{2} \left(\bar{\Theta} - \frac{P_B}{s(1+d)}\right) (\bar{\pi} - P_S)\ [s\,(1+d)\,\left(\bar{\Theta} + \frac{P_B}{s(1+d)}\right) + (\bar{\pi} + P_S) - 2d]$$

$$- \ \frac{\alpha}{2} s^2 - \frac{\beta\lambda^2}{2}$$

The above equation will define social welfare as a function of four endogenous variables (P_B, P_S, s and λ) of our model.

$$W\,(P_B, P_S, s, \lambda) = \frac{\lambda}{2} \left(\bar{\Theta} - \frac{P_B}{s(1+d)}\right) (\bar{\pi} - P_S)\ [s\,(1+d)\,\left(\bar{\Theta} + \frac{P_B}{s(1+d)}\right) + (\bar{\pi} + P_S) - 2d]$$

$$- \ \frac{\alpha}{2} s^2 - \frac{\beta\lambda^2}{2}$$

Setting values of P_B, P_S similar to monopoly price structure (derived in (11) and (12)), we obtain social welfare as a function of Qos (s) and level of advertisement (λ).

So, above equation can be expressed as,

$$W(s, \lambda) = \frac{\lambda}{2}[\bar{\theta} - \frac{(2\bar{\theta}s(1+d)+d-\bar{\pi})}{3s(1+d)}]\ [\bar{\pi}\ - \frac{2\bar{\pi}+d-\bar{\theta}s(1+d)}{3}]$$

$$[s\ (1+d)\ (\bar{\theta} + \frac{(2\bar{\theta}s(1+d)+d-\bar{\pi})}{3s(1+d)}) + (\bar{\pi} + \frac{2\bar{\pi}+d-\bar{\theta}s(1+d)}{3}) - 2d]$$

$$-\ \frac{\alpha}{2}s^2 - \frac{\beta\lambda^2}{2}$$

$$= \frac{\lambda}{2}\ [\frac{\{\bar{\theta}s(1+d)+\bar{\pi}-d\}}{3s(1+d)}]\ [\frac{[\bar{\theta}s(1+d)+\bar{\pi}-d]}{3}][\ \frac{(5\bar{\theta}s(1+d)+d-\bar{\pi})}{3} + \frac{5\bar{\pi}+d-\bar{\theta}s(1+d)}{3} - 2d]$$

$$-\ \frac{\alpha}{2}s^2 - \frac{\beta\lambda^2}{2}$$

$$\text{Or, } W(s, \lambda) = \frac{2\lambda}{27}\frac{[\bar{\theta}s(1+d)+\bar{\pi}-d]^3}{s(1+d)} - \frac{\alpha}{2}s^2 - \frac{\beta\lambda^2}{2} \qquad (23)$$

We derive social welfare as function of four endogenous variables of our model.

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