Improving Abatement Levels and Welfare by Coarse Correlation in an Environmental Game*

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Abstract

Coarse correlated equilibria (CCE, Moulin and Vial, 1978) can be used to substantially improve upon the Nash equilibrium solution of the well-analysed *abatement game* (Barrett, 1994). We show this by computing successively the CCE with the largest total utility, the one with the highest possible abatement levels and finally, the one with maximal abatement level while maintaining at least the level of utility from the Nash outcome.

Keywords: Abatement game, Coarse correlated equilibrium, Efficiency gain.

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1 INTRODUCTION

Mediated communication is a true and tested way to achieve incentive-compatible coordination on efficient outcomes in many non-cooperative games. Surely, the most influential among such schemes in the literature is *correlated equilibrium* (CE), as proposed by Aumann (1974, 1987). We note that applications of CE and related concepts to economic modeling have been few and far between; we here offer a modest step towards filling this obvious gap.

We focus on the simplest version of the very popular game of environmental-abatement, introduced by Barrett (1994), in which two agents (two countries or two firms) choose how much to abate that involves a personal (abatement) cost but the total abatement does generate a benefit to both agents.[1] To keep computations tractable, we postulate a symmetric two-person game with quadratic utilities, where the Nash equilibrium outcome is inefficient however cannot be improved by any CE outcome, because we are in a potential game, where the unique Nash equilibrium is known to be the only CE (Liu (1996), Neyman (1997), Yi (1997) and Ui (2008)).

We instead examine the performance of *coarse correlated equilibrium* (CCE), a less-demanding variant of CE, introduced in Moulin and Vial (1978), in our abatement game. In a CCE, a (fictitious) mediator asks the players, before running a lottery (a correlation device), to either commit to the future outcome of the lottery or to play any strategy of their own without learning anything about the outcome of the lottery. The equilibrium property is that each player finds it optimal to commit ex ante to use the strategy selected by the lottery.[2]

In the context of climate change negotiation, in particular for the abatement game, a (coarse) correlation device can be interpreted as an independent agency working with all relevant countries towards the ultimate goal of global emission reduction. In a CCE of the abatement game, each country remains free to revert to a non-cooperative emission, but does not benefit from doing so as long as other countries commit to the policy selected by the agency.[3]

[1] Several non-cooperative as well as cooperative solutions have already been analysed for this abatement game, such as, Barrett (2001), Finus (2003), McGinty (2007), Rubio and Ulph (2006). To the best of our knowledge, in the whole literature on game theoretic applications in environmental economics, correlation has been mostly ignored, with the possible exceptions of Forgó, Fülöp and Prill (2005) and Forgó (2011) who used (modified versions of) CE in other environmental games. Baliga and Maskin (2003) surveyed some models of mechanisms in this literature.

[2] In their paper, Moulin and Vial (1978) called this equilibrium concept a correlation scheme. Young (2004) and Roughgarden (2009) introduced the terminology of coarse correlated equilibrium that was later adopted by Ray and Sen Gupta (2013) and Moulin et al. (2014), while Forgó (2010) called it a weak correlated equilibrium.

[3] The randomisation device in a CCE can be seen as mediating institutions like government agencies, international bodies (as analysed in Arce (1995), Arce (1997) and Awaya and Krishna (2018), for example), such as European Union (EU), World Trade Organisation (WTO), United Nations Framework Convention on Climate Change (UNFCCC) who can provide recommendations to all the signatories towards the ultimate goal of global emission reduction (see, for example, Forgo et al. (2005), Forgo (2010), Forgo (2011) and Slechten (2013)).

We first build on results of Moulin et al. (2014), who showed, in a more general class of quadratic symmetric two-person games, how coarse correlation can improve the efficiency of the Nash outcome, that is, raise the common utility of the two agents in such a game. We thus have characterised the best, in terms of payoffs, CCE (that turns out to be just a $50-50$ lottery over two outcomes, as in Moulin et al. (2014)) for our abatement game.[4]

Undoubtedly, in the environmental game we study here, the improvement of the polluters' welfare is only one of many normative goals of interest; however perhaps more important is the social concern for reducing the total level of physical pollution, i. e., increasing the (equilibrium) abatement levels. Not surprisingly, there is a tension between these two goals: the optimal CCE (in terms of payoffs) may not provide the optimal abatement level; on the other hand, we observe that the abatement-maximising CCE may generate low, even well below the Nash, payoffs.

For example, consider a baseline abatement game in which the utility of agent i is given by the function $q - bq^2 - cq_i^2$, where q_i is the choice of abatement level by agent i and thus q $(= q_1 + q_2)$ is the total abatement, with the *benefit* and *cost* parameters, respectively, $b = 2$ and $c = 1$. One can show that for this game, the optimal CCE (total) payoff is $\pi^{CCE} = \frac{23}{104} \approx 0.2211$, while the Nash (total) payoff is $\pi^N = \frac{11}{50} \approx 0.22$ and hence, $\frac{\pi^{CCE}}{\pi^N} \approx 1.0052$ (an improvement over the Nash payoff by 0.5%, which seems small but can be a significant amount if one thinks of real-life magnitudes); in this case, $\frac{q^{CCE}}{q^N} \approx 1.057$, an improvement over the Nash abatement by 5.7%. If one wished to maximise just the abatement level, then the best possible scenario would have been associated with another similar lottery for which $\frac{q^{CCE}}{q^N} \approx 1.53$, i.e., 53% improvement over the Nash abatement level; however, the associated payoff falls by about 35% from the Nash payoff level. Of course, one may now ask what if we wish to achieve at least the Nash payoff while maximising the abatement level; the answer (for this baseline game) is that we can have a maximum of 11.5% improvement in the abatement maintaining such a restriction, with a payoff just above (0.05% more than) the Nash payoff.

Keeping this tug-of-war spirit in mind, we, in this paper, prove three general results for our abatement game, by computing successively: 1. the most efficient (in terms of payoffs) symmetric CCE (irrespective of abatement levels); 2. the CCE with the maximal abatement level (that may force the players' utilities below their Nash equilibrium levels) and 3.the CCE with the largest abatement level among the ones maintaining (at least) the Nash equilibrium utility level. Our answers are indeed in closed forms; these are illustrated using the above baseline game, based on the (ratio of) the cost and the benefit parameters.

[4]Note that the class of public good provision (as analysed in Moulin et al. (2014)) and that of the abatement game differ only in the cost term which is linear there and quadratic here, however this difference changes the entire analysis; for instance, in the public good provision game, the support of the optimal CCE is on the axis, which never happens here.

2 MODEL

We first present here the notations and definitions used in Moulin et al. (2014), for the sake of consistency and completeness.

2.1 Coarse Correlated Equilibrium

Consider a two-person normal form game, $G = [X_1, X_2; u_1, u_2]$, where the strategy sets, X_1 and X_2, are closed real intervals and the payoff functions $u_i : X_1 \times X_2 \to \mathbb{R}$, $i = 1, 2$, are continuous. We write $\mathbb{C}(X_1 \times X_2)$ for the set of such continuous functions and similarly, $\mathbb{C}(X_i)$ for the set of continuous functions on X_i.

Let $\mathcal{L}(X_1 \times X_2)$ with generic element L and $\mathcal{L}(X_i)$ with generic element ℓ_i denote the sets of probability measures on $X_1 \times X_2$ and X_i respectively. Let the mean of $u_i(x_1, x_2)$ with respect to L be denoted by $u_i(L)$.

The deterministic distribution at z is denoted by δ_z, and for product distributions such as $\delta_{x_1} \otimes \ell_2$ we write $u_i(\delta_{x_1} \otimes \ell_2)$ simply as $u_i(x_1, \ell_2)$.

Given $L \in \mathcal{L}(X_1 \times X_2)$, we write L^i for the marginal distribution of L on X_i, defined as follows: $\forall f \in \mathbb{C}(X_i)$, $f(L^i) = f^*(L)$, where $f^*(x_1, x_2) = f(x_1)$ for all $x_1, x_2 \in X_1 \times X_2$.

Definition 1 *A coarse correlated equilibrium (CCE) of the game G is a lottery $L \in \mathcal{L}(X_1 \times X_2)$ such that $u_1(L) \geq u_1(x_1, L^2)$ and $u_2(L) \geq u_2(L^1, x_2)$ for all $(x_1, x_2) \in X_1 \times X_2$.*

2.2 Abatement Game

We present below the model proposed in Barrett (1994) with two agents ($n = 2$).

The payoff function of an agent is a function of the abatement level chosen by both agents q_1 and q_2. Let us write the total abatement as q ($q = q_1 + q_2$) and the benefit function of agent i as $B_i(q) = \frac{B}{2}(Aq - \frac{q^2}{2})$. The cost function of each agent is a function of its own abatement level q_i and is given as $C_i(q_i) = \frac{Cq_i^2}{2}$. The payoff function of agent i is thus given by $u_i(q_1, q_2) = \frac{ABq}{2} - \frac{Bq^2}{4} - \frac{Cq_i^2}{2}$, where A, B and C are all positive.[5]

For our purposes, we now call $a = \frac{AB}{2}$, $b = \frac{B}{4}$, $c = \frac{C}{2}$ with the assumption that $a > 0$, $b > 0$ and $c \geq 0$; we also set the ratio $r = \frac{c}{b}$ with $0 \leq r \leq 1$. The above payoff function can then be written as: $u_1(q_1, q_2) = aq - bq^2 - cq_1^2$; $u_2(q_1, q_2) = u_1(q_2, q_1)$.

We call the above game an *abatement game* in the rest of this paper.

Given q_2, the best response of agent 1 (symmetrically, for agent 2) is $BR_1(q_2) = \frac{\partial u_1(q_1, q_2)}{\partial q_1} = a - 2bq - 2cq_1$. Thus, the Nash equilibrium (q_1^N, q_2^N) is given by $q_1^N = q_2^N = \frac{a}{2(2b+c)}$; the corresponding

[5]Note that the benefit function in the published version of Barrett (1994) has a typo that we have corrected here.

total abatement q^N and total payoff π^N are given by $q^N = \frac{a}{b}\frac{1}{(2+r)}$ and $\pi^N = \frac{a^2}{2b}\frac{(4+3r)}{(2+r)^2}$.

One may compute the efficient abatement levels (q_1^{eff}, q_2^{eff}) by maximising the total payoff $u_1(q_1,q_2)+u_2(q_1,q_2) = 2aq - 2bq^2 - c(q_1^2+q_2^2)$; using $q_1 = q_2$, it is easy to prove that $q_1^{eff} = q_2^{eff} = \frac{a}{4b+c}$ with $\pi^{eff} = \frac{2a^2}{4b+c} = \frac{a^2}{b}\frac{2}{4+r}$.

As is well-known, the abatement game is a potential game with the potential function $P(q_1,q_2) = aq - bq^2 - c(q_1^2+q_2^2)$, which is smooth and concave. Therefore, the only correlated equilibrium is the Nash equilibrium q^N. One can however use CCE for this game to improve upon the Nash equilibrium.

We borrow the characterization of the CCEs in the more general class of quadratic symmetric two-person games in Moulin et al. (2014), as our game here is a special instance in that class.

The equilibrium condition in Definition 1 translates to a condition linking the three moments of L. If lottery L is the distribution of the symmetric random variable (Z_1,Z_2), these moments are the expected values of Z_i, Z_i^2, and $Z_1 \cdot Z_2$ that we denote by α, β and γ, where $\alpha = E_L[Z_1]$, $\beta = E_L[Z_1^2]$ and $\gamma = E_L[Z_1 \cdot Z_2]$.

Proposition 1 *A symmetric lottery $L \in \mathcal{L}^{sy}(\mathbb{R}_+^2)$ is a CCE of the abatement game if and only if $\max_{z\geq 0}\{(a-2b\alpha)z - (b+c)z^2\} \leq a\alpha - (b+c)\beta - 2b\gamma$. The corresponding utility (for an agent i) is $u_i(L) = 2a\alpha - (2b+c)\beta - 2b\gamma$.*

The proof of Proposition 1 is in the Appendix. We now state a couple of technical Lemmata that will be used in our main results. The first is due to Moulin et al. (2014). It identifies the range of the vector (α,β,γ) when $L \in \mathcal{L}^{sy}(\mathbb{R}_+^2)$ and also shows that this range is covered by two families of very simple lotteries with at most four strategy profiles in their support.

Let $\mathcal{L}^*$ be the subset of $\mathcal{L}^{sy}(\mathbb{R}_+^2)$ containing the simple lotteries of the form $L = \frac{q}{2}(\delta_{z,z} + \delta_{z',z'}) + \frac{p}{2}(\delta_{z,z'} + \delta_{z',z})$, where z, z', q and p are non-negative and $q+p=1$. Let $\mathcal{L}^{**}$ be the subset of $\mathcal{L}^{sy}(\mathbb{R}_+^2)$ of the form $L = q\cdot\delta_{z,z} + q'\cdot\delta_{0,0} + \frac{p}{2}(\delta_{0,z}+\delta_{z,0})$, where z, q, q' and p are non-negative and $q+q'+p=1$.

Lemma 1 *$i)$ For any $L \in \mathcal{L}^{sy}(\mathbb{R}_+^2)$ and the corresponding random variable (Z_1,Z_2), $\alpha,\gamma \geq 0$; $\beta \geq \gamma$; $\beta+\gamma \geq 2\alpha^2$;*

$ii)$ Equality $\beta = \gamma$ in $i)$ holds if and only if L is diagonal: $Z_1 = Z_2$ (a.e.);

$iii)$ Equality $\beta+\gamma = 2\alpha^2$ in $i)$ holds if and only if L is anti-diagonal: Z_1+Z_2 is constant (a.e.);

$iv)$ For any $(\alpha,\beta,\gamma) \in \mathbb{R}_+^3$ satisfying inequalities in $i)$, there exists $L \in \mathcal{L}^* \cup \mathcal{L}^{**}$ with precisely these parameters.

Note that Lemma 1 implies $\beta \geq \alpha^2$, with equality $\beta = \alpha^2$ if and only if L is deterministic, because $\beta = \alpha^2$ implies both $\beta = \gamma$ and $\beta+\gamma = 2\alpha^2$. The proof of Lemma 1 can be found in Moulin et al. (2014) and thus is omitted here.

For a given lottery L with the parameters α, β, γ the corresponding (total) abatement level is $q^L = 2\alpha$. Now, we observe the following.

Lemma 2 *A lottery $L(\alpha, \beta, \gamma)$ is a CCE if and only if*

$$\text{either } \alpha > \frac{a}{2b} \text{ and } a\alpha \geq (b+c)\beta + 2b\gamma$$

$$\text{or } \alpha \leq \frac{a}{2b} \text{ and } a\alpha \geq (b+c)\beta + 2b\gamma + \frac{1}{4}\frac{(a-2b\alpha)^2}{b+c}.$$

The proof of Lemma 2 is straightforward, developing the statement in Proposition 1 and thus is omitted.

Remark 1 *Given Lemma 1, the first case (involving case $\alpha > \frac{a}{2b}$) in Lemma 2 is impossible. This can be shown easily by fixing α and considering the two conditions on the vector (β, γ), namely, $\beta + \gamma \geq 2\alpha^2$ (from Lemma 1) and $a\alpha \geq (b+c)\beta + 2b\gamma$; note that the line $(b+c)\beta + 2b\gamma = a\alpha$ is flatter than the line $\beta + \gamma = 2\alpha^2$ in the (β, γ)-plane and therefore, the two corresponding half-spaces intersect in the positive orthant if and only if $\frac{a\alpha}{b+c} \geq 2\alpha^2$ which contradicts $\alpha > \frac{a}{2b}$.*

3 RESULTS

3.1 Payoff-maximising CCE

The focus of this section is CCEs that maximize the total payoff $u_1 + u_2$. Recall that the baseline example in the Introduction illustrates a CCE which achieves a higher payoff compared to the Nash equilibrium. We will now provide the formal result that forms the basis for this example.

First, as the abatement game is symmetric, we can limit our search to symmetric lotteries L only (as explained in Moulin et al. (2014), when one identifies an optimal symmetric CCE, one also captures an optimal CCE among all CCEs, symmetric or otherwise). We denote the set of symmetric lotteries by $\mathcal{L}^{sy}(\mathbb{R}^2_+)$. The following result characterises the payoff-maximising CCE for the abatement game.

Proposition 2 *i) If $b \leq c$, the Nash equilibrium of the abatement game is its only CCE.*

ii) If $b > c$, the optimal values of the three moments of the payoff-maximising L, $\widetilde{L}$, are given by $(\widetilde{\alpha}, \widetilde{\beta}, \widetilde{\gamma})$:

$$\widetilde{\alpha} = \frac{a}{b}\frac{2+2r-r^2}{2(4+5r)},$$

$$\widetilde{\beta} = \frac{a^2}{b^2}\frac{4+8r+r^2-4r^3}{4(4+5r)^2} \text{ and } \widetilde{\gamma} = \frac{a^2}{b^2}\frac{4+8r-r^2-4r^3+2r^4}{4(4+5r)^2};$$

while the optimal CCE is $\widetilde{L} = \frac{1}{2}\delta_{(z,z')} + \frac{1}{2}\delta_{(z',z)}$, with

$$z, z' = \frac{a}{b}\frac{2+2r-r^2 \pm r\sqrt{1-r^2}}{2(4+5r)}.$$

Lemma 3 below states a two-step algorithm to find the payoff-maximising CCEs using Lemma 1 and Proposition 1.

Lemma 3 *Given the abatement game, the following nested programmes generate the payoff-maximising CCEs:*

Step 1*: Fix* α *non-negative, and solve the linear programme*

$$\min_{\beta,\gamma}\{(2b+c)\beta+2b\gamma\} \textit{ under constraints}$$

$$\beta \geq \gamma \geq 0;\ \beta+\gamma \geq 2\alpha^2;\ (b+c)\beta+2b\gamma \leq a\alpha - \max_{z\geq 0}\{(a-2b\alpha)z-(b+c)z^2\}.$$

Step 2*: With the solutions* $\beta(\alpha),\gamma(\alpha)$ *found in Step* 1*, solve*

$$\max_{\alpha}\{2a\alpha-(2b+c)\beta(\alpha)-2b\gamma(\alpha)\} \textit{ under constraints}$$

$$\alpha \geq 0;\ \max_{z\geq 0}\{(a-2b\alpha)z-(b+c)z^2\} \leq a\alpha-(b+c)\beta(\alpha)-2b\gamma(\alpha).$$

Moreover, there is a payoff-maximising CCE in $\mathcal{L}^* \cup \mathcal{L}^{**}$*.*

Lemma 3 is similar to Theorem 1 in Moulin et al. (2014) and hence the proof is omitted here. The rest of the proof of Proposition 2 is postponed to the Appendix.

Remark 2 *Using Proposition 2, one can easily compute the maximum payoff obtained by the CCE* $\widetilde{L}$ *(when* $b > c$*); the payoff of agent* i *at* $\widetilde{L}$ *is* $u_i(\widetilde{L}) = \frac{1}{b^2-c^2}[\frac{a^2}{b^2}\frac{(2+2r-r^2)^2}{4(4+5r)} - \frac{a^2c}{4b^2}] = \frac{a^2}{b}\frac{4+4r-r^2}{4(4+5r)}$*. The computation is straightforward and hence is omitted.*

Remark 3 *Observe that from Proposition 2, the ratio* $\frac{q^{CCE}}{q^N}$ *for payoff-maximizing CCE is equal to* $\frac{(2+2r-r^2)(2+r)}{(4+5r)}$ *which is a concave function when* r *is between* 0 *and* 1 *with its maximum achieved close to* $r=\frac{1}{2}$ *(*≈ 0.489 *to be more specific).*

3.1.1 Baseline example

We now illustrate Proposition 2 by revisiting the example in the Introduction more formally.

Consider an abatement game with $a = 1$, $b = 2$ and $c = 1$; hence, $r = \frac{c}{b} = \frac{1}{2} < 1$ and the payoff function is given by $u_i(q_1, q_2) = q - 2q^2 - q_i^2$, with Nash equilibrium (total) abatement level, $q^N = \frac{1}{5}$.

From Proposition 2, the payoff-maximising CCE corresponds to the optimal values of the moments: $\widetilde{\alpha} = \frac{11}{104} \approx 0.1057$, $\widetilde{\beta} = \frac{31}{2704} \approx 0.0114$ and $\widetilde{\gamma} = \frac{59}{5408} \approx 0.0109$. The optimal CCE is the lottery $\widetilde{L} = \frac{1}{2}\delta_{(z,z')} + \frac{1}{2}\delta_{(z',z)}$, where $z, z' = \frac{11\pm\sqrt{3}}{104}$, that chooses two outcomes $(\frac{11+\sqrt{3}}{104}, \frac{11-\sqrt{3}}{104})$ and $(\frac{11-\sqrt{3}}{104}, \frac{11+\sqrt{3}}{104})$ each with probability $\frac{1}{2}$.

The corresponding expected payoff (for one agent) derived by playing this CCE is $u_i(\widetilde{L}) = \frac{299}{2704} \approx 0.1105$. The optimal CCE (total) payoff is $\pi^{CCE} = 2u_i(\widetilde{L}) = \frac{23}{104} \approx 0.2211$, while the Nash equilibrium (total) payoff is $\pi^N = \frac{11}{50} \approx 0.22$. We have $\frac{\pi^{CCE}}{\pi^N} = \frac{575}{572} \approx 1.0052$ and $\frac{q^{CCE}}{q^N} \approx 1.057$. The CCE in this case improves over the Nash payoff by 0.5% and over the Nash abatement by 5.7%.

3.2 Abatement-maximising CCE

Remark 3 shows that both utility and abatement levels are improved (from the Nash outcome) in a payoff maximizing CCE. However, the interesting question from abatement point of view is, is this (also) the best abatement improvement possible over Nash abatement? The answer is *no* as shown by our next result, Proposition 3, characterizing abatement maximizing CCEs which achieves an abatement improvement of as much as 150%.

First, for convenience, we adopt the following notational simplification:

$$\alpha = \frac{a}{b}\alpha',\ \beta = \frac{a^2}{b^2}\beta' \text{ and } \gamma = \frac{a^2}{b^2}\gamma'.$$

Using this new notation, our main result of this subsection is the following Proposition.

Proposition 3 *For a fixed $r < 1$, the optimal values of the three moments of the optimal abatement maximising CCE are $\alpha' = \frac{1}{2(2+r-\sqrt{1-r^2})}$, $\beta' = 2\alpha'^2$ and $\gamma' = 0$; the optimal lottery is $L^*(r) = \frac{1}{2}\delta_{(z,z')} + \frac{1}{2}\delta_{(z',z)}$, with $z, z' = \frac{1}{(2+r-\sqrt{1-r^2})}$.*

To prove Proposition 3, we will need the following supporting lemma.

Observe that the CCE lotteries $L(r)$ correspond to triples $(\alpha', \beta', \gamma')$ satisfying $\beta' \geq \gamma' \geq 0$, $\alpha' \geq 0$ and $\beta' + \gamma' \geq 2\alpha'^2$, $\alpha' \leq \frac{1}{2}$ and

$$(1+r)\beta' + 2\gamma' \leq \alpha' - \frac{(1-2\alpha')^2}{4(1+r)} = \frac{1}{1+r}((2+r)\alpha' - \alpha'^2 - \frac{1}{4}).$$

We then ask for which values of the positive α', r and Δ, we can find non-negative parameters β', γ' such that $\beta' \geq \gamma' \geq 0$, $\beta' + \gamma' \geq 2\alpha'^2$ and $(1+r)\beta' + 2\gamma' \leq \Delta$.

As the latter line is flatter than the former one, we see (in the (β, γ)-plane) that it is possible if and only if $\frac{1}{1+r}\Delta \geq 2\alpha'^2 \iff (2(1+r)^2 + 1)\alpha'^2 - (2+r)\alpha' + \frac{1}{4} \leq 0$.

This observation leads to the following lemma.

Lemma 4 *Given $\alpha' > 0$, there exist β', γ' such that $L(\alpha', \beta', \gamma')$ is a CCE if and only if $\alpha' \leq \frac{1}{2}$ and $(2(1+r)^2 + 1)\alpha'^2 - (2+r)\alpha' + \frac{1}{4} \leq 0$. The corresponding relative abatement level is given by $\frac{q^L}{q^N} = 2(2+r)\alpha'$.*

The proof of Lemma 4 follows directly from the above arguments. The proof of Proposition 3 is in the Appendix that uses Lemmata 2 and 4.

3.2.1 Baseline example contd.

We revisit our baseline example and illustrate Proposition 3. The abatement-maximising CCE has the associated values of α, β and γ as follows: $\alpha = 0.153$, $\beta = .049$ and $\gamma = 0$. The corresponding expected payoff (for one agent) derived by playing this CCE is $u_i(L) \approx 0.0719$. The optimal CCE (total) payoff is $\pi^{CCE} = 2u_i(L) \approx 0.1438$, while the Nash equilibrium (total) payoff is $\pi^N \approx 0.22$. We thus have $\frac{\pi^{CCE}}{\pi^N} \approx 0.653$ and $\frac{q^{CCE}}{q^N} \approx 1.53$. The CCE in this case improves over the Nash abatement by 53% but has less payoff compared to the Nash outcome.

3.2.2 Comments

The relative gain in total abatement level at $L^*(r)$ is $\frac{q^{L^*(r)}}{q^N} = \frac{2+r}{2+r-\sqrt{1-r^2}}$ which decreases in r from 2 (at $r=0$) to 1 at ($r=1$). The total payoff at $L^*(r)$ is

$$u^{L^*(r)} = 2\frac{a^2}{b}(2\alpha - (2+r)\beta - 2\gamma) = \frac{a^2}{b}2\alpha(2-(2+r)2\alpha) = \frac{a^2}{b}\frac{2+r-2\sqrt{1-r^2}}{(2+r-\sqrt{1-r^2})^2}$$

$$\Longrightarrow u^{L^*(r)} = \frac{a^2}{b}\frac{1}{2+r-\sqrt{1-r^2}}(2-\frac{q^{L^*(r)}}{q^N})$$

So, $u^{L^*(r)}$ starts at 0 for $r=0$ and is increasing in r. Therefore,

$$\frac{u^{L^*(r)}}{u^N} = 2\frac{(2+r)^2(2+r-2\sqrt{1-r^2})}{(4+3r)(2+r-\sqrt{1-r^2})^2}$$

increases from 0 (at $r=0$) to $\frac{6}{7}$ (at $\ell = 1$).

Using the baseline example, at $r = \frac{1}{2}$, we find $\frac{q^{L(\frac{1}{2})}}{q^N} \simeq 150\%$ and $\frac{u^{L(\frac{1}{2})}}{u^N} \simeq 67\%$. This implies that the CCE corresponding to the optimal abatement level always achieves a payoff which is lower than the Nash payoff. This is in contrast with the payoff-maximising CCE that also improves upon the Nash abatement levels (although, they are far from the optimal abatement levels). This clearly shows the conflicting nature of these measures, and thereby leads us to our analysis in the next subsection.

3.3 Abatement vs. Payoff

We first impose the additional constraint on $L(\alpha', \beta', \gamma')$

$$2(2\alpha' - (2+r)\beta' - 2\gamma') \geq \frac{4+3r}{2(2+r)^2}$$

$$\Longleftrightarrow (2+r)\beta' + 2\gamma' \leq 2\alpha' - \frac{4+3r}{4(2+r)^2} \tag{1}$$

Recalling the earlier constraints

$$(1+r)\beta' + 2\gamma' \leq \frac{1}{1+r}((2+r)\alpha' - {\alpha'}^2 - \frac{1}{4}) \tag{2}$$

$$\beta' \geq \gamma' \geq 0 \text{ and } \beta' + \gamma' \geq 2{\alpha'}^2 \tag{3}$$

We now fix α' and look for β', γ' meeting (3) and below the lines Γ_1 given by (1) and Γ_2 given by (2). As Γ_1 is steeper than the diagonal $\beta' + \gamma' = 2{\alpha'}^2$ while Γ_2 is flatter than this diagonal (see Figure 1),

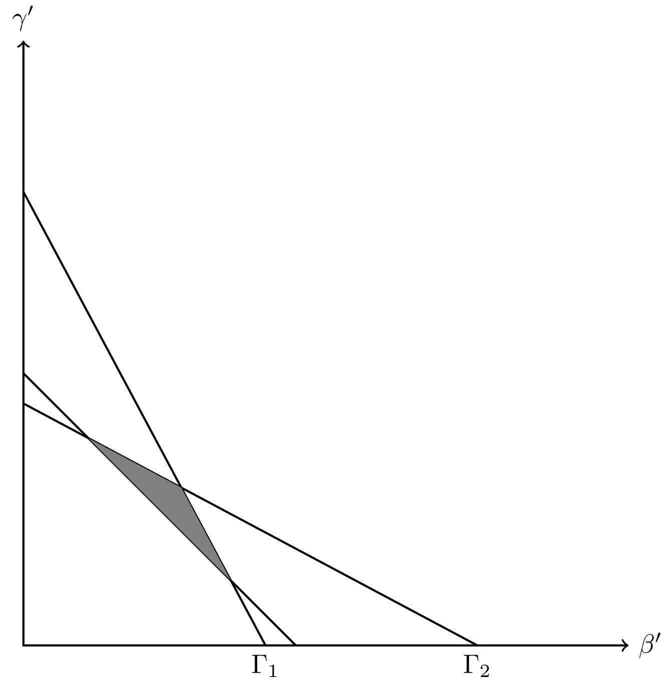

Figure 1: Feasible region of abatement maximizing CCEs subject to utility constraint.

we get that, for any fixed α', the existence of a solution β', γ' to this system is equivalent to the fact that the point $\beta' = \gamma' = {\alpha'}^2$ is below Γ_1 while the intersection of Γ_1 and Γ_2 is above the diagonal.

The former constraint is:

$$(4+r){\alpha'}^2 - 2\alpha' + \frac{4+3r}{4(2+r)^2} \leq 0 \tag{4}$$

For the second one, we compute first the intersection ${\beta'}^*, {\gamma'}^*$ of Γ_1 and Γ_2. Straightforward computations give:

$${\beta'}^* = \frac{1}{1+r}(r\alpha' + {\alpha'}^2 - \frac{r(3+2r)}{4(2+r)^2});\ {\gamma'}^* = \frac{1}{2(1+r)}((2-r^2)\alpha' - (2+r){\alpha'}^2 - \frac{4+r-4r^2-2r^3}{4(2+r)^2}) \tag{5}$$

Hence, the additional constraint on α' is

$$\frac{1}{2(1+r)}((2+2r-r^2)\alpha' - r{\alpha'}^2 - \frac{4+7r-2r^3}{4(2+r)^2}) \geq 2{\alpha'}^2$$

$$\iff (4+5r){\alpha'}^2 - (2+2r-r^2)\alpha' + \frac{4+7r-2r^3}{4(2+r)^2} \leq 0 \tag{6}$$

The two roots of the equation derived from (4) are $\frac{1}{2(2+r)}$ and $\frac{4+3r}{2(2+r)(4+r)}$, while the two roots of the equation derived from (6) are $\frac{1}{2(2+r)}$ and $\frac{-2r^3+7r+4}{2(5r^2+14r+8)}$. Note that for both $r=0$ and $r=1$, there is a unique root and it corresponds to the Nash equilibrium.

In our next result, we give an upper bound on α'.

Proposition 4 *With payoff at least that much of Nash, α' in any CCE is at most* 0.25*, that is, the maximum improvement in the abatement level over Nash is at most* 25%*.*

The proof of Proposition 4 is postponed to the Appendix.

By plotting the two areas defined by (4) and (6) in the (r, α')-plane, we can find out how much abatement is compatible with preserving the Nash payoffs, by simply looking for the greatest α' at each given r.

We conclude by illustrating our baseline example with (4) and (6).

3.3.1 Illustration with the baseline example

Consider our baseline example with $r = \frac{1}{2}$. The inequalities become

$$\frac{9}{2}\alpha'^2 - 2\alpha' + \frac{11}{50} \leq 0 \text{ and } \frac{13}{2}\alpha'^2 - \frac{11}{4}\alpha' + \frac{29}{100} \leq 0$$

The first inequality gives $\alpha' \in [0.2, 0.244]$ while the second suggests $\alpha' \in [0.2, 0.223]$; so the largest feasible increase in abatement is for $\alpha' = 0.233$ (the exact value is $\frac{29}{130}$).

The values of α, β and γ, associated with the (constrained) optimal CCE here are as follows: $\alpha = 0.1115$, $\beta = .0135$ and $\gamma = .01136$. The expression for the lottery in this case can be calculated in the similar way as in Proposition 3. However, the expression for α' being slightly complicated, we omit the presentation of the exact closed form expression for the lottery. We note that the lottery in this case also will be anti-diagonal as specified in Moulin et al. (2014).

The corresponding expected payoff (for one agent) derived by playing this CCE is $u_i(L) \approx 0.11006$. The optimal CCE (total) payoff is $\pi^{CCE} = 2u_i(L) \approx 0.22012$, while the Nash equilibrium (total) payoff is $\pi^N \approx 0.22$. We thus have $\frac{\pi^{CCE}}{\pi^N} \approx 1.00054$ (improvement by 0.05%) and $\frac{q^{CCE}}{q^N} \approx 1.115$, an increase in abatement of 11.5%, not spectacular, but still significant (more than double the improvement in abatement compared to payoff-maximising CCE).

The above example provides (almost) the best bound on the improvement (in terms of the abatement level, with at least the Nash payoff) obtained by a CCE over the Nash levels. To see this, it is enough to observe that as r increases, the best α' decreases faster than the improvement brought about by the increase in r. Similarly, as r decreases, α' increases but this increase is outweighed by the decrease in r.

4 REMARKS

We have analysed coarse correlated equilibria in a class of two-person symmetric games called the abatement game; we have characterised the payoff and abatement maximising CCEs and have shown a very simple way of achieving the maximum level of abatement keeping at least the Nash payoff. Such a computation is the first of its kind for coarse correlated equilibria for the abatement game and, this is why we regard this exercise as an interesting first step towards more sophisticated computations to understand mediation in general for such games.

The contribution of this paper is two-fold. First as a theoretical exercise, our result is perhaps the first attempt of characterising the benefit from (coarse) correlation in choosing abatement levels by countries. Second, as the importance of enforcing agreements is an important theme in the environmental literature, our characterisation suggests why and how a mediator (an independent agency) could be used for agreements and commitments in abatement games in practice; a mediator can improve upon the Nash equilibrium outcome by using the optimal CCE which is just a lottery over two outcomes that the countries would agree to commit to.

There is a huge recent literature in the algorithmic game theory that focuses on the popular ratios, known as the price of anarchy (PoA) and price of stability (PoS) in similar framework. While the analysis of both PoA and PoS do apply to the situation we study here, the questions we consider in this paper are different. The existing literature focuses on measuring the loss of efficiency with respect to one measure only; PoA with respect to one measure (say, utility) is not studied conditional on PoA on another measure (say, welfare). To the best of our knowledge, we are the first to provide such results in a small but economically relevant class of games.

Of course, there are quite a few limitations of our results. We have used a quadratic payoff function, and not any general differentiable concave function. This is not just because it enables us to use the techniques identified in Moulin et al. (2014); this choice has been justified in the literature (such as the RICE model in Nordhaus et al. (2000) that tries to set up abatement cost functions fitting real data). Quadratic approximation is indeed a natural choice for payoffs as shown in the models by Bosetti et al. (2009), Finus et al. (2005), Klepper et al. (2006). Also, we have worked with the assumption of identical agents for simplicity. Our characterisation is only for a two-player game. Although it is unclear how our main result could be generalised in a game with n players, our conjecture is that CCE can improve upon the Nash equilibrium outcome in an abatement game with n countries. It is of course true that the efficiency of the results depends heavily upon the number of nations. Consequently, our paper does not address the important issues of participation decisions and abatement levels.[6]

Finally, we do not relate our work to the important issues of coalition formation and applications of coalitional form games, which are now perhaps standard approaches in the literature on the International Environmental Agreements (IEAs).[7] We are also aware of the issues on the structures of (self-enforcing) IEAs to analyse the interaction among countries and their behaviors to arrive at a final outcome (Barrett (2003), McGinty (2007), Finus (2008), for example) which are beyond the scope of our current paper.

[6]Finus (2003) showed that full participation and the efficient outcome is obtained with only two players.

[7]See, for example, Tulkens (1998) and the references therein.

5 APPENDIX: PROOFS

Proof of Proposition 1. First note that the expected payoff (for agent 1, say) from any lottery $L \in \mathcal{L}^{sy}(\mathbb{R}_+^2)$ can be written as

$$u_1(L) = aE_L[Z_1] + aE_L[Z_2] - bE_L[Z_1^2] - bE_L[Z_2^2] - 2bE_L[Z_1 \cdot Z_2] - cE_L[Z_1^2],$$

which by symmetry is

$$\begin{aligned} u_1(L) &= 2aE_L[Z_1] - (2b+c)E_L[Z_1^2] - 2bE_L[Z_1 \cdot Z_2] \\ &= 2a\alpha - (2b+c)\beta - 2b\gamma. \end{aligned}$$

We write the expected payoff when agent 1 plays a pure strategy z and agent 2 commits to L, as

$$\begin{aligned} u_1(z, L^2) &= az + aE_L[Z_2] - bz^2 - bE_L[Z_2^2] - 2bzE_L[Z_2] - cz^2 \\ &= (a - 2b\alpha)z - (b+c)z^2 + a\alpha - b\beta. \end{aligned}$$

Hence, L is a CCE if and only if

$$\max_{z \geq 0}\{(a - 2b\alpha)z - (b+c)z^2\} + a\alpha - b\beta \leq 2a\alpha - (2b+c)\beta - 2b\gamma,$$

which, after rearranging, gives us the condition in the statement. ■

Proof of Proposition 2. Consider the equilibrium condition in Proposition 1. Note that if $a - 2b\alpha < 0 \iff \alpha > \frac{a}{2b}$, the L.H.S. of that inequality (the maximum over $z \geq 0$) is zero; therefore, the equilibrium condition in Proposition 1 becomes

$$a\alpha \geq (b+c)\beta + 2b\gamma = b(\beta + \gamma) + c\beta + b\gamma > b(\beta + \gamma) \geq 2b\alpha^2,$$

which is a contradiction. So, we must have $\alpha \leq \frac{a}{2b}$; then the L.H.S. of the equilibrium condition is $\frac{(a-2b\alpha)^2}{4(b+c)}$ and the condition is now

$$(b+c)\beta + 2b\gamma \leq a\alpha - \frac{(a - 2b\alpha)^2}{4(b+c)} = -\frac{b^2\alpha^2 - a(2b+c)\alpha + \frac{a^2}{4}}{b+c}. \tag{7}$$

We now fix α and solve Step 1 in Lemma 3: we must minimise $(2b+c)\beta + 2b\gamma$ in the polytope $\Psi = \{(\beta, \gamma) | \beta \geq \gamma, \beta + \gamma \geq 2\alpha^2\}$ under the additional constraint (7). Note that Ψ is unbounded from above and bounded from below by the interval $[P, Q]$, where $P = (\alpha^2, \alpha^2)$ and $Q = (2\alpha^2, 0)$. We distinguish two cases here.

Case 1 ($b \leq c$): In this case, the minimum in Ψ of both $(2b+c)\beta + 2b\gamma$ and $(b+c)\beta + 2b\gamma$ is achieved at P. Therefore, if P meets (7) it is our optimal pair $(\beta(\alpha), \gamma(\alpha))$; otherwise, there is no CCE for this choice of α. Now, P meets (7) if and only if $(3b+c)\alpha^2 \leq -\frac{b^2\alpha^2 - a(2b+c)\alpha + \frac{a^2}{4}}{b+c}$, which reduces to

$[a-(2b+c)\alpha)]^2 \leq 0 \iff \alpha = \frac{a}{2(2b+c)} = q_i^N$. By Lemma 1, the optimal CCE L is diagonal ($\beta = \gamma$) and deterministic ($\beta = \alpha^2$). It is simply the Nash equilibrium $L = \delta_{q^N}$ of our game.

Case 2 ($b > c$): Here, the minimum of $(b+c)\beta + 2b\gamma$ in Ψ is achieved at Q; so, if Q fails to meet the constraint (7) there is no hope to meet it anywhere in Ψ. Thus, we must choose α such that

$$2(b+c)\alpha^2 \leq -\frac{b^2\alpha^2 - a(2b+c)\alpha + \frac{a^2}{4}}{b+c} \iff \Lambda(\alpha) = (3b^2+4bc+2c^2)\alpha^2 - a(2b+c)\alpha + \frac{a^2}{4} \leq 0 \quad (8)$$

The discriminant of the right-hand polynomial $\Lambda(\alpha)$ is $a^2(b^2-c^2)$; therefore, (8) restricts α to an interval $[\alpha_-, \alpha_+]$, between the two positive roots of $\Lambda(\alpha)$. For such a choice of α, the constraint (7) cuts a subinterval $[R, Q]$ of $[P, Q]$, where R meets (7) as an equality. Note that $R = P$ only if $\alpha = q_i^N$ (from Case 1 and the fact that $\Lambda(q_i^N) < 0$), otherwise $R \neq P$. Clearly, R is our optimal choice $(\beta(\alpha), \gamma(\alpha))$ and it solves the system

$$\beta + \gamma = 2\alpha^2;\ (b+c)\beta + 2b\gamma = -\frac{b^2\alpha^2 - a(2b+c)\alpha + \frac{a^2}{4}}{b+c}.$$

Therefore,

$$\beta(\alpha) = \frac{1}{b^2-c^2}\left[b(5b+4c)\alpha^2 - a(2b+c)\alpha + \frac{a^2}{4}\right] \text{ and}$$

$$\gamma(\alpha) = \frac{1}{b^2-c^2}\left[-(3b^2+4bc+2c^2)\alpha^2 + a(2b+c)\alpha - \frac{a^2}{4}\right].$$

Now in Step 2 of Lemma 3, we must maximise $2a\alpha - (2b+c)\beta(\alpha) - 2b\gamma(\alpha)$ under the constraints $\alpha \geq 0$ and $\Lambda(\alpha) \leq 0$. Developing this objective function yields the programme

$$\frac{1}{b^2-c^2}\max_{\alpha}\{-b^2(4b+5c)\alpha^2 + a(2b^2+2bc-c^2)\alpha - \frac{a^2c}{4}\} \quad (9)$$

under the constraints

$$\alpha \geq 0 \text{ and } \Lambda(\alpha) = (3b^2+4bc+2c^2)\alpha^2 - a(2b+c)\alpha + \frac{a^2}{4} \leq 0.$$

The unconstrained maximum of the objective function is achieved at $\widetilde{\alpha} = \frac{a(2b^2+2bc-c^2)}{2b^2(4b+5c)}$.

We now show that $\Lambda(\widetilde{\alpha}) \leq 0$. With the change of variable $r = \frac{c}{b}$, this amounts to

$$\frac{(3+4r+2r^2)(2+2r-r^2)^2}{4(4+5r)^2} - \frac{(2+r)(2+2r-r^2)}{2(4+5r)} + \frac{1}{4} \leq 0$$
$$\iff 4 + 8r - 5r^2 - 12r^3 + 3r^4 + 4r^5 - 2r^6 \geq 0$$

The above polynomial is 0 at $r = 1$; it is also easy to check, numerically, that it is non-negative on $[0, 1]$. The proof of Proposition 2 is now complete if we express $\widetilde{\alpha}$, $\widetilde{\beta}$ and $\widetilde{\gamma}$ in terms of r. This is indeed easy for $\widetilde{\alpha}$. One may also verify, using the expression for $\widetilde{\alpha}$ that

$$\begin{aligned}\widetilde{\beta} &= \beta(\widetilde{\alpha}) = \frac{1}{b^2-c^2}\left[b(5b+4c)\widetilde{\alpha}^2 - a(2b+c)\widetilde{\alpha} + \frac{a^2}{4}\right] \\ &= \frac{a^2}{b^2}\frac{4+8r+r^2-4r^3}{4(4+5r)^2} \text{ and}\end{aligned}$$

$$\begin{aligned}\widetilde{\gamma} &= \gamma(\widetilde{\alpha}) = \frac{1}{b^2 - c^2}\left[-(3b^2 + 4bc + 2c^2)\widetilde{\alpha}^2 + a(2b + c)\widetilde{\alpha} - \frac{a^2}{4}\right] \\ &= \frac{a^2}{b^2}\frac{4 + 8r - r^2 - 4r^3 + 2r^4}{4(4 + 5r)^2}.\end{aligned}$$

Finally, we construct the optimal CCE $\widetilde{L}$. From $\widetilde{\beta} + \widetilde{\gamma} = 2\widetilde{\alpha}^2$ and Lemma 1*iii*), we see that $\widetilde{L}$ is an anti-diagonal lottery of the form $\widetilde{L} = \frac{1}{2}\delta_{(z,z')} + \frac{1}{2}\delta_{(z',z)}$, where z and z' are non-negative numbers such that $z + z' = 2\widetilde{\alpha}$ and $z^2 + z'^2 = 2\widetilde{\beta}$. This implies $2zz' = (2\widetilde{\alpha})^2 - (2\widetilde{\beta}) = 2\widetilde{\gamma}$, hence z, z' solve $Z^2 - 2\widetilde{\alpha}Z + \widetilde{\gamma} = 0$. The discriminant is $\widetilde{\alpha}^2 - \widetilde{\gamma} = \widetilde{\beta} - \widetilde{\alpha}^2 = \frac{a^2}{b^2}\frac{r^2(1-r^2)}{4(4+5r)^2}$; thus, the expressions for z and z' follow. ■

Proof of Proposition 3. The system of inequalities in Lemma 4 characterise the set of CCEs of this game; to find the optimal abatement level, we simply need to take the largest solution of this system. The polynomial has two real roots and the largest one is

$$\alpha' = \frac{1}{2(2(1+r)^2+1)}(2 + r + \sqrt{1-r^2}) = \frac{1}{2(2 + r - \sqrt{1-r^2})}$$

which is clearly below $\frac{1}{2}$.

We write $L^*(r)$ for the largest abatement CCE at r. The corresponding values of (β', γ') are $\gamma' = 0$ and $\beta' = 2{\alpha'}^2$ (to see this, use Figure 2).

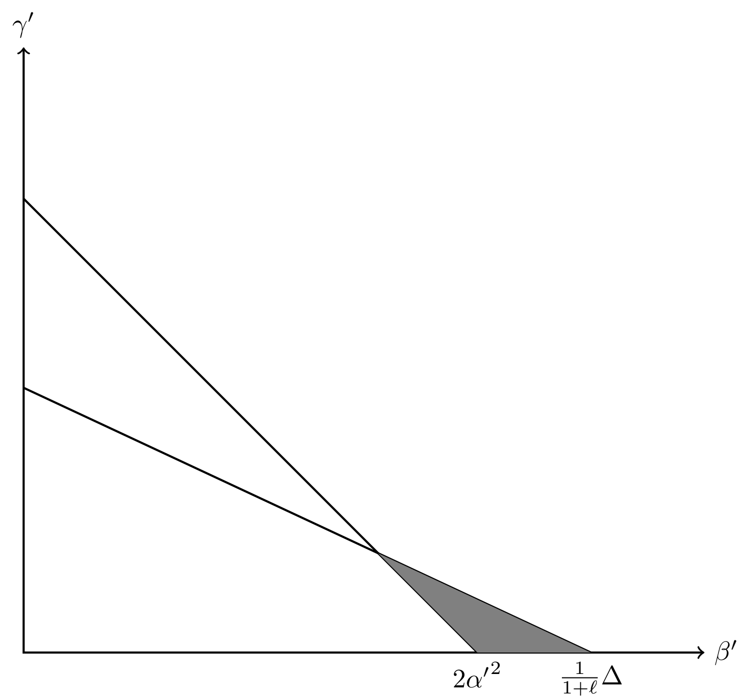

Figure 2: Feasible region for abatement maximizing CCEs

Now, from $\beta' + \gamma' = 2{\alpha'}^2$ and Lemma 1iii) above, we observe that $L^*(r)$ is an anti-diagonal lottery of the form $L^*(r) = \frac{1}{2}\delta_{(z,z')} + \frac{1}{2}\delta_{(z',z)}$, where z and z' are non-negative numbers such that $z + z' = 2\alpha'$ and $z^2 + z'^2 = 2\beta'$. Hence z, z' solve $Z^2 - 2\alpha' Z + \gamma' = 0$. Since $\gamma' = 0$, we get $Z = 2\alpha'$, which gives us the result. ■

Proof of Proposition 4. From (4) the upper bound on α' can be derived as:

$$\begin{aligned}
\alpha' &\leq \frac{4+3r}{2(2+r)(4+r)} \\
&= \frac{4+2r}{2(2+r)(4+r)} + \frac{r}{2(2+r)(4+r)} \\
&= \frac{2(2+r)}{2(2+r)(4+r)} + \frac{r}{2(2+r)(4+r)} \\
&= \frac{1}{4+r} + \frac{r}{2(2+r)(4+r)} \qquad (10)
\end{aligned}$$

Right hand side of (10) is a decreasing function in r when $r \in [0, 1]$ with a maximum of 0.25 at $r = 0$. Since $r = 0$ is not interesting, the best candidate is $0.25 - \epsilon$, for a very small $\epsilon > 0$. It is not hard to show that for every choice of $\epsilon < 0.25$ we can find r possibly very close to 0 depending on the value of ϵ for which (4) and (6) are satisfied. So the maximum improvement in abatement over Nash is at most 25%. ■

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