# SKILL NEUTRAL TECHNICAL CHANGES, GROWTH, AND WAGE INEQUALITY

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ABSTRACT. This paper explains the wage inequality in a closed economy. The paper employs a 'Product Cycle' based Endogenous growth Model to show how a growth that is skill neutral can have the same impact on income distribution (Wage Inequality) as international trade and skill-biased technical change. The paper shows that an increase in the size of unskilled worker will decrease its relative wage, while an increase in the size of skilled workers which is required for both production and research might or might not increase its relative wage. The relative wage of skilled workers will increase with an increase in their population only when the research effect dominates the population effect. The replacement of endogenous growth by semi-endogenous growth in the model wipes out the scale effect and suggests that an increase in skilled workers' population decreases its relative wage in a closed economy.

### 1. INTRODUCTION

The pattern of income distribution in developed as well as in developing countries has changed considerably in recent past. Katz and Autor (1999) report 29 percent increase in the gap between the 90th percentile of earner and the 10 percentile of earner from the late 1970s to the mid 1990s in the United States. The gap increased by 27 percent in the same period in the United Kingdom and by 9 percent in the Canada. Feenstra and Hanson (1997) and Moretti (2013) report a similarly high increase in wage inequality for Mexico, a developing country. Observations such as these have motivated a large body of literature that aims to understand the determination of an economy's distribution of income at a given point in time and in the long run. These explanation can be categorized broadly in two groups based on two different theories used to explain the problem at hand. The first emphasizes international trade with other countries (Wood, 1995; Dinopoulos and Segerstrom, 1999). The second centers on skill-biased-technical-change (STBC) hypothesis (Berman et al., 1998; Acemoglu, 2002).

The rise in wage inequality has coincided with the gradual removal of trade barriers. So, it was natural, at least initially, to hypothesize that increase in international trade leads to the rise in wage inequality. This hypothesis found firm theoretical grounding in the work of Stolper and Samuelson (1941). Specifically the Stolper-Samuelson theorem, when applied to the Heckscher-Ohlin model, postulates that a decrease in the relative price of a good reduces the real return to the factor used intensively in its production. Usually with a larger relative supply of skilled woker, a developed country engaged in trade with a developing country specializes in the production of goods which use skilled workers intensively. Since international trade leads to decrease in the cost of imported good, the removal of trade barriers pushes the relative wage of skilled workers in developed countries upward. Borjas and Ramey (1994) and many others report empirical evidence correlating the decrease in relative wage of unskilled workers with net imports of durable goods.

The view that international trade leads to wage inequality has not gone unchallenged. Some empirical papers have suggested that inequality is rising not only in developed economies but also in developing economies (Feenstra and Hanson, 1997). Even when trade barriers were lowered, the domestic relative prices of imported good in developed countries remained roughly constant (Lawrence and Slaughter, 1993). Since the Stolper Samuelson theorem predicts a change in relative wage of unskilled worker in opposite directions for a developed and a developing country and the theorem's prediction about the change in relative wage works through the changes in product prices, this evidence casts serious doubts on the international trade based explanation for increasing wage inequality. Krugman and Lawrence (1994) argued very forcefully that, even though US trade with the rest of the world has increased manifold in the past, the living standards in the economy are still determined by domestic factors. In their view, technological change plays a major role in explaining the current wage inequality.

The second type of explanation for the rise in wage inequality is based on "skill-biased-technological-change (SBTC)" hypothesis. This type of explanation is largely shaped by the observation of skill biased technological changes taking place in the economy along with the rise in wage inequality. As the name suggests, SBTC indicates a particular type of technological change in the economy that is biased in favor of one particular type of worker. Typically, the bias is for skilled workers as it is assumed that technical advances eliminate or reduce the need for unskilled workers. This leads to a change in labor composition in favor of skilled workers. A common feature of models based on the SBTC hypothesis is that, as technical progress occurs, the relative marginal productivity of different inputs change. SBTC based explanations are also not free from criticism. The main criticism comes from the fact that a rise in relative wage makes use of unskilled labor relatively cheap. Thus, an incentive exists to develop technologies which favors unskilled workers rather than skilled workers. Still why do STBC favor skilled workers? Very few SBTC based models try to answer this question. Acemoglu (2002) claims and shows empirically that a positive supply shock can lead to a technical change that is biased towards a particular factor of production.

The debate over what leads to rise in wage inequality and thus influences the income distribution is hardly settled, Dinopoulos and Segerstrom (1999) describe a product cycle based Schumpeterian growth model to argue that the role of international trade in rising wage inequality has been underestimated. This underestimation is because a traditional Heckscher-Ohlin trade model, which focuses on trade driven by differences in relative factor endowments between countries, is hardly suitable to explain the international trade that is driven by differences in knowledge between countries. According to them, knowledge-difference based trade can explain observed rise in wage inequality. The new knowledge based trade explanations, and the SBTC hypothesis suggest that a differences in level of knowledge/technology has emerged as the main explanation for the rise in wage inequality.

Building on the established trend in the literarure, I provide a technology based explanation to understand the distribution of income which is different from existing explanations. My intention is not to point out weaknesses of existing model, but to augment them. My model, based on a product cycle with standardization, suggests that a product cycle with standardization has the same impact on distribution of income as international trade or skill biased technical change. The term 'product cycle' was first used by Vernon (1966) to describe a phenomena that most new goods are manufactured first in the countries where they were originally discovered and developed, and later in countries where production costs are lower, when products have been standardized. Vernon's implicit assumption was that, in the beginning of cycle the production function is not clearly specified such that production can only take place under the supervision of skilled engineers. As time progresses, the manufacturer gradually gains knowledge on how to produce the good without such assistance, and gradually production becomes less skill intensive.

Vernon's (1966) description of the "product cycle hypothesis" led to a large body of empirical research pointing to a richer implication than what Vernon envisioned in his paper. Apart from suggesting the existence of the product cycle in the shift of production of standardized goods to countries where production costs are lower, the empirical findings also suggested the presence of an entire product cycle inside an economy. Heckman (1980) provides evidence on the shift in production of textiles from mature industrial region (New England) to the low wage worker abundant south in United States after 1880.

Vernon's (1966) work also pushed theorists to formalize the theory behind the cycle. The first notable formalization came from Krugman (1979). In his model, a developed country (industrialized north) innovates and produces new goods, and a developing country (south) produces old goods. Since agents in both economies have a 'love of variety' type of preferences, there is trade between the north and the south. In his model, a new good becomes an old good over time with a lag specified exogenously. Since the south can imitate old goods and produce them more cheaply, southern manufacturers drive northern manufacturers of old goods out of the market. Grossman and Helpman (1991) developed a model of product cycle based on endogenous growth theory. In endogenous growth models, whenever the discounted present value of the expected profits exceeds the current cost of development resources (skilled labor), entrepreneurs spend resources to bring new products to the market. The cost of developing a new product decreases in real terms as the number of already developed products increases in the economy. The reasoning is that available products represent disembodied knowledge in the economy. As disembodied knowledge in the economy increases, development costs decrease.

Krugman (1979), Grossman and Helpman (1991), and most if not all other all other existing analyses of product cycles study the various implications of international trade. All these models have only one type of factor of production. Hence they are not suitable to study the distribution of income in a closed economy. These models abstract from Vernon's description of continuous standardization and use the same production function through the whole product life cycle. In a recent paper, Antras (2005) describes a model, much closer to Vernon's description of standardization. He uses a standardization process to describe the change in factor requirements to produce the same good. Although, his model is based on two factors of production (skilled and unskilled workers), both factors of production are assumed to be paid the same wage, thereby making the model unsuitable to study the distribution of income. The main focus of Antras (2005) is to describe how an endogenous product cycle can arise due to incomplete contracts. I develop a model of endogenous product cycle with process standardization to explain the income distribution in a closed economy.<sup>1</sup> Unlike the traditional product cycle literature, in my model, production of standardized goods does not move to developing countries. Instead, the standardization of production frees up skilled workers tied in the production of existing goods. The freed skilled workers contribute to endogenous growth and contribute to the creation of new products. Over time, technological innovation itself requires fewer resources which implies an increasing number of newly developed goods over any given interval of time. The production of newly develop products are skill intensive. Given a fixed ratio of skilled and unskilled workers in the economy, this leads to a typical distribution of income in a closed economy.

My paper makes two contribution to the literature. First, it describes a endogenous product cycle with production process standardization. Second, it presents a product cycle based mechanism to describe the income distribution in a closed economy.

## 2. The Model

In this section, I present my model of endogenous product cycle. The model builds upon Grossman-Helpman (1991), yet differs in two important ways. First, my model incorporates two types of labor, skilled and unskilled, which is important to consider intra economy income distribution. Second, my formulation allows for the standardization of production function in the product cycle that represents product development over the life cycle.

<sup>&</sup>lt;sup>1</sup>My paper is much closer in spirit to Ranjan (2005). My model differs from his model in two ways. First, he uses a random standardization process. Second, His model assumes perfectly competitive market

2.1. Consumers. Consider an economy populated by two types of infinitely lived workers, skilled (h) and unskilled (l), with populations equal to H and L, respectively. At time t, there are n(t) differentiated goods available in the economy. A worker of type  $k \in \{h, l\}$ , has a timeseparable intertemporal lifetime utility function,  $U_k(t)$ , with a common discount rate as  $\rho$ . Worker k's lifetime utility function depends on her instantaneous sub-utility function,  $u_k(\tau)$ , which, in turn, depends on her instantaneous consumptions,  $C_{kj}(\tau)$  of  $(j \in n(\tau))$  products, of  $n(\tau)$ differentiated goods, available at time  $\tau$ .

(1) 
$$U_k(t) = \int_t^\infty e^{-\rho\tau} \log[u_k(\tau)] d\tau$$

(2) 
$$u_k(\tau) = \left[ \int_{j \in n(\tau)} C_{kj}^{\alpha}(\tau) dj \right]^{\frac{1}{\alpha}}, \qquad \alpha \in (0,1)$$

The assumption of CES (constant elasticity of substitution) implies that consumers have 'love of variety'. It also implies the elasticity of substitution between any two products is constant and equal to  $\sigma = \frac{1}{1-\alpha} > 1.$ 

The consumer k can solve her maximization problem in two stages. In first stage, she chooses the consumption of good i at time t so as to maximize  $u_k(t)$  given prices of all available goods and expenditure  $E_k(t)$ , where  $E_k(t) = \int_{j \in n(\tau)} p_j(t) C_{kj}(t) dj$ .

(3) 
$$C_{kj}(t) = \frac{p_j^{-\sigma}(t)}{\int_{j' \in n(t)} p_{j'}^{1-\sigma}(t) dj'} E_k(t)$$

The instantaneous demand function of good i in the economy is

(4) 
$$Y_j(t) = L \cdot C_{jl}(t) + H \cdot C_{jh}(t) = \frac{p_j^{-\sigma}(t)}{\int_{j' \in n(t)} p_{j'}^{1-\sigma} dj'} E(t)$$
  
$$= \lambda p_j^{-\sigma}(t), \quad \text{where } \lambda = \frac{E(t)}{\int_{j' \in n(t)} p_{j'}^{1-\sigma} dj'} \text{ and } \sigma = \frac{1}{1-\alpha}$$

 $E(t) = (L \cdot E_l(t) + H \cdot E_h(t))$ , is economy's total expenditure at time t.

In the second stage, consumer  $k \in \{h, l\}$  chooses the path of her expenditures,  $E_k(t)$  to maximize  $U_k$ . While maximizing  $U_k$ , she needs to satisfy her intertemporal budget constraint. The budget constraint depends on her wages  $\{w_k(\tau)\}_{\tau=t}^{\infty}$ , asset holding at time t,  $\mathcal{A}_j(t)$ , and instantaneous interest rates,  $\{\dot{R}(\tau)\}_{\tau=t}^{\infty}$ , prevailing in the capital market. The cumulative interest factor from time 0 to time t that a worker faces in the capital market is given by  $R(t) = \int_0^t \dot{R}(t)$ 

Assuming a consumer can lend and borrow freely in the capital marker, her, k's, budget constraint is

(5) 
$$\int_{t}^{\infty} e^{-[R(\tau)-R(t)]} E_{k}(\tau) d\tau = \int_{t}^{\infty} e^{-[R(\tau)-R(t)]} w_{k}(\tau) d\tau + \mathcal{A}(t)$$

Consumption of good js for the worker k, obtained in first stage and given by  $\{C_{kj}\}_{j\in n(\tau)}$  lead to an indirect utility function  $u_k(\tau)$  that is weakly separable in the level of k's expenditure,  $E_k(\tau)$ , and in a function of prices of differentiated goods. It implies that  $u_k(\tau)$  can be written as  $u_k[p(\tau), E(\tau)] = E_k(\tau)f(p(\tau))$ . I can rewrite the life time utility function, equation (1), as

(6) 
$$U_k(t) = \int_t^\infty e^{-\rho\tau} [\log E_k(.) + \log f(p(.))] d\tau$$

The Lagrangian expression using equation (6) and lifetime budget constraint (5) is given by

$$\mathcal{L} = \int_{t}^{\infty} e^{-\rho\tau} [\log E(\tau) + \log f(p(\tau))] d\tau$$
$$-\mu_{t} \left[ \int_{t}^{\infty} e^{-[R(\tau) - R(t)]} (E_{k}(\tau) - W(\tau)) d\tau - \mathcal{A}_{j}(t) \right]$$

where  $\mu_t$  denotes the Lagrangian multiplier on the budget constraint. The first-order condition for maximizing  $U_k(t)$  with respect to  $E_k(\tau)$  can be written as

(7) 
$$e^{-\rho\tau} \frac{1}{E_k(\tau)} - \mu_t e^{-[R(\tau) - R(t)]} = 0$$

Taking logs on both of the sides and differentiating with respect to t gives

$$\frac{\dot{E}_k}{E_k} = \dot{R} - \rho, \qquad k \in (L, H)$$

The economy's total expenditure in period t is  $E(t) = L \cdot E_l(t) + H \cdot E_k(t)$ which gives

(8) 
$$\frac{\dot{E}}{E} = \frac{\dot{E}_L}{E_L} = \frac{\dot{E}_H}{E_H} = \dot{R} - \rho$$

The above condition implies that the individual's ( (skilled or unskilled), and the economy's expenditure, all, grow at the same instantaneous rate equal to the instantaneous interest rate corrected by future discount rate.

2.2. **Producers.** The number of potential products is infinite. To begin production of one of the potential differentiated goods, the producer needs to learn how to produce that good. All new producers incur a development cost to start production. A new producer does not want to develop an already existing type, as this leads to Bertrand competition between two identical products. In Bertrand competition, competitors have to set price of the good equal to the marginal cost. Since the learning process is costly, a new producer would never able to recover the development cost of the good, had she developed an already existing product.

After a new producer learns how to produce a good, production takes place under the constant return to scale technology. In the beginning, only skilled laborers are capable of producing the new good. Following the 'product cycle' literature, any new product goes through a standardization process, and, once standardized, the production shifts to unskilled laborers. For convenience, I call the good "new good" when produced solely by skilled workers, and "old good" when production uses unskilled hand. For good j, the production function is given as  $Y_j$ :

(9) 
$$Y_j = \begin{cases} h & \text{When good } j \text{ is new} \\ l & \text{When good } j \text{ is old} \end{cases}$$

The process standardization in this model is taken as exogenous and discrete in nature. After producing a new good for an exogenously given period of time, say T, the producer accumulates enough information regarding the production process, that it can be undertaken by unskilled workers.<sup>2</sup>

$$Y = \zeta h^{1-z} l^z, \quad 0 \le z \le 1,$$

 $<sup>^{2}</sup>$ One can envision a more general production function such as Cobb-Douglas function with continuous standardization process given as

where  $\zeta = z^{-z}(1-z)^{-(1-z)}$ . Since  $\lim_{z\to 1} z^{-z}(1-z)^{1-z} = 1$  &  $\lim_{z\to 0} z^{-z}(1-z)^{1-z} = 1$ , the production function is continuous in z. z as a function of time span  $(\tau)$  captures the standardization process. The standardization implies that output elasticity of unskilled workers increase as product becomes older, and of skilled workers decreases. The basic characteristics of continuous standardization can be given as

Consumers' CES type of preference lead to an iso-elastic demand curve for a unique differentiated good  $j, \forall j \in n(t)$ .

(10) 
$$Y_j(t) = \lambda [p_j(t)]^{-\frac{1}{(1-\alpha)}}, \quad 0 < \alpha < 1$$

Where  $\lambda$  is a parameter given in equation (4) that the producer takes as given. Such producers maximizes profit by setting a price  $p_j(t)$  that is a fixed mark up over marginal cost of production.

Since the production function is linear, the marginal cost of production is equal to unit cost of production. At time t, the cost of production of good j,  $c_j(t)$ , can be given in terms of unit factor prices, wage of skilled  $w_h(t)$  and unskilled workers  $w_l(t)$ .

(11) 
$$c_j(t) = \begin{cases} w_h(t) & \text{for new goods} \\ w_l(t) & \text{for old goods} \end{cases}$$

A monopolist has unique ability to produce good j. The demand she faces for good j at time t,  $Y_j(t)$ , is given in equation (10). To maximize her profit, she solves the following optimization problem:

(12) 
$$\max_{p_j(t)} [p_j(t) - c_j(t)] \cdot Y_j(t)$$

It is straightforward to check that to optimize her profit she sets up price,  $p_j(t) = \frac{c_j(t)}{\alpha}$  as the optimal price for good j. The price  $p_j(t)$  is a

Antras (2005) envisions one such standardization process in which productdevelopment intensity of the good is inversely related to product maturity. For this, he proposes a exponential standardization process,  $z = e^{-\frac{\tau}{\theta}}$ . My model can accommodate this special standardization, or a more general standardization process. However, to keep exposition simple I use a discrete and tractable standardization process.

fixed mark-up over marginal cost,  $c_j(t)$ .

(13) 
$$p_j(t) = \begin{cases} \frac{w_h(t)}{\alpha} & \text{for new goods} \\ \frac{w_l(t)}{\alpha} & \text{for old goods} \end{cases}$$

And, the producer of a good j earns instantaneous profit at time t given as

$$\pi_{j}(t) \quad (\ddagger 4) \max_{p_{j}(t)} \quad [p_{j}(t) - c_{j}(t)] \cdot Y_{j}(t) = (1 - \alpha) \cdot p_{j}(t) \cdot Y_{i}(t)$$

$$= \begin{cases} (1 - \alpha) \frac{\left[\frac{w_{h}(t)}{\alpha}\right]^{1 - \sigma}}{n_{N}(t) \left[\frac{w_{h}(t)}{\alpha}\right]^{1 - \sigma} + n_{O}(t) \left[\frac{w_{l}(t)}{\alpha}\right]^{1 - \sigma}} E(t), & \text{for a new product } j \\ (1 - \alpha) \frac{\left[\frac{w_{h}(t)}{\alpha}\right]^{1 - \sigma}}{n_{N}(t) \left[\frac{w_{h}(t)}{\alpha}\right]^{1 - \sigma} + n_{O}(t) \left[\frac{w_{l}(t)}{\alpha}\right]^{1 - \sigma}} E(t), & \text{for an old product } j \end{cases}$$

where the expression for  $Y_i(t)$  is given in the equation (4).  $n_N(t)$  and  $n_O(t)$  are number of new and old goods respectively at time t. Let the relative wage of a skilled worker at time t be denoted by  $\omega(t) = \frac{w_h(t)}{w_l(t)}$ , the instantaneous profit for a good i at time t can be rewritten as:

$$\pi_{j}(t) = \begin{cases} \pi_{N}(t) = (1-\alpha) \frac{1}{\frac{n_{N}(t)}{n(t)} + \frac{n_{O}(t)}{n(t)} [\omega(t)]^{\sigma-1}} \frac{E(t)}{n(t)}, & \text{for a new product } j \\ \pi_{O}(t) = (1-\alpha) \frac{[\omega(t)]^{\sigma-1}}{\frac{n_{N}(t)}{n(t)} + \frac{n_{O}(t)}{n(t)} [\omega(t)]^{1-\sigma}} \frac{E(t)}{n(t)}, & \text{for an old product } j. \end{cases}$$
(15)

2.3. Labor Market Clearing Conditions. At any time t, as given in equation (9), one skilled worker produces one unit of a new good and one unskilled worker produces one unit of an old good. The derived demand for labor for each differentiated good is simply equal to the demand of that good. Total demand for skilled labor,  $H_p(t)$ , (unskilled labor,  $L_p(t)$ ,) engaged in productive activities at time t can be obtained by integrating the demand for skilled (unskilled) labor over all new (old) goods available in the economy at time t.

(16) 
$$L_p(t) = \int_{j \in n_O(t)} Y_j(t) dj.$$
$$H_p(t) = \int_{j \in n_N(t)} Y_j(t) dj.$$

 $n_N(t)$  and  $n_O(t)$  are number of new goods and old goods available in the economy at time t.

2.4. **Product Development.** Following the endogenous growth literature, particularly Romer (1986, 1990), Grossman and Helpman (1991), I assume that the resources dedicated to research lead to two types of outputs. First, a direct output that is the ability to produce a new differentiated product from the pool of infinitely feasible products. It gives the developer a monopoly over the production of the new good and earns her a stream of monopolistic profit. Second is an indirect and unintended output. The development of each new good leads to the addition of general knowledge available in the economy. The underlying assumption is that such knowledge has widespread scientific applicability and it increases the productivity of any such development efforts in the future.

Following Grossman and Helpman (1991), if  $\mathcal{K}$  denotes the level of disembodied knowledge capital in the economy and  $a_d$  denotes a fixed productivity parameter in the product development sector, the resources required to come up with a new product could be given as  $\frac{a_d}{\mathcal{K}}$  units of skilled labor.<sup>3</sup> The total available number of products in the economy can be used as proxy for the disembodied knowledge capital. If  $H_d$ ,

<sup>&</sup>lt;sup>3</sup>Grossman and Helpman (1991) describe the requirement to come up with a new product in very similar fashion. Since, their model has only one type of factor of production, the same factor is used for the development. Here, I shy away from using both factors of production as this will unnecessarily complicate the model without adding any extra insight.

where  $H_d + H_p = H$ , is the number of high skilled laborers involved in the development work, the rate of development  $\dot{n}$  can be given as

(17) 
$$\dot{n} = \frac{n \cdot H_d}{a_d}$$

The model allows for free entry. Accordingly, the discounted value of the cumulative profit for an individual producer should equal to her development cost at time t, V(t).<sup>4</sup>

$$V(t) = \frac{a_d w_h(t)}{n(t)} = \int_t^\infty e^{-[R(\tau) - R(t)]} \pi_j(\tau) d\tau$$
(18) 
$$= \int_t^{t+T} e^{-[R(\tau) - R(t)]} \pi_N(\tau) d\tau + \int_{t+T}^\infty e^{-[R(\tau) - R(t)]} \pi_O(\tau) d\tau$$

where subscript N denotes a new and O denotes an old product, and T is the time span that a newly innovated product takes for standardization.

Differentiating equation (18) with respect to t, I can write,

$$(\dot{I} \mathfrak{N}) = \frac{dV}{dt} = \left[ \frac{\dot{w}_h}{w_h} - \frac{\dot{n}}{n} \right] V(t)$$
  
=  $\dot{R}(t) \cdot V(t) - \pi_N(t) - e^{-[R(t+T) - R(t)]} [\pi_O(t+T) - \pi_N(t+T)]$ 

which implies,

$$\dot{R} = \frac{\pi_N(t) + e^{-[R(t+T) - R(t)]} \left[\pi_O(t+T) - \pi_N(t+T)\right]}{a_d w_h(t) / n(t)} + \left(\frac{\dot{w}_h}{w_h} - \frac{\dot{n}}{n}\right)$$
(20)

The equations derived above can completely determine the evolution of the economy from any initial conditions. Provided E(0) is consistent with long term convergence, the economy attains a steady state.

<sup>&</sup>lt;sup>4</sup>It is clear from equation (15) that  $\pi_j(\tau)$  is independent of j, and depends only on the production process (old or new) employed at time  $\tau$  to produce j.

# 3. Steady-State Analysis

I am interested in showing and characterizing long term rate of product development and the distribution of income in the economy. I denote the growth rate of number of products  $(\frac{\dot{n}}{n})$  by g in the steady state.<sup>5</sup>

In the model, there is no monetary authority. So, I am free to give an arbitrary value to one of variables in the model. Following Grossman and Helpman (1991), I fix the economy's expenditure at every time equal to the number of products available at that time, E(t) = n(t). Using this normalization and equation (8) the instantaneous interest in the economy can be given as

(21) 
$$\dot{R} = \rho + \frac{\dot{n}}{n}$$

Since, total expenditure in the economy, E(t), is equal to the number of products in the economy, in the steady state, the economy's expenditure grows at constant rate g. The growth rate for both wages (skill and unskilled) would also be the same as g. The equation (21) implies:

$$\dot{R} = g + \rho$$

The equation (22) gives the instantaneous interest rate as sum of future discount factor and the growth rate of number of varieties. In the previous section, equation (20) also gives instantaneous interest rate in the economy. Equating these two instantaneous interest rates provides the no-arbitrage condition in research and development sector. To calculate the value of instantaneous interest rate in the economy using equation (20), one need to know the proportion of old and new goods  $\left(\frac{n_N(t)}{n(t)} \text{ and } \frac{n_N(t)}{n(t)}\right)$  in the economy. In the steady state, the proportion

<sup>&</sup>lt;sup>5</sup>In the steady state, a fixed fraction of skilled workers participates in product development making  $\frac{H_d}{a_d}$  constant.

of new and old goods can be expressed in terms of g, which is the growth rate of number of products in the economy  $(\frac{\dot{n}}{n})$ , and T, which is the time span after which production of a newly innovated good shifts from skilled workers to unskilled workers.



FIGURE 1. Timeline in The Economy.

t = current time.

n(t) = the number for products innovated in the economy up to time t.

$$n_O(t) = n(t-T) = n(t)e^{-gT}$$
 = the number of old products in the economy at time t.

 $n_N(t) = n(t)(1 - e^{-gT})$  the number of new products at time t. (Grows at the rate of g).

 $dn(t-s) = g \cdot n(t)e^{-g \cdot s}ds$  = the number of products with maturity s < T(innovated between t-s to t-(s+ds) time).

Notice that in the steady state, the ratios  $\frac{n_N(t)}{n(t)}$  and  $\frac{n_O(t)}{n(t)}$  are independent of time t. I have taken E(t) = n(t) as the numéraire in the economy. The number of products in the economy, n(t), grows at rate g in the steady state, therefore, E(t) also grows at rate g in the steady state. Further, to achieve the steady state,  $w_k, k \in \{h, l\}$ , must grow at the same rate as  $E_k$ . From equation (8),  $\frac{\dot{E}}{E} = \frac{\dot{E}_k}{E_k}, \forall k \in \{h, l\}$ , which implies  $\frac{\dot{E}}{E} = \frac{\dot{w}_h}{w_h} = \frac{\dot{w}_l}{w_l} = g$ . Also notice that the expressions for  $\pi_N(t)$  and  $\pi_O(t)$  given in equation (15) depend only on  $\frac{n_N(t)}{n(t)}, \frac{n_O(t)}{n(t)}, \omega(t)$  and  $\frac{E(t)}{n(t)}$ ; all of these are constant in the steady state, which make  $\pi_N(t)$  and  $\pi_O(t)$  independent of time in the steady state.

In the steady state, as growth rate in wage is equal to the growth rate in the number of products and profit from producing a good is independent of time t, I can rewrite the expression for  $\dot{R}$  given in equation (20) as

(23) 
$$\dot{R} = \frac{\pi_N + e^{-[R(t+T) - R(t)]} [\pi_O - \pi_N]}{a_d w_h(t) / n(t)}$$

Using expressions for  $\dot{R}$ , given in equation (22),  $\pi_N$  and  $\pi_O$ , given in equation (15), steady state equilibrium values of ratios  $\frac{n_N(t)}{n(t)}$  and  $\frac{n_O(t)}{n(t)}$ , derived earlier, and numeraire E(t) = n(t) I can give the no-arbitrage condition in research and development sector as:

(24) 
$$\rho + g = (1 - \alpha) \frac{n(t)}{a_d \cdot w_h(t)} \left[ \frac{1 + e^{-(\rho + g)T} \left[ \omega^{\sigma - 1} - 1 \right]}{1 + e^{-gT} \left[ \omega^{\sigma - 1} - 1 \right]} \right]$$

As  $\frac{n(t)}{w_h(t)}$  is constant in the steady state, the above expression is independent of time t.

To explore the equilibrium condition in research and development sector, one need to know the steady state value of  $\frac{n(t)}{w_h(t)}$ . It can be obtained from labor market clearing condition in the equilibrium. No one in the economy is unemployed. It means that both skilled and unskilled labor market clear. Equation (16) gives the expression for labor involved in productive activities. I can obtain the expression for labor involved in development activities from equation (17). The labor market claering condition can be written as<sup>6</sup>.

$$(25) H = H_p + H_d$$

$$L = L_p$$

<sup>&</sup>lt;sup>6</sup>Labor supply is inelastic in the economy, therefor,  $L_p(t)$  is always equal to L. Further,  $H_d(t)$  is constant in the steady state. So,  $H_p(t) = H_p$ 

where  $H_p$  and  $L_p$  are given in the equation (16). Using the expression for  $Y_j(t)$  given in equation (4),  $H_p$  and  $L_p$  can be expressed in terms of  $g, \omega$  and system parameters such as T and  $\rho$ .

(26) 
$$H_p = H - H_d = \frac{\alpha}{w_h(t)} \frac{1 - e^{-gT}}{1 - e^{-gT} + e^{-gT} \omega^{\sigma-1}} n(t)$$

(27) 
$$L_p = L = \frac{\alpha}{w_l(t)} \frac{e^{-gT} \omega^{\sigma-1}}{1 - e^{-gT} + e^{-gT} \omega^{\sigma-1}} n(t)$$

Using equation (17), I obtain number of skilled workers engaged in development activity.

(28) 
$$H_d = g \cdot a_d$$

Using equations (26), (27), and (28), one can obtain the expression for  $\frac{n(t)}{w_h(t)}$  in the steady state. By multiplying equation (26) with  $w_h(t)$  and equation (27) with  $w_l(t)$ , adding them up and using equation (28) for  $H_d$ , I obtain:

(29) 
$$\frac{n(t)}{w_h(t)} = \frac{1}{\alpha} \left[ H - a_d g + \frac{L}{\omega} \right]$$

Using equation (29), the equilibrium condition in the research and development sector can be rewritten as:

$$\dot{R} = \rho + g = \frac{1}{(\sigma - 1)a_d} \left[ H - a_d g + \frac{L}{\omega} \right] \left[ \frac{1 + e^{-(\rho + g)T} \left[ \omega^{\sigma - 1} - 1 \right]}{1 + e^{-gT} \left[ \omega^{\sigma - 1} - 1 \right]} \right];$$
(30)

As, 
$$\sigma = \frac{1}{1-\alpha}$$
.

Now, similar to the equilibrium condition for research and development sector, I can write an equilibrium condition for labor market clearing in terms of variables relative wage ( $\omega$ ), growth rate in number of products,

g, and parameters, standardization period, T, elasticity of substitution  $\sigma$  and productivity parameter in research sector,  $a_d$ . Diving equation (26) by equation (27) and rearranging, I get the equilibrium condition in labor market.

(31) 
$$\omega^{\sigma} = (e^{gT} - 1)\frac{L}{H - a_d g}$$

# 4. Determinants of Growth Rate g and Relative Wage $\omega$ in the Long Run

In the previous section, I obtained the two conditions, labor market clearing condition (equation 31) and research no-arbitrage condition (equation 30). A pair of  $\{g, \omega\}$  that satisfies both the condition gives the steady state growth rate  $g^*$  and relative wage  $\omega^*$  in the economy. I am interested in the determinants of these long run values of growth rate and relative wage.

In figure (2), curve M-M represents the labor market clearing condition given in equation (31). This curve is upward slopping and represents the combinations of steady-state rates of growth and relative wage that are consistent with both the labor market clearing conditions in the economy. An increase in g takes skilled workers away from productive activities to research activities, that creates scarcity of skilled workers in production, which in turn, raises the skilled wage for market clearing condition. The equation representing the equilibrium condition in research and development sector is not as straight forward as the other equilibrium condition. To get better understanding of this equation, consider a special case:  $\rho = 0$ , implying that consumers are very patient; they do not discount the future consumption. With this condition, the research and development equilibrium condition given in equation (30) reduces to the following form:

(32) 
$$\omega = \frac{L}{\sigma a_d g - H}$$

The modified curve is downward slopping and represents the combinations of steady state growth rate and relative wage that are consistent with the equilibrium condition in research and development sector. The curve describing this equilibrium condition in  $\{g, \omega\}$  plane is denoted by R-R in figure (2). For low values of future discount factor,  $\rho$ , the equilibrium condition in research and development sector (equation 30) behaves in the same fashion in  $\{g, \omega\}$  as the curve R-R (representing equation 32) does.



FIGURE 2. Equilibrium Conditions and Effects of Change in Labor Supply

4.1. Effects of an Increase in Labor Supply. An increase in H, as shown by equations (30 and 31) and in figure (2) shifts M-M curve down and R-R curve up, leading to an increase in the equilibrium value of  $g^*$ . The increased value of  $g^*$  is denoted as  $g'^*$  in the figure (2). From the

same figure, it is seen that the increase in H has an ambiguous effect on the equilibrium value of  $\omega$ . An increase in H increases the supply of skilled workers participating in productive activities causing their relative wage to fall, which in turn, shifts the M-M curve down. One the other side, an increase in H leads to two different effects on R-R. First, an increase in H increases the demand for all goods in the economy increasing the profit per differentiated varieties, as a consequence research and development activities in the economy increases. This effect is known as 'scale effect'.<sup>7</sup> Second, an increase in H also increases the number of innovations fueling the demand for skilled worker to manufacture newly developed new goods. I call this effect innovation effect. The scale effect and the innovation effect work in the same direction and shift the R-R curve up. The magnitudes of effects of an increased H on M-M and R-R curves depend on the various parameters in the economy making the overall effect of a change in H on the equilibrium value of  $\omega^*$  ambiguous.

An increase in L as shown by equation (31) shifts both M-M and R-R curves up, implying a clear increase in the equilibrium value of relative wage,  $\omega^*$  and a clear decrease in the equilibrium value of growth rate,  $g^*$ . An increase in L increases the supply of unskilled workers involved in the production of old goods causing their relative wage,  $\frac{1}{\omega}$ , to fall. This effect leads to an upward shift in M-M curve. Also, an increase in L, like an increase in H, generates a scale effect implying an increased profit for a differentiated good. The scale effect increases the return from a new innovation, thus, moves the R-R curve up. 4.2. Effects of an Increase in Productivity Parameter.  $a_d$  is the productivity parameter in research and development sector. An increase in  $a_d$  increases the cost of a new innovation that creates a disincentive to participate in research and development sector, which in turn, leads to a downward shift in R-R curve. Similarly, an increase in  $a_d$  necessitates an increased number of skilled workers per innovation that implies a decreased supply of skilled workers in production sector, which in turn, raises the wage for a skilled worker and leads to an upward shift in M-M curve. Overall, an increase in  $a_d$  decreases the equilibrium value of economy wide growth rate  $g^*$ . The overall effect of an increased  $a_d$  on the equilibrium value of relative wage  $\omega^*$  is ambiguous as the change in the productivity parameter affects the two equilibrium condition for the determination of  $\omega^*$  in an opposite way.



FIGURE 3. Equilibrium Conditions and Effects of Change in Research Productivity Parameter.

4.3. Effects of an Increase in Standardization Time. When I take the future discount factor as 0, the standardization period T does not appear in the expression denoting the equilibrium condition in

	Effects on			
	Curves		Equilibrium Values	
Parameters	M-M	R-R	$g^*$	$\omega^*$
H↑	$\downarrow$	$\uparrow$	1	-
$L\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$
$a_d \uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	-
$T\uparrow$	$\uparrow$	NC	$\downarrow$	$\uparrow$

TABLE 1. Effects of changes in parameters (With Scale Effect)

research and development sector. However, an increase in T shifts M-M curve up. A higher T means that skilled workers' service is required for longer period to produce the same good, and with the same token, unskilled worker's service is required for lesser period. Therefore, an increase in T shifts M-M curve up and does not affect R-R curve. It implies that an increase in T, when future discount factor is low, increases the equilibrium value of relative wage,  $\omega$  and decreases the equilibrium value of growth rate  $g^*$ .

Table (1) shows the effects of changes in parameters on curves representing equilibrium conditions and equilibrium values of long term growth rate and relative wage in the economy. Dash, '-', denotes ambiguous effect and NC implies no change.

## 5. EXTENSION: PRODUCT CYCLE WITHOUT THE SCALE EFFECT

The source of economic growth, in the model described above, is knowledge creation. Knowledge is very unique as it is non-rival in nature: the use of a piece of knowledge by one economic agent does not preclude the simultaneous use of the same piece by another agent. In knowledge based growth models, the growth rate of the economy is directly related to the amount of knowledge created in the economy. When the rate of knowledge creation is linearly dependent on the available knowledge in the economy and labor employed in research, as described in the earlier model, any increase in labor supply raises the resources devoted to knowledge creation linearly and leads to an exponential increase in per capita growth rate. This effect is known as scale effect. It implies an accelerating per capita income growth in presence of population growth. Jones (1995) was first to point out the lack of evidence in support of such accelerated growth in presence of population growth. Subsequently, endogenous models with scale effect came under heavy criticism. Later theorist have attempted to remove the scale effect from endogenous growth models.<sup>8</sup> I follow the original prescription by Jones (1995) to remove the unintended scale effect from my model.

In the previous model, I suggested that number of products in the economy can be a proxy for the disembodied knowledge ( $\mathcal{K}$ ) in the economy. Jones (1995) shows that this particular choice of the proxy leads to the scale effect in models based on Grossman and Helpman (1991). To remove the effect, Jones suggested an alternative formulation which differs from Grossman-Helpman's formulation in two important ways. First, the productivity of a labor is negatively affected by the total number of skilled labor devoted to research and development. This relationship can be attributed to the possibility of duplication in research. Second, the knowledge in the economy is a concave function of the total number of goods available in the economy. With assumption that the technology of the research exhibits constant returns to scale with respect to the number of labor devoted to research at the firm level, Jones gives the following formulation for the product development at the firm level:

(33) 
$$\frac{1}{a_d} h_d n^{\phi} H_d^{\lambda - 1}$$

<sup>&</sup>lt;sup>8</sup>See Jones (1999) for an excellent survey of such models

where  $h_d$  is unit of labor employed by a firm,  $a_d$ , a labor productivity parameter, n, number of good available in the economy, and  $H_d$  is the total skilled workers in the research and development sector.  $0 < \phi < 1$ and  $0 < \lambda < 1$ . In equilibrium, by aggregating over all firm in research and development, one can get  $\sum h_d = H_d$  which gives the total number of new products developed in the economy at date t is given by  $\dot{n}$ .

(34) 
$$\dot{n} = \frac{n^{\phi} \cdot H_d^{\lambda}}{a_d}$$

The rate of innovation is given by  $g = \frac{\dot{n}}{n}$ . The growth rate in g can be written as

(35) 
$$\frac{\dot{g}}{g} = -(1-\phi)g + \lambda \frac{\dot{H}_d}{H_d}$$

Unlike the model in the previous section, here growth rate, g, is not endogenous and depends on population growth rate and other exogenously given parameters. If there is no population growth in the economy, there will be no growth in this model. Therefore, I introduce an exogenous population growth with rate  $N_p$  in the economy. Both populations, of skilled workers and unskilled workers, grow at the rate  $N_p$ . In the steady state the fraction of skilled worker employed in the research and development work would be a constant that implies  $\frac{\dot{H}_d}{H_d} = \frac{\dot{H}}{H} = N_P$  that, in turn, implies a constant g in the steady state.

$$(36) g = \frac{\lambda N_P}{1-\phi}$$

Population growth also affects the optimal condition on the consumer side given in equation (8) as the evolution of economy wide expenditure will no longer follow the same path as the evolution of any individuals' expenditure. The economy's total expenditure in period t is E(t) =  $L(t).E_l(t) + H(t).E_k(t)$ . This gives:

(37) 
$$\frac{\dot{E}}{E} = N_p + \frac{\dot{E}_k}{E_k} = N_P + \dot{R} - \rho$$

where where,  $N_P = \frac{\dot{L}}{L} = \frac{\dot{H}}{H}$ , and  $k \in \{h, l\}$ . The rate of total spending in the economy grows at the rate equal to individuals' spending growth rate corrected by the population growth rate.

I define labor productivity,  $\frac{\dot{n}}{H_d}$ , in research and development sector as  $\frac{m}{a_d}$  expression for which is given in the following equation:

(38) 
$$\frac{m}{a_d} = \frac{\dot{n}}{H_d} = \frac{n^{\phi} \cdot H_d^{\lambda-1}}{a_d},$$

m follows the growth path of labor productivity in the r & d sector, and its steady state growth rate can be given by

$$\frac{\dot{m}}{m} = \phi g - (1 - \lambda) N_P = \psi g, \text{ where } g = \frac{\lambda N_P}{1 - \phi}, \text{ and } \psi = 1 - \frac{1 - \phi}{\lambda}$$
(39)

Like the previous model model, there is no monetary authority in this model. So, I am free to give an arbitrary value to one of variables in the model. For convenience I follow ? and normalize the labor productivity in the r & d sector, m(t) by making it equal to a skilled worker's wage,  $w_h(t)$ . This particular specification implies that the wage paid to a skilled worker is always proportional to labor productivity in r & d sector. It also implies that an innovator incurs a fixed cost equal to  $a_d$  to come up with a new product as innovation cost for a new good is  $a_d \frac{w_h(t)}{m(t)}$ . In other word, this particular normalization makes the cost of a new design the numeraire in the economy. This also implies that wages and expenditures for skilled and unskilled workers grow at the same rate as labor productivity in the r & d sector, while the economy wide expenditure grows at the same rate as number of products in the economy.<sup>9</sup>

The new specification for product development would imply that new equilibrim condition in r & d sector differs from the condition given in equation (20). The new equilibrium condition would be:

(40) 
$$\dot{R} = \frac{\pi_N + e^{-[R(t+T) - R(t)]} [\pi_O - \pi_N]}{a_d}$$

In the steady state, the growth rate of number of products  $(\frac{\dot{n}}{n})$  is given by g. In the steady state, the economy's expenditure grows at constant rate g. The equation (37) implies:

(41) 
$$\dot{R} = g + \rho - N_P$$

Using expressions for  $\dot{R}$ , given in equation (41),  $\pi_N$  and  $\pi_O$ , given in equation (15), and steady state equilibrium values of ratios  $\frac{n_N(t)}{n(t)}$  and  $\frac{n_O(t)}{n(t)}$ , derived earlier, I can rewrite the above equation as:

(42) 
$$g + \rho - N_P = \frac{(1-\alpha)}{a_d} \left[ \frac{1 + e^{-(g+\rho-N_P)T} \left[\omega^{\sigma-1} - 1\right]}{1 + e^{-gT} \left[\omega^{\sigma-1} - 1\right]} \right] \frac{E(t)}{n(t)}$$

The above equation gives the equilibrium condition in the research and development market. It is important to note that equation (42) is not a no-arbitrage condition, as the economy's growth rate g is no longer an endogenous variable. Instead, g is determined by population growth rate  $N_p$ . The relationship between g and  $N_p$  is given in equation (36). Equation (42) gives the number of skilled workers engaged in and r & d activities. Using equations (42) and (26), I obtain the expression for

<sup>&</sup>lt;sup>9</sup>Rate of growth in economy wide expenditure is equal to the sum of the rate of growth in population and an individual's expenditure.  $\frac{\dot{E}}{E} = \frac{\dot{m}}{m} + \frac{\dot{H}}{H}$ . As,  $\frac{\dot{H}}{H} = \frac{\dot{L}}{L}$  = Population growth rate.

$$H_{p}(t) = H(t) - H_{d}(t), \text{ in terms of } H(t), g, T \text{ and } \omega.^{10}$$

$$H_{p}(t) = \frac{\alpha[g + \rho - N_{P}][1 - e^{-gT}]H(t)}{\alpha[g + \rho - N_{P}][1 - e^{-gT}] + (1 - \alpha)g[1 + e^{-(g + \rho - N_{P})T}[\omega^{\sigma - 1} - 1]]}$$
(43)

Now, I can write the equilibrium condition for labor market clearing in terms of variables relative wage ( $\omega$ ), growth rate in number of products, g, and parameters, maturity period, elasticity of substitution  $\sigma$ . Diving equation (43), by equation (46), given in appendix, and rearranging, I get the equilibrium condition in labor market.

$$\omega^{*\sigma} = \left[ \left( e^{gT} + 1 \right) + \frac{(1-\alpha)}{\alpha} \frac{g}{[g+\rho-N_P]} \left[ e^{gT} + e^{-(\rho-N_P)T} \left( \omega^{*(\sigma-1)} - 1 \right) \right] \right] \frac{L(t)}{H(t)}$$
(44)

where  $g^* = \frac{\lambda N_P}{1-\phi}$ .

5.1. Comparative Statistics. In the steady state of this model, the number of varieties grows at the constant rate  $g^* = \frac{\lambda N_P}{1-\phi}$  which is exogenous to the model and relative wage remains at the level  $\omega^*$  which is given in equation (44). Unlike the previous model, there is only one endogenous parameter, relative wage  $\omega$ , in this model.

5.1.1. Effects of an Increase in Labor Supply. An increase in H(t) at time t, keeping all other parameters constant, lowers the steady state relative wage  $\omega^*$  in the economy and does not affect the growth rate  $g^*$  in the economy.<sup>11</sup> Equation (44) is an identity in the steady state. From equation (43),  $\frac{H_p(t)}{H(t)}$  is independent of H(t) in the steady state, therefore, an increase in H(t) does not affect the proportion of skilled workers employed in research and development sector. However, it raises the ratio of skilled workers employed in productive activities

 $<sup>^{10}</sup>$ See appendix for formulation

<sup>&</sup>lt;sup>11</sup>See the derivation in appendix

	Ef Equilit	Effects on Equilibrium Values		
Parameters	$g^*$	$\omega^*$		
H↑	NC	$\downarrow$		
$L\uparrow$	NC	$\uparrow$		
$a_d \uparrow$	$\mathbf{NC}$	NC		
$\mathrm{T}\uparrow$	NC	$\uparrow$		

 TABLE 2. Effects of changes in parameters (In absence of Scale Effect)

with respect to unskilled worker, which in turn lowers the equilibrium relative wage. Similarly, an increase in L(t), keeping all parameters constant, increases the steady state relative wage and does not affect the steady state growth rate  $g^*$  in the economy.

5.1.2. Effects of an Increase in Standardization Period. An increase in T does not affect the steady state growth rate and increase the relative wage  $\omega^*$ . A higher T implies longer service from skilled workers which increases the relative wage in the steady state.

The effect of economy wide parameters on  $\omega *$  and  $g^*$  is given in table (2).

### 6. CONCLUSION

Grossman and Helpman (1991) show that relative wage between north and south varies directly with the size of the labor in the these region, and the long run rate of innovation in north and imitation in south are determined endogenously. By assuming an innovation process in the north similar to Grossman Helpman and replacing imitation by south in Grossman-helpman model by product standardization in the north (No trade), I find that an increase in the size of unskilled worker will decrease its relative wage. An increase in the size if skilled workers which is required for both production and research may or may not increase its relative wage. The relative wage of skilled workers will increase with increase in their population only when research effect dominates the population effect. A replacement of Grossman Helpman type of innovation by Jones (1995) type of innovation in the model wipes out the scale effect and suggest that an increase in skilled workers' population decreases its relative wage in a closed economy. Further, the paper, using the insight from product cycle literature, shows how an endogenous growth that is skill neutral can have the same impact on income distribution as international trade and skill biased technical change.

### 7. Appendix

Using the expression for  $Y_j(t)$  given in equation (4),  $H_p$  and  $L_p$  can be expressed in terms of g,  $\omega$  and system parameters such as T and  $\rho$ .

(45) 
$$H_p(t) = H(t) - H_d(t) = \frac{\alpha}{w_h(t)} \frac{1 - e^{-gT}}{1 - e^{-gT} + e^{-gT} \omega^{\sigma-1}} E(t)$$

(46) 
$$L_p(t) = L(t) = \frac{\alpha}{w_l(t)} \frac{e^{-gT} \omega^{\sigma-1}}{1 - e^{-gT} + e^{-gT} \omega^{\sigma-1}} E(t)$$

Using equations (45), (46), one can obtain the expression for  $\frac{E(t)}{w_h(t)}$  in the steady state. By multiplying equation (45) with  $w_h(t)$  and equation (46) with  $w_l(t)$ , adding them up and using equation (28) for  $H_d(t)$ , I obtain:

(47) 
$$\frac{E(t)}{w_h(t)} = \frac{1}{\alpha} \left[ H(t) - H_d(t) + \frac{L(t)}{\omega} \right]$$

Using  $w_h(t) = m(t)$ ,  $m(t) = a_d \frac{\dot{n}}{H_d(t)}$  from equation (38), and  $\frac{\dot{n}}{n} = g$  in equation (47), I obtain:

(48) 
$$\frac{E(t)}{n(t)} = \frac{a_d g}{H_d(t)\alpha} \left[ H(t) - H_d(t) + \frac{L(t)}{\omega} \right]$$

Using equations (42) and (48), I can write: (#9)- $H_d + \frac{L}{\omega} = \frac{\alpha}{(1-\alpha)} \frac{[g+\rho-N_P]}{g} \frac{1-e^{-gT}}{1+e^{-(g+\rho-N_P)T} [\omega^{\sigma-1}-1]} H_d$ 

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