Free licensing of patents: A theoretical analysis

Debapriya Sen* Giorgos Stamatopoulos†

December 19, 2022

Abstract

This paper considers a simple general equilibrium model to explore the optimality of free licensing. In a model of an economy where the representative consumer has a CES utility function over two goods which are produced by two different firms and one of the firms has a patent on an efficient production technology, we show that free licensing is superior to no licensing, but it is optimal to set positive royalties when royalty licensing is allowed. In an extended model with a basic good that follows a hierarchical demand (Matsuyama, 2002) and that is produced by a competitive fringe of firms, we show that free licensing to all firms can be superior even to royalty licensing.

Keywords: patent licensing; free licensing; unit royalties; CES utility; hierarchial demand
JEL Classification: D01, D21, D45, D50

1 Introduction

A patent grants an innovator monopoly rights over its innovation for a given period of time. By licensing a patent to other firms, the patent holder gives the patented innovation to those firms in return for payments based on some agreed upon policy such as a royalty or a fixed fee. A frequently observed phenomenon in recent years is free licensing, where a patent holder makes its patents freely available without charging any royalties or fees. This is particularly common in high technology industries in the form of open source softwares, but free licenses through "patent pledges" are observed in other industries as well.

A broad, across-the-board release of patents was recently offered by Samsung. According to the announcement made in May 2021 by the Ministry of Trade, Industry and Energy of South Korea, Samsung would share 505 of its patented technologies with local small and medium enterprises (SMEs). These technologies are across different sectors such as mobile, semiconductor, telecommunications and medical equipments (see Woo-hyun, 2021).

*Toronto Metropolitan University, Toronto, Canada.

†University of Crete, Crete, Greece.

Firms may give out free patents for different possible reasons. When the same firm produces two related components, giving away the technology of one component for free may enable it to charge a high price for the other component. As a specific example of this pricing strategy, Vertinsky (2018) points out that open source softwares of Google are mostly related to technologies that facilitate free access to the Internet and the company "...has been accused of exploiting its dominant position in Internet search to favor its own applications and to extract higher prices from advertisers and from manufacturers that want to install Google applications."

Another reason for free licensing can be the long term goal of promoting a new technology that has future potential. For example, in 2010 Hewlett-Packard pledged its patents to the green technology patent pool Eco-Patent Commons (see Ziegler, 2014). Yet another objective of free licensing could be to establish a specific technology in the industry by encouraging future investments in compatible technologies. For instance, in the clean energy vehicle industry, both Tesla and Toyota have a number of patent pledges. Given that these two companies have different kinds of technologies (electric for Tesla and hydrogen fuel for Toyota), each company might have offered patent pledges to create incentives to develop related technologies (e.g., vehicle recharging) that are aligned to its specific technology (see Vertinsky, 2018).

Given the prevalence of free licensing of patents in different contexts, it is useful to have a clear theoretical understanding of the possible optimality of free licensing for a patent holder. There is a large theoretical literature (onwards Arrow, 1962; Katz and Shapiro, 1986; Kamien and Tauman, 1986) that has studied different patent licensing policies and their effects on market prices, profits and incentives to innovate, but the question of free licensing is rather under-explored in the literature. A literature in international trade has looked at free technology transfer across countries (e.g., McCulloch and Yellen, 1982; Jones and Ruffin, 2008), but the key considerations there are issues such as comparative advantage.

To understand the optimality of free licensing, in this paper we consider a simple general equilibrium model where the representative consumer has a constant elasticity of substitution (CES) utility function (Arrow et al., 1961) over two goods that are produced by two firms $1, 2$. The income of the consumer is the sum of an exogenous component and a positive fraction of the sum of the profits of two firms. Firm 1 holds a patent for an efficient technology that lowers the marginal cost of production.

In this setting we show that for firm 1, giving the patent to firm 2 for free is superior to not licensing (Proposition 1). Giving the efficient technology for free ensures that the economy as a whole is more productive, which raises the demands and profits of both firms. It should be noted that superiority of free licensing over no licensing can never occur in partial equilibrium models of duopoly or oligopoly (such models have been extensively studied in the large literature of patent licensing).

Next we allow firm 1 to license its technology to firm 2 using a per unit royalty, so that free licensing to firm 2 becomes equivalent to setting a zero royalty. We show that it is optimal for firm 1 to set a positive royalty, which implies royalty licensing is superior to free licensing (Proposition 2). Thus, although free licensing is superior to no licensing, it is no longer optimal when the patentee can set royalties.

To further explore the optimality of free licensing, we consider an extended model that more adequately captures the situation in which the patented technology is made more broadly available to the whole economy. Our initial two-good model is extended to three goods, where the third good (called good 0) corresponds to the basic good. Following Matsuyama (2002), we consider a "hierarchial demand" utility function, where the representative consumer exclusively cares about the basic good if its consumption falls below a certain threshold (which can be interpreted as the subsistence requirement). Once this threshold level of consumption is reached for the basic good, consuming more of it is no longer useful and the consumer cares about goods $1, 2$ as imperfect substitutes with constant elasticity of substitution.

On the production side, as before goods $1, 2$ are produced by firms $1, 2$, with firm 1 having a patent on an efficient production technology. The basic good 0 is produced by a competitive fringe of firms. This fringe, in a simple way, seeks to approximate the SMEs (small and medium enterprises) of an economy that can potentially benefit from the wide availability of patented technologies released by a large firm such as Samsung.

We assume that patent enforcement is weak in the fringe, which means if one firm in the fringe has the patented technology, all other firms can have it at no cost.[1] This implies that firm 1 cannot obtain a licensing revenue from the fringe. It can be noted that firm 1 always has the option of excluding the fringe and give the technology exclusively to firm 2 (by either setting a royalty or for free).

As before the income of the representative consumer is the sum of an exogenous component and a fraction of the sum of profits of all firms. In this extended model we show that free licensing for all (the fringe as well as firm 2) is superior to no licensing (Proposition 3). Furthermore, if the exogenous component of the consumer's income is relatively small, free licensing for all is superior to royalty licensing as well (Proposition 4), that is, it is optimal to give the technology for free to all firms even when firm 1 has the option of licensing it exclusively to firm 2 using royalties. We also show that out of these different options, free licensing for all gives the highest utility to the representative consumer.

When the technology is given for free to all firms, the fringe has the efficient technology and the price of the basic good falls. Because the demand of the basic good is highly inelastic, a fall in its price implies the consumer spends less on the basic good and devotes more of its income towards the consumption of the non-basic goods. Then the interdependent general equilibrium effects result in higher demands and higher profits for the patentee firm.

There are two effects behind the optimality of free licensing: (i) the net income effect driven by lower expenditure for the basic good and (ii) the general equilibrium effect of higher profits of firms. One important point to observe is that free licensing is optimal even in the polar case in which profits are not part of the consumer's income at all and the income consists only of the exogenous component (Proposition 4), that is, the net income effect on its own can lead to optimality of free licensing even when the

[1] One reason for this could be that the fringe is part of the informal sector of the economy, which is outside the regulatory framework of patents and once the technology is available to some firms in the fringe, its imitation cannot be prevented. A large proportion of SMEs are often part of the informal sector, especially in developing and emerging economies (see, e.g., Qiang and Ghossein, 2020; World Bank, 2020).

general equilibrium effect is absent. This means for free licensing to be optimal, the patentee firm does not necessarily have to be large in relation to the whole economy. As long as the exogenous component of the consumer's income is relatively small, there are gains from free sharing of the new technology with the fringe.

A key implication of our results is that for an economy with a relatively low starting income whose basic good sectors are technologically nascent and dominated by a large number of small firms (a situation that closely approximates many developing or emerging economies), free sharing of a patented technology can be optimal even when the patentee is not large with respect to the whole economy; if the patentee is large, then of course there is the additional general equilibrium effect of interdependent profits, demands and income.

The paper is organized as follows. We present the initial model and results in Section 2. The extended model with hierarchial demand and the results are presented in Section 3. We conclude in Section 4.

2 The model

Consider an economy with two goods $1, 2$. The economy has one representative consumer and two firms $1, 2$. Firm 1 produces good 1 and firm 2 produces good 2. Firms compete in prices.

For $i = 1, 2$, let x_i be the amount of good i. The representative consumer of the economy has a constant elasticity of substitution (CES) utility function (Arrow et al., 1961) given by

$$u(x_1, x_2) = [\alpha x_1^\rho + (1-\alpha)x_2^\rho]^{1/\rho} \tag{1}$$

where $0 < \alpha < 1$ and $\rho \in (-\infty, 0) \cup (0, 1)$, i.e., $\rho < 1$ and $\rho \neq 0$. The CES utility function includes several standard utility functions as special cases: (i) $\rho \to 1$ corresponds to perfect substitutes, (ii) $\rho \to 0$ gives Cobb-Douglas and (iii) $\rho \to -\infty$ gives Leontief utility functions.

The constant elasticity of substitution of the utility function (1) is given by $\sigma \equiv 1/(1-\rho)$. Note that $\sigma > 0$ and $\sigma \neq 1$. If $0 < \rho < 1$, then $\sigma > 1$ and the goods are "good substitutes"; if $\rho < 0$, then $0 < \sigma < 1$ and the goods are "poor substitutes" (see, e.g., Black et al., 2009).

Denote by y the income of the representative consumer and for $i = 1, 2$, let p_i be the price of good i. The consumer's utility maximization problem is to choose $x_1, x_2 \geq 0$ to maximize $u(x_1, x_2)$ given in (1) subject to the budget constraint $p_1 x_1 + p_2 x_2 \leq y$. Denote

$$g(p_1, p_2) := \alpha^\sigma p_1^{1-\sigma} + (1-\alpha)^\sigma p_2^{1-\sigma} \tag{2}$$

This utility maximization problem has a unique solution $(x_1^*(p_1, p_2, y), x_2^*(p_1, p_2, y))$ given by

$$x_1^*(p_1, p_2, y) = y\alpha^\sigma / p_1^\sigma g(p_1, p_2) \text{ and } x_2^*(p_1, p_2, y) = y(1-\alpha)^\sigma / p_2^\sigma g(p_1, p_2) \tag{3}$$

Income of the consumer: The income y of the consumer is the sum of two components: (i) an exogenously given minimum income $\hat{y} > 0$ and (ii) a fraction t of the total profit Π of the economy. Thus

$$y = \hat{y} + t\Pi \tag{4}$$

where $0 \leq t < 1$. The total profit Π is the sum of the profits of two firms $1, 2$. Observe that prices p_1, p_2 as well as the total profit Π (which in turn determines y) are determined in equilibrium.

Remark 1 The fraction t is a reduced form parameter that captures the interrelation between profits, income and demands for the goods. This reduced form approach helps us to clearly present our main point on technology transfer. It is possible to make the same point by explicitly modeling the interdependence between profits and incomes. For instance, as in Murphy et al. (1989), if firms employ labor inputs that the representative consumer supplies, then higher profits and labour earnings due to the adoption of an efficient production technology by one firm raises the income of the consumer which in turn raises demands for all goods.

Initial and new production technologies: Initially both firms $1, 2$ have the same constant marginal cost of production $\overline{c}$. Firm 1 has a patent on a new technology that lowers the marginal cost from $\overline{c}$ to $\underline{c}$, where $0 < \underline{c} < \overline{c}$.

Assumption 1 The prices p_1, p_2 that firms can set must be in the closed interval $[\underline{p}, \overline{p}]$, where $0 < \underline{p} < \underline{c} < \overline{c} < \overline{p}$.

Imposing the lower bound $\underline{p}$ on prices is not a crucial assumption, because firms have positive constant marginal costs of production and it will not be optimal for any firm to set a price below its marginal cost. The requirement of the upper bound $\overline{p}$ is needed to ensure existence of equilibrium.

2.1 No licensing versus free licensing of the new technology

Firm 1 has the new technology, so its own marginal cost is $\underline{c}$. If firm 1 does not license the new technology to firm 2, the marginal cost of firm 2 will be the initial high cost $\overline{c}$. On the other hand, if firm 1 licenses the new technology to firm 2 for free, the marginal cost of firm 2 will be $\underline{c}$.

It will be useful to study the duopoly in which firm 1 has marginal cost c_1 and firm 2 has marginal cost c_2, where $c_1, c_2 \in [\underline{p}, \overline{p}]$. Under no licensing, $c_1 = \underline{c}$, $c_2 = \overline{c}$ and under free licensing, $c_1 = c_2 = \underline{c}$.

If firm 1 sets price p_1, firm 2 sets price p_2 and the income of the representative consumer is y, the consumer demands $x_i^*(p_1, p_2, y)$ units of good i (given by (3)). Therefore the operating profits of firms $1, 2$ are

$$\pi_1(p_1, p_2, c_1, y) = (p_1 - c_1)x_1^*(p_1, p_2, y) = (p_1 - c_1)y\alpha^\sigma / p_1^\sigma g(p_1, p_2) \text{ and}$$

$$\pi_2(p_1, p_2, c_2, y) = (p_2 - c_2)x_2^*(p_1, p_2, y) = (p_2 - c_2)y(1-\alpha)^\sigma / p_2^\sigma g(p_1, p_2) \tag{5}$$

Determining equilibrium income at p_1, p_2: To analyze the duopoly interaction between firms $1, 2$, at all prices p_1, p_2, using (4) first we find income y that can support the profits given in (5). The total profit Π of the economy is the sum of the operating profits of firms $1, 2$. Thus by (4), the income of the representative consumer is

$$\begin{aligned} y &= \hat{y} + t\Pi = \hat{y} + t\pi_1(p_1, p_2, c_1, y) + t\pi_2(p_1, p_2, c_2, y) \\ &= \hat{y} + t(p_1 - c_1)y\alpha^\sigma / p_1^\sigma g(p_1, p_2) + t(p_2 - c_2)y(1-\alpha)^\sigma / p_2^\sigma g(p_1, p_2) \end{aligned} \quad (6)$$

Observe from equation (6) that profits on the right side depend on the income y, while income y depends on the profits, reflecting the general equilibrium nature of the problem. Solving equation (6) for y, the unique solution is

$$y = \hat{y} p_1^\sigma p_2^\sigma g(p_1, p_2) / h^t(p_1, p_2, c_1, c_2) \equiv y^t(p_1, p_2, c_1, c_2, \hat{y}) \quad (7)$$

where

$$h^t(p_1, p_2, c_1, c_2) = \alpha^\sigma p_2^\sigma [tc_1 + (1-t)p_1] + (1-\alpha)^\sigma p_1^\sigma [tc_2 + (1-t)p_2] \quad (8)$$

Note that $h^t > 0$ for any positive p_1, p_2. The (unique) equilibrium income at price p_1, p_2 under marginal costs c_1, c_2 is $y^t(p_1, p_2, c_1, c_2, \hat{y})$, given by (7). Note that $h^0(p_1, p_2, c_1, c_2) = p_1^\sigma p_2^\sigma g(p_1, p_2)$, so when $t = 0$, the equilibrium income is $\hat{y}$.

Determining Nash Equilibrium of the duopoly: Taking $y = y^t(p_1, p_2, c_1, c_2, \hat{y})$ from (7) in (5), the profit functions at prices p_1, p_2 under marginal costs c_1, c_2 are

$$\begin{aligned} \Pi_1^t(p_1, p_2, c_1, c_2, \hat{y}) &\equiv \pi_1(p_1, p_2, c_1, y^t) = (p_1 - c_1)\hat{y}\alpha^\sigma p_2^\sigma / h^t(p_1, p_2, c_1, c_2), \\ \Pi_2^t(p_1, p_2, c_1, c_2, \hat{y}) &\equiv \pi_2(p_1, p_2, c_2, y^t) = (p_2 - c_2)\hat{y}(1-\alpha)^\sigma p_1^\sigma / h^t(p_1, p_2, c_1, c_2) \end{aligned} \quad (9)$$

By (4), the consumer gets fraction t of profit of each firm $i = 1, 2$. Therefore the net payoff of firm i is $(1-t)\Pi_i^t$. Since $0 \le t < 1$, firms $1, 2$ interact in a standard duopoly of price competition where firm i seeks to maximize its profit Π_i^t given in (9). Denote this duopoly by $\mathbb{D}^t(c_1, c_2, \hat{y})$.

Lemma 1 *Suppose firms $1, 2$ have constant marginal costs $c_1, c_2 \in (\underline{p}, \overline{p})$ and either $0 < \sigma < 1$ or $1 < \sigma < \overline{p}/(\overline{p} - c_i)$ for $i = 1, 2$. Then for any $i, j = 1, 2$ and $i \neq j$, Π_i^t given in (9) is increasing in p_i for any positive p_j, so the unique best response of firm i to any $p_j \in [\underline{p}, \overline{p}]$ is to choose $p_i = \overline{p}$. Consequently the unique NE of $\mathbb{D}^t(c_1, c_2, \hat{y})$ is $(p_1 = \overline{p}, p_2 = \overline{p})$. At the NE, for $i = 1, 2$, firm i obtains profit $\Pi_1^t(\overline{p}, \overline{p}, c_1, c_2, \hat{y})$ and net payoff $(1-t)\Pi_i^t(\overline{p}, \overline{p}, c_1, c_2, \hat{y})$.*

Proof See the Appendix. ■

Lemma 1 shows that when the two goods produced by two firms $1, 2$ are either poor substitutes ($0 < \sigma < 1$) or not sufficiently good substitutes ($\sigma > 1$, but bounded above by $\overline{p}/(\overline{p} - c_i)$ for $i = 1, 2$), then the unique equilibrium outcome of the duopoly interaction has both firms setting the maximum permissible price $\overline{p}$. Using Lemma 1, we compare no licensing and free licensing.

Proposition 1 *Suppose either $0 < \sigma < 1$ or $1 < \sigma < \overline{p}/(\overline{p} - \underline{c})$.*

(i) *For any* $0 < t < 1$, *licensing the new technology to firm* 2 *for free is superior for firm* 1 *compared to no licensing and for* $t = 0$, *firm* 1 *is indifferent between free licensing and no licensing.*

(ii) *For any* $0 \leq t < 1$, *receiving the license for free is superior for firm* 2 *compared to no licensing.*

(iii) *For any* $0 < t < 1$, *the representative consumer has higher income and higher utility under free licensing compared to no licensing. For* $t = 0$, *the consumer has the same income and same utility under free licensing and no licensing.*

Proof See the Appendix. ■

Proposition 1 shows that for firm 1, licensing the new technology for free to firm 2 is superior to no licensing. This is because free transfer of the new technology makes firm 2 more productive, thus raising the income of the representative consumer, demands for both goods and profits of firms. This why there are gains from technology transfer for firm 1 even in the absence of any licensing revenue.

2.2 Licensing by per unit royalty

In this section we consider the *per unit royalty* policy of licensing. Under this policy, in return for licensing of the new technology, for every unit of good 2 that firm 2 produces, firm 2 pays a unit royalty $r \geq 0$ to firm 1. Note that $r = 0$ corresponds to free licensing.

Under per unit royalty r, firm 2 gets the new technology and in return pays r for every unit it produces. Since the marginal cost under the new technology is $\underline{c}$, with royalty r, the *effective marginal cost* of firm 2 is $\underline{c} + r$. Since the maximum price is $\overline{p}$ and firm 2 obtains positive profit without a license, it will not accept a royalty r that has $\underline{c} + r \geq \overline{p}$, so we can restrict $r < \overline{p} - \underline{c}$.

If firm 1 sets price p_1, firm 2 sets price p_2 and the income of the representative consumer is y, the consumer demands $x_i^*(p_1, p_2, y)$ units of good i (given by (3)). Under per unit royalty r, the total profit of firm 1 is the sum of its own operating profit $(p_1 - \underline{c})x_1^*(p_1, p_2, y)$ and the licensing revenue $rx_2^*(p_1, p_2, y)$. Since the unit royalty is included the marginal cost of firm 2, the profit of firm 2 is $[p_2 - (\underline{c} + r)]x_2^*(p_1, p_2, y)$. These profits are

$$\phi_1(p_1, p_2, \underline{c}, y, r) = (p_1 - \underline{c})x_1^*(p_1, p_2, y) + rx_2^*(p_1, p_2, y) \text{ and}$$

$$\phi_2(p_1, p_2, \underline{c}, y, r) = [p_2 - (\underline{c} + r)]x_2^*(p_1, p_2, y) \tag{10}$$

Determining equilibrium income at p_1, p_2**:** The representative consumer receives fraction t of the operating profit of each firm and in addition gets fraction t of the royalty revenue $rx_2^*(p_1, p_2, y)$. Given this, the income of the representative consumer is

$$y = \hat{y} + t(p_1 - \underline{c})x_1^*(p_1, p_2, y) + t[p_2 - (\underline{c} + r)]x_2^*(p_1, p_2, y) + trx_2^*(p_1, p_2, y)$$

Since the royalty revenue $rx_2^*(p_1, p_2, y)$ is a transfer from firm 2 to firm 1, this term does not directly affect the consumer's income. So we have

$$y = \hat{y} + t(p_1 - \underline{c})x_1^*(p_1, p_2, y) + t(p_2 - \underline{c})x_2^*(p_1, p_2, y)$$

Using the expression of $x_i^*(p_1, p_2, y)$ from (3), the income of the representative consumer is

$$y = \hat{y} + t(p_1 - \underline{c})y\alpha^\sigma / p_1^\sigma g(p_1, p_2) + t(p_2 - \underline{c})y(1-\alpha)^\sigma / p_2^\sigma g(p_1, p_2) \quad (11)$$

Note that equation (11) is the same as equation (6) with $c_1 = c_2 = \underline{c}$. Solving equation (11) for y, the unique solution is

$$y = \hat{y} p_1^\sigma p_2^\sigma g(p_1, p_2) / h^t(p_1, p_2, \underline{c}, \underline{c}) \equiv y^t(\hat{y}, p_1, p_2, \underline{c}, \underline{c}) \quad (12)$$

where the function h^t is given in (8). Using (12) in (3), we have

$$x_1^*(p_1, p_2, y^t(\hat{y}, p_1, p_2, \underline{c}, \underline{c})) = \hat{y}\alpha^\sigma p_2^\sigma / h^t(p_1, p_2, \underline{c}, \underline{c}),$$

$$x_2^*(p_1, p_2, y^t(\hat{y}, p_1, p_2, \underline{c}, \underline{c})) = \hat{y}(1-\alpha)^\sigma p_1^\sigma / h^t(p_1, p_2, \underline{c}, \underline{c}) \quad (13)$$

Determining Nash Equilibrium of the duopoly: Under unit royalty r, firm 1 obtains operating profit $(p_1 - \underline{c})x_1^*$ and licensing revenue rx_2^*. Since fraction t of both its operating profit and licensing revenue is left with the representative consumer, the net payoff of firm 1 is $(1-t)[(p_1 - \underline{c})x_1^* + rx_2^*]$. Since $0 \le t < 1$, firm 1 seeks to maximize $(p_1 - \underline{c})x_1^* + rx_2^*$. Using (13) this payoff is

$$\Phi_1^t(p_1, p_2, \underline{c}, r, \hat{y}) = (p_1 - \underline{c})\hat{y}\alpha^\sigma p_2^\sigma / h^t(p_1, p_2, \underline{c}, \underline{c}) + r\hat{y}(1-\alpha)^\sigma p_1^\sigma / h^t(p_1, p_2, \underline{c}, \underline{c}) \quad (14)$$

Under unit royalty r, firm 2 obtains operating profit $[p_2 - (\underline{c} + r)]x_2^*$. Since fraction t of this profit is left with the consumer, the net payoff of firm 2 is $(1-t)[p_2 - (\underline{c} + r)]x_2^*$. Noting that $0 \le t < 1$, firm 2 seeks to maximize its operating profit. Using (13), this profit is

$$\Phi_2^t(p_1, p_2, \underline{c}, r, \hat{y}) = [p_2 - (\underline{c} + r)]\hat{y}(1-\alpha)^\sigma p_1^\sigma / h^t(p_1, p_2, \underline{c}, \underline{c}) \quad (15)$$

Denote by $\mathbb{D}^t(\underline{c}, \underline{c} + r, \hat{y})$ the two-person game in which firms 1, 2 choose prices $p_1, p_2 \in [\underline{p}, \overline{p}]$ and the payoffs are given in (14)-(15). The next lemma is immediate from Lemma 1.

Lemma 2 *Suppose either* $0 < \sigma < 1$ *or* $1 < \sigma < \overline{p}/(\overline{p} - \underline{c})$. *For any* $t \in [0, 1)$ *and* $r \in [0, \overline{p} - \underline{c}]$, *the game* $\mathbb{D}^t(\underline{c}, \underline{c} + r, \hat{y})$ *has a unique NE:* $(p_1 = \overline{p}, p_2 = \overline{p})$. *At the NE, for* $i = 1, 2$, *firm* i *obtains net payoff* $(1-t)\Phi_i^t(\overline{p}, \overline{p}, \underline{c}, r, \hat{y})$.

Determining optimal royalty for firm 1: Using the result of Lemma 2, under royalty r, firm 2 obtains net payoff $(1-t)\Phi_i^t(\overline{p}, \overline{p}, \underline{c}, r, \hat{y})$. Recall that under no licensing firm 2 obtains net payoff $(1-t)\Pi_2^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}, \hat{y})$ where

$$\Pi_2^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}, \hat{y}) = (\overline{p} - \overline{c})\hat{y}(1-\alpha)^\sigma \overline{p}^\sigma / h^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}) \quad (16)$$

Note from (15) and (16) that

$$\Phi_i^t(\overline{p}, \overline{p}, \underline{c}, r, \hat{y}) \ge \Pi_2^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}, \hat{y})$$

$$\Leftrightarrow r \leq (\overline{p} - \underline{c}) - (\overline{p} - \overline{c})h^t(\overline{p}, \overline{p}, \underline{c}, \underline{c})/h^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}) \equiv r^t(\overline{p}, \underline{c}, \overline{c}) \tag{17}$$

Thus a unit royalty r is acceptable to firm 2 if and only if $r \leq r^t(\overline{p}, \underline{c}, \overline{c})$. By (8), the ratio $h^t(\overline{p}, \overline{p}, \underline{c}, \underline{c})/h^t(\overline{p}, \overline{p}, \underline{c}, \overline{c})$ is decreasing in t, $h^t(\overline{p}, \overline{p}, \underline{c}, \underline{c}) < h^t(\overline{p}, \overline{p}, \underline{c}, \overline{c})$ for $0 < t < 1$ and $h^0(\overline{p}, \overline{p}, \underline{c}, \underline{c}) = h^0(\overline{p}, \overline{p}, \underline{c}, \overline{c})$. This shows that $r^t(\overline{p}, \underline{c}, \overline{c})$ is increasing in t, $\overline{c} - \underline{c} < r^t(\overline{p}, \underline{c}, \overline{c}) < \overline{p} - \underline{c}$ for $0 < t < 1$ and $r^0(\overline{p}, \underline{c}, \overline{c}) = \overline{c} - \underline{c}$. By (14), $\Phi_1^t(\overline{p}, \overline{p}, \underline{c}, r, \hat{y})$ is increasing in r, so the unique optimal unit royalty for firm 1 is to set $r = r^t(\overline{p}, \underline{c}, \overline{c})$ given in (17).

Proposition 2 *Suppose either $0 < \sigma < 1$ or $1 < \sigma < \overline{p}/(\overline{p} - \underline{c})$. The following hold for any $0 \leq t < 1$.*

(i) *The unique optimal per unit royalty policy for firm 1 is to set royalty $r^t(\overline{p}, \underline{c}, \overline{c})$ given in (17) and this policy is superior to free licensing as well as no licensing for firm 1.*

(ii) *Firm 2 is indifferent between no licensing and receiving the license with royalty $r^t(\overline{p}, \underline{c}, \overline{c})$ and both are inferior to free licensing for firm 2.*

(iii) *The representative consumer has same income and same utility under free licensing and licensing with royalty $r^t(\overline{p}, \underline{c}, \overline{c})$. For $0 < t < 1$, income and utility under these policies are higher than no licensing. For $t = 0$, income and utility of all three policies are the same.*

Proof See the Appendix. ■

Proposition 2 shows that when firm 1 can transfer the technology using per unit royalty, it is optimal to set a royalty that makes firm 2 just indifferent between having a license or not. This optimal royalty policy is superior to free licensing. From Propositions 1,2, we can see that although free licensing is superior to no licensing, once firm 1 has the options of setting royalties for licensing, selling the technology for free is no longer optimal. In view of the evidence that firms do give away free patents, a natural question is: is it possible that even when the patentee can set royalties, it is optimal to set no royalties and give licenses for free? We consider an extended model to explore this question.

3 A model of hierarchical demand

In this section we consider another good (good 0) in addition to goods $1, 2$. Good 0 can be viewed as a basic good that has two features: (i) necessity (the consumer requires a specific minimum amount of that good; if that requirement is not met, the other goods are not useful) and (ii) saturation (once the minimum threshold consumption of the basic good is attained, consuming more of it does not give additional utility). We consider the "hierarchical demand" model developed by Matsuyama (2002).

Consider an economy with one representative consumer and three goods $0, 1, 2$. Good 0 is the basic good, while goods $1, 2$ are non-basic goods (for instance, manufacturing goods). We assume that the basic good 0 is produced by a competitive fringe of firms. For goods $1, 2$ as before we assume there are two distinct firms $1, 2$, firm 1 producing good 1 and firm 2 producing good 2. Firms compete in prices.

For $i = 0, 1, 2$, let x_i be the amount of good i. The representative consumer of the economy has a utility function that has the feature of "hiererachial demand" with respect to good 0 (Matsuyama, 2002) and constant elasticity of substitution between goods $1, 2$. Specifically the utility function is given by

$$U(x_0, x_1, x_2) = \begin{cases} x_0 & \text{if } x_0 \leq \underline{x}_0 \\ \underline{x}_0 + u(x_1, x_2) & \text{if } x_0 > \underline{x}_0 \end{cases} \tag{18}$$

where $u(x_1, x_2)$ is the CES utility function given in (1). All assumptions of the last section are maintained.

The threshold $\underline{x}_0$ correponds to the level of minimum requirement of the basic good 0. The specification (18) implies that the non-basic goods $1, 2$ are not useful when the consumption of the basic good does not exceed the minimum required level. Beyond the minimum level, saturation for the basic good is reached and non-basic goods are useful.

The income of the representative consumer is denoted by Y. For $i = 0, 1, 2$, let p_i be the price of good i. The consumer's utility maximization problem is to choose $x_i \geq 0$ to maximize $U(x_0, x_1, x_2)$ given in (18) subject to the budget constraint $p_0x_0 + p_1x_1 + p_2x_2 \leq Y$.

If $Y \leq p_0\underline{x}_0$ (that is, the income is not sufficient to afford the minimum required level $\underline{x}_0$ of good 0), it is optimal for the consumer to buy only good 0 and not buy goods $1, 2$ at all. In that case the unique solution to the utility maximization problem is given by

$$\tilde{x}_0(p_0, p_1, p_2, Y) = Y/p_0, \tilde{x}_1(p_0, p_1, p_2, Y) = 0 \text{ and } \tilde{x}_2(p_0, p_1, p_2, Y) = 0 \tag{19}$$

By (18), satiation for good 0 is reached at $\underline{x}_0$. For this reason, if $Y > p_0\underline{x}_0$ (the income is sufficient to afford $\underline{x}_0$), it is optimal for the consumer to buy exactly $\underline{x}_0$ units of good 0 and use the remaining income $Y - p_0\underline{x}_0$ to buy goods $1, 2$ to maximize $u(x_1, x_2)$. Using (3), in this case the unique solution to the utility maximization problem is

$$\tilde{x}_0(p_0, p_1, p_2, Y) = \underline{x}_0, \tilde{x}_1(p_0, p_1, p_2, Y) = (Y - p_0\underline{x}_0)\alpha^\sigma/p_1^\sigma g(p_1, p_2) \text{ and}$$

$$\tilde{x}_2(p_0, p_1, p_2, Y) = (Y - p_0\underline{x}_0)(1-\alpha)^\sigma/p_2^\sigma g(p_1, p_2) \tag{20}$$

where $g(p_1, p_2)$ is given in (2). Note that in this case for goods $1, 2$, the quantity demanded by the consumer are the same as (3) with $y = Y - p_0\underline{x}_0$, that is, the consumer demands for goods $1, 2$ are obtained on the basis of the net income $Y - p_0\underline{x}_0$, which is the income net of the expenditure for the minimum required level of the basic good. Specifically, the solutions of (3) and (20) are related as follows:

$$\tilde{x}_1(p_0, p_1, p_2, Y) = x_1^*(p_1, p_2, Y - p_0\underline{x}_0), \tilde{x}_2(p_0, p_1, p_2, Y) = x_2^*(p_1, p_2, Y - p_0\underline{x}_0) \tag{21}$$

Initial and new production technologies: We assume that initially both firms $1, 2$, as well as all firms in the competitive fringe that produce good 0 have the same constant marginal cost of production $\overline{c}$ (this assumption is made for computational convenenience; the key points of our analysis go through even if firms have different initial costs). Firm 1 has a new technology that lowers the marginal cost from $\overline{c}$ to $\underline{c}$, where $0 < \underline{c} < \overline{c}$.

New technology and the competitive fringe: We assume that the competitive fringe that produces the basic good 0 has weak enforcement of patent rights. As a result, if one or more firms in the fringe has the new technology, all remaining firms (both inside the fringe as well as outside) also have the technology and all of them operate with the same marginal cost, yielding zero profits for all firms in the fringe. For this reason no firm in the fringe is willing to make any licensing payment for the new technology.

Different options of technology transfer: Note that once the technology is made available to the fringe, it is freely available to firm 2 as well. This implies that making the technology available to the fringe *and* asking a licensing payment from firm 2 is not a viable option. For this reason, firm 1 has three distinct options of transferring the new technology:

(i) No licensing: In this case the marginal cost of the fringe stays $\overline{c}$ and the price of good 0 is $p_0 = \overline{c}$. The marginal cost of firm 2 is also $c_2 = \overline{c}$.

(ii) Inclusive free licensing: When the technology is given for free to all firms (firms in the fringe as well as firm 2), we call it *inclusive free licensing*. In this case the marginal cost of the fringe is $\underline{c}$, so $p_0 = \underline{c}$. The marginal cost of firm 2 is $c_2 = \underline{c}$.

(iii) Licensing the technology exclusively to firm 2 using unit royalty $r \geq 0$: In this case the marginal cost of the fringe stays $\overline{c}$, so $p_0 = \overline{c}$. Since firm 2 has to pay r for every unit it produces, the effective marginal cost of firm 2 is $c_2 = \underline{c} + r$.

(iv) Exclusive free licensing: The particular case of licensing to firm 2 using royalty $r = 0$ corresponds to *exclusive free licensing* of the technology. This is the case where the technology is given for free only to firm 2 and not to the firms in the fringe. In this case the marginal cost of the fringe stays $\overline{c}$, so $p_0 = \overline{c}$ and $c_2 = \underline{c}$.

Income of the consumer: As before the income Y of the consumer is the sum of two components: (i) an exogenously given minimum income, denoted by $\hat{Y} > 0$ and (ii) a fraction t of the total profit Π of the economy, where $0 \leq t < 1$. Thus

$$Y = \hat{Y} + t\Pi \tag{22}$$

As discussed before, regardless of the nature of technology transfer, firms in the competitive fringe always make zero profit. So the total profit Π is the sum of the profits of two firms $1, 2$.

The representative consumer's income is always at least $\hat{Y}$. Since p_0 is either $\overline{c}$ or $\underline{c}$ and $\underline{c} < \overline{c}$, to buy $\underline{x}_0$ units of the basic good, the consumer needs at most $\overline{c}\underline{x}_0$.

Assumption 2 $\hat{Y} > \overline{c}\underline{x}_0$.

The assumption above ensures that the consumer always has sufficient income to buy $\underline{x}_0$ units of the basic good, so the solution to the utility maximization problem is given by (20)-(21), where $p_0 = \bar{c}$ if the fringe does not have the new technology and $p_0 = \underline{c}$ if it does.

3.1 No licensing versus inclusive free licensing

As before, it will be useful to consider the general situation in which the fringe has marginal cost c_0 (so that $p_0 = c_0$), firm 1 has marginal cost c_1 and firm 2 has marginal cost c_2, where $c_1, c_2 \in [\underline{p}, \bar{p}]$. Under no licensing, $p_0 = \bar{c}$, $c_1 = \underline{c}$, $c_2 = \bar{c}$ and under inclusive free licensing, $p_0 = \underline{c}$, $c_1 = c_2 = \underline{c}$.

When the fringe has marginal cost c_0, the net income of the representative consumer after spending for $\underline{x}_0$ units of good 0 is $Y - c_0\underline{x}_0$ (since $c_0 \leq \bar{c}$ and $Y \geq \hat{Y}$, by Assumption 2, this net income is positive).

Taking $p_0 = c_0$ in (20)-(21), note that if firm 1 sets price p_1, firm 2 sets price p_2 and the income of the consumer is Y, the consumer demands $x_i^*(p_1, p_2, Y - c_0\underline{x}_0)$ units of good $i = 1, 2$ (where x_i^* is given by (3)). Taking $y = Y - c_0\underline{x}_0$ in (5), the profits of firms $1, 2$ are

$$\pi_1(p_1, p_2, c_1, Y - c_0\underline{x}_0) = (p_1 - c_1)(Y - c_0\underline{x}_0)\alpha^\sigma / p_1^\sigma g(p_1, p_2) \text{ and}$$

$$\pi_2(p_1, p_2, c_2, Y - c_0\underline{x}_0) = (p_2 - c_2)(Y - c_0\underline{x}_0)(1-\alpha)^\sigma / p_2^\sigma g(p_1, p_2) \tag{23}$$

Determining equilibrium income at p_1, p_2: As before, the representative consumer receives fraction t of the profit of each firm. So the income of the representative consumer is $Y = \hat{Y} + t\pi_1 + t\pi_2$, which implies

$$Y - c_0\underline{x}_0 = \hat{Y} - c_0\underline{x}_0 + t\pi_1(p_1, p_2, c_1, Y - c_0\underline{x}_0) + t\pi_2(p_1, p_2, c_2, Y - c_0\underline{x}_0)$$

Using (23) in the equation above, we have

$$Y - c_0\underline{x}_0 = \hat{Y} - c_0\underline{x}_0 + t(p_1 - c_1)(Y - c_0\underline{x}_0)\alpha^\sigma / p_1^\sigma g(p_1, p_2)$$
$$+ t(p_2 - \bar{c})(Y - c_0\underline{x}_0)(1-\alpha)^\sigma / p_2^\sigma g(p_1, p_2) \tag{24}$$

Observe that equation (24) is the same as equation (6) with $y = Y - c_0\underline{x}_0$ and $\hat{y} = \hat{Y} - c_0\underline{x}_0$. Using (7) (the unique solution of (6)), we conclude that the unique solution of (24) satisfies

$$Y - c_0\underline{x}_0 = (\hat{Y} - c_0\underline{x}_0)p_1^\sigma p_2^\sigma g(p_1, p_2)/h^t(p_1, p_2, c_1, c_2) \tag{25}$$

where the function h^t is given in (8).

Determining Nash Equilibrium of the duopoly: Using the solution (25) in (23), the profit functions at prices p_1, p_2 under marginal costs c_1, c_2 are

$$\Pi_1^t(p_1, p_2, c_1, c_2, \hat{Y} - c_0\underline{x}_0) = (p_1 - c_1)(\hat{Y} - c_0\underline{x}_0)\alpha^\sigma p_2^\sigma / h^t(p_1, p_2, c_1, c_2),$$

$$\Pi_2^t(p_1, p_2, c_1, c_2, \hat{Y} - c_0\underline{x}_0) = (p_2 - c_2)(\hat{Y} - c_0\underline{x}_0)(1-\alpha)^\sigma p_1^\sigma / h^t(p_1, p_2, c_1, c_2) \tag{26}$$

Since the consumer gets fraction t of profit of each firm, for $i = 1, 2$, the net payoff of firm i is $(1-t)\Pi_i^t$. Since $0 \leq t < 1$, firms $1, 2$ interact in a standard duopoly of price competition where firm i seeks to maximize its profit Π_i^t given in (26). Denote this duopoly by $\mathbb{D}^t(c_1, c_2, \hat{Y} - c_0\underline{x}_0)$. Noting that the profit functions in (26) are same as in (9) with $\hat{y} = \hat{Y} - c_0\underline{x}_0$, the following lemma is immediate from Lemma 1.

Lemma 3 *Suppose firms* $1, 2$ *have constant marginal costs* $c_1, c_2 \in (\underline{p}, \overline{p})$ *and either* $0 < \sigma < 1$ *or* $1 < \sigma < \overline{p}/(\overline{p} - c_i)$ *for* $i = 1, 2$. *The unique NE of* $\mathbb{D}^t(c_1, c_2, \hat{Y} - c_0\underline{x}_0)$ *is* $(p_1 = \overline{p}, p_2 = \overline{p})$. *At the NE, for* $i = 1, 2$, *firm* i *obtains profit* $\Pi_i^t(\overline{p}, \overline{p}, c_1, c_2, \hat{Y} - c_0\underline{x}_0)$ *and net payoff* $(1-t)\Pi_i^t(\overline{p}, \overline{p}, c_1, c_2, \hat{Y} - c_0\underline{x}_0)$.

As in Lemma 1, Lemma 3 shows that when the two goods produced by two firms $1, 2$ are either poor substitutes ($0 < \sigma < 1$) or not sufficiently good substitutes ($\sigma > 1$, but bounded above by $\overline{p}/(\overline{p} - c_i)$ for $i = 1, 2$), then the unique equilibrium outcome of the duopoly interaction has both firms setting the maximum permissible price $\overline{p}$. Using Lemma 3, we can compare no licensing and free licensing.

Proposition 3 *Suppose either* $0 < \sigma < 1$ *or* $1 < \sigma < \overline{p}/(\overline{p} - \underline{c})$.

(i) *For any* $0 \leq t < 1$, *inclusive free licensing (that is, the new technology is given for free to firm* 2 *as well as all firms in the fringe) is superior for firm* 1 *compared to no licensing and receiving the license for free is superior for firm* 2 *compared to no licensing.*

(ii) *For any* $0 < t < 1$, *the representative consumer has higher income and higher utility under inclusive free licensing compared to no licensing. For* $t = 0$, *the consumer has the same income and same utility under inclusive free licensing and no licensing.*

Proof See the Appendix. ■

Proposition 3 shows that conclusions of Proposition 1 continues to hold in the extended model in that for firm 1, licensing the technology for free to all firms is superior to not licensing.

3.2 Licensing exclusively to firm 2 by per unit royalty

Now consider the policy of licensing the new technology exclusively to firm 2 with unit royalty $r \geq 0$ (in particular, $r = 0$ means the technology is given for free exclusively to firm 2). The effective marginal cost of firm 2 under this policy is $\underline{c} + r$. Since the maximum price that can be set is $\overline{p}$ and firm 2 obtains positive profit without a license, it will not accept a royalty r with $\underline{c} + r \geq \overline{p}$, so as before we restrict $r < \overline{p} - \underline{c}$.

In this case the competitive fringe producing good 0 does not have the new technology, so $p_0 = \overline{c}$. Therefore the net income of the representative consumer after spending for $\underline{x}_0$ units of good 0 is $Y - \overline{c}\underline{x}_0$ (since $Y \geq \hat{Y}$, by Assumption 2, this net income is positive).

Taking $p_0 = \overline{c}$ in (20)-(21), if firm 1 sets price p_1, firm 2 sets price p_2 and the income of the representative consumer is y, the consumer demands $x_i^*(p_1, p_2, Y - \overline{c}\underline{x}_0)$ units of good $i = 1, 2$ (where x_i^* is given by (3)). Taking $y = Y - \overline{c}\underline{x}_0$ in (10), the profits of firms $1, 2$ are

$$\phi_1(p_1, p_2, \underline{c}, Y - \overline{c}\underline{x}_0, r) = (p_1 - \underline{c})x_1^*(p_1, p_2, Y - \overline{c}\underline{x}_0) + rx_2^*(p_1, p_2, Y - \overline{c}\underline{x}_0) \text{ and}$$

$$\phi_2(p_1, p_2, \underline{c}, Y - \overline{c}\underline{x}_0, r) = [p_2 - (\underline{c} + r)]x_2^*(p_1, p_2, Y - \overline{c}\underline{x}_0) \tag{27}$$

Note that the profit of firm 1 is the sum of its operating profit and royalty revenue rx_2^*. The *effective* marginal cost of firm 2 is $\underline{c} + r$, so for firm 2, the royalty payments rx_2^* are included as part of its cost in its profit function.

Determining equilibrium income at p_1, p_2: By (22), the income of the representative consumer is $Y = \hat{Y} + t\phi_1 + t\phi_2$, which implies $Y - \overline{c}\underline{x}_0 = \hat{Y} - \overline{c}\underline{x}_0 + t\phi_1 + t\phi_2$ and by (27) we have

$$Y - \overline{c}\underline{x}_0 = \hat{Y} - \overline{c}\underline{x}_0 + t(p_1 - \underline{c})x_1^*(p_1, p_2, Y - \overline{c}\underline{x}_0) + t(p_2 - \underline{c})x_2^*(p_1, p_2, Y - \overline{c}\underline{x}_0)$$

Taking $y = Y - \overline{c}\underline{x}_0$ in the expression of x_i^* in (3) and using it in the equation above, we have

$$Y - \overline{c}\underline{x}_0 = \hat{Y} - \overline{c}\underline{x}_0 + t(p_1 - \underline{c})(Y - \overline{c}\underline{x}_0)\alpha^\sigma / p_1^\sigma g(p_1, p_2) + t(p_2 - \underline{c})(1 - \alpha)^\sigma / p_2^\sigma g(p_1, p_2) \tag{28}$$

Observe that equation (28) is the same as (11) with with $y = Y - \overline{c}\underline{x}_0$ and $\hat{y} = \hat{Y} - \overline{c}\underline{x}_0$. Using (12) (the unique solution of (11)), we conclude that the unique solution of (28) satisfies

$$Y - \overline{c}\underline{x}_0 = (\hat{Y} - \overline{c}\underline{x}_0)p_1^\sigma p_2^\sigma g(p_1, p_2)/h^t(p_1, p_2, \underline{c}, \underline{c}) \tag{29}$$

where the function h^t is given in (8).

Determining Nash Equilibrium of the duopoly: The solution (29) is the same as (12) with $\hat{y} = \hat{Y} - \overline{c}\underline{x}_0$. So the demands at income (29) are the same as (13) with $\hat{y} = \hat{Y} - \overline{c}\underline{x}_0$ and the profit functions at prices p_1, p_2 under unit royalty r are given by (14)-(15) with $\hat{y} = \hat{Y} - \overline{c}\underline{x}_0$.

The net payoff of each firm is fraction $(1 - t)$ of its profit. Since $0 \leq t < 1$, each firm seeks to maximize its profit. Denote by $\mathbb{D}^t(\underline{c}, \underline{c} + r, \hat{Y} - \overline{c}\underline{x}_0)$ the two-person game in which firms $1, 2$ choose prices $p_1, p_2 \in [\underline{p}, \overline{p}]$ and the payoffs are given in (14)-(15) with $\hat{y} = \hat{Y} - \overline{c}\underline{x}_0$.

Taking $\hat{y} = \hat{Y} - \overline{c}\underline{x}_0$ in Lemma 2, if either $0 < \sigma < 1$ or $1 < \sigma < \overline{p}/(\overline{p} - \underline{c})$, then the game $\mathbb{D}^t(\underline{c}, \underline{c} + r, \hat{Y} - \overline{c}\underline{x}_0)$ has a unique NE: $(p_1 = \overline{p}, p_2 = \overline{p})$. At the NE, for $i = 1, 2$, firm i obtains net payoff $(1 - t)\Phi_i^t(\overline{p}, \overline{p}, \underline{c}, r, \hat{Y} - \overline{c}\underline{x}_0)$ (where $\Phi_i^t(p_1, p_2, \underline{c}, r, \hat{y})$ is given in (14)-(15)).

Determining optimal royalty for firm 1: Under no licensing firm 2 obtains net payoff $(1 - t)\Pi_2^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}, \hat{Y} - \overline{c}\underline{x}_0)$. Noting that

$$\Phi_i^t(\overline{p}, \overline{p}, \underline{c}, r, \hat{Y} - \overline{c}\underline{x}_0) \geq \Pi_2^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}, \hat{Y} - \overline{c}\underline{x}_0) \Leftrightarrow \Phi_i^t(\overline{p}, \overline{p}, \underline{c}, r, \hat{y}) \geq \Pi_2^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}, \hat{y}),$$

from (17) we conclude that the unique optimal unit royalty for firm 1 is to set $r = r^t(\overline{p}, \underline{c}, \overline{c})$ given in (17) that makes firm 2 indifferent between having a license or not.

Proposition 4 *Suppose either* $0 < \sigma < 1$ *or* $1 < \sigma < \overline{p}/(\overline{p} - \underline{c})$*. The following hold for any* $0 \leq t < 1$.

(i) *The unique optimal per unit royalty policy for firm* 1 *is to set royalty* $r^t(\overline{p}, \underline{c}, \overline{c})$ *given in* (17). *For firm* 1, *this policy is superior to no licensing as well as exclusive free licensing.*

(ii) *There exists a threshold* $\kappa^t(\overline{p}, \underline{c}, \overline{c}, \underline{x}_0) \equiv \alpha^\sigma(\overline{p} - \underline{c})(\overline{c} - \underline{c})\underline{x}_0/(1-\alpha)^\sigma r^t(\overline{p}, \underline{c}, \overline{c})$, *which is decreasing in* t, *such that for firm* 1, *inclusive free licensing is superior to royalty licensing if* $\overline{c}\underline{x}_0 < \hat{Y} < \overline{c}\underline{x}_0 + \kappa^t(\overline{p}, \underline{c}, \overline{c}, \underline{x}_0)$ *and royalty licensing is superior if* $\hat{Y} > \overline{c}\underline{x}_0 + \kappa^t(\overline{p}, \underline{c}, \overline{c}, \underline{x}_0)$.

(iii) *In particular, for* $t = 0$ (*that is, when the consumer's income consists only of* $\hat{Y}$ *and does not depend on profits at all*), *inclusive free licensing is superior to royalty licensing for firm* 1 *if* $\overline{c}\underline{x}_0 < \hat{Y} < \overline{c}\underline{x}_0 + \kappa^0(\overline{p}, \underline{c}, \overline{c}, \underline{x}_0)$ *and royalty licensing is superior if* $\hat{Y} > \overline{c}\underline{x}_0 + \kappa^0(\overline{p}, \underline{c}, \overline{c}, \underline{x}_0)$, *where* $\kappa^0(\overline{p}, \underline{c}, \overline{c}, \underline{x}_0) \equiv \alpha^\sigma(\overline{p} - \underline{c})\underline{x}_0/(1-\alpha)^\sigma$.

(iv) *The utility of the representative consumer is highest at inclusive free licensing. Specifically, inclusive free licensing gives higher utility than royalty licensing, exclusive free licensing and no licensing.*

Proof See the Appendix. ■

It was shown in Propositions 1,2 that although free licensing is superior to no licensing for firm 1, free licensing is no longer optimal when it can charge unit royalties from firm 2. By contrast, Proposition 4 shows that in the extended model of hierarchial demand, inclusive free licensing (that is, giving the technology for free to firm 2 as well as to all firms in the fringe) can be the optimal choice even when firm 1 can license exclusively to firm 2 by using royalties. This happens provided $\hat{Y}$ (the exogenous part of the income of the representative consumer) is relatively small.

The intuitive explanation of this result is as follows. Firm 1 can earn a royalty revenue by licensing exclusively to firm 2, but in that case the fringe continues to operate under the high cost and the consumer has to pay a high price for the basic good 0. The policy of inclusive free licensing ensures that firm 2 as well as the firms in the fringe operate under the low cost. In this case the consumer has to pay a lower price for the basic good and has a higher net income available to spend for goods $1, 2$. This raises the demand and profit of firm 1.

The optimality of inclusive free licensing is determined by two effects: (i) the effect of higher net income due to lower expenditure for the basic good and (ii) the general equilibrium effect of higher profits. These effects compensate for the loss of revenue in royalties when $\hat{Y}$ (the exogenous part of the income of the representative consumer) is relatively small. Part (iii) of Proposition 4 shows that even when $t = 0$ (so that there is no general equilibrium effect on the consumer's income), the net income effect on its own can result in the optimality of inclusive free licensing. This implies inclusive free licensing can be optimal even when firm 1 is not large relative to the whole economy.

A larger value of t (the fraction of profits that contribute to the consumer's income) better mitigates the loss in the consumer's net income arising from the high price of the basic good. For this reason, with a larger value of t, optimality of inclusive free licensing requires the exogenous component $\hat{Y}$ to be lower. A lower $\hat{Y}$ ensures that the high price of the basic good that results from the exclusion of the fringe significantly affects the net income of the consumer. This explains why the threshold κ^t of Proposition 4 is decreasing in t.

4 Concluding remarks

Although free licensing of patents is widely observed, the issue of optimality of this practice has remained unexplored in the theoretical literature of patent licensing. In this paper we consider a simple general equilibrium model to see if free licensing can be the optimal choice for a patent holder. In a hierarchial demand model (Matsuyama, 2002) with one basic good that is produced by a competitive fringe and two non-basic goods with constant elasticity of substitution that are produced by two distinct firms, we show that free licensing to all firms can be the optimal choice for the patentee firm even when it can charge royalties by exclusive licensing. The demand for the basic good is completely inelastic in the hierarchial demand model, which helps to present our results in particular clarirty. With some added complications in the analysis, our main conclusions will continue to hold when the demand for the basic good is not completely inelastic. The key intuition of our results is also likely to hold in more general scenarios such as multiple basic and non-basic goods.

In this paper we have approached free licensing in the framework of optimal policy choice of the patent holder. Alternatively one can approach free licensing in a public policy framework. Suppose the sector that produces good 1 in our model is a large "innovating sector" of the economy and the policymaker (e.g., the government) decides on the patent protection policy with the objective of maximizing the utility of the representative consumer. In view of the conclusion of Proposition 4(iv) that the consumer has the highest utility under inclusive free licensing, it seems plausible that free patents, or at least a weak patent protection policy may be optimal for the policymaker.

Appendix

Proof of Lemma 1 By (9) we have

$$\partial\Pi_1^t/\partial p_1 = [\hat{y}\alpha^\sigma p_2^\sigma/(h^t)^2][h^t - (p_1 - c_1)(\partial h^t/\partial p_1)],$$

$$\partial\Pi_2^t/\partial p_2 = [\hat{y}(1-\alpha)^\sigma p_1^\sigma/(h^t)^2][h^t - (p_2 - c_2)(\partial h^t/\partial p_2)] \tag{30}$$

From (8),

$$\partial h^t/\partial p_1 = (1-t)\alpha^\sigma p_2^\sigma + \sigma(1-\alpha)^\sigma p_1^{\sigma-1}[tc_2 + (1-t)p_2]$$
$$\partial h^t/\partial p_2 = (1-t)(1-\alpha)^\sigma p_1^\sigma + \sigma\alpha^\sigma p_2^{\sigma-1}[tc_1 + (1-t)p_1] \tag{31}$$

Using (8) and (31) in (30) we have

$$\partial\Pi_1/\partial p_1 = [\hat{y}\alpha^\sigma p_2^\sigma/(h^t)^2][\alpha^\sigma p_2^\sigma c_1 + (1-\alpha)^\sigma p_1^{\sigma-1}\{tc_2 + (1-t)p_2\}\{\sigma c_1 + (1-\sigma)p_1\}]$$

$$\partial\Pi_2/\partial p_2 = [\hat{y}p_1^\sigma/(h^t)^2][(1-\alpha)^\sigma p_1^\sigma c_2 + \alpha^\sigma p_2^{\sigma-1}\{tc_1 + (1-t)p_1\}\{\sigma c_2 + (1-\sigma)p_2\}] \quad (32)$$

Observe that if $0 < \sigma < 1$, then both expressions of (32) are positive for any positive p_1, p_2. Next consider $\sigma > 1$. If $1 < \sigma < \overline{p}/(\overline{p} - c_1)$, then $\sigma c_1 > (\sigma - 1)\overline{p}$. Hence for all $p_1 \leq \overline{p}$, we have $\sigma c_1 > (\sigma - 1)p_1$, so that $\sigma c_1 + (1-\sigma)p_1 > 0$. Therefore for any positive p_2, $\partial\Pi_1^t/\partial p_1 > 0$ for all $p_1 \in [\underline{p}, \overline{p}]$. Similarly if $1 < \sigma < \overline{p}/(\overline{p} - c_2)$, then for any positive p_1, $\partial\Pi_2^t/\partial p_2 > 0$ for all $p_2 \in [\underline{p}, \overline{p}]$. This shows that for any positive p_j, Π_i^t is maximum at $p_i = \overline{p}$, which proves the result. ■

Proof of Proposition 1 We are given either $0 < \sigma < 1$ or $1 < \sigma < \overline{p}/(\overline{p} - \underline{c})$. Under no licensing, $c_1 = \underline{c}$, $c_2 = \overline{c}$ and under free licensing, $c_1 = c_2 = \underline{c}$. Noting that $\overline{p}/(\overline{p} - \underline{c}) < \overline{p}/(\overline{p} - \overline{c})$, under both no licensing and free licensing we have either $0 < \sigma < 1$ or $1 < \sigma < \overline{p}/(\overline{p} - c_i)$ for $i = 1, 2$, so we can apply the results of Lemma 1. Thus under both no licensing and free licensing, the unique NE of the duopoly is $(p_1 = \overline{p}, p_2 = \overline{p})$.

(i) By Lemma 1 and (9), firm 1 has profit $\Pi_1^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}, \hat{y}) = (\overline{p} - \underline{c})\hat{y}\alpha^\sigma\overline{p}^\sigma/h^t(\overline{p}, \overline{p}, \underline{c}, \overline{c})$ under no licensing and $\Pi_1^t(\overline{p}, \overline{p}, \underline{c}, \underline{c}, \hat{y}) = (\overline{p} - \underline{c})\hat{y}\alpha^\sigma\overline{p}^\sigma/h^t(\overline{p}, \overline{p}, \underline{c}, \underline{c})$ under free licensing.

By (8), for $0 < t < 1$, h^t is increasing in c_2, so $h^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}) > h^t(\overline{p}, \overline{p}, \underline{c}, \underline{c})$ and for $t = 0$, $h^0(\overline{p}, \overline{p}, \underline{c}, \overline{c}) = h^0(\overline{p}, \overline{p}, \underline{c}, \underline{c}) = \overline{p}^{2\sigma}g(\overline{p}, \overline{p})$. The results of (i) follows by using these properties in the profit function of firm 1.

(ii) Firm 2 has profit $\Pi_2^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}, \hat{y}) = (\overline{p} - \overline{c})\hat{y}(1-\alpha)^\sigma\overline{p}^\sigma/h^t(\overline{p}, \overline{p}, \underline{c}, \overline{c})$ under no licensing and $\Pi_2^t(\overline{p}, \overline{p}, \underline{c}, \underline{c}, \hat{y}) = (\overline{p} - \underline{c})\hat{y}(1-\alpha)^\sigma\overline{p}^\sigma/h^t(\overline{p}, \overline{p}, \underline{c}, \underline{c})$ under free licensing. Since $\overline{p} - \underline{c} > \overline{p} - \overline{c}$, the results of (ii) follow by using the properties of h^t in the profit function of 2.

(iii) For $t = 0$, the representative consumer has the same income $\hat{y}$ under both no licensing and free licensing. Since in both cases the price of each good is $\overline{p}$, by (4), the quantity consumed for each good stays the same and so the consumer gets the same utility under no licensing and free licensing.

Let $0 < t < 1$. The consumer has income $y^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}, \hat{y}) = \hat{y}\overline{p}^{2\sigma}g(\overline{p}, \overline{p})/h^t(\overline{p}, \overline{p}, \underline{c}, \overline{c})$ under no licensing and $y^t(\overline{p}, \overline{p}, \underline{c}, \underline{c}, \hat{y}) = \hat{y}\overline{p}^{2\sigma}g(\overline{p}, \overline{p})/h^t(\overline{p}, \overline{p}, \underline{c}, \underline{c})$ under free licensing (by (6)). Again using the properties of h^t, the income is higher under free licensing. In both cases the price of each good is $\overline{p}$. So by (4), the quantity consumed for each good is higher under free licensing and by (1), the consumer gets higher utility under free licensing. ■

Proof of Proposition 2 The first statement of (i) follows by noting that the royalty $r = r^t(\overline{p}, \underline{c}, \overline{c})$ given in (17) is positive and $r = 0$ corresponds to free licensing. The last statement of (i) follows from Proposition 1(i). Part (ii) is immediate from (17) and Proposition 1(ii). The first statement of part (iii) follows by noting that under any royalty policy r, the price of each good is $\overline{p}$ and the consumer has the same income $y^t(\hat{y}, \overline{p}, \overline{p}, \underline{c}, \underline{c})$. So prices, income and utility under any royalty policy are the same as in the case of free licensing ($r = 0$). The last two statements follow by Proposition 1(iii). ■

Proof of Proposition 3 We are given either $0 < \sigma < 1$ or $1 < \sigma < \overline{p}/(\overline{p} - \underline{c})$. Under no licensing, $c_0 = \overline{c}$, $c_1 = \underline{c}$, $c_2 = \overline{c}$ and under free licensing, $c_0 = c_1 = c_2 = \underline{c}$. In both cases, either $0 < \sigma < 1$ or $1 < \sigma < \overline{p}/(\overline{p} - c_i)$ for $i = 1, 2$, so we can apply Lemma

3. Thus under both no licensing and free licensing, the unique NE of the duopoly is $(p_1 = \overline{p}, p_2 = \overline{p})$. By Lemma 3, for $i = 1, 2$, firm i obtains profit $\Pi_i^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}, \hat{Y} - \overline{c}\underline{x}_0)$ under no licensing and $\Pi_i^t(\overline{p}, \overline{p}, \underline{c}, \underline{c}, \hat{Y} - \underline{c}\,\underline{x}_0)$ under free licensing.

(i) First let $0 < t < 1$. By (8), h^t is increasing in c_2. Using this in (26), for $i = 1, 2$, $\Pi_i^t(p_1, p_2, c_1, c_2, \hat{Y} - c_0\underline{x}_0)$ is decreasing in both c_0, c_2. This shows that each of firms $1, 2$ obtains a higher profit under free licensing compared to no licensing.

For $t = 0$, by (8), h^0 does not depend on c_2. So by (26), $\Pi_1^0(p_1, p_2, c_1, c_2, \hat{Y} - c_0\underline{x}_0)$ is decreasing in c_0 and unaffected by c_2 and $\Pi_2^0(p_1, p_2, c_1, c_2, \hat{Y} - c_0\underline{x}_0)$ is decreasing in both c_0, c_2. This shows that each of firms $1, 2$ obtains a higher profit under free licensing compared to no licensing for $t = 0$ as well.

(iii) First let $0 < t < 1$. Since both firms $1, 2$ obtain a higher profit under free licensing compared to no licensing, by (22), the consumer has a higher income under free licensing compared to no licensing. In both cases, the consumer buys $\underline{x}_0$ units of the basic good 0. Under free licensing, the price of good 0 is lower ($\underline{c}$ compared to $\overline{c}$) and so the consumer has a higher net income to buy goods $1, 2$. Since price of each of the goods $1, 2$ are $\overline{p}$ in both cases, by (20), the consumer purchases a higher amount of each of these goods under free licensing, which proves the the utility is also higher under free licensing.

For $t = 0$, the consumer's income is $\hat{Y}$ under both free licensing and no licensing, but as before under free licensing the consumer has to spend less to buy good 0, so it has a higher net income for purchasing goods $1, 2$ and obtains a higher utility under free licensing. ■

Proof of Proposition 4 Since a firm gets the same positive fraction $(1 - t)$ of its profit at any policy, for comparing different policies for any firm $i = 1, 2$ we compare the profits of the firm at those policies.

(i) Because setting $r = 0$ (exclusive free licensing to firm 2) is a feasible royalty policy and the unique optimal royalty is to set $r > 0$, the optimal royalty policy is superior to exclusive free licensing to firm 2.

Taking $\hat{y} = \hat{Y} - \overline{c}\underline{x}_0$, $p_1 = p_2 = \overline{p}$ and $r = r^t(\overline{p}, \underline{c}, \overline{c})$ in (14), under the optimal royalty policy, firm 1 obtains profit

$$\Phi_1^t(\overline{p}, \overline{p}, \underline{c}, r^t(\overline{p}, \underline{c}, \overline{c}), \hat{Y} - \overline{c}\underline{x}_0) = (\hat{Y} - \overline{c}\underline{x}_0)\overline{p}^{\sigma}[\alpha^{\sigma}(\overline{p} - \underline{c}) + (1 - \alpha)^{\sigma} r^t(\overline{p}, \underline{c}, \overline{c})]/h^t(\overline{p}, \overline{p}, \underline{c}, \underline{c})$$

Taking $\hat{y} = \hat{Y} - \overline{c}\underline{x}_0$, $p_1 = p_2 = \overline{p}$, $c_1 = \underline{c}$ and $c_0 = c_2 = \overline{c}$ in (26), under no licensing, firm 1 obtains profit

$$\Pi_1^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}, \hat{Y} - \overline{c}\underline{x}_0) = (\hat{Y} - \overline{c}\underline{x}_0)\overline{p}^{\sigma}\alpha^{\sigma}(\overline{p} - \underline{c})/h^t(\overline{p}, \overline{p}, \underline{c}, \overline{c})$$

Noting that $\alpha^{\sigma}(\overline{p} - \underline{c}) + (1 - \alpha)^{\sigma} r^t(\overline{p}, \underline{c}, \overline{c}) > \alpha^{\sigma}(\overline{p} - \underline{c}) > \alpha^{\sigma}(\overline{p} - \overline{c})$ and $h^t(\overline{p}, \overline{p}, \underline{c}, \overline{c}) \geq h^t(\overline{p}, \overline{p}, \underline{c}, \underline{c})$ it follows that firm 1 obtains a higher profit at royalty licensing compared to no licensing.

(ii) Taking $\hat{y} = \hat{Y} - \underline{c}\,\underline{x}_0$, $p_1 = p_2 = \overline{p}$ and $c_0 = c_1 = c_2 = \underline{c}$ in (26), under inclusive free licensing, firm 1 obtains profit

$$\Pi_1^t(\overline{p}, \overline{p}, \underline{c}, \underline{c}, \hat{Y} - \underline{c}\,\underline{x}_0) = (\hat{Y} - \underline{c}\,\underline{x}_0)\overline{p}^{\sigma}\alpha^{\sigma}(\overline{p} - \underline{c})/h^t(\overline{p}, \overline{p}, \underline{c}, \underline{c})$$

The result follows by noting that $\Phi_1^t(\overline{p},\overline{p},\underline{c},r^t(\overline{p},\underline{c},\overline{c}),\hat{Y}-\overline{c}\underline{x}_0) \gtreqless \Pi_1^t(\overline{p},\overline{p},\underline{c},\underline{c},\hat{Y}-\underline{c}\,\underline{x}_0) \Leftrightarrow \hat{Y} \gtreqless \overline{c}\underline{x}_0+\kappa^t(\overline{p},\underline{c},\overline{c},\underline{x}_0)$. Noting that r^t (given in (17)) is increasing in t, it follows that κ^t is decreasing in t.

(iii) This is immediate from (ii).

(iv) Note that under any royalty policy r, the price of each of the goods $1,2$ is $\overline{p}$, the price of good 0 is $\overline{c}$. Taking $p_1=p_2=\overline{p}$, $c_0=\overline{c}$ and $c_1=c_2=\underline{c}$ in (29), the income Y of the representative consumer under any royalty policy satisfies

$$Y-\overline{c}\underline{x}_0=(\hat{Y}-\overline{c}\underline{x}_0)\overline{p}^{2\sigma}g(\overline{p},\overline{p})/h^t(\overline{p},\overline{p},\underline{c},\underline{c}) \tag{33}$$

So prices, income and utility under any royalty policy are the same as in the case of exclusive free licensing (which corresponds to $r=0$).

Under inclusive free licensing, the price of each of the goods $1,2$ is $\overline{p}$ and the price of good 0 is $\underline{c}$. Taking $p_1=p_2=\overline{p}$, $c_0=\underline{c}$ and $c_1=c_2=\underline{c}$ in (29), the income Y of the representative consumer under inclusive free licensing

$$Y-\underline{c}\underline{x}_0=(\hat{Y}-\underline{c}\underline{x}_0)\overline{p}^{2\sigma}g(\overline{p},\overline{p})/h^t(\overline{p},\overline{p},\underline{c},\underline{c}) \tag{34}$$

By (33) and (34), the net income of the consumer to purchase goods $1,2$ after spending for good 0 is higher under inclusive free licensing. Because the consumer buys the same amount $\underline{x}_0$ of good 0 and prices of goods $1,2$ are $\overline{p}$ in both cases, the consumer purchases more of goods $1,2$ and has higher utility under inclusive free licensing.

Under no licensing, the price of each of the goods $1,2$ is $\overline{p}$ and the price of good 0 is $\overline{c}$. Taking $p_1=p_2=\overline{p}$, $c_0=c_2=\overline{c}$ and $c_1=\underline{c}$ in (29), the income Y of the representative consumer under no licensing satisfies

$$Y-\overline{c}\underline{x}_0=(\hat{Y}-\overline{c}\underline{x}_0)\overline{p}^{2\sigma}g(\overline{p},\overline{p})/h^t(\overline{p},\overline{p},\underline{c},\overline{c}) \tag{35}$$

For $0<t<1$, $h^t(\overline{p},\overline{p},\underline{c},\overline{c})>h^t(\overline{p},\overline{p},\underline{c},\underline{c})$, so by (33) and (35), the net income of the consumer to purchase goods $1,2$ after spending for good 0 is higher under royalty licensing compared to no licensing. Because the consumer buys the same amount $\underline{x}_0$ of good 0 and prices of goods $1,2$ are $\overline{p}$ in both cases, the consumer purchases more of goods $1,2$ and has higher utility under royalty licensing compared to no licensing. Therefore inclusive free licensing has a higher utility than no licensing.

For $t=0$, $h^0(\overline{p},\overline{p},\underline{c},\overline{c})=h^0(\overline{p},\overline{p},\underline{c},\underline{c})$. By (33) and (35), the net income of the consumer to purchase goods $1,2$ after spending for good 0 is the same and so the consumer has the same utility under royalty licensing and no licensing. Hence inclusive free licensing has a higher utility than no licensing. ■

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