Integrating a Free Online Service: Competition and Welfare

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Abstract

In digital markets, big technology firms like Google, Microsoft, Amazon, etc., have expanded across multiple products and services. Some of these online services differ from traditional services in the sense that they have a platform market structure, offered for free to users, and generate advertising revenue by placing advertisements in them. In this setting, using a game-theoretic model, we study firms' incentives to integrate an online service with the hardware product. We find that the difference in hardware product qualities/functionalities leads to different market outcomes as it helps firms strategically focus on a different component's user base to increase profits. It is shown that both firms adopt service integration (independent selling) for a large (small) difference in product qualities, whereas for an intermediate level of difference in product qualities, the firm with a better product adopts service integration and the rival firm adopts independent selling. Moreover, we also examine the implications of service integration for the user and social welfare. Importantly, an anti-competitive service integration can increase social welfare but at the cost of reduced user welfare.

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JEL classification: D21, D42, L12, L13, L42, L51

1 Introduction

Big technology firms have expanded into unrelated markets in the digital sphere, both through developing their own complementary services or through a series of mergers and acquisitions. For example, Motta and Peitz (2021) document the number of acquisitions by the "Big Five" over the last five years.[1] A notable feature of the resulting platform ecosystem is that these firms are selling both the hardware products and complementary online services taking competition to the system

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[1]42 by Amazon, 43 by Apple, 21 by Facebook, 48 by Google, and 53 by Microsoft.

level. With the rise of system competition between big technology firms, a prevalent business strategy is to adopt a tighter *integration* between the firm's service and hardware product. We call it *service integration*. Such integration results in an incompatibility between a firm's product and service with the rival firm's system. In other words, digital firms are embracing a "*walled gardens*" approach.

We provide a theory which shows that competing firms selling hardware products and free online services embrace service integration/incompatibility. Consider the smartphone market. In this market, the two main competitors are Apple and Huawei. They produce and sell both smartphones as well as own applications/services. For instance, Apple provide News service (Apple News) to access news on iPhone. Similarly, Huawei offers Petal Search for searching news, etc. in its ecosystem. However, neither service can be accessed on the rival's device, making the two firms as two incompatible systems. As another example, consider the market smart display market with two main competitors: Google Nest Hub, and Amazon Echo Show. Moreover, both owns voice assistant services to access the content through these smart display: Google Assistant and Amazon Alexa. At present, both Google and Amazon have integrated their smart displays with their proprietary voice assistants, making the two systems incompatible with each other. For accessing content and services on Google Nest, by default users are required to use Google Assistant and Amazon Alexa is incompatible with Nest Hub. Similarly, Amazon relies on Alexa to provide content and services on its Echo Show devices and Google Assistant is incompatible with it.

This service integration strategy can potentially raise anti-trust concerns as it may adversely affect the market competition. One possible way, as discussed in leverage theory of bundling (Whinston (1990)), is through deterring entry of the new services in the market or to making it unprofitable for existing services to compete in the market. Another possible way is to soften competition between the mutli-product firms and increase prices for the products.

The service integration in our paper is implicitly similar to a pure bundling/ incompatibility strategy, in which a firm's multiple products can only be consumed as a system. The earlier literature on competitive bundling examining competition between systems (e.g., Matutes and Regibeau (1988)) has shown that incompatibility tends to intensify price competition between firms, and higher prices is not a concern. However, few recent papers have advanced our understanding of competitive bundling by extending the symmetric duopoly two-dimensional Hotelling model of Matutes and Regibeau (1988) in different directions such as firm's dominance across products (Hurkens et al. (2019)), more than two firms (Zhou (2017)), oligopolistic market structure (Shuai et al. (2022)), etc. One of the central conclusion of these papers is that competitive bundling, by enforcing competition between systems, can raise price charged to the consumer for the system.

However, these papers do not consider the market structure illustrated in the earlier examples, i.e. *a market with a paid hardware product and a free online service with advertisements in it*.

These online services differ from traditional services (e.g., maintenance, repair, and training services) because they are free to use for the users however they help the firm to earn advertising revenue either directly through selling space for advertisements in it or indirectly enabling the firm to reach and monetize the users by directing them to its own other services. In the examples illustrated above, the online services fits the two-sided (platform) market structure that we have introduced. For instance, Apple News service allows users to freely read news but monetizes by selling advertising space in it to the advertisers. Likewise, Huawei Petal Search offers free news results to users but places advertisements along with its search results. In the smart display market, both Google Assistant and Amazon Alexa have started monetizing the search results based on users' queries either through placement of advertisements in the search results or in the smart display screen.[2] Moreover, these voice assistants also, by default, direct users to the proprietary services such as shopping comparison, music services, etc., which are monetized using advertisements in them. As a result, these voice assistants also indirectly enable the firms to earn additional advertising revenue. Thus, these services exhibit positive indirect network effects on the advertiser side. That is, the value of placing an advertisement to an advertiser increases with the number of users consuming the service.

This new market structure leads to a mismatch between the results from the existing literature and market reality. For instance, an extension in Hurkens et al. (2019) examining product integration when firm has a dominance only over a single product finds that integration is not a profitable strategy for same unit transportation costs across products.[3] However, as the anecdotal evidence mentioned above suggests that in certain technology-driven markets, their result does not hold true, and the point of our work is to show why the result breaks down. In particular, we introduce a specific market setting with one of the component (service) as an advertising financed platform. Understanding the effect of integrating a free online service (with advertisements in it) with a hardware product can provide new insights on platform strategy as well as concerns about rising concentration among dominant platforms.

This paper contributes to the theory of competitive bundling and incompatibility choice by addressing the following research questions: Under what market conditions service integration is a profitable strategy? How will this affect consumer and social welfare? To answer these questions, we develop a game-theoretic model examining competition between two firms with both having two components: a hardware product and a free online service, generating profits from both hardware sales to the users and advertising revenues from advertisements in the services. Moreover, one firm has a "quality advantage" for the product in the sense that consumers systematically value

[2] See, https://www.marketingdive.com/news/google-brings-ads-to-assistant-search-results/552206/ and https://voicebot.ai/2022/05/02/amazon-uses-alexa-to-target-ads-study/

[3] Refer the model in section IV of Hurkens et al. (2019).

more highly the vertical dimension of the firm's hardware product. Both firms make service integration decisions first and then set their hardware prices and advertising quantities, and finally advertisers make adoption decisions and consumers purchase hardware product and service. If either firm adopts service integration then competition takes place between two incompatible systems, each having a product and a service.

We find that the incentive to adopt service integration depends on its effect on each firm's market share for the product and service and also the resulting price competition. We find that service integration helps firms to strategically focus on a different component's user base. For firm with better product, market share for the service becomes more important but less important for the rival firm. First, consider the case when the level of quality advantage is small. Then, service integration is not profitable for either firm, because neither firm gains sufficient market share but price competition intensifies between the firms, resulting in lower profits. Whereas, when quality advantage is sufficiently large, then both firms adopt service integration as they gain sufficient market shares and price competition also softens because of increase in market power, raising prices charged by both firms and benefits both firms. However, for an intermediate level of quality advantage, integration strategy has an asymmetric effect on the profit of two firms. The firm with better quality product adopts service integration because it gains sufficient users for its service, compensating for any profit loss from fewer product sales. Whereas the rival firm does not adopt service integration because its loss of user base in the service dominates any increase in its hardware sales, lowering its profits.

Our analysis provides insights to some of the dynamics that we can observe in the digital market. Our results shows that when the difference in product qualities is sufficiently large, then both firms benefit from service integration. In the smartphone market, Apple phone provides better functionalities due to its very efficient integration with other Apple devices and applications, large number of popular apps, etc., relative to Huawei phone. As a result, there is a significant difference in the functionalities of the two smartphones, giving Apple a quality advantage. Our model predicts that in this scenario, both smartphones would be better off keeping their systems incompatible with each other. A tighter integration between Apple and Apple News helps Apple to strategically expand its news user base for monetizing the smartphone sales. Similarly, keeping Huawei phone and Petal Search better integrated helps it to focus on improving its device sales through improving its smartphone user base. Likewise, in the smart displays market, Google Nest Hub provides more features/functionalities, due to its integration with the Google ecosystem, relative to Amazon Echo Show, giving Google Nest Hub a quality advantage. Embracing service integration enables Google to strategically focus on expanding Google Assistant's user base for monetizing its smart display sales. Similarly, keeping Echo Show and Alexa better integrated helps Amazon to improve its profits by expanding Echo show user base.

Another interesting implication of our results is that it helps explain when leveraging of market power by a firm with better product quality is profitable. We find that it is profitable for an intermediate level of quality advantage. This result is contrary to the one present in e.g., Hurkens et al. (2019) in which leveraging market power through integration always reduces both firms' profit when the per unit transportation costs are same across products. By introducing a specific platform market structure, i.e., a purely advertising financed platform for the online services, we have demonstrated that the profitability of leveraging of dominance cannot be inferred from the previous literature.

A final contribution of our paper is to show the social optimality of service integration. However, the answer to this question depends on the welfare standard considered. Our main result is that service integration could be socially optimal but users would suffer. This is most likely with an intermediate to large difference in the product qualities of the two firms. Thus, depending on whether an antitrust authority considers user or social welfare standard, the intervention would be different and raises a dilemma for antitrust policy.

We then examine several extensions of the baseline model. We first examine an alternate payment system for advertisers under which they pay for each click on the advertisements in a service and show that the results under the baseline framework go through qualitatively. A similar conclusion is reached when we also introduce pricing for the service with one important caveat: integration always improves social welfare. Intuitively, with services that are both user and advertising financed, the advertising revenues remain unchanged under both independent pricing and integration. Thus, social welfare comparison would depend only on the net change in users' surplus, which increases with integration because more users consume the system with a larger standalone value. Next, we examine an extension where a single firm has a better quality product and service and find that our results go through qualitatively. Finally, we consider an extension with competition between a generalist firm and two specialist firms, one specialist selling the hardware product and other providing online service. We find that service integration is always profitable for generalist firm, but it can also increase specialist hardware firm's profit for sufficiently large difference in the product qualities, and always reduces profit of the specialist firm offering online service. Intuitively, service integration leads to an externality problem, where each specialist would choose its price/ advertising quantity ignoring the effect of its decision on the total demand and profit of the other firm. As a result, specialist selling hardware would set a higher price and specialist providing online service would choose a higher advertising level, reducing their market demand. This would benefit the generalist firm, and its profit will increase.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 discusses the model. Section 4 discusses the equilibrium outcomes. Section 5 conducts a welfare analysis. Section 6 discusses a few extensions to the baseline model. Section 7 discusses managerial implications and concludes. All proofs are in the appendix.

2 Related Literature

Our research is related to the literature on networks effects and platform competition, platform envelopment and competitive bundling.

Network Effects and Platform Competition

Our work contributes to literature on competition between firms in the presence of network effects. It can be further sub-divided into two streams. The first stream discusses how direct (positive) network effects affect platform competition. A crucial insight from this literature is that network effects intensify competition (see, e.g., Lee and Mendelson (2007), etc.). Our paper differs from this strand because we consider an online service with advertisements in it, generating cross-side (indirect) network effects on the advertiser side. A recent paper by Etzion and Pang (2014) considers competition between firms when they can offer a complementary service that exhibits positive network effects and examines the incentive of firms to offer this service. They find that symmetric network effects for the complementary services can lead to a prisoner's dilemma situation and both firms' profit would decrease if they offer the service. Our paper also considers competition when firms offer a complementary service. However, there are significant differences. First, unlike them, we look at advertising network benefits as the source of intensified price competition. Second, we endogenize the decision whether to integrate the complementary service with the product or not. Third, contrary to their result, we find that even with symmetric network effects (on the advertising side), service integration can make both firms' better off.

The second strand looks at the role of indirect network effects on firm competition. A few papers have dealt with the question of market fragmentation that can arise in a platform market either due to a difference in consumer tastes over which product is best suited to them (see, e.g., Armstrong (2006), Rochet and Tirole (2003), etc.) or even without a difference in consumer preferences (see, e.g., Ambrus and Argenziano (2009), Ambrus et al. (2016)). Our paper also shows that service integration results in market fragmentation and improves both firms' profit. However, our mechanism is different. Integration results in strategic differentiation with each firm focussing on a different component's user base to improve its profits, i.e. firm with better quality improves its market share for the service, whereas the rival firm gains market share for its product. Another key insight from this literature is the role of indirect network effects in shaping price competition. Armstrong (2006) finds that network effects intensify price competition on the one single-homing side when the other side is mutli-homing. Stealing business from a rival on the single-homing side reduces the ability of platforms to extract rent. The larger the indirect network effects the more intense is the price competition on the single-homing side. Our paper also highlights a similar price competition mechanism under service integration. It leads to a system with a single-homing user side and mutli-homing advertising side. As a result, competition between firms intensifies to attract more users for the advertisers, resulting in pass-through of advertising revenue to users in the form of lower prices. However, there are significant differences with this paper. Importantly, we explicitly consider two different components: product and a service, which allows us to consider a different set of research questions. In particular, we look at the incentive for both firms to adopt service integration and also, an incentive to leverage market power over a hardware product to an online service.

Platform Envelopment

The focus on leverage of market power through service integration makes our paper close to the literature on platform envelopment. Eisenmann et al. (2011) defines platform envelopment as a entry strategy where the entrant capitalizes on network effects in its original market to enter the network market dominated by an incumbent firm. It is done by combining product functionalities across markets and leveraging the shared user relationships. Our work complements theirs as we do not rely on network effects to explain the leverage of market power. Building on this, a few more theoretical papers have focussed on examining the leveraging of the existing user base to compete in a complementary market. Dou and Wu (2021) study "piggybacking", i.e. using the exclusive users from an external network to compete effectively in a platform market, and examine the interplay between piggybacking, price competition and network effects. Our work is distinct from theirs in many ways. For one, unlike Dou and Wu (2021), there is a single market, and the firm with better product quality can leverage its product user base. Moreover, the user adoption for both the product and service is an endogenous decision. Finally, we consider a specific platform structure for the online service, i.e. advertising financed services offered for free to the users.

Another focus of platform envelopment literature has been on understanding the strategic and welfare effects of bundling. Our work is linked and contributes to this stream of literature as well. The leverage theory of bundling has a well established intellectual history and many papers have studied bundling as an entry deterrence device (see, e.g., Whinston (1990), Choi and Stefanadis (2001), Carlton and Waldman (2002), Nalebuff (2004), Dewan and Freimer (2003), etc.). Our analysis differs from this literature since we consider leveraging of market power in a distinct market structure: a market with paid hardware products and free online services with advertisements in them, and examines its welfare implications. However, a few papers have also focussed on bundling in platform markets. Amelio and Jullien (2012) and Choi and Jeon (2016) considered models with platforms that are unable to charge negative prices. Choi (2010) studied tying in two-sided markets when each platform has some exclusive content to offer to consumers. Corniere and Taylor (2017) set up a slightly different model in which applications derive benefits for their developers, and developers can offer payments to the device manufacturers in exchange for being installed.

Our paper is closely related to Choi and Jeon (2016) who examine the leverage theory of tying in two-sided markets with non-negative price constraints. However, the two papers differ substantially. First, Choi and Jeon (2016) analyze leveraging of market power in a monopoly context (monopoly over the product), whereas we examine service integration when both firms offer product and service. Second, while Choi and Jeon (2016) put a non-negative restriction on the tied product price, we examine a scenario where the online service is offered for free and is purely advertising financed. Finally, there are significant differences in the results as well. Choi and Jeon (2016) highlight the presence of non-negative constraints as the mechanism that makes tying profitable. Whereas we identify different channels, i.e. the effect of change in market shares, demand elasticities, and competition for viewers to attract advertisers, on the profitability of service integration. Thus, our results are complementary and show that whether service integration is profitable or not depends on the underlying assumptions about the market structure.

Finally, Pang and Etzion (2012) consider leveraging market power form one market to another market with direct network effects and find that bundling is profitable with stronger network effects. Our model set up and results are significantly different. First, our focus is on competition between firms offering both product and service. Second, unlike Pang and Etzion (2012), we show that integration may not be profitable even with strong advertising network benefits.

Product Compatibility and Competitive Bundling

Our paper is also related to the literature on product compatibility and competitive bundling that allows for system competition between firms, and consumer demand systems made of complementary components. The earlier literature (see, for e.g., Matutes and Regibeau (1988), Economides (1989), etc.) shows an alignment between private and social incentives for compatibility. Both papers consider competition between firms producing system components and making their compatibility decisions prior to competition in prices. They show that the equilibrium entails full compatibility between component products of the two rival firms because compatibility leads to weaker internalization of the complementary between their products, raising prices relative to the incompatibility case. Moreover, compatibility increases social welfare due to the increased variety of systems available. Our paper differs, both on theoretical modelling and central results. They have examined a symmetric set up with neither firm having a better component. However, we have introduced asymmetry with one firm offering a better product quality. Moreover, in our set up, the online service is offered for free to users with advertisements in it. Using this market structure, we find that, contrary to the results in Matutes and Regibeau (1988), and Economides (1989), service integration can benefit both firms and the market incentive for integration may diverge from the social optimum. In particular, for an intermediate level of quality advantage over the product, there can be insufficient integration in the market. In this region, only firm with better product or neither firm opts for integration, whereas social optimality would require both firms to adopt service integration.

Few recent papers (see, for e.g., Zhou (2017), Kim and Choi (2015), Hurkens et al. (2019), Shuai et al. (2022), etc.) have extended the analysis of Matutes and Regibeau (1988) and Economides (1989) by examining competitive bundling under different modelling assumption. Hurkens et al. (2019) introduce firm asymmetry into Matutes and Regibeau (1988) with a single firm dominating all markets, and find that bundling is profitable for both firms for a sufficiently large level of dominance of the dominant firm as it leads to a change in selection of marginal consumer resulting in higher prices. Kim and Choi (2015) and Zhou (2017) both examine bundling in a symmetric oligopoly framework with n symmetric firms. They show that when the number of firms is large enough, bundling raises prices and thus benefits firms. Shuai et al. (2022) consider a market structure: an oligopoly market with a dominant firm and several small firms, and find that the dominant firm will bundle if and only if its dominance level is relatively high. We contribute to the theory of competitive bundling by examining a different market structure: a market with a paid hardware product and a free online service with advertisements in it, and provide an intuition for integration based on how it affects the market shares, demand elasticities and competition for viewers to attract advertisers. Moreover, we identify conditions under which a profitable leverage of market power over the product dimension into the service dimension is possible. Unlike Hurkens et al. (2019), we show that an anti-competitive leverage of market power is possible by the firm with better product quality even when users face same per-unit transportation costs for both product and service. A recent paper by Adner, Chen, and Zhu (2020) also consider compatibility decisions of two competing platform owners that generate profits through both hardware sales and royalties from content sales. They show that market outcome would entail one-way incompatibility which can increase both firms' profit because it helps each firm to strategically focus on a different profit base. Our paper differs from this study, in two main ways. First, our set up includes online services with advertisements in it. Second, as explained above, we provide different channels to understand the incompatibility decisions of two firms.

3 The Model

We consider a market with two competing firms, G and S selling a differentiated hardware product, G1 and S1, two differentiated online services G2 and S2 offered by each of the firm, a unit mass of users, and a unit mass of advertisers. The services offered are free to users and purely advertising financed by placing advertisements in it (connecting advertisers to a service users). To fix ideas, in the smartphone market, the hardware product is smartphone: Apple and Huawei devices, whereas online service is news: Apple News and Petal Search.

Firms

We consider a competitive setting in which firms G and S offer both a hardware product (G1 and S1) and an online service (G2 and S2). In addition, firms G and S can also decide whether to sell the product and service independently or integrate the online service with the hardware product. In case of service integration by firm G and/or firm S, a consumer either purchases the system G1G2 or S1S2.

Under independent pricing, firm $i, i = G, S$, charges a product price p_{i1} to the users. Whereas, for the service, users are charged a zero price. However, on the advertising side, firms set advertising quantities, a_{G2} and a_{S2} respectively.[4] Let r_{i2} be the price per unit of an advertisement in firm i's service, $i = G, S$. Let N_{i1} be the total number of users who consume product i and N_{i2} be the total number of users who consume service i, for $i = G, S$. The profit of the firms are

$$\pi_G = p_{G1}N_{G1} + r_{G2}a_{G2} : \text{ Firm G's profit,} \tag{1}$$

$$\text{and } \quad \pi_S = p_{S1}N_{S1} + r_{S2}a_{S2} : \text{ Firm S' profit.} \tag{2}$$

Next, consider the cases when either firm G or firm S or both integrate the service with the product. For instance, if firm G integrates the service G2 with its product G1 then users can either consume the system G1G2 or firm S' product S1 and service S2. In other words, service S2 cannot be consumed with product G1 and service G2 cannot be consumed with product S1. Effectively, competition between firms takes place as competition between the two systems G1G2 and S1S2.

Under service integration by firm G and/or firm S, firm G charges a price $\widetilde{p}_G$ for the hardware product, and firm S charges a price $\widetilde{p}_S$ for its hardware product. On the advertising side, firm i sets advertising quantity, $\widetilde{a}_i$, $i = G, S$. Let $\widetilde{r}_i$ be the price per unit of an advertisement in firm i's service, $i2$, where $i = G, S$. Let $\widetilde{N}_G$ be the total number of users who consume the system G1G2,

[4]The modelling set up in which firms choose advertising quantities is similar to that in de Cornière and Taylor (2014).

and $\widetilde{N}_S$ be the total number of users who consume the system S1S2. The profit of the firms are

$$\widetilde{\pi}_G = \widetilde{p}_G \widetilde{N}_G + \widetilde{r}_G \widetilde{a}_G : \text{ Firm G's profit,} \tag{3}$$

$$\text{and } \widetilde{\pi}_S = \widetilde{p}_S \widetilde{N}_S + \widetilde{r}_S \widetilde{a}_S : \text{ Firm S' profit.} \tag{4}$$

Users

There is a unit mass of users who demand both products and services. They are willing to consume at most one unit of the hardware product and one unit of the service and have a reservation value equal to zero for both. We capture differentiation between the products and services via horizontal and vertical differentiation. Cremer and Thisse (1991) has defined horizontal differentiation as "if all the variants of a product are sold at the same price, there is a positive demand for each of them." In other words, if the products G1 and S1 or services G2 and S2 are offered at the same price, then each of them would receive a positive demand. We can map this to the unique features that attract users for specific product and service. First, consider the demand for hardware products. Our motivating example is of smartphones sold by Apple and Huawei. The two smartphones differ in terms of size, design, camera quality, battery life, etc. One feature that distinguishes the two is battery life. Huawei phone has huge battery life relative to Apple.[5] However, Apple is better at offering ease of use, user interface, screen display, and facial recognition features. The consumers would differ in terms of their phone usability. The consumers who want a very good battery life would prefer Huawei phone, whereas those who mainly want ease of user interface would prefer Apple phone. Next consider the services G2 and S2. Using our motivating example, G2 is Apple news and S2 is Huawei petal search. Both services differ in their focus, user interface and design. As a result, each service would attract certain consumers depending on their tastes. In other words, this may lead to heterogeneous consumer preferences for products and services. We model this as firms competing à la Hotelling for both hardware product and the service (see figure 1). The consumer preferences for the products and services are represented using a 1×1 unit square with Firm G located at the origin (0, 0) and firm S is located at the coordinate (1, 1). A user is characterized by a pair (x_1, x_2) (refer figure 1), where x_1 (x_2) is her location on Hotelling line representing preference for ideal product (service). If a firm's product (service) location does not match her preference, then she incurs a misfit cost or transportation cost from consuming the product (service), and it is increasing in the distance between the firm's location and her location. Let t be the unit transportation cost for consuming the product/service. Thus, she faces a transportation cost of tx_1 ($t(1 - x_1)$), if she consumes product G1 (S1). Similarly, she

[5]See, https://www.stuff.tv/news/apple-iphone-11-pro-vs-huawei-p30-pro-which-best/

incurs a transportation cost of tx_2 ($t(1-x_2)$), if she consumes service G2 (S2).

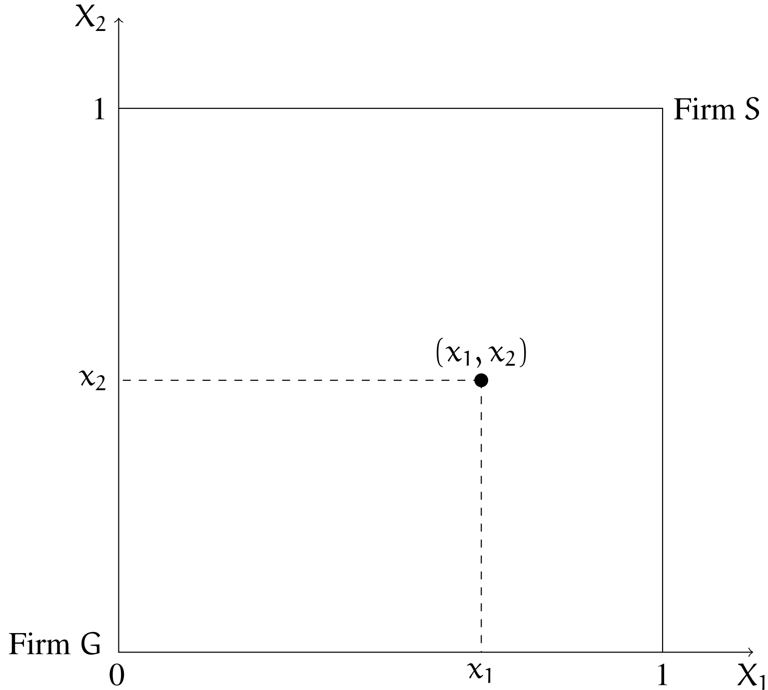

Figure 1: Locations of Firms and Users on the Unit Square

Moreover, they obtain a standalone value V_{i1} (V_{i2}) from consuming product $i1$ (service $i2$), for $i = G, S$. The product and service are independent in the sense that consumers' preferences for the product and service are independent.

■ **Independent pricing.** Consider the market regime when both firms sell products and services independently. Following Etzion and Pang (2014), we assume that firms' products are also vertically differentiated with $V_{G1} > V_{S1}$. If firm G is Apple and firm S is Huawei, then an important component of the vertical dimension of smartphones on which all consumers prefer iPhone to Mate 40 Pro is the value derived from functionalities of the product. For instance, iPhone is more efficiently connected with basic applications like alarms, calender, etc., has access to popular third-party apps like Google Search, YouTube,etc., which at present are unavailable on Huawei phone.[6] Since Google Services are dominant in search, video streaming, etc. iPhone has an upper hand because it can provide access to Google Search app, whereas Huawei relies on Petal Search. Let $\Delta = V_{G1} - V_{S1} > 0$ measure the quality advantage of firm G's product.

Thus, a user's net utility from consuming product $i1$, U_{i1}, $i = G, S$, equals

[6]See, https://www.theverge.com/2020/3/25/21193639/huawei-mate-30-google-apps-services-appgallery-p40-preview

$$\begin{cases} V_{G1} - p_{G1} - tx_1, & \text{if she consumes product G1, and} \\ V_{S1} - p_{S1} - t(1-x_1), & \text{if she consumes product S1.} \end{cases} \tag{5}$$

For the service, in the baseline model, we assume that there is no vertical differentiation with $V_{G2} = V_{S2} = W$. However, we consider a more general scenario in section 6. In addition to transportation costs, she incurs disuility from viewing advertisements. She is exposed to an advertising level of a_{G2} (a_{S2}), with a total disutility of δa_{G2} (δa_{S2}), if she consumes service G2 (S2), where δ is the per unit nuisance cost of advertisement. Thus, her net utility from consuming service $i2$, $U_{i2}, i = G, S$, equals

$$\begin{cases} W - \delta a_{G2} - tx_2, & \text{if she consumes service G2, and} \\ W - \delta a_{S2} - t(1-x_2), & \text{if she consumes service S2.} \end{cases} \tag{6}$$

■ **Service Integration.** Next, consider the market regimes in which firm G or/and firm S integrate the service with the product and offers them as a system. Note that service integration by firm i would imply that a user cannot use service (product) of firm i, where $i = 1, 2$, with the product (service) of firm j, where $j = 1, 2$, and $j \neq i$.

In this scenario the user choice would be restricted to choose between the two systems G1G2 and S1S2. In the case of smartphone market, both Apple and Huawei have integrated their news services with their smartphones which has restricted them to use Apple News with iPhone or Petal Search with Huawei Phone.

A user's net utility, $U_i, i = G1G2$, or S1S2, equals

$$\begin{cases} V_{G1} + W - \delta \tilde{a}_G - \tilde{p}_G - t(x_1 + x_2), & \text{if she consumes firm G's system G1G2, and} \\ V_{S1} + W - \delta \tilde{a}_S - \tilde{p}_S - t[(1-x_1) + (1-x_2)], & \text{if she consumes firm S' system S1S2.} \end{cases} \tag{7}$$

Advertisers

On the advertising side, there is a unit mass of advertisers, all of whom want to generate attention for their product or service through placing advertisements in firms G's service and S'service. A successful transaction for an advertiser is dependent on a multitude of factors. One important factor is the reach of an advertiser's product to the audience, i.e. whether it will attract a broad audience or a narrow audience. In other words, audience targeting is one of the most important factor in digital advertising. An advertiser can have multitude of targeting options in a firm's service such as demographic, location, interests, etc. Based on it's nature of product it can select specific parameters within these options. For e.g., baby products such as diapers, clothes, etc. would attract a narrow set of audience, i.e., parents of new born babies or toddlers. Whereas, a television or personal computers will have a broader audience range with a wider age group. We use α to define the nature of advertiser's product. Since advertiser's differ in their product characteristics, we allow α to be uniformly distributed over the interval [0, 1]. Thus, an advertiser with higher α would mean that its product has a wider audience appeal and can reach a larger audience in a firm's service. Whereas, a lower α would imply that the product is targeted to a specific audience with a lower audience reach on a platform. Moreover, let β denote the probability that a user would view and purchase the product, where $\beta \in (0, 1]$. In other words, β is the advertising targeting rate. Next, we consider the modeling of payments by advertisers. We consider a pricing model in which an advertiser pays a price for a fixed number of impressions from an advertisement in firm i's service. This approach is similar to that assumed in recent papers on online advertising (see, e.g., Gal-Or et al. (2018), Reisinger (2012), etc.) As a result, the net payoff of an advertiser α from advertising in firm i's service, $i = G, S$, is

$$\pi_\alpha = \begin{cases} \alpha.\beta N_{i2} - r_{i2}: & \text{Independent pricing, and} \\ \\ \alpha.\beta \widetilde{N}_i - \widetilde{r}_i: & \text{Servcie Integration.} \end{cases} \tag{8}$$

The first term of the payoff function measures the gross expected benefit to the advertiser from advertising in firm i, and the second term r_{i2} ($\widetilde{r}_i$) is fixed access fee under independent pricing (service integration). Note that given a certain number of users who join firm i, an advertiser with a higher α will be able to reach a larger audience, and will more be able to reach higher potential customers for its product, obtaining a higher network benefit from placing an advertisement in firm i. In equilibrium, the advertising prices are determined so as to equate demand for advertising slots and supply (determined by firm i's choice of advertising level).

Note that since our focus is on understanding the integration strategy, we work with a reduced form way of modeling interactions between advertisers and users, ignoring the details of the auction process through which ads are placed in a firm's service platform. This is a standard approach in the literature on two-sided markets (see, e.g., Armstrong (2006), Rochet and Tirole (2003), Dimakopoulos and Sudaric (2018), Peitz and Valletti (2008)).

Timing of the Game

The game proceeds as follows:

Stage 1: Firms G and S decide whether to adopt independent pricing (N) or service integration (I). As a result, four market regimes are possible. In the first regime, case NN, both firms adopt independent pricing. In the second regime, case IN and the third regime, case NI, only firm G or firm S adopts service integration, while the other firm does not. In the fourth regime, case II, both firms adopt the service integration.

Stage 2: Based on stage 1 decisions, firms compete in prices and advertising quantities. Under case NN, i.e. when both firms adopt independent pricing, firms G and S simultaneously choose i) user prices p_{G1} and p_{S1}, and ii) the quantity of advertisements, i.e. a_{G2} and a_{S2}. Whereas, under cases IN, NI or II, firms G and S simultaneously choose i) user prices $\widetilde{p}_G$ and $\widetilde{p}_S$, and ii) the quantity of advertisements, i.e. $\widetilde{a}_G$ and $\widetilde{a}_S$.

Stage 3: Advertisers decide whether to advertise in the service of firm G or S or both.[7] Advertising prices adjust so that the demand for advertisements equals its supply.

Stage 4: Observing firms' choices, under independent pricing (case NN), users decide which firm i's ($i = G, S$) product and service to consume. Whereas, under cases IN, NI or II, users decide whether to consume i) firm G's system $G1G2$, or ii) firm S' system $S1S2$.

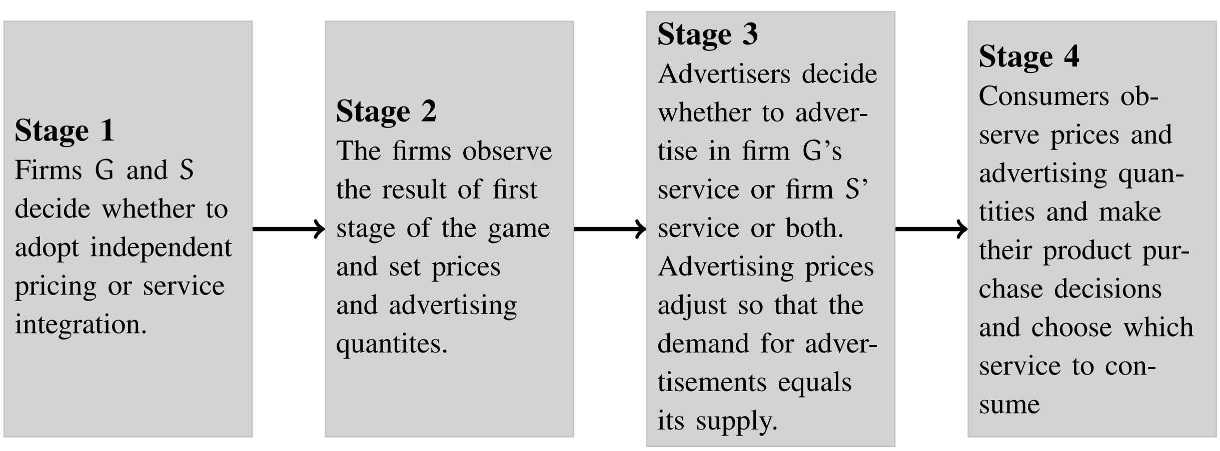

Figure 2: Timing of the Game

The solution concept used is the subgame perfect Nash equilibrium (henceforth equilibrium).

[7] An advertiser's strategy is to decide whether to join firm G 's service or firm S' service or both. Since advertisers are price takers in the model, they will place an advertisement in firm i, $i = G, S$, if the marginal benefit of advertising in firm i is at least as large as the price of an advertisement.

Notation	Description
V_{i1}	standalone/intrinsic value of firm i's product.
W	standalone/intrinsic value of firm i's service.
t	per unit transportation cost.
δ	per unit nuisance cost of an advertisement.
β	probability that a user would purchase the advertiser's product.
p_{i1}	price paid by a user to consume firm i's product under independent pricing (case NN).
$\widetilde{p}_i$	price paid by a user to consume firm i's system under service integration (case IN, NI or II).
r_{i2}	price per unit of an advertisement under independent pricing (case NN).
$\widetilde{r}_i$	price per unit of an advertisement under service integration (case IN, NI or II).
N_{ij}	demand for firm i's product in market j under independent pricing (case NN).
$\widetilde{N}_i$	demand for firm i's system under service integration (case IN, NI or II).
a_{i2}	level of advertisements in firm i under independent pricing (case NN).
$\widetilde{a}_i$	level of advertisements in firm i under service integration (case IN, NI or II).

Table 1: Summary of Key Notations

4 Equilibrium Analysis

We begin by solving the second, third and fourth stages of the game, i.e. we characterize the equilibrium prices, advertising quantities, and profits under the four different market regimes, i.e. cases NN, IN, NI and II, as given in table 2. The detailed derivations are provided in the appendix. Finally, we solve the 2×2 payoff matrix (given in Table 2) to determine the sub-perfect Nash equilibrium in the first stage of the game as specified in Proposition 2.

		Firm S: N	Firm S: I
Firm G	N	$\pi_G^* = \frac{(3t+\Delta)^2}{18t} + \frac{\beta t}{\delta}\left[\sqrt{\frac{1}{4}+\frac{t^2}{\delta^2}} - \frac{t}{\delta}\right]$ $\pi_S^* = \frac{(3t-\Delta)^2}{18t} + \frac{\beta t}{\delta}\left[\sqrt{\frac{1}{4}+\frac{t^2}{\delta^2}} - \frac{t}{\delta}\right]$	$\widetilde{\pi}_G^* = \frac{[32t^2-(t-\Delta/2+\sqrt{9t^2+\Delta^2/4-t\Delta})^2]^2}{128t^2(t-\Delta/2+\sqrt{9t^2+\Delta^2/4-t\Delta})}$ $\widetilde{\pi}_S^* = \frac{(t-\Delta/2+\sqrt{9t^2+\Delta^2/4-t\Delta})^3}{128t^2}$
	I	$\widetilde{\pi}_G^* = \frac{[32t^2-(t-\Delta/2+\sqrt{9t^2+\Delta^2/4-t\Delta})^2]^2}{128t^2(t-\Delta/2+\sqrt{9t^2+\Delta^2/4-t\Delta})}$ $\widetilde{\pi}_S^* = \frac{(t-\Delta/2+\sqrt{9t^2+\Delta^2/4-t\Delta})^3}{128t^2}$	$\widetilde{\pi}_G^* = \frac{[32t^2-(t-\Delta/2+\sqrt{9t^2+\Delta^2/4-t\Delta})^2]^2}{128t^2(t-\Delta/2+\sqrt{9t^2+\Delta^2/4-t\Delta})}$ $\widetilde{\pi}_S^* = \frac{(t-\Delta/2+\sqrt{9t^2+\Delta^2/4-t\Delta})^3}{128t^2}$

Table 2: The Equilibrium Profits for the Four Possible Market Regimes

To ensure that there are net gains from trade in market 2, we assume that the net surplus from placing an advertisement is positive for each firm. This ensures that they will rely on advertising financing as the equilibrium business model. So, the advertising targeting rate is sufficiently large relative to per unit nuisance cost of advertisement.

Assumption 1. $\beta \geq \delta$.

Second, we assume that a user's gross utility in market $j = 1, 2$, i.e. $V_{i1}, i = G, S$, and W are sufficiently large relative to the transportation cost parameter t to ensure that, under independent pricing, there is full market coverage. Moreover, we focus on the scenario where both firms have positive market shares in market 1 under independent pricing.[8]

Assumption 2. $\frac{V_{G1}+V_{S1}}{3} \geq t,\ W - \frac{3t}{2} - \frac{\delta}{2} + \delta\left[\frac{1}{4} + \frac{t^2}{\delta^2}\right]^{1/2} \geq 0,$ *and* $\Delta < 3t$.

Finally, we assume that competition for users is sufficiently weak in both markets. This assumption guarantees that there are gains from trade in market 2 such that service integration can be a profitable strategy.

Assumption 3. $t \geq \max\{1.2, 5.73\beta - .011\delta^2/\beta\}$.

[8]The inequality $\Delta < 3t$ simplifies the analysis. Allowing for $\Delta \geq 3t$ would not yield additional insights on equilibrium outcome and welfare. The details are available from the author.

Notice that if in Stage 2 at least one firm has chosen service integration (i.e. case IN, NI or II), then competition at Stages 3 and 4 occurs between the two pure systems. For instance, suppose that firm G has chosen I and firm S has chosen N. Then the choice set of a user is restricted to choosing firm G's system G1G2 or consuming firm S' product S1 and service S2, which can be viewed as a system S1S2. Thus, when calculating equilibrium profits at stage 2, we can consider two sub-cases: one in which both firms choose independent pricing or no service integration, i.e. case NN, and second in which either firm G or S or both have chosen service integration, i.e. case IN, NI or II.

Independent Pricing

We characterize equilibrium under the case NN, i.e. both firms choose independent pricing.

■ **Stages 3 and 4**: A type (x_1, x_2) user can make her purchase decision independently for product and service. Therefore. her choice set comprises of four options:

i. G1G2: Consume product G1 and service G2.

ii. G1S2: Consume product G1 and service S2.

iii. S1G2: Consume product S1 and service G2.

iv. S1S2: Consume product S1 and service S2.

Let $U(x_1, x_2; Y)$ be the user (x_1, x_2) utility from opting for option Y, where $Y \in \{G1G2; G1S2; S1G2; S1S2\}$. Using equations (5) and (6), we can write

$$U(x_1, x_2; G1G2) = V_{G1} - p_{G1} - tx_1 + W - \delta a_{G2} - tx_2, \tag{9}$$

$$U(x_1, x_2; G1S2) = V_{G1} - p_{G1} - tx_1 + W - \delta a_{S2} - t(1 - x_2), \tag{10}$$

$$U(x_1, x_2; S1G2) = V_{S1} - p_{S1} - t(1 - x_1) + W - \delta a_{G2} - tx_2, \text{ and} \tag{11}$$

$$U(x_1, x_2; S1S2) = V_{S1} - p_{S1} - t(1 - x_1) + W - \delta a_{S2} - t(1 - x_2). \tag{12}$$

At *stage 4*, users make participation decisions. Since x_1 and x_2 are independently and identically distributed the demand for product and service can be analysed separately.[9] For the product, given

[9] Another approach to obtain demand for each firm's product and service is by finding the cutoffs $\hat{x}_1, \hat{x}_2, x_1 = h(x_2)$, and $x_1 = z(x_2)$ where $h(x_2)$ is decreasing in x_2 with $h(\hat{x}_2) = \hat{x}_1$ and $z(x_2)$ is increasing in x_2 with $z(\hat{x}_2) = \hat{x}_1$ such that $u(\hat{x}_1, x_2; G1i2) = u(\hat{x}_1, x_2; S1i2)$, $u(x_1, \hat{x}_2; i1G2) = u(x_1, \hat{x}_2; i1S2)$, for

prices p_{G1} and p_{S1}, an indifferent user is defined by the location $\hat{x}_1$ such that

$$V_{G1} - p_{G1} - t\hat{x}_1 = V_{S1} - p_{S1} - t(1-\hat{x}_1) \Rightarrow \hat{x}_1 = \frac{1}{2} + \frac{\Delta}{2t} + \frac{p_{S1} - p_{G1}}{2t}, \tag{13}$$

where $\Delta = V_{G1} - V_{S1} > 0$ by assumption. Using this, the demand for product $i1, i = G, S$, is

$$N_{G1} = \hat{x}_1 = \frac{1}{2} + \frac{\Delta}{2t} + \frac{p_{S1} - p_{G1}}{2t}, \text{ and} \tag{14}$$

$$N_{S1} = 1 - \hat{x}_1 = \frac{1}{2} - \frac{\Delta}{2t} + \frac{p_{G1} - p_{S1}}{2t}. \tag{15}$$

For the service in the market, given advertising levels a_{G2} and a_{S2}, the user indifferent between consuming service G2 and S2 is defined by the location $\hat{x}_2 \in [0, 1]$ such that

$$W - \delta a_{G2} - t\hat{x}_2 = W - \delta a_{S2} - t(1-\hat{x}_2) \Rightarrow \hat{x}_2 = \frac{1}{2} + \frac{\delta a_{S2} - \delta a_{G2}}{2t}. \tag{16}$$

Using this, the demand for service $i2, i = G, S$, is

$$N_{G2} = \hat{x}_2 = \frac{1}{2} + \frac{\delta a_{S2} - \delta a_{G2}}{2t}, \text{ and} \tag{17}$$

$$N_{S2} = 1 - \hat{x}_2 = \frac{1}{2} + \frac{\delta a_{G2} - \delta a_{S2}}{2t}. \tag{18}$$

At *stage 3*, advertisers make the participation decision. Given advertising prices $r_{i2}, i = G, S$, an advertiser α would advertise in firm i's service, $i = G, S$, as long as the marginal benefit of an advertisement is at least as large as its marginal cost, i.e. r_{i2}. Using equation (8), the marginal advertiser $\hat{\alpha}_i$ indifferent between advertising and not advertising in firm i's service, $i = G, S$, is

$$\hat{\alpha}_i = \frac{r_{i2}}{\beta N_{i2}}, i = G, S. \tag{19}$$

$i = G, S$, and $u(h(x_2), x_2; G1G2) = u(h(x_2), x_2; S1S2)$, and $u(z(x_2), x_2; G1S2) = u(z(x_2), x_2; S1G2)$. Then a user would consume

i) G1G2 if a) $0 \leq x_1 \leq \hat{x}_1$, b) $0 \leq x_2 \leq \hat{x}_2$ and c) $0 \leq x_1 \leq h(x_2)$,

ii) G1S2 if a) $0 \leq x_1 \leq \hat{x}_1$, b) $\hat{x}_2 \leq x_2 \leq 1$ and c) $0 \leq x_1 \leq z(x_2)$,

iii) S1G2 if a) $\hat{x}_1 < x_1 \leq 1$, b) $0 \leq x_2 \leq \hat{x}_2$ and c) $z(x_2) < x_1 \leq 1$, and

iv) S1S2 if a) $\hat{x}_1 < x_1 \leq 1$, b) $\hat{x}_2 < x_2 \leq 1$ and c) $h(x_2) < x_1 \leq 1$.

It can be easily shown that the demand for firm G's (firm S') product is $N_{G1}(N_{S1}) = \hat{x}_1(1 - \hat{x}_1)$. Similarly, the demand for firm G's (firm S') service is $N_{G2}(N_{S2}) = \hat{x}_2(1 - \hat{x}_2)$. Thus, we get the same demand systems.

Using the preceding expression, the level of advertisement in service i is

$$a_{i2} = 1 - \frac{r_{i2}}{\beta N_{i2}}, i = G, S. \tag{20}$$

Note that advertising quantity in firm i's service, $i = G, S$, is not affected by the advertising decision of the competing firm. This gives the inverse advertising demand function of service $i, i = G, S$, as

$$r_{i2} = (1 - a_{i2})\beta N_{i2}. \tag{21}$$

■ **Stage 2**: Using the inverse advertising demand function defined in equation (21), the user demand functions defined in equations (14), (15) ,(17) and (18), and putting the values for them in the profit functions (1) and (2), firm $i, i = G, S$, chooses the user price p_{i1} and advertising quantity a_{i2} to maximize its profits. Let the second stage equilibrium prices, advertising quantities and demands be $p_{i1} = p^*_{i1}$, $a_{i2} = a^*_{i2}$, , $N_{i1} = N^*_{i1}$ and $N_{i2} = N^*_{i2}$, $i = G, S$. The following lemma characterizes the equilibrium.

Lemma 1. *Consider the case* NN*, i.e. when both firms sell the products* G1 *and* S1*, and services* G2 *and* S2 *independently. Then stage 2 equilibrium satisfies the following:*

i. the equilibrium advertising levels are characterized by

$$a^*_{G2} = a^*_{S2} = \frac{1}{2} + \frac{t}{\delta} - \sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}}, \tag{22}$$

ii. the equilibrium prices, and market shares are

$$p^*_{G1} = \frac{3t + \Delta}{3},\ p^*_{S1} = \frac{3t - \Delta}{3},\ \textit{and} \tag{23}$$

$$N^*_{G1} = \frac{3t + \Delta}{6t},\ N^*_{S1} = \frac{3t - \Delta}{6t},\ N^*_{S2} = N^*_{G2} = \frac{1}{2}. \tag{24}$$

The equilibrium profit of the firms are

$$\pi^*_G = \frac{(3t + \Delta)^2}{18t} + \frac{\beta t}{\delta}\left[\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}} - \frac{t}{\delta}\right],\ \textit{and}\ \pi^*_S = \frac{(3t - \Delta)^2}{18t} + \frac{\beta t}{\delta}\left[\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}} - \frac{t}{\delta}\right]. \tag{25}$$

Note that, for the hardware product, the equilibrium price charged by firm G (firm S) increases (decreases) with an increase in the level of firm G's quality advantage. Also, the weaker the competition (larger t), higher the prices charged by both firms. Moreover, at the equilibrium prices, p^*_{G1} and p^*_{S1}, the price elasticity of demand is equal to -1 for each firm, i.e.

$$\frac{p^*_{i1}}{N^*_{i1}}.\frac{\partial N_{i1}(.)}{\partial p_{i1}} = -1.$$

Intuitively, each firm operates as a monopolist over the residual demand it faces (taking as given the other firm's price). Since marginal costs are zero, it maximizes profit over the hardware product w.r.t. price using the inverse elasticity rule.

Now consider the online service. The advertising quantities depend on both the intensity of competition between firms' services and the nuisance cost of advertisements. This is because advertisements are implicit prices charged to users for accessing firm i's service, $i = G, S$. Thus, advertising competition between firms would depend on the ability of users to switch (measured by the transportation cost parameter, t). If users can switch easily between the two firms' services, then advertising competition would be intense and firms would set smaller equilibrium advertising levels to attract users. Moreover, as nuisance cost parameter δ increases, users react negatively to advertisements and thus, equilibrium advertising quantities would decrease.

Service Integration

We focus on the market regimes when, at stage 2, at least one firm has chosen service integration, i.e. cases IN, NI or II. In either of the three cases, competition at stages 3 and 4 occur between the two pure systems G1G2 and S1S2.

■ **Stages 3 and 4**: The users' choice set is now reduced to choosing between the two systems, i.e.

i. G1G2: consume firm G's product G1 and service G2.

ii. S1S2: consume firm S' product S1 and service S2.

Let $U(x_1, x_2; Y)$ be the user (x_1, x_2) utility from opting for option Y, where $Y \in \{G1G2; S1S2\}$. Using equation (7) , we can write

$$U(x_1, x_2; G1G2) = V_{G1} - \widetilde{p}_G - tx_1 + W - \delta\widetilde{a}_G - tx_2, \text{ and} \tag{26}$$

$$U(x_1, x_2; S1S2) = V_{S1} - \widetilde{p}_S - t(1 - x_1) + W - \delta\widetilde{a}_S - t(1 - x_2). \tag{27}$$

Now, at *stage 4*, given prices $\widetilde{p}_G$ and $\widetilde{p}_S$, and advertising quantities $\widetilde{a}_G$ and $\widetilde{a}_S$, the users decide which of the above two systems to consume. A user indifferent between consuming system G1G2 and S1S2 is defined by a pair (x_1, x_2) such that

$$V_{G1} + W - \delta\widetilde{a}_G - \widetilde{p}_G - tx_1 - tx_2 = V_{S1} + W - \delta\widetilde{a}_S - \widetilde{p}_S - t(1 - x_1) - t(1 - x_2). \tag{28}$$

Let $\widetilde{y} = (x_1 + x_2)/2$ denote the average location of a user type (x_1, x_2).[10] Therefore, using the preceding expression, $\widetilde{y}$ denote the average location of the indifferent user which is given by

$$\widetilde{y} = \frac{x_1 + x_2}{2} = \frac{1}{2} + \frac{\Delta}{4t} + \frac{\widetilde{p}_S - \widetilde{p}_G}{4t} + \frac{\delta\widetilde{a}_S - \delta\widetilde{a}_G}{4t}. \tag{29}$$

The departure from independent pricing regime (case NN) is that we have to use the density function of the average location $\widetilde{y}$ to find the demand for firm i's product, $i = G, S$. Let $\widetilde{F}(.)$ and $\widetilde{f}(.)$ denote the distribution and probability density functions of the average location $\widetilde{y}$. They are

$$\widetilde{F}(y) = \begin{cases} 2y^2, & \text{if } 0 \leq y \leq \frac{1}{2}, \text{ and} \\ 1 - 2(1-y)^2, & \text{if } \frac{1}{2} < y \leq 1. \end{cases} \qquad \widetilde{f}(y) = \begin{cases} 4y, & \text{if } 0 \leq y \leq \frac{1}{2}, \text{ and} \\ 4(1-y), & \text{if } \frac{1}{2} < y \leq 1. \end{cases} \tag{30}$$

Using the preceding expression, the demand for each firm's system is

$$\widetilde{N}_G = \widetilde{F}(\widetilde{y}), \text{ and } \widetilde{N}_S = 1 - \widetilde{F}(\widetilde{y}). \tag{31}$$

Similar to independent pricing regime, using equation (8), the inverse advertising demand function of firm $i, i = G, S$, is

$$\widetilde{r}_i = (1 - \widetilde{a}_i)\beta\widetilde{N}_i. \tag{32}$$

■ **Stage 2**: Using the user demand functions defined in (31), and the inverse advertising demand function given in equation (32), and putting the values for them in the profit functions (3) and (4), firm $i, i = G, S$, chooses the user price $\widetilde{p}_i$ and advertising quantity $\widetilde{a}_i$ to maximize its profits. Let the second stage equilibrium prices, advertising quantities and demands be $\widetilde{p}_i = \widetilde{p}_i^*$, $\widetilde{a}_i = \widetilde{a}_i^*$, and $\widetilde{N}_i = \widetilde{N}_i^*$, $i = G, S$. The following lemma characterizes the equilibrium.

Lemma 2. *When either firm G or firm S or both integrate the services with their products, then stage 2 equilibrium satisfies the following:*

[10] A similar approach to obtain the demand functions under pure bundling regime has been followed in Hurkens et al. (2019).

i. the equilibrium advertising levels are

$$\widetilde{a}_G^* = \widetilde{a}_S^* = \frac{\beta - \delta}{2\beta}, \tag{33}$$

ii. the equilibrium prices and market shares are

$$\widetilde{p}_G^* = \frac{32t^2 - (t - \Delta/2 + C)^2}{4(t - \Delta/2 + C)} - \frac{(\beta^2 - \delta^2)}{4\beta}, \; \widetilde{p}_S^* = \frac{t - \Delta/2 + C}{4} - \frac{(\beta^2 - \delta^2)}{4\beta}, \tag{34}$$

$$\widetilde{N}_G^* = \frac{32t^2 - (t - \Delta/2 + C)^2}{32t^2}, \;\; \textit{and} \; \widetilde{N}_S^* = \frac{(t - \Delta/2 + C)^2}{32t^2} \; \textit{with} \; C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}. \tag{35}$$

The equilibrium profit of the firms are

$$\widetilde{\pi}_G^* = \frac{[32t^2 - (t - \Delta/2 + C)^2]^2}{128t^2(t - \Delta/2 + C)}, \;\; \textit{and} \;\; \widetilde{\pi}_S^* = \frac{(t - \Delta/2 + C)^3}{128t^2}. \tag{36}$$

The preceding lemma shows interesting implications of adopting the integration strategy. First, consider the change in the market scenario. Since users are single-homing (join only one service) and advertisers are multi-homing (advertising in both services), the firms become "bottlenecks" or "gatekeepers" providing exclusive access to the single-homing users. This gives them market power over the advertisers who wish to interact with the users and is known as a "competitive bottleneck" situation in platform markets literature (see, e.g., Armstrong and Wright (2007), Peitz and Valletti (2008)). In this context, the firms will likely compete intensively for the single-homing users. By doing so and ensuring users' participation they will then enjoy monopoly power over the multi-homing advertisers with regards to granting access to their (exclusive) single-homing users. Put it another way, price competition on the user side will be intense. In return, firms will be able to extract higher rents from the advertisers who want to reach users and generate attention for their products. As a result, this gives rise to a biased pricing structure in which profits extracted from the advertising side are used to compete aggressively for the single-homing users, and they may end up paying positive, zero, or negative prices. A firm's profit maximization can be looked at as a two-step process. First, they maximize profits with respect to advertisements in its service. They do so by maximizing the joint advertising surplus between the firm and the user. Note that the advertising surplus is affected by the gross expected advertising revenue (refer equation (8)), which depends on the advertising targeting rate β and the cost to users, which depends on the nuisance cost of advertisements δ. This implies that the advertising levels are chosen to maximize the difference between the two and explains the equilibrium advertising quantities in lemma 2(i).

Next, the firm splits the advertising surplus with the users, determining the equilibrium prices as shown in lemma 2(ii). The first term captures the effect of the intensity of competition (measured by the transportation cost parameter, t) on the user prices. The second term captures the role of advertising in generating revenues. The greater the advertising targeting rate β or the lower the nuisance cost of advertisements δ, the greater the advertising surplus that can be generated. Hence, prices are lowered to attract more users for the advertisers. In fact, as lemma 2(ii) shows, prices are lowered by the total amount of advertising profits generated. Thus, in equilibrium, there is a full pass-through of advertising profits to the users in the form of lower prices. Consequently, equilibrium profits (shown in (36)) are independent of the advertising profits.

Next, consider the price elasticity of demand for the system i, $i = G1G2, S1S2$. At equilibrium prices, it will be greater than -1, i.e.

$$\frac{\widetilde{p}_i^*}{\widetilde{N}_i^*} \cdot \frac{\partial \widetilde{N}_i}{\partial \widetilde{p}_i} > -1.$$

This is counter-intuitive to the standard approach that, when marginal costs are zero, a profit maximizing firm would set price such that price elasticity of demand is equal to -1. In order to understand this, we should consider the instruments that a firm has to maximize profits. With zero marginal costs and absence of advertising side, a profit maximizing price would lead to unitary elastic demand. However, it must be noted that in our model each firm has two instruments to maximize profits: price and advertising quantity. As discussed above, in our model, service integration results in a "competitive bottleneck" situation and in equilibrium there is a full pass-through of advertising profits to the users in the form of lower prices. Thus, we have equilibrium prices less than the prices that set demand elasticity equal to -1. Hence, we would have price elasticity of demand at $(\widetilde{p}_{G1}^*, \widetilde{p}_{S1}^*)$ greater than -1, i.e. inelastic demand.

Comparison of Pricing Regimes

Comparison of pricing and advertising levels under independent pricing and service integration

Comparing lemmas 1 and 2 shows that equilibrium prices under independent pricing and service integration are different. This occurs because of changes in the distribution and density functions. Under independent pricing, the demand for each firm's product and service is obtained by using the distribution function $F(x) = x$ of the user's location in the product and service dimension. Thus, the demand for product G1 (S1) is $N_{G1} = F(x_1^*) = x_1^*$ ($N_{S1} = 1 - F(x_1^*) = 1 - x_1^*$) and demand for service G2 (S2) is $N_{G2} = F(x_2^*) = x_2^*$ ($N_{S2} = 1 - F(x_2^*) = 1 - x_2^*$). Whereas, under service integration, the demand for the system is obtained by using the distribution function of the user's average location, i.e. $\widetilde{F}(.)$ as shown in (30). This brings a change in the demand elasticities under the two regimes. In order to understand how pricing strategy will change, we evaluate the demand elasticities at price pair (p^*_{G1}, p^*_{S1}) under the two regimes,[11] i.e. elasticities $(p_{i1}/N_{i1}).(\partial N_{i1}/\partial p_{i1})$ and $(\widetilde{p}_i/\widetilde{N}_i).(\partial \widetilde{N}_i/\partial \widetilde{p}_i)$, for $i = G, S$, at (p^*_{G1}, p^*_{S1}). Since (p^*_{G1}, p^*_{S1}) are optimal prices under independent pricing, we know that $(p^*_{i1}/N^*_{i1}).(\partial N_{i1}/\partial p_{i1})$ = -1. Now, consider the service integration regime. The demand elasticity $(\widetilde{p}_i/\widetilde{N}_i).(\partial \widetilde{N}_i/\partial \widetilde{p}_i)$ at (p^*_{G1}, p^*_{S1}) can be written as

$$\frac{\widetilde{p}_G}{\widetilde{N}_G}.\frac{\partial \widetilde{N}_G}{\partial \widetilde{p}_G}\bigg|_{(\widetilde{p}_G,\widetilde{p}_S)=(p^*_{G1},p^*_{S1})} = -\frac{2.(3t+\Delta)(6t-\Delta)}{36t^2-\Delta^2+12t\Delta}, \text{ and}$$

$$\frac{\widetilde{p}_S}{\widetilde{N}_S}.\frac{\partial \widetilde{N}_S}{\partial \widetilde{p}_S}\bigg|_{(\widetilde{p}_G,\widetilde{p}_S)=(p^*_{G1},p^*_{S1})} = -\frac{2.(3t-\Delta)(6t-\Delta)}{36t^2+\Delta^2-12t\Delta}.$$

Straightforward calculations will show that the preceding two expressions are greater than -1. Hence, demand becomes inelastic with service integration by either or both firms. It induces each firm $i, i = G, S$, to charge a system price $\widetilde{p}_i$ greater than p^*_{i1}. Intuitively, a consumer considers the utility from both the product and service while deciding to switch in response to a price increase. The presence of a free service in a system makes her less sensitive to price changes, making demand inelastic. Moreover, demand is more inelastic at larger values of Δ. Thus, for sufficiently large Δ, this effect dominates the competitive bottleneck effect (discussed below), softening competition between firms and thus, the prices under service integration will be greater than the prices under independent pricing.

Next, consider the advertising levels under independent pricing (given by equation (22)) and service integration (given by (33)). Only at $\delta = 0$, the advertising levels under independent pricing and service integration would be the same and equals $1/2$. If $\delta > \beta$ then service integration would lead to zero advertising levels, whereas under independent pricing it would be strictly positive. Intuitively, under service integration users pay both a price and incur disutility from viewing advertisements, whereas under independent pricing platform services are free. Thus, for a strong aversion to advertisements, firms would set lower advertising levels under service integration. Now, consider $\delta \in (0, \beta)$. Then the comparison of advertising levels would depend on equilibrium values given by (22) and (33). If $\frac{\beta-\delta}{2\beta} > \frac{1}{2} + \frac{t}{\delta} - \sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}}$, then service integration equilibrium, i.e., case IN, NI or II would have more advertising. This depends on the nuisance costs of advertisements, targeting rate, and transportation cost. While transportation cost does not affect the service integration equilibrium advertising, it does affect it under independent pricing. Whereas targeting rate affects the advertising under service integration but not under independent pricing. If competition between firms is sufficiently weak (large t), then advertising under independent pricing is

[11]Note that under the independent pricing, the distribution function is $F(x)$ and under bundling, it is $\widetilde{F}(x)$.

greater than under service integration. Under assumption 3, this scenario would hold, and service integration equilibrium would always have lower advertising levels. The preceding discussion is summarized in the following proposition.

Proposition 1. *When either firm* G *or* S *or both can adopt service integration, then we have that, for each firm, service integration*

i. reduces prices charged to users for a) sufficiently small Δ*, and b) increases it otherwise, and*

ii. reduces advertising levels.

Comparison of profits under independent pricing and service integration

We next turn to one of the central results of this paper, namely how service integration by either or both firms affects the equilibrium profit of each firm. Proposition 2 below shows that depending on the level of firm G's quality advantage for the product (Δ), service integration can increase or decrease the profit of firm $i, i = G, S$. In particular, we find that for low level of quality advantage for the product, service integration decreases both firms' profit, whereas for intermediate level of quality advantage, it increases firm G's profit but decreases firm S' profit. For sufficiently large quality advantage for the product, it increases both firms' profit.

Proposition 2. *When either firm* G *or firm* S *or both can adopt service integration, then there exist thresholds* $\Delta_G(\delta)$ *and* $\Delta_S(\delta)$*, where* $0 < \Delta_G(\delta) < \Delta_S(\delta) < 3t$ *and* $\Delta'_G(\delta), \Delta'_S(\delta) < 0$ *such that given a market characterized by* (δ, Δ) *we have that*

i. both firms' adopt independent pricing for $0 < \Delta \leq \Delta_G(\delta)$*, i.e. case* NN *is an equilibrium,*

ii. firm G *adopts service integration whereas firm* S *adopts independent pricing for* $\Delta_G(\delta) < \Delta \leq \Delta_S(\delta)$*, i.e. case* IN *is an equilibrium and*

iii. both firms' adopt service integration for $\Delta_S(\delta) < \Delta < 3t$*, i.e. case* II *is an equilibrium.*

Figure 3 represents the market equilibrium as a function of the nuisance cost of advertisement (δ) and level of firm G's quality advantage for the product (Δ).

To understand the intuition behind the preceding proposition, we decompose the effect of when either firm G or S or both adopt service integration on profitability into three different effects: *market share effect*, *demand elasticity effect*, and *competitive bottleneck effect*. We discuss each of the following channels in detail:

i. The *market share effect* arises because service integration changes each firm's market share for both product and service. First, consider demand for firm G's system G1G2. Service integration

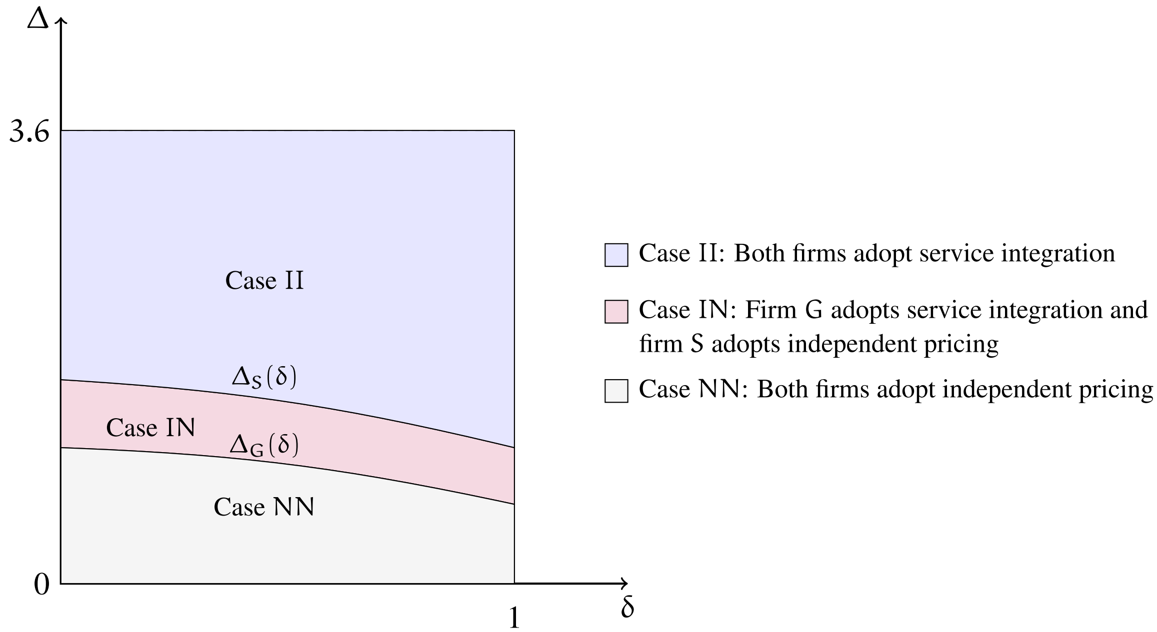

Figure 3: Comparison of Profits under Different Market Regimes

The figure is drawn for parameter values $t = 1.2$, and $\beta = 1$. On the horizontal axis, the parameter δ ranges from 0 to 1. The thresholds $\Delta_G(\delta)$ ($\Delta_S(\delta)$) represents the loci of points along which firm G's (firm S') profit under independent pricing and service integration are the same.

increases the demand for the service G2 and reduces the demand for the product G1. To see this, consider the demand for the system at price (p^*_{G1}, p^*_{S1}). Using the location of the average consumer and the distribution function given in (30), it is $\frac{1}{2} + \frac{\Delta}{6t}\left(1 - \frac{\Delta}{12t}\right)$. Since, for $\Delta > 0$, it is strictly greater than 1/2, the demand for service G2 increases with integration. Moreover, at price (p^*_{G1}, p^*_{S1}), the demand for the product is $\frac{1}{2} + \frac{\Delta}{6t}\left(1 - \frac{\Delta}{12t}\right) < \frac{3t+\Delta}{6t}$. Since firm G earns a positive margin through product sales, reduced sales would decrease its profit. Next, consider the demand for firm S' system S1S2. Evaluated at prices (p^*_{G1}, p^*_{S1}), it is $\frac{1}{2} - \frac{\Delta}{6t}\left(1 - \frac{\Delta}{12t}\right)$. For all $\Delta > 0$, it is less than 1/2, but greater than $\frac{3t-\Delta}{6t}$. Thus, the number of users joining firm S' service S2 decreases but more users are now purchasing its product S1. The sign of market share effect for each firm would depend on the strength of change in demand in the two markets.

ii. The *demand elasticity effect* arises because of the change in distribution function of consumer's average location, and more importantly, a consumer considers the utility from both paid and free products while making the decision to switch in response to a price increase. Hence, in response to a price increase, it is less likely to switch. As a result, demand becomes inelastic with integration relative to independent pricing, raising each firm's market power over the users and softening price competition between firms. Thus, it induces each firm to charge a higher price.

iii. Finally, the *competitive bottleneck effect* arises because with integration, firm $i, i = G, S$, cannot expropriate the advertising gains and there is a full pass-through of advertising revenues to the users in the form of lower prices, reducing profits. The magnitude of the competitive bottleneck effect depends on the level of nuisance cost of advertisement parameter, δ. For small δ, there is a large pass-through of advertising revenues, whereas for large δ it is the opposite. Thus, it would be large mainly for small to intermediate values of δ and negligible for $\delta \to 1$.

First, consider the case when δ is small to intermediate. For weak level of quality advantage (i.e. $0 < \Delta \leq \Delta_G(\delta)$), the competitive bottleneck dominates demand elasticity effect, intensifying price competition between firms. Moreover, the market share effect is negative for each firm. This occurs because the average consumer will be located close to 1/2. For firm G, the increment in demand for its service S2 is small, whereas positive profit margins are lost for it product G1. Similarly, firm S' increment in market share for the product S1 is small relative to lost market share for the service S2. This generates a negative market share effect which reinforces the competitive bottleneck effect, reducing each firm's profits. Next, consider a large level of quality advantage (i.e. $\Delta_S(G) < \Delta < 3t$). Then each firm's demand becomes sufficiently inelastic and dominates competitive bottleneck effect, softening price competition between firms. Moreover, market share effect is positive for each firm because firm G gains sufficient users for its service G2, and firm S' increase in demand for product S1 is also sufficiently large. Thus, each firm benefits from service integration because it helps in *better market segmentation*, i.e. firm G is able to improve its profits through expanding its service $G2'$s user base, whereas firm S gains because of increase in product S1's user base. However, for intermediate level of quality advantage (i.e. $\Delta_G(\delta) < \Delta < \Delta_S(\delta)$), service integration has an asymmetric effect on firms' profit. This is because of asymmetric market share effect dominating any incentive to change price. For firm G, the increase in demand for the service G2 is sufficient to generate a positive market share effect, increasing its profits. Whereas, for firm S, the increase in demand for the product S1 is insufficient to compensate for reduce demand for service S2, leading to a negative market share effect and reducing its profit.

Next, consider the case when δ is large (close to 1). In this case, competitive bottleneck effect is negligible, and the effect of service integration on firm i's profit depends on the interplay of market share and demand elasticity effects. Using a similar argument, as for the previous case, we can argue that integration would increase the profit of firm $i, i = G, S$, only for large level of quality advantage, and decreases it otherwise.

Note that, as in the previous literature examining competitive bundling/compatibility choices in markets without an advertising side (e.g., Hurkens et al. (2019)), in our framework as well, firms choose service integration considering the change in price competition on the user side. However, when adopting a integration strategy in these markets, firms consider only the preferences of the user side. By contrast, when competing in the platform market, firms have to consider the impact of the advertising side on the price competition on the user side. The decision to integrate the service would depend on maximizing profit considering both sides, i.e. user and advertiser sides in our case. In particular, integrating a free online service introduces novel trade-offs that are absent in the markets with no advertisements. First, as discussed above, a firm would operate as a bottleneck that would enjoy market power over advertisers and compete for users. To illustrate the change in prices, consider equation (35). The first term captures the price component due to the integration of product and service, similar to that in one-sided markets. However, we have an additional second component, i.e. $(\beta^2 - \delta^2)/4\beta$, which captures the intensified price competition due to the specific nature of the platform market, i.e. lowering prices to attract users for the advertisers. Thus, incorporating the advertising side in our environment and the resulting intensified price competition leads to moderation in the incentives of firms to adopt integration in comparison to competition in markets with no advertising side. Next, consider the novel effect of introducing a free service due to the zero-price nature of the service. Integrating a free service makes consumer preferences more inelastic, and thus firms' market power increases, raising the ability to charge higher prices due to softened competition. As a result, the results differ substantially from the literature studying competitive bundling/compatibility decisions in markets without advertisements. For instance, unlike Hurkens et al. (2019), we have shown profitable leveraging of quality advantage through service integration by firm G even when per-unit transportation costs are the same for the product and service sold in a market (refer case IN in figure 3).

5 Welfare Analysis

Social Welfare

Based on the equilibrium analysis done in the last section, we now examine and compare the social welfare under independent pricing and service integration. It is defined as the sum of users' surplus, advertisers' profit and firms' profit. Since prices are just transfers in the model, it equals the sum of *net* users' surplus (net of transportation costs) and *net* advertisers' surplus (net of nuisance costs). In order to understand the welfare effects of service integration, we need to look at the various trade-offs involved. First, service integration raises the users' surplus as more users will be consuming the product G1G2 that has a larger standalone value. Second, it raises the total transportation cost because of the redistribution of users across the two firms. Now, users are restricted from choosing the most preferred system, and thus there is a larger mismatch between their preference and the system location. Third, service integration reduces the total nuisance costs of advertisements because of lower advertisements. Fourth, it also decreases the advertising revenue. The following proposition summarizes the main results on welfare analysis.

Proposition 3. *When either firm* G *or firm* S *or both can adopt service integration, then we have that service integration*

i) *decreases social welfare for sufficiently small level of quality advantage, and*

ii) *increases it otherwise.*

In order to understand the preceding proposition, we consider the change in welfare in $\delta - \Delta$ space. It could be understood by analysing the change in net user surplus (net of transportation costs) and net advertising surplus (net of nuisance costs). Two important points need to be highlighted. First, note that the change in net advertising surplus (net of nuisance costs) is independent of dominance level Δ. It depends only on the magnitude of nuisance costs, δ. For small δ, it decreases with service integration, whereas for large δ, it increases. Second, the change in social welfare is dominated by the change in user surplus and transportation costs. For small to intermediate level of quality advantage (small Δ), the increase in user surplus is small. However, transportation costs increases and would dominate the change in user surplus and net advertising surplus, decreasing social welfare. Now, consider intermediate to large level of quality advantage for the product. The gain in user surplus from consuming the system G1G2 dominates the rise in transportation costs, and change in net advertising surplus, increasing social welfare. Figure 4 illustrates the preceding discussion for $t = 1.2$ and $\beta = 1$. As can be seen from the figure, there is a threshold $\Delta_{sw}(\delta)$ such that welfare is same under independent pricing and service integration. For $0 < \Delta \leq \Delta_{sw}(\delta)$, social welfare decreases, and for $\Delta_{sw}(\delta) < \Delta < 3t$, social welfare increases. Also, the threshold is weakly decreasing in δ. This is because for larger δ, net advertising surplus increases with service integration and thus, we would require a smaller value of Δ for social welfare to increase.

Moreover, figure 4 also highlights the regions in which the number of firms adopting service integration is socially sub-optimal (under-provision of the service integration) or socially excessive (over-provision of the service integration).

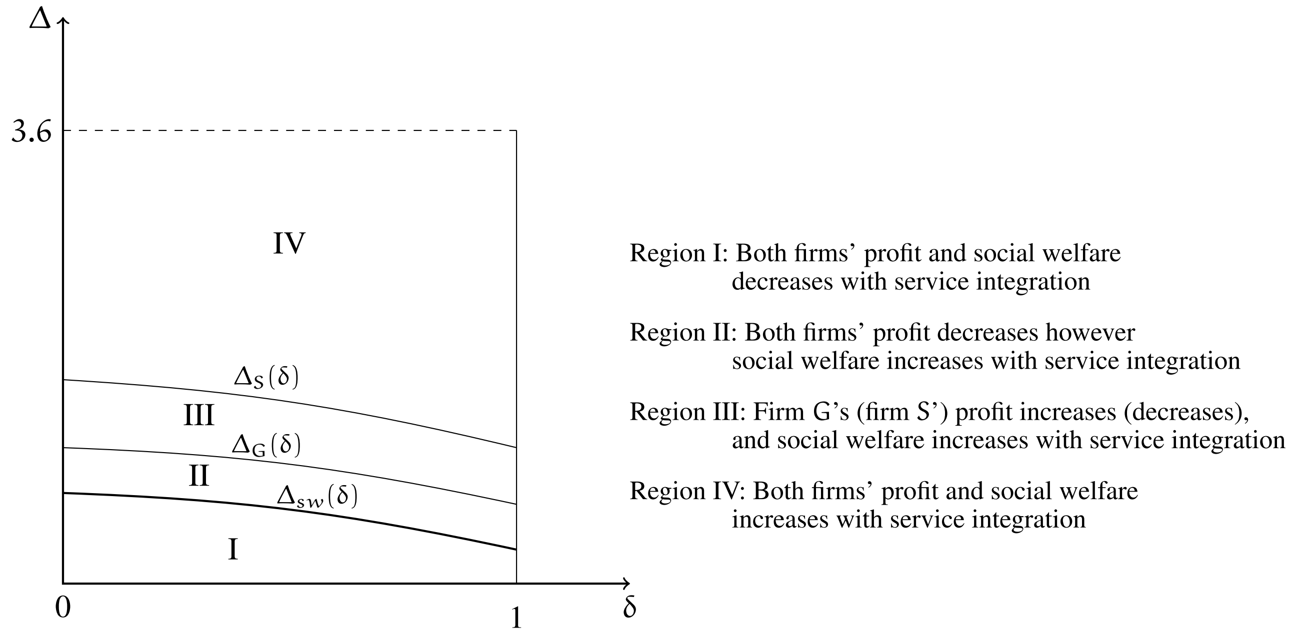

Figure 4: Comparison of Firms' Profit and Social Welfare under Different Market Regimes
The figure is drawn for parameter values $t = 1.2$, and $\beta = 1$. The thresholds $\Delta_G(\delta)$ and $\Delta_S(\delta)$ are as defined previously. The threshold $\Delta_{sw}(\delta)$ represents the loci of points along which social welfare under independent pricing and service integration are the same. In regions II and III, there is under-provision of the service integration.

From the figure, it is clear that there can be a divergence between private and social incentive to bundle. This holds in regions II and III, where there can be under-provision of the service integration. In region II, quality advantage is not large enough ($\Delta_{sw}(\delta) \leq \Delta \leq \Delta_G(\delta)$), generating a negative market share effect and that jointly with competitive bottleneck effect reduces both firms' profit from service integration. In region III, due to asymmetric market share effects, firm G gains sufficient market share for its service and finds it optimal to adopt service integration, however firm S' increment in market share for its product is insufficient and adopts independent pricing. However, in both regions II and III, from a social point of view, quality advantage is large enough to raise users' surplus sufficiently, increasing social welfare. Thus, in these two regions, while it is socially optimal that both firms adopt service integration, in market equilibrium either none of the firms adopt service integration (region II) or only firm G adopts service integration (region III). Hence, there is under-provision of the service integration.

Corollary 1. *When firm* G*'s quality advantage is at an intermediate level, then adopting service integration by both firms increases social welfare, however in market equilibrium, either none of the firms or only firm* G *adopts service integration. There is under-provision of the service integration.*

User Welfare

Competition authorities dealing with cases related to bundling/tying/platform incompatibility in two sided markets are interested not just in how regulation affects social welfare, but also how user welfare changes with antitrust intervention. In fact, major antitrust decisions on tying cases in platform markets have focussed on understanding the user welfare components to regulate the firm behaviour.[12] If we reinterpret service integration as competitive bundling or incompatibility, then our model can help explain changes in user welfare with policy intervention. In order to do that, we need to delineate the various trade-offs. First, service integration makes more users consume the system $G1G2$ that has a larger standalone value. However, user welfare change also depends on the price change. As discussed in previous section, service integration makes demand inelastic, inducing firms to raise products prices and thus can reduce user welfare. Moreover, service integration reduces user welfare because of redistribution of users across two the firms raising the total transportation cost. However, it leads to a decrease in the total nuisance costs because of lower advertising levels. So, the net effect on aggregate user welfare would depend on the strength of these trade-offs. The main insight is summarized in the following proposition.

Proposition 4. *When firm G or firm S or both can adopt service integration, then we have that service integration*

i) increases user welfare for small level of quality advantage, and

ii) decreases it otherwise.

Intuitively, when Δ is small to intermediate, then prices under service integration either decrease or increase by a small magnitude. Moreover, nuisance costs decreases and jointly with rise in user surplus from consuming the system $G1G2$ dominates the increase in prices and transportation cost, increasing user welfare. Whereas, when Δ is large, service integration makes demand sufficiently inelastic, raising the price paid for the system by the users, and which, in turn, reduces user welfare. Figure 5 illustrates this for $t = 1.2$ and $\beta = 1$. It highlights the threshold $\Delta_{uw}(\delta)$ such that user welfare under independent pricing and service integration are the same. For $0 < \Delta \leq \Delta_{uw}(\delta)$, user welfare increases and for $\Delta_{uw}(\delta) < \Delta < 3t$, it decreases. Also, it can be

[12] The investigation of Microsoft by the European Commission (EC) was one of the biggest antitrust tying cases in the European Union. Briefly, Microsoft was accused of abusing its dominant position in the PC Operating System market in two ways. The first related to compatibility issues in the work group server market, and the second to Microsoft's practice of tying its Windows Media Player (WMP) to the Windows OS. On 24th May 2004, European Commission, in its decision on Microsoft case, required Microsoft to offer a version of its Windows client PC OS without WMP to PC manufacturers. Similarly, mandatory unbundling has been decided upon in the Google Android case where the European Commission in its decision has required, among other things, Google to stop forcing manufacturers to pre-install Chrome and Google search in order to offer the Google Play Store on their handsets.

seen that $\Delta_{uw}(\delta)$ is weakly increasing in δ because as δ increases, the system price increases and thus, we need a higher value of Δ for service integration to increase user welfare.

Next, a comparison with social welfare shows that there can be parameter regions where service integration improves social welfare but at the cost of reduced user welfare. Figure 5 below highlights this result. There are five subregions differentiated based on how service integration affects firms' profit, user, and social welfare. In region I, competitive bottleneck and market share effects reinforce each other, reducing both firms' profit. Moreover, since the quality advantage Δ is small, it generates an insufficient increase in users' surplus, reducing social welfare. However, due to lower quality advantage, the change in system prices is dominated by a reduction in the nuisance costs of advertisements and an increase in user surplus, raising user welfare. In region II, service integration still reduces both firms' profit and social welfare. Moreover, system prices increase sufficiently to reduce user welfare as well. In region III, social welfare rises because user surplus increases as a sufficient number of users consume the higher quality system $G1G2$. However, both firms' profit and user welfare fall. In region IV, firm G adopts service integration and firm S adopts independent pricing with different effects on social and user welfare. The former increases because of increase in users' surplus with service integration, whereas the latter decreases because demand becomes sufficiently inelastic, raising the price paid for the system. Thus, in this region, an anti-competitive service integration by firm G would reduce firm S' profit, however it will have different policy implications depending on the welfare standard followed by the antitrust authorities. Finally, in region V both firms adopt service integration reducing user welfare but increasing social welfare.

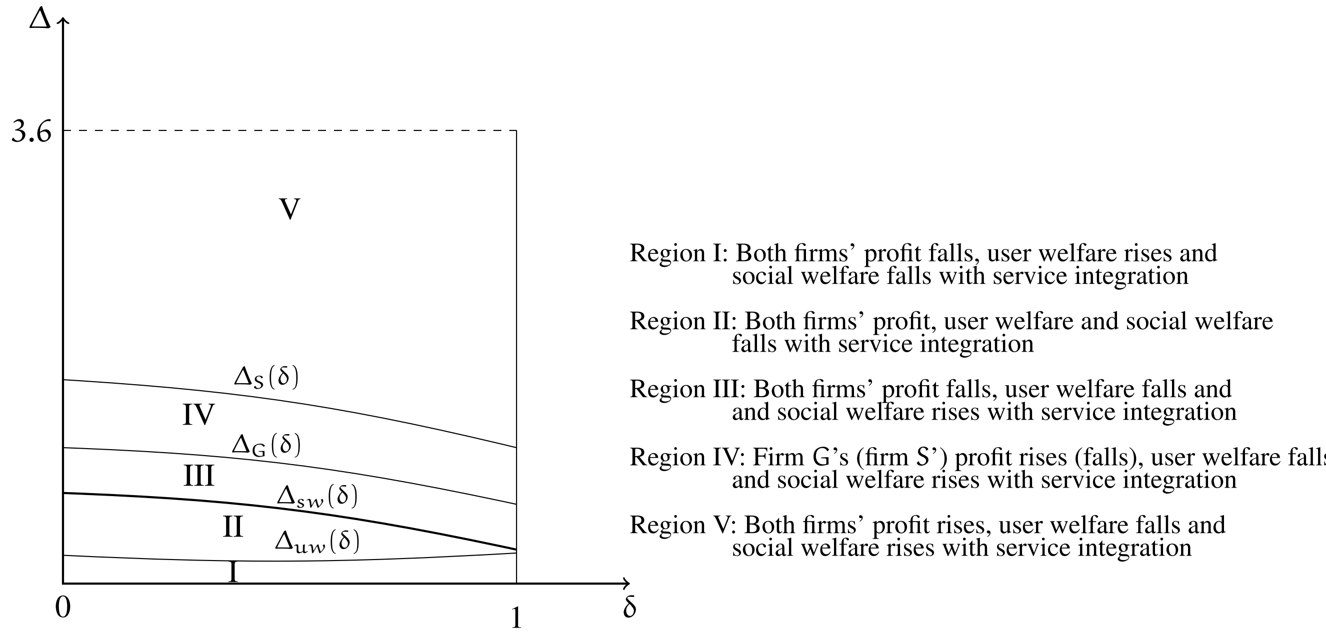

Figure 5: Comparison of Firms' Profit, User Welfare and Social Welfare under Different Market Regimes

The figure is drawn for parameter values $t = 1.2$, and $\beta = 1$. The thresholds $\Delta_G(\delta)$, $\Delta_S(\delta)$ and $\Delta_{sw}(\delta)$ are as defined previously. The threshold $\Delta_{uw}(\delta)$ represents the loci of points along which user welfare under independent pricing and service integration are the same.

From figure 5, it is clear that the parameter region where the firm(s) and society would disagree depends on the welfare standard. Consider regions IV and V. In this case, if competition authorities follow a social welfare standard, service integration would not be prohibited, but users would lose. Whereas, if a user welfare standard is followed, then service integration would be prohibited, but society would lose.

6 Model Extensions

We next extend the baseline framework in several directions, examining if the results go through qualitatively.

6.1 Pay Per Click Model

In the baseline model, we assumed that an advertiser pays an access fee for advertising in firm i's service, $i = G, S$, irrespective of whether or not a user clicks and purchases the product. This might be at deviation from the pricing scheme in certain industry. For instance, "per click pricing" under which search engines collect fees from advertisers every time a consumer clicks on their link. However, this fact can be easily introduced by considering a slight variant of our model (see, e.g., de Cornière and Taylor (2014), Dimakopoulos and Sudaric (2018)). Suppose an advertiser pays a price r_i ($\widetilde{r}_i$) under independent pricing (service integration) for advertising in firm i's service only if a user clicks on its advertisement. Then, under this interpretation, an advertiser α's expected profit will be

$$\pi_\alpha = \begin{cases} (\alpha - r_{i2})\beta N_{i2} : \text{ Independent Pricing, and} \\ \\ (\alpha - \widetilde{r}_i)\beta \widetilde{N}_i : \text{ Service Integration.} \end{cases} \tag{37}$$

At *stage 3*, advertisers make the participation decision. An advertiser would advertise in firm $i, i = G, S$, as along as the marginal benefit of an advertisement is greater than its marginal cost. Using equation (37), the inverse advertising demand function of firm $i, i = G, S$, under independent pricing (case NN) is $r_i = 1 - a_i$, whereas under service integration (case NI, IN or II) it is $\widetilde{r}_i = 1 - \widetilde{a}_i$.

A firm i's profit will be

$$\pi_i = p_{i1}N_{i1} + a_{G2}\beta N_{i2} r_i : \text{ Independent Pricing,} \tag{38}$$

$$\text{and } \widetilde{\pi}_i = \widetilde{p}_i \widetilde{N}_i + \widetilde{a}_G \beta \widetilde{N}_i \widetilde{r}_i : \text{ Service Integration.} \tag{39}$$

The second component of firm i's profit function means that a_i ($\widetilde{a}_i$) advertisers at firm i pay r_i ($\widetilde{r}_i$) whenever N_{i2} ($\widetilde{N}_i$) users click on an advertisement with probability β. Next, using the inverse advertising demand functions, the user demand functions defined in equations (14), (15) ,(17), (18), and (31) and putting the values for them in the profit functions (38) and (39), firm $i, i = G, S$, chooses the user price and advertising quantity to maximize its profits under the different market regimes at stage 2. At stage 1, firms simultaneously decide whether to adopt independent pricing or service integration. Relegating the details to the appendix, proposition 5 summarizes the main findings.

Proposition 5. *When either firm* G *or firm* S *or both can adopt service integration, then we have that*

i. both firms' adopt independent pricing for small level of quality advantage, i.e. case NN *is an equilibrium,*

ii. firm G *adopts service integration whereas firm* S *adopts independent pricing for intermediate level of quality advantage, i.e. case* IN *is an equilibrium and* *iii. both firms' adopt service integration for sufficiently large level of quality advantage, i.e. case* II *is an equilibrium.*

The preceding proposition shows that the main result on market equilibrium remains unchanged.

6.2 Service Pricing

In our main model, we considered a setting with online services that are purely advertising financed. However, the issue of integration is also important for services that are both user and advertising financed. In this extension, we allow firms to charge prices for the services as well. Our main result on the profitability of service integration goes through qualitatively. However, with service pricing, integration always improves social welfare. Note that the equilibrium levels of advertisements are symmetric and equal under the different market regimes, i.e. cases NN, IN, NI and II. This is because of the competitive bottleneck effect that, in contrast to the baseline model, will be present under both regimes. As a result, the net advertising surplus (net of nuisance costs) remains unchanged. Therefore, the change in social welfare would depend only on how net users' surplus changes. Since with service integration, more users would consume the product $G1G2$ which has a larger standalone value, the increase in users' surplus would dominate any increase in mismatch costs because of the redistribution of users across the two firms, increasing social welfare. The main result on social welfare is summarized in the following proposition.

Proposition 6. *When either firm* G *or firm* S *or both can adopt service integration, then we have that service integration always improves social welfare.*

6.3 Quality Advantage for Both Product and Service

In our main analysis, we assumed that service quality is symmetric across two firms. However, a single firm can have a better service quality. We consider this possibility in this extension. Assume that firm G owns both a better quality product and service. Let V_{i2} be firm i's service quality with $V_{G2} > V_{S2}$ and $\Delta = V_{G2} - V_{S2} > 0$. For tractability, we restrict attention to the numerical simulations with $t = 1.2$, $\delta = 0.5$, $\beta = 1$ and consider three possible scenarios with different level of quality advantage, i.e. $\Delta = 0.2, 1$ & 2.8. Relegating the details to the appendix, we find that our qualitative conclusions on firm i's profit comparison remains unchanged, i.e. service integration decreases the equilibrium profit of both firms when the extent of quality advantage is sufficiently small, increases the equilibrium profit of both firms when the level of quality advantage is sufficiently large. For intermediate values of quality advantage, service integration increases firm

G's profit and reduces rival firm S' profit. The preceding discussion is summarized in the following proposition.

Proposition 7. *The main results from the baseline model remain qualitatively unchanged when firm* G *has quality advantage for both product and service.*

Competition with Specialist Firms

In the main model, we have considered competition between two generalist (multi-product) firms G and S. In this section, we consider competition between a multi-product firm G and two specialist firms S1 and S2, where S1 (S2) specialises in product (service). Under independent pricing, competition between firms would be same as described in lemma 1.

Under service integration, firm G's system would compete against two separate firms S1 and S2. Intuitively, this gives rise to an externality problem: each specialist firm $Sj, j = 1, 2$, would choose its price/advertising level ignoring the effect of its decision on the total demand and hence, profit of the other specialist firm Sk, $k = 1, 2$, and $k \neq j$. Consequently, firm S1 would set a higher price and firm S2 would choose a higher advertising level, reducing their market share. As a result, firm G would benefit from this negative externality problem between the specialist firms and its profit will increase. Since firm S2's demand decreases with service integration, it becomes worse off. However, the effect of service integration on firm S1's profitability would depend on the level of firm G's quality advantage for the product. For small level of quality advantage, the decrease in market share would make firm S1 worse off. Whereas for sufficiently large level of firm G's quality advantage, softened price competition (due to sufficiently inelastic demand) would make firm S1 better off with service integration. We examine this using a numerical simulation with $\beta = 1$, $t = 1.2$, $\delta = 0.5$ and three different values of Δ, i.e. $\Delta = 0.2, 1,$ & 2.9. The following proposition summarizes the main result.

Proposition 8. *Consider competition between a generalist firm* G *and two specialist firms* S1 *and* S2*. When firm* G *could adopt service integration, then we have that service integration*

i. always increases firms G*'s profit,*

ii. reduces firm S1*'s profit for small level of quality advantage, and increases it otherwise, and*

iii. always reduces firm S2*'s profit.*

7 Discussion and Conclusion

Our paper theoretically examines the trade-offs that firms would face when considering to adopt service integration in a setting with each firm having a paid hardware product and a free online service with advertisements in it. An important feature of the setting is the asymmetric firm competition with one firm having a superior product and indirect network effects on the advertising side. A key contribution of the paper is to highlight the market conditions under which firms will adopt independent pricing or service integration. We find that both firms' profit decreases (increases) with service integration when the level of quality advantage is small (large). With intermediate level of quality advantage, we find asymmetric effects of service integration on firms' profit. Firm G with better product quality gains and the rival firm S becomes worse off with service integration. In this region, firm G's integration strategy can lead to anti-competitive effects, i.e. firm G leveraging market power in the product space through service integration can have foreclosure effects for the rival firm S' service. In doing so, we complement the study by Hurkens et al. (2019), by opening up the service to a platform setting, and show that platform market structure can explain the variation in leveraging of dominance that we observe in reality. Moreover, we also highlight the tensions that rival firm would face, and highlights the importance for platforms to take strategic differentiation into account when planning to compete with a dominant firm in the presence of network effects.

Managerial Implications

Implications for Compatibility Incentives

Our model explains why would firms open up their services to the rival firms' hardware. This is most likely when the two products are offering similar qualities/functionalities. It is interesting to note the change in Google's strategy as its market share in the streaming media player declined over time. Since it's introduction in 2013, Google Chromecast's market share has declined in the streaming devices market. Associated with this is a shift in Google's startegy to make its video streaming app YouTube compatible with Amazon Fire stick. Similarly, Amazon also offers Amazon Prime and Twitch on Chromecast. This behaviour is consistent with our model's predictions, i.e. firms selling similar hardware devices would embrace independent pricing to increase profits. Since, neither Google nor Amazon has a quality advantage for the streaming media player devices, integrating their proprietary services with their devices and making it unavailable on rival devices is not profitable. This is because, with service integration, their product sales would marginally increase, whereas price competition would intensify, reducing both firms' profit.

Our results also shed light on why many firms choose to adopt an integration between their hardware devices and online services, embracing a closed ecosystem approach. As difference in hardware functionalities increases, there is a greater incentive to strategically differentiate and focus on a different component's user base. In the smart phone, Apple and Huawei have significant differences in the functionalities. Hence, both prefer a closed ecosystem approach with tighter integration between their devices and free services (generating advertising revenue through advertisements in it) such as voice assistants, news, maps, etc.

When is Leveraging of Market Power through Service Integration Strategy Profitable?

Another interesting implication of our result is regarding identifying the market conditions under which a firm with a superior product in digital markets can leverage its market power to the service dimension. Contrary to the knowledge of prior works that suggests that network effects intensify price competition and reduces the profitability of leveraging market power (e.g., Etzion and Pang (2014), Lee and Mendelson (2007), Derdenger and Kumar (2013), etc.), we find that *strong network effects does not deter a dominant firm from leveraging its market power*. In fact, when the quality differentiation between two firms' products is intermediate, then service integration can have asymmetric effects on firms' profit and can help the firm with better quality product to sustain dominance across both product and service. This holds true even when the strength of advertising network benefits is sufficiently large (i.e. β is sufficiently large). Moreover, we find that leveraging is more likely when the competition between online services is strong.

Implications for Optimal Design of the Online Services

Our results also suggest a number of strategies that a rival online service can consider in the face of threat of leveraging by a dominant firm. From proposition 7, it is clear that rival online services are less likely to face a foreclosure risk if they *strategically maintain a quality differentiation by offering a lower quality for the product and service*. In terms of our result , this would help it cover losses through a market segmentation (expanding its hardware sales) and better pricing power over users of its device. Another approach, based on our results, is to invest and match the product quality/ functionalities of the firm G. In this case, service integration would not be profitable and foreclosure risk will be eliminated. Our result complements the result by Zhu and Liu (2018) which shows that complementors can avoid platform owner's entry threat by focussing on platform specific investment in non-popular products where greater seller effort is required. Whereas, we analyse a different market setting and show that the inferior product quality firm can strategically maintaining a lower product quality to enforce quality differentiation or match the rival firm's quality to sustain its service in the presence of leverage threat.

In our model, leveraging is less likely for smaller level of nuisance costs of advertisements (refer figure 3). Proposition 2 shows that the derivative of the threshold for the profitability of firm G's integration strategy ($\Delta_G(\delta)$) is decreasing in the nuisance costs of advertisements, i.e. $\Delta'_G(\delta) < 0$. This suggests that competing online services can mitigate the risk of foreclosure by investing more heavily to reduce nuisance costs for the users. One way is to make advertisements more relevant for them. This require better investments in data processing abilities. Another way is to better inform users about the privacy risks and help them gain better control over the data collection. This again would help in reducing the privacy costs associated with joining a platform and, in turn, make users feel advertisements being less intrusive.

Limitations and Future Research

Future research could be directed to study variants of the market structure in this paper. For instance, we have considered competition between two firms. A more general setting can consider a market structure with more than two multi-product firms and study the service integration decisions of the firms. In addition, our analysis is completely static in that it does not consider quality competition between firms. If integration reduces profit of the rival firm, then it could reduce innovation in the future, dynamic consumer surplus can be lower with integration. Our model (for tractability) assumed that consumers single-home in the platform market. However, one can analyze a market setting where consumers multi-home (see, e.g., Choi (2010), Choi and Jeon (2016), etc.). In this setting, service integration strategy also has to consider how it affects the total multi-homing user base which, in turn, would affect the advertising revenue collected over them. Next, in the main analysis, we assumed that the market is fully covered for the product. If we allow uncovered markets, then independent pricing can also expand the market, as some users didn't purchase the integrated system due to higher mismatch costs would also purchase, raising firm's profits. Both firms would have higher incentives to adopt independent pricing. An extension of the paper can look at system competition in the present setting with uncovered market. Finally, our setting can also be used to study alternative strategies that a firm can use to extend market power in the complementary markets. On way to is to limit and degrade the quality of a rival service when used with its product (see, Miao (2009)).

Appendix A: Proofs of the Baseline Model

Proof of Lemma 1

Under independent pricing, the profit of the firms are

$$\pi_G = p_{G1}N_{G1} + r_{G2}a_{G2} : \text{ Firm G's profit,} \tag{40}$$

$$\text{and } \pi_S = p_{S1}N_{S1} + r_{S2}a_{S2} : \text{ Firm S' profit.} \tag{41}$$

Since profit maximization for the hardware product and online service is independent of each other, we consider each component separately.

First, consider the hardware product. Since profit functions are continuously differentiable, any optimal pair of prices must satisfy the first-order necessary conditions of firms' optimization problem. Using equations (40) and (41) and differentiating w.r.t. prices, they are

$$\frac{\partial \pi_G}{\partial p_{G1}} = N_{G1} + p_{G1}.\frac{\partial N_{G1}}{\partial p_{G1}} = 0, \text{ and} \tag{42}$$

$$\frac{\partial \pi_S}{\partial p_{S1}} = N_{S1} + p_{S1}.\frac{\partial N_{S1}}{\partial p_{S1}} = 0. \tag{43}$$

Note that the first-order conditions (42) and (43) must hold with equality since we allow for negative prices to be charged to the users. Solving them would give us equilibrium prices as defined in lemma 1(ii). They are

$$p^*_{G1} = \frac{3t + \Delta}{3}, \text{ and } p^*_{S1} = \frac{3t - \Delta}{3}. \tag{44}$$

Now, consider online service. Any optimal pair of advertising quantities must satisfy the first-order necessary conditions of firms' optimization problem. Putting the value of $r_{i2} = (1 - a_{G2})\beta N_{G2}$ in equations (1) and (2), and differentiating w.r.t. advertising quantities, they are

$$\frac{\partial \pi_G}{\partial a_{G2}} = (1 - 2a_{G2})\beta N_{G2} + (1 - a_{G2}).a_{G2}.\beta.\frac{\partial N_{G2}}{\partial a_{G2}} \leq 0, \text{ and} \tag{45}$$

$$\frac{\partial \pi_S}{\partial a_{S2}} = (1 - 2a_{S2})\beta N_{S2} + (1 - a_{S2}).a_{S2}.\beta.\frac{\partial N_{S2}}{\partial a_{S2}} \leq 0. \tag{46}$$

First, we argue that the advertising levels in both firms are positive, which would imply that the first-order conditions (45) and (46) bind. This follows since in any equilibrium with both firms having a positive market share, i.e. $0 < N_{G2}; N_{S2} < 1$, and zero advertising levels, i.e. $a^*_{G2} = a^*_{S2} = 0$ would violate (45) and (46). Thus, (45) and (46) would bind, and at symmetric equilibrium, i.e.

$a^*_{G2} = a^*_{S2} > 0$ would give us the advertising levels as defined in lemma 1(i). They are

$$a^*_{G2} = a^*_{S2} = \frac{1}{2} + \frac{t}{\delta} - \sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}}. \tag{47}$$

Second-Order Conditions

Next, we evaluate the Hessian matrix of each firm, denoted by $H_i, i = G, S$. We have

$$H_i = \begin{bmatrix} -\frac{1}{\tau} & 0 \\ 0 & -2\beta N_{G2} - \frac{\beta\delta}{t}.(1-2a_{i2}) \end{bmatrix}$$

From the preceding expression we can see that principal minors of order 1 is negative and straightforward calculations will show that principal minor of order 2 (i.e. determinant of H_i) is positive. Thus, principal minors alternate in sign and $H_i, i = G, S$, is negative semi-definite. Therefore, the solution constitutes a global maximum. Hence proved. □

Proof of Lemma 2

First-Order Conditions

As defined in the main text, $\widetilde{y}$ denote the average location of the indifferent user which is given by

$$\widetilde{y} = \frac{x_1 + x_2}{2} = \frac{1}{2} + \frac{\Delta}{4t} + \frac{\widetilde{p}_S - \widetilde{p}_G}{4t} + \frac{\delta\widetilde{a}_S - \delta\widetilde{a}_G}{4t}. \tag{48}$$

Let $\widetilde{F}(.)$ and $\widetilde{f}(.)$ denote the distribution and probability density functions of the average location $\widetilde{y}$. They are

$$\widetilde{F}(y) = \begin{cases} 2y^2, & \text{if } 0 \le y \le \frac{1}{2}, \text{ and} \\ 1 - 2(1-y)^2, & \text{if } \frac{1}{2} < y \le 1. \end{cases} \qquad \widetilde{f}(y) = \begin{cases} 4y, & \text{if } 0 \le y \le \frac{1}{2}, \text{ and} \\ 4(1-y), & \text{if } \frac{1}{2} < y \le 1. \end{cases} \tag{49}$$

The profit of the firms are

$$\widetilde{\pi}_G = \widetilde{p}_G\widetilde{N}_G + \widetilde{r}_G\widetilde{a}_G : \text{ Firm G's profit,} \tag{50}$$

$$\text{and } \widetilde{\pi}_S = \widetilde{p}_S\widetilde{N}_S + \widetilde{r}_S\widetilde{a}_S : \text{ Firm S' profit.} \tag{51}$$

Since profit functions are continuously differentiable, any optimal pair of prices and advertising quantities must satisfy the first-order necessary conditions of firms' optimization problem. Putting the value of $\widetilde{r}_i = (1 - \widetilde{a}_i)\beta\widetilde{N}_i, i = G, S$, in equations (50) and (51) and differentiating w.r.t. prices and advertising quantities, they are

$$\frac{\partial \widetilde{\pi}_G}{\partial \widetilde{p}_G} = \widetilde{N}_G + (\widetilde{p}_G + \beta.\widetilde{a}_G.(1-\widetilde{a}_G))\frac{\partial \widetilde{N}_G}{\partial \widetilde{p}_{G2}} = 0, \tag{52}$$

$$\frac{\partial \widetilde{\pi}_G}{\partial \widetilde{a}_G} = (\beta.(1-2\widetilde{a}_G))\widetilde{N}_G + (\widetilde{p}_G + \beta.\widetilde{a}_G.(1-\widetilde{a}_G))\frac{\partial \widetilde{N}_G}{\partial \widetilde{a}_{G2}} \leq 0, \tag{53}$$

$$\frac{\partial \widetilde{\pi}_S}{\partial \widetilde{p}_S} = \widetilde{N}_S + (\widetilde{p}_S + \beta.\widetilde{a}_S.(1-\widetilde{a}_S))\frac{\partial \widetilde{N}_S}{\partial \widetilde{p}_{S2}} = 0, \text{ and} \tag{54}$$

$$\frac{\partial \widetilde{\pi}_S}{\partial \widetilde{a}_S} = (\beta.(1-2\widetilde{a}_S))\widetilde{N}_S + (\widetilde{p}_S + \beta.\widetilde{a}_S.(1-\widetilde{a}_S))\frac{\partial \widetilde{N}_S}{\partial \widetilde{a}_{S2}} \leq 0. \tag{55}$$

First, consider the equilibrium advertising levels. If the advertising levels are positive then equations (53) and (55) would bind. Moreover, equation (53) together with (52) or equation (55) together with (54) imply

$$\beta(1-2\widetilde{a}_i) = \delta, \quad i = G, S.$$

Solving the preceding equation gives us the equilibrium advertising level for firm $i, i = G, S$, as defined in lemma 2(i). Note that if $\delta > \beta$, then $\widetilde{a}_G^* = \widetilde{a}_S^* = 0$.

Next, consider the equilibrium prices. Using equations (52) and (54), the equilibrium advertising levels, and the fact that $\widetilde{N}_G = \widetilde{F}(\widetilde{y})$, and $\partial\widetilde{N}_G/\partial\widetilde{p}_G$ = - $\widetilde{f}(\widetilde{y})/4t$, we can obtain

$$\widetilde{p}_G^* = \frac{4t.\widetilde{F}(\widetilde{y})}{\widetilde{f}(\widetilde{y})} - \frac{(\beta^2-\delta^2)}{4\beta}, \text{ and}$$

$$\widetilde{p}_S^* = \frac{4t.(1-\widetilde{F}(\widetilde{y}))}{\widetilde{f}(\widetilde{y})} - \frac{(\beta^2-\delta^2)}{4\beta}.$$

From the preceding equations, it can be seen that we need to solve for equilibrium value of $\widetilde{y}$ to obtain $\widetilde{p}_G^*$ and $\widetilde{p}_S^*$. Putting the value for $\widetilde{p}_S^* - \widetilde{p}_G^*$ and $\widetilde{a}_S^* - \widetilde{a}_G^*$ in (48), we get

$$\widetilde{y} = \frac{1}{2} + \frac{\Delta}{4t} + \frac{1-2\widetilde{F}(\widetilde{y})}{\widetilde{f}(\widetilde{y})}.$$

First. we establish the uniqueness and interiority of $\widetilde{y}^*$ and then we derive the equilibrium $\widetilde{y}$.
Step i. There exists a unique and an interior $\widetilde{y}^*$*, i.e.* $1/2 < \widetilde{y}^* < 1$.

Consider the function $h(\widetilde{y})$ defined as

$$h(\widetilde{y}) = \frac{1}{2} + \frac{\Delta}{4t} + \frac{1 - 2\widetilde{F}(\widetilde{y})}{\widetilde{f}(\widetilde{y})} - \widetilde{y}. \tag{56}$$

From the preceding equation, and using equation (49), we can see that $h(0) > 0$ and $h(1) \to -\infty$. Moreover,

$$dh(.)/d\widetilde{y} = \begin{cases} \frac{-1-8\widetilde{y}^2}{4\widetilde{y}^2} < 0 \text{ if } 0 \leq \widetilde{y} \leq 1/2, \\ \\ \frac{-1-8(1-\widetilde{y})^2}{4(1-\widetilde{y})^2} < 0 \text{ if } 1/2 < \widetilde{y} \leq 1. \end{cases}$$

Therefore, using intermediate value theorem, there exists $\widetilde{y}^* \in (0,1)$ such that $h(\widetilde{y}^*) = 0$, and i) $h(\widetilde{y}) > 0$ for $0 \leq \widetilde{y} \leq \widetilde{y}^*$ and ii) $h(\widetilde{y}) < 0$ for $\widetilde{y}^* < \widetilde{y} \leq 1$. Moreover, at $\widetilde{y} = 1/2$, we have $h(1/2) = \Delta/4t > 0$, implying that $\widetilde{y}^* \in (1/2, 1)$.

Step ii. equilibrium $\widetilde{y}^*$.

Using (49) in (56) and equating $h(\widetilde{y}^*) = 0$, we will get the value $\widetilde{y}^*$ by solving

$$\widetilde{y}^* = \frac{1}{2} + \frac{\Delta}{4t} + \frac{4(1-\widetilde{y}^*)^2) - 1}{4(1-\widetilde{y}^*)^2}.$$

This gives

$$\widetilde{y}^* = \frac{7t + \Delta/2 - \sqrt{9t^2 + \Delta^2/4 - t\Delta}}{8t}.$$

Now using the value of $\widetilde{y}^*$ in (30), we get

$$f(\widetilde{y}^*) = \frac{t - \Delta/2 + C}{2t}, \quad \text{and } \widetilde{F}(\widetilde{y}^*) = \frac{32t^2 - (t - \Delta/2 + C)^2}{32t^2},$$

where $C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}$. Finally, using the preceding values in optimal price functions, we get the bundling prices as defined in lemma 2(ii). They are

$$\widetilde{p}_G^* = \frac{32t^2 - (t - \Delta/2 + C)^2}{4(t - \Delta/2 + C)} - \frac{(\beta^2 - \delta^2)}{4\beta}, \text{ and } \widetilde{p}_S^* = \frac{t - \Delta/2 + C}{4} - \frac{(\beta^2 - \delta^2)}{4\beta}. \tag{57}$$

Second-Order Conditions

Next, we evaluate the Hessian matrix of each firm, denoted by $H_i, i = G, S$. We have

$$H_i = \begin{bmatrix} -\frac{1}{t} & -\frac{\delta}{2t} - \frac{\beta}{2t}(1-2\widetilde{a}_i) \\ -\frac{\delta}{2t} - \frac{\beta}{2t}(1-2\widetilde{a}_i) & -2\beta\widetilde{N}_i - \frac{\beta\delta}{t}.(1-2\widetilde{a}_i) \end{bmatrix}$$

From the preceding expression we can see that principal minors of order 1 is negative. Moreover, evaluated at the equilibrium values defined in lemma 2, straightforward calculations will show that principal minor of order 2, i.e. determinant of $H_i = 2\beta\widetilde{N}_i/t$ is positive. Thus, principal minors alternate in sign and $H_i, i = G, S$, is negative semi-definite. Therefore, the solution constitutes a global maximum. Hence proved. □

Proof of Proposition 1

First, consider firm G's optimal prices under independent pricing (case NN) and service integration (case IN, NI ot II). Using lemmas 1 and 2, the equilibrium product prices under independent pricing and service integration are

$$p^*_{G1} = \frac{3t+\Delta}{3} \quad \text{and} \quad \widetilde{p}^*_G = \frac{32t^2 - (t - \Delta/2 + C)^2}{4(t-\Delta/2+C)} - \frac{(\beta^2-\delta^2)}{4\beta},$$

where $C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}$. Now, define $z(\Delta) = \widetilde{p}^*_G - p^*_{G1}$. We show that i) $z(\Delta)$ is monotonically increasing in Δ and ii) $z(\Delta)$ is negative for small values of Δ and positive otherwise.

Step i. $z(\Delta)$ is monotonically increasing in Δ.

Taking derivative of $z(.)$ w.r.t. Δ gives

$$\frac{\partial z(.)}{\partial \Delta} = \left[\frac{8t^2}{(t-\Delta/2+C)^2} + \frac{1}{4}\right]\left[\frac{1}{2} - \frac{(\Delta/2 - t)}{2\sqrt{9t^2+\Delta^2/4 - t\Delta}}\right] - \frac{1}{3}.$$

Straightforward calculations will show that the preceding expression is positive if

$$44t^2 - \Delta^2 + 4t\Delta + (t - \Delta/2 + C)^2 > 0.$$

Since $\Delta < 3t$ (assumption 2), the preceding inequality always holds. Thus, $z(\Delta)$ is monotonically increasing.

Step ii. $z(\Delta)$ is negative for small values of Δ and positive otherwise.

As $\Delta \to 0$, we have $z(\Delta) \to -(\beta^2 - \delta^2)/4\beta < 0$. Whereas when $\Delta \to 3t$, we have $z(\Delta) \to (16t - t.(\sqrt{33}-1)/8)/(\sqrt{33}-1) - (\beta^2-\delta^2)/4\beta$. It is positive if

$$t > 0.307\left(\frac{\beta^2-\delta^2}{4\beta}\right).$$

Given assumptions 1-3, the preceding inequality holds for all $\delta, \beta \in [0,1]$.

Hence, by intermediate value theorem, there exists a threshold $\Delta_{pG}(\delta) \in (0, 3t)$ such that $z(\Delta_{pG}(\delta)) = 0$. In other words, i) for $0 < \Delta \leq \Delta_{pG}(\delta)$, we have $z(\Delta(\delta)) \leq 0$ implying $\widetilde{p}_G^* \leq p_{G1}^*$, and ii) for $\Delta_{pG}(\delta) < \Delta < 3t$, we have $z(\Delta) > 0$ implying $\widetilde{p}_G^* > p_{G1}^*$.

Now, consider firm S' optimal product prices under independent pricing (case NN) and service integration (case NI, IN, or II). Using lemmas 1 and 2, the equilibrium prices are

$$p_{S1}^* = \frac{3t-\Delta}{3} \text{ and } \widetilde{p}_S^* = \frac{t-\Delta/2+C}{4} - \frac{\beta^2-\delta^2}{4\beta},$$

where $C = \sqrt{9t^2+\Delta^2/4-t\Delta}$. Now, define $h(\Delta) = \widetilde{p}_S^* - p_{S1}^*$. We show that i) $h(\Delta)$ is monotonically increasing in Δ and ii) $h(\Delta)$ is negative for small values of Δ and positive otherwise.

Step i. $h(\Delta)$ *is monotonically increasing in* Δ.
Define $h(\Delta) = \widetilde{p}_S^* - p_{S1}^*$. Taking derivative of $h(.)$ w.r.t. Δ gives

$$\frac{\partial h(.)}{\partial \Delta} = -\frac{1}{4}\left[\frac{1}{2} - \frac{(\Delta/2-t)}{2\sqrt{9t^2+\Delta^2/4-t\Delta}}\right] + \frac{1}{3}.$$

Straightforward calculations will show that the preceding expression is positive if $216t^2 + 4\Delta^2 - 16t\Delta > 0$. Since $\Delta < 3t$ (assumption 2), this always holds and thus, $h(\Delta)$ is monotonically increasing.

Step ii. $h(\Delta)$ *is negative for small values of* Δ *and positive otherwise.*
As $\Delta \to 0$, we have $h(\Delta) \to -(\beta^2-\delta^2)/4\beta < 0$. Whereas as $\Delta \to 3t$, $h(\Delta) \to (\sqrt{33}-1)t/8 - (\beta^2-\delta^2)/4\beta$. Given assumption 3, it is positive.

Hence, by intermediate value theorem, there exists a threshold $\Delta_{pS}(\delta) \in (0, 3t)$ such that $h(\Delta_{pS}(\delta)) = 0$. In other words, i) for $0 < \Delta \leq \Delta_{pS}(\delta)$, we have $h(\Delta) \leq 0$ implying $\widetilde{p}_S^* \leq p_{S1}^*$, and ii) for $\Delta_{pS}(\delta) < \Delta < 3t$, we have $h(\Delta) > 0$ implying $\widetilde{p}_S^* > p_{S1}^*$.

Next, we compare the advertising levels under independent pricing (case NN) and service integration (case NI, IN, or II). Let us define $a_{G2}^* = a_{S2}^* = a^*$, and $\widetilde{a}_G^* = \widetilde{a}_S^* = \widetilde{a}^*$. Using lemmas 1 and

2, they are

$$\alpha^* = \frac{1}{2} + \frac{t}{\delta} - \sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}}, \text{ and } \widetilde{\alpha}^* = \frac{\beta - \delta}{2\beta}.$$

From the preceding equation it can be seen that if $\delta = \beta$, then $\alpha^* > \widetilde{\alpha}^*$ always. If $\delta < \beta$, then α^* is greater than $\widetilde{\alpha}^*$ depending on t and δ. We can find a threshold t_α where

$$t_\alpha = \frac{\beta^2 - \delta^2}{4\beta}$$

such that i) for $t \leq t_\alpha$, $\alpha^* \leq \widetilde{\alpha}^*$ and ii) for $t > t_\alpha$, $\alpha^* > \widetilde{\alpha}^*$. Straightforward calculations will show that the preceding threshold is less than 1 for all $\delta, \beta \in [0, 1]$. Hence, under assumption 3, we have $\alpha^* > \widetilde{\alpha}^*$. Hence proved. □

Proof of Proposition 2

First, we compare firm G's profit under independent pricing (case NN) and service integration (case NI, IN, or II). Using lemmas 1 and 2, the equilibrium profits under the two regimes are

$$\pi_G^* = \frac{(3t + \Delta)^2}{18t} + \frac{\beta t}{\delta}\left[\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}} - \frac{t}{\delta}\right], \text{ and } \widetilde{\pi}_G^* = \frac{[32t^2 - (t - \Delta/2 + C)^2]^2}{128t^2(t - \Delta/2 + C)},$$

where $C = \sqrt{9t^2 + \Delta^2/4 - t\Delta}$. Now, define $g(\Delta) = \widetilde{\pi}_G^* - \pi_G^*$. We show that i) $g(\Delta)$ is monotonically increasing in Δ and ii) $g(\Delta)$ is negative for small values of Δ and positive otherwise.

step i. $g(\Delta)$ is monotonically increasing in Δ.
It can be written as

$$g(\Delta) = \frac{8t^2}{(t - \Delta/2 + C)} + \frac{(t - \Delta/2 + C)^3}{128t^2} - \frac{(t - \Delta/2 + C)}{2} - \frac{(3t + \Delta)^2}{18t} - \frac{\beta t}{\delta}\left[\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}} - \frac{t}{\delta}\right]. \quad (58)$$

Taking the derivative of the preceding expression w.r.t. Δ gives

$$\frac{\partial g(\Delta)}{\partial \Delta} = \frac{8t^2}{2C(t - \Delta/2 + C)} - \frac{3(t - \Delta/2 + C)^3}{256t^2 C} + \frac{(t - \Delta/2 + C)}{4C} - \frac{(3t + \Delta)}{9t}.$$

Using the numerical simulation (with $t = 1.2, \beta = 1, \delta = 0.5$), we find that the preceding expression is positive for all $\Delta > 0$. Since, deriving an analytical solution is not possible, we assume it to hold in the general framework.

step ii. $g(\Delta)$ is negative for small values of Δ and positive otherwise.
Consider $\Delta \to 0$. Then, $g(\Delta) = -\frac{\beta t}{\delta}\left[\sqrt{\frac{1}{4}+\frac{t^2}{\delta^2}}-\frac{t}{\delta}\right]$. Whereas, when $\Delta \to 3t$, then we have $g(\Delta) = 0.3t - \frac{\beta t}{\delta}\left[\sqrt{\frac{1}{4}+\frac{t^2}{\delta^2}}-\frac{t}{\delta}\right]$. It is positive if $t > 0.416\beta - \frac{.15\delta^2}{\beta}$. Under assumption 3, this always hold. Hence, by intermediate value theorem, there exists $\Delta_G(\delta) \in (0, 3t)$ such that $g(\Delta_G(\delta)) = 0$. In other words, a) for $0 < \Delta \leq \Delta_G(\delta)$, we have $g(\Delta) \leq 0$ implying $\widetilde{\pi}^*_G \leq \pi^*_G$ and b) for $\Delta_G(\delta) < \Delta < 3t$, we have $g(\Delta) > 0$ implying $\widetilde{\pi}^*_G > \pi^*_G$. Moreover, since π^*_G is strictly decreasing in δ, the derivative $\Delta'_G(\delta) < 0$.

Next, consider firm S' profit under independent pricing (case NN) and service integration (case NI, IN, or II). Using lemmas 1 and 2, the equilibrium profits are

$$\pi^*_S = \frac{(3t-\Delta)^2}{18t} + \frac{\beta t}{\delta}\left[\sqrt{\frac{1}{4}+\frac{t^2}{\delta^2}}-\frac{t}{\delta}\right], \text{ and } \widetilde{\pi}^*_S = \frac{(t-\Delta/2+C)^3}{128t^2},$$

where $C = \sqrt{9t^2+\Delta^2/4-t\Delta}$. We show that i) $s(\Delta)$ is monotonically increasing for small Δ and decreasing otherwise, and ii) $s(\Delta)$ is negative for small values of Δ and positive otherwise.

step i. $s(\Delta)$ is monotonically increasing for small Δ and decreasing otherwise.
It can be written as

$$s(\Delta) = \frac{(t-\Delta/2+C)^3}{128t^2} - \frac{(3t-\Delta)^2}{18t} - \frac{\beta t}{\delta}\left[\sqrt{\frac{1}{4}+\frac{t^2}{\delta^2}}-\frac{t}{\delta}\right].$$

Taking the derivative of the preceding expression w.r.t. Δ gives

$$\frac{\partial s(\Delta)}{\partial \Delta} = -\frac{3(t-\Delta/2+C)^3}{256Ct^2} + \frac{(3t-\Delta)}{9t}.$$

Consider $\Delta \to 0$. This gives $\partial s(\Delta)/\partial\Delta = 1/12$. Now, consider $\Delta \to 3t$. We have $\partial s(\Delta)/\partial\Delta = -\frac{0.156t}{C}$. Moreover, we conduct a numerical simulation (with $t = 1.2, \beta = 1, \delta = 0.5$) and find that $\partial s(\Delta)/\partial\Delta$ is strictly positive for small Δ and strictly negative otherwise. Since, deriving an analytical solution is not possible, we assume it to hold in the general framework.

step ii. $s(\Delta)$ is negative for small Δ and positive otherwise.

We show this step in two parts.

part a. As $\Delta \to 3t$, $s(\Delta) > 0$.

For $\Delta \to 3t$, we have

$$s(\Delta) = .104t - \frac{\beta t}{\delta}\left[\sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}} - \frac{t}{\delta}\right].$$

Under assumption 3, straightforward calculations will show that the preceding expression will be positive for all $\delta, \beta \in [0, 1]$.

part b. There exists a unique $\Delta_S(\delta) \in (0, 3t)$ such that $s(\Delta) \leq 0$ for $0 < \Delta \leq \Delta_S(\delta)$ and is positive otherwise.

Suppose not, i.e. there exists $\Delta_S(\delta), \Delta_{S'}(\delta) \in (0, 3t)$ such that $s(\Delta_S(\delta)) = s(\Delta_{S'}(\delta)) = 0$. Also, w.l.o.g., assume that $\Delta_S(\delta) < \Delta_{S'}(\delta)$. Then, using the fact that $s(\Delta) < 0$ as $\Delta \to 0$ and step i) claim, we can argue that

bi). for $0 < \Delta \leq \Delta_S(\delta)$, we have $\widetilde{\pi}_S^*(\Delta) \leq \pi_S^*(\Delta)$,

bii). for $\Delta_S(\delta) < \Delta \leq \Delta_{S'}(\delta)$, we have $\widetilde{\pi}_S^*(\Delta) \geq \pi_S^*(\Delta)$, and

biii). for $\Delta_{S'}(\delta) < \Delta < 3t$, we have $\widetilde{\pi}_S^*(\Delta) < \pi_S^*(\Delta)$.

This contradicts our claim from part (a) that $\widetilde{\pi}_S^*(\Delta) > \pi_S^*(\Delta)$ as $\Delta \to 3t$. Thus, there exists a unique $\Delta_S(\delta) \in (0, 3t)$ such that $\widetilde{\pi}_S^*(\Delta_S(\delta)) = \pi_S^*(\Delta_S(\delta))$. In other words, for $0 < \Delta \leq \Delta_S(\delta)$, we have $s(\Delta) \leq 0$ implying $\widetilde{\pi}_S^*(\Delta) \leq \pi_S^*(\Delta)$, and for $\Delta_S(\delta) < \Delta < 3t$, we have $s(\Delta) > 0$ implying $\widetilde{\pi}_S^*(\Delta) > \pi_S^*(\Delta)$. Moreover, since π_S^* is strictly decreasing in δ, the derivative $\Delta_S'(\delta) < 0$.

Next, we argue that $\Delta_G(\delta) < \Delta_S(\delta)$. We know that $\widetilde{\pi}_S^*(\Delta_S(\delta)) = \pi_S^*(\Delta_S(\delta))$. Now, evaluating $g(\Delta) = \widetilde{\pi}_G^* - \pi_G^*$ defined in (58) at $\Delta_S(\delta)$ and using the equality $\widetilde{\pi}_S^*(\Delta_S(\delta)) = \pi_S^*(\Delta_S(\delta))$, we have

$$g(\Delta_S) = \frac{8t^2}{(t - \Delta_S/2 + C)} - \frac{(t - \Delta_S/2 + C)}{2} - \frac{2\Delta_S}{3}.$$

Note that we have suppressed the argument δ and written $g(\Delta_S(\delta))$ as $g(\Delta_S)$ for simplicity. Straightforward calculations will that show that the preceding expression equals $16t^2\Delta^2/6(t - \Delta_S/2 + C)$ which is always positive. Thus, using the fact that $g(\Delta)$ is monotonically increasing in Δ and $g(\Delta_G(\delta)) = 0$, it must be that $\Delta_G(\delta) < \Delta_S(\delta)$ for all $\delta \in [0, 1]$. Hence proved. $\square$

Proof of Proposition 3

Consider the independent pricing regime (case NN). When $0 < N^*_{Gj}, N^*_{Sj} < 1, j = 1, 2$, then social welfare is

$$SW^* = \int_0^{x_1^*} [V_{G1} - tx_1] dx_1 + \int_{x_1^*}^1 [V_{S1} - t(1 - x_1)] dx_1 + \int_0^{x_2^*} [W - \delta a^*_{G2} - tx_2]\, dx_2 + \int_{x_2^*}^1 [W - \delta a^*_{S2} - t(1 - x_2)]\, dx_2 + \beta \int_{\alpha^*}^1 \alpha N^*_{G2} d\alpha + \beta \int_{\alpha^*}^1 \alpha N^*_{S2} d\alpha.$$

Using the stage 2 equilibrium values defined in lemma 1, the preceding equation equals

$$SW^* = \frac{V_{G1} + V_{S1}}{2} + \frac{5\Delta^2}{36t} - \frac{t}{2} + W + \left[\beta \left(a^* - \frac{(a^*)^2}{2}\right) - \delta a^*\right]. \tag{59}$$

Next consider service integration (case NI, IN, or II). For each of these regimes, social welfare is defined as

$$\widetilde{SW}^* = \int_0^{1/2} [V_{G1} + W - \delta \tilde{a}^*_{G2} - ty]\, 4y\ dy + \int_{1/2}^{\tilde{y}^*} [V_{G1} + W - \delta \tilde{a}^*_{G2} - ty]\, 4(1 - y)\ dy + \int_{\tilde{y}^*}^1 [V_{S1} + W - \delta \tilde{a}^*_{S2} - t(1 - y)]\, 4(1 - y)\ dy + \beta \int_{\tilde{\alpha}^*}^1 \alpha \tilde{N}^*_{G2} d\alpha + \beta \int_{\tilde{\alpha}^*}^1 \alpha \tilde{N}^*_{S2} d\alpha.$$

Using the stage 2 equilibrium values defined in lemma 2, the preceding equation equals

$$\widetilde{SW}^* = V_{G1} + W - 2(1 - \tilde{y}^*)^2 \Delta - \frac{7t}{6} + 8t(\tilde{y}^*)^3 - 6t(\tilde{y}^*)^2 + 4t\tilde{y}^* + \left[\beta \left(\tilde{a}^* - \frac{(\tilde{a}^*)^2}{2}\right) - \delta \tilde{a}^*\right]. \tag{60}$$

where $\tilde{y}^*$ and $\tilde{a}^*$ are as defined in lemma 2 and the rest of the notations are the same as defined in earlier sections.

Using (59) and (60), the change in social welfare, i.e. $\Delta SW = \widetilde{SW}^* - SW^*$, is defined as

$$\Delta SW = \underbrace{\Delta \left[\frac{1}{2} - 2(1 - \tilde{y}^*)^2\right] - \frac{5\Delta^2}{36t} - \frac{2t}{3} + 8t(\tilde{y}^*)^3 - 6t(\tilde{y}^*)^2 + 4t\tilde{y}^*}_{\text{net user surplus}} + \underbrace{\left[\beta \left(\tilde{a}^* - \frac{(\tilde{a}^*)^2}{2}\right) - \delta \tilde{a}^*\right] - \left[\beta \left(a^* - \frac{(a^*)^2}{2}\right) - \delta a^*\right]}_{\text{net advertising surplus}}.$$

Using the example ($t = 1.2, \beta = 1$) considered, it can be argued that net surplus advertising surplus increases with service integration for large values of δ and decreases otherwise. The net user surplus (net of transportation costs) will increase only for sufficiently large Δ. This is because only then the user gain is sufficient to counter rise in transportation costs and decrease in advertising surplus (if any). Hence proved. $\square$

Proof of Proposition 4

Consider the independent pricing regime (case NN). When $0 < N^*_{Gj}, N^*_{Sj} < 1, j = 1, 2$, then user welfare is

$$UW^* = \int_0^{x_1^*}[V_{G1} - p^*_{G1} - tx_1]dx_1 + \int_{x_1^*}^1 [V_{S1} - p^*_{S1} - t(1-x_1)]dx_1 + \int_0^{x_2^*}[W - \delta a^*_{G2} - tx_2]\,dx_2 + \int_{x_2^*}^1 [W - \delta a^*_{S2} - t(1-x_2)]\,dx_2.$$

Using the stage 2 equilibrium values defined in lemma 1, the preceding equation equals

$$UW^* = \frac{V_{G1} + V_{S1}}{2} + \frac{5\Delta^2}{36t} - \frac{3t}{2} + W - \delta a^*, \tag{61}$$

where a^* is as defined in lemma 1. Next, consider service integration regime (case NI, IN, or II). User welfare is

$$\widetilde{UW}^* = \int_0^{1/2}[V_{G1} + W - \tilde{p}^*_G - \delta\tilde{a}^*_{G2} - ty]\,4y\;dy + \int_{1/2}^{\tilde{y}^*}[V_{G1} + W - \tilde{p}^*_G - \delta\tilde{a}^*_{G2} - ty]\,4(1-y)\;dy + \int_{\tilde{y}^*}^1 [V_{S1} + W - \tilde{p}^*_S - \delta\tilde{a}^*_{S2} - t(1-y)]\,4(1-y)\;dy.$$

Using the stage 2 equilibrium values defined in lemma 2, the preceding equation equals

$$\widetilde{UW}^* = V_{G1} + W - 2(1-\tilde{y}^*)^2\Delta - \frac{7t}{6} + \frac{8t(\tilde{y}^*)^3}{3} - 6t(\tilde{y}^*)^2 + 4t\tilde{y}^* + 2(1-\tilde{y}^*)^2(\tilde{p}^*_G - \tilde{p}^*_S) - \tilde{p}^*_G - \delta\tilde{a}^*, \tag{62}$$

where $\tilde{y}^*, \tilde{p}^*_G, \tilde{p}^*_S$, and $\tilde{a}^*$ are as defined in lemma 2 and the rest of the notations are the same as defined in earlier sections.

Using (61) and (62), change in user welfare, i.e. $\Delta UW = \widetilde{UW}^* - UW^*$ is

$$\Delta UW = \Delta\left[\frac{1}{2} - 2(1-\tilde{y}^*)^2\right] - \frac{5\Delta^2}{36t} + \frac{t}{3} + \frac{8t(\tilde{y}^*)^3}{3} - 6t(\tilde{y}^*)^2 + 4t\tilde{y}^* + 2(1-\tilde{y}^*)^2(\tilde{p}^*_G - \tilde{p}^*_S) - \tilde{p}^*_G - \delta(\tilde{a}^* - a^*).$$

Using the example ($t = 1.2, \beta = 1$) considered, it can be argued that for small Δ, $\tilde{p}^*_G$ and $\tilde{p}^*_S$

are not large, and thus, gain in user surplus from consuming the system $G1G2$ and fall in nuisance costs (measured by $\delta(a^* - \widetilde{a}^*)$) will dominate the increase in price and transportation costs, increasing user welfare. However, for large Δ, increase in $\widetilde{p}_G^*$ and $\widetilde{p}_S^*$ dominates the positive effects, decreasing user welfare. Hence proved. □

Appendix B: Proofs of the Extensions

B.1. Proofs of the Extension with Pay Per Click Pricing Model

Proof of Proposition 5

With pay per click pricing, the advertiser α payoff function is

$$\pi_\alpha = \begin{cases} (\alpha - r_{i2})\beta N_{i2}: & \text{Independent pricing, and} \\ (\alpha - \widetilde{r}_i)\beta \widetilde{N}_i: & \text{Service Integration.} \end{cases} \tag{63}$$

Now, using the preceding expression, the inverse advertising demand function of firm $i, i = G, S$, under independent pricing (case NN) is $r_i = 1 - a_i$, whereas under service integration (case NI, IN, or II) it is $\widetilde{r}_i = 1 - \widetilde{a}_i$. Now, using these demand functions in firm's profit function defined in equations (1), (2), (3) and (4), we can obtain firm i's profit function as

$$\pi_i = p_{i1}N_{i1} + a_{G2}\beta N_i r_i: \text{ Independent Pricing,} \tag{64}$$

$$\text{and } \widetilde{\pi}_i = \widetilde{p}_{i1}\widetilde{N}_{i1} + \widetilde{a}_{G2}\beta \widetilde{N}_i \widetilde{r}_i: \text{ Service Integration.} \tag{65}$$

The user demand functions would remain unchanged. Maximizing the preceding profit functions w.r.t. prices and advertising quantities would lead to same results as described in lemmas 1 and 2 in the main text. We then follow the same procedure as described in our main analysis to derive the main propositions 1 and 2, and find that our key results remains unchanged. Similarly, we can use the pricing and levels defined in lemmas 1 and 2, and use them in social welfare functions (59) and (60), and find that they remain unchanged. Thus, the welfare analysis also remains identical. Hence proved. □

B.2. Proofs of the Extension with Service Pricing

Proof of Proposition 6

First, we consider market equilibrium under independent pricing (case NN). The profit of the firms are

$$\pi_G = p_{G1}N_{G1} + p_{G2}N_{G2} + r_{G2}a_{G2} : \text{ Firm G's profit,} \tag{66}$$

$$\text{and } \pi_S = p_{S1}N_{S1} + p_{S2}N_{S2} + r_{S2}a_{S2} : \text{ Firm S' profit.} \tag{67}$$

Since profit maximization for the product and service is independent of each other, we consider each component separately.
First, consider the hardware product. Since profit functions are continuously differentiable, any optimal pair of prices must satisfy the first-order conditions of firms' optimization problem. Using equations (66) and (67) and differentiating w.r.t. prices, and solving the first-order conditions would give us equilibrium prices as defined in lemma 1(ii) in the baseline model. They are

$$p^*_{G1} = \frac{3t+\Delta}{3}, \text{ and } p^*_{S1} = \frac{3t-\Delta}{3}. \tag{68}$$

Now, consider the online service. Any optimal pair of prices and advertising quantities must satisfy the first-order necessary conditions of firms' optimization problem. Putting the value of $r_{i2} = (1-a_{G2})\beta N_{G2}$ in equations (66) and (67), and differentiating w.r.t. prices and advertising quantities, they are

$$\frac{\partial \pi_G}{\partial p_{G2}} = N_{G2} + (p_{G2} + \beta.a_{G2}.(1-a_{G2}))\frac{\partial N_{G2}}{\partial p_{G2}} = 0, \tag{69}$$

$$\frac{\partial \pi_G}{\partial a_{G2}} = (\beta.(1-2a_{G2}))N_{G2} + (p_{G2} + \beta.a_{G2}.(1-a_{G2}))\frac{\partial N_{G2}}{\partial a_{G2}} \leq 0, \tag{70}$$

$$\frac{\partial \pi_S}{\partial p_{S2}} = N_{S2} + (p_{S2} + \beta.a_{S2}.(1-a_{S2}))\frac{\partial N_{S2}}{\partial p_{S2}} = 0, \tag{71}$$

$$\frac{\partial \pi_S}{\partial a_{S2}} = (\beta.(1-2a_{S2}))N_{S2} + (p_{S2} + \beta.a_{S2}.(1-a_{S2}))\frac{\partial N_{S2}}{\partial a_{S2}} \leq 0. \tag{72}$$

Using the preceding equations, the symmetric equilibrium advertising level, i.e. $a^*_{G2} = a^*_{S2} > 0$ is

$$a^*_{G2} = a^*_{S2} = \frac{\beta-\delta}{2\beta}. \tag{73}$$

Using the preceding expression in equations (69) and (71), we obtain the symmetric equilibrium prices as

$$p^*_{G2} = p^*_{S2} = t - \frac{\beta^2 - \delta^2}{4\beta}. \tag{74}$$

Now, using the equilibrium pricing and advertising levels defined in equations (68), (73) and (74) in firms' profit functions defined in (66) and (67), we obtain equilibrium firms' profit as

$$\pi^*_G = \frac{(3t + \Delta)^2}{18t} + \frac{t}{2} : \text{ Firm G's profit,} \tag{75}$$

$$\text{and } \ \pi^*_S = \frac{(3t - \Delta)^2}{18t} + \frac{t}{2} : \text{ Firm S' profit.} \tag{76}$$

Next, we consider service integration (case NI, IN, or II). The equilibrium pricing and advertising levels would remain the same as given in lemma 2 in the main text, and thus, the equilibrium profits will be the same.

We can follow the same procedure as under the proof of proposition 2 and show that the main result on profit comparison would remain qualitatively unchanged. We also conduct a numerical numerical simulation with $t = 1.2$, $\delta = 0.5$, and confirm the same.

Next, we conduct a welfare analysis. First, consider independent pricing regime (case NN). Using the equilibrium values defined in (68), (73) and (74), and putting them in equation (59), we obtain social welfare, SW^*, as

$$SW^* = \frac{V_{G1} + V_{S1}}{2} + \frac{5\Delta^2}{36t} - \frac{t}{2} + W + \frac{3(\beta - \delta)^2}{8\beta}. \tag{77}$$

Next, consider service integration (case NI, IN, or II)regime. Since equilibrium remains unchanged, the social welfare, $\widetilde{SW}^*$, would also remain unchanged (as defined in (60)).

Note that since advertising levels are the same under the both independent pricing (case NN) and service integration (case NI, IN, or II), social welfare comparison would depend only on how net user surplus changes with service integration. Using the example with $t = 1.2$, and $\beta = 1$, it can be argued that the net user surplus increases with service integration. This is because the gain in user surplus is sufficient to compensate for the increase in transportation costs. Hence proved. $\square$

B.3. Proofs of the Extension with Firm G's Quality Advantage for Both Product and Service

Proof of Proposition 7

In this extension, we consider firm G's symmetric dominance for both product and product. For both components, we assume that there is a vertical differentiation with $\Delta = V_{G1} - V_{S1} = V_{G2} - V_{S2} > 0$. For the hardware product, a user's net utility, U_{i1}, $i = G, S$, equals

$$\begin{cases} V_{G1} - p_{G1} - tx_1, & \text{if she consumes product G1, and} \\ V_{S1} - p_{S1} - t(1 - x_1), & \text{if she consumes product S1.} \end{cases} \tag{78}$$

For the online service, a user's net utility, $U_{i2}, i = G, S$, equals

$$\begin{cases} V_{G2} - \delta a_{G2} - tx_2, & \text{if she consumes service G2, and} \\ V_{S2} - \delta a_{S2} - t(1 - x_2), & \text{if she consumes service S2.} \end{cases} \tag{79}$$

First, we consider independent pricing regime (case NN). At stage 4, users make participation decisions. The demand for firm G's (firm S') product is $N_{G1}(N_{S1}) = \hat{x}_1(1 - \hat{x}_1)$. Similarly, the demand for firm G's (firm S') service is $N_{G2}(N_{S2}) = \hat{x}_2(1 - \hat{x}_2)$. For the product, given prices p_{G1} and p_{S1}, an indifferent user is defined by the location $\hat{x}_1$ such that

$$V_{G1} - p_{G1} - t\hat{x}_1 = V_{S1} - p_{S1} - t(1 - \hat{x}_1) \Rightarrow \hat{x}_1 = \frac{1}{2} + \frac{\Delta}{2t} + \frac{p_{S1} - p_{G1}}{2t}, \tag{80}$$

where $\Delta = V_{G1} - V_{S1} > 0$ by assumption. Using this, the demand for product $i1, i = G, S$, is

$$N_{G1} = \hat{x}_1 = \frac{1}{2} + \frac{\Delta}{2t} + \frac{p_{S1} - p_{G1}}{2t}, \quad \text{and} \tag{81}$$

$$N_{S1} = 1 - \hat{x}_1 = \frac{1}{2} - \frac{\Delta}{2t} + \frac{p_{G1} - p_{S1}}{2t}. \tag{82}$$

For the online service, given advertising levels a_{G2} and a_{S2}, the user indifferent between consuming product G2 and S2 is defined by the location $\hat{x}_2 \in [0, 1]$ such that

$$V_{G2} - \delta a_{G2} - t\hat{x}_2 = V_{S2} - \delta a_{S2} - t(1 - \hat{x}_2) \Rightarrow \hat{x}_2 = \frac{1}{2} + \frac{\Delta}{2t} + \frac{\delta a_{S2} - \delta a_{G2}}{2t}. \tag{83}$$

Using this, the demand for service $i2, i = G, S$, is

$$N_{G2} = \hat{x}_2 = \frac{1}{2} + \frac{\Delta}{2t} + \frac{\delta a_{S2} - \delta a_{G2}}{2t}, \text{ and} \tag{84}$$

$$N_{S2} = 1 - \hat{x}_2 = \frac{1}{2} - \frac{\Delta}{2t} + \frac{\delta a_{G2} - \delta a_{S2}}{2t}. \tag{85}$$

From stage 3, the inverse advertising demand function of firm $i, i = G, S$, is given as

$$r_{i2} = (1 - a_{i2})\beta N_{i2}. \tag{86}$$

Using the inverse advertising demand function defined in equation (86), the user demand functions defined in equations (81), (82) ,(84) and (85), and putting the values for them in the firms' profit functions, firm $i, i = G, S$, chooses the user price p_{i1} and advertising quantity a_{i2} to maximize its profits. The stage 2 equilibrium satisfies the following:

i. the equilibrium advertising levels a^*_{G2} and a^*_{S2} are characterized by the following system of equations

$$\frac{\partial \pi_G}{\partial a_{G2}} = (1 - 2a^*_{G2})\beta N_{G2} + (1 - a^*_{G2}).a^*_{G2}.\beta.\frac{\partial N_{G2}}{\partial a_{G2}} \leq 0, \text{ and} \tag{87}$$

$$\frac{\partial \pi_S}{\partial a_{S2}} = (1 - 2a^*_{S2})\beta N_{S2} + (1 - a^*_{S2}).a^*_{S2}.\beta.\frac{\partial N_{S2}}{\partial a_{S2}} \leq 0. \tag{88}$$

ii. the equilibrium prices are

$$p^*_{G1} = \frac{3t + \Delta}{3}, \text{ and } p^*_{S1} = \frac{3t - \Delta}{3}. \tag{89}$$

The equilibrium profit of the firms are

$$\pi^*_G = \frac{(3t + \Delta)^2}{18t} + \frac{\beta}{2}(1 - a^*_{G2})a^*_{G2}, \text{ and } \pi^*_S = \frac{(3t - \Delta)^2}{18t} + \frac{\beta}{2}(1 - a^*_{S2})a^*_{S2}. \tag{90}$$

Next, we consider the service integration regime (case IN, NI or II). At stage 4, given prices $\widetilde{p}_G$ and $\widetilde{p}_S$, and advertising quantities $\widetilde{a}_G$ and $\widetilde{a}_S$, the users decide which of the above two systems to consume. A user indifferent between consuming system G1G2 and S1S2 is defined by a pair

(x_1, x_2) such that

$$V_{G1} + V_{G2} - \delta\widetilde{a}_G - \widetilde{p}_G - tx_1 - tx_2 = V_{S1} + V_{S2} - \delta\widetilde{a}_S - \widetilde{p}_S - t(1 - x_1) - t(1 - x_2). \quad (91)$$

Let $\widetilde{y} = (x_1 + x_2)/2$ denote the average location of a user type (x_1, x_2). Therefore, using the preceding expression, $\widetilde{y}$ denote the average location of the indifferent user which is given by

$$\widetilde{y} = \frac{x_1 + x_2}{2} = \frac{1}{2} + \frac{\Delta}{2t} + \frac{\widetilde{p}_S - \widetilde{p}_G}{4t} + \frac{\delta\widetilde{a}_S - \delta\widetilde{a}_G}{4t}. \quad (92)$$

Let $\widetilde{F}(.)$ and $\widetilde{f}(.)$ denote the distribution and probability density functions of the average location $\widetilde{y}$. They are

$$\widetilde{F}(y) = \begin{cases} 2y^2, & \text{if } 0 \le y \le \frac{1}{2}, \text{ and} \\ 1 - 2(1-y)^2, & \text{if } \frac{1}{2} < y \le 1. \end{cases} \qquad \widetilde{f}(y) = \begin{cases} 4y, & \text{if } 0 \le y \le \frac{1}{2}, \text{ and} \\ 4(1-y), & \text{if } \frac{1}{2} < y \le 1. \end{cases} \quad (93)$$

The profit of the firms are

$$\widetilde{\pi}_G = \widetilde{p}_G\widetilde{N}_G + \widetilde{r}_G\widetilde{a}_G : \text{ Firm G's profit}, \quad (94)$$

$$\text{and } \widetilde{\pi}_S = \widetilde{p}_S\widetilde{N}_S + \widetilde{r}_S\widetilde{a}_S : \text{ Firm S' profit}. \quad (95)$$

Since profit functions are continuously differentiable, any optimal pair of prices and advertising quantities must satisfy the first-order necessary conditions of firms' optimization problem. Putting the value of advertising prices, i.e. $\widetilde{r}_i = (1 - \widetilde{a}_i)\beta\widetilde{N}_i, i = G, S$, in equations (94) and (95) and differentiating w.r.t. prices and advertising quantities, they are

$$\frac{\partial\widetilde{\pi}_G}{\partial\widetilde{p}_G} = \widetilde{N}_G + (\widetilde{p}_G + \beta.\widetilde{a}_G.(1 - \widetilde{a}_G))\frac{\partial\widetilde{N}_G}{\partial\widetilde{p}_{G2}} = 0, \quad (96)$$

$$\frac{\partial\widetilde{\pi}_G}{\partial\widetilde{a}_G} = (\beta.(1 - 2\widetilde{a}_G))\widetilde{N}_G + (\widetilde{p}_G + \beta.\widetilde{a}_G.(1 - \widetilde{a}_G))\frac{\partial\widetilde{N}_G}{\partial\widetilde{a}_{G2}} \le 0, \quad (97)$$

$$\frac{\partial\widetilde{\pi}_S}{\partial\widetilde{p}_S} = \widetilde{N}_S + (\widetilde{p}_S + \beta.\widetilde{a}_S.(1 - \widetilde{a}_S))\frac{\partial\widetilde{N}_S}{\partial\widetilde{p}_{S2}} = 0, \text{ and} \quad (98)$$

$$\frac{\partial\widetilde{\pi}_S}{\partial\widetilde{a}_S} = (\beta.(1 - 2\widetilde{a}_S))\widetilde{N}_S + (\widetilde{p}_S + \beta.\widetilde{a}_S.(1 - \widetilde{a}_S))\frac{\partial\widetilde{N}_S}{\partial\widetilde{a}_{S2}} \le 0. \quad (99)$$

Solving the preceding system of first-order conditions, the stage 2 equilibrium satisfies the following:

i. the equilibrium advertising levels $\widetilde{a}^*_{G2}$ and $\widetilde{a}^*_{S2}$ are characterized by the following system of equations

$$\widetilde{a}^*_{G2} = \widetilde{a}^*_{S2} = \frac{\beta - \delta}{2\delta}. \tag{100}$$

ii. the equilibrium prices $\widetilde{p}^*_{G2}$ and $\widetilde{p}^*_{S2}$ are

$$\widetilde{p}^*_G = \frac{4t.\widetilde{F}(\widetilde{y})}{\widetilde{f}(\widetilde{y})} - \frac{(\beta^2 - \delta^2)}{4\beta}, \text{ and}$$

$$\widetilde{p}^*_S = \frac{4t.(1 - \widetilde{F}(\widetilde{y}))}{\widetilde{f}(\widetilde{y})} - \frac{(\beta^2 - \delta^2)}{4\beta}.$$

From the preceding equations, it can be seen that we need to solve for equilibrium value of $\widetilde{y}$ to obtain $\widetilde{p}^*_G$ and $\widetilde{p}^*_S$. Putting the value for $\widetilde{p}^*_S - \widetilde{p}^*_G$ and $\widetilde{a}^*_S - \widetilde{a}^*_G$ in (92), we get

$$\widetilde{y} = \frac{1}{2} + \frac{\Delta}{2t} + \frac{1 - 2\widetilde{F}(\widetilde{y})}{\widetilde{f}(\widetilde{y})}.$$

Using the same process as in the proof of lemma 2, we get equilibrium $\widetilde{y}$ as

$$\widetilde{y}^* = \frac{7t + \Delta - \sqrt{9t^2 + \Delta^2 - 2t\Delta}}{8t}.$$

Now, using the value of $\widetilde{y}^*$ in (93), we get

$$\widetilde{f}(\widetilde{y}^*) = \frac{t - \Delta + C}{2t}, \quad \text{and } \widetilde{F}(\widetilde{y}^*) = \frac{32t^2 - (t - \Delta + C)^2}{32t^2},$$

where $C = \sqrt{9t^2 + \Delta^2 - 2t\Delta}$. Finally, using the preceding values in optimal price functions, we get the bundling prices. They are

$$\widetilde{p}^*_G = \frac{32t^2 - (t - \Delta + C)^2}{4(t - \Delta + C)} - \frac{(\beta^2 - \delta^2)}{4\beta}, \text{ and } \widetilde{p}^*_S = \frac{t - \Delta + C}{4} - \frac{(\beta^2 - \delta^2)}{4\beta}. \tag{101}$$

Using the equilibrium values, we get equilibrium profits as

$$\widetilde{\pi}_G^* = \frac{[32t^2 - (t - \Delta + C)^2]^2}{128t^2(t - \Delta + C)}, \text{ and } \widetilde{\pi}_S^* = \frac{(t - \Delta + C)^3}{4}, \tag{102}$$

where $C = \sqrt{9t^2 + \Delta^2 - 2t\Delta}$.

Next, we compare profits under the independent pricing (case NN) and service integration (case IN, NI or II). For tractability, we conduct a numerical simulations with $t = 1.2$, $\delta = 0.5$, $\beta = 1$ and three different levels of quality advantage, i.e. $\Delta = 0.2, 1$ & 2.8. Tables 3 and 4 show the profit of the firms under the numerical simulations. From the tables, we can see that service integration increases firm i's profit for sufficiently large level of quality advantage, and decreases it otherwise. Hence proved. □

Table 3: Firm G's profit under independent pricing and service integration

Regime	$\Delta = 0.2$	$\Delta = 1$	$\Delta = 2.8$
Independent Pricing	0.82	1.21	2.14
Service Integration	0.7	1.5	4

Table 4: Firm S' profit under independent pricing and service integration

Regime	$\Delta = 0.2$	$\Delta = 1$	$\Delta = 2.8$
Independent Pricing	0.63	0.32	0 (approx.)
Service Integration	0.46	0.25	0.1

B.4. Proofs of the Extension with Competition against Specialist Firms

Proof of Proposition 8

In this extension, we consider the scenario when firm G is competing with two independent specialists, S1 and S2. First, consider independent pricing (case NN). The profit of the firms are

$$\pi_G = p_{G1}N_{G1} + r_{G2}a_{G2} : \text{ Firm G's profit,} \tag{103}$$

$$\pi_{S1} = p_{S1}N_{S1} : \text{ Firm S1's profit,} \tag{104}$$

$$\text{and } \pi_{S2} = r_{S2}a_{S2} : \text{ Firm S2's profit.} \tag{105}$$

Since profit maximization for product and service is independent of each other, we consider each component separately. Consider hardware product. Using equations (103) and (104) and differentiating w.r.t. prices, the first-order conditions are

$$\frac{\partial \pi_G}{\partial p_{G1}} = N_{G1} + p_{G1}.\frac{\partial N_{G1}}{\partial p_{G1}} = 0, \text{ and} \tag{106}$$

$$\frac{\partial \pi_{S1}}{\partial p_{S1}} = N_{S1} + p_{S1}.\frac{\partial N_{S1}}{\partial p_{S1}} = 0. \tag{107}$$

Solving the preceding first-order conditions would give us equilibrium prices as defined in lemma 1(ii). They are

$$p^*_{G1} = \frac{3t+\Delta}{3}, \text{ and } p^*_{S1} = \frac{3t-\Delta}{3}. \tag{108}$$

Now, consider online service. Putting the value of $r_{i2} = (1-a_{G2})\beta N_{G2}$ in equations (103) and (105), and differentiating w.r.t. advertising quantities, the first-order conditions are

$$\frac{\partial \pi_G}{\partial a_{G2}} = (1-2a_{G2})\beta N_{G2} + (1-a_{G2}).a_{G2}.\beta.\frac{\partial N_{G2}}{\partial a_{G2}} \leq 0, \text{ and} \tag{109}$$

$$\frac{\partial \pi_{S2}}{\partial a_{S2}} = (1-2a_{S2})\beta N_{S2} + (1-a_{S2}).a_{S2}.\beta.\frac{\partial N_{S2}}{\partial a_{S2}} \leq 0. \tag{110}$$

At symmetric equilibrium, i.e. $a^*_{G2} = a^*_{S2} > 0$ would give us the advertising levels as defined in lemma 1(i). They are

$$a^*_{G2} = a^*_{S2} = \frac{1}{2} + \frac{t}{\delta} - \sqrt{\frac{1}{4} + \frac{t^2}{\delta^2}}. \tag{111}$$

Next, consider service integration (case IN, NI or II). The profit of the firms are

$$\widetilde{\pi}_G = \widetilde{p}_{G1}\widetilde{N}_{G1} + \widetilde{r}_{G2}\widetilde{a}_{G2} : \text{ Firm G's profit,} \tag{112}$$

$$\widetilde{\pi}_{S1} = \widetilde{p}_{S1}\widetilde{N}_{S1} : \text{ Firm S1's profit,} \tag{113}$$

$$\text{and } \widetilde{\pi}_{S2} = \widetilde{r}_{S2}\widetilde{a}_{S2} : \text{ Firm S2's profit.} \tag{114}$$

Using the preceding profit functions and maximizing them w.r.t. equilibrium prices and advertising levels, we get first-order conditions as

$$\frac{\partial \widetilde{\pi}_G}{\partial \widetilde{p}_G} = \widetilde{N}_G + (\widetilde{p}_G + \beta.\widetilde{a}_G.(1-\widetilde{a}_G))\frac{\partial \widetilde{N}_G}{\partial \widetilde{p}_{G2}} = 0, \tag{115}$$

$$\frac{\partial \widetilde{\pi}_G}{\partial \widetilde{a}_G} = (\beta.(1-2\widetilde{a}_G))\widetilde{N}_G + (\widetilde{p}_G + \beta.\widetilde{a}_G.(1-\widetilde{a}_G))\frac{\partial \widetilde{N}_G}{\partial \widetilde{a}_{G2}} \leq 0, \tag{116}$$

$$\frac{\partial \widetilde{\pi}_{S1}}{\partial \widetilde{p}_S} = \widetilde{N}_S + \widetilde{p}_S\frac{\partial \widetilde{N}_S}{\partial \widetilde{p}_S} = 0, \text{ and} \tag{117}$$

$$\frac{\partial \widetilde{\pi}_{S2}}{\partial \widetilde{a}_S} = (\beta.(1-2\widetilde{a}_S))\widetilde{N}_S + \beta.\widetilde{a}_S.(1-\widetilde{a}_S).\frac{\partial \widetilde{N}_S}{\partial \widetilde{a}_{S2}} \leq 0. \tag{118}$$

Solving the preceding system of first-order conditions, the stage 2 equilibrium satisfies the following:

i. the equilibrium advertising levels $\widetilde{a}^*_{G2}$ and $\widetilde{a}^*_{S2}$ are characterized by the following system of equations

$$\widetilde{a}^*_G = \frac{\beta-\delta}{2\beta}, \text{ and}$$

$$\widetilde{a}^*_S = \left[\left((1-\widetilde{F}(\widetilde{y})) + \frac{\delta\widetilde{f}(\widetilde{y})}{8t}\right) - \left((1-\widetilde{F}(\widetilde{y}))^2 + \frac{\delta^2(\widetilde{f}(\widetilde{y}))^2}{64t^2}\mathrm{right}\right)^{1/2}\right] \cdot \left[\frac{4t}{\delta\widetilde{f}(\widetilde{y})}\right]^{-1}.$$

ii. the equilibrium prices $\widetilde{p}^*_{G2}$ and $\widetilde{p}^*_{S2}$ are

$$\widetilde{p}^*_G = \frac{4t.\widetilde{F}(\widetilde{y})}{\widetilde{f}(\widetilde{y})} - \frac{(\beta^2-\delta^2)}{4\beta}, \text{ and}$$

$$\widetilde{p}^*_S = \frac{4t.(1-\widetilde{F}(\widetilde{y}))}{\widetilde{f}(\widetilde{y})}.$$

From the preceding equations, it can be seen that we need to solve for the equilibrium value of $\widetilde{y}$ to obtain $\widetilde{p}^*_G$ and $\widetilde{p}^*_S$. Putting the value for $\widetilde{p}^*_S - \widetilde{p}^*_G$ and $\widetilde{a}^*_S - \widetilde{a}^*_G$ in (92), we get

$$\widetilde{y} = \frac{1}{2} + \frac{\Delta}{4t} + \frac{2-3\widetilde{F}(\widetilde{y})}{\widetilde{f}(\widetilde{y})} + \frac{\beta^2+\delta^2}{16t\beta} - \frac{1}{\widetilde{f}(\widetilde{y})}\left[(1-\widetilde{F}(\widetilde{y}))^2 + \frac{\delta^2(\widetilde{f}(\widetilde{y}))^2}{64t^2}\right]^{1/2}.$$

Using the value of $\widetilde{F}(\widetilde{y})$ and $\widetilde{f}(\widetilde{y})$ as defined in (30), we get

$$\widetilde{y} = \frac{1}{2} + \frac{\Delta}{4t} + \frac{6(1\widetilde{y})^2 - 1}{4(1-\widetilde{y})} + \frac{\beta^2 + \delta^2}{16t\beta} - \frac{1}{2}\left[(1-\widetilde{y})^2 + \frac{\delta^2}{16t^2}\right]^{1/2}.$$

Using the preceding expression we can derive $\widetilde{y}^*$. Then we can use $\widetilde{y}^*$ to derive equilibrium profits.

Next, we compare the equilibrium profits. Since deriving closed form solutions is not possible, we conduct numerical simulations to compare the profit of the firms under independent pricing (case NN) and service integration (case IN, NI or II). We use example with $t = 1.2$, $\beta = 1$, $\delta = 0.5$ and three different levels of quality advantage, i.e. $\Delta = 0.2, 1$, and 2.9. Tables 5, 6 and 7 show the equilibrium values of the profit of firm i, $i = G, S$, under three different scenarios.

Table 5: Firm G's profit underindependent pricing and service integration

Regime	$\Delta = 0.2$	$\Delta = 1$	$\Delta = 2.9$
Independent Pricing	0.6	0.8	1.7
Service Integration	1.2	1.38	1.8

Table 6: Firm S1's profit under independent pricing and service integration

Regime	$\Delta = 0.2$	$\Delta = 1$	$\Delta = 2.9$
Independent Pricing	0.52	0.35	0.05
Service Integration	0.5	0.3	0.2

Table 7: Firm S2's profit under independent pricing and service integration

Regime	$\Delta = 0.2$	$\Delta = 1$	$\Delta = 2.9$
Independent Pricing	0.15	0.15	0.15
Service Integration	0.113	0.08	0.0811

From the preceding tables, we can see that service integration always increases firm G's profit, increases firm S1's for sufficiently large values of Δ, and always decreases firm S2's profit. Hence proved. $\square$

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