Parents–Children Teams in School Contest

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Abstract

More wealth inequality produces better talents. The result is derived by considering talent as the accumulated human capital at the end of a two-round contest in schools and colleges. In the baseline model, parental investments in children's education help the children compete harder in schools without any long-term benefits.

The model is then extended to allow for investment to be myopic that hinders children's independence. In response, wealthier parents exercise self-control and withhold investments. Parental strategies balance between early gains from investment improving the chance of success in school competition and the loss due to children not developing enough independence. For children born to poor parents, while the odds of progressing to college are small, conditional on crossing this initial hurdle one can dream the *American dream* of outshining others. Affirmative action serves dual roles: improve job prospects for the poor and enhance the quality of top talents.

Keywords: Two-stage race, school contest, college admissions, tug of war, parental investments, student independence, wealth inequality, class-based affirmative action, level playing field, total efforts, top talent, dynamic inconsistency, American dream.

JEL Classification: C79, D64, I21, I23

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1 Introduction

A much debated topic in the United States concerns college admissions. Parents–children teams in schools compete for admission in colleges and the subsequent race in colleges. The academic race contributes to the development of talent pool and the top talents. Affirmative action in college admissions is a major part of the debate. Wealth-based affirmative actions, directly or indirectly using other proxies such as race or ethnicity, are practised in countries such as India, Fiji, Malaysia, South Africa and the USA to help historically disadvantaged groups, facilitating entry of their children into colleges.[1] Recent legal rulings in the USA confirm that universities may use race among other criteria for admission decisions.[2]

With the ultimate goal of producing talents, in the academic race the question of relationships between contestants' resources, contest intensity and the quality of talents is an important one. Many existing models study winner-take-all and multi-prize contests (e.g., Lazear and Rosen, 1981; Moldovanu and Sela, 2001 and 2006; Che and Gale, 2003; Fang et al., 2020), analyzing the incentive properties and design aspects of general contests, labor tournaments, R&D tournaments, etc., for various objectives such as performance maximization (suitably defined), maximization of expected highest effort, or just to motivate efforts by the participants. None of these models apply to the academic race in the way we formulate, for the type of issues we are going to raise. On the other hand, although there is an extensive empirical literature on early stage family investments in children, inequality and skill formation (e.g., Cunha et al., 2010; Heckman et al., 2013; Heckman and Mosso, 2014), higher education (i.e., high schools rather than early investment in primary schools) has not received a similar attention.[3] In particular, the mechanism of parents-children team contest for college admissions and any interventions therein have not been modelled theoretically.[4] We offer this missing link between these two

[1] See Bertrand et al. (2010) for affirmative action admissions in engineering colleges in India, Arcidiacono (2005) and Fryer et al. (2008) for affirmative actions in higher education in the USA, and Ratuva (2013) on affirmative action and ethnic conflict in Fiji, Malaysia and South Africa.

[2] See Oct. 1, 2019 *The New York Times* (NYT) article, "Harvard Does not Discriminate Against Asian-Americans in Admissions, Judge Rules" (https://www.nytimes.com/2019/10/01/us/harvard-admissions-lawsuit.html). The article reports, "The judge, Allison D. Burroughs, rejected the plaintiff's argument, and said that the university met the strict constitutional standard for considering race in its admissions process." See a similar ruling by the Justice Department for alleged bias against Asian Americans and White applicants in undergraduate admissions by Yale University, a case brought by the Trump administration (Feb. 3, 2021 report, "US drops discrimination lawsuit against Yale" at https://us.cnn.com/2021/02/03/politics/yale-university-affirmative-action-lawsuit/index.html). An article in the Vox (2019) noted that in the 1980s several schools including Brown, Princeton, Stanford, and the University of California Berkeley had conducted in-house investigations of their admissions policies in response to criticisms of bias against Asian Americans; see also Takagi (1990).

[3] Barlevy and Neal (2012) is an exception, studying a problem of effort incentive provision to teachers using a tournament mechanism. A student's talent production function depends on her initial talent but not effort, and on the teacher's effort targeted to the specific student and a common effort aimed at all students in the class. By applying an ordinal performance ranking of the teachers based on their students' performance, the teachers are induced to exert socially optimal efforts. The academic race in this paper is a race among students, and the human capital (or talents) generated is the result of the students' efforts in the race.

[4] Chade et al. (2014) studied decentralized college admissions with the main focus being on matching between colleges and students in terms of college quality and student preferences.

dominant but disparate paradigms.

Given high costs of higher education and the growing income and wealth inequality in the USA,[5] it is pertinent to ask how does inequality affect the academic race? The power of wealth can give one head-start as much as abilities. Should the government intervene? A reduction of income and wealth inequality through taxes and transfers should imply the children of poorer wealth backgrounds are able to compete better with the children of more well-off parents. While this is certainly true, it is not necessarily the case that the contest becomes more intense. On the contrary, total efforts need not increase and its composition can become less spread out, so the best talents who are more likely to win the race could suffer in quality. If true, this will be bad news for the society in areas where only the very best talents matter – having top surgeons, cardiologists, scientists and engineers, all with hard skills.

Our paper is the first attempt approaching the issue of parental investment in children in high schools and its implications for the development of human capital in a more well-rounded manner, in a contest theory framework. We also hope to contribute to the related policy debates. We develop a model of a two-stage race by focusing on certain aspects and abstracting away from others, to make the contest theory tractable. Along the way, we will address the following related issues:

(1) Do high parental investments in children's education help the children achieve better long-term outcomes? Or is a competitive culture counter-productive, lessening the quality of talents produced from the children's lack of independence? If the latter is true, could there be some strategic voluntary restraint on parental investments?

(2) Natural progression from the school phase to the race in colleges and beyond requires fresh thinking. Any myopic classification of technological substitutability in parents' and children's joint initiatives may miss out on the persistent effect of early stage education.[6]

(3) Producing 'top talents' and maximization of total student efforts are two different objectives. While the contest literature has mostly focused on the latter,[7] our interest in top talent goal as another alternative resonates with the earlier r&d race literature where what matters are the *best skills*.[8]

(4) Increasing representation of certain groups who are disadvantaged is a well-recognized

[5]See Ehrenberg (2012), Piketty and Saez (2003) and Kopczuk and Saez (2004). The increasing trajectory of both college costs and income and wealth inequality have continued since the publications of these works.

[6]Chevalier et al. (2004), and Heckman et al. (2013), Chetty et al. (2014), offer empirical support, respectively, for the role of education and human capital in wage determination, and the impact of early childhood training/education on future earnings. Becker (1962) and Schultz (1963) are the early proponents of the theory of wages due to education enhancing productivity.

[7]See, for example, Moldovanu and Sela (2001), and Fang et al. (2020).

[8]School contest starts the process of creation of knowledge which is considered as capital good by Romer (1986) and earlier by Arrow (1962) in his theory of learning by doing. Generation of top talent and maximization of knowledge base (with total students efforts as its proxy) are equally plausible missions of academic contest. A similar objective, maximization of expected highest effort, has been studied in Modovanu and Sela (2006) for multi-round contests with varied applications.

goal. In a theoretical model, Fryer and Loury (2013) study the economics and diversity-enhancing policies. Earlier, Bertrand et al. (2010) studied affirmative action in engineering colleges in India targeting the financially disadvantaged, reserving slots for "non-creamy layer".[9] Do such affirmative actions compromise the academic talent pool? In the contest setting the answer is not clear a priori. And this is especially so because school competition may push the wealthier parents into an overdrive, compromising their children's independent development. Affirmative action in college admission provides a counter-balance to this negative effect and smoothen the entry of the less privileged into the more advanced phase, college contest, where they can outshine.

We build a contest model involving three parents-children pairs spanning two rounds. In the first round, students (or children) compete in schools to come out among top two performers in an exam. Then these two students progress to college (and the larger world) where they participate in a more mature, second round race, based on skills they have developed, where only a single winner is picked for the ultimate prize given exogenously. The first round race comprises of observable investments by parents followed by efforts by the children.[10] While these investments and efforts occur in separate stages, within the relevant stage the actions are simultaneous. The sequentiality of moves helps us to capture the strategic interactions between rival parents and their children. The combined profile of investments and efforts determine the top performers. The second round race is a stochastic draw based on the children's efforts alone, i.e., the human capital accumulated in the first round; we abstract away from additional efforts by the contestants in this round which we justify in Section 2.

Throughout the paper we will focus on a team production technology where investment substitutes one-for-one child's effort (additive technology), and the contest success function is a stochastic version of 'tug of war' (Lazear and Rosen, 1981; Harris and Vickers, 1987).

■ **Results and relation to the literature.** We start by establishing a supermodularity result on parental investment and child's effort. Despite the investment substituting one-for-one the child's effort, by helping improve the prospect of moving onto the second round of the contest a marginal increase in parental investment enhances the child's marginal payoff from effort. This draws the parents into an investment war (Proposition 1, Corollary 1).

With the full investment result in place, in Section 3 we engage in two policy analysis. Our first policy, already discussed above, concerns the effect of wealth inequality. What we find is surprising: Irrespective of the type of inequality – uniform, negatively skewed (more wealthy in greater numbers) or positively skewed (less wealthy in greater numbers) – as wealth spread is reduced the children of the wealthiest parents lower their efforts by at least as much as, if not more than, the increase in the efforts of the children from poor wealth backgrounds; the

[9]The Supreme Court of India coined the term, "creamy layer," in a public litigation case, *Indra Sawhney & Others v. Union of India* (also known as the Mandal verdict), to refer to backward class above a threshold income.

[10]Parents can commit to less than maximal investments so that the children are induced to be self-reliant.

top quality talent thus suffers– *bad egalitarianism* (Proposition 2).[11]

Given that the students' efforts form the foundations of the future talent pool through 'learning by doing', what the result suggests is that a society should tolerate greater inequality if it wants to have top talents. This contrasts with the findings of Fudenberg et al. (1983) and Harris and Vickers (1985). These authors had observed that as a rival in an R&D (patent) race gets ahead, others should drop out – *ϵ-preemption*; so inequality among the race participants is harmful to innovation. Their results thus suggest that more symmetry among the contestants is good for the contest. But in the academic race, any mean-preserving lowering of the distribution of efforts due to reduction of wealth and income inequality hurts the chances of winning for contestants with higher efforts more because of the *persistent effect* of efforts in school extending to the college race. Formally, the overall probability of winning the two-round contest is quadratic in one's effort while just for the school race taken in isolation it is linear (for the 'tug of war' contest that we assume for the analysis). So any effort reduction by the more hard working students lower their winning odds in a two-stage contest disproportionately more than the increase in the winning probability of students with below average efforts. This creates mediocre talents.

The idea that competition can be bad for contests also appears in Fang et al. (2020). The authors' modeling of competition, however, is different from ours: For homogeneous contestants, increasing prize inequality, scaling up of competition (promotion tournament held at the entire organization level rather than being division specific), and entry of new contestants, all lower the expected efforts of the contestants. Instead, we treat "closeness" of contestants symbolising competition. Other details also differ.[12]

Rather than inequality reducing intervention through taxes and/or transfers, one way to help the students of poorer wealth backgrounds would be by direct affirmative action. This can take the form of a *quota* in college admission for students coming from families with low parental resources – *means tested admission* (or based on a combination of means and merit).[13] This creates a *level playing field* in the scheme of the two-round race, much like bid discrimination in favor of the weak bidder in Myersonian (1981) auction.[14] In Proposition 3,

[11] Ensuring greater participation of poor income families in post-secondary, higher education is a familiar promise by political parties competing for the support of median voters. But with high and rising tuition fees for college enrollment, one way to achieve such an objective would be through inequality reduction. This should then inevitably lead to a weakening of the top end of the talent pool. There is thus the familiar conflict between populism and elitism. Elitism, however, need not be a bad thing given that the value of extra skills of the very best talents is of much higher significance for the society given the restricted entry, in medicine for example.

[12] For example, ours is a two-round team contest specifically in the academic setting. The parents, who are as important as players as the children/students, take a long-term view when making their investments, giving rise to new problems of *dynamic inconsistency* as hinted in point **(1)** above (Strotz, 1955; Kydland and Prescott, 1977; Thaler and Shefrin, 1981). One broad message that we want to highlight is that team competition in a dynamic setting gives rise to new issues that go beyond the design-theoretic analysis of static contests, started by Moldovanu and Sela (2001) and in that tradition pursued in Fang et al. (2020).

[13] This is also known as *class-based* affirmative action. See Bridges (2017) on the controversial issue with regard to admissions competition in schools and colleges.

[14] While means tested affirmative action may not be an acknowledged admission policy in colleges, affirmative action of a different kind is well documented. In India, reservation of seats at higher educational institutions

we show that such a policy would depress all contestants' efforts, thus total efforts, and the expected top talent. This happens because out of the three contestants, the first round race reduces to 'one out of two' (as opposed to 'two out of three') and thus hurts the effort incentives of the children with higher parental wealths: in equilibrium their odds of succeeding in the school race and progressing to college under affirmative action falls relative to the ones in the contest without the means tested admission. This induces them to exert lower efforts. This, in turn, slackens the effort incentive of the student benefiting from the quota. The race thus turns into *domino effect*, causing the level-playing-field policy intervention to fail. Just like inequality reduction, making the contestants closer through the affirmative action hurts the positive objectives mentioned in point **(3)**.

Finally, in Section 4 we take up some of the most pressing issues concerning college admissions. Whether race, ethnicity and different orientation of the applicants (e.g., exam focused, personality, social skills etc.) should be taken into account in university admissions have featured in policy debates in the USA, with some cases even making to the courts. We will start with the question, do overeager parents hurt their children in the long term (our point **(1)**)?

A related question has been analyzed, framed as a *nature vs. nurture* debate, in a theoretical model by Lizzeri and Siniscalchi (2008). The main issue concerns whether the talent is genetic or parents can improve their child's talent through guided learning. The child has to perform a number of tasks and learn in the process. The parents face the tradeoff between sheltering the child from making costly mistakes or letting the child to learn from mistakes. The authors argue that optimal parenting involves partial sheltering.

In a variant model that we propose (variant of our model in Section 2), parental investment lowers the potency of children's development by reducing the return from efforts when they leave school and enter college. Because talent in our setting is a product of effort by the student in school competition, the parents do not directly *nurture* talent in the way Lizzeri and Siniscalchi frame. Instead, parents intervene in the school contest, influence their own child's and the rival children's effort incentives. Implicitly, when parents invest heavily, the children do not learn to be independent (*negative effect*). Worrying this, wealthy parents may hold back part of their investments despite such a choice compromising their children's chances of success at the school stage. So the nurturing decision is equivalent to lowering of investment, but the

for students from scheduled castes and scheduled tribes (SCSTs) is mandatory. Historically, SCSTs have been on the lowest tier of social and economic hierarchy. So the original intent of reservations in Indian education and government jobs can be considered also linked to economic inequality. As Bertrand et al. (2010) have noted in the context of affirmative actions for entry into engineering colleges in India, "governments may intend to target poor households, and choose to use race or minority status as a proxy for income" (see section 4, "Does affirmative action target the poor?"). In the United States, universities are permitted to take into consideration the applicants' race, among others, while making admission decisions. When the average earnings of a particular race, say Black or African American, is much lower than that of another race, say White American or Asian American, a policy of diversity and affirmative action in admissions indirectly links to particular groups' incomes or wealths. Pertinent to the issue under consideration are also the recent rulings by The Justice Department declaring that Harvard and Yale, two top universities, did not practise discrimination against Asian American and White applicants in undergraduate admissions. See February 3, 2021 report, "US drops discrimination lawsuit against Yale" at https://us.cnn.com/2021/02/03/politics/yale-university-affirmative-action-lawsuit/index.html. See also footnote 2.

main point is that it is *strategic contest driven* as opposed to a single parent-child learning experimentation in Lizzeri and Siniscalchi.

Now the parents compete in two dimensions: help the children cross the first hurdle (school contest) or not make them over-reliant on parental support so that they grow up to be more independent? The following predictions are obtained (Lemma 1, Proposition 4): (i) despite the negative long-term impact if parents have identical wealth they invest fully; under wealth heterogeneity, (ii) wealthier parents are likely to cut back investment, (iii) investment is bounded below by that of the poorest parents who make full investments. All in all, the parents suffer from the dynamic inconsistency problem mentioned earlier. The end product is that under the negative effect of investment the academic race makes all parties worse off; a Pareto improvement is possible if all parents could be persuaded to cut back investments uniformly.

Following on, we also bring in three novel insights. First is the real possibility of the underprivileged dreaming *the American dream* (Proposition 5). It captures the idea that if a child born to poor parents can cross the initial hurdle of school competition, he or she can be the front-runner in the college race. This is so due to the fact that someone who manages to enter college without parental support must have done so through own independent initiatives that won't be subject to any adverse long-term impact of parental investment. So the student's talent is possibly higher than that of her peers generously supported by parental investments. Second, one should expect more wealth inequality to hurt the chances of the poor for their American dream materializing. If one cannot get off the base due to wealth constraint, what the odds are of outshining others conditional on progressing to college becomes a theoretical curiosity. Here affirmative action in college admissions becomes very relevant. Not only does it help the poor to have a realistic shot at excelling in the second round of the race, the intervention may even boost the cause of top talents (Proposition 8; Examples 2 and 3). This result lends further support to the policy of affirmative action practised indirectly using race and socio-economic background information in admission decisions by some of the top universities in the United States.[15] The contrast between the negative implications of affirmative action (Proposition 3) and the positive implications (Proposition 8) relies critically on whether parental investment compromises child's independence. In the first when there is no negative effect of investment, affirmative action takes some of the wind out of competition due to reduced investments and works against children's effort incentives, whereas in the latter the same investment mitigation curbs its harmful long-term effects on children's potency. Plus, the children of poor income households whose efforts, and thus their acquired skills/talents, tend to be higher than that of more wealthy parents are now guaranteed, through affirmative action, to go to college. This improves their odds of winning the final race and the (average) top talents. If one were to believe in the hypothesis that overbearing parents actually harm

[15] Harvard and Yale have been involved in recent court cases for alleged discrimination against Asian American applicants and leaning towards race based admissions. Prior to 1996, public universities in California practised affirmative action that was banned with the passing of *Proposition 209*. *Proposition 16* favoring affirmative action resurfaced on the ballot in November 2020 election but failed to get the majority support.

children's development,[16] the advocacy in favor of affirmative action gains more ground. The core American belief that the children be allowed to be more independent while growing up is a precursor to affirmative actions; see the discussion following Proposition 4.

Finally, without any intervention such as affirmative action we find that more wealth inequality often boosts both top talents and total talents (Propositions 6 and 7), but it is also possible that an inegalitarian distribution of wealth comes to hurt the top talent objective (Proposition 6 and Example 1). The ambiguity in the relationship between wealth inequality and top talent derives from the negative impact of parental investments on children's independence. If the negative impact is in an intermediate range (see Proposition 6), more wealth inequality improves the odds of the more wealthy children progressing to college and eventually winning the two-round race but their overall talents will suffer due to the investment war waged in the college admission race.

■ **Robustness of results, modelling choice etc.** Our choice of tug-of-war contest, rooted in the prominent early literature on R&D tournaments, allows us to build the analysis by explicitly deriving the equilibrium efforts in the initial model and its extension in Section 4 and parental investments in the extended model (Section 4).

We also settled on a simplified model of the two-stage race with (i) parental investments taking place only in the first stage, (ii) student efforts during the school contest seen as forming the core talent pool that translates into college success stochastically, (iii) parental investments and student efforts acting as substitutes in the first-stage race, and (iv) number of parents-children pairs assumed to be three. The restriction (i) is intended to capture the idea that at the college phase parents will have very little role to influence the outcome; after all college is where students grow into independent adults. The restriction (ii) can be relaxed by having the admitted students exert another round of efforts in the contest. But given that the foundations for talents have already been laid at the school stage, there will be a natural ordering of the contestants in the college phase and efforts will be influenced by the talents in a predictable manner: less talented ones will already have fallen behind and lesser will be their incentives to exert effort. So the ordering of the chances of success in the college race, if an additional effort stage were added, will preserve the ordering of talents developed in the school phase. Adding the extra effort stage will complicate the analysis with no clear change in the insights for the results developed in the paper. As for the restriction (iii), note that despite the technological substitution assumption, the two-round race makes parental investment and student effort strategic complements. So the alternative formulation of making investment and effort complementary in the first-stage contest success function does not give us any different result from what we already have. Besides, we have to keep the model simple yet reasonable enough in approximating the functional relation between parental investment and students' efforts in the overall two-stage race. Generalizing restriction (iv) does not change our basic results except that the analysis becomes cumbersome.

[16] Only an empirical analysis using panel data can settle the merit of such hypothesis.

The rest of the paper is structured as follows. The contest model is set up in Section 2. The analysis of parental investment with no long-term impact on skill enhancement is contained in Section 3. In Section 4 we present the extended model with negative effect of investment. Only the proofs of Lemma 1 and Propositions 4, 5 and 8 are included in the main appendix (Appendix A). An online supplementary Appendix B contains the proofs of the remaining propositions (see Section B.1); supporting derivations of all proofs (including proofs in Appendix A) appear in Sections B.2–B.10. In a working paper version (Bag and Chakraborty, 2022), we include an additional Appendix C supplying the Mathematica programs for the lengthy derivations omitted from Appendices A and B. The working paper, together with the executable codes for Appendix C, can be downloaded from https://profile.nus.edu.sg/fass/ecsbpk/ (unpublished reports under Publications).

2 Basic Framework

We consider a model of an academic race involving three parents-child pairs and two rounds of contest. Parents have only one child each. Round 1 is the pre-college/school contest for two admission slots in a prestigious college. At the end of Round 2, the college phase, a single winner is determined.[17] Children's efforts in the first round help them develop independent learning skills critical for success in the later phase.

Both the child who emerges as the eventual winner, and his/her parents, receive a fixed positive payoff V in the form of monetary benefits, while the losers and their parents receive zero payoff. We normalize $V = 1$.

Denote the wealth of child i's $(= 1, 2, 3)$ parents by $\omega_i \geq 0$.

Assumption 1 *(i) The parents have no other use for their wealth, besides investing in their children's education;*

(ii) the wealth cannot be transferred onto children;

(iii) the parents cannot borrow to spend on their children's education beyond the wealth they possess.

In the pre-college school phase, parents of child i invest $0 \leq I_i \leq \omega_i$. Each child also chooses to exert effort, $e_i \geq 0$, to perform well in academic studies in school. The cost of effort to child i is $d_i e_i^2$, where d_i can be interpreted as the child's 'learnability skill' or sometimes referred to as 'merit' or 'smartness'. We will assume throughout $d_i > 3$.[18]

Child's effort and parental investment, (e_i, I_i), determine a composite input

$$\xi_i = e_i + I_i, \tag{1}$$

[17] We lump together college and post-college phases and call it college race to imply that ultimately the realization of success/failure may come in the long run.

[18] Assuming $d_i > 3$ keeps our analysis and the proofs clean. All but one of our proofs will use $d_i > 2$ or lower.

where $\xi_i \in [0, \frac{1}{2}]$. The upperbound of the composite input is a normalization and its purpose will become clear soon. The profile of composite inputs (ξ_1, ξ_2, ξ_3) determines the chance of child i becoming one of two successful college entrants at the end of the school contest.

The probability that a pair of students, $\{i, j\}$, will receive college admissions is

$$\frac{(\xi_i + \xi_j + 1 - 2\xi_l)}{3}, \quad l \neq i, j. \tag{2}$$

This is a stochastic version of the 'tug of war' contest (Lazear and Rosen, 1981; Harris and Vickers, 1987).

Given successful college entry by a pair of students $\{i, j\}$, the probability that student i will be the winner in the college race is

$$\frac{(e_i + 1 - e_j)}{2}. \tag{3}$$

As a child puts in more effort in Round 1, one learns to be independent – 'learning by doing'.[19] This equips a student with essential skills such as critical thinking and problem solving that are key to success in Round 2.

We abstract away from a second-round of efforts by the students in the college stage primarily to keep the analysis of the multi-stage contest tractable and develop a number of intuitive insights. Allowing for efforts in the second contest would bring in more complex interactions – first-round effort would then consider the additional boost to the marginal productivity of the second-round effort. While we do not deny the importance of this richness, we want to build around the force of the persistent effect of skills developed in the initial school phase. It is to be expected that a good base skill developed during twelve years of schooling (high school and Advancement Placement or IB level) makes students mature enough so that the college phase largely turns into a race among the better talents where the average incremental effort in the second round is monotonically increasing in the average talent of the successful college entrants. Interpretation of children's efforts in the academic race contributing to their long-term talents, rather than assuming some exogenous intrinsic talents, is consistent with the costly skill acquisition/investment description of human capital formation in the seminal work of Coate and Loury (1993).

It is also worth emphasizing that the one-shot parental investment is just a supplementary means to boost the child's exam-taking (cognitive) skills in the school contest, a short-term gain. A much richer model of multi-round investment in different phases of a child's development would bring in additional issues such as timing, distribution and types (cognitive vs. non-cognitive) of investment. These have been studied elsewhere in the literature but without the complication of contest that is quite central and a distinctive feature of this paper (e.g., Caucutt and Lochner, 2020). An excellent expository survey of parental investment in child's

[19] The macro literature on learning by doing starting from Arrow (1962) is extensive. We take the same spirit at the micro, individual agent level.

skill formation in a life-cycle model is by Heckman and Mosso (2014).

To summarize: The two-round contest defines a sequential game of observable parental investments followed by children's efforts. Parents invest simultaneously and the children exert their efforts simultaneously. The general approach in solving the game and its variants will be by backward induction with the solution concept of *subgame perfect equilibrium* (or simply, equilibrium).

2.1 Contest equilibrium

In contests involving sequential investments (Dixit, 1987), an increase in investment by any player in the early stage of the contest may increase or lower investment by the rival contestant later in the game. In our setting, parents and children move sequentially that determine the success probabilities not only in the first round, school contest but also the success probabilities in the college contest. So looking at only the school contest would be myopic.

What we will argue is that with the overall two-round contest in mind, parental investments and children's efforts work as complements even though parental investments have no direct bearing on the second round contest, and in the first round contest parental investments and child's efforts substitute out one for one.

We assume that wealth $\omega_i \leq \frac{1}{4}$ and effort $e_i \leq \frac{1}{4}$ to keep the probability of winning a contest well-defined. In what follows we calculate the equilibrium efforts for any exogenous profile of investments, assuming interior solution. This solution will be used in different applications throughout the paper.

■ **Equilibrium Efforts.** Since the payoff of any child is obtained from winning only the final contest, the expected payoff to child i, $i \neq j, l$, and $i, j, l = 1, 2, 3$ can be written as

$$\pi_i = \frac{(e_i + I_i) + (e_j + I_j) + 1 - 2\,(e_l + I_l)}{3} \times \frac{e_i + 1 - e_j}{2} + \frac{(e_i + I_i) + (e_l + I_l) + 1 - 2\,(e_j + I_j)}{3} \times \frac{e_i + 1 - e_l}{2} - d_i e_i^2.$$

The first-order conditions

$$\begin{aligned}
\frac{\partial \pi_1}{\partial e_1} &= \frac{1}{6}\left[4 + (4 - 12d_1)\,e_1 - 2e_2 - 2e_3 + 2I_1 - I_2 - I_3\right] = 0, \\
\frac{\partial \pi_2}{\partial e_2} &= \frac{1}{6}\left[4 - 2e_1 + (4 - 12d_2)\,e_2 - 2e_3 - I_1 + 2I_2 - I_3\right] = 0, \\
\frac{\partial \pi_3}{\partial e_3} &= \frac{1}{6}\left[4 - 2e_1 - 2e_2 + (4 - 12d_3)e_3 - I_1 - I_2 + 2I_3\right] = 0,
\end{aligned} \tag{4}$$

can be solved to derive the equilibrium efforts (see Appendix B.2):

$$
\begin{aligned}
e_1^* &= \frac{2-4d_2-4d_3+8d_2d_3-(d_2+d_3-4d_2d_3)I_1+d_2(1-2d_3)I_2+d_3(1-2d_2)I_3}{2(d_1+d_2+d_3-4d_1d_2-4d_1d_3-4d_2d_3+12d_1d_2d_3)},\\
e_2^* &= \frac{2-4d_1-4d_3+8d_1d_3+d_1(1-2d_3)I_1-(d_1+d_3-4d_1d_3)I_2+d_3(1-2d_1)I_3}{2(d_1+d_2+d_3-4d_1d_2-4d_1d_3-4d_2d_3+12d_1d_2d_3)},\\
e_3^* &= \frac{2-4d_1-4d_2+8d_1d_2+d_1(1-2d_2)I_1+d_2(1-2d_1)I_2-(d_1+d_2-4d_1d_2)I_3}{2(d_1+d_2+d_3-4d_1d_2-4d_1d_3-4d_2d_3+12d_1d_2d_3)}.
\end{aligned}
\tag{5}
$$

Proposition 1 (Comparative statics) *A child's equilibrium effort increases in her own parents' investment but decreases in other parents' investment.*

Given the increasing property of children's efforts with respect to their own parents' investment (*supermodularity*), and zero opportunity cost of investment (Assumption 1), the following result is immediate:

Corollary 1 (Full investment) *In equilibrium the parents invest all their wealth in their children.*

3 Policy Interventions

In this section, we explore two types of policy interventions in the school contest/college admissions race with two goals in mind: to increase total efforts in the contest and enhance the expected quality of the eventual winner in the race which we call the *(expected) top talent*.

3.1 Wealth inequality

We start with the question of optimal intervention in the presence of wealth inequality.

Proposition 2 (Bad Egalitarianism) *Consider students who are identical in their learnability skills,* $d_1 = d_2 = d_3 = d > 0$*. The total effort does not change but the expected top talent increases as wealth inequality* ($x \geq 0$) *increases for each of the following three types of inequalities:*

(i) $\omega_1 = w - x,\ \omega_2 = w,\ \omega_3 = w + x$*;*

(ii) $\omega_1 = w - x,\ \omega_2 = \omega_3 = w$*;*

(iii) $\omega_1 = \omega_2 = w - x,\ \omega_3 = w$.

The first inequality spreads the wealth uniformly (low, medium and upper income class in equal proportion), the second one is negatively skewed and the third inequality is positively skewed. The general result here has striking policy implication: reduction of inequality would lower the development of top talent. The result is reminiscent of a similar tension in the growth literature – that lowering inequality reduces saving and investment and thus growth. However, the mechanism in our case is very different. Inequality reduction lessens the starting lead in the race of the children of wealthier backgrounds, so their incentives to exert effort are dampened while the incentives of the poorer wealth background students will improve.[20] Even if the winning probabilities had remained the same, the expected top talent would come down because the wealthier children have an overall better odds of winning. Indeed the winning probabilities of the wealthier children are reduced and those of the poorer children are increased, and the two combined puts further downward pressure on the top talent. If a society wants to develop top doctors and scientists, it should be willing to tolerate wealth inequality and not interfere on the margin through redistributive tax and transfers.

As a corollary of Proposition 2, we may think of abolition of SAT scores as one of the criteria for college admissions to be inequality reducing. In May, 2020 the governing body of the University of California approved a plan to suspend testing requirements (SAT and ACT) till 2024. See The Washington Post article, "University of California takes huge step toward dropping SAT and ACT from admissions" (source: `https://www.washingtonpost.com/education/2020/05/21/sats-university-california-system/`). The article further reports, "Researchers have long found a link between family income and test scores. Students from affluent backgrounds, who have more access to test preparation and academic resources, tend to score higher on the exams." Our result suggests abolition of SAT and ACT is likely to have an adverse effect on top talents.

3.2 Affirmative action in college admission

With inequality reduction not doing any better for the two positive goals set out in this paper, we now look at another intervention, the affirmative action (or AA), traditionally deployed as a means to achieving equity – correction for inequality between different income classes. If there is anything to go by the result in Proposition 2, one should expect a similar adversarial performance for affirmative action. In this section, we analyze this issue formally.[21]

Consider the first two types of wealth distributions considered in Proposition 2. In the contest without any intervention, that we call the *benchmark contest*, the child of the lowest wealth parents is clearly disadvantaged. So we consider affirmative action that guarantees student 1 college entry. Only students 2 and 3 will have to compete for the college admission. Student 1 has to only compete in college to have a chance of winning the contest. In what follows, we compare this policy to the benchmark contest.

[20] Recall, the children's best-response functions are positively related to own parent's wealth and negatively related to the wealth of the parents of rival children.

[21] Estimating empirically how affirmative action favoring the financially disadvantaged impacts on the overall labor market outcomes, i.e., the gains and losses, is very hard, as Bertrand et al. (2010) have noted.

Proposition 3 (AA for the financially disadvantaged) *A policy of reservation of college seats for students of poorer wealth background, i.e.,* quota*, lowers all students' efforts and thus the total effort in the contest. It also lowers the expected top talent.*

Compared to the efforts in the benchmark contest, it is easy to verify that affirmative action induces all students to lower their efforts. The asymmetry in the benchmark contest ($w-x < w < w+x$ or just $w-x < w$) is replaced by a different kind of asymmetry now. The two students who had the initial advantage due to superior parental wealth are now depressed having been leapfrogged in the initial race. This dissuades their efforts. On the other hand, the student to benefit from affirmative action (student 1) will have one less hurdle to cross. Given the fact that the rival students are going to slacken, the assurance of a place in college for student 1 tends to lower his marginal benefit of effort, thus lowering equilibrium effort. As a result, both total efforts and the expected top talent decline. In a way the result parallels the bad egalitarian result of Proposition 2, suggesting the first asymmetry (under no affirmative action) biasing the contest in favor of the wealthy parents' children creates enough optimism for them to pursue the contest with more intensity, which goes away under affirmative action.

We thus see a clear conflict between the goal of equity and the positive goals being considered in this paper. The affirmative action in the form of reservation of seats in colleges is implemented in India – for entry into prestigious medical and engineering colleges (such as the Indian Institute of Technologies) for students from the scheduled castes and scheduled tribes background whose parental incomes are lower on average. In their empirical studies of affirmative actions in selective engineering colleges in India, Bertrand et al. (2010) indicate the losses that result for the policy: "Our estimates, however, also suggest that these gains may come at an absolute cost because the income losses experienced by displaced upper-caste applicants are larger than the income gains experienced by displacing lower-caste students." We want to add two remarks here: (1) As we have discussed at length in the Introduction, the wealth-based affirmative action considered in this paper can be likened to caste-based reservation in India; (2) the income losses discussed in Bertrand et al. can be used as a proxy for the lost talent of wealthier backgrounds, i.e., total talent as well as the expected top talent.

Proposition 3 also highlights the problem of affirmative action particularly for the middle income class who may not be that strong financially but can be disenchanted and disincentivized as a result. In the context of the post-apartheid policy shift in South Africa in the mid 1990s, the group that suffered from affirmative action was labelled as the new "victim class" (of white professionals) due to "racism in reverse"; see chapter 9 titled, *"Black empowerment" policies: Dilemmas of affirmative action in South Africa*, in Ratuva (2013). See also the New York Times (2015) article, "India's Middle-Class Revolt."

The negative effect of affirmative action on top talent will be reversed if parental investments make the children's efforts less potent in translating into talents. This positive role of affirmative action, an important message of this paper, is developed Section 4.2.

4 Parental investments: Short term vs. long term

So far we assumed parental investments to have only a positive short-term impact on children's contestability in school but without any direct effect in the long term. In this section, we modify this assumption by allowing investments to lower the long-term development of children's human capital.[22] The following narrative should clarify its basis.

Consider two hypothetical contestants for good grades in high school exams: Ashley and Becky each studies 20 hours/week in learning maths, physics and english. Ashley has a private tutor who spends 5 hours in the weekends to help her solve test papers. Becky, who has no such help, spends a fraction of her study hours to (re)search answers by doing google search (of similar exercises) and assimilates it by her own backward reasoning. Clearly, Ashley is likely to be ahead of Becky in high school exams due to more test preparation. However, Becky's experience of searching solutions on her own initiative and her thinking process builds a skill/independence that will be more valuable when she starts taking lessons in college that requires lot of self-study. Ashley, due to close tutorial supervision in school, is not as efficient in college as Becky despite putting in the same number of hours. In short, while getting parental help eases the burden of competing in school, the children do not learn to utilise their efforts as effectively, i.e., by a method of *sink and swim* and thus becoming more independent. So when they proceed to the later stage contest, the potency of human capital (i.e., learning capability) acquired through the early stage efforts gets diluted. It should also be noted that our specific position on the role of parental help applies to high school students, as opposed to the evidence of positive impact of early childhood intervention (e.g., Heckman et al., 2013).

The modelling also speaks to recent debates on the informativeness of high school grades and similar standardized tests (SAT) for college admissions ('informativeness principle' of Holmstrom (1979)). As to be expected, with disparity in parental resources admissions authorities are likely to question the independence of applicants with perfect grades and perfect SAT scores. Because independence, or differently sometimes called personalities, could be a useful criterion in assessing the applicants' overall merit, it is reasonable to subtract the contribution of wealth.[23] This, in essence, motivates our modelling of wealth impacting negatively on talent/independence. We aim to clarify why with no clear way of disentangling the applicants' potential talents from school grades and SAT scores, universities may want to use affirmative

[22]Cunha et al. (2006), and Cunha and Heckman (2007) formulate dynamic models of parental investments allowing for complementarity between early and late stage of the child's development under credit constraints. These authors focus on how college subsidies encouraging college attendance might not be fully taken advantage of by poor families who are unable to invest in their children in the early phase. While we acknowledge that parental investments have a positive role in human capital development, we take a complementary view that parental investments that are meant to give the children an immediate short-term advantage in the school competition and thus substitute out children's efforts (as captured by our tug-of-war contest success function) are likely to also compromise children's *independence*. Our modelling choice is motivated to capture this compromising effect.

[23]See Dixon-Roman et al. (2013), Washington Post (2014), CNBC (2019), New York Times (2021), and Alvero et al. (2021) on class bias (or influence of wealth and the educated class) in the evaluation of college applications through essays and standardized tests such as SAT.

action to rectify the imperfections with standard metric-based admission procedures.

4.1 American dream under non-intervention

Assumption 2 *Investment* I *by the parents lowers the effective effort of the child by* kI *where* $0 < k \leq 1$.[24] *That is, a child's effort* e *in the school contest is equivalent to* $e - kI$ *in the second round contest in college.*

Note that if the child has not put in any effort in school and parents have invested a positive I then the effective effort, $e - kI$, is negative. The negative effective effort can arise even when the child puts in a positive e. This seeming technical glitch is not really a problem since we can interpret the negative effective effort as low levels of effective effort (as a result of the origin being set at a negative number). We could also address the issue by assuming that all children must make a minimum time commitment, so many hours per week that one must attend school. These hours are the same for all contestants and we normalize the cost of these hours to be zero. This amounts to adding a constant K to all the efforts which cancel out in tug of war contest success probability functions, e.g., for the second stage probability we have

$$\frac{(K + e_i) - kI_i + 1 - (K + e_j) + kI_j}{2} = \frac{e_i - kI_i + 1 - e_j + kI_j}{2}.$$

Our analysis remains unchanged under the second interpretation since K cancels out in all the expressions. So we can safely ignore the fixed term K.

For the rest of this section, we will assume that the children are of identical abilities: $d_1 = d_2 = d_3 = d$.

We assume the parental wealths are $\omega_1 = w - m_1$, $\omega_2 = w$ and $\omega_3 = w + m_3$, where $m_1, m_2 > 0$. Recall that when $k = 0$ the parents make full investments (Corollary 1). In this section, we consider the possibility of less than full investments. If parents 1, 2 and 3 decide to lower investments to $I_1 = w - x$, $I_2 = w - y$ and $I_3 = w + z$, then they are underinvesting by $x - m_1$, y and $m_3 - z$ respectively. It should be easy to see, from the earlier studied case of $k = 0$, that in equilibrium parents will continue to make full investments ($x = m_1$, $y = 0$ and $z = m_3$), if k, the factor of depreciation of human capital e, is low enough. What will be new and interesting is that for larger values of k the wealthiest parents, parents 3, would underinvest in equilibrium by choosing $z < m_3$. Further, if k is even larger, closer to 1, then the middle income parents would also underinvest by choosing $y > 0$.

Let $\Pi_1(x, y, z, k)$ be the expected payoff to parents 1 when they underinvest by $x - m_1$, and parents 2 and 3 underinvest by y and $m_3 - z$, respectively. Similarly, define $\Pi_2(x, y, z, k)$ and $\Pi_3(x, y, z, k)$. We assume that $m_1 \leq x \leq w$, $y \leq x$ and $-y \leq z \leq m_3$.[25] The full investment result for $k = 0$ is due to the fact that $\Pi_1(x, 0, m_3, 0)$ and $\Pi_2(m_1, y, m_3, 0)$ are decreasing in x

[24] In fact, k may be allowed to exceed 1.

[25] Allowing $y > x$ makes parents 2 invest the least. In that case, we can switch the roles of parents 1 and 2 and show that parents 2 will never invest less than parents 1.

and y at $x = m_1$ and $y = 0$, respectively, and $\Pi_3(m_1, 0, z, 0)$ is increasing in z at $z = m_3$.

In fact, it can be shown that for any k the payoff $\Pi_1(x, y, z, k)$ is decreasing in x, i.e., given parental investments $I_1 = w - x$, $I_2 = w - y$ and $I_3 = w + z$, the poorest parents have no incentive to reduce investment from a given level $w - x$ regardless of the extent of depreciation of the child's human capital, e. In contrast, the richest parents' payoff at $k = 1$, $\Pi_3(x, y, z, 1)$, is decreasing in z but the payoff $\Pi_3(x, y, z, 0)$ is increasing in z. In fact, there is a cutoff k_3 (that depends on x, y and z) such that whether $\Pi_3(x, y, z, k)$ decreases with z or increases depends on whether $k > k_3$ or $k < k_3$. In other words, whether the richest parents have an incentive to decrease investment from $w + z$ or not depends on if the rate of depreciation is high or low. Similarly, it can also be shown that there is a cutoff k_2 (that depends on x, y and z) such that whether $\Pi_2(x, y, z, k)$ is increasing in y or decreasing, i.e., whether parents 2 have incentive to lower investment from $w - y$ or not, depends on whether $k > k_2$ or $k < k_2$. Furthermore, we can show that $k_2 > k_3$. Thus, for intermediate depreciation rates only parents 3 will have an incentive to underinvest and for high depreciation rates both parents 2 and 3 will have incentives to underinvest. Parents 1 have no incentive to underinvest. We summarize these observations in the following lemma. The detailed proof, which is quite subtle, appears in Appendix A.

Lemma 1 (Investment best responses) *Consider any exogenous profile of parental investments $I_1 = w - x$, $I_2 = w - y$ and $I_3 = w + z$ where $m_1 \leq y \leq x$, $0 \leq z \leq m_3$. There are critical values k_2 and k_3,*

$$\widehat{k}_2 \equiv \frac{x - 2y - z - 4d(1 + x - 2y - z) + 8d^2(1 + x - 2y - z) - 4\sqrt{(2d-1)^2 d\left\{-x + 2y + d(1 + x - 2y - z)^2 + z\right\}}}{(4d-1)(x - 2y - z)} \tag{6}$$

and

$$\widehat{k}_3 \equiv \frac{x + y + 2z - 4d(1 + x + y + 2z) + 8d^2(1 + x + y + 2z) - 4\sqrt{(2d-1)^2 d\left\{-x - y - 2z + d(1 + x + y + 2z)^2\right\}}}{(4d-1)(x + y + 2z)}, \tag{7}$$

satisfying $0 < \widehat{k}_3 < \widehat{k}_2 < 1$ such that, given the other parents' investments as specified,

(i) *if $x > m_1$, parents 1 would raise investment to $w - m_1$, i.e., parents 1 never underinvest;*

(ii) *parents 2 would lower investment from $w - y$ if $k > \widehat{k}_2$;*

(iii) *parents 3 would lower investment from $w + z$ if $k > \widehat{k}_3$.*

The best response properties indicate that only parents 2 and 3 might be inclined towards choosing investments below the levels permitted by their wealths for the adverse long-term impacts on their children's talents. The characterization of equilibrium investments and the cutoffs will be addressed in Proposition 4.

■ **Equilibrium investments.** Recall that the parents' wealth levels are heterogenous: $w - m_1$, w and $w + m_3$. In determining the equilibrium investments, let us make the cutoffs k_2 and k_3 given by (6) and (7) more precise as follows: (i) derive k_2 by setting $x = m_1$, $y = z = 0$ in (6) so that parents 1 and 2 make full investments and parents 3 invest w, (ii) derive k_3 by setting $x = m_1$, $y = 0$ and $z = m_3$ in (7) so that all three parents make full investments, as follows:

$$k_2 \equiv \frac{m_1 - 4d(1 + m_1) + 8d^2(1 + m_1) - 4\sqrt{(2d-1)^2 d\left\{d(1 + m_1)^2 - m_1\right\}}}{(4d-1)\, m_1}, \tag{8}$$

$$k_3 \equiv \frac{m_1 + 2m_3 - 4d(1 + m_1 + 2m_3) + 8d^2(1 + m_1 + 2m_3) - 4\sqrt{(2d-1)^2 d\left\{d(1 + m_1 + 2m_3)^2 - m_1 - 2m_3\right\}}}{(4d-1)(m_1 + 2m_3)}. \tag{9}$$

Before presenting Proposition 4 we want to lay out the underlying core economic logic somewhat informally. In a way, the argument is an explanation of Lemma 1 as well because the proposition is a simple extension of the lemma.

Fix parental investments at any (I_1, I_2, I_3). Denote by

$$p_1(e_1, I_1, e_2, I_2, e_3, I_3) = \frac{(e_1^* + I_1) + (e_2^* + I_2) + 1 - 2(e_3^* + I_3)}{3}$$
$$\text{and} \qquad p_2(e_1, I_1, e_2, I_2) = \frac{(e_1^* - kI_1) + 1 - (e_2^* - kI_2)}{2},$$

the probability of children 1 and 2 winning the first contest and the probability of child 1 winning the second contest against child 2, respectively. Parents 1's payoff in the equilibrium of the continuation game is given by

$$p_1(e_1^*, I_1, e_2^*, I_2, e_3^*, I_3)\, p_2(e_1^*, I_1, e_2^*, I_2) + p_1(e_1^*, I_1, e_3^*, I_3, e_2^*, I_2)\, p_3(e_1^*, I_1, e_3^*, I_3),$$

where the equilibrium efforts depend on the parental investments (with arguments suppressed for brevity).

The derivative of parents 1's payoff w.r.t. I_1 given the investments of the other parents can be written as

$$\begin{aligned}
&\left[\frac{1}{3}\frac{(e_1^*-kI_1)+1-(e_2^*-kI_2)}{2}-\frac{k}{2}\frac{(e_1^*+I_1)+(e_2^*+I_2)+1-2(e_3^*+I_3)}{3}\right.\\
&\quad\left.+\frac{1}{3}\frac{(e_1^*-kI_1)+1-(e_3^*-kI_3)}{2}-\frac{k}{2}\frac{(e_1^*+I_1)+(e_3^*+I_3)+1-2(e_2^*+I_2)}{3}\right]\\
&+\left[\frac{1}{3}\frac{(e_1^*-kI_1)+1-(e_2^*-kI_2)}{2}+\frac{1}{2}\frac{(e_1^*+I_1)+(e_2^*+I_2)+1-2(e_3^*+I_3)}{3}\right]\frac{\partial e_1^*}{\partial I_1}\\
&+\left[\frac{1}{3}\frac{(e_1^*-kI_1)+1-(e_3^*-kI_3)}{2}+\frac{1}{2}\frac{(e_1^*+I_1)+(e_3^*+I_3)+1-2(e_2^*+I_2)}{3}\right]\frac{\partial e_1^*}{\partial I_1}\\
&+\left[\begin{array}{c}\frac{1}{3}\frac{(e_1^*-kI_1)+1-(e_2^*-kI_2)}{2}-\frac{1}{2}\frac{(e_1^*+I_1)+(e_2^*+I_2)+1-2(e_3^*+I_3)}{3}\\-\frac{1}{3}\frac{(e_1^*-kI_1)+1-(e_3^*-kI_3)}{2}\end{array}\right]\frac{\partial e_2^*}{\partial I_1}\\
&+\left[\begin{array}{c}\frac{1}{3}\frac{(e_1^*-kI_1)+1-(e_3^*-kI_3)}{2}-\frac{1}{2}\frac{(e_1^*+I_1)+(e_3^*+I_3)+1-2(e_2^*+I_2)}{3}\\-\frac{1}{3}\frac{(e_1^*-kI_1)+1-(e_2^*-kI_2)}{2}\end{array}\right]\frac{\partial e_3^*}{\partial I_1}.
\end{aligned}$$

In this case $\frac{\partial e_2^*}{\partial I_1}=\frac{\partial e_3^*}{\partial I_1}$, so we can simplify the above expression to

$$\begin{aligned}
&\left[\frac{1}{3}(1-k)\left(\frac{e_1^*+1-e_2^*}{2}+\frac{e_1^*+1-e_3^*}{2}\right)-k\frac{2I_1-I_2-I_3}{3}\right]\\
&+\left[\frac{1}{3}\frac{(e_1^*-kI_1)+1-(e_2^*-kI_2)}{2}+\frac{1}{2}\frac{(e_1^*+I_1)+(e_2^*+I_2)+1-2(e_3^*+I_3)}{3}\right]\frac{\partial e_1^*}{\partial I_1}\\
&+\left[\frac{1}{3}\frac{(e_1^*-kI_1)+1-(e_3^*-kI_3)}{2}+\frac{1}{2}\frac{(e_1^*+I_1)+(e_3^*+I_3)+1-2(e_2^*+I_2)}{3}\right]\frac{\partial e_1^*}{\partial I_1}\\
&-\frac{1}{2}\frac{(e_1^*+I_1)+(e_2^*+I_2)+1-2(e_3^*+I_3)}{3}\frac{\partial e_2^*}{\partial I_1}-\frac{1}{2}\frac{(e_1^*+I_1)+(e_3^*+I_3)+1-2(e_2^*+I_2)}{3}\frac{\partial e_3^*}{\partial I_1}.
\end{aligned}$$

Consider the indirect effect of increasing I_1:

$$\begin{aligned}
&\left[\frac{1}{3}\frac{(e_1^*-kI_1)+1-(e_2^*-kI_2)}{2}+\frac{1}{2}\frac{(e_1^*+I_1)+(e_2^*+I_2)+1-2(e_3^*+I_3)}{3}\right]\frac{\partial e_1^*}{\partial I_1}\\
&+\left[\frac{1}{3}\frac{(e_1^*-kI_1)+1-(e_3^*-kI_3)}{2}+\frac{1}{2}\frac{(e_1^*+I_1)+(e_3^*+I_3)+1-2(e_2^*+I_2)}{3}\right]\frac{\partial e_1^*}{\partial I_1}\\
&-\frac{1}{2}\frac{(e_1^*+I_1)+(e_2^*+I_2)+1-2(e_3^*+I_3)}{3}\frac{\partial e_2^*}{\partial I_1}-\frac{1}{2}\frac{(e_1^*+I_1)+(e_3^*+I_3)+1-2(e_2^*+I_2)}{3}\frac{\partial e_3^*}{\partial I_1}.
\end{aligned}$$

We have $\frac{\partial e_1^*}{\partial I_1}>0$ and the bracketed expressions in the coefficients of $\frac{\partial e_1^*}{\partial I_1}$ involve probabilities of winning the different contests. The coefficients are thus positive. We also have that $\frac{\partial e_2^*}{\partial I_1}=\frac{\partial e_3^*}{\partial I_1}<0$ and the coefficients are school contest winning probabilities. Thus, these terms are also positive, so the indirect effect on parents 1's payoff is positive through increase in child 1's effort and decrease in the efforts of the other children.

Now consider the direct effect

$$\begin{aligned}
&\frac{1}{3}\frac{(e_1^*-kI_1)+1-(e_2^*-kI_2)}{2}-\frac{k}{2}\frac{(e_1^*+I_1)+(e_2^*+I_2)+1-2(e_3^*+I_3)}{3} \\
&\quad+\frac{1}{3}\frac{(e_1^*-kI_1)+1-(e_3^*-kI_3)}{2}-\frac{k}{2}\frac{(e_1^*+I_1)+(e_3^*+I_3)+1-2(e_2^*+I_2)}{3} \\
&=\frac{1}{3}(1-k)\left(\frac{e_1^*+1-e_2^*}{2}+\frac{e_1^*+1-e_3^*}{2}\right)-k\frac{(2I_1-I_2-I_3)}{3}.
\end{aligned}$$

Suppose parents 1 are the least wealthy and their investment is less than that of the other more wealthy parents. Then we have $2I_1-I_2-I_3<0$. The bracketed expression is positive by virtue of being college contest winning probabilities. So if k is between 0 and 1, the direct effect of an increase in investment by parents 1 will be positive. Since both the direct and indirect effects are positive, in this case parents 1 will not want to reduce their investment I_1 when the other parents are investing more.[26]

If parents 1 are the richest, investing more than the other parents, then $2I_1-I_2-I_3>0$ and the direct effect is negative if k is not too close to zero. In this case, if the negative direct effect which arises from the trade-off between present and future probabilities of winning dominates the positive indirect effect, then they will underinvest. Now apply this logic to parents 3 who are the richest. For parents 2 with intermediate wealth, the direct effect will involve the term $-k\frac{(2I_2-I_1-I_3)}{3}$, the sign of which is indeterminate and so parents 2 may or may not reduce investment.

So far in the above discussion we held k fixed. Next we vary k and turn to why the decision to invest below the level permitted by the wealth follows the order of wealth in reverse – from the most wealthy parents to the least – as the intensity with which parental investment damages child's independence becomes stronger. Starting from $k=0$ and all parents investing fully, as k increases parents who invest the maximal (parents 3) would suffer the most in the second round contest as their high investment hurts their child's potency for any induced profile of efforts (e_1, e_2, e_3). So these parents start to consider the merit of the last dollar of their investment. As k becomes large,

(i) the direct marginal hurt caused by investment in terms of the probability of success in second round follows the wealth order, $w+m_3>w>w-m_1$, whereas the direct marginal effect on the probability of first round success is zero (k is not relevant for the first round contest success);

(ii) in addition, there will be indirect adverse effects on the children's marginal effort incentives with the child of the most wealthy suffering the highest dilution to her potency in the long term (i.e., in the second round).

(iii) Overall ((i) and (ii) combined), given full investments by all three parents, the payoff

[26] Note that if k is sufficiently larger than 1, then the direct effect may be negative and parents 1 may have an incentive to reduce investment, as well.

drops the most for the wealthiest parents.

Therefore as k rises, the marginal incentives of parental investments start to decline the most for the most wealthy (parents 3), and thus they will be the first to invest less than their full capacity. Parents 2 with wealth w below that of parents 3 $(w + m_3)$ would still have positive marginal incentives at the value k_3 (where parents 3 start to underinvest), simply because their last dollar of investment at w inflicts less marginal damage on their child's future potency. This implies k will still have to climb up to push parents 2 to start underinvesting: $k_2 > k_3$. However, marginal investment incentives of parents 1 remain strictly positive throughout for two reasons: first, the assumed upper limit of $k = 1$ is not big enough; second, the parents cannot afford to further compromise the low chances of their child succeeding in the first round contest due to low wealth $(w - m_1 < w < w + m_3)$.

Proposition 4 (Sink and swim strategy) *Suppose the parents have wealth levels $\omega_1 = w - m_1$, $\omega_2 = w$ and $\omega_3 = w + m_3$, where $m_1, m_3 > 0$. Then parents 3 are the most likely to underinvest (relative to their wealth), followed by parents 2. More specifically,*

(i) *For $k \leq k_3$, the parents invest $w - m_1, w$ and $w + m_3$, respectively.*

(ii) *For $k_3 < k \leq k_2$, parents 1 and 2 invest $w - m_1$ and w, and parents 3 invest $w + z^*$ where*

$$0 < z^* = \frac{16d^2(1 - k - km_1) + 4d\left\{-2 + m_1 + k^2 m_1 + 2k(1 + m_1)\right\} - (1 + k)^2 m_1}{2\left[16d^2 k - 4d(1 + k)^2 + (1 + k)^2\right]} < m_3.$$

(iii) *For $k_2 < k < 1$, parents 1 invest $w - m_1$ and parents 2 and 3 both invest $w - y^*$ where*

$$0 < y^* = \frac{(1 + k)^2 m_1 + 16d^2(k + km_1 - 1) + 4d\left\{2 - m_1 - k^2 m_1 - 2k(1 + m_1)\right\}}{16d^2 k + (1 + k)^2 - 4d(1 + k)^2} < m_1.$$

This result should be seen at the backdrop of the debate on different parental approaches to children's education and upbringing. In a 2011 *Wall Street Journal* article, "Why Chinese Mothers Are Superior," Yale Law School Professor Amy Chua extolled the virtue of being tiger mums. These parents prioritise their children's academic achievements even at the expense of their development of independence or personality (Chua, 2011). As a counterpoint, Natalia Nedzhvetskaya wrote, "Why 'Chinese Mothers' Are Not Superior" and how being tiger mums could be counterproductive hindering children's independence critical for future success (Nedzhvetskaya, 2011). The main issue here is the concern that too much parental supervision is counter-productive. At the same time the society gets drawn into an unwanted investment war even when everyone recognizes the potential negative effect of such investments on children's development.

There are several implications of Proposition 4. First and foremost, with the negative effect of investment parents are caught between countervailing incentives – helping their children to cross the first hurdle, to ensure their admission to college, and not helping too much so that the children's dependence on parental support in the early stage dilutes their sharpness when it comes to the more independent race later on. This incentivizes some of the more wealthy parents to exercise self-control and not invest fully.[27] But when the parents are on an even keel ($m_1 = m_3 = 0$), they still engage in an all-out unproductive investment war just like in Proposition 1 and Corollary 1. With unequal wealths, the poorest parents are pressed into full investment for their children to have the best shot at the race by crossing the school hurdle. Of course more wealthy parents will have (weakly) better odds in the initial race because their investments are bounded below by the poorest parents' investment. As the negative effect of investment, k, increases, more parents from the upper end of the wealth distribution recognize the value of letting their children learn to be independent through trial and error and not overcoach them; this way children develop some non-cognitive skills as well such as patience, discipline etc. While our modelling does not explicitly capture the dynamic process of learning through mistakes, its essence is condensed in the aggregative index k.

At this stage, two points about our modelling are worth emphasizing. The first is about the choice of only one-time parental investment. Second, one can alternatively consider investment to be of two categories, one to improve children's skills (or ability to keep learning) and another to help them improve their college application. The tradeoffs that parents face for the elaborate choices noted in the second point are already captured in our investment formulation for the following reason. Choosing between application-boosting investment and skill-improving investment can be replicated in our model by the choice of how much, if any, to underinvest. The higher the underinvestment, the lower the chances of success in the school race and the higher the child's ultimate talent (for any given effort by the child). For any given wealth budget, parents can never improve both the odds of school success and child's skill, which is also the case in our model. However, regarding the first point, relaxing the restriction of one-time parental investment and accommodating multi-round investments to choose between timing, distribution and types (cognitive vs. non-cognitive) of investment (Heckman and Mosso, 2014), as we have discussed in Section 2, would introduce additional complications. We sacrifice this generality in the bargain for an explicitly fleshed-out model of contest that we feel captures better some fine elements of real-life academic race that are not analyzed in the empirical literature cited in the Introduction.

■ **Social mobility and American dream.** We are now ready to analyze an issue on education that cuts across all countries interested in the development of the poor and underprivileged. Can children from disadvantaged backgrounds entertain a realistic hope to rise to the top? Seeing education as nothing but a contest breeding talents from the pool of which top talents will be picked, in the following proposition we argue that there is a path for the less privileged to move to the top.

[27]This result has some parallel in the *partial sheltering* result of Lizzeri and Siniscalchi (2008), discussed in the Introduction.

Proposition 5 (The American dream) *Suppose the parental wealths are $\omega_1 = w - x$, $\omega_2 = w$ and $\omega_3 = w + x$, where $x > 0$. There exists a k^* (specifically $k^* = \frac{1}{4d-1}$) such that the effective effort by the poorest child is the largest among the three competitors if and only if $k > k^*$.*

The positive result without any normative connotation (yet) is a logical prediction of our hypothesis that parental investments can deprive the children of their independence. Just because wealthy parents can give their children an "unfair" advantage in the school race does not mean that the less fortunate families are completely out of it. This ray of hope is due to the long-term negative effect of such investments. Of course for Proposition 5 to hold, the adversarial effect of hyper-competitive parents, i.e. k, has to be significant. The more the employers look beyond school grades and have the means to accurately evaluate potential recruits' talents, the better the chances of the American dream coming to fruition. In this regard, university admissions play an important role. The current practice of relying on multiple indicators of student abilities, not just academic grades, is a case in point. For example, in Section 4.2 we will see the potential of wealth-based affirmative action to help the objective of developing talents in an unequal race due to wealth inequalities. With this in mind we now turn to the relation between inequality and top talent under no intervention.

Proposition 6 (American dream and top talent) *Suppose the parents have wealths $\omega_1 = w - x$, $\omega_2 = w$ and $\omega_3 = w + x$, and $w > x > \frac{1}{3}$. Increasing wealth inequality reduces expected top effective talent for $k^* < k < k_3$, and increases expected top effective talent otherwise, i.e, if either $0 < k < k^*$ or $k > k_3$, where k^* is as defined in Proposition 5.*

Thus more inequality leads to improved expected top (effective) talent when the negative effect of parental investment is either small or large, and lowers it in the intermediate range. For small k, the result is a natural extension of part (i) of Proposition 2. For $k^* < k < k_3$, as x increases it can be verified that the probability of the richer children winning the two-round race will stay above that of the poorest child but their effective efforts worsen due to the negative effect of (full) parental investments, and by Proposition 5 the poorest child's effective effort is the largest. Thus, the more wealthy students who win more often end up having their effective efforts lowered due to inequality (x) increasing. In expectation, effective top talent starts declining in this range of k. Finally, when k surpasses k_3, underinvestment of parents 3 and subsequently similar underinvestment by parents 2 neutralize the negative effects of investment. So any increase in inequality (x) pulls up the effective top talent pretty much for the same reasons as discussed following Proposition 2 (for part (i)).

Example 1. In Fig. 1 we have k on the horizontal axis, the derivative of the expected top effective talent w.r.t. x (when the wealths are $w - x, w, w + x$) on the vertical axis. The derivative, given by (B.8) in the Appendix, is plotted for $x = \frac{1}{16}$ and $d = 4$. Note that the derivative is independent of w. The plots are only for the ranges $0 < k < k^*$ and $k^* < k < k_3$. For $k > k_3$ the derivative is positive and large that does not display in the figure. Mathematica yields $k^* = 1/15$ and $k_3 = 0.859274$.

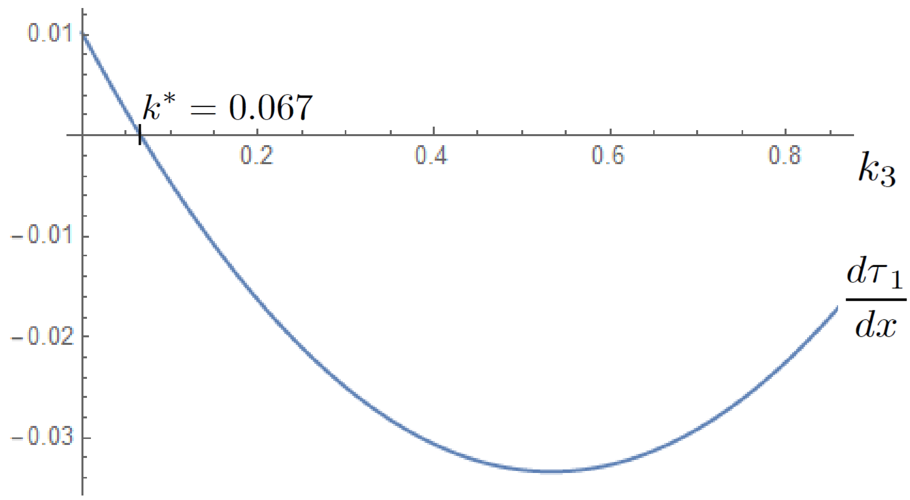

Figure 1: Expected top talent may fall as wealth inequality rises (Proposition 6)

Proposition 7 (Inequality & total talent) *Suppose that the parents have wealths* $\omega_1 = w - x$*,* $\omega_2 = w$ *and* $\omega_3 = w + x$*. Then as* x *increases the total effective effort (or talent) remains unchanged if* $k \leq k_3$*, but increases if* $k > k_3$*.*

For negative effect of investment sufficiently small ($k \leq k_3$), increases in wealth inequality leaves total efforts (or talent) unchanged because of the symmetric effects on efforts: the wealthiest child increases her effort by the same amount as that by which the poorest child lowers her effort; the effort of the child with parental wealth w does not change with x. For large negative effect ($k > k_3$), more inequality improves effective total effort due to the wealthier parents lowering their investments.

It is also to be noted here that total (effective) efforts maximization and maximization of (expected) top talent, as objectives, do not necessarily go hand in hand. Total effective efforts do not vary as inequality increases in the entire range $0 \leq k \leq k_3$, but expected top talent increases in inequality in the range $0 \leq k < k^*$ and then it decreases in the range $k^* < k < k_3$. So in the first interval (i.e., $0 \leq k \leq k_3$) inequality reduction hurts the societal objective of developing top talents, whereas in the second interval ($k^* < k < k_3$) inequality reduction serves to enhance the top talent objective.

4.2 Affirmative action may boost top talents

Proposition 5 is a statement about what the children from underprivileged backgrounds can hope to achieve. It does not say, however, that the dream would indeed materialize. If the extent of wealth inequality is large, the underprivileged stands little chance of crossing the first hurdle and enter college. In such situations there is a clear role for the government or the college authorities: AA accounting for the handicap on the applicants from poor income families serves the redistribution goal. But here we are going to argue that the intervention often produces better top talents (on average). This important final message we develop next.[28] We will start with an example.

4.2.1 Example 2

All derivations for this example are included in Appendix B.10 and Appendix-C-10-0.

Suppose parental wealths are $\omega_1 = 0$, $\omega_2 = \frac{1}{8}$ and $\omega_3 = \frac{1}{4}$, and the children (students) have identical costs with marginal cost $d > 0$. In this case the students' equilibrium efforts in the benchmark contest (i.e., in the absence of affirmative action) are as follows:

$$e_1^* = \frac{29d + 3dk - 16}{48d(2d-1)}, \quad e_2^* = \frac{1}{3d}, \quad e_3^* = \frac{35d - 3dk - 16}{48d(-1+2d)}$$

where the parents make full investments. It can be checked that for $k < k_3$,

$$k_3 = \frac{3 - 44d + 88d^2 - 4\sqrt{484d^4 - 580d^3 + 217d^2 - 24d}}{3(4d-1)},$$

the parents indeed make full investments in equilibrium.

Under AA with a quota guaranteeing student 1's college entry, the equilibrium efforts are:

$$e_1^{AA} = \frac{1}{4d}, \quad e_2^{AA} = \frac{120d^2 - 8d^2k - 63d + 4dk + 6}{8d(8d-3)(8d-1)}, \quad e_3^{AA} = \frac{136d^2 - 16d^2k - 65d + 5dk + 6}{8d(8d-3)(8d-1)}$$

when parents 2 and 3 make full investments. Again, it can be checked that for $k < k_3^{AA}$,[29]

$$k_3^{AA} = \frac{1408d^3 - 832d^2 + 122d - 1 - 2\sqrt{495616d^6 - 634880d^5 + 300416d^4 - 62528d^3 + 5073d^2 - 72d}}{96d^2 - 36d + 1},$$

parents 2 and 3 would make full investments.

So we have:

$$e_1^{AA} - e_1^* = -\frac{(5+3k)\,d - 4}{(96d - 48)\,d}, \qquad e_2^{AA} - e_2^* = -\frac{8d^2(19+3k) - d(67+12k) + 6}{24d(8d-3)(8d-1)},$$
$$e_3^{AA} - e_3^* = -\frac{608d^3 - d^2(548+60k) + d(155+21k) - 12}{48d(2d-1)(8d-3)(8d-1)}.$$

It can be shown that each of the expressions on the RHS is negative for our range of k and d,[30] so that every student lowers effort under affirmative action. Note that the difference in

[28] A reader should recognize that the objective of developing top talents is best served, in our setting, by facilitating college entry of the children of the wealthiest parents ahead of the poorer parents. That way, the parents who would otherwise have invested the most are taken out from heating up school competition too much. But clearly such a policy will not be politically viable. So we adopt the more conventional route of affirmative action.

[29] k_3^{AA} is a cutoff k for full investments by parents 2 and 3, analogous to k_3, under AA. It is defined formally in Section 4.2.2.

[30] The first difference is negative because $(5+3k)\,d - 4 > 0$ for all $d > 3$ and $k \geq 0$.

The second difference is negative because $8d^2(19+3k) - d(67+12k) + 6$ is quadratic (convex) with both the efforts are equal to the difference in effective efforts.

Now consider the difference in expected top talents $\tau_1^{AA} - \tau_1$ (τ_1^{AA} is expected top talent under AA): it is positive for all $k^* < k < \min\{k_3, k_3^{AA}\}$. Fig. 2 shows this difference against k on the horizontal axis for various values of d, using materials developed in Appendix B.10. (k_3 and k_3^{AA} both exceed 0.7.)

Thus, while all students lower their efforts under AA, the expected top talent increases. While all students are of equal merit (uniform d), in the benchmark (no AA) contest the first-round race in school determines who goes to college. Wealthier parents, by investing fully (in our example), keeps the odds of the poorest student (student 1) progressing to college small. Notice, however, it is the poorest student whose effective effort (equal to his effort as his parents have no wealth) is the highest – see Fig. 3. This means in the standard two-round race the person who would have lifted the top talent is the least likely to get off the ground: the American dream isn't allowed to materialize.

Affirmative action rectifies the above deficiency for reasons explained below:

First, the poorest student is given a guaranteed admission in college, thus correcting a fundamental shortcoming of the benchmark contest.

Second, while the intervention (AA) lowers all students' efforts, still the effective effort of the poorest student remains the highest, for the range $k^* < k < \min\{k_3, k_3^{AA}\}$, as can be seen from Fig. 4. This means, the most talented (ex post) person would also have the highest odds of winning the contest (college race). If the lowered effort of the poorest student is still higher than the efforts of the wealthier students in the benchmark contest, affirmative action can improve the expected top talent.

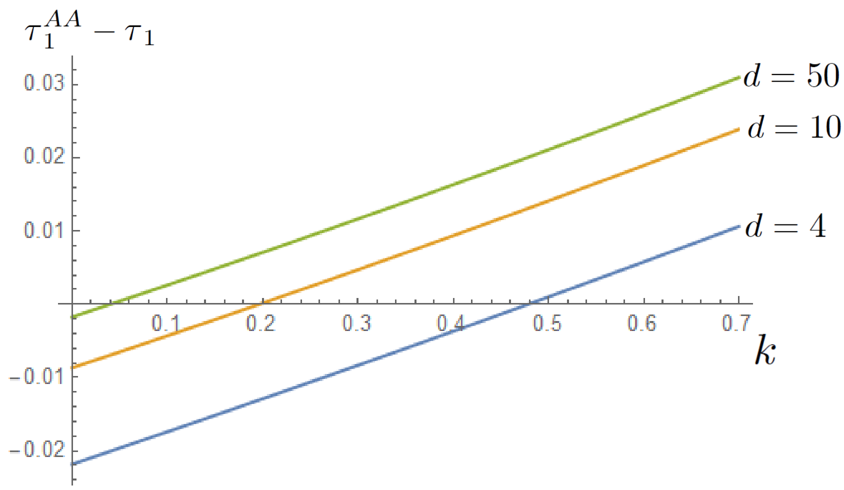

Figure 2

roots being smaller than 1 for all $0 \leq k \leq 1$.

The third difference is negative because $608d^3 - d^2(548 + 60k) + d(155 + 21k) - 12$ is increasing in d (the derivative is quadratic with both roots smaller than 1) and at $d = 3$ it takes a value $11937 - 477k$ which is positive for $0 \leq k \leq 1$.

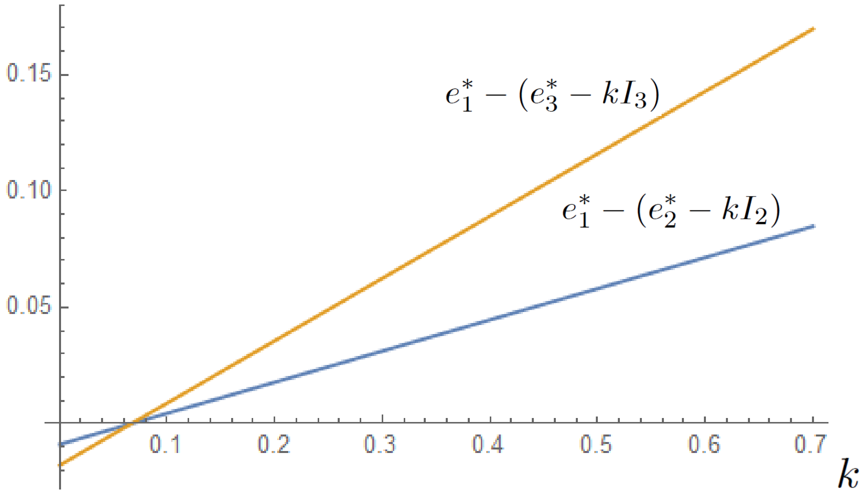

Figure 3

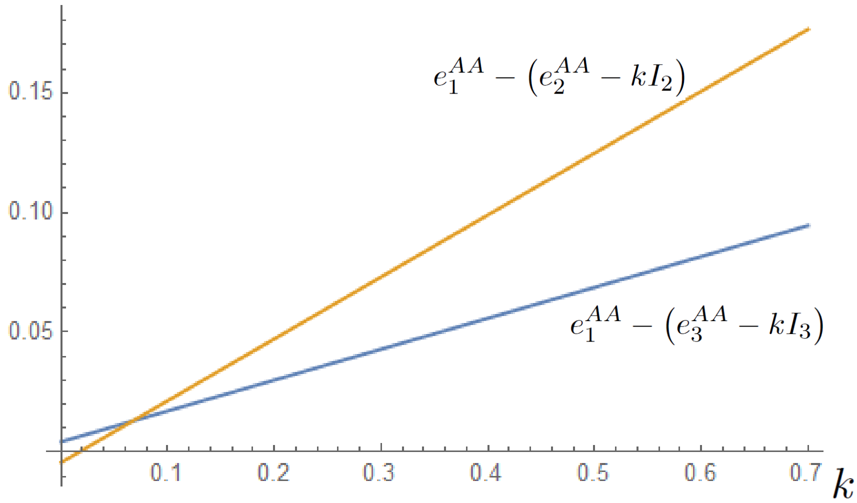

Figure 4

Third, as k increases wealthier parents exert more and more damages given their full investments. This dissuades their children more on the margin (under both the benchmark contest and AA); student 1, however, does not suffer from this negative influence. This translates into an increasing difference between student 1's effective effort and a wealthier student's effective effort as k increases. Affirmative action, by shifting the winning probability to student 1, allows the higher margin of his effective effort (over the other two) to count more: the expected top talent tends to increase.

Fourth, because AA induces all students to lower their efforts, there is also a negative effect on top expected talent. If k is small enough then the difference in the effective efforts of student 1 and a wealthier student is not enough to overcome the loss due to the decrease in efforts under AA. In particular, if $k = 0$, AA reduces the expected top talent.

Fifth, in an alternative scenario if parents 1 had some wealth, then in the absence of AA they would invest fully. This damages student 1's effective effort, which is avoided under AA. So the case for affirmative action is likely to strengthen for $I_1 > 0$.

Finally, a comparison with Proposition 3 is instructive. There too affirmative action had lowered all students' efforts, but unlike in the current example the expected top talent had decreased. An obvious difference between the two setups is the long-term damaging effect of investment on talents vs. neutral effect of investment. It is easy to see that if k is set equal to 0 or very small (see Fig. 2), we get back the result that AA would lower the expected top talent (see also the fourth point above). Thus, the negative effect of investment is critical to the positive role of affirmative action. ||

By this example the main point that we want to convey is that by not giving the (equally) meritorious underprivileged students a shot at college, primarily due to the heated competition in schools, the society is deprived of a talent base that it can ill afford. When one combines the negative effect of the parental race on the children's long-term development, the cost of missing out on these students from poor income families is even more significant. That this sentiment is shared by educational institutions is reflected in a report on admissions by Ivy league and elite colleges in the United States, highlighting how they have managed to admit a high percentage of first generation college applicants, and person of color.[31]

4.2.2 Generalizing the example

We next turn to extend the above example by allowing for $\omega_1 \geq 0$ and large values of k.

Fix the parents' wealth levels at $\omega_1 = w - x,\ \omega_2 = w,\ \omega_3 = w + x$.[32] Start with the no intervention benchmark of Section 4.1 when the students are subjected to uniform contest rules. Denote by τ_1 and τ_2 the expected top (effective) talent, respectively, when no parents underinvest, and both parents 2 and 3 underinvest but parents 1 invest fully, i.e. $k < k_3$ and $k > k_2$, with k_2 and k_3 as in (8) and (9) after substituting $m_1 = m_3 = x$ (Proposition 4). Under affirmative action, let k_3^{AA} and k_2^{AA} be such that parents 2 and 3 invest fully for $k < k_3^{AA}$, and for $k > k_2^{AA}$ parents 2 and 3 underinvest. The cutoff k_2^{AA} is determined in the proof of Proposition 8, but the determination of k_3^{AA} is relegated to Appendix B.10. So we can define τ_1^{AA} and τ_2^{AA} to be the expected top talents respectively for $k < k_3^{AA}$ and $k > k_2^{AA}$.[33]

The general solution of equilibrium efforts under non-intervention is presented in (10)–(12) in the Appendix. For wealth distribution $w - x, w, w + x$, these efforts are given by (16) for $k < k_3$ (which we will need for Example 2 in Section 4.2.1) and by (19) and (20) for $k > k_2$

[31]"Acceptance Rates at Ivy League & Elite Colleges – Class of 2025," published on April 9, 2021, reports the following data: first generation college students are 35% for Middlebury, 22% for Princeton and Amherst, 17% for Brown and Dartmouth, 15% for Penn, and 13% for Notre Dame. Cornell reports that underrepresented minorities increased to 34.2% from 33.7% last year, and 59.3% self-identify as students of color; self-identified admits of color for Penn is 56% (up from 53% last year), and for Amherst it is 60%. Notre Dame reported that 40% of those admitted were students of color. **Source:** `https://www.collegetransitions.com/blog/ivy-league-acceptance-rates-class-of-2025/` .

[32]A non-uniform spread of wealth should not alter the results qualitatively. We do not consider the wealth distribution $(w-x,\ w,\ w)$ because we want to include a middle income class in the starting wealth distribution to study how they fare in the redistribution. They are likely to be the strongest voice opposing affirmative action.

[33]Other combinations of (under)investments are also possible but we use the specific two cases to highlight our main points effectively.

(proof of Proposition 5). Nash equilibrium efforts under affirmative action in the continuation game, for $k > \max\{k_2, k_2^{AA}\}$, are derived in the proof of Proposition 8. These efforts are then used to derive the expected top talents under no intervention and under affirmative action, a comparison of which yields the result.

Proposition 8 (Affirmative action: top talent) *For $k > \max\{k_2, k_2^{AA}\}$ and the marginal cost parameter d sufficiently large, affirmative action would improve the expected top talent: $\tau_2^{AA} > \tau_2$. Also, parents 2 and 3 would lower investment under affirmative action relative to the benchmark contest.*

Allowing for positive investment by the poorest parents in the benchmark contest adds an extra reason to favor affirmative action, as the damaging investment will be avoided. Also, large k makes affirmative action more potent: $y^*|_{AA} > y^*|_{\text{benchmark}}$, implying wealthier parents (parents 2 and 3) would lower their investment more under AA. Parents 2 and 3 are now caught between conflicting incentives: under AA they face stiffer hurdle in school competition (one out of two to win) relative to the benchmark contest (two out of three to win). At the same time, their children would also face a rival who has not been weakened by parental investment. With k large, the long-term objective dominates, inducing lower investments.

The key message here contrasts with the pessimistic projection of affirmative action presented in Proposition 3. The difference arises from the adverse effect of investment on children's independence. Through affirmative action the poorest child is guaranteed college admission, so parents 1 abstain from making the investment $w - x$; parents 2 and 3 lower investments as the negative effect is sufficiently strong. Below we plot two figures to illustrate how the cutoff $\max\{k_2, k_2^{AA}\}$ declines with the increase in the effort cost parameter d.

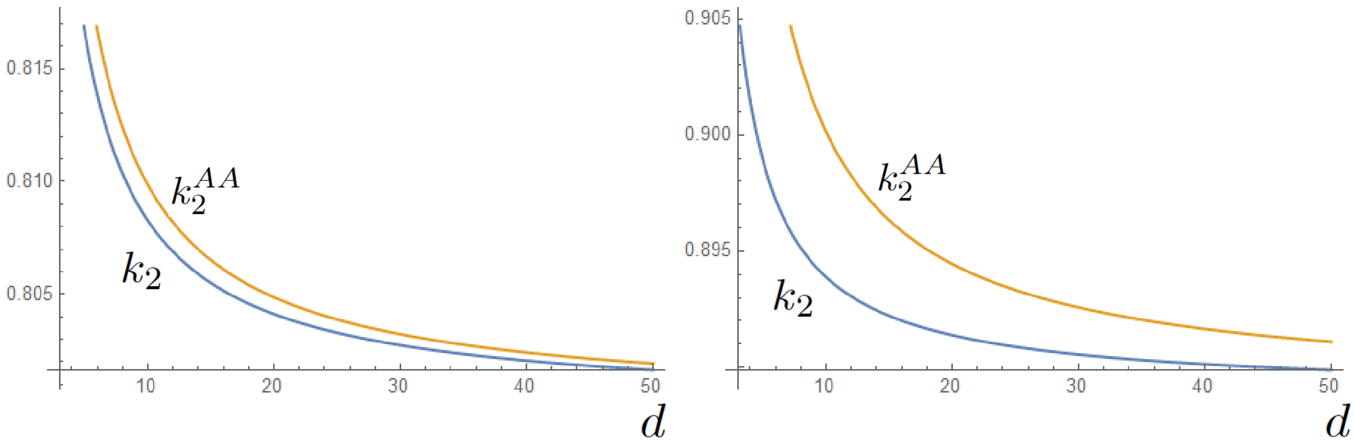

Figure 5: Left panel: $w = 1/4, x = 1/4, d \in [3, 50]$; Right panel: $w = 1/4, x = 1/8, d \in [3, 50]$

4.2.3 Example 3: $\tau^{AA} - \tau > 0$ over a large range of k

Example 2 and Proposition 8 show the dominance of affirmative action over the benchmark contest for (expected) top talents for the ranges $k^* < k < \min\{k_3, k_3^{AA}\}$ and $k > \max\{k_2, k_2^{AA}\}$, respectively. For the analytical tools developed, while comparing top talents is possible for any profile of parameters (w, x, d) and for any k, we do not attempt a complete characterization; this would require partitioning the interval $(0, 1]$ of k finely with precise ranking of the cutoffs $k_3, k_3^{AA}, k_2, k_2^{AA}$ that also depend on the parameters, specifically d.[34] Instead, we report the following construction.

For the wealth distribution $(w - x = 0,\ w = 1/8,\ w + x = 1/4)$, we constructed the following three plots for $\tau^{AA} - \tau$ for $d = 5, 10, 50$. The cutoff ranking is uniform for all three plots, $0 < k_3 < k_3^{AA} < k_2 < k_2^{AA} < 1$, although the specific values differ. Two main points may be noted specific to this construction: (1) As k increases the appeal of affirmative action becomes stronger; (2) with the increase in d, the cutoff k above which affirmative action dominates falls. The reason for the first point was already explained in the third point in the discussion following Example 2 but with one small difference – for high k values parents 2 and 3 may voluntarily restrain investment somewhat even in the benchmark contest. This difference merely softens the advantage of AA without overturning it. The second point is also intuitive: as marginal cost increases, (net) marginal return from efforts in the contest, under the benchmark as well as AA, will decrease. But then freeing the poorest student from the hurdle of school competition mitigates the aggravation due to higher d, whereas under the benchmark contest the aggravation would have applied uniformly to all students. As a result, the advantage of AA over the benchmark contest increases, shifting the $\tau^{AA} - \tau$ curve upwards and lowers the cutoff k.

The construction of Fig. 6 and all relevant derivations appear in Appendix-C-10 that partly utilises materials developed in Appendix B.10.

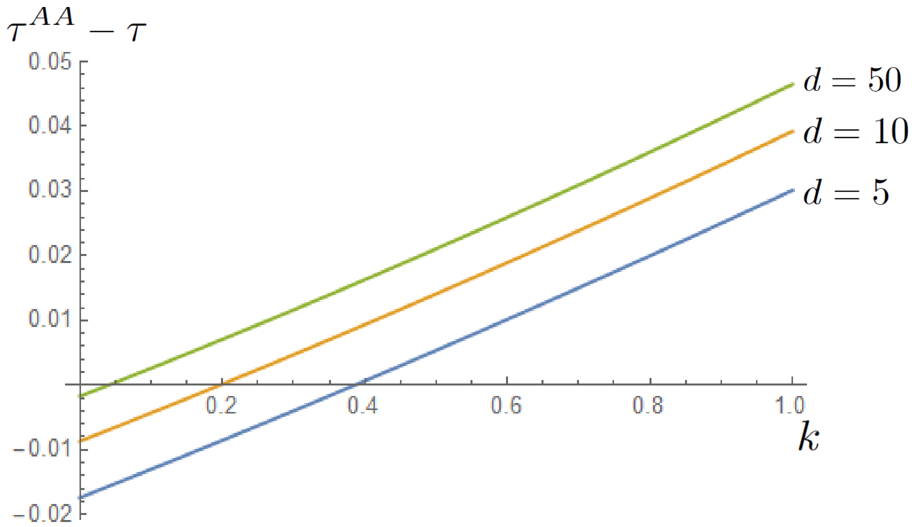

Figure 6

[34]Thus, the cutoffs are moving cutoffs.

A Appendix A: Proofs of Lemma 1 and Propositions 4, 5 and 8

Proof of Lemma 1. Consider any exogenous profile of parental investments $I_1 = w - x$, $I_2 = w - y$ and $I_3 = w + z$, as stated in the Lemma. Our initial derivations will be written in terms d_1, d_2, d_3, but subsequently we will set $d_1 = d_2 = d_3 = d$; recall, children are of identical abilities (Section 4.1).

The expected payoff to child $i \neq j, l, \ j \neq l,$ and $i, j, l = 1, 2, 3$ is

$$\begin{aligned}\pi_i =& \frac{(e_i + I_i) + (e_j + I_j) + 1 - 2(e_l + I_l)}{3} \times \frac{(e_i - kI_i) + 1 - (e_j - kI_j)}{2} \\ &+ \frac{(e_i + I_i) + (e_l + I_l) + 1 - 2(e_j + I_j)}{3} \times \frac{(e_i - kI_i) + 1 - (e_l - kI_l)}{2} - d_i e_i^2.\end{aligned}$$

Taking derivatives with respect to the respective efforts we have the first-order conditions:

$$\begin{aligned}\frac{\partial \pi_1}{\partial e_1} &= \frac{1}{6}[4 + (4 - 12d_1)e_1 - 2e_2 - 2e_3 + 2I_1 - I_2 - I_3 - 2I_1k + I_2k + I_3k] = 0, \\ \frac{\partial \pi_2}{\partial e_2} &= \frac{1}{6}[4 - 2e_1 + (4 - 12d_2)e_2 - 2e_3 - I_1 + 2I_2 - I_3 + I_1k - 2I_2k + I_3k] = 0, \\ \frac{\partial \pi_3}{\partial e_3} &= \frac{1}{6}[4 - 2e_1 - 2e_2 + 4e_3 - 12d_3e_3 - I_1 - I_2 + 2I_3 + I_1k + I_2k - 2I_3k] = 0.\end{aligned}$$

The solution gives the equilibrium efforts in the subgame following parental investments as follows (see Appendix B.5):

$$e_1^* = \frac{\begin{array}{c}2 - 4d_2 - 4d_3 + 8d_2d_3 - (d_2 + d_3 - 4d_2d_3)I_1(1-k) \\ + (d_2 - 2d_2d_3)I_2(1-k) + (d_3 - 2d_2d_3)I_3(1-k)\end{array}}{\Delta}, \tag{10}$$

$$e_2^* = \frac{\begin{array}{c}2 - 4d_1 - 4d_3 + 8d_1d_3 + (d_1 - 2d_1d_3)I_1(1-k) \\ - (d_1 + d_3 - 4d_1d_3)I_2(1-k) + (d_3 - 2d_1d_3)I_3(1-k)\end{array}}{\Delta}, \tag{11}$$

$$e_3^* = \frac{\begin{array}{c}2 - 4d_1 - 4d_2 + 8d_1d_2 + (d_1 - 2d_1d_2)I_1(1-k) \\ + (d_2 - 2d_1d_2)I_2(1-k) - (d_1 + d_2 - 4d_1d_2)I_3(1-k)\end{array}}{\Delta}, \tag{12}$$

where

$$\Delta = 2(d_1 + d_2 + d_3 - 4d_1d_2 - 4d_2d_3 - 4d_1d_3 + 12d_1d_2d_3).$$

We have

$$\frac{\partial e_1^*}{\partial x} = \frac{(d_2 + d_3 - 4d_2d_3)(1-k)}{\Delta}, \quad \frac{\partial e_1^*}{\partial y} = -\frac{(d_2 - 2d_2d_3)(1-k)}{\Delta}, \quad \frac{\partial e_1^*}{\partial z} = \frac{(d_3 - 2d_2d_3)(1-k)}{\Delta}$$

$$\frac{\partial e_2^*}{\partial x} = -\frac{(d_1 - 2d_1d_3)(1-k)}{\Delta}, \quad \frac{\partial e_2^*}{\partial y} = \frac{(d_1 + d_3 - 4d_1d_3)(1-k)}{\Delta}, \quad \frac{\partial e_2^*}{\partial z} = \frac{(d_3 - 2d_1d_3)(1-k)}{\Delta}$$

$$\frac{\partial e_3^*}{\partial x} = -\frac{(d_1 - 2d_1 d_2)(1-k)}{\Delta}, \; \frac{\partial e_3^*}{\partial y} = -\frac{(d_2 - 2d_1 d_2)(1-k)}{\Delta}, \; \frac{\partial e_3^*}{\partial z} = -\frac{(d_1 + d_2 - 4d_1 d_2)(1-k)}{\Delta}.$$

The expected payoff to parents $i \neq j, l, \; j \neq l, \; i, j, l = 1, 2, 3$ is given by

$$\begin{aligned}\Pi_i =& \frac{(e_i^* + I_i) + \left(e_j^* + I_j\right) + 1 - 2\left(e_l^* + I_l\right)}{3} \times \frac{(e_i^* - kI_i) + 1 - \left(e_j^* - kI_j\right)}{2} \\ &+ \frac{(e_i^* + I_i) + (e_l^* + I_l) + 1 - 2\left(e_j^* + I_j\right)}{3} \times \frac{(e_i^* - kI_i) + 1 - (e_l^* - kI_l)}{2}.\end{aligned}$$

Taking derivatives of the parents' payoffs w.r.t. x, y and z, respectively, substituting the expressions for the derivatives from above and simplifying we have

$$\begin{aligned}&\frac{\partial \Pi_1}{\partial x} \\ &= \frac{(e_1^* + I_1) + (e_2^* + I_2) + 1 - 2(e_3^* + I_3)}{3} \times \frac{1}{2}\left(\frac{\partial e_1^*}{\partial x} + k - \frac{\partial e_2^*}{\partial x}\right) + \frac{(e_1^* - kI_1) + 1 - (e_2^* - kI_2)}{2} \times \frac{1}{3}\left(\frac{\partial e_1^*}{\partial x} - 1 + \frac{\partial e_2^*}{\partial x} - 2\frac{\partial e_3^*}{\partial x}\right) \\ &+ \frac{(e_1^* + I_1) + (e_3^* + I_3) + 1 - 2(e_2^* + I_2)}{3} \times \frac{1}{2}\left(\frac{\partial e_1^*}{\partial x} + k - \frac{\partial e_3^*}{\partial x}\right) + \frac{(e_1^* - kI_1) + 1 - (e_3^* - kI_3)}{2} \times \frac{1}{3}\left(\frac{\partial e_1^*}{\partial x} - 1 + \frac{\partial e_3^*}{\partial x} - 2\frac{\partial e_2^*}{\partial x}\right) \\ &= \frac{\begin{array}{c}(8d - 16d^2 - 2x + 8dx + y - 4dy - z + 4dz) + \left[8d(2x - y + z - 1) - 16d^2(2x - y + z - 1) - 2(2x - y + z)\right]k \\ + [-2x + y - z + 4d(2x - y + z)]k^2\end{array}}{12(2d-1)^2},\end{aligned} \tag{13}$$

$$\begin{aligned}&\frac{\partial \Pi_2}{\partial y} \\ &= \frac{(e_1^* + I_1) + (e_2^* + I_2) + 1 - 2(e_3^* + I_3)}{3} \times \frac{1}{2}\left(\frac{\partial e_2^*}{\partial y} + k - \frac{\partial e_1^*}{\partial y}\right) + \frac{(e_1^* - kI_1) + 1 - (e_2^* - kI_2)}{2} \times \frac{1}{3}\left(\frac{\partial e_2^*}{\partial y} - 1 + \frac{\partial e_1^*}{\partial y} - 2\frac{\partial e_3^*}{\partial y}\right) \\ &+ \frac{(e_2^* + I_2) + (e_3^* + I_3) + 1 - 2(e_1^* + I_1)}{3} \times \frac{1}{2}\left(\frac{\partial e_2^*}{\partial y} + k - \frac{\partial e_3^*}{\partial y}\right) + \frac{(e_2^* - kI_2) + 1 - (e_3^* - kI_3)}{2} \times \frac{1}{3}\left(\frac{\partial e_2^*}{\partial y} - 1 + \frac{\partial e_3^*}{\partial y} - 2\frac{\partial e_1^*}{\partial y}\right) \\ &= \frac{\begin{array}{c}(8d - 16d^2 + x - 4dx - 2y + 8dy - z + 4dz) + \left[2(x - 2y - z) - 8d(1 + x - 2y - z) + 16d^2(1 + x - 2y - z)\right]k \\ + [x - 2y - z + 4d(-x + 2y + z)]k^2\end{array}}{12(2d-1)^2},\end{aligned} \tag{14}$$

$$\begin{aligned}&\frac{\partial \Pi_3}{\partial z} \\ &= \frac{(e_3^* + I_3) + (e_2^* + I_2) + 1 - 2(e_1^* + I_1)}{3} \times \frac{1}{2}\left(\frac{\partial e_3^*}{\partial z} - k - \frac{\partial e_2^*}{\partial z}\right) + \frac{(e_3^* - kI_3) + 1 - (e_2^* - kI_2)}{2} \times \frac{1}{3}\left(\frac{\partial e_3^*}{\partial z} + 1 + \frac{\partial e_2^*}{\partial z} - 2\frac{\partial e_1^*}{\partial z}\right) \\ &\frac{(e_1^* + I_1) + (e_3^* + I_3) + 1 - 2(e_2^* + I_2)}{3} \times \frac{1}{2}\left(\frac{\partial e_3^*}{\partial z} - k - \frac{\partial e_1^*}{\partial z}\right) + \frac{(e_3^* - kI_3) + 1 - (e_1^* - kI_1)}{2} \times \frac{1}{3}\left(\frac{\partial e_3^*}{\partial z} + 1 + \frac{\partial e_1^*}{\partial z} - 2\frac{\partial e_2^*}{\partial z}\right) \\ &= \frac{\begin{array}{c}(-8d + 16d^2 - x + 4dx - y + 4dy - 2z + 8dz) + \left[-2(x + y + 2z) + 8d(1 + x + y + 2z) - 16d^2(1 + x + y + 2z)\right]k \\ + [-x - y - 2z + 4d(x + y + 2z)]k^2\end{array}}{12(2d-1)^2}.\end{aligned} \tag{15}$$

(The simplifications are tedious, so we skip the steps here and, instead, verify that these expressions are indeed the correct expressions in Appendix B.5.) Note that the denominators are all positive. Hence, we need to examine only the numerator expressions for these derivatives to see the effect of investment.

(**i**) Consider the numerator of (13), the derivative of parents 1's (expected) payoff w.r.t. x. The second derivative of this numerator expression w.r.t. k is

$$2(4d-1)(2x-y+z) \geq 0,$$

given $0 \leq y \leq x$, $z \geq 0$, so the first derivative w.r.t. k is increasing in k. The first derivative w.r.t. k, at $k=0$, is equal to

$$\begin{aligned}&2\left[-2x+y-z+4d(-1+2x-y+z)-8d^2(-1+2x-y+z)\right]\\&=2\left[8d^2-4d+(4d-1)(2x-y+z)-8d^2(2x-y+z)\right],\end{aligned}$$

which is positive since $0 \leq (2x-y+z) \leq \frac{1}{2}$.[35] At $k=1$ the first derivative is equal to

$$-4(2d-1)\left[-2x+y-z+2d(-1+2x-y+z)\right]=4(2d-1)\left[2d-(2d-1)(2x-y+z)\right],$$

which is also positive by the same reasoning as above. Thus using the convexity property established in the first step, the numerator expression of (13) is throughout increasing in k.

At $k=0$ the numerator is equal to

$$-16d^2-2x+y-z+4d(2+2x-y+z)=-16d^2+4d+(4d-1)(2x-y+z),$$

hence it is negative. At $k=1$ it is equal to

$$-4(2d-1)^2(2x-y+z) \leq 0.$$

Again using convexity of (13) in k, we can conclude that the expression (13) is negative for all k implying parents 1 would never underinvest.

(**ii**) Consider the numerator of (14), the derivative of parents 2's payoff w.r.t. y. The second derivative of this numerator expression w.r.t. k is independent of k:

$$-2(4d-1)(x-2y-z),$$

so the first derivative (w.r.t. k) is monotonic in k. At $k=0$ the first derivative is positive:

$$\begin{aligned}&2\left[x-2y-4d(1+x-2y-z)+8d^2(1+x-2y-z)-z\right]\\&=8d^2+4d(2d-1)+(x-2y-z)\left[1+4d(2d-1)\right]\\&\geq 8d^2+4d(2d-1)-\frac{1}{2}\left[1+4d(2d-1)\right] \qquad \text{because } (x-2y-z) \geqq -\frac{1}{2}\\&=(8d^2-\frac{1}{2})+4d(2d-1)(1-\frac{1}{2})>0.\end{aligned}$$

[35] Recall, $0 \leq w-x \leq w-y \leq w+z \leq 1/4$.

At $k = 1$ the first derivative is positive, as well:

$$\begin{aligned}
&4\,(2d-1)\,[-x+2y+2d\,(1+x-2y-z)+z] = 4\,(2d-1)\,[2d+(2d-1)\,(x-2y-z)] \\
&\geq 4\,(2d-1)\left[2d+(2d-1)\,(-\frac{1}{2})\right] \quad \text{(see above)} \\
&= 4\,(2d-1)\,(d+\frac{1}{2}) > 0.
\end{aligned}$$

Hence, using the monotonicity result established above, for all values of k the first derivative is positive and thus (14) is increasing in k.

At $k = 0$ the numerator of (14) takes the value

$$\begin{aligned}
-16d^2+x-2y-z+4d\,(2-x+2y+z) &= -8d(2d-1)+(4d-1)\,(-x+2y+z) \\
&\leq -8d(2d-1)+(4d-1)\,(x+z) \leq -8d(2d-1)+(4d-1)\,(\frac{1}{2}) \\
&= (2d-1)(-8d+1)+\frac{1}{2} < 0,
\end{aligned}$$

given $d > 3$. At $k = 1$ the numerator takes the value

$$4\,(2d-1)^2\,(x-2y-z)\,,$$

which is non-positive if and only if $x \leq 2y+z$. Hence, for $x > 2y+z$ there exists a $\widehat{k}_2 \in (0,\ 1)$, by intermediate value theorem, such that eq. (14)=0. Moreover, $\widehat{k}_2$ is unique, given that (14) is increasing in k.

We can therefore conclude that $\frac{\partial \Pi_2}{\partial y} > 0$ for $k > \widehat{k}_2$, so parents 2 will reduce investment $w - y$ for all $k > \widehat{k}_2$.

By solving eq. (14)=0 we obtain two roots:

$$\widehat{k}_2 = \frac{(x-2y-z)-4d\,(1+x-2y-z)+8d^2\,(1+x-2y-z) \pm 4\sqrt{(2d-1)^2\,d\left\{d\,(1+x-2y-z)^2-x+2y+z\right\}}}{(4d-1)\,(x-2y-z)}.$$

Note that

$$\begin{aligned}
&\frac{(x-2y-z)-4d\,(1+x-2y-z)+8d^2\,(1+x-2y-z)}{(4d-1)\,(x-2y-z)} \\
&= \frac{8d^2-4d}{(4d-1)\,(x-2y-z)}+\frac{8d^2-4d+1}{4d-1} \geq \frac{8d^2-4d}{4d-1}+\frac{8d^2-4d+1}{4d-1} \\
&\qquad = \frac{16d^2-4d}{4d-1}+\frac{1-4d}{4d-1} = 4d-1 > 1.
\end{aligned}$$

The first inequality follows from the observation that $0 \leq x-2y-z \leq \frac{1}{4}$. The second inequality follows from the fact that $d > 3$. So we must choose the smaller root of $\widehat{k}_2$, yielding the solution (6).

(**iii**) Consider the numerator of the derivative of parents 3's payoff w.r.t. z, i.e., (15). The second derivative w.r.t. k is given by

$$2\,(4d-1)\,(x+y+2z)\,,$$

which is positive. At $k=0$ the first derivative is equal to

$$\begin{aligned}&-2\,(x+y+2z)+8d\,(1+x+y+2z)-16d^2\,(1+x+y+2z)\\&=-16d^2-16d^2\,(x+y+2z)+8d+(8d-2)\,(x+y+2z)\,,\end{aligned}$$

which is negative. At $k=1$ it is equal to

$$-4\,(2d-1)\,\{-x-y-2z+2d\,(1+x+y+2z)\}=-4\,(2d-1)\,\{2d+(2d-1)\,(x+y+2z)\}\,,$$

which is negative, as well. Given the positive second derivative, it thus follows that the first derivative is negative for all k. Hence the numerator of (15) is decreasing in k. At $k=0$ the numerator is equal to

$$16d^2-x-y-2z+4d\,(-2+x+y+2z)=16d^2-8d+(4d-1)\,(x+y+2z)\,,$$

which is positive. At $k=1$ the numerator is equal to

$$-4\,(2d-1)^2\,(x+y+2z)\,,$$

which is negative. Hence, there exists a $\widehat{k}_3\in(0,\ 1)$, by intermediate value theorem, such that eq. (15)=0. Moreover, $\widehat{k}_3$ is unique, given that (15) is increasing in k.

We can therefore conclude that $\frac{\partial\Pi_3}{\partial z}<0$ for $k>\widehat{k}_3$, so parents 3 will reduce investment $w+z$ for all $k>\widehat{k}_3$.

By solving eq. (15)=0 we obtain two roots:

$$\widehat{k}_3=\frac{x+y+2z-4d\,(1+x+y+2z)+8d^2\,(1+x+y+2z)\pm4\sqrt{(1-2d)^2\,d\left\{-x-y-2z+d\,(1+x+y+2z)^2\right\}}}{(4d-1)\,(x+y+2z)}.$$

We have

$$\begin{aligned}&\frac{x+y+2z-4d\,(1+x+y+2z)+8d^2\,(1+x+y+2z)}{(4d-1)\,(x+y+2z)}\\&=\frac{8d^2-4d}{(4d-1)\,(x+y+2z)}+\frac{8d^2-4d+1}{4d-1}\geq\frac{8d^2-4d}{4d-1}+\frac{8d^2-4d+1}{4d-1}>1.\end{aligned}$$

So again we must choose the smaller root of $\widehat{k}_3$, yielding the solution as in (7).

The proof is completed upon showing that $\widehat{k}_2>\widehat{k}_3$. Refer Fig. 7. Consider the *negative of the derivative* of parents 3's payoff w.r.t. z, i.e., negative of (15). It is negative if $k<\widehat{k}_3$ and

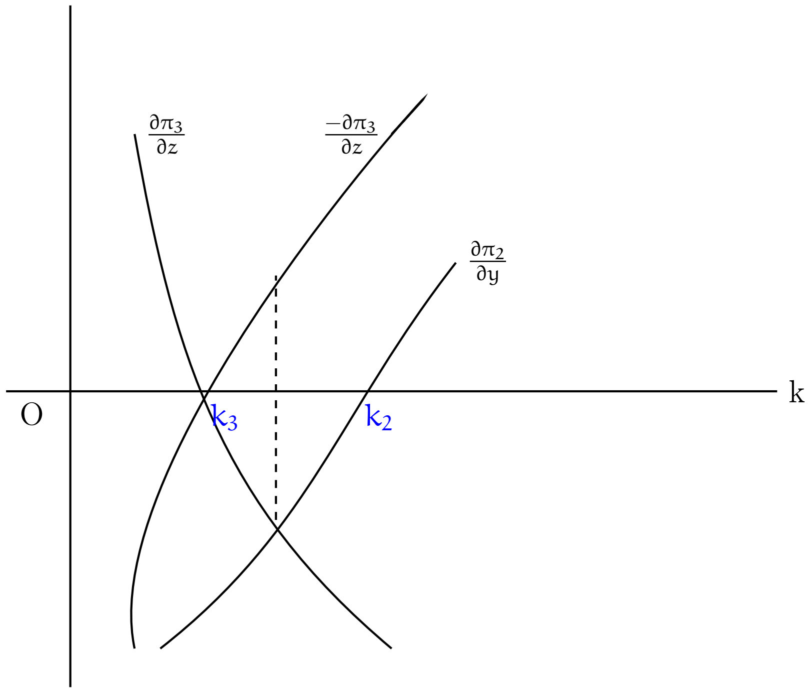

Figure 7: Graphical proof that $\widehat{k}_2 > \widehat{k}_3$

positive if $k > \widehat{k}_3$. The derivative of parents 2's payoff w.r.t. y, i.e. (14), is also negative if $k < \widehat{k}_2$ and positive if $k > \widehat{k}_2$. The two expressions vanish at k equal to $\widehat{k}_3$ and $\widehat{k}_2$, respectively. Consider the first expression minus the second, $-(15)-(14)$ (in Fig. 7, $-\frac{\partial \pi_3}{\partial z} - \frac{\partial \pi_2}{\partial y}$). It is given by

$$\frac{\left\{16d^2k + (1+k)^2 - 4d(1+k)^2\right\}(y+z)}{4(2d-1)^2},$$

which is positive (see the dotted difference in Fig. 7). Hence, it must be the case that the first expression vanishes at a value of k that is smaller than the value at which the second expression vanishes, i.e., $\widehat{k}_3 < \widehat{k}_2$. ■

Proof of Proposition 4. Suppose the parents are investing $I_1 = w - x$, $I_2 = w - y$ and $I_3 = w - z$. Consider in that case the derivatives of the expected payoffs of parents 2 and 3 w.r.t. y and z, respectively, given by (14) and (15).

(**i**) The claim follows directly from Lemma 1.

(**ii**) For $k_3 < k \leq k_2$ only parents 3 underinvest, by Lemma 1. Specifically, we have equilibrium investments $w - m_1$, w, and $w + z^*$, where (see Appendix B.6)

$$z^* = \frac{16d^2(1-k-km_1) - 4d\left\{2 - m_1 - k^2m_1 - 2k(1+m_1)\right\} - (1+k)^2 m_1}{2\left\{16d^2k - 4d(1+k)^2 + (1+k)^2\right\}},$$

solves the first-order condition:

$$(15) = 0,$$

fixing $x = m_1, y = 0$. It can be checked that $z^* > 0$ if and only if $k < k_2$. See Appendix B.6.

(iii) Similarly, again by applying Lemma 1, for $k > k_2$ both parents 2 and 3 underinvest. In this case the level of investments are given by $w - m_1, w - y^*$ and $w - y^*$, where (see Appendix B.6)

$$y^* = \frac{(1+k)^2 m_1 + 16d^2(-1+k+km_1) - 4d\left\{-2+m_1+k^2m_1+2k(1+m_1)\right\}}{16d^2k+(1+k)^2-4d(1+k)^2},$$

is obtained by solving the first-order condition:

$$(14) = 0,$$

after fixing $x = m_1$ and setting $z = -y$. Equivalently, $z = -y^* < 0$ is solved using the first-order condition (15)$= 0$, after fixing $m_1 = x$ and setting $y = -z$. Thus, parents 2 and 3 invest the same amount in this case.

Note that y^* is increasing in k (see Appendix B.6). At $k = 1$, $y^* = m_1$, so all parents invest the same amount, $w - m_1$. For $k < 1$ parents 2 and 3 invest more than parents 1. ■

Proof of Proposition 5. For $k < k_3$, after solving the equilibrium efforts as in (10)–(12), the effective efforts of children 1, 2 and 3 can be solved respectively as follows:[36]

$$\begin{aligned} & e_1^* - k(w-x) = \tfrac{d(4+6kw-3x-3kx)-2-12d^2k(w-x)}{6d(2d-1)}, \\ & e_2^* - kw = \tfrac{1}{3d} - kw, \\ \text{and} \quad & e_3^* - k(w+x) = \tfrac{d\{4+3x+3k(2w+x)\}-2-12d^2k(w+x)}{6d(2d-1)}. \end{aligned} \tag{16}$$

The effective effort by child 1 minus that of child 2 is

$$\left\{e_1^* - k(w-x)\right\} - (e_2^* - kw) = \left[\frac{(4d-1)k-1}{4d-2}\right]x. \tag{17}$$

This is also equal to the effective effort by child 2 minus that of child 3. Setting this difference equal to zero and solving for k we have

$$k^* = \frac{1}{(4d-1)}.$$

The expression (17) is increasing in k. At $k = 0$ it is negative. At $k = k_3$ it is equal to

$$2\left[\frac{2d^2(1+3x) - d(1+3x) - \sqrt{(2d-1)^2 d\{d(1+3x)^2 - 3x\}}}{3\,(2d-1)}\right]. \tag{18}$$

[36]Note that the effective efforts are negative. This is not inconsistent with our model since we could add the same constant to all the children's efforts so that they become positive.

The numerator inside the bracket can be expressed as

$$(1+3x)d(2d-1)-\sqrt{(2d-1)^2 d\{d(1+3x)^2-3x\}}>0,$$

so (18) is positive. Thus, the effective effort of a poorer child is greater than that of any wealthier child if $k^* < k \leq k_3$. At $k = k^*$, the effective efforts of all three competitors are equal, and for $k < k^*$ the poorest child's effective effort is the lowest amongst the three.

For $k_3 < k < k_2$, after solving the equilibrium efforts as in (10)–(12), the effective efforts by children 1, 2 and 3 can be obtained respectively as follows (see Appendix B.7):

$$e_1^* - k(w-x) = \frac{\begin{array}{c} -4(1+k)^2 + 16d^3\left\{-1+k^2(-1+24w-21x)+k(10+6w-9x)+6k^3(w-x)\right\} \\ -384d^4k^2(w-x)+3d(1+k)^2(8+4kw-x-3kx) \\ -12d^2\left\{2+k^2(2+12w-11x)+k(12+6w-7x)+k^3(6w-5x)-x\right\} \end{array}}{12d(2d-1)\left\{16d^2k+(1+k)^2-4d(1+k)^2\right\}},$$

$$e_2^* - kw = \frac{\begin{array}{c} -4(1+k)^2 - 384d^4k^2w + 3d(1+k)^2(8+4kw+x-kx) \\ -12d^2\left\{2+k^3(6w-x)+k^2(2+12w-x)+x+k(12+6w+x)\right\} \\ +16d^3\left\{-1+6k^3w+k^2(-1+24w-3x)+k(10+6w+3x)\right\} \end{array}}{12d(2d-1)\left\{16d^2k+(1+k)^2-4d(1+k)^2\right\}},$$

$$e_3^* - k(w+z^*) = \frac{\begin{array}{c} 2(1+k)^2 + 4d^2\left\{2+2k^2(-2+6w-3x)+k^3(6w-3x)+k(10+6w-3x)\right\} \\ -48d^3k\{1+k(-1+2w-x)\} - d(1+k)^2(8+6kw-3kx) \end{array}}{6d\left\{16d^2k+(1+k)^2-4d(1+k)^2\right\}},$$

where z^* is as in Proposition 4 with m_1 set equal to x.

The effective effort by child 1 minus that of child 2 is (see Appendix B.7):

$$\left\{e_1^* - k(w-x)\right\} - (e_2^* - kw) = \frac{(4dk-1-k)x}{2(2d-1)}.$$

It is increasing in k, and at $k = k_3$ it is equal to the earlier derived expression (18), which is positive. So child 1's effective effort is higher than that of child 2 for $k_3 < k < k_2$.

The effective effort by child 2 minus that of child 3 is (see Appendix B.7):

$$(e_2^* - kw) - \left\{e_3^* - k(w+z^*)\right\} = \frac{(1+k)^2x + 16d^2(k+kx-1) - 4d\{x+k^2x+2k(1+x)-2\}}{4(2d-1)(4d-1-k)}.$$

The denominator is positive. The derivative of the numerator w.r.t. k is

$$-16d^2(1+x) + 8d(1+x+kx) - 2(1+k)x < 0,$$

so the numerator is decreasing in k. At $k = k_2$ the difference is equal to 0. Hence it is always positive for $k_3 < k < k_2$. Hence, the effective effort by child 2 is larger than the effective effort by child 3. Combining the result derived above comparing child 1 and child 2's effective efforts, it follows that the poorest child's effective effort is again the largest of the three for $k_3 < k < k_2$.

For $k > k_2$, as before solving the equilibrium efforts as in (10)–(12), the effective efforts by child 1 and child 3 (or child 2) can be obtained respectively as follows (see Appendix B.7):

$$e_1^* - k(w-x) = \frac{\begin{array}{c}(1+k)^2 - d(1+k)^2\{4+3k(w-x)\} - 48d^3k^2(w-x) \\ +4d^2\left\{-2+k^2(-2+6w-6x)+k(8+3w-3x)+3k^3(w-x)\right\}\end{array}}{3d\left\{16d^2k+(1+k)^2-4d(1+k)^2\right\}}, \tag{19}$$

$$e_2^* - k(w-y^*) = e_3^* - k(w-y^*) = \frac{\begin{array}{c}(1+k)^2 - 48d^3k\{1+k(-1+w-x)\} - d(1+k)^2\{4+3k(w-x)\} \\ +4d^2\left\{1+k^2(-5+6w-6x)+k(8+3w-3x)+3k^3(w-x)\right\}\end{array}}{3d\left\{16d^2k+(1+k)^2-4d(1+k)^2\right\}}, \tag{20}$$

where y^* is as in Proposition 4 with m_1 set equal to x.

The effective effort by the poorest child (child 1) minus that of child 3 (or child 2) is

$$\left\{e_1^* - k(w-x)\right\} - \left\{e_3^* - k(w-y^*)\right\} = \frac{4d(1-k)}{4d-1-k},$$

which is positive. This completes the proof. ■

Proof of Proposition 8. In order to compare expected top talents, we first need to solve the sequential investment and efforts game under affirmative action using backward induction. In the process we will be able to determine investment cutoffs.

Fix $I_1 = 0$, $I_2 = w - y$, and $I_3 = w + z$, where $0 \le y \le 0$ and $-w \le z \le x$. That $I_1 = 0$ is easy to explain: parents 1, whose child is guaranteed college admission, can only lose by making a positive investment due to its adverse impact on child 1's long-term development ($k > 0$). The payoffs to the students are given by

$$\begin{aligned}
\pi_1^{AA} =& \frac{(e_2+w-y)+1-(e_3+w+z)}{2} \times \frac{e_1+1-\{e_2-k(w-y)\}}{2} \\
&+ \frac{(e_3+w+z)+1-(e_2+w-y)}{2} \times \frac{e_1+1-\{e_3-k(w+z)\}}{2} - de_1^2, \\
\pi_2^{AA} =& \frac{(e_2+w-y)+1-(e_3+w+z)}{2} \times \frac{\{e_2-k(w-y)\}+1-e_1}{2} - de_2^2, \\
\pi_3^{AA} =& \frac{(e_3+w+z)+1-(e_2+w-y)}{2} \times \frac{\{e_3-k(w+z)\}+1-e_1}{2} - de_3^2.
\end{aligned}$$

The first-order conditions

$$
\begin{aligned}
\frac{\partial \pi_1^{AA}}{\partial e_1} &= \frac{1}{2} - 2de_1 = 0,\\
\frac{\partial \pi_2^{AA}}{\partial e_2} &= \frac{1}{4}\left[2 - e_1 + (2-8d)e_2 - e_3 + (w-y) - (w+z) - k(w-y)\right] = 0,\\
\frac{\partial \pi_3^{AA}}{\partial e_3} &= \frac{1}{4}\left[2 - e_1 - e_2 + 2e_3 - 8de_3 - (w-y) + (w+z) - k(w+z)\right] = 0,
\end{aligned}
$$

can be solved to obtain the following equilibrium efforts:

$$
\begin{aligned}
e_1^{AA} &= \tfrac{1}{4d},\\
e_2^{AA} &= \tfrac{32d^2\{2-k(w-y)-y-z\}-4d\{8-y-z-k(3w-2y+z)\}+3}{4d(8d-3)(8d-1)},\\
e_3^{AA} &= \tfrac{32d^2\{2+y+z-k(w+z)\}-4d\{8+y-k(3w-y+2z)+z\}+3}{4d(8d-3)(8d-1)}.
\end{aligned}
\tag{21}
$$

The solution is verified in Appendix-C-9-1 and Appendix-C-9-2.

Now consider parents' investments. Parents 3's payoff can be written as

$$
\Pi_3^{AA} = \frac{\left(e_3^{AA} + w + z\right) + 1 - \left(e_2^{AA} + w - y\right)}{2} \times \frac{\left\{e_3^{AA} - k(w+z)\right\} + 1 - e_1^{AA}}{2},
$$

When parents 3 underinvest but parents 2 invest fully, we can derive parents 3's optimal investment $w + z^*$ by setting

$$
\frac{\partial\left(\Pi_3^{AA}\big|_{y=0}\right)}{\partial z} = 0,
$$

and solving for z (see Appendix-C-9-3 and Appendix-C-9-4) as follows:

$$
z^* = -\frac{4d(8d-3)\{1 - k^2w - k(2+w) - 8d(1-k-kw)\}}{512d^3k - (1+k)^2 - 64d^2(1+4k+k^2) + 8d(2+5k+3k^2)}. \tag{22}
$$

The solution of k in $z^* = 0$ gives the critical value k_2^{AA} so that for $k > k_2^{AA}$ both parents 2 and 3 underinvest. Note that in that case parents 2 and 3 invest equal amounts $w - y^*$. k_2^{AA} is given by (see Appendix-C-9-5):

$$
k_2^{AA} = \frac{\begin{array}{c} 6 + 3w + 64d^2(1+w) - 8d(5+4w) \\ -\sqrt{(8d-3)^2\{4 + 8w + w^2 + 64d^2(1+w)^2 - 16d(2+5w+w^2)\}} \end{array}}{2(8d-3)w}.
$$

To calculate y^* we set

$$
\frac{\partial \Pi_3^{AA}}{\partial z} = 0,
$$

and solve for y at $z = -y$. This yields

$$
y^* = \frac{1 - k^2w - k(2+w) + 8d(k + kw - 1)}{k(8d-1-k)}, \tag{23}
$$

as verified in Appendix-C-9-6.

Now we turn to proving the top talent comparison claim for $k > \max\{k_2, k_2^{AA}\}$.

Suppose there is no affirmative action. Then both parents 2 and 3 underinvest, and parents 1 invest fully. The expected top effective talent, τ_2, is derived in (B.10) in the proof of Proposition 6.

Consider the case of affirmative action. Again parents 1 make zero investment. Parents 2 and 3 invest $I_2 = I_3 = w - y^*$ where y^* is given in (23).

Using $y = y^*$ and $z = -y^*$ in (21), we have the equilibrium efforts which can be substituted into the expected effective top talent:

$$\begin{aligned}
\tau_2^{AA} = & \left[\frac{(e_2^{AA} + I_2) + 1 - (e_3^{AA} + I_3)}{2} \times \frac{(e_1^{AA} - kI_1) + 1 - (e_2^{AA} - kI_2)}{2} \right. \\
& \left. + \frac{(e_3^{AA} + I_3) + 1 - (e_2^{AA} + I_2)}{2} \times \frac{(e_1^{AA} - kI_1) + 1 - (e_3^{AA} - kI_3)}{2} \right] (e_1^{AA} - kI_1) \\
& + \left[\frac{(e_2^{AA} + I_2) + 1 - (e_3^{AA} + I_3)}{2} \times \frac{(e_2^{AA} - kI_2) + 1 - (e_1^{AA} - kI_1)}{2} \right] (e_2^{AA} - kI_2) \\
& + \left[\frac{(e_3^{AA} + I_3) + 1 - (e_2 + I_2)}{2} \times \frac{(e_3^{AA} - kI_3) + 1 - (e_1^{AA} - kI_1)}{2} \right] (e_3^{AA} - kI_3) \,.
\end{aligned}$$

The explicit derivation appears in Appendix-C-9-8 .

Consider $\tau_2 - \tau_2^{AA}$. After tedious steps it is shown to be equal to (see Appendix-C-9-8 for verification):

$$\frac{\left(\begin{array}{c}
-(1+k)^5 \\
+4d(1+k)^4 \left\{10 + k(5 + 3w - 3x) + 3k^2(w - x)\right\} \\
-16d^2(1+k)^3 \left\{38 + k(49 + 30w - 30x) + 5k^2(2 + 9w - 9x) + 15k^3(w - x)\right\} \\
+64d^3(1+k)^2 \left\{70 + 3k^2(29 + 83w - 83x) + 2k^3(17 + 75w - 75x) + 41k(4 + 3w - 3x) + 24k^4(w - x)\right\} \\
-256d^4(1+k) \left\{67 + k^3(130 + 591w - 591x) + k(271 + 264w - 264x) + 3k^2(62 + 229w - 229x) + 4k^4(19 + 45w - 45x) + 12k^5(w - x)\right\} \\
+1024d^5 \left\{44 + k^2(151 + 1032w - 1032x) + 3k(93 + 104w - 104x) + k^5(68 + 72w - 72x) + 8k^4(25 + 66w - 66x) + 84k^3(1 + 14w - 14x)\right\} \\
-8192d^6 \left\{26 + 8k^5 + k^3(7 + 300w - 300x) + k(78 + 96w - 96x) + k^4(64 + 78w - 78x) + 9k^2(-7 + 36w - 36x)\right\} \\
+65536d^7 \left\{16 + 4k^4 + k^3(13 + 36w - 36x) + 3k^2(-7 + 20w - 20x) + k(-5 + 12w - 12x)\right\} \\
-524288d^8 \left\{4 - 8k - k^3 + k^2(5 + 6w - 6x)\right\}
\end{array} \right)}{12(8d-1)^2 d(8d-k-1)^2(4d-k-1)^2\{(4d-1)k-1\}}.$$

The largest power of d in the expression is d^8 which has a coefficient

$$-524288\left\{4 - 8k - k^3 + k^2(5 + 6w - 6x)\right\}.$$

Since $x \le w$, we have

$$4 - 8k - k^3 + k^2(5 + 6w - 6x) \ge 4 - 8k - k^3 + 5k^2 = (2-k)^2(1-k) > 0$$

for $k < 1$. Hence in this case for all large d we have $\tau_2 < \tau_2^{AA}$.

To complete the proof we show that as d increases the critical values k_2 and k_2^{AA} will decrease. First, consider the benchmark contest. We have from Proposition 4 (substituting $m_1 = x$):

$$y^* = \frac{(1+k)^2x - 16d^2(1-k-kx) + 4d\left\{2 - x - k^2x - 2k(1+x)\right\}}{16d^2k + (1+k)^2 - 4d(1+k)^2}. \tag{24}$$

We claim that y^* is increasing in k. The derivative of y^* w.r.t. d is equal to

$$\frac{8(1-k)\left\{(1+k)^2 - 4d(1+k)^2 + 8d^2(1+k^2)\right\}}{\{16d^2k + (1+k)^2 - 4d(1+k)^2\}^2}.$$

The denominator is positive, and the expression $(1+k)^2 - 4d(1+k)^2 + 8d^2(1+k^2)$ in the numerator is quadratic (convex) with a negative discriminant $-16\left(1-k^2\right)^2$. Hence, y^* is increasing in d for all $0 < k < 1$ which implies that the solution k_2 of k in $y^* = 0$ is also decreasing.[37]

Next consider y^* under affirmative action from (23). Its derivative w.r.t. k is positive, i.e., y^* is increasing in k. The derivative w.r.t. d simplifies to:

$$\frac{8\left(2-k\right)}{\left(8d-1-k\right)^2} > 0.$$

Hence, the solution k_2^{AA} of k in $y^* = 0$ in this case is decreasing in k.[38]

Finally, subtracting (24) from (23) we obtain:

$$\begin{aligned}
&y^*|_{AA} - y^*|_{\text{benchmark}} \\
&= \frac{1 - k^2w - k(2+w) + 8d(k+kw-1)}{k(8d-1-k)} - \frac{(1+k)^2x - 16d^2(1-k-kx) + 4d\left\{(2 - x - k^2x - 2k(1+x))\right\}}{16d^2k + (1+k)^2 - 4d(1+k)^2} \\
&= \frac{\begin{array}{c}(1+k)^2\left\{1 - k\left(2+w-x\right) - k^2\left(w-x\right)\right\} - 4d(1+k)\left\{3 - 3k\left(1+w-x\right) - k^2\left(2+4w-4x\right) - k^3\left(w-x\right)\right\} \\ +16d^2\left\{2 - 2k\left(1+w-x\right) - 5k^2\left(w-x\right) - k^3\left(1+3w-3x\right)\right\} + d^3128k^2\left(w-x\right)\end{array}}{k(8d-1-k)\left[16d^2k + (1+k)^2 - 4d(1+k)^2\right]}.
\end{aligned}$$

The denominator is positive and the coefficient of d^3 in the numerator is positive. Hence, for all large d the investments of parents 2 and 3 under AA is smaller than that under the benchmark contest. ■

[37] That y^* is increasing in k follows by similar steps from the observation that the derivative of y^* w.r.t. k ,

$$\frac{8d(2d-1)\left[3 + 16d^2 + 2k - k^2 - 4d(3+2k-k^2)\right]}{\left[16d^2k + (1+k)^2 - 4d(1+k)^2\right]^2},$$

is positive since the quadratic factor in the numerator has a negative discriminant, $-3 + 4k + 2k^2 - 4k^3 + k^4$, for $0 < k < 1$.

[38] Again, y^* in this case is increasing in k because the derivative of y^*, $\frac{64d^2+2k-2k^2-8d(2+2k-k^2)+1}{k^2(8d-1-k)^2}$, is positive (the numerator is convex quadratic with the larger root of d, $\frac{1}{16}(2+2k-k^2+\sqrt{8k^2-4k^3+k^4})$, being smaller than 1).

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B Online Appendix – Supplement to "Parents–Children Teams in School Contest" (not for publication)

B.1 Proofs of Propositions 1–3 and Propositions 6 and 7

Proof of Proposition 1. The derivative of e_1^* in (5) w.r.t. I_1 is given by

$$\frac{2d_2d_3 + d_2(d_3-1) + d_3(d_2-1)}{2\{d_1 + d_2 + d_3 + 4d_1d_2(d_3-1) + 4d_1d_3(d_2-1) + 4d_2d_3(d_1-1)\}},$$

which is positive for $d_1, d_2, d_3 > 1$.

The derivative of e_2^* w.r.t. I_1 is given by

$$-\frac{d_1(2d_3-1)}{2\{d_1 + d_2 + d_3 + 4d_1d_2(d_3-1) + 4d_1d_3(d_2-1) + 4d_2d_3(d_1-1)\}},$$

which is negative for $d_1, d_2, d_3 > 1$.

Thus, we have that the equilibrium effort of child 1 is increasing in I_1 and those of the other children are decreasing in I_1. ■

Proof of Proposition 2. (**i**) In this case the equilibrium efforts of the children are given by (5), as follows (see Appendix B.3):

$$e_1^* = \frac{4d - 3dx - 2}{6d(2d-1)}, \quad e_2^* = \frac{1}{3d}, \quad e_3^* = \frac{4d + 3dx - 2}{6d(2d-1)}.$$

In order that the equilibrium efforts are well defined they must be less than $\frac{1}{4}$. In this case, the efforts are less than $\frac{1}{4}$ if $e_3 < \frac{1}{4}$ for $x = \frac{1}{4}$, i.e.,

$$4d + 3d \times \frac{1}{4} - 2 < \frac{3}{2}(2d-1)d \quad \text{or,} \quad \left(3d - \frac{25}{4}\right)d + 2 > 0,$$

which holds in our case. Hence the equilibrium efforts are interior and all the quantities are well defined. The total effort is given by $\frac{1}{d}$ which does not change with wealth inequality.

The expected top talent is given by

$$\sum_{\substack{i,j,l=1 \\ i \neq j,l \text{ and } j \neq l}}^{3} \frac{(e_i + I_i) + (e_j + I_j) + 1 - 2(e_l + I_l)}{3} \times \left(\frac{e_i + 1 - e_j}{2}\right) \times e_i, \tag{B.1}$$

which after substituting $I_1 = w - x$, $I_2 = w$, $I_3 = w + x$ and the equilibrium efforts e_1^*, e_2^*, e_3^* equals

$$\frac{1 - 4d + d^2(4 + 3x^2)}{3(1-2d)^2 d}. \tag{B.2}$$

Taking the derivative of this last expression w.r.t. x yields $\frac{2dx}{(2d-1)^2} > 0$. Hence the expected top talent increases as wealth inequality increases.

(**ii**) The equilibrium efforts are given by (5), as follows:

$$e_1^* = \frac{2d - dx - 1}{3d(2d-1)}, \quad e_2^* = \frac{4d + dx - 2}{6d(2d-1)}, \quad e_3^* = \frac{4d + dx - 2}{6d(2d-1)}.$$

It follows from the proof of part (i) that $e_3^* < \frac{1}{4}$ (since it is smaller than the $e_3^* = \frac{4d+3dx-2}{6d(2d-1)}$ solved there). Furthermore, $e_1^* < e_3^*$, hence all efforts are smaller than $\frac{1}{4}$.

The total effort is given by $\frac{1}{d}$, so it does not change with x. Thus the total effort is unaffected by the specific change in inequality holding the wealth of the two more wealthy parents fixed.

The expected top talent, using $I_1 = w - x,\ I_2 = I_3 = w$ and the equilibrium efforts in the expression (B.1), equals

$$\frac{16d\left(4d^2 - 6d + 3\right) - 8 + 8d^2(2d-1)x^2 - d(4d-1)x^3}{24d(2d-1)^3}.$$

The derivative of the expected top talent w.r.t. x is given by

$$\begin{aligned}\frac{x[32d^2 + 3x - 4d(4+3x)]}{24(2d-1)^3} &> \frac{x[32d^2 + 3(0) - 4d(4 + 3(\frac{1}{4}))]}{24(2d-1)^3} \\ &= \frac{xd(-19+32d)}{24(2d-1)^3} > 0.\end{aligned}$$

Hence, an increase in inequality improves the expected top talent.

(**iii**) In this case the equilibrium efforts are

$$e_1^* = \frac{4d - dx - 2}{6d\,(2d-1)}, \qquad e_2^* = \frac{4d - dx - 2}{6d\,(2d-1)}, \qquad e_3^* = \frac{2d + dx - 1}{3\,(2d-1)\,d}.$$

The total effort is $\frac{1}{d}$ which is independent of the inequality. The expected top talent, using $I_1 = I_2 = w - x, I_3 = w$ and the equilibrium efforts in (B.1), is equal to

$$\frac{16d\left(4d^2 - 6d + 3\right) - 8 + 8d^2(2d-1)x^2 + d(4d-1)x^3}{24d(2d-1)^3},$$

and its derivative w.r.t. x equals

$$\frac{x\left[32d^2 - 3x + 4d\,(3x-4)\right]}{24\,(2d-1)^3}.$$

Replacing x in the negative term of $\left(32d^2 - 3x + 4d\,(3x-4)\right)$ by $\frac{1}{4}$ and that in the positive term by 0 we have that

$$\begin{aligned} 32d^2 - 3x + 4d(3x-4) &> 32d^2 - 3(1/4) + 4d(3 \times 0 - 4) \\ &= 32d^2 - 16d - \frac{3}{4} \\ &= (d-3)(80+32d) + \frac{957}{4} > 0. \end{aligned}$$

Hence, the expected top talent is increasing in inequality. ∎

Proof of Proposition 3. In parts (i) and (ii), with student 1 given a guaranteed entry into college, the school contest is a shoot-out between students 2 and 3 whose parents invest fully. In (iii), one of students 1 and 2 will be chosen with equal probability for a guaranteed entry into college whereas the other student will have to compete with student 3 for college admission. We analyze the three cases separately.

(i) Consider the wealth distribution $\omega_1 = w - x, \omega_2 = w, \omega_3 = w + x$. Using (B.2), we have

$$\tau = \frac{1 - 4d + d^2\left(4 + 3x^2\right)}{3\left(2d-1\right)^2 d}.$$

Under affirmative action student 1 chooses effort to maximize

$$\frac{(e_2+I_2)+1-(e_3+I_3)}{2} \times \frac{e_1+1-e_2}{2} + \frac{(e_3+I_3)+1-(e_2+I_2)}{2} \times \frac{e_1+1-e_3}{2} - de_1^2,$$

and students 1 and 2 choose efforts to maximize respectively

$$\frac{(e_2+I_2)+1-(e_3+I_3)}{2} \times \frac{e_2+1-e_1}{2} - de_2^2,$$
$$\text{and} \quad \frac{(e_3+I_3)+1-(e_2+I_2)}{2} \times \frac{e_3+1-e_2}{2} - de_3^2.$$

For $I_1 = w - x$, $I_2 = w$ and $I_3 = w + x$ equilibrium efforts are given by

$$e_1^{AA} = \frac{1}{4d}, \quad e_2^{AA} = \frac{4d(2-x)-3}{4d(8d-3)}, \quad e_3^{AA} = \frac{4d(2+x)-3}{4d(8d-3)}.$$

Using these efforts the expected top talent is given by

$$\tau^{AA} = \frac{9 - 48d + 16d^2(4+x^2)}{4(8d-3)^2 d}.$$

We have

$$\tau - \tau^{AA} = \frac{(16d^2-14d+3)^2 + 12d^2(48d^2-32d+5)x^2}{12(8d-3)^2(2d-1)^2 d}.$$

The denominator is positive and it is straightforward to see that for $d > 3$ the quadratic expression $(48d^2 - 32d + 5) > 0$. Hence, $\tau > \tau^{AA}$, i.e., the expected top talent is higher under the benchmark contest than under affirmative action.

From Proposition 2 we have

$$e_1^* = \frac{4d - 3dx - 2}{6d(2d-1)}, \quad e_2^* = \frac{1}{3d}, \quad e_3^* = \frac{4d + 3dx - 2}{6d(2d-1)}.$$

Taking the difference we have

$$e_1^* - e_1^{AA} = \frac{2d\,(1-3x) - 1}{12d\,(2d-1)}, \quad e_2^* - e_2^{AA} = \frac{4d\,(2+3x) - 3}{12d\,(8d-3)},$$
$$e_3^* - e_3^{AA} = \frac{2d(8d-7) + 3 + 6dx\,(4d-1)}{12d\,(2d-1)\,(8d-3)}.$$

It is straightforward to see that $e_1^* - e_1^{AA} > 0$, $e_2^* - e_2^{AA} > 0$ and $e_3^* - e_3^{AA} > 0$ for $0 \le x \le \frac{1}{4}$.

(**ii**) Consider the wealth distribution $\omega_1 = w - x,\ \omega_2 = \omega_3 = w$. Students $i, j = 2, 3$ solve the following problems:

$$\max_{0 \le e_i \le 1/4} \frac{e_i + w + 1 - e_j - w}{2} \times \frac{e_i + 1 - e_1}{2} - de_i^2, \quad j \ne i$$

and student 1 solves:

$$\max_{0 \le e_1 \le 1/4} \frac{e_1 + 1 - e_i}{2} - de_1^2.$$

From the second problem it is easy to see that $e_1^{AA} = \frac{1}{4d}$, which if plugged into the first problem yields the equilibrium efforts $e_2^{AA} = e_3^{AA} = \frac{1}{4d}$ $(< 1/4)$ as well (see Appendix B.4).[39] The total effort is given by $\frac{3}{4d}$ and the expected top talent is given by $\frac{1}{4d}$.

In the benchmark contest of part (ii) of Proposition 2, $e_1^* = \frac{2d - dx - 1}{3d(2d-1)}$, whereas under affirmative action $e_1^{AA} = \frac{1}{4d}$. So,

$$e_1^{AA} - e_1^* = \frac{1}{4d} - \frac{2d - dx - 1}{3d(2d-1)} < 0 \Leftrightarrow 2d(2x-1) < -1,$$

which is true for $0 \le x \le 1/4$. This implies affirmative action lowers student 1's effort. Similarly for $i = 2, 3$, using $e_2^* = e_3^* = \frac{4d + dx - 2}{6d(2d-1)}$ from the proof of Proposition 2, affirmative action lowers efforts if

$$e_i^{AA} - e_i^* = \frac{1}{4d} - \frac{4d + dx - 2}{6d(2d-1)} < 0 \Leftrightarrow -2d(1+x) + 1 < 0,$$

which is true for $d \ge 3$ and $x \ge 0$.

Clearly the total effort under affirmative action falls.

Let us now turn to a comparison of the expected effort of the eventual winner. We first

[39]The identical efforts solution, despite students 2 and 3 facing an additional hurdle of school contest, is due to the fact that both know that only one of them will go through and face student 1 in the college race. Because the problem they will face in the second round must be seen in advance sitting in the first round itself, and the two problems – first and second round – are identical, the effort solution is essentially the same as that of student 1's one-shot solution.

verify that $e_1^* < e_2^*$ $(= e_3^*)$:

$$\frac{2d - dx - 1}{3d(2d-1)} - \frac{4d + dx - 2}{6d(2d-1)} < 0, \quad \text{i.e.,} \quad 2(2d - dx - 1) < 4d + dx - 2 \ \text{ for } \ x > 0.$$

Also verify that in the benchmark contest,

$$\underbrace{\Pr(\text{student 1 wins})}_{\equiv\alpha_1} < \underbrace{\Pr(\text{student 2 wins})}_{\equiv\alpha_2} \quad (= \underbrace{\Pr(\text{student 3 wins})}_{\equiv\alpha_3})$$

using (B.1) as follows:

$$\left[\frac{(e_1^* + w - x) + (e_2^* + w) + 1 - 2\,(e_3^* + w)}{3} \times \frac{e_1^* + 1 - e_2^*}{2} + \frac{(e_1^* + w - x) + (e_3^* + w) + 1 - 2\,(e_2^* + w)}{3} \times \frac{e_1^* + 1 - e_3^*}{2}\right]$$
$$< \left[\frac{(e_2^* + w) + (e_1^* + w - x) + 1 - 2\,(e_3^* + w)}{3} \times \frac{e_2^* + 1 - e_1^*}{2} + \frac{(e_2^* + w) + (e_3^* + w) + 1 - 2\,(e_1^* + w - x)}{3} \times \frac{e_2^* + 1 - e_3^*}{2}\right]$$

i.e.,

$$\left[\frac{(e_1^* + e_2^* - x + 1 - 2e_3^*)}{3} \times \frac{e_1^* + 1 - e_2^*}{2} + \frac{(e_1^* + e_3^* - x + 1 - 2e_2^*)}{3} \times \frac{e_1^* + 1 - e_3^*}{2}\right]$$
$$< \left[\frac{(e_2^* + e_1^* - x + 1 - 2e_3^*)}{3} \times \frac{e_2^* + 1 - e_1^*}{2} + \frac{(e_2^* + e_3^* + 1 - 2\,(e_1^* - x))}{3} \times \frac{e_2^* + 1 - e_3^*}{2}\right], \tag{B.3}$$

by comparing term by term and using the fact that $e_1^* < e_2^*$ $(= e_3^*)$ and $x > 0$.

Under affirmative action,

$$\underbrace{\Pr(\text{student 1 wins})}_{\equiv\alpha_1'} = \frac{e_1^{AA} + 1 - e_i^{AA}}{2} = \frac{1}{2},$$

$$\underbrace{\Pr(\text{student 2 wins})}_{\equiv\alpha_2'} = \underbrace{\Pr(\text{student 3 wins})}_{\equiv\alpha_3'} = \frac{e_i^{AA} + w + 1 - e_j^{AA} - w}{2} \times \frac{e_i^{AA} + 1 - e_1^{AA}}{2} = \frac{1}{4},$$

for $i, j \neq 1$.

Given that on the LHS of (B.3) both $\frac{e_1^*+1-e_2^*}{2}$ and $\frac{e_1^*+1-e_3^*}{2}$ are less than $1/2$, it follows that

$$\alpha_1' > \alpha_1,$$

and because $\alpha_2 = \alpha_3$ and $\alpha_2' = \alpha_3'$, therefore

$$\alpha_2' < \alpha_2, \quad \text{and} \quad \alpha_3' < \alpha_3.$$

Now take the difference between the expected talents of the winners in the benchmark contest and under affirmative action:

$$\begin{aligned}
&\alpha_1 e_1^* - \alpha_1' e_1^{AA} + (\alpha_2 + \alpha_3) e_2^* - (\alpha_2' + \alpha_3') e_2^{AA} \\
&= \alpha_1'(e_1^* - e_1^{AA}) + (\alpha_2' + \alpha_3')(e_2^* - e_2^{AA}) + (\alpha_1 - \alpha_1') e_1^* + 2(\alpha_2 - \alpha_2') e_2^* \\
&= \underbrace{[\alpha_1'(e_1^* - e_1^{AA}) + (\alpha_2' + \alpha_3')(e_2^* - e_2^{AA})]}_{>0} + \underbrace{[(\alpha_1 e_1^* + 2\alpha_2 e_2^*) - (\alpha_1' e_1^* + 2\alpha_2' e_2^*)]}_{>0} > 0,
\end{aligned}$$

where the second $\underbrace{[.]} > 0$ because $e_1^* < e_2^*$ and $\alpha_1' > \alpha_1$ together with $\alpha_1 + 2\alpha_2 = \alpha_1' + 2\alpha_2' = 1$.

Hence, the affirmative action policy will lower the expected talent of the winner.

(**iii**) Suppose that the wealths of parents 1 and 2 are $w - x$ and that of parents 3 is w. Student 1's objective:

$$\begin{aligned}
&\max_{0 \le e_1 \le 1/4} \pi_1 \\
&= \frac{1}{2} \frac{(e_1 + w - x) + 1 - (e_3 + w)}{2} \times \frac{e_1 + 1 - e_2}{2} \\
&\quad + \frac{1}{2}\left[\frac{(e_2 + w - x) + 1 - (e_3 + w)}{2} \times \frac{e_1 + 1 - e_2}{2} + \frac{(e_3 + w) + 1 - (e_2 + w - x)}{2} \times \frac{e_1 + 1 - e_3}{2}\right] \\
&\quad - de_1^2.
\end{aligned} \tag{B.4}$$

Student 2's objective:

$$\begin{aligned}
&\max_{0 \le e_2 \le 1/4} \pi_2 \\
&= \frac{1}{2} \frac{(e_2 + w - x) + 1 - (e_3 + w)}{2} \times \frac{e_2 + 1 - e_1}{2} \\
&\quad + \frac{1}{2}\left[\frac{(e_1 + w - x) + 1 - (e_3 + w)}{2} \times \frac{e_2 + 1 - e_1}{2} + \frac{(e_3 + w) + 1 - (e_1 + w - x)}{2} \times \frac{e_2 + 1 - e_3}{2}\right] \\
&\quad - de_2^2.
\end{aligned} \tag{B.5}$$

Student 3's objective:

$$\max_{0 \le e_3 \le 1/4} \pi_3 = \frac{(e_3 + w) + 1 - (e_i + w - x)}{2} \times \frac{e_3 + 1 - e_j}{2} - de_3^2, \quad j \neq i. \tag{B.6}$$

The equilibrium efforts without AA are given by

$$e_1 = \frac{d(4 - x) - 2}{6d(2d - 1)}, \quad e_2 = \frac{d(4 - x) - 2}{6d(2d - 1)}, \quad e_3 = \frac{(d(2 + x) - 1)}{3d(2d - 1)}.$$

The equilibrium efforts with AA are given by

$$e_1^{AA} = \frac{d(32 - 8x) - 10 + x}{8d(16d - 5)}, \quad e_2^{AA} = \frac{32d - 8dx - 10 + x}{8d(16d - 5)}, \quad e_3^{AA} = \frac{32d + 16dx - 10 + x}{8d(16d - 5)}.$$

The expected talent without AA is given by

$$\frac{16d^3\left(4+x^2\right)-4d^2\left(24+2x^2-x^3\right)+d\left(48-x^3\right)-8}{24d\left(2d-1\right)^3}.$$

The expected talent under AA is given by

$$\frac{+512d^3\left(16-x+3x^2\right)-96d^2\left(80-6x+4x^2-3x^3\right)+6d\left(400-35x-5x^2-6x^3\right)-25\left(10-x\right)}{8d\left(16d-5\right)^3}.$$

The first minus the second is given by

$$\frac{\begin{array}{c} d\left[\left(82928+29664d+9472d^2\right)\left(d-2\right)^2+212868d-331695\right]x^3 \\ +2d\left[\left(298332+117224d+42496d^2+14336d^3\right)\left(d-2\right)^2+724626d-1193373\right]x^2 \\ +\left[3\left(16d-5\right)^2\left(2d-1\right)^4\right]x+2\left[\left(32d^2-26d+5\right)^3\right]\end{array}}{24d\left(2d-1\right)^3\left(16d-5\right)^3}.$$

That the expected talent under AA is lower follows from the observation that the bracketed terms in the above difference are positive for $d > 2$.

The total talent without AA is given by $\frac{1}{d}$, whereas the total talent with AA is given by

$$\frac{3\left(32d+x-10\right)}{8d\left(16d-5\right)}.$$

The first minus the second is given by

$$\frac{32d-3x-10}{8d\left(16d-5\right)}.$$

It follows straightforwardly that the above expression is positive for $d > 2$. ■

Proof of Proposition 6. Suppose the parents have wealth levels $\omega_1 = w - x$, $\omega_2 = w$ and $\omega_3 = w + x$. Consider the expected top effective talent, denoted as τ_1, when there is full investment, i.e., $k < k_3(w-x, w, w+x)$. It is given by, using (e_1^*, e_2^*, e_3^*) from (16),

$$\begin{aligned}
\tau_1 = &\left[\begin{array}{c} \frac{(e_1^*+w-x)+(e_2^*+w)+1-2(e_3^*+w+x)}{3} \times \frac{\{e_1^*-k(w-x)\}+1-(e_2^*-kw)}{2} \\ +\frac{(e_1^*+w-x)+(e_3^*+w+x)+1-2(e_2^*+w)}{3} \times \frac{\{e_1^*-k(w-x)\}+1-\{e_3^*-k(w+x)\}}{2} \end{array}\right] \{e_1^* - k(w-x)\} \\
&+ \left[\begin{array}{c} \frac{(e_2^*+w)+(e_1^*+w-x)+1-2(e_3^*+w+x)}{3} \times \frac{(e_2^*-kw)+1-\{e_1^*-k(w-x)\}}{2} \\ +\frac{(e_2^*+w)+(e_3^*+w+x)+1-2(e_1^*+w-x)}{3} \times \frac{(e_2^*-kw)+1-\{e_3^*-k(w+x)\}}{2} \end{array}\right] (e_2^* - kw) \\
&+ \left[\begin{array}{c} \frac{(e_3^*+w+x)+(e_1^*+w-x)+1-2(e_2^*+w)}{3} \times \frac{\{e_3^*-k(w+x)\}+1-\{e_1^*-k(w-x)\}}{2} \\ +\frac{(e_3^*+w+x)+(e_2^*+w)+1-2(e_1^*+w-x)}{3} \times \frac{\{e_3^*-k(w+x)\}+1-(e_2^*-kw)}{2} \end{array}\right] \{e_3^* - k(w+x)\} \\
&= \frac{1-d(4+3kw)-12d^3k\left\{w+(1-k)x^2\right\}+d^2\left(4+12kw+3x^2-3k^2x^2\right)}{3(2d-1)^2 d} \qquad \text{(B.7)}
\end{aligned}$$

(this last step is verified in Appendix B.8), and its derivative w.r.t. x is given by

$$-\frac{2d(1-k)\{(4d-1)k-1\}x}{(2d-1)^2}. \qquad \text{(B.8)}$$

The first derivative of this expression w.r.t. k is given by

$$-\frac{4d\{k+d(2-4k)\}x}{(2d-1)^2},$$

and the second derivative w.r.t. k is given by

$$\frac{4d(4d-1)x}{(2d-1)^2}.$$

The second derivative is positive and the first derivative is

$$-\frac{8d^2x}{(2d-1)^2} < 0$$

at $k=0$, and

$$\frac{4dx}{2d-1} > 0$$

at $k=1$. Hence (B.8) is decreasing in k and then increasing. At $k=0$, the derivative (B.8) is equal to

$$\frac{2dx}{(2d-1)^2},$$

and at $k=1$, the derivative (B.8) is equal to 0. Hence, there is a k^* such that the derivative (B.8) is negative if and only if $k^* < k < 1$. Specifically, it can be shown that

$$k^* = \frac{1}{4d-1},$$

same as in Proposition 5. We have from (9),

$$k_3 = \frac{3x - 4d\,(1+3x) + 8d^2\,(1+3x) - 4\sqrt{d\,(2d-1)^2\left\{d\,(1+3x)^2 - 3x\right\}}}{3\,(4d-1)\,x}.$$

So,

$$k_3 - k^* = \frac{4\left((1+3x)(2d^2-d) - \sqrt{d\,(2d-1)^2\left\{d\,(1+3x)^2 - 3x\right\}}\right)}{3\,(4d-1)\,x},$$

which is positive because

$$\left(\underbrace{(1+3x)(2d^2-d)}_{>0\text{ for }d>1/2}\right)^2 - \left(d\,(2d-1)^2\underbrace{\left\{d\,(1+3x)^2-3x\right\}}_{>0\text{ for }x>1/3}\right) = 3xd\,(2d-1)^2 > 0.$$

This ensures that increasing inequality (i.e., x) increases or decreases expected top effective talent in the case $k < k_3$ depending on whether $0 < k < k^*$ or $k^* < k < k_3$.

Next consider the case where $k_3 < k < k_2$. The expected top effective talent in this case is given by, using (e_1^*, e_2^*, e_3^*) and z^* from the proof of Proposition 5,

$$\begin{aligned}
&\left[\begin{array}{c}\frac{(e_1^*+w-x)+(e_2^*+w)+1-2(e_3^*+w+z^*)}{3}\times\frac{\{e_1^*-k(w-x)\}+1-(e_2^*-kw)}{2}\\ +\frac{(e_1^*+w-x)+(e_3^*+w+z^*)+1-2(e_2^*+w)}{3}\times\frac{\{e_1^*-k(w-x)\}+1-\{e_3^*-k(w+z^*)\}}{2}\end{array}\right]\{e_1^*-k\,(w-x)\}\\
&+\left[\begin{array}{c}\frac{(e_2^*+w)+(e_1^*+w-x)+1-2(e_3^*+w+z^*)}{3}\times\frac{(e_2-kw)+1-\{e_1^*-k(w-x)\}}{2}\\ +\frac{(e_2^*+w)+(e_3^*+w+z^*)+1-2(e_1^*+w-x)}{3}\times\frac{(e_2^*-kw)+1-\{e_3^*-k(w+z^*)\}}{2}\end{array}\right](e_2^*-kw)\\
&+\left[\begin{array}{c}\frac{(e_3^*+w+z^*)+(e_1^*+w-x)+1-2(e_2^*+w)}{3}\times\frac{\{e_3^*-k(w+z^*)\}+1-\{e_1^*-k(w-x)\}}{2}\\ +\frac{(e_3^*+w+z^*)+(e_2^*+w)+1-2(e_1^*+w-x)}{3}\times\frac{\{e_3^*-k(w+z^*)\}+1-(e_2^*-kw)}{2}\end{array}\right]\{e_3^*-k\,(w+z^*)\}
\end{aligned}$$

$$= \frac{\left(\begin{array}{c}-8\,(1+k)^3 + 4d\,(1+k)^2\left\{24 + k^2\,(6w-3x) + k\,(16+6w-3x)\right\}\\ +256d^6\left\{-1-k+k^2\left(1-24w+12x-15x^2\right)+k^3\left(1+15x^2\right)\right\}\\ -128d^5\left\{-2+15k^4x^2+k^2\left(6-84w+42x-15x^2\right)-3k\left(8+4w-2x+5x^2\right)+k^3\left(4-24w+12x+15x^2\right)\right\}\\ -8d^3\left(\begin{array}{c}15k^5x^2-3\left(32+5x^2\right)-2k^2\left(88+210w-105x+15x^2\right)+k^3\left(8-324w+162x+30x^2\right)\\ -k\left(376+156w-78x+45x^2\right)+k^4\{-60w+15x\,(2+3x)\}\end{array}\right)\\ +d^2\,(1+k)\left(\begin{array}{c}-416-16k^2\,(8+30w-15x)-15x^2+15k^4x^2\\ -2k\left(368+144w-72x+15x^2\right)+k^3\{-192w+6x\,(16+5x)\}\end{array}\right)\\ +16d^4\left(\begin{array}{c}15k^5x^2-3\left(12+5x^2\right)-4k^2\left(7+126w-63x+15x^2\right)\\ +4k^3\left(7-72w+36x+15x^2\right)-k\left(284+144w-72x+75x^2\right)+k^4\{-24w+3x\,(4+25x)\}\end{array}\right)\end{array}\right)}{\left(24\,(2d-1)^2\,d\,(4d-1-k)^2\{(4d-1)\,k-1\}\right)}$$

(this last step is verified in Appendix B.8), and the derivative w.r.t. x is

$$\frac{k\left\{2-8d+d^2(8-20x)\right\}+5dx+5d(4d-1)k^2x}{4(2d-1)^2}. \tag{B.9}$$

The first derivative of this expression w.r.t. k is

$$\frac{1-d(4+5kx)+2d^2\{2+5(2k-1)x\}}{2(2d-1)^2},$$

which is increasing in k since the second derivative

$$\frac{5d(4d-1)x}{2(2d-1)^2}$$

is positive. At $k=0$, the derivative (B.9) is equal to

$$\frac{1-4d+d^2(4-10x)}{2(2d-1)^2} \geq \frac{1-4d+d^2\left(4-\frac{10}{4}\right)}{2(2d-1)^2} = \frac{d(3d-8)+2}{4(2d-1)^2} > 0,$$

and at $k=1$, the derivative (B.9) is equal to

$$\frac{d(2+5x)-1}{4d-2} > 0.$$

Hence, the derivative of the expected top effective talent w.r.t. x, i.e., (B.9), is increasing in k. At $k=0$, (B.9) is equal to

$$\frac{5dx}{4(2d-1)^2},$$

and at $k=1$, (B.9) is equal to $1/2$, so both are positive. Hence, as income inequality increases the expected top effective talent increases.

Finally, consider the case where $k>k_2$. The expected top effective talent is given by, using

(e_1^*, e_2^*, e_3^*) and y^* from the proof of Proposition 5,

$$\tau_2 = \left[\begin{array}{c} \frac{(e_1^*+w-x)+(e_2^*+w-y^*)+1-2(e_3^*+w-y^*)}{3} \times \frac{(e_1^*-k(w-x))+1-(e_2^*-k(w-y^*))}{2} \\ +\frac{(e_1^*+w-x)+(e_3^*+w-y^*)+1-2(e_2^*+w-y^*)}{3} \times \frac{(e_1^*-k(w-x))+1-(e_3^*-k(w-y^*))}{2} \end{array} \right] \{e_1^* - k(w-x)\}$$

$$+ \left[\begin{array}{c} \frac{(e_2^*+w-y^*)+(e_1^*+w-x)+1-2(e_3^*+w-y^*)}{3} \times \frac{(e_2^*-k(w-y^*))+1-(e_1^*-k(w-x))}{2} \\ +\frac{(e_2^*+w-y^*)+(e_3^*+w-y^*)+1-2(e_1^*+w-x)}{3} \times \frac{(e_2^*-k(w-y^*))+1-(e_3^*-k(w-y^*))}{2} \end{array} \right] \{e_2^* - k(w-y^*)\}$$

$$+ \left[\begin{array}{c} \frac{(e_3^*+w-y^*)+(e_1^*+w-x)+1-2(e_2^*+w-y^*)}{3} \times \frac{(e_3^*-k(w-y^*))+1-(e_1^*-k(w-x))}{2} \\ +\frac{(e_3^*+w-y^*)+(e_2^*+w-y^*)+1-2(e_1^*+w-x)}{3} \times \frac{(e_3^*-k(w-y^*))+1-(e_2^*-k(w-y^*))}{2} \end{array} \right] \{e_3^* - k(w-y^*)\}$$

$$= \frac{\begin{array}{c} -(1+k)^3 + d(1+k)^2 \left[8 + k(4+3w-3x) + 3k^2(w-x)\right] \\ -4d^2(1+k)\left[4 + k^2(-4+9w-9x) + 3k^3(w-x) + 6k(2+w-x)\right] \\ +16d^3k\left[10+3w+k(-4+9w-9x)+k^2(-2+6w-6x)-3x\right] \\ +64d^4\left[-2+4k+2k^3+k^2(-4-3w+3x)\right] \end{array}}{3d(1-4d+k)^2\left[(4d-1)k-1\right]}. \tag{B.10}$$

(See Appendix B.8 for verification of this last step.) The derivative w.r.t. x simplifies to $k > 0$. Hence, as inequality increases the expected top effective talent increases. ■

Proof of Proposition 7. Suppose $k \le k_3$. Then the parental investments are $I_1 = w - x$, $I_2 = w$ and $I_3 = w + x$. The individual effective efforts have been derived at the start of the proof of Proposition 5. The resulting total effective effort is

$$e_1^* + e_2^* + e_3^* - k(w - x + w + w + z) = \frac{1}{d} - 3kw,$$

which does not change as x increases.

Next suppose that $k_3 < k \le k_2$. Using the individual effective efforts derived in Proposition 5, the total effective effort (or talent) can be shown to be equal to

$$\frac{\begin{array}{c} 2(1+k)^2 - 16d^3k[1+k(-1+6w-3x)] + 4d^2k\left[10+6w+2k(-1+6w-3x)+k^2(6w-3x)-3x\right] \\ -d(1+k)^2(8+6kw-3kx) \end{array}}{2d\left[16d^2k + (1+k)^2 - 4d(1+k)^2\right]},$$

the derivative of which w.r.t. x is given by

$$\frac{3k}{2}.$$

Hence, the total effective effort increases as the inequality increases.

Finally, for $k > k_2$ the total effective talent (using effective efforts derived in Proposition 5) is given by

$$\frac{\begin{array}{c}(1+k)^2-16d^3k\,[2+k\,(-2+3w-3x)]-d\,(1+k)^2\,[4+3k\,(w-x)]\\ +4d^2k\left[8+3w+k\,(-4+6w-6x)+3k^2\,(w-x)-3x\right]\end{array}}{d\left[16d^2k+(1+k)^2-4d\,(1+k)^2\right]}.$$

The derivative w.r.t. x simplifies to $3k$. Hence, total effective talent increases as the inequality increases. ■

B.2 Derivation of equilibrium efforts (5)

Let π_i denote the payoff to child i given the parental investments and efforts of the children (we suppress the arguments in the notation for brevity). We have, for $i \neq j, l, j \neq l$, and $i, j, l = 1, 2, 3$,

$$\pi_i = \frac{(e_i+I_i)+(e_j+I_j)+1-2\,(e_l+I_l)}{3} \times \frac{e_i+1-e_j}{2} + \frac{(e_i+I_i)+(e_l+I_l)+1-2\,(e_j+I_j)}{3} \times \frac{e_i+1-e_l}{2}$$
$$-\, d_i e_i^2.$$

By solving the first-order conditions

$$\frac{\partial \pi_1}{\partial e_1} = \frac{1}{6}\,[4+(4-12d_1)\,e_1-2e_2-2e_3+2I_1-I_2-I_3]=0,$$
$$\frac{\partial \pi_2}{\partial e_2} = \frac{1}{6}\,[4-2e_1+(4-12d_2)\,e_2-2e_3-I_1+2I_2-I_3]=0,$$
$$\frac{\partial \pi_3}{\partial e_3} = \frac{1}{6}\,[4-2e_1-2e_2+4e_3-12d_3e_3-I_1-I_2+2I_3]=0,$$

we have

$$e_1^* = \frac{2-4d_2-4d_3+8d_2d_3-(d_2+d_3-4d_2d_3)\,I_1+d_2\,(1-2d_3)\,I_2+(1-2d_2)\,d_3I_3}{2\,(d_1+d_2-4d_1d_2+d_3-4d_1d_3-4d_2d_3+12d_1d_2d_3)},$$

$$e_2^* = \frac{2-4d_1-4d_3+8d_1d_3+d_1\,(1-2d_3)\,I_1-(d_1+d_3-4d_1d_3)\,I_2+(1-2d_1)\,d_3I_3}{2\,(d_1+d_2-4d_1d_2+d_3-4d_1d_3-4d_2d_3+12d_1d_2d_3)},$$

$$e_3^* = \frac{2-4d_1-4d_2+8d_1d_2+d_1\,(1-2d_2)\,I_1+(1-2d_1)\,d_2I_2-(d_1+d_2-4d_1d_2)\,I_3}{2\,(d_1+d_2-4d_1d_2+d_3-4d_1d_3-4d_2d_3+12d_1d_2d_3)}.$$

To verify that these efforts indeed solve the first-order conditions, we substitute the expressions for the efforts in the expressions $\frac{\partial \pi_1}{\partial e_1}$, $\frac{\partial \pi_2}{\partial e_2}$ and $\frac{\partial \pi_3}{\partial e_3}$ and simplify the equations in Mathematica (see Appendix-C-1) to observe that the derivatives vanish at the solution stated above.

B.3 Supplementary material for the Proof of Proposition 2

We verify that the equilibrium efforts given for the three cases (i) $I_1 = w - x, I_2 = w, I_3 = w + x$, (ii) $I_1 = w - x, I_2 = w, I_3 = w$, and (iii) $I_1 = w - x, I_2 = w - x, I_3 = w$, indeed solve the first-order conditions by the same method as above. We substitute the efforts in the three cases for the corresponding investment levels in Mathematica (see Appendix-C-2) and see that the derivatives indeed vanish in each of these three cases.

The expressions for expected talent in the three cases indeed simplify to the given expressions, as verified in Appendix-C-2.

B.4 Supplementary material for the Proof of Proposition 3

(**i**) First we verify that

$$e_1^{AA} = \frac{1}{4d}, \quad e_2^{AA} = \frac{4d\,(2-x)-3}{4d(8d-3)}, \quad e_3^{AA} = \frac{4d(2+x)-3}{4d(8d-3)}$$

satisfy the first-order conditions in Appendix-C-3-1.

Using these efforts the expected top talent is given by

$$\tau^{AA} = \frac{9-48d+16d^2(4+x^2)}{4(8d-3)^2 d}.$$

We have

$$\tau - \tau^{AA} = \frac{(16d^2-14d+3)^2+12d^2(48d^2-32d+5)x^2}{12(8d-3)^2(2d-1)^2 d}.$$

The expression is verified in Appendix-C-3-2.

From Proposition 2 we have

$$e_1^* = \frac{4d-3dx-2}{6d(2d-1)}, \quad e_2^* = \frac{1}{3d}, \quad e_3^* = \frac{4d+3dx-2}{6d(2d-1)}.$$

The expressions for the difference in children's efforts, $e_i^* - e_i^{AA}$, are verified in Appendix-C-3-3.

(**ii**) The first-order conditions for the problem

$$\max_{0 \le e_i \le \frac{1}{4}} \frac{e_i + w + 1 - e_j - w}{2} \times \frac{e_i + 1 - \frac{1}{4d}}{2} - de_i^2$$

$(i, j = 2, 3; i \neq j)$ are given by

$$\frac{e_i + w + 1 - e_j - w}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{e_i + 1 - \frac{1}{4d}}{2} - 2de_i = 0$$

$$\text{or,} \quad 2e_i + 2 - e_j - \frac{1}{4d} - 8de_i = 0.$$

It is straightforward to verify that $e_2^{AA} = e_3^{AA} = \frac{1}{4d}$ satisfy the first-order conditions.

B.5 Supplementary material for the Proof of Lemma 1

First we show that (10)–(12) generate the equilibrium efforts by substituting the efforts in the LHS of the first-order conditions and verifying that they indeed vanish. We show this in Appendix-C-4-1.

We show that the derivatives of the efforts are indeed the same as those given in the expressions, using Mathematica in Appendix-C-4-2.

We verify (13)–(15) as follows: first we take derivatives of parents' payoffs w.r.t. investments (exp1 in Mathematica), and then show that these are equal to the RHS expressions of (13)–(15) (exp2 in Mathematica). See Appendix-C-4-3.

To show that the solution of k in equation (14)=0 is the given expression, we substitute $k = \widehat{k}_2$ (we take only one root, since for quadratic equations the two roots differ only by the sign of the square root of the discriminant) into expression (14) and verify that it vanishes in Mathematica in Appendix-C-4-4.

To show that the solution of k in equation (15)=0 is the given expression, we substitute $k = \widehat{k}_3$ (we take only one root) into expression (14) and verify that it vanishes in Mathematica in Appendix-C-4-5.

B.6 Supplementary material for the Proof of Proposition 4

Part (ii). To show that the solution of z^* in equation (15)=0 is the given expression, we substitute $z = z^*$ into expression (15) and verify that it vanishes in Mathematica in Appendix-C-5-1.

Next we check that $z^* > 0$ if and only if $k < k_2$. The derivative of z^* w.r.t. k is

$$-\frac{4d(-1+2d)[3+16d^2+2k-k^2+4d(-3-2k+k^2)]}{[16d^2k+(1+k)^2-4d(1+k)^2]^2}.$$

We have

$$\begin{aligned}
3+16d^2+2k-k^2+4d(-3-2k+k^2) &> 3+16d^2+2\times 0-1^2+4d(-3-2\times 1+0^2)\\
&= 2-20d+16d^2\\
&= (d-3)(28+16d)+86 > 0.
\end{aligned}$$

The other factors in the numerator and the denominator are positive. Hence, z^* is decreasing in k. Solving for k in $z^* = 0$ we have

$$k_2 = \frac{m_1 - 4d(1+m_1) + 8d^2(1+m_1) - 4\sqrt{(1-2d)^2 d\{-m_1 + d(1+m_1)^2\}}}{(4d-1)m_1}.$$

Part (iii): y^* increasing in k. To show that the solution of y^* in equation (14)=0 is the given expression, we substitute $y = y^*$ into expression (14) and verify that it vanishes in

Mathematica in Appendix-C-5-2.

Finally we verify that y^* is increasing in k. The derivative of y^* w.r.t. k is

$$\frac{8d\left(-1+2d\right)\left[3+16d^2+2k-k^2+4d\left(-3-2k+k^2\right)\right]}{\left[16d^2k+\left(1+k\right)^2-4d\left(1+k\right)^2\right]^2}.$$

We have

$$\begin{aligned}3+16d^2+2k-k^2+4d\left(-3-2k+k^2\right) &> 3+16d^2+2\times 0-1^2+4d\left(-3-2\times 1+0^2\right)\\ &= 2-20d+16d^2\\ &= \left(d-3\right)\left(28+16d\right)+86>0.\end{aligned}$$

The other factors in the numerator and the denominator are positive. Hence, y^* is increasing in k.

B.7 Supplementary material for the Proof of Proposition 5

First we verify the expression in equations (16). We use the benchmark contest efforts to evaluate $e_i^* - kI_i$ in Mathematica (Appendix-C-6-1). Similarly we verify equations (17) and (18) (Appendix-C-6-2).

Next we verify the expressions for effective equilibrium efforts for $k_3 < k < k_2$, so that parents 1 and 2 invest fully but parents 3 (under)invests an amount $w + z^*$. We use the expression for z^* from Proposition 4 after setting $m_1 = m_3 = x$, and use $I_1 = w - x, I_2 = w, I_3 = w + z^*$ to calculate the effective efforts (Appendix-C-6-3). The expressions for the differences in effective efforts between child 1 and child 2, and then child 2 and child 3 are then verified to be equal to the given expressions in the proof (Appendix-C-6-4).

Finally, we verify the expressions for effective equilibrium efforts for $k > k_2$, so that parents 1 invest fully but parents 2 and 3 (under)invest an amount $w - y^*$. We use the expression for y^* from Proposition 4 after setting $m_1 = m_3 = x$, and use $I_1 = w - x$ and $I_2 = w - y^*$ (which is the same as $I_3 = w - y^*$) to calculate the effective efforts (Appendix-C-6-5). The expressions for the differences in effective efforts between child 1 and child 2 (child 2 and child 3 have the same effective efforts) are then verified to be equal to the given expressions in the proof (Appendix-C-6-6).

B.8 Supplementary material for the Proof of Proposition 6

The expression in the proof for expected top (effective) talent when $k < k_3$ is verified using Mathematica in Appendix-C-7-1.

The expression in the proof for expected top (effective) talent when $k_3 < k < k_2$ is verified in Appendix-C-7-2.

The expression in the proof for expected top (effective) talent when $k > k_2$ is verified in Appendix-C-7-3.

B.9 Supplementary material for the Proof of Proposition 7

The expression in the proof for total (effective) talent when $k < k_3$ is verified using Mathematica in Appendix-C-8-1.

The expression in the proof for total (effective) talent when $k_3 < k < k_2$ is verified in Appendix-C-8-2.

The expression in the proof for total (effective) talent when $k > k_2$ is verified in Appendix-C-8-3.

B.10 Supplementary material for Examples 2 and 3

The solution of k in $z^* = x$, plugged in (22), gives the critical value k_3^{AA} so that for $k < k_3^{AA}$ parents 2 and 3 fully invest. In this case k_3^{AA} is given by (see Appendix-C-9-5):

$$k_3^{AA} =$$

$$-\frac{\begin{array}{c}\{x - 128d^3(1+w+2x) + 16d^2(5+4w+8x) - 2d(6+3w+10x)\} \\ +2\sqrt{(3-8d)^2 d\left[-x+64d^3(1+w+2x)^2 - 16d^2\{2+w^2+10x+4x^2+w(5+4x)\} + d\{w^2+4w(2+x)+4(1+6x+x^2)\}\right]}\end{array}}{x - 12d(w+2x) + 32d^2(w+2x)}.$$

■ **Comparison of top talent for** $k < \min\{k_3, k_3^{AA}\}$. Recall that when there is *no affirmative action* for $k < k_3$, no parents underinvest and so $I_1 = w - x, I_2 = w, I_3 = w + x$. The expected effective top talent in this case is given by τ_1 derived in (B.7) in the proof of Proposition 6.

Next consider affirmative action. For $k < k_3^{AA}$, we already know that parental investments $I_1 = 0, I_2 = w$ and $I_3 = w + x$. In this case, the students' payoffs are given by

$$\begin{aligned}
\pi_1^{AA} =& \frac{(e_2+w)+1-(e_3+w+x)}{2} \times \frac{e_1+1-(e_2-kw)}{2} \\
&+ \frac{(e_3+w+x)+1-(e_2+w)}{2} \times \frac{e_1+1-\{e_3-k(w+x)\}}{2} - de_1^2, \\
\pi_2^{AA} =& \frac{(e_2+w)+1-(e_3+w+x)}{2} \times \frac{(e_2-kw)+1-e_1}{2} - de_2^2, \\
\pi_3^{AA} =& \frac{(e_3+w+x)+1-(e_2+w)}{2} \times \frac{\{e_3-k(w+x)\}+1-e_1}{2} - de_3^2.
\end{aligned}$$

The first-order conditions:

$$\begin{aligned}
\frac{\partial \pi_1^{AA}}{\partial e_1} =& \frac{1}{2} - 2de_1 = 0, \\
\frac{\partial \pi_2^{AA}}{\partial e_2} =& \frac{1}{4}\left[2 - e_1 + (2-8d)\, e_2 - e_3 + (1-k)w - (w+x)\right] = 0, \\
\frac{\partial \pi_3^{AA}}{\partial e_3} =& \frac{1}{4}\left[2 - e_1 - e_2 + 2e_3 - 8de_3 - w + (1-k)(w+x)\right] = 0,
\end{aligned}$$

yield equilibrium efforts:

$$
\begin{aligned}
e_1^{AA} &= \frac{1}{4d}, \\
e_2^{AA} &= \frac{32d^2(2-kw-x)-4d[8-x-k(3w+x)]+3}{4d(8d-3)(8d-1)}, \\
e_3^{AA} &= \frac{32d^2[2+x-k(w+x)]-4d(8-3kw+x-2kx)+3}{4d(8d-3)(8d-1)}.
\end{aligned}
$$

The expected effective top talent is given by

$$
\begin{aligned}
\tau_1^{AA} = &\left[\begin{array}{c} \frac{\left(e_2^{AA}+w\right)+1-\left(e_3^{AA}+w+x\right)}{2} \times \frac{e_1^{AA}+1-\left(e_2^{AA}-kw\right)}{2} \\ +\frac{\left(e_3^{AA}+w+x\right)+1-\left(e_2^{AA}+w\right)}{2} \times \frac{e_1^{AA}+1-\left\{e_3^{AA}-k(w+x)\right\}}{2} \end{array}\right] e_1^{AA} \\
&+ \left[\frac{\left(e_2^{AA}+w\right)+1-\left(e_3^{AA}+w+x\right)}{2} \times \frac{\left(e_2^{AA}-kw\right)+1-e_1^{AA}}{2}\right]\left(e_2^{AA}-kw\right) \\
&+ \left[\frac{\left(e_3^{AA}+w+x\right)+1-\left(e_2^{AA}+w\right)}{2} \times \frac{\left\{e_3^{AA}-k(w+x)\right\}+1-e_1^{AA}}{2}\right]\left[e_3^{AA}-k(w+x)\right].
\end{aligned}
$$

The precise expression for τ_1^{AA}, after substituting $(e_1^{AA}, e_2^{AA}, e_3^{AA})$, is included in Appendix-C-9-7.

Now consider $\tau_1 - \tau_1^{AA}$. After tedious steps it can be shown to be as follows (verified in Appendix-C-9-7):

$$
\frac{\begin{array}{c}
9-12d\,(19+9kw)+4d^2\left[553+15x^2+12k^3x^2\,(2w+x)+3k^2x^2\,(-7+16w+8x)+6k\left\{4w\left(24+x^2\right)+x\left(-9-x+2x^2\right)\right\}\right] \\
-16d^3\left[656+84x^2+48k^3x^2\,(2w+x)+27k^2\left\{8w^2+8wx\,(1+x)+x^2\,(-3+4x)\right\}+3k\left\{x\left(-78-5x+20x^2\right)+w\left(397+40x^2\right)\right\}\right] \\
+96d^4\left[272+110x^2+42k^3x^2\,(2w+x)+k^2\left\{336w^2+8wx\,(42+31x)+x^2\,(-135+124x)\right\}+k\left\{2w\left(415+66x^2\right)+x\left(-241+23x+66x^2\right)\right\}\right] \\
-256d^5\left[33k^3x^2\,(2w+x)+4\left(32+33x^2\right)+3k^2\left\{146w^2+x^2\,(-89+55x)+2wx\,(73+55x)\right\}+3k\left\{x\left(-87+45x+20x^2\right)+w\left(234+40x^2\right)\right\}\right] \\
+2048d^6\left[8+18x^2+3k^3x^2\,(2w+x)+k^2\left\{84w^2+9x^2\,(-9+4x)+12wx\,(7+6x)\right\}+3k\left\{x\left(-15+21x+2x^2\right)+w\left(34+4x^2\right)\right\}\right] \\
-49152d^7k\left[2w-x+3x^2+k\left\{2w^2-3x^2+x^3+2wx\,(1+x)\right\}\right]
\end{array}}{12\,(8d-1)^2\,(8d-3)^2\,d\,(2d-1)^2}. \tag{B.11}
$$

For $k > \min\{k_3, k_3^{AA}\}$ (relevant for Example 3), the expressions for the difference in top talents are available in Appendix-C-10-5. ||