When to merge with a lower quality producer?

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Abstract

This paper studies the possibility of different types of mergers when firms produce vertically and horizontally differentiated products. The two firms produce a better quality product, while the third firm produces a lower quality product and the firms compete in quantities. The merger of two firms that produce better-quality products is possible if the quality difference (net of cost) or the horizontal product differentiation are high. However, if the quality difference (net of cost) and the horizontal product differentiation are neither too high nor too low, then the firm that produces the better quality product will merge with the firm that produces a lower quality product. Welfare may increase after the merger between the two firms that produce different quality products. However, if the two firms that produce the better quality product merge then welfare always falls.

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1 Introduction

Literature on merger in oligopolistic markets is quite vast given its significance for competition policy. The competition-reducing effect of a merger is often highlighted and competition policy mostly describes the restrictive conditions under which a merger should be permitted. In an important contribution, Salant et al. (1983) have shown that a merger is privately profitable only if a relatively high fraction of previously existing firms engage in the merging process. Specifically, they show that if demand and cost functions are linear, then an exogenous merger followed by Cournot competition is only profitable if at least 80 per cent of the firms engage in the merger. Kamien and Zang (1990) modelled mergers as an endogenous process through acquisitions and showed with a linear demand function and constant returns to scale there is no merger in the equilibrium, when initially there are more than two firms.

In reality, mergers are observed to be very significant phenomena in many industries. According to the Thomson Financial Securities Data, there were 87,804 mergers and acquisitions recorded for Europe in the period 1993–2001. In monetary terms, the total value of these deals adds up to USD 5.6 trillion (see Belleflamme and Peitz (2015)). Mueller (1985) shows that out of the 1,000 largest manufacturing companies of 1950, 384 companies had disappeared through mergers by 1973. In a seminal contribution, Deneckere and Davidson (1985) showed that the possibility of mergers can significantly increase under Bertrand price competition with differentiated products. Also, Perry and Porter (1985) focused on cost synergies due to mergers and claimed that aggregate profits of the merging firms will increase if the cost synergies are sufficiently large.

While a merger reduces competition, does it necessarily reduce welfare?[1] In a seminal contribution, Farrell and Shapiro (1990) considered Cournot oligopolies with homogeneous goods and provided a procedure for analyzing the effect of a merger and focused on when the merger would be profitable and welfare-increasing. They showed that the equilibrium price increases under linear cost structures without cost synergies. Moreover, they derived a necessary and sufficient condition for a price increase under certain classes of non-linear cost functions and possible cost synergies. Farrell and Shapiro (1990) mainly focus on synergistic gains as the reason for welfare increase.

In an interesting contribution, Ashenfelter et al. (2013) studies Whirlpool's purchase of Maytag. The merger of Whirlpool and Maytag reduced the number of major appliance manufacturers (Whirlpool, Maytag, GE, and Electrolux) in the United States from four to three. They mention that "Both Maytag and Whirlpool had sales in seven of the eight major home appliance categories: dishwashers, clothes dryers, refrigerators, clothes washers, cooktops, ovens, and ranges. Within each appliance category there is substantial product differentiation. There are two leading sources of differentiation. The first comes from measurable product characteristics. Second, products are also differentiated by brand marketing." Moreover, pre-merger Whirlpool was the largest manufacturer in each of these appliance categories, and Maytag was the second largest producer of washers and dryers. However, Maytag had relatively little market share for cooktops, freezers, ovens, and ranges. Thus,

[1] Ashenfelter et al. (2013) mentions that in U.S. each year thousands of mergers are proposed to the U.S. Federal Trade Commission and U.S. Department of Justice. After filing, the merging firms must wait while the antitrust authority attempts to identify and block mergers that would increase consumer prices and thereby consumer surplus.

it seems that players in the oligopolistic market, may prefer to merge even when their quality (as well as market share) difference is high. To the best of our knowledge, this aspect of mergers has not been given adequate attention in the literature. Ashenfelter et al. (2013) also argues that "The degree of pre-merger competition between Maytag and Whirlpool, however, likely varied across product categories. Maytag and Whirlpool were major competitors in four appliance categories: dishwashers, refrigerators, clothes washers, and clothes dryers. In each of these categories, the markets appear to be highly concentrated with Whirlpool typically having about 40 percent of sales and Maytag having a share between 9 percent and 16 percent in these categories." Maytag was also a relatively less important producer of cooktops, ovens, and ranges. The difference in the market shares depends heavily on the observed product characteristics. In this case study of Maytag-Whirlpool, Ashenfelter et al. (2013) has measured a product's quality using observed product characteristics, which is a common approach used in the matching of new and discontinued products in price measurement. Thus, they have used measured product characteristics to control for product quality. Therefore, in these four appliance categories mentioned above, as Maytag and Whirlpool have different market shares, it can be argued that the quality of the products sold by Maytag and Whirlpool must be different in the pre-merger stage.

The present model derives its motivation from such examples (where firms producing higher quality product may prefer to merge with a firm producing a lower quality product) and contributes to the literature on mergers with the possibility of both horizontal and vertical product differentaition, which depend both on product characteristics and brand value respectively. In this paper, we consider a merger between two firms in a three firm oligopolistic setting and allow for both vertical and horizontal product differentiation of the products in a market. We focus on the situations where profitable mergers occur and then look into the possibility of cases where welfare increases (while firms compete in quantities). In the pre-merger situation, the market share driven by brand value and product characteristics may differ. Thereby, we try to answer the following two interesting questions: a) With whom a firm with highest brand value or product quality (and thereby highest market share) in the pre-merger stage will merge? and b) When will a firm with highest brand value or product quality merge with a firm that produces a lower quality product? *Interestingly, we discuss for the first time in the literature that a firm producing a better quality product may prefer to merge with a firm that produces a lower quality product than to merge with another firm with a better quality product.*

There are only a few works in the literature that analyze the possibility of a merger in a setting of both horizontal and vertical product differentiation. Kao and Menezes (2010) examine the merger issue in a duopoly setting with both vertical and horizontal product differentiation and found that a merger is always profitable. However, the model ignores the classic externality effect of the firms that are outside the merger process. In a vertical differentiation model with three incumbent firms, Norman et al. (2005) show that if a cost-reducing (due to cost synergies) merger occurs between two firms that produce lower quality products, then the merger leads to a reduction in the number of products available in the market resulting into higher prices.

Ebina and Shimizu (2009) study the sequential merger incentives under the presence of horizontal product differentiation and show that merger incentives under product differentiations are found to be stronger for two firms producing closely related goods than more differentiated goods. Hsu and

Wang (2010) in a model with four firms producing horizontally differentiated products shows that the merger of two firms is profitable provided that goods are sufficiently distant substitutes. Toshimitsu and Nakajima (2021) also founds a similar result. Cellini (2021) in a model of three firms under horizontal differentiation shows that a merger between firms that supply more similar product is more profitable as compared to the merger between firms supplying more differentiated goods. However, these papers consider that all the firms produce the goods at the same unit cost and there is no vertical product differentiation (quality difference). This is in contract with our paper where we assume that the production of a better quality product is associated with a higher unit cost.

Sen and Narula (2021) is the only paper in the literature that discusses the possibility of a two firm merger in a setup of three firms with both horizontal and vertical product differentiation. The costs of the firms are different and the better quality goods require a higher unit cost of production and the firms compete in quantities. They show that the merger between the firms that produce the higher-quality product and the other firm next in the quality ranking is possible: if either the horizontal product differentiation or the quality difference (net of cost) is high. It also shows that the merger between the firms that produce the lower-quality products is never possible and it is always more profitable for a firm to merge with a firm that produces a higher-quality product than to merge with a firm that produces a lower-quality product.

In Sen and Narula (2021) one firm produces the high-quality product and the other two firms produce the low-quality product. It discusses whether there will be a merger between the two firms that produce the low-quality product or between the high-quality producer and one of the low-quality producing firms. However, in the present paper, there are two firms that produce the high-quality products and there also exists another firm that produces the low-quality product. The current paper discusses whether there will be a merger between the two high-quality producing firms or between the low-quality producing firm and one of the high-quality producing firms. Thus, despite the difference between this paper and Sen and Narula (2021) as discussed here, the present paper also complements Sen and Narula (2021).

In this paper, we closely follow the structure of Sen and Narula (2021) which discusses the possibility of a two firm merger in a setup of three firms with both horizontal and vertical product differentiation. In a three firm Cournot setting where only two firms can merge, we show that not only merger between firms that produce the better quality products is possible, but also between the firms which produce different quality products. In contrast to Sen and Narula (2021), we assume that the two firms produce the better quality product whereas the third firm produces a lower quality product. We show that the merger of two firms that produce better-quality products is possible if the quality difference (net of cost) or the horizontal product differentiation are high. On the other hand, the merger of firms producing different quality products is possible if either the horizontal product differentiation is high or the quality difference (net of cost) is high or both. These types of mergers however are not possible in Sen and Narula (2021).

It is also shown that the low-quality product may be withdrawn post-merger and there can be an overlap of parameter regions (regions defined in terms of product differentiation and cost) where the different combinations of mergers coexist and the merging parties will choose the most profitable

one. In such cases, we show that if the quality difference (net of cost) and the horizontal product differentiation are neither too high nor too low, then the firm that produces the better quality product will merge with the firm that produces a lower quality product, but not with the better quality producer. We do find that there are circumstances where a merger can be welfare improving and a necessary (but not sufficient) condition for this to happen is that the two firms that produce different qualities merge. On the other hand, if the two firms that produce the better quality product merge, then welfare always reduces.

The empirical motivation of the paper comes from most of the examples of mergers around us, e.g. Whirlpool's purchase of Maytag, merger of InBev (itself a merger between Interbrew from Belgium and AmBev from Brazil) and Anheuser-Busch from the United States.[2] It is very important to note that products of different firms are always differentiated either vertically or horizontally or on both dimensions. So real life merger story is to be captured through the lens of our theoretical model rather than other existing theoretical models focusing on either horizontal or vertical differentiation. Shapiro (1996) emphasized the need for a better understanding of mergers under differentiated products though the area is "confusing". In general in such a merger context there would be product re-positioning and entry into the market and he advocated the measure of "diversion ratio" from one brand to another post-merger as a guiding principle for price rise which can be partly counteracted by rivals' product re-positioning, entry and available synergy. Though we do not explicitly model the product re-positioning, entry or synergy in our model, we shed some important theoretical insights about the pattern of merging brands and its effect on competition, availability of brands post-merger and welfare under the Cournot market structure.[3]

Norman et al. (2005) studies mergers under vertical product differentiation and follows the discrete choice approach as in Mussa and Rosen (1978) and Shaked and Sutton (1982).[4] In Norman et al. (2005) each consumer buys exactly one unit of the product that offers the highest utility from a specific firm out of three firms present in the market, provided that the utility is non-negative and not buy at all otherwise. An alternative approach is to model each consumer with variable demand for all products but to assume that all consumers are identical. This approach is called the representative consumer approach.[5] Deneckere and Davidson (1985), Ebina and Shimizu (2009), Kao and Menezes

[2]Gugler et al. (2003) analyzes the effects of mergers around the world over the past 15 years.

[3]In India around the mid-1990s, soft-drink giant the Coca-Cola company bought out a well known domestic soft drink brand Thums Up and immediately after this takeover, the company cut down on the advertisement of Thums Up, cut down on it's supplies to markets and tried to promote Coco-Cola brand more aggressively leading to loss of market share of the Thums Up brand quite substantially. The consumers were not happy to see this dilution of the brand and none of the soft drinks variety was close to the taste of Thums Up as perceived by the consumers (Thumps Up "taste the thunder" which was the tag line of the advertisement). After several years the company re position the brand and it is now a flagship brand of the Coco-Cola company at least in the Indian market (see Dutta (2022)). Brand rationalization after merger is a common feature whenever the merging firms have competing brand in the same market. In 2005, the merger of Proctor and Gamble, a US consumer goods company and Gillette, the world's largest manufacturer of shaving products made them one of the largest consumer products company. However, the product portfolio was rationalized over time and some under-performing brands were sold off.

[4]Under the discrete choice approach there exists the "spokes model" of non-localized spatial competition as a tool for oligopoly analysis developed by Chen and Riordan (2007).

[5]Studies of product differentiation that follow the representative consumer model are Spence (1976), Dixit and Stiglitz (1977), Dixit (1979), Singh and Vives (1984), Hackner (2000) etc. For a detailed review of literature following

(2010), Cellini (2021) and Sen and Narula (2021) have followed this tradition for studying mergers under product differentiation. Moreover, to study merger under horizontal and vertical product differentiation the representative consumer approach, as followed by Kao and Menezes (2010) and Sen and Narula (2021), is tractable and allows us to show the effect of business cannibalization which depends not only on the degree of horizontal but also on vertical product differentiation. Bos and Vermeulen (2022) also shows that the same demand structure as in Hackner (2000) and can be easily derived directly from a population of heterogeneous consumers making discrete choices. Brekke et al. (2017) studies the effects of a horizontal merger when firms compete on price and quality in a Salop framework with three symmetric firms. Brekke et al. (2017) derives the demand from individual preferences which depend on price, quality and distance, which implies that products are horizontally and (potentially) vertically differentiated. They consider a pre-merger market structure with three identical firms symmetrically located on the Salop circle where two of the three firms can merge.

The structure of the paper is as follows. In Section 2, we discuss the basic model. Section 3 discusses the possibility of a merger between the firms that produce the better quality product. In Section 4, we discuss the merger of the firms that produce the different quality products. In Section 5 we make a comparison in terms of the surplus of the different types of mergers when different types of mergers are possible and finally, we conclude in Section 6.

2 The Model

The preferences of the consumers are described by the utility functions of the representative consumer as in Sen and Narula (2021) following Hackner (2000) by equation (1).

$$U(q_1, q_2, q_3, I) = \alpha_1 q_1 + \alpha_2 q_2 + \alpha_3 q_3 - \frac{1}{2}[q_1^2 + q_2^2 + q_3^2 + 2\gamma q_1 q_2 + 2\gamma q_1 q_3 + 2\gamma q_2 q_3] + I. \tag{1}$$

As in Sen and Narula (2021) the consumer derives utility from the consumption of three differentiated goods, good 1, good 2 and good 3; $q_i = i, 2, 3$ at prices $p_i, i = 1, 2, 3$ respectively and a composite good marked as I, with the price of the composite good normalized to one. In equation (1) $\gamma \in (0, 1)$ is the measure of the substitutability between the products horizontally. If $\gamma = 0$, then the products are independent, however for $\gamma = 1$, the products are perfect substitutes if $\alpha_1 = \alpha_2 = \alpha_3$. Note that, α_is in equation (1) captures the quality of the product. Therefore, if $\alpha_i \neq \alpha_j$ for all i, j and $i \neq j$ then there also exists vertical (or, quality) product differentiation. Otherwise, if $\alpha_1 = \alpha_2 = \alpha_3$ then there exists only horizontal product differentiation in the absence of vertical product differentiation. For a detailed description of product differentiation, one can see Breton and Sbragia (2021) and Sen et al. (2021).[6]

this tradition as well as the linear demand system, one can see Chone and Linnemer (2020) and Martin (2009). Shubik and Levitan (1980) however considers an aggregate linear demand, differentiated product specification in which total demand at identical prices is constant with respect to changes in the number of varieties. Belleflamme and Peitz (2015) mentions that "Also preferences in these representative consumer models can be specified on an underlying characteristics space. The idea here is that a product is a bundle of different characteristics. A consumer has preferences over these bundles."

[6]There is another demand system developed by Sutton (1997) and later used by Symeonidis (1999) that characterizes horizontally and vertically differentiated product markets simultaneously. However, they follow the representative

The consumers maximize the utility subject to the budget constraint $\Sigma p_i q_i + I \leq M$, where M denotes income and therefore, the first-order condition determining the optimal consumption of good i is,

$$\frac{\partial U}{\partial q_i} = \alpha_i - q_i - \gamma \sum_{i \neq j} q_j - p_i = 0. \tag{2}$$

Following Sen and Narula (2021) we assume that there are three firms: firm 1, firm 2 and firm 3, in the market and they compete in quantities. As in Norman et al. (2005), Mazzeo (2018) and Sen and Narula (2021) we discuss the possibility of the merger between any two firms, while the third represents the experience of non-merging parties. Firm i produce a specific brand (say brand i) where $q_i,\ i = 1, 2, 3$ is the output produced by the firm i and α_i is interpreted as a measure of brand strength or brand image. It is also assumed that once established, firms can't change their choice of α_i. The inverse demand function following equation (2) is

$$p_i(q_i, q_j) = \alpha_i - q_i - \gamma \sum_{i \neq j} q_j, \tag{3}$$

where q_i is the output produced by firm i and q_j is the output produced by firm j.

Following Sen and Narula (2021) we also assume that the cost function of firm i is given by $C(q_i) = c_i q_i$, where $i = 1, 2, 3$ and c_i is the constant marginal cost which is increasing in α_i.[7] This means that the production of a better quality product is associated with a higher unit cost. In contrast to Sen and Narula (2021)[8], we assume that firm 1 and firm 2 are identical and produce a similar quality product as $\alpha_1 = \alpha_2$ and thereby $c_1 = c_2$. However, there is a quality difference (net of cost) between the third firm and the first two firms as $0 < \beta = \frac{\alpha_3 - c_3}{\alpha_1 - c_1} < 1$. Here, β is the measure of the quality-cost difference between firm 3 and the other two firms, such that the higher is β lower is the quality difference (net of cost). Thus, the first two firms produce a better quality product, while the third firm produces a lower quality product and the firms compete in quantities.

2.1 Cournot-Nash Outcome

Here we present the competitive outcome when the firms make quantity decisions simultaneously. The profit function of firm i is $\Pi_i = [\alpha_i - q_i - \gamma \sum_j q_j - c_i] q_i$, $i = 1, 2, 3$, $j \neq i$; and the reaction functions of firm 1, firm 2 and firm 3 are

$$q_1 = \frac{\alpha_1 - \gamma q_2 - \gamma q_3 - c_1}{2}, \quad q_2 = \frac{\alpha_2 - \gamma q_1 - \gamma q_3 - c_2}{2} \quad \textit{and} \quad q_3 = \frac{\alpha_3 - \gamma q_1 - \gamma q_2 - c_3}{2} \tag{4}$$

respectively. From the reaction functions of firm 1, firm 2 and firm 3 we get the equilibrium quantities of firm 1, firm 2 and firm 3 as

$$q_1^* = q_2^* = \frac{2(\alpha_1 - c_1) - \gamma(\alpha_3 - c_3)}{2(2 - \gamma)(\gamma + 1)} \quad \textit{and} \quad q_3^* = \frac{(2 + \gamma)(\alpha_3 - c_3) - 2\gamma(\alpha_1 - c_1)}{2(2 - \gamma)(\gamma + 1)} \tag{5}$$

consumer model and there too the demand is linear. For analytical simplicity Symeonidis (1999) also considers that the marginal costs are constant in a study on collusion. This approach is quite similar to Hackner (2000) as followed in our paper.

[7] These costs also include the promotional costs of marketing the brand as well as the production cost of the specific quality associated with the brand.

[8] Sen and Narula (2021) assumes that $\alpha_1 > \alpha_2 = \alpha_3$ and $c_1 > c_2 = c_3$.

and the profits as

$$\Pi_1^* = (q_1^*)^2 = \Pi_2^* = (q_2^*)^2 \; and \; \Pi_3^* = (q_3^*)^2 \tag{6}$$

respectively. We assume that

$$\beta > \frac{2\gamma}{(2+\gamma)} \tag{7}$$

such that $q_i^* > 0 \; \forall \; i = 1, 2, 3$. Hence, firm 1 and firm 2, the producers of the higher quality product (good 1 and good 2) are more efficient than firm 3, which produces the lower quality product (good 3) as $\Pi_1^* = \Pi_2^* > \Pi_3^*$. In the following sections we study the possibility of different types of merger.

3 Merger between firm 1 and firm 2

We study here the possibility of merger of the two firms that produce the better quality product, i.e. firm 1 and firm 2. After merger, the profit function of the merged firms is given by $\Pi_{12} = (\alpha_1 - c_1 - q_1 - \gamma q_2 - \gamma q_3)q_1 + (\alpha_2 - c_2 - q_2 - \gamma q_1 - \gamma q_3)q_2$. The profit function of firm 3 is $\Pi_3 = (\alpha_3 - c_3 - q_3 - \gamma q_1 - \gamma q_2)q_3$. The reaction functions of the merged firms and firm 3 are

$$q_1 = \frac{\alpha_1 - 2\gamma q_2 - \gamma q_3 - c_1}{2}, \;\; q_2 = \frac{\alpha_2 - 2\gamma q_1 - \gamma q_3 - c_2}{2}, \; and \;\; q_3 = \frac{\alpha_3 - \gamma q_1 - \gamma q_2 - c_3}{2} \tag{8}$$

respectively. Thus the equilibrium outputs are

$$\begin{aligned} q_1 = q_2 &= \frac{(\alpha_1 - c_1)(4 - 4\gamma) + (\alpha_3 - c_3)(2\gamma^2 - 2\gamma)}{4(2 - 3\gamma^2 + \gamma^3)} (> 0) \;\; and \\ q_3 &= \frac{(\alpha_3 - c_3)(4 - 4\gamma^2) + 2(\alpha_1 - c_1)(2\gamma^2 - 2\gamma)}{4(2 - 3\gamma^2 + \gamma^3)} (> 0). \end{aligned} \tag{9}$$

Moreover, we observe that $q_1 = q_2 < q_1^* = q_2^*$. As the goods are substitute the merged firms take care of the adverse effect of more production on the price of good 1 and good 2 (which are substitutes of each other) as well as on the total profit it earns. Hence, the merged firm reduces the production of both good 1 and good 2 and thus the other firm (firm 3) increases the production of good 3, $q_3 > q_3^*$. The profit earned by the merged firms and firm 3 are

$$\begin{aligned} &\Pi_{12} = \frac{2}{Z}\Big(\big[(\alpha_1 - c_1)(4 - 4\gamma^2) - (\alpha_3 - c_3)(2\gamma - 2\gamma^3)\big] \times \; \big[(\alpha_1 - c_1)(4 - 4\gamma) + (\alpha_3 - c_3)(2\gamma^2 - 2\gamma)\big] \Big) \\ &and \;\; \Pi_3 = (q_3)^2 = \Big[\frac{(\alpha_3 - c_3)(4 - 4\gamma^2) + 2(\alpha_1 - c_1)(2\gamma^2 - 2\gamma)}{4(2 - 3\gamma^2 + \gamma^3)} \Big]^2 \end{aligned} \tag{10}$$

respectively, where $Z = 16(2 - 3\gamma^2 + \gamma^3)^2$. Therefore, firm 1 and firm 2 will merge if $\Pi_{12} > {\Pi_1}^* + {\Pi_2}^*$ or

$$(2 - \gamma)^2(1 + \gamma)^3 > (2 + 2\gamma - \gamma^2)^2. \tag{11}$$

The above inequality holds in the region B_1 of Figure 1.

Lemma 1 *Merger between firm 1 and firm 2 is possible in zone B_1 (see Figure 1).*

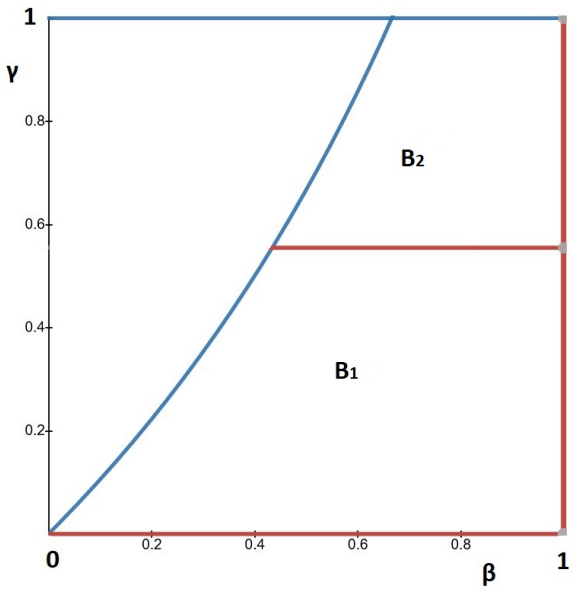

Figure 1: Possibility of merger between firm 1 and firm 2

Therefore, the merger is possible in zone B_1, but it is not possible in B_2 of Figure 1.[9] Post-merger the amount of good 1 and good 2 produced by the merged firms decreases (first effect) to avoid business cannibalization and this is conducive to its profit. However, firm 3 reacts by producing more output (compared to the no-merger situation). This effect (second effect) is detrimental to the profit of the merged firms. From Figure 1 we observe in zone B_2 where γ is very high, i.e. the horizontal product differentiation is low, the latter effect dominates and hence the profit of the merged firms does not increase after the merger. In contrast, if the horizontal product differentiation is high (γ is low), the opposite happens and then the merger is profitable as the second effect becomes weaker. Hence, the merger is profitable if the horizontal product differentiation is high irrespective of the quality difference (net of cost).

Proposition 1 *The firms producing better quality products will merge if the horizontal product differentiation is high.*

3.1 Welfare analysis

We discuss here the impact of the merger on the welfare. The welfare (W) is the sum of the consumer surplus (CS) and industry profits (IP). From the utility function (see equation 1), one gets $CS = \frac{1}{2}[q_1^2 + q_2^2 + q_3^2 + 2\gamma q_1 q_2 + 2\gamma q_1 q_3 + 2\gamma q_2 q_3]$. In the pre-merger stage, the consumer surplus is $CS^* = \frac{1}{2}[(q_1^*)^2 + (q_2^*)^2 + (q_3^*)^2 + 2\gamma q_1^* q_2^* + 2\gamma q_1^* q_3^* + 2\gamma q_2^* q_3^*]$. Therefore, using equation (5), we get

$$CS^* = \frac{(\alpha_1 - c_1)^2}{8(2-\gamma)^2(1+\gamma)^2}\Big(2(1+\gamma)(2-\gamma\beta)^2 + (2\beta + \gamma\beta - 2\gamma)^2 + 4\gamma(2-\gamma\beta)(2\beta+\gamma\beta-2\gamma)\Big). \quad (12)$$

[9] It is always better to produce both the goods in comparison to not producing either of the two products.

After merger between firm 1 and firm 2 the consumer surplus is $CS_{12} = \frac{1}{2}[(q_1)^2+(q_2)^2+(q_3)^2+2\gamma q_1q_2+2\gamma q_1q_3+2\gamma q_2q_3]$ and using equation (9), we get

$$CS_{12} = \frac{(\alpha_1-c_1)^2}{32(2-3\gamma^2+\gamma^3)^2}\Big(2(1+\gamma)\big[(4-4\gamma)+\beta(2\gamma^2-2\gamma)\big]^2+\big[\beta(4-4\gamma^2)+2(2\gamma^2-2\gamma)\big]^2 + 4\gamma\big[(4-4\gamma)+\beta(2\gamma^2-2\gamma)\big]\big[\beta(4-4\gamma^2)+2(2\gamma^2-2\gamma)\big]\Big). \quad (13)$$

Using equations (12) and (13), we observe that $CS^* > CS_{12}$ in zone B_1 of Figure 1 where merger between firm 1 and firm 2 is possible. Thus after the merger consumer surplus always falls. Post-merger the total output (good 1 + good 2) produced by the merged firms falls. Even though post-merger firm 3 produces a higher output, as the total amount of output produced by the merged firms reduces the consumer surplus always falls.

The welfare (W) is the sum of the consumer surplus (CS) and industry profits (IP). In the pre-merger stage, the welfare is $W^* = CS^* + ({\Pi_1}^* + {\Pi_1}^* + {\Pi_3}^*)$ and using equations (12) and (6) we get

$$W^* = \frac{(\alpha_1-c_1)^2}{8(2-\gamma)^2(1+\gamma)^2}\Big(2(3+\gamma)(2-\gamma\beta)^2+3(2\beta+\gamma\beta-2\gamma)^2+4\gamma(2-\gamma\beta)(2\beta+\gamma\beta-2\gamma)\Big). \quad (14)$$

However, after merger between firm 1 and firm 2 the welfare is $W_{12} = CS_{12} + \Pi_{12} + \Pi_3$ and using equations (13) and (10) we get

$$W_{12} = \frac{(\alpha_1-c_1)^2}{32(2-3\gamma^2+\gamma^3)^2}\Big(2(1+\gamma)\big[(4-4\gamma)+\beta(2\gamma^2-2\gamma)\big]^2+3\big[\beta(4-4\gamma^2)+2(2\gamma^2-2\gamma)\big]^2 + 4\gamma\big[(4-4\gamma)+\beta(2\gamma^2-2\gamma)\big]\big[\beta(4-4\gamma^2)+2(2\gamma^2-2\gamma)\big]\Big) + \Big[\frac{(1+\gamma)(2-\gamma\beta)^2}{2(2+2\gamma-\gamma^2)^2}\Big](\alpha_1-c_1)^2 \quad (15)$$

for zone B_1 of Figure 1 where merger between firm 1 and firm 2 is possible. Using equations (14) and (15), we see that $W^* > W_{12}$. Hence, welfare always reduces after merger in the region B_1 of Figure 1 where merger between firm 1 and firm 2 is possible. The profit of the merged firms and the non-merged firm always increase after merger. But the increase in the industry profit can't compensate the loss of consumer surplus. Thus welfare falls if firm 1 and firm 2 merge.

Proposition 2 *After the merger consumer surplus and welfare always fall if firm 1 and firm 2 merge.*

4 Merger between firm 1 and firm 3

This section studies the possibility of merger between firm 1 that produces the better quality product and firm 3 who produces the lower quality product. This is equivalent to the merger between firm 2 and firm 3. After merger, the profit function of the merged firms is given by $\Pi_{13} = (\alpha_1 - c_1 - q_1 - \gamma q_2 -$

$\gamma q_3)q_1+(\alpha_3-c_3-q_3-\gamma q_1-\gamma q_2)q_3$. The profit function of firm 2 is $\Pi_2=(\alpha_2-c_2-q_2-\gamma q_1-\gamma q_3)q_2$. The first-order conditions for profit maximization give us the following reaction functions for the merged firms and firm 2 as

$$q_1=\frac{\alpha_1-2\gamma q_3-\gamma q_2-c_1}{2},\quad q_3=\frac{\alpha_3-2\gamma q_1-\gamma q_2-c_3}{2},\ and\ \ q_2=\frac{\alpha_2-\gamma q_1-\gamma q_3-c_2}{2} \tag{16}$$

respectively and zero otherwise. The merged firms can produce both the goods ($q_1>0$, $q_3>0$) or can stop the production of good 3 and produce only good 1 ($q_1>0$ and $q_3=0$) depending on the profitability of each action.

4.1 Good 3 is produced

We observe from the first-order condition of the merged firms and firm 2 that the outputs produced (given $q_3>0$) will be as follows:

$$\begin{aligned}
q_1 &= \frac{(\alpha_1-c_1)(4-2\gamma+\gamma^2)+(\alpha_3-c_3)(\gamma^2-4\gamma)}{4(2-3\gamma^2+\gamma^3)}(>0),\\
q_3 &= \frac{(\alpha_3-c_3)(4-\gamma^2)+(\alpha_1-c_1)(3\gamma^2-6\gamma)}{4(2-3\gamma^2+\gamma^3)}(<q_3^*)\ \ and\\
q_2 &= \frac{(\alpha_1-c_1)(4-2\gamma-2\gamma^2)+(\alpha_3-c_3)(2\gamma^2-2\gamma)}{4(2-3\gamma^2+\gamma^3)}(>q_2^*).
\end{aligned} \tag{17}$$

Hence, the merged firms will produce good 3 ($q_3>0$) if and only if

$$(4-\gamma^2)\beta>6\gamma-3\gamma^2. \tag{18}$$

The above inequality holds, i.e. good 3 is produced by the merged firms in zone A_1 in Figure 2. Otherwise, in the zone, A_2 good 3 will not be produced. Moreover, in the zone, A_1, the merged firms reduce the production level of good 3 than in the no-merger case. Post-merger the production of good 1 may either increase or decrease in zone A_1. In zone A_1 for every γ, if β is relatively high (quality difference net of cost is low), then the production of good 1 reduces or else it increases. Moreover, in the zone, A_1, the total output produced by the merged firms (good 1 + good 3) is lower than the output produced in the pre-merger stage. Therefore, the production of good 2 always increases in the post-merger stage in zone A_1.

The profits earned given condition (18) holds, (i.e. when $q_3>0$) by the merged firms and firm 2 are

$$\begin{aligned}
\Pi_{13}=\frac{1}{Z}\Big(&\big[(\alpha_1-c_1)(3\gamma^3-5\gamma^2-2\gamma+4)-(\alpha_3-c_3)(\gamma^3-\gamma^2)\big]\big[(\alpha_1-c_1)(4+\gamma^2-2\gamma)+(\alpha_3-c_3)(\gamma^2-4\gamma)\big]\\
&+\big[(\alpha_3-c_3)(\gamma^3-5\gamma^2+4)-(\alpha_1-c_1)(2\gamma-\gamma^3-\gamma^2)\big]\big[(\alpha_3-c_3)(4-\gamma^2)+(\alpha_1-c_1)(3\gamma^2-6\gamma)\big]\Big)\\
&and\ \ \Pi_2=(q_2)^2=\Big[\frac{(\alpha_1-c_1)(4-2\gamma-2\gamma^2)+(\alpha_3-c_3)(2\gamma^2-2\gamma)}{4(2-3\gamma^2+\gamma^3)}\Big]^2
\end{aligned} \tag{19}$$

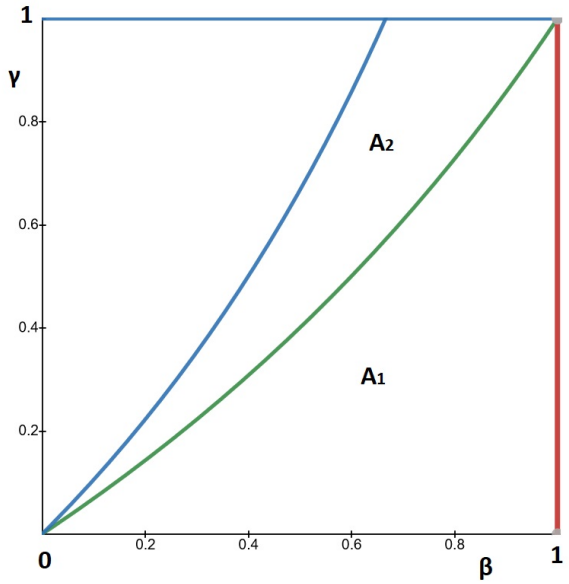

Figure 2: Production of good 3 if firm 1 and firm 3 merge

respectively, where $Z = 16(2-3\gamma^2+\gamma^3)^2$. Therefore, in this context merger is possible if $\Pi_{13} > \Pi_1{}^* + \Pi_3{}^*$ or

$$\begin{aligned} 4(2-\gamma)^2(\gamma+1)^2\Big(&\big[(3\gamma^3-5\gamma^2-2\gamma+4)-\beta(\gamma^3-\gamma^2)\big]\times\big[(4+\gamma^2-2\gamma)+\beta(\gamma^2-4\gamma)\big] \\ &+\big[\beta(\gamma^3-5\gamma^2+4)-(2\gamma-\gamma^3-\gamma^2)\big]\times\big[\beta(4-\gamma^2)+(3\gamma^2-6\gamma)\big]\Big) \\ &> 16(2-3\gamma^2+\gamma^3)^2\Big(\big[(2-\gamma\beta)\big]^2+\big[(2+\gamma)\beta-2\gamma\big]^2\Big). \end{aligned} \tag{20}$$

The above inequality holds in the region L_1 of Figure 3, which is a subset of zone A_1 of the previous diagram (see Figure 2).

Lemma 2 *Merger between firm 1 and firm 3 given $q_3 > 0$ is possible in zone L_1 of Figure 3.*

Merger between firm 1 and firm 3 is possible only in zone L_1, but not in zone L_2 of Figure 3 when $q_3 > 0$. In zone L_2 merger is not possible as β and γ are high or the quality difference (net of cost), as well as horizontal product differentiation, are low. Post-merger the total amount of good 1 and good 3 produced by the merged firms decreases (first effect), which is conducive to its profit. However, firm 2 reacts by producing higher output (compared to the no-merger case). This reduces the profit of the merged firms. In the zone, L_2 as the horizontal product differentiation is low (γ is very high) and quality difference net of cost is low (β is high), thus the latter effect dominates and the profit of the merged firms does not increase after the merger. In contrast, if β is low (quality difference net of cost is high) irrespective of the degree of horizontal product differentiation, the merger is profitable as the second effect becomes weaker.

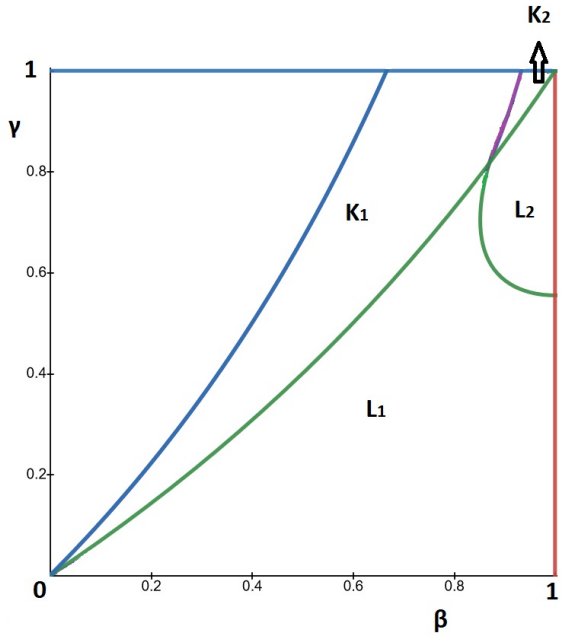

Figure 3: Possibility of merger between firm 1 and firm 3

4.2 Good 3 is not produced

The merged firms will not produce good 3, i.e. $q_3 = 0$, in zone A_2 of Figure 2. Under such circumstances the output produced by the merged firms and firm 3 are

$$q_1^0 = q_2^0 = \frac{(2-\gamma)(\alpha_1 - c_1)}{4-\gamma^2} (> q_1^* = q_2^* > 0). \tag{21}$$

Contrarily, post-merger the production of good 1 increases in zone A_2 (as good 3 is not produced). Moreover, in the zone, A_2 the total output produced by the merged firms (good 1 + good 3) is lower than the output produced in the no-merger case. Therefore, the production of good 2 always increases in the post-merger stage in zone A_2.

The profits earned by the merged firms and firm 3 are

$$\Pi_{13}^0 = (q_1^0)^2 \quad and \quad \Pi_2^0 = (q_2^0)^2 \tag{22}$$

respectively. Hence, merger is possible if $\Pi_{13}^0 > {\Pi_1}^* + {\Pi_3}^*$ or

$$\left(\frac{2-\gamma}{4-\gamma^2}\right)^2 > \left[\frac{(2-\gamma\beta)}{2(2-\gamma)(\gamma+1)}\right]^2 + \left[\frac{(2+\gamma)\beta - 2\gamma}{2(2-\gamma)(\gamma+1)}\right]^2. \tag{23}$$

The above inequality holds in the region K_1 of Figure 3 which is a subset of zone A_2 of the previous diagram (see Figure 2).

Lemma 3 *Merger between firm 1 and firm 3 given $q_3 = 0$ is possible in zone K_1 of Figure 3.*

Merger is possible in zone K_1, but not in zone K_2 of Figure 3. As discussed before post-merger the total amount of good 1 and good 3 produced by the merged firms decreases (first effect), and firm 2 reacts by producing higher output (second effect). The second effect results in a lower profit for the merged firms while the first effect increases the profit of the merged firms as the inter-firm rivalry reduces. In zone K_2 where β is relatively high (close to 1) or the quality difference (net of cost) is low, the total pre-merger profit of firm 1 and firm 3 ($\Pi_1^* + \Pi_3^*$) increases in β. Then the increase in profit of the merged firm (firm 1 and firm 3) cannot compensate for the loss of profits when good 3 is not produced post-merger as $\Pi_1^* + \Pi_3^*$ is very high. The post-merger profit of the merged firm (firm 1 and firm 3) is less as firm 2 increases the supply of good 2 in a higher amount. This thereby obstructs the merger of firm 1 and firm 3. On the other hand, in the zone, K_1, β is relatively less or the quality difference (net of cost) is high and thus the total output (good 1 + good 3) in the no-merger case is less. Therefore, the reduction in the total output (good 1 + good 3) of the merged firm is less and thus the increase in the output of firm 2 is also less. Thus, in the zone, K_1 the firms that produce different quality products (firm 1 and firm 3) will merge as the second effect becomes weak and the profit of the merged firms increases. In other words, we can also say that when the quality difference is large then stopping the production of good 3 gets compensated easily by the increase in the quantity of good 1.

4.3 Result

Therefore, from Lemma 2 and Lemma 3, we can say that merger between firm 1 and firm 3 is possible in zone L_1 and in zone K_1 of Figure 3. This shows that the merger of firms producing different quality products is possible if either the horizontal product differentiation is high or the quality difference (net of cost) is high or both.

Proposition 3 *The merger of firms producing different quality products is possible if either the horizontal product differentiation is high or the quality difference (net of cost) is high or both.*

This result is in contrast to Sen and Narula (2021) as we show that firm 1 may merge with firm 3 which produces a lower quality product. In Sen and Narula (2021) the merger between the firm that produces the higher-quality product and the other firm next in the quality ranking is possible if either the horizontal product differentiation or the quality difference (net of cost) is high, but the merger between the firms that (both) produce the lower-quality products is never possible. This is true in Sen and Narula (2021), because in their set-up firm 1 produces the better quality product, whereas firm 2 and firm 3 both produce a lower quality product. Thus, if firm 2 and firm 3 merge they reduce the production of good 2 and good 3, while the quantity of good 1 produced by firm 1 increases in comparison to the no-merger situation. The increase in the production of good 1 by firm 1 reduces the profit of the merged firm and thus the merger of firm 2 and firm 3 is not possible as post-merger the joint profit falls. Thus, merger of firms producing lower quality products is never possible. Interestingly, in the present paper as discussed in the previous paragraphs, we show that as in our set-up firm 1 and firm 2 produce a better quality product and firm 3 produces a lower quality product, merger of firm 1 and firm 3 is possible here.

4.4 Welfare analysis

After merger between firm 1 and firm 3 if good 3 is produced post merger the consumer surplus is $CS_{13} = \frac{1}{2}[(q_1)^2 + (q_2)^2 + (q_3)^2 + 2\gamma q_1 q_2 + 2\gamma q_1 q_3 + 2\gamma q_2 q_3]$, and using equation (17), we get

$$\begin{aligned} CS_{13} = \frac{(\alpha_1 - c_1)^2}{32(2-3\gamma^2+\gamma^3)^2}\Big(& \big[(4-2\gamma+\gamma^2)+\beta(\gamma^2-4\gamma)\big]^2 + \big[(4-2\gamma-2\gamma^2)+\beta(2\gamma^2-2\gamma)\big]^2 \\ & + \big[\beta(4-\gamma^2)+(3\gamma^2-6\gamma)\big]^2 \\ & + 2\gamma\big[(4-2\gamma+\gamma^2)+\beta(\gamma^2-4\gamma)\big]\big[(4-2\gamma-2\gamma^2)+\beta(2\gamma^2-2\gamma)\big] \\ & + 2\gamma\big[(4-2\gamma+\gamma^2)+\beta(\gamma^2-4\gamma)\big]\big[\beta(4-\gamma^2)+(3\gamma^2-6\gamma)\big] \\ & + 2\gamma\big[(4-2\gamma-2\gamma^2)+\beta(2\gamma^2-2\gamma)\big]\big[\beta(4-\gamma^2)+(3\gamma^2-6\gamma)\big]\Big). \end{aligned} \tag{24}$$

However, after the merger between firm 1 and firm 3 if good 3 is not produced post-merger the consumer surplus and using equation (21) is

$$CS_{13} = \frac{1}{2}[(q_1^0)^2 + (q_2^0)^2 + 2\gamma q_1^0 q_2^0] = \frac{(\alpha_1 - c_1)^2(1+\gamma)}{(2+\gamma)^2}. \tag{25}$$

Moreover, i) using equations (12) and (24), we found that $CS^* > CS_{13}$ in the region L_1 of Figure 3 where merger between firm 1 and firm 3 is possible and post-merger good 3 is produced and ii) using equations (12) and (25), we found that $CS^* > CS_{13}$ in the region K_1 of Figure 3 where merger between firm 1 and firm 3 is possible and post-merger good 3 is not produced. Therefore, after merger the consumer surplus always falls. Post-merger consumer surplus always falls irrespective of whether good 3 is produced or not in the post-merger stage. Post-merger the amount of good 1 produced by the merged firms increases (if good 3 is not produced) in comparison to the no-merger case. However, the total output (good 1 + good 3) produced by the merged firms falls post-merger. Even though post-merger firm 2 produces a higher output, the consumer surplus falls. If good 3 is produced post-merger, then also consumer surplus falls.

After merger between firm 1 and firm 3 such that good 3 is produced the welfare is $W_{13} = CS_{13} + \Pi_{13} + \Pi_2$. Using equations (24) and (19) we get

$$\begin{aligned} W_{13} = \frac{(\alpha_1 - c_1)^2}{32(2-3\gamma^2+\gamma^3)^2}\Big(& \big[(4-2\gamma+\gamma^2)+\beta(\gamma^2-4\gamma)\big]^2 + 3\big[(4-2\gamma-2\gamma^2)+\beta(2\gamma^2-2\gamma)\big]^2 \\ & + \big[\beta(4-\gamma^2)+(3\gamma^2-6\gamma)\big]^2 \\ & + 2\gamma\big[(4-2\gamma+\gamma^2)+\beta(\gamma^2-4\gamma)\big]\big[(4-2\gamma-2\gamma^2)+\beta(2\gamma^2-2\gamma)\big] \\ & + 2\gamma\big[(4-2\gamma+\gamma^2)+\beta(\gamma^2-4\gamma)\big]\big[\beta(4-\gamma^2)+(3\gamma^2-6\gamma)\big] \\ & + 2\gamma\big[(4-2\gamma-2\gamma^2)+\beta(2\gamma^2-2\gamma)\big]\big[\beta(4-\gamma^2)+(3\gamma^2-6\gamma)\big]\Big) \\ + \frac{(\alpha_1 - c_1)^2}{16(2-3\gamma^2+\gamma^3)^2}\Big(& \big[(3\gamma^3-5\gamma^2-2\gamma+4)-\beta(\gamma^3-\gamma^2)\big] \times \big[(4+\gamma^2-2\gamma)+\beta(\gamma^2-4\gamma)\big] \\ & + \big[\beta(\gamma^3-5\gamma^2+4)-(2\gamma-\gamma^3-\gamma^2)\big] \times \big[\beta(4-\gamma^2)+(3\gamma^2-6\gamma)\big]\Big) \end{aligned} \tag{26}$$

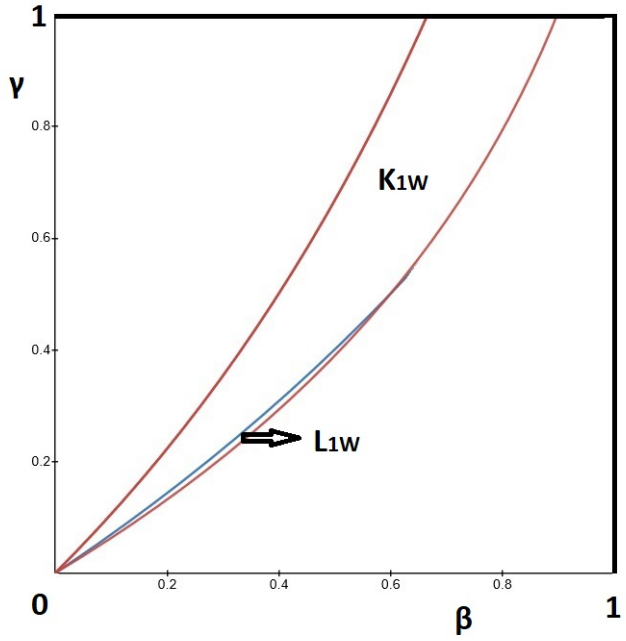

Figure 4: Welfare analysis

in the region L_1 of Figure 3 where merger between firm 1 and firm 3 is possible and post-merger good 3 is produced. After merger between firm 1 and firm 3 such that good 3 is not produced the welfare is $W_{13} = CS_{13} + \Pi_{13} + \Pi_2$ and using equations (25) and (22) we get

$$W_{13} = \frac{(\alpha_1 - c_1)^2(3+\gamma)}{(2+\gamma)^2} \tag{27}$$

in the region K_1 of Figure 3 where merger between firm 1 and firm 3 is possible and post-merger good 3 is not produced.

Moreover, i) if firm 1 and firm 3 merge and good 3 is not produced then using equations (14) and (27) we observe that welfare increases ($W_{13} > W^*$) in the region K_{1W} of Figure 4, where the region K_{1W} is a subset of region K_1 of Figure 3 and ii) if firm 1 and firm 3 merge and good 3 is produced then using equations (14) and (26) we observe that welfare increases ($W_{13} > W^*$) in the region L_{1W} of Figure 4, where the region L_{1W} is a subset of region L_1 of Figure 3. Therefore, from Figure 4 we can say that for any γ if β is low, which means that the quality difference (net of cost) is high, then the increase in the industry profits can compensate for the fall in the consumers surplus. Thus welfare increases.

Proposition 4 *Consumer surplus always reduces post merger, but if the quality difference (net of cost) is high and firm 1 and firm 3 merge then welfare will increase.*

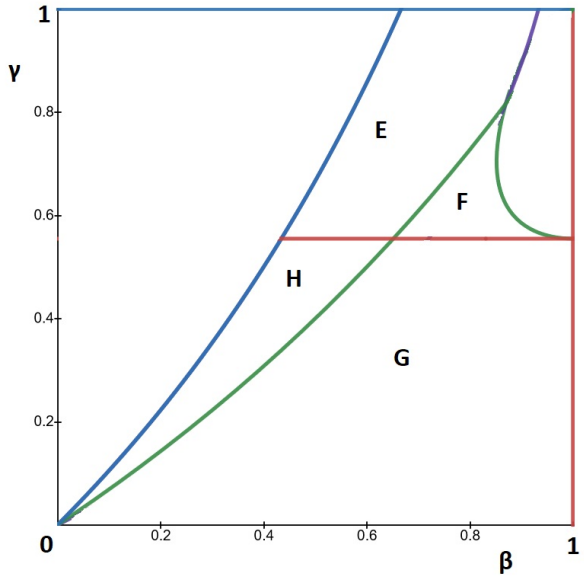

Figure 5: Overlap

5 Comparison

Interestingly, from Lemma 1, Lemma 2 and Lemma 3, we observe that in the region G and H of Figure 5, merger of firm 1 and firm 2 as well as merger of firm 1 and firm 3 are possible. Zone G is a subset of zone B_1 and L_1 of the previous diagrams (see Figure 1 and Figure 5 respectively for zone B_1 and L_1) and zone H is a subset of zone B_1 and K_1 of the previous diagrams (see Figure 1 and Figure 5 respectively for zone B_1 and K_1). However, in the region E and F of Figure 5 only merger of firm 1 and firm 3 is possible

Lemma 4 *In zones G and H of Figure 5 merger between firm 1 and firm 2 as well as merger between firm 1 and firm 3 are possible.*

In the zones where the merger between firm 1 and firm 2 as well as the merger between firm 1 and firm 3 are possible, we compare the surplus the merged firm acquires to identify whether firm 1 and firm 2 will merge or firm 1 and firm 3 will merger. The merger that generates greater surplus in these scenarios will only take place and the other type of merger will not occur in equilibrium.[10] Hence, in the following discussion we compare the surplus in the two scenarios.

The surplus if firm 1 and firm 2 merge, using equation (6) and equation (10), is

$$\Pi_{12} - ({\Pi_1}^* + {\Pi_2}^*) = \Big[\frac{(1+\gamma)(2-\gamma\beta)^2}{2(2+2\gamma-\gamma^2)^2} - \frac{(2-\gamma\beta)^2}{2(2-\gamma)^2(1+\gamma)^2}\Big](\alpha_1 - c_1)^2. \tag{28}$$

[10]It is also to be noted in this context that merger between firm 1 and firm 3 and merger of firm 2 and firm 3 are equivalent to each other.

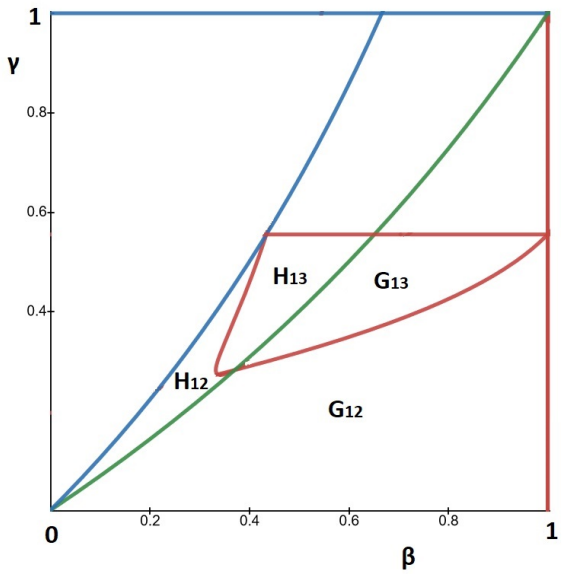

Figure 6: Type of merger

On the other hand, the surplus if firm 1 and firm 3 merge such that good 3 is produced, using equation (6) and equation (19), is

$$\Pi_{13} - ({\Pi_1}^* + {\Pi_3}^*) = \frac{(\alpha_1 - c_1)^2}{Z}\Big(\big[(3\gamma^3 - 5\gamma^2 - 2\gamma + 4) - \beta(\gamma^3 - \gamma^2)\big]\big[(4 + \gamma^2 - 2\gamma) + \beta(\gamma^2 - 4\gamma)\big] + \big[\beta(\gamma^3 - 5\gamma^2 + 4) - (2\gamma - \gamma^3 - \gamma^2)\big]\big[\beta(4 - \gamma^2) + (3\gamma^2 - 6\gamma)\big]\Big) - \frac{(\alpha_1 - c_1)^2\Big[(2 - \gamma\beta)^2 + (2\beta + \gamma\beta - 2\gamma)^2\Big]}{4(2 - \gamma)^2(1 + \gamma)^2}, \quad (29)$$

where $Z = 16(2 - 3\gamma^2 + \gamma^3)^2$. However, the surplus if firm 1 and firm 3 merge when good 3 is not produced, using equation (6) and equation (22), is

$$\Pi_{13}^0 - ({\Pi_1}^* + {\Pi_3}^*) = \frac{(\alpha_1 - c_1)^2}{(2 + \gamma)^2} - \frac{(\alpha_1 - c_1)^2\Big[(2 - \gamma\beta)^2 + (2\beta + \gamma\beta - 2\gamma)^2\Big]}{4(2 - \gamma)^2(1 + \gamma)^2}. \quad (30)$$

Now, i) in zone G_{12} of Figure 6, after comparing equation (28) and equation (29) we observe that the surplus is more if firm 1 and firm 2 merge, i.e. $\Pi_{12} - ({\Pi_1}^* + {\Pi_2}^*) > \Pi_{13} - ({\Pi_1}^* + {\Pi_3}^*)$,

ii) but in zone G_{13} the surplus is more if firm 1 and firm 3 merge, i.e. $\Pi_{12} - ({\Pi_1}^* + {\Pi_2}^*) < \Pi_{13} - ({\Pi_1}^* + {\Pi_3}^*)$,

iii) in zone H_{12} of Figure 6 after comparing equation (28) and equation (30), we observe that the surplus is more if firm 1 and firm 2 merge, i.e. $\Pi_{12} - ({\Pi_1}^* + {\Pi_2}^*) > \Pi_{13}^0 - ({\Pi_1}^* + {\Pi_3}^*)$ and

iv) but in zone H_{13} the surplus is more if firm 1 and firm 3 merge, i.e. $\Pi_{12} - ({\Pi_1}^* + {\Pi_2}^*) < \Pi^0_{13} - ({\Pi_1}^* + {\Pi_3}^*)$.

Therefore, after comparing the surplus we conclude that in zone G_{12} as well as in zone H_{12} of Figure 6, firm 1 and firm 2 will merge, as they acquire higher surplus in comparison to what firm 1 and firm 3 get after the merger. On the other hand, in zone G_{13} as well as in zone H_{13}, firm 1 and firm 3 will merge as it generates a higher surplus.

Lemma 5 *In zone G_{12} as well as in zone H_{12} of Figure 6, firm 1 and firm 2 will merge, whereas in zone G_{13} as well as in zone H_{13}, firm 1 and firm 3 will merge.*

Therefore, in zone G_{13} as well as in zone H_{13} of Figure 6 for firm 1 it is better to merge with the firm 3 which produces a lower quality product than to merge with firm 2 that produces the better quality product. In these zones both γ and β are relatively high. This means that if the horizontal product differentiation, as well as quality difference (net of cost), are less then firm 1 will merge with firm 3, otherwise firm 1 will merge with firm 2. If γ and β are high, then the impact of the increase of output by firm 3 on the post-merger profit of firm 1 and firm 2 is larger. The increase in the output by firm 3 reduces the profit of the merged firms (firm 1 and firm 2) and reduces their surplus. Contrarily, when either γ or β or both are low then the effect of the increase of output by firm 3 on the profit of the merged firms becomes weak. Therefore, the surplus in zone G_{12} as well as in zone H_{12} is more when firm 1 and firm 2 merge. Thus, it is better to merge with firm 2 than to merge with firm 3, if either the horizontal product differentiation or the quality difference (net of cost) or both are high as shown in zone G_{12} and zone H_{12}.

Proposition 5
i) For firm 1 it is better to merge with the firm that produces the lower quality product than to merge with the firm that produces the better quality product, if the horizontal product differentiation and quality difference (net of cost) are relatively low.

ii) Otherwise, it is better for firm 1 to merge with the firm that produces the better quality product, if either the horizontal product differentiation or the quality difference (net of cost) or both are high.

Finally, we conclude using the previous propositions that in zones G_{12} and zone H_{12} of Figure 5, firm 1 and firm 2 will merge. Thus, the merger of two firms that produce better quality products is possible, if the quality difference (net of cost) or the horizontal product differentiation or both are high. On the other hand, in the regions E, F, G_{13} and H_{13} of Figure 5 firm 1 and firm 3 will merge. Thus, if the quality difference (net of cost) or the horizontal product differentiation is neither too high nor too low, then the firm that produces the better quality product will merge with the firm that produces the lower quality product. Otherwise, the merger is never possible. The final proposition discusses the types of merger.

Proposition 6
i) The merger of two firms that produce two different higher quality products is possible if the quality difference (net of cost) or the horizontal product differentiation are high.

ii) If the quality difference (net of cost) or the horizontal product differentiation is neither too high nor too low, then a firm producing higher quality product will merge with another firm that produces lower quality product.

6 Conclusion

This paper discusses the possibility of a two firm merger in a setup of three firms with both horizontal and vertical product differentiation (quality difference). We show that not only the firms that produce the better quality products may merge but also the firms that produce two different quality products may merge. It is shown that the merger of two firms that produce better-quality products is possible if the quality difference (net of cost) or the horizontal product differentiation are high. Moreover, the merger of firms producing two different quality products is possible if either the horizontal product differentiation is high or the quality difference (net of cost) is high or both. We also show that the low-quality product may be withdrawn if the firms that produce different quality products merge. Considering the degree of horizontal as well as quality difference (net of cost), there are cases where both i) the merger of firms producing the same quality products as well as ii) the merger of firms producing different qualities are possible. In such cases we show that if the quality difference (net of cost) and the horizontal product differentiation are neither too high nor too low, then the firm that produces the better quality product will merge with the firm that produces a lower quality product, otherwise, it will merge with the better quality producer. Consumer surplus always reduces due to merger of any two firms, but if the quality difference (net of cost) is high and the two firms that produce different quality products merge then welfare will increase. However, if the two firms that produce the better quality products merge, then welfare always reduces post-merger.

The objective of the antitrust authority is to scrutinise the proposal of merger and block them if it is welfare-reducing. In our model, if the two high-quality producing firms merge, welfare is inevitably reduced. Moreover, in our model, the firms that sell the high-quality products have a higher market share than the third firm which sells the lower-quality product. Therefore, the antitrust authority can prevent such proposals of merger by a simple policy that firms with a larger market share cannot merge in any industry. On the other hand, if the firm that produces the high-quality product merges with the firm that sells the low-quality product, then the welfare would increase if the quality difference (net of cost) is high even though the consumer surplus falls. Hence, the antitrust authority can permit such mergers between a firm having a higher market share and another firm that has a lower market share.

In the present paper, we have assumed that both in the pre-merger stage as well as in the post-merger stage the firms compete in quantities. Thus, as a possible extension of this work, it is important to re-examine our finding if the firms compete in prices, both in the pre-merger stage as well as in the post-merger stage. The linear demand system as used in our analysis is standard in the literature and the reason for this is the analytical convenience. Thus, as a possible extension of this paper, the current analysis can be conducted in some of the frameworks that use a non-linear set up.

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