Non-linear regret: Menu effects in risk attitudes and behavioral anomalies

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Abstract

We introduce a decision theoretic model that captures regret. Our model features an environment of uncertainty. Regret influences behavior when ex ante, the decision maker's choice of which gamble to choose is influenced by a concern that, ex post, once uncertainty resolves, her choice may turn out to be an inferior one. We behaviorally characterize the model. We also highlight "behavioral anomalies" and menu effects in risk attitudes that result because of a concern about regret.

JEL codes: D81, D91

Keywords: regret, menu effect, menu-dependent risk attitudes, behavioral anomalies

1 Introduction

This paper introduces a decision theoretic model of regret. In the model, regret arises from the fact that our decision maker (DM) lives in an uncertain world. As such, when choosing acts or gambles from a menu ex ante, she does so with the concern that, ex post, once uncertainty has resolved, she may regret her choice. This is because in the state that realizes, some other act that she could have chosen but did not is more attractive. We use our model to throw further light on menu effects and non-standard patterns in risk attitudes owing to regret.

To illustrate this concretely, consider an investor Cathie who has to decide whether to invest an amount of $10,000 for a period of two years in Crypto or government bonds. Assume that two year government bonds are risk-free and give a pre-determined amount

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on maturity. On the other hand, the return on her investment in Crypto depends on US Fed policy and interest rates over the next two years about which there is substantial uncertainty. Let L(ow) denote the state that the yield on 10 year US T-bills stays below 3% and H(igh) the state that it is above 3%. Suppose the probabilities that can be reasonably assigned to these states are 0.3 and 0.7, respectively. Further, assume that the return on crypto goes in the opposite direction as yields. As such, return on crypto is higher than that on bonds in state L, but the opposite is true in state H. Given this, at the time of investing, Cathie may be subject to the fear that if she did choose to invest in bonds and state L realizes then, ex post, she may experience the unpleasant emotion of missing out on the higher return on crypto. At the same time, if she were to invest in crypto, then regret may arise with respect to state H.

To understand how our model proposes Cathie will decide, assume that payoffs resulting from the initial amount of \$10,000 under the two investments in each of the states is as shown below.

	Low	**High**
Crypto	17000	8000
Bond	11000	11000

Table 1

Let u denote Cathie's utility function over money and for concreteness assume that $u(x) = \sqrt{x}$. Under our model, she evaluates the two investments based on not just their expected utility but also a non-linear regret cost that captures the utility loss she experiences whenever, ex post, her choice turns out to be the inferior one. Denote by ϕ the regret cost function and suppose $\phi(y) = y^2$. Then her assessment of the two investments in this two alternative menu according to our model is given by:

$$
\begin{aligned}
v(\text{Crypto}) &= \overbrace{0.3u(17000) + 0.7u(8000)}^{\textit{Expected Utility}} - \overbrace{0.7\phi\big(u(11000) - u(8000)\big)}^{\textit{Regret cost}} \\
&= 0.3\sqrt{17000} + 0.7\sqrt{8000} - 0.7\big(\sqrt{11000} - \sqrt{8000}\big)^2 \\
&= 101.73 - 166.83 = -65.11 \\
v(\text{Bond}) &= \overbrace{0.3u(11000) + 0.7u(11000)}^{\textit{Expected Utility}} - \overbrace{0.3\phi\big(u(17000) - u(11000)\big)}^{\textit{Regret cost}} \\
&= 0.3\sqrt{11000} + 0.7\sqrt{11000} - 0.3\big(\sqrt{17000} - \sqrt{11000}\big)^2 \\
&= 104.88 - 195.12 = -90.24
\end{aligned}
$$

Therefore, Cathie ends up investing in crypto. Clearly regret has a bite for Cathie, for if she were to assess these investments as an expected utility maximizer, as can be verified from the calculations above, she would have chosen to invest in government bonds. Instead, she goes for the choice of crypto as it gives a disproportionately high reward in comparison to bonds, even though state L is less likely to occur.

Now suppose along with Crypto and Bonds, Cathie also has the option to invest in a portfolio of Bank Stocks. Further assume that return on bank stocks goes in the same direction as the yield on the US T-bills. That is, Bank Stocks give a higher return in state H than in State L.

	Low	**High**
Crypto	17000	8000
Bonds	11000	11000
Bank Stocks	10000	13000

Table 2

Continuing with the functional forms from above, under our model, Cathie's assessment of the options in this three-alternative menu is given by:

$$
\begin{aligned}
v(\text{Crypto}) &= \overbrace{0.3u(17000) + 0.7u(8000)}^{\textit{Expected Utility}} - \overbrace{0.7\phi\big(u(13000) - u(8000)\big)}^{\textit{Regret cost}} \\
&= 0.3\sqrt{17000} + 0.7\sqrt{8000} - 0.7\big(\sqrt{13000} - \sqrt{8000}\big)^2 \\
&= 101.73 - 422.74 = -321.02 \\
v(\text{Bond}) &= \overbrace{0.3u(11000) + 0.7u(11000)}^{\textit{Expected Utility}} - \overbrace{0.3\phi\big(u(17000) - u(11000)\big)}^{\textit{Regret cost}} \\
&\quad - \overbrace{0.7\phi\big(u(13000) - u(11000)\big)}^{\textit{Regret cost}} \\
&= 0.3\sqrt{11000} + 0.7\sqrt{11000} - 0.3\big(\sqrt{17000} - \sqrt{11000}\big)^2 \\
&\quad -0.7\big(\sqrt{13000} - \sqrt{11000}\big)^2 \\
&= 104.88 - 253.55 = -148.67 \\
v(\text{Bank Stocks}) &= \overbrace{0.3u(10000) + 0.7u(13000)}^{\textit{Expected Utility}} - \overbrace{0.3\phi\big(u(17000) - u(10000)\big)}^{\textit{Regret cost}} \\
&= 0.3\sqrt{10000} + 0.7\sqrt{13000} - 0.3\big(\sqrt{17000} - \sqrt{10000}\big)^2 \\
&= 109.81 - 276.95 = -167.14
\end{aligned}
$$

In other words, Cathie ends up investing in the safe option of Bonds. This highlights the menu dependence in her choice of risky alternatives as, in the absence of bank stocks, she was willing to invest in the riskier but potentially high reward giving crypto, but after the introduction of bank stocks her choice switches to the risk-free alternative of bonds.

More generally, we consider a world with n states, denoted $1, \dots, n$, whose probabilities are given by $\pi(1), \dots, \pi(n)$, respectively. An act, $f = (f_1, \dots, f_n)$, is a vector that specifies for each state s, the outcome f_s that f gives in that state. Faced with any menu A of such acts, the DM's choice is captured by the choice correspondence

$$c(A) = \underset{f\in A}{\operatorname{argmax}} \sum_{s=1}^{n} \pi(s) \Big[u(f_s) - \phi\big(\max_{g\in A} u(g_s) - u(f_s) \big) \Big]$$

Further, when comparing between menus, her evaluation of any such menu A is given by

$$U\big(A\big) = \max_{f} \left(\sum_{s=1}^{n} \pi(s) \Big[u(f_s) - \phi\big(\max_{g\in A} u(g_s) - u(f_s) \big) \Big] \right)$$

Taking as primitives, the DM's preferences over lotteries of such menus and her choice correspondence over menus, the main theoretical task that we accomplish in this paper is to provide a behavioral foundation of the non-linear regret model. Besides this, we also highlight the empirical content of the model. We highlight how regret may influence a DM's risk attitudes. Specifically, we show that regret may introduce a long shot bias in her behavior, and may result in her risk-attitudes being menu-dependent. In addition, regret may make her prefer mean preserving spreads and may result in her behavior violating first order stochastic dominance.

The rest of the paper is organized as follows. In the next section, we point out some connections to the literature. In Section 3, we state the primitives. Thereafter, in Section 4, we formally define a non-linear regret representation. Section 5 provides a behavioral foundation for the model. Section 6 highlights empirical content of the model. Proofs of results appear in the Appendix.

2 Related literature

The idea of ex-post inferior outcomes affecting the DM's consideration of acts ex-ante has been established in the regret theory literature. Classic models of regret like Loomes and Sugden (1982)[1] and Bell (1982) introduced the notion that utility of an act can be affected by what would have happened if some other act had been chosen, under a scenario where the actual state of the world is unknown. Many other papers[2] extend the framework laid down by Loomes and Sugden (1982). Most of these papers take preference over acts or lotteries as the primitive and then go on to show how anticipated regret can affect these preferences and the subsequent choice from a given set of alternatives. We, however,

[1]Bleichrodt and Wakker (2015) provides a detailed review of Loomes and Sugden (1982).

[2]Quiggin (1994) provides a framework which extends the scope of Loomes and Sugden (1982) from pairwise choices to general choice sets. Loomes and Sugden (1987) provides implications of regret theory in terms of non-transitive pairwise choices, Sugden (1993) axiomatizes regret theory by dropping the transitivity axiom.

take a different approach that looks at preferences over (lottery of) menus containing various acts,[3] and how anticipated ex-post regret affects agents' assessment of these menus. Anticipated ex-post regret, thus, affects the choice of the menu and, subsequently, also the act which is finally chosen.

To begin with, we point out a key difference between our model of regret and prominent axiomatic models of regret like those in Sarver (2008) and Hayashi (2008), among others. In most such models, the regret cost of choosing an ex-post inferior act is assessed before the final level of uncertainty is resolved. That is, the cost of regret in these models is calculated as the difference between the 'expected utilities' of the chosen act and the best act, instead of the 'prizes or outcomes' that these acts end up realizing. To make things clearer, consider the following scenario: suppose there are two equally likely events and two acts f and g where act f gives the degenerate lottery of 5000 in both the events and act g gives the degenerate lottery of 5000 in the first event and a lottery of the form $(10000, 0.5; 0, 0.5)$ in the second event. Further assume that the utility u over outcomes is linear. Now as per models which take the cost of regret as the difference between expected utilities of lotteries, there will not be any regret cost while evaluating the choice between these two acts as the expected utility of both these acts in either of the events is 5000. However, if the agents ends up choosing act g, for instance, and event 2 occurs, and the lottery $(10000, 0.5; 0, 0.5)$ ends up giving 0 as the prize, there will be a sense of regret of not choosing f which would have given 5000 for sure. Therefore, in our representation, the regret cost is calculated as the difference between the utilities of the outcome under the chosen act and that under the best act in the state under consideration.

In terms of modeling regret by studying preference over menus, Sarver (2008) comes very close to our approach of analysing ex-post regret. His model, however, considers a linear cost of regret which restricts it from explaining choice anomalies or menu effects in the second-stage choice. Since our model doesn't assume a linear regert cost, the choice correspondence in our model is capable of accommodating violations of WARP. Another point of difference to note from Sarver (2008) is that, in his representation, uncertainty is introduced in the form of subjective probabilistic beliefs that the agent has about her possible ex-post tastes (utility functions). In our representation, the agent has a single utility function and uncertainty is introduced through objective probabilistic beliefs over a given set of possible states, with different acts giving state contingent outcomes.

Hayashi (2008) also presents a model of regret which uses a choice correspondence from a menu of acts as the primitive. He characterizes two representations, one where the agent has multiple priors over the states and the choice from the menu is the act which minimizes the maximum ex-post regret from these priors. The other representation, which is closer to ours, is where the agent has a single prior over states and the choice is determined by

[3] Preferences over menus have been commonly used in the literature on temptation and self-control (Gul and Pesendorfer (2001), Noor and Takeoka (2010), Noor and Takeoka (2015)).

minimizing the expected ex-post regret. Ex-post regret in this representation is a non-linear function of the utility differences of the ex-post best act and the chosen alternative in a state. Our model differs from his model primarily because our DM not only tries to minimize ex-post regret but also maximizes the expected utility of the chosen alternative.

3 Primitives

Let $S = \{1, 2, ..., n\}$ be a finite set of states, with $\pi = (\pi_1, \ldots, \pi_n)$ the probability measure on S. Z is a set of outcomes, with typical elements denoted by x, y, etc. An act f is a mapping, $f : S \to Z$, that specifies an outcome in each state. We denote the outcome that results under an act f in state s by f_s. An act can, accordingly, be specified an an n-tuple, $f = (f_1, \ldots, f_n)$. For any two acts f and g, (f_s, g_{-s}) denotes the act that pays according to f in state s and g in all other states. We abuse notation by denoting the constant act that gives the outcome, say, x in every state by x itself. Specifically, (x_s, y_{-s}) will denote that act that gives x in state s and y in every other state. F denotes the set of all acts and $\mathcal{A}$ the set of all non-empty subsets of F. Any element of $\mathcal{A}$ which we refer to as a menu and denote by A, B, etc. is, therefore, a collection of acts. $\Delta(\mathcal{A})$ denotes the set of all simple lotteries over $\mathcal{A}$, i.e., lotteries with finite support. Typical elements of $\Delta(\mathcal{A})$ are denoted by p, q, etc., and for any such lottery p, $p(A)$ denotes the probability of A realizing under p. For any $\alpha \in [0, 1]$ and menus A and B, the mixture $\alpha A + (1 - \alpha)B \in \Delta(\mathcal{A})$ has the usual interpretation of a lottery which gives A with probability α probability and B with probability $(1 - \alpha)$. We abuse notation by denoting the degenerate lottery that gives the menu, say, A with probability 1 by A itself.

The decision-maker (DM) that we study has preferences, $\succcurlyeq \subseteq \Delta(\mathcal{A}) \times \Delta(\mathcal{A})$, over these lotteries. The symmetric and asymmetric components of $\succcurlyeq$ capturing indifferences and strict preferences, respectively, are denoted by $\sim$ and $\succ$ and defined in the usual way. In addition, $c : \mathcal{A} \to \mathcal{A}$, with $c(A) \subseteq A$, represents a choice correspondence that captures the set of acts from any given menu that the DM is happy choosing. In the way of interpretation, the DM engages in a two-stage decision making process wherein, in the first stage, she chooses between lotteries over menus. Then, in the second stage, for any menu that realizes from the first stage choice, she chooses an act from that menu.

4 Non-linear regret representation

We can now formally define our representation which captures how the DM's preferences over menu-lotteries and her choice from menus are influenced by regret.

Definition 4.1. *$(\succcurlyeq, c)$ has a non-linear regret representation if there exists functions u :*

$Z \to \mathbb{R}$ and $\phi : \{|u(x) - u(y)| : x, y \in Z\} \to \mathbb{R}_+$ that is strictly increasing, with $\phi(0) = 0$, such that defining the function $U : \mathcal{A} \to \mathbb{R}$ by

$$U\big(A\big) = \max_{f} \left(\sum_{s=1}^{n} \pi(s) \Big[u(f_s) - \phi\big(\max_{g \in A} u(g_s) - u(f_s) \big) \Big] \right),$$

we have:

1. *for any $p, q \in \Delta(\mathcal{A})$, $p \succcurlyeq q$ iff $\sum_{A \in \mathcal{A}} p(A)U(A) \geq \sum_{A \in \mathcal{A}} q(A)U(A)$; and*
2. *for any $A \in \mathcal{A}$, $c(A) = \operatorname{argmax}_{f \in A} \sum_{s=1}^{n} \pi(s) \Big[u(f_s) - \phi\big(\max_{g \in A} u(g_s) - u(f_s) \big) \Big]$*

The representation proposes that the DM goes about her decision making in the following way. First, in the second stage, faced with any menu A, the DM evaluates any act $f \in A$ by the following criterion:

$$\sum_{s=1}^{n} \pi(s) \Big[u(f_s) - \phi\big(\max_{g \in A} u(g_s) - u(f_s) \big) \Big]$$

Here u has the interpretation of a Bernoulli utility function over prizes and $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ of a cost function capturing regret concerns. Her evaluation of f is composed of two parts. The first is a standard expected utility evaluation of the act using the utility function u and probability measure π on the state space. The second is a regret component. The DM realizes that in any state s, choosing $f \in A$ may produce ex post regret if a better outcome than the one under f could have been achieved by choosing some other act from A. In utility terms this regret is given by $\max_{g \in A} u(g_s) - u(f_s)$ and the cost of this regret is captured by $\phi\big(\max_{g \in A} u(g_s) - u(f_s) \big)$. Since regret is an uncomfortable emotion for the DM to experience, therefore, in assessing any such act $f \in A$, she attaches a penalty to it given by the expected regret cost, $\sum_{s=1}^{n} \pi(s)\phi\big(\max_{g \in A} u(g_s) - u(f_s) \big)$. In any such menu, she is happy choosing those acts that come out on top by this evaluation criterion.

From an ex ante perspective, her utility of any menu A is, accordingly, given by

$$U\big(A\big) = \max_{f \in A} \sum_{s=1}^{n} \pi(s) \Big[u(f_s) - \phi\big(\max_{g \in A} u(g_s) - u(f_s) \big) \Big]$$

When it comes to her preferences over lotteries on menus, she is simply an expected utility maximizer and she prefers lottery p to q if the expected utility of p is greater than the expected utility of q, i.e., $p \succcurlyeq q$ iff $\sum_{A \in \mathcal{A}} p(A)U(A) \geq \sum_{A \in \mathcal{A}} q(A)U(A)$.

5 Behavioral foundation

We now introduce a set of axioms that characterizes a non-linear regret representation. In terms of terminology note that we refer to the preference relation $\succcurlyeq$ on $\Delta(\mathcal{A})$ as vNM if it satisfies the three vonNeumann-Morgenstern axioms of weak order, Archimedean continuity and independence that together characterize an expected utility representation under risk.

Axiom 1: $\succcurlyeq$ on $\Delta(\mathcal{A})$ is vNM

Observe that any singleton menu, say, $\{f\}$ involves no regret costs. We maintain that such a menu should be assessed in a state separable manner, given the probabilities of the different states.

Axiom 2: For all $f = (f_1, f_2, ..., f_n) \in F$,

$$\{f\} \sim \pi_1\{f_1\} + \pi_2\{f_2\} + ... + \pi_n\{f_n\}$$

Our next axiom captures the idea that regret considerations impose a cost on the DM. Specifically, if f is the chosen act from a menu A, then the DM is not as well-off as choosing f when no other acts are present. This is because when chosen in the presence of other acts the DM is susceptible to experiencing ex post regret, which being forward looking she accounts for in her assessments at the ex ante stage.

Axiom 3: For all $A \in \mathcal{A}$ and $f \in c(A)$, $\{f\} \succcurlyeq A$

To state our final two axioms, we make use of the following definition.

Definition 5.1. *An act $f^s = (x_s, z_{-s})$ imposes regret on act f in state s if $\{x\} \succcurlyeq \{f_s\}$, $\{f_t\} \succcurlyeq \{z\}$, $\forall t \neq s$, and $f = c(\{f, f^s\})$.*

Observe that between the act f and f^s, the former is chosen. Further, f^s yields worse outcomes than f in all states other than s. Accordingly, f^s can potentially impose regret costs on f in state s.

Axiom 4: If $f^s = (x_s, z_{-s})$ and $g^t = (\hat{x}_t, \hat{z}_{-t})$ impose regret on f and g in states s and t, respectively, then

$$\frac{1}{2}\{x\}+\frac{1}{2}\{g_t\} \succcurlyeq \frac{1}{2}\{\hat{x}\}+\frac{1}{2}\{f_s\} \text{ iff } \frac{\pi_t}{\pi_s+\pi_t}\{f\}+\frac{\pi_s}{\pi_s+\pi_t}\{g, g^t\} \succcurlyeq \frac{\pi_t}{\pi_s+\pi_t}\{f, f^s\}+\frac{\pi_s}{\pi_s+\pi_t}\{g\}$$

Observe that the preference $\frac{1}{2}\{x\} + \frac{1}{2}\{g_t\} \succcurlyeq \frac{1}{2}\{\hat{x}\} + \frac{1}{2}\{f_s\}$ reveals that the preference or utility difference between the outcomes x and f_s is at least as big as that between $\hat{x}$ and g_t. Hence, the ex post regret experienced in state s when f is chosen from the menu $\{f, f^s\}$ is at least as high as that experienced in state t when g is chosen from the menu $\{g, g^t\}$.

This is what explains the preference for the lottery $\frac{\pi_t}{\pi_s + \pi_t}\{f\} + \frac{\pi_s}{\pi_s + \pi_t}\{g, g^t\}$ over the lottery $\frac{\pi_t}{\pi_s + \pi_t}\{f, f^s\} + \frac{\pi_s}{\pi_s + \pi_t}\{g\}$.

To state our final axiom, we introduce the following piece of notation. For any menu A, denote by $\bar{f}_s^A$ any element of the set $\{f_s : f \in A, f_s \succcurlyeq g_s, \forall g \in A\}$. Our final axiom essentially makes the point that the DM's regret concerns are assessed in an event-wise separable manner.

Axiom 5: For all $A \in \mathcal{A}$, $f \in A$ and collection of acts $\{f^s = (\bar{f}_s^A, z_{-s}) : s \in S\}$ that impose regret on f,

$$\frac{1}{n}A + (1 - \frac{1}{n})\{f\} \succcurlyeq \frac{1}{n}\{f, f^1\} + \cdots + \frac{1}{n}\{f, f^n\},$$

and if $f \in c(A)$,

$$\frac{1}{n}A + (1 - \frac{1}{n})\{f\} \sim \frac{1}{n}\{f, f^1\} + \cdots + \frac{1}{n}\{f, f^n\}$$

For our representation result we require the following richness condition.

Richness: For all $f \in A$, $s \in S$ and $x \in Z$, if $\{x\} \succcurlyeq \{f_s\}$ then there exists $z \in Z$ such that the act (x_s, z_{-s}) imposes regret on f in state s.

Essentially, what this condition requires is that for any act f and outcome x, there exists a "bad enough" outcome y such that $f = c(\{f, (x_s, y_{-s})\})$.

Theorem 5.1. *Suppose the richness condition holds. Then $(\succcurlyeq, c)$ satisfies axioms 1-5 if and only if it has a non-linear regret representation.*

Proof: Please refer to Section A.1

6 Empirical content of the model

In this section, we discuss some implications of regret for behavior in risky environments. Our goal is to show how regret may result in patterns of behavior that from the perspective of standard economics, may be described as "anomalies". For the purpose of this exercise, in this section, we will mostly assume that the set of outcomes is a subset of the real line. We begin by presenting a result that helps to throw light on the channel through which these behavioral anomalies manifest themselves under regret. The result looks at two-outcome acts, which give some outcome x on an event E and y on its complement.[4] We will denote such acts by the notation xEy.

[4] An event E is defined as a collection of states, i.e. $E \subseteq S$.

Proposition 6.1. *Let $E \subseteq S$ be such that $0 < \pi(E) < \frac{1}{2}$, and $f = xEy$, $g = x^{'}Ey'$ be two acts with $\{f\} \sim \{g\}$ and $\{x\} \succ \{x^{'}\}$. If ϕ is convex (resp., strictly convex), then $f \in c(\{f, g\})$ (resp., $f = c(\{f, g\})$).*

Proof: Please refer to Section A.2

The above result demonstrates that for two-outcome acts, where essentially, the choice is between a higher advantage with a lower probability and a lower advantage with a higher probability, regret can make agents choose the former. Regret makes agents focus more on the higher outcome difference, and the fear of missing out on it draws their attention away from the fact that it is less likely to occur. In our representation, a convex ϕ does this job of making the outcome difference, and the possible regret of missing out on it seem greater than it actually is.

6.1 Long shot bias

Preference for gambles which give a disproportionately high prize with a relatively low probability of success is known as the long shot bias. Simply, it is the tendency to go for for lotteries which give sizeable payoffs with a lower probability. Gollier (2020) documents a link between regret-risk aversion and the long shot bias by showing that the risk of regret can induce people to have a bias for such risky long shots. In this section, we show that our model of non-linear regret can also explain this rather "risk seeking" behavior.

Consider a binary menu of the form $A = \{xEy, z\}$ with $\{x\} \succ \{z\} \succ \{y\}$ and $\{xEy\} \sim \{z\}$. Consequently, xEy is better than z in one event and worse than z in the other. Accordingly, $\pi(E)(u(x) - u(z)) = \pi(E^c)(u(z) - u(y))$. Consider the case where $\pi(E) < \pi(E^c)$. It implies $u(x) - u(z) > u(z) - u(y)$. Therefore, agents who choose xEy from $\{xEy, z\}$ in this case tend to focus more on the higher prize of x over z in E than the lower likelihood of this advantage being realized. Therefore, when $\pi(E)$ is small choosing xEy implies a preference for a long shot where the low chance of a higher comparative advantage seems very lucrative. Drawing on Proposition 6.1, our model can explain this bias for long shots with a convex ϕ.

More generally, this preference for long shots corresponds to agents becoming less risk averse due to regret. Consider again the case where z is the certainty equivalent of xEy in the sense of $\{xEy\} \sim \{z\}$. Because of the anticipated ex-post regret, the agent can have a clear choice between the two. If the choice in $\{xEy, z\}$ is xEy, then it implies that the agent no longer considers z as good as the gamble of xEy. Because of potential regret, the sure amount that the agent considers as good as xEy is now something higher than z. In that sense, under regret the agent may behave in a less risk averse manner. Indeed, it is possible that her behavior may be risk loving in the sense of choosing a gamble over a sure prospect giving the expected value of the gamble.

Finally, we present the following proposition that extends the essence of long shot bias to general acts.

Proposition 6.2. *Suppose for $f, g \in A$ such that $\{f\} \sim \{g\}$, there exists $E \subset S$ such that $\{f_s\} \succ \{g_s\}$ $\forall s \in E$, and $\{g_s\} \succ \{f_s\}$ $\forall s \in E^c$, and $\pi(E) \leq \min\{\pi_s | s \in E^c\}$. If ϕ is convex (resp. strictly convex), then $f \in c(\{f, g\})$ (resp. $f = c(\{f, g\})$).*

Proof: Please refer to Section A.3

6.2 Menu effects: Menu-dependent risk attitudes

Menu effects refer to the phenomenon wherein adding decoy alternatives to a menu result in choice reversals. Formally, such choice behavior violate the independence of irrelevant alternatives condition or, equivalently, the weak axiom of revealed preferences.

Carrying from our result of preference for long shots, we now show that regret, through an appropriate decoy, can lead to choice reversals wherein the DM switches from her choice of long shot to a safer alternative. For this purpose, consider the setup in Proposition 6.1 with one change that g is a degenerate act giving a fixed outcome, say in both both events. That is, $f = xEy$ and $g = x^{'}$ Given a strictly convex ϕ, from Proposition 6.1, we know that $f = c(\{f, g\})$.

Consider another act $h = zEx$ such that $\{f\} \sim \{g\} \sim \{h\}$. By construction, we have that $\{y\} \succ \{z\}$. For acts f and h, the "riskiness" is running in opposite directions. Act f offers the best outcome in the menu but is also less likely to be realized, and with act h, for the same event, there is a possibility of ending up with the absolute worst outcome (even though the best outcome is more likely to occur with h). Notice that, the lucrative outcome in both f and h is the same. We propose that on adding h in the menu of $\{f, g\}$, the choice for the agent flips from f to g.

Proposition 6.3. *Let $E \subseteq S$ such that $\pi_E < \pi_{E^c}$. For any outcome $x^{'} \in Z$ and two-event acts $f = xEy$ and $h = zEx$ such that $\{f\} \sim \{x^{'}\} \sim \{h\}$, $\{x\} \succ \{x^{'}\} \succ \{y\} \succ \{z\}$, we have $f = c(\{f, x^{'}\})$ and $x^{'} = c(\{f, x^{'}, h\})$ if ϕ is strictly convex.*

Proof: Please refer to Section A.4

The above result highlights the ideas that, first, regret can lead to menu dependent risk preferences where the choice between a gamble and sure outcome is dependent on what else is available in the menu; and second, that these effects can be generalized to a particular class of gambles where the "riskiness" is running in opposite directions. More concretely, in act f the better outcome is offered in a less likely event and in act h the same outcome is offered in the other event. These acts maybe classified as non-comonotonic in the spirit of Schmeidler (1989). This, in some sense, evens out DM's consideration of both events since the best outcome in the menu is now possible in either of the two events.

6.3 Other behavioral anomalies

6.3.1 Preference for mean preserving spreads

Just as regret can produce a preference for long shots, we now show that following the same pattern of creating a bias for higher but less likely payoffs regret can also create a preference for mean-preserving spreads. Consider a constant act $g = 2000$ and a mean preserving spread of it, $f = 4000E1000$, with $\pi(E) = \frac{1}{3}$.

Assume that $u(x) = \sqrt{x}$. With a strictly concave u, f has a lower expected utility than g. However, under non-linear regret, in the menu $\{f, g\}$, these acts are evaluated as follows

$$v(g, \{f, g\}) := \frac{1}{3}\Big[u(2000) - \phi\big(u(4000) - u(2000)\big)\Big] + \frac{2}{3}\Big[u(2000) - \phi\big(u(2000) - u(2000)\big)\Big]$$

$$v(f, \{f, g\}) := \frac{1}{3}\Big[u(4000) - \phi\big(u(4000) - u(4000)\big)\Big] + \frac{2}{3}\Big[u(1000) - \phi\big(u(2000) - u(1000)\big)\Big]$$

With a convex ϕ of the form $\phi(y) = y^\alpha$ where $\alpha > 2.06$, we get $v(f, \{f, g\}) > v(g, \{f, g\})$. Hence, $c(\{f, g\}) = f$. Therefore, regret can create a preference for mean preserving spreads.

6.3.2 Reversal of stochastic dominance preferences

Define the following menu-dependent regret preferences:

Definition 6.1. *For any menu A and $f, h \in A$, $f \succsim_A h$ if*

$$\sum_{s=1}^{n} \pi(s)\Big[u(f_s) - \phi\big(\max_{g\in A} u(g_s) - u(f_s)\big)\Big] \geq \sum_{s=1}^{n} \pi(s)\Big[u(h_s) - \phi\big(\max_{g\in A} u(g_s) - u(h_s)\big)\Big]$$

Clearly, $f \in c(A)$ iff $f \succsim_A h$, for all $h \in A$.

Consider the menu $A = \{f, g, h\}$ where the acts f, g, h over four equally likely events E_1, E_2, E_3, E_4 are as shown in the Table below.

	E_1	E_2	E_3	E_4
f	5000	2000	1000	850
g	5000	850	2000	950
h	2000	850	900	5000

Table 3

Observe that f first order stochastically dominates g and h, and g first order stochastically dominates h.

Under non-linear regret, in menu A, we show that preferences can violate stochastic dominance. The acts in A would be evaluated as follows under non-linear regret:

$$v(f, A) = \pi(E_1)\Big[u(5000) - \phi\big(u(5000) - u(5000)\big)\Big] + \pi(E_2)\Big[u(2000) - \phi\big(u(2000) - u(2000)\big)\Big] +$$

$$\pi(E_3)\Big[u(1000) - \phi\big(u(2000) - u(1000)\big)\Big] + \pi(E_4)\Big[u(850) - \phi\big(u(5000) - u(850)\big)\Big]$$

$$v(g, A) = \pi(E_1)\Big[u(5000) - \phi\big(u(5000) - u(5000)\big)\Big] + \pi(E_2)\Big[u(850) - \phi\big(u(2000) - u(850)\big)\Big] +$$

$$\pi(E_3)\Big[u(2000) - \phi\big(u(2000) - u(2000)\big)\Big] + \pi(E_4)\Big[u(950) - \phi\big(u(5000) - u(950)\big)\Big]$$

$$v(h, A) = \pi(E_1)\Big[u(2000) - \phi\big(u(5000) - u(2000)\big)\Big] + \pi(E_2)\Big[u(850) - \phi\big(u(2000) - u(850)\big)\Big] +$$

$$\pi(E_3)\Big[u(900) - \phi\big(u(2000) - u(900)\big)\Big] + \pi(E_4)\Big[u(5000) - \phi\big(u(5000) - u(5000)\big)\Big]$$

Assume that $u(x) = \sqrt{x}$ and $\phi(y) = y^{\alpha}$ where $\alpha > 1.44$. Using these parameters, we get $v(h, A) > v(g, A) > v(f, A)$, which implies $h \succ_A g \succ_A f$, leading to a violation of stochastic dominance.

The results of long shot bias and preference for mean preserving spreads are similar to the *possibility effect*, where people tend to overestimate the likelihood of a low probability event. The possibility effect can be explained by theories like cumulative prospect theory (Tversky and Kahneman, 1992) and rank-dependent utility (Quiggin, 1982) that draw on the theoretical construct of non-linear probability weighting. However, it is worth noting that such models cannot accommodate violations of stochastic dominance. Therefore, even though the effects look similar, the empirical content of non-linear regret is not a special case of these models.

A Appendix

A.1 Proof of Theorem 5.1

Given the richness condition, we prove that axioms 1 - 5 are sufficient for the representation.

Step 1: Defining the objects of the representation

Defining the function $u : Z \to \mathbb{R}$: First note that by virtue of axiom 1, we have by the vNM expected utility theorem that there exists a function $U : \mathcal{A} \to \mathbb{R}$ such that for any $p, q \in \Delta\mathcal{A}$,

$$p \succcurlyeq q \iff \sum_{A \in \mathcal{A}} p(A)U(A) \geq \sum_{A \in \mathcal{A}} q(A)U(A)$$

Now, define the function $u : Z \to \mathbb{R}$ by $u(x) = U(\{x\})$. The representation for singleton menus is completely specified in terms of the function u. This is because, by axiom 2, for any $f = (f_1, \ldots, f_n) \in F$, $\{f\} \sim \pi_1\{f_1\} + \cdots + \pi_n\{f_n\}$. Accordingly,

$$U(\{f\}) = \pi_1 U(\{f_1\}) + \cdots + \pi_n U(\{f_n\}) = \sum_{s \in S} \pi_s u(f_s)$$

Defining the function $\phi : \{|u(x) - u(y)| : x, y \in Z\} \to \mathbb{R}_+$: Fix any state $s \in S$ and consider any $x, y \in Z$ such that $u(x) \geq u(y)$, i.e., $x \succcurlyeq y$. Let f be any act with $f_s = y$. By the richness condition, there exists $z \in Z$ such that (x_s, z_{-s}) imposes regret on f. Define the function $\phi : \{|u(x) - u(y)| : x, y \in Z\} \to \mathbb{R}_+$ by:

$$\phi(u(x) - u(y)) = \frac{U(\{f\}) - U(\{f, (x_s, z_{-s})\})}{\pi_s}$$

First, note that since by axiom 3, $\{f\} \succcurlyeq \{f, (x_s, z_{-s})\}$, $U(\{f\}) - U(\{f, (x_s, z_{-s})\}) \geq 0$. Accordingly, the range of ϕ is a subset of $\mathbb{R}_+$. Next, using axiom 4, we establish that ϕ is well defined and is strictly increasing. For that consider $x, y, x', y' \in Z$ such that $u(x') - u(y') \geq u(x) - u(y)$. By what we established above, $u(x') - u(y') \geq u(x) - u(y)$ iff $\frac{1}{2}u(x') + \frac{1}{2}u(y) \geq \frac{1}{2}u(x) + \frac{1}{2}u(y')$ iff $\frac{1}{2}\{x'\} + \frac{1}{2}\{y\} \succcurlyeq \frac{1}{2}\{x\} + \frac{1}{2}\{y'\}$. Now, let f and f' be acts with $f_s = y$ and $f'_s = y'$. Further, let (x_s, z_{-s}) and (x'_s, z'_{-s}) impose regret on f and f', respectively, in state s. We know that such acts exist from the richness condition.

Then by axiom 4, we have that:

$$\begin{aligned}
u(x') - u(y') \geq u(x) - u(y) \quad &\iff \quad \frac{1}{2}\{x'\} + \frac{1}{2}\{y\} \succcurlyeq \frac{1}{2}\{x\} + \frac{1}{2}\{y'\} \\
&\iff \quad \frac{1}{2}\{f'\} + \frac{1}{2}\{f, (x_s, z_{-s})\} \succcurlyeq \frac{1}{2}\{f\} + \frac{1}{2}\{f', (x'_s, z'_{-s})\} \\
&\iff \quad \frac{U(\{f'\}) - U(\{f', (x'_s, z'_{-s})\})}{\pi_s} \geq \frac{U(\{f\}) - U(\{f, (x_s, z_{-s})\})}{\pi_s} \\
&\iff \quad \phi(u(x') - u(y')) \geq \phi(u(x) - u(y))
\end{aligned}$$

Therefore, if in particular, $u(x) = u(x')$ and $u(y) = u(y') = u(f_s) = u(f'_s)$, then $\phi(u(x') - u(y')) = \phi(u(x) - u(y))$. Hence, the definition of ϕ is independent of the particular acts chosen. Finally, to establish that $\phi(0) = 0$, consider x, y with $u(x) - u(y) = 0$. Then, the act $x = (x_s, x_{-s})$ trivially imposes regret on y. Accordingly, $\phi(u(x) - u(y)) = 0$.

Next, consider any $f \in F$ and $x \in Z$ such that $x \succcurlyeq f_t$, $t \neq s$. By the richness condition there exists an act (x_t, z_{-t}) that imposes regret on f in state t. Further, there exists an act (x_s, z'_{-s}) that imposes regret on f_t in state s. Then, since $\frac{1}{2}\{x\} + \frac{1}{2}\{f_t\} \sim \frac{1}{2}\{x\} + \frac{1}{2}\{f_t\}$, using axiom 4 again, we have $\frac{\pi_s}{\pi_s+\pi_t}\{f\} + \frac{\pi_t}{\pi_s+\pi_t}\{f_t, (x_s, z'_{-s})\} \succcurlyeq \frac{\pi_s}{\pi_s+\pi_t}\{f, (x_t, z_{-t})\} + \frac{\pi_t}{\pi_s+\pi_t}\{f_t\}$. Accordingly,

$$\frac{U(\{f\}) - U(\{f, (x_t, z_{-t})\})}{\pi_t} = \frac{U(\{f_t\}) - U(\{f_t, (x_s, z'_{-s})\})}{\pi_s} = \phi(u(x) - u(f_t))$$

Step 2: Establishing the representation

We can now proceed to show that with respect to the functions $u : Z \to \mathbb{R}$ and $\phi : \{|u(x) - u(y)| : x, y \in Z\} \to \mathbb{R}_+$ defined in Step 1, $(\succcurlyeq, c)$ have a Non-linear regret representation. Consider any menu $A \in \mathcal{A}$. Define the act $\bar{f}^A = (\bar{f}^A_1, \ldots, \bar{f}^A_n)$, where $\bar{f}^A_s$ is any element of the set $\{f_s : f \in A, f_s \succcurlyeq g_s, \forall g \in A\}$. Now, for any $f \in A$, by the richness condition, we can define a collection of acts $\{f^s = (\bar{f}^A_s, z_{-s}) : s \in S\}$ that impose regret on f in each of the states $s \in S$. Then, by axiom 5, we have

$$\frac{1}{n}A + (1 - \frac{1}{n})\{f\} \succcurlyeq \frac{1}{n}\{f, f^1\} + \cdots + \frac{1}{n}\{f, f^n\}$$

Accordingly,

$$\begin{aligned}
& U\left(\frac{1}{n}A + (1 - \frac{1}{n})\{f\}\right) \geq U\left(\frac{1}{n}\{f, f^1\} + \cdots + \frac{1}{n}\{f, f^n\}\right) \\
\implies \quad & \frac{1}{n}U(A) + (1 - \frac{1}{n})U(\{f\}) \geq \frac{1}{n}U(\{f, f^1\}) + \cdots + \frac{1}{n}U(\{f, f^n\}) \\
\implies \quad & U(A) \geq U(\{f\}) - \big(U(\{f\}) - U(\{f, f^1\})\big) - \cdots - \big(U(\{f\}) - U(\{f, f^n\})\big) \\
\implies \quad & U(A) \geq \sum_{s=1}^{n} \pi(s)u(f_s) - \pi_1\phi\Big(u(\bar{f}^A_1) - u(f_1)\Big) - \cdots - \pi_n\phi\Big(u(\bar{f}^A_n) - u(f_n)\Big)
\end{aligned}$$

Further, if $f \in c(A)$, then

$$\frac{1}{n}A + (1-\frac{1}{n})\{f\} \sim \frac{1}{n}\{f, f^1\} + \cdots + \frac{1}{n}\{f, f^n\}$$

and, accordingly,

$$U(A) = \sum_{s=1}^{n} \pi(s)u(f_s) - \pi_1\phi\Big(u(\bar{f}_1^A) - u(f_1)\Big) - \cdots - \pi_n\phi\Big(u(\bar{f}_n^A) - u(f_n)\Big)$$

Therefore,

$$U\big(A\big) = \max_{f}\left(\sum_{s=1}^{n}\pi(s)\Big[u(f_s) - \phi\big(\max_{g\in A} u(g_s) - u(f_s)\big)\Big]\right)$$

and

$$c(A) = \operatorname*{argmax}_{f\in A}\sum_{s=1}^{n}\pi(s)\Big[u(f_s) - \phi\big(\max_{g\in A} u(g_s) - u(f_s)\big)\Big]$$

This completes the proof establishing the sufficiency of the axioms for a Non-linear regret representation.

A.2 Proof of Proposition 6.1

Proof. As $\{f\} \sim \{g\}$, to show that $f \in c(\{f, g\})$ it is sufficient to show that f imposes a higher regret cost on g in $\{f, g\}$ than g does on f. That is, $\pi(E)\phi[u(x) - u(x')] \geq \pi(E^c)\phi[u(y') - u(y)]$.

Notice that as $\{f\} \sim \{g\}$ and $\{x\} \succ \{x'\}$ with $\pi(E) < \pi(E^c)$, it must be the case that $u(x) - u(x') > u(y') - u(y) > 0$. Now, convexity of ϕ implies $\phi[\alpha(u(x) - u(x')) + (1-\alpha)\cdot 0]$ $\leq \alpha\phi[u(x) - u(x')] + (1-\alpha)\phi(0)$ for all $\alpha \in (0, 1)$.

As $u(y') - u(y) = \alpha'(u(x) - u(x')) + (1-\alpha')\cdot 0$, we have

$$\begin{aligned} \phi[u(y') - u(y)] &\leq \alpha'\phi[u(x) - u(x')] + (1-\alpha')\phi(0) \\ \iff \phi[u(y') - u(y)] &\leq \frac{u(y') - u(y)}{u(x) - u(x')}\phi[u(x) - u(x')] \\ \iff \frac{\phi[u(y') - u(y)]}{u(y') - u(y)} &\leq \frac{\phi[u(x) - u(x')]}{u(x) - u(x')} \end{aligned}$$

As $\{f\} \sim \{g\}$, the above inequality holds iff

$$\pi(E^c)(u(y') - u(y))\frac{\phi[u(y') - u(y)]}{u(y') - u(y)} \leq \pi(E)(u(x) - u(x'))\frac{\phi[u(x) - u(x')]}{u(x) - u(x')}$$

$$\iff \pi(E^c)\phi[u(y^{'}) - u(y)] \leq \pi(E)\phi[u(x) - u(x^{'})]$$

This completes the proof for $f \in c(\{f, g\})$. Observe that $f = c(\{f, g\})$ if ϕ is *strictly* convex.

$\square$

A.3 Proof of Proposition 6.2

Proof. As $\{f\} \sim \{g\}$, choice from $\{f, g\}$ depends on the regret cost imposed by f on g and the regret cost imposed by g on f. $f \in c(\{f, g\}) \iff \sum_{s \in E} \pi_s \phi\big(u(f_s) - u(g_s)\big) \geq \sum_{s \in E^c} \pi_s \phi\big(u(g_s) - u(f_s)\big)$.

$$\begin{aligned}
\{f\} \sim \{g\} \quad &\iff \quad \sum_s \pi_s u(f_s) = \sum_s \pi_s u(g_s) \\
&\iff \quad \sum_{s \in E} \pi_s \big(u(f_s) - u(g_s)\big) = \sum_{s \in E^c} \pi_s \big(u(g_s) - u(f_s)\big) \\
&\iff \quad \sum_{s \in E} \frac{\pi_s}{\pi(E)} \big(u(f_s) - u(g_s)\big) = \sum_{s \in E^c} \frac{\pi_s}{\pi(E)} \big(u(g_s) - u(f_s)\big)
\end{aligned}$$

As ϕ is convex, we get

$$\sum_{s \in E} \left(\frac{\pi_s}{\pi(E)} \phi\big(u(f_s) - u(g_s)\big) \right) \quad \geq \phi\left(\sum_{s \in E^c} \frac{\pi_s}{\pi(E)} \big(u(g_s) - u(f_s)\big) \right)$$

Further, since $\phi(0) = 0$, ϕ is also superadditive. That is,

$$\phi\left(\sum_{s \in E^c} \frac{\pi_s}{\pi(E)} \big(u(g_s) - u(f_s)\big) \right) \quad \geq \sum_{s \in E^c} \phi\left(\frac{\pi_s}{\pi(E)} \big(u(g_s) - u(f_s)\big) \right)$$

Also, as ϕ is convex with $\phi(0) = 0$, ϕ also has the property that $\phi(tx) \geq t\phi(x)$ $\forall t > 1$. This implies

$$\sum_{s \in E^c} \phi\left(\frac{\pi_s}{\pi(E)} \big(u(g_s) - u(f_s)\big) \right) \quad \geq \sum_{s \in E^c} \left(\frac{\pi_s}{\pi(E)} \phi\big(u(g_s) - u(f_s)\big) \right)$$

Finally, from the above inequalities we get,

$$\begin{aligned}
\sum_{s \in E} \left(\frac{\pi_s}{\pi(E)} \phi\big(u(f_s) - u(g_s)\big) \right) \quad &\geq \sum_{s \in E^c} \phi\left(\frac{\pi_s}{\pi(E)} \big(u(g_s) - u(f_s)\big) \right) \\
\iff \sum_{s \in E} \pi_s \phi\big(u(f_s) - u(g_s)\big) \qquad &\geq \sum_{s \in E^c} \pi_s \phi\big(u(g_s) - u(f_s)\big)
\end{aligned}$$

Hence, $f \in c(\{f, g\})$. Observe that the above inequalities would be strict if ϕ is strictly convex, implying $f = c(\{f, g\})$.

$\square$

A.4 Proof of Proposition 6.3

Proof. In the menu $\{x^{'}, f\}$ we have $0 < \pi(E) < \frac{1}{2}$, where $\{x\} \succ \{x^{'}\}$ and $\{f\} \sim \{x^{'}\}$. Hence, by Proposition 6.1, we have $c(\{x^{'}, f\}) = f$.

Now, in the menu $\{x^{'}, f, h\}$, the three acts are evaluated as follows:

$$v(f, \{x^{'}, f, h\}) = \pi(E)u(x) + \pi(E^c)u(y) - \pi(E^c)\phi\big(u(x) - u(y)\big)$$

$$v(x^{'}, \{x^{'}, f, h\}) = u(x^{'}) - \pi_E\phi\big(u(x) - u(x^{'})\big) - \pi(E^c)\phi\big(u(x) - u(x^{'})\big) = u(x) - \phi\big(u(x) - u(x^{'})\big)$$

$$v(h, \{x^{'}, f, h\}) = \pi(E)u(x) + \pi(E^c)u(z) - \pi(E)\phi\big(u(x) - u(z)\big)$$

Since $\{x^{'}\} \sim \{f\} \sim \{h\}$, the act with the least regret cost is the best in this menu. Consider the regret costs imposed on f and $x^{'}$. Cost on f is $\pi(E^c)\phi\big(u(x) - u(y)\big)$. Cost on $x^{'}$ is

$$\begin{aligned}
\phi\big(u(x) - u(x^{'})\big) &= \phi\big(u(x) - \pi(E)u(x) - \pi(E^c)u(y)\big) \\
&= \phi\big((1 - \pi(E))u(x) - \pi(E^c)u(y)\big) \\
&= \phi\big(\pi(E^c)(u(x) - u(y))\big)
\end{aligned}$$

Since ϕ is strictly convex, $\phi\big(\pi(E^c)(u(x) - u(y) + \pi(E) \cdot 0\big) < \pi(E^c)\phi\big(u(x) - u(y)\big) + \pi(E)\phi\big(0\big)$ $= \pi(E^c)\phi\big(u(x) - u(y)\big)$. Therefore, $v(x^{'}, \{x^{'}, f, h\}) > v(f, \{x^{'}, f, h\})$.

Now comparing $v(f, \{x^{'}, f, h\})$ and $(h, \{x^{'}, f, h\})$. First, notice that since $\{x\} \succ \{y\} \succ \{z\}$, we have $u(x) - u(z) > u(x) - u(y)$. That is, there exists a $\beta \in (0, 1)$ such that $u(x) - u(y) = \beta(u(x) - u(z)) + (1 - \beta) \cdot 0$. Further, since ϕ is strictly convex, it must be the case that

$$
\begin{aligned}
\phi\big(u(x)-u(y)\big) &< \beta\phi\big(u(x)-u(z)\big)+(1-\beta)\phi\big(0\big) \\
\iff \phi\big(u(x)-u(y)\big) &< \frac{u(x)-u(y)}{u(x)-u(z)}\phi\big(u(x)-u(z)\big) \\
\iff \frac{\phi\big(u(x)-u(y)\big)}{u(x)-u(y)} &< \frac{\phi\big(u(x)-u(z)\big)}{u(x)-u(z)} \\
\iff \pi(E^c)(u(x)-u(y))\frac{\phi\big(u(x)-u(y)\big)}{u(x)-u(y)} &< \pi(E)(u(x)-u(z))\frac{\phi\big(u(x)-u(z)\big)}{u(x)-u(z)} \\
\iff \pi(E^c)\phi\big(u(x)-u(y)\big) &< \pi(E)\phi\big(u(x)-u(z)\big)
\end{aligned}
$$

Hence, $v(f,\{x^{'},f,h\}) > v(h,\{x^{'},f,h\})$. That is, $x^{'}$ is the best act in $\{x^{'},f,h\}$. Therefore, $c(\{x^{'},f,h\}) = x^{'}$.

$\square$

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