Public, Private, or a bit of both (Mixed)?

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Outline

Question

- Consider a mixed duopoly where a private firm R competes with a public firm U.
  - R maximizes profit
  - U maximizes *welfare*
  - R is *more efficient* than U
- Government has three options:
  - 1. Nationalize R
  - 2. Privatize U
  - 3. Do nothing: retain mixed duopoly
- Q: When is consumer surplus highest?
- A: 1 or 2, but (almost) never 3.

Two key ingredients

- **Efficiency**

  Private firms are typically **more or at least as efficient** and profitable as their **public** counterparts (Tirole, 1993; Ehrlich et. al, 1994; Shleifer, 1998; Berglof & Roland, 1998; Majumder, 1996; Megginson & Netter, 2002, Matsumura & Matsushima, 2004)

- **Objective function**

  Welfare maximization is routinely considered as **public firms' objective** in the literature on **mixed markets** (De Fraja & Delbono, 1989; Matsumura, 1998; Anderson et. al., 1997)

Literature

- **Privatization can improve welfare** (even when public firms are equally efficient) by
  - reducing the scale of production for public firms (De Fraja and Delbono, 1989) in the presence of increasing marginal costs
  - reducing product concentration (Cremer et. al. 1991)
  - encouraging entry (Anderson et. al, 1997)
- A common theme across these papers: *despite welfare improvement, post-privatization prices are higher and consumer surplus typically lower*

Literature

- In developing countries where mixed markets are quite common, **distributional concerns** are important for privatization (Estrin and Pelletier, 2018)
- Lack of well-functioning private markets and absence of strong regulatory authorities in developing countries often lead to less competition, higher prices, and increased income inequality after privatization (Stiglitz, 2002; Piketty, 2014).
- Privatization has no perceptible benefit for the consumers unless **it improves access or consumer surplus** (Birdsall and Nellis, 2003; Mckenzie et. al., 2003)

Consumer surplus

- Our focus is not so much on whether privatization increases or reduces consumer surplus but rather on mixed arrangement. In particular, how does mixed market compare with fully public or fully private environment when it comes to consumer surplus?
- **Result:** Let CS_r, CS_u, and CS_m denote consumer surplus under private, public, and mixed arrangement respectively. A common theme across several standard environments (e.g.,homogeneous Cournot, differentiated Bertrand)

  Consumer Surplus is never highest under mixed oligopoly

$$\min\{CS_u, CS_r\} \leq CS_m \leq \max\{CS_u, CS_r\}$$

Outline

Model

- n firms, out of which m are private and $n - m$ are public
- Each private firm produces output: x_i
- Each public firm produces output: y_k
- Cost function for private firm: $C_r(x_i)$
- Cost function for public firm: $C_u(y_k)$
- $C_r(x_i)$ and $C_u(y_k)$ are twice-differentiable, strictly increasing and convex in output
- Public firm relatively inefficient: $C_u'(z) > C_r'(z)$ for $z > 0$

Model (Continued)

- Inverse demand function: $P(Q)$ where $P'(Q) < 0$
- Aggregate output: $Q = \sum_{i=1}^{m} x_i + \sum_{k=m+1}^{n} y_k$
- **Assumption 1:** Strictly logconcave demand function:

$$P'(Q) + QP''(Q) < 0;\ \forall Q > 0$$

- **Assumption 2:**

$$P_0 > \max\{C_r'(0), C_u'(0)\} > P_\infty = 0$$

where $P_0 = \lim_{Q\to 0} P(Q)$ and $P_\infty = \lim_{Q\to\infty} P(Q)$

Private Firm

- Private firm i chooses x_i to maximize its own profit:

$$\pi_i = P(Q)x_i - C_r(x_i)$$

- First order condition:

$$P(Q) + P'(Q)x_i = C_r'(x_i)$$

Public Firm

- Profit of public firm k:

$$\pi_k = P(Q)y_k - C_u(y_k)$$

- Consumer surplus and welfare:

$$CS = \int_0^Q P(y)dy - P(Q)Q$$

$$W = CS + \sum_{i=1}^{m} \pi_i + \sum_{k=m+1}^{n} y_k$$

$$= \int_0^Q P(y)dy - \sum_{i=1}^{m} C_r(x_i) - \sum_{k=m+1}^{n} C_u(y_k)$$

Public Firm (contd.)

- Public firm chooses y_k to maximize

$$(1-\lambda)W + \lambda\pi_k$$

  - $\lambda = 0$: welfare maximization
  - $\lambda = 1$: profit maximization

- First order condition:

$$P(Q) + \lambda P'(Q)y_k = C_u'(y_k)$$

Cournot equilibrium

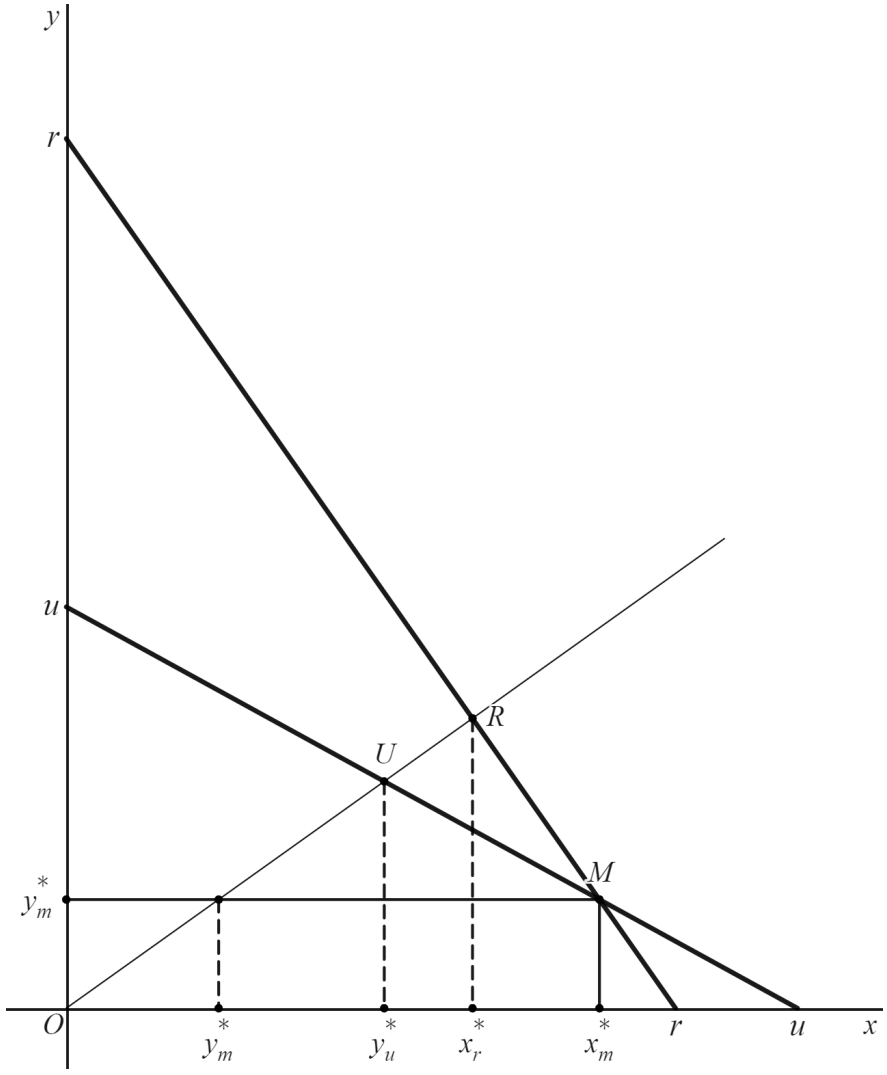

Figure 1: Equilibrium in Cournot Duopoly ($Q_r > Q_u$)

Cournot equilibrium (contd.)

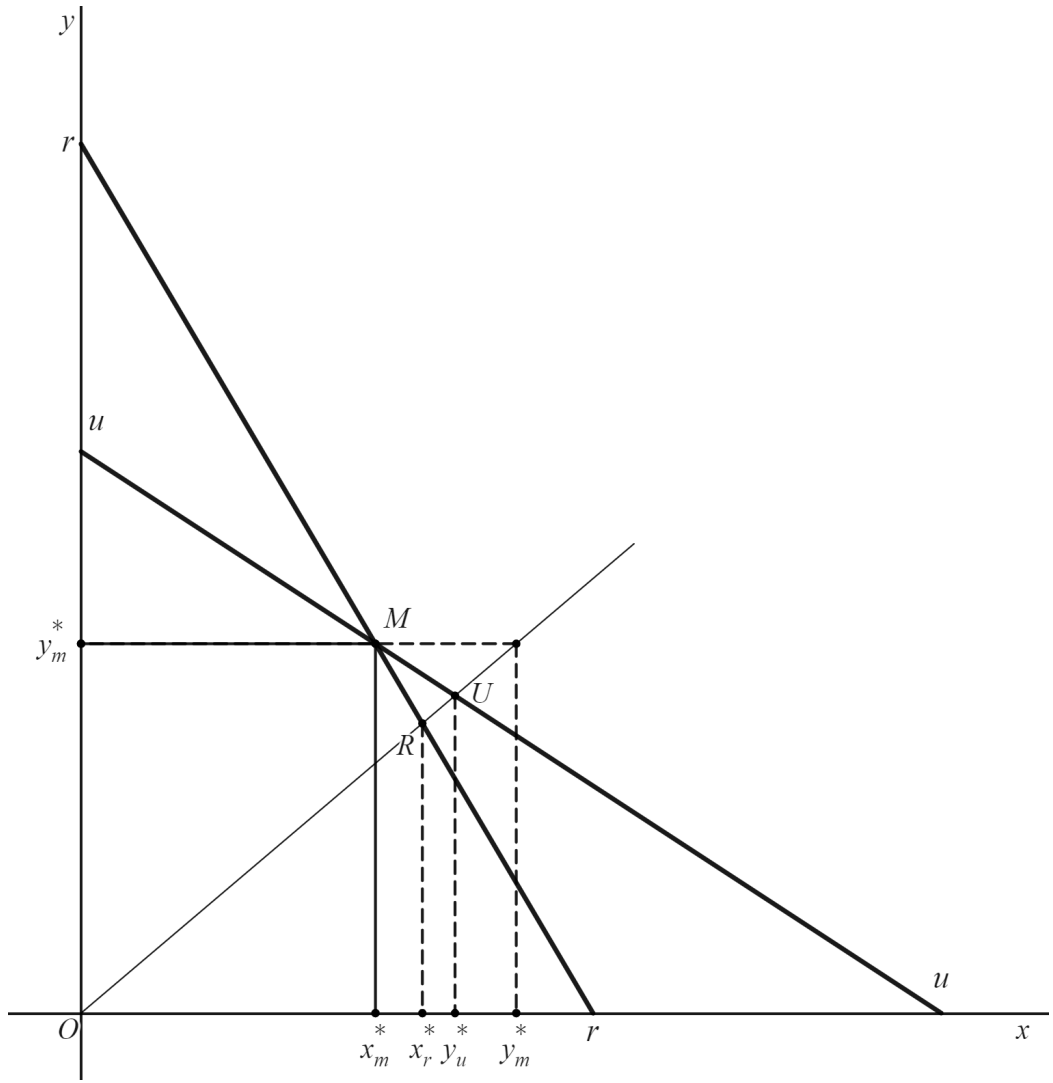

Figure 2: Equilibrium in Cournot Duopoly ($Q_u > Q_r$)

Result

Proposition

Let CS_u^*, CS_r^*, and CS_m^* refer to equilibrium consumer surplus in public, private, and mixed oligopoly regimes, respectively. Consumer surplus is the highest/lowest either when all firms are public or when all firms are private. Consumer surplus is never (uniquely) maximized under mixed oligopoly. More formally,

$$\min\{CS_u^*, CS_r^*\} \leq CS_m^* \leq \max\{CS_u^*, CS_r^*\}$$

where the equality holds if and only if $CS_u^* = CS_r^*$.

Outline

Product Differentiation

- Product differentiation features prominently in several mixed markets with public firms (e.g. telecommunication, banking etc)
- **Differentiated product** lends itself naturally to **price competition**. In particular private and public firm can charge different prices
- Aggregate output and consumer surplus do not necessarily move in the same direction

Model

- Quasilinear preferences: $U(x, y) + z$
- **Example:**

$$U(x, y) = a(x + y) - \frac{x^2 + y^2}{2} - bxy$$

where $a > 0$, and $b \in (0, 1)$ captures the degree of substitutability between x and y.

- Direct demand functions:

$$x = \frac{a(1 - b) - p^x + bp^y}{1 - b^2} \equiv x(p^x, p^y)$$

$$y = \frac{a(1 - b) - p^y + bp^x}{1 - b^2} \equiv y(p^x, p^y)$$

Model (contd.)

- Profits of private firm and public firm are given by π_r and π_u respectively

$$\pi_r = (p^x - c_r)x(p^x, p^y)$$
$$\pi_u = (p^y - c_u)y(p^x, p^y)$$

- Consumer surplus and welfare:

$$CS = U(x(p^x, p^y), y(p^x, p^y)) - p^x x(p^x, p^y) - p^y y(p^x, p^y)$$
$$W = U(x(p^x, p^y), y(p^x, p^y)) - c_r x(p^x, p^y) - c_u y(p^x, p^y)$$

Model (contd.)

- Private firm chooses p^x to maximize π_r
- Public firm chooses p^y to maximize

$$(1-\lambda)W + \lambda\pi_u$$

Consumer Surplus and Prices

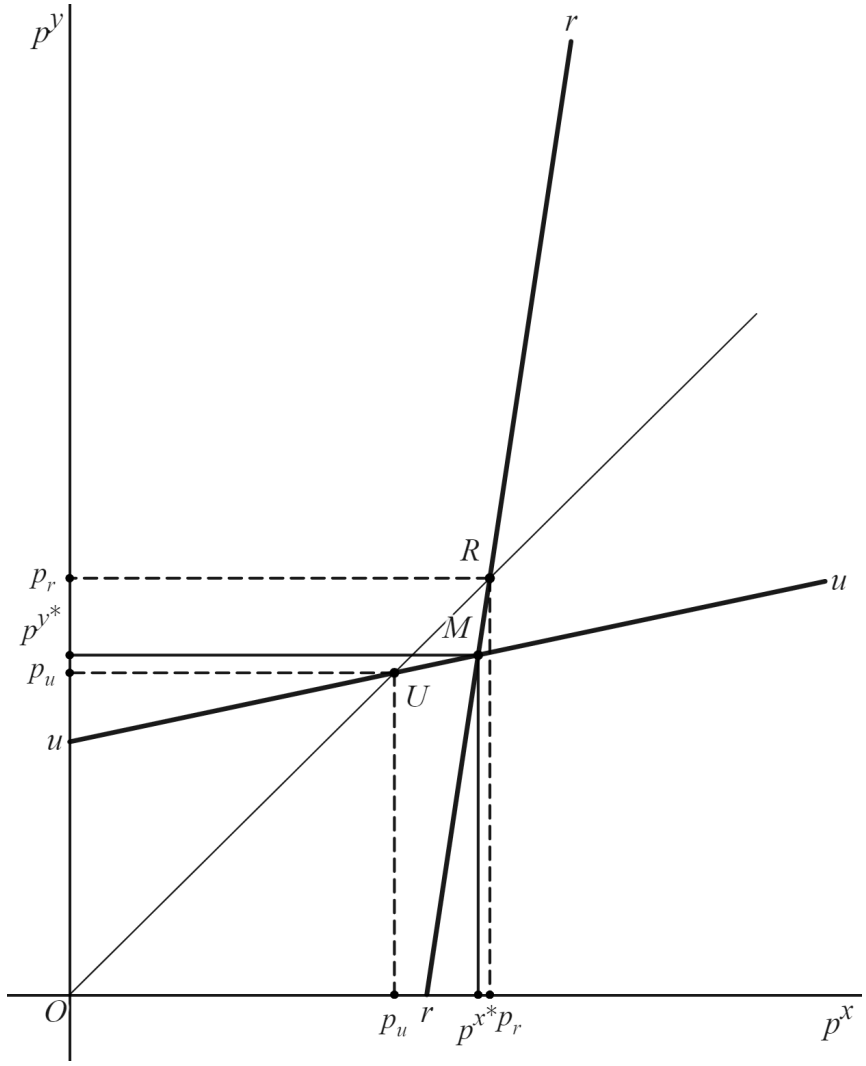

Figure 3: Differentiated Duopoly with Price Competition ($p_r > p_u$)

Consumer Surplus and Prices (contd.)

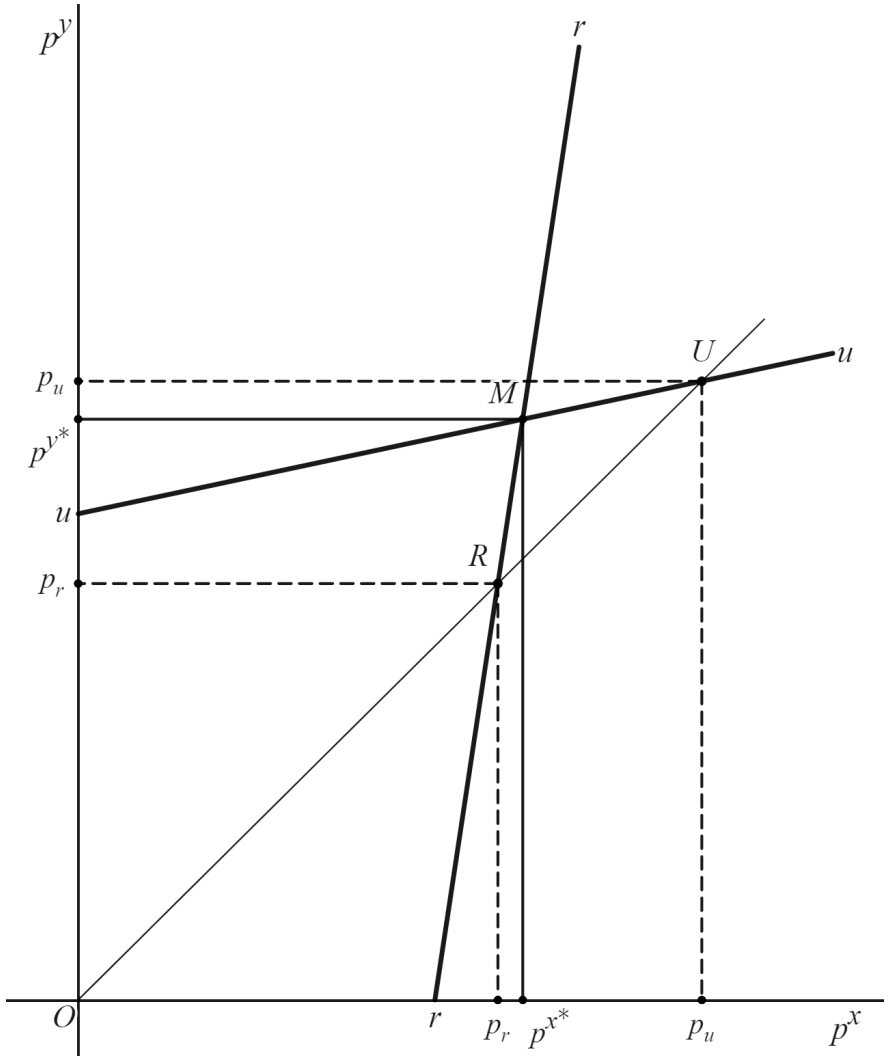

Figure 4: Differentiated Duopoly with Price Competition ($p_u > p_r$)

Consumer Surplus and Prices (contd.)

- The ranking of prices across three market regimes:

$$\min\{p_u, p_r\} < p^x < \max\{p_u, p_r\}$$
$$\min\{p_u, p_r\} < p^y < \max\{p_u, p_r\}$$

- Consumer surplus is decreasing in prices
- The ranking of consumer surplus is given by the reverse ranking of prices

Result

Proposition

Consider a differentiated duopoly which can be public, private, or mixed. Let CS_u^*, CS_r^* and CS_m^* denote the equilibrium consumer surplus corresponding to public, private and mixed duopoly, respectively. Then,

$$\min\{CS_u^*, CS_r^*\} \leq CS_m^* \leq \max\{CS_u^*, CS_r^*\}$$

where the equality holds if and only if $CS_u^* = CS_r^*$

Outline

Prevalence of Mixed Duopoly

- Consumer Surplus is not the metric, maybe welfare or **private firms' profits**? Individual firm-level incentive can block the higher consumer surplus generating privatization
- Success of privatization often depends on the competitive atmosphere post privatization. **Lack of competition** following privatization may limit its benefit

Outline

Individual Firm Size and Profitability

- Aggregate output is never the highest in mixed duopoly but it is not true for firm-level output
- **Firm-level output** is the **highest** in **mixed duopoly** for the **private firm** when private duopoly yields higher consumer surplus compared to public duopoly
- **Higher firm-level output** in **mixed duopoly** for private firm translates into **higher profit** for **private firm** in mixed duopoly compared to private firm in private duopoly

Individual Firm Size and Profitability (Continued)

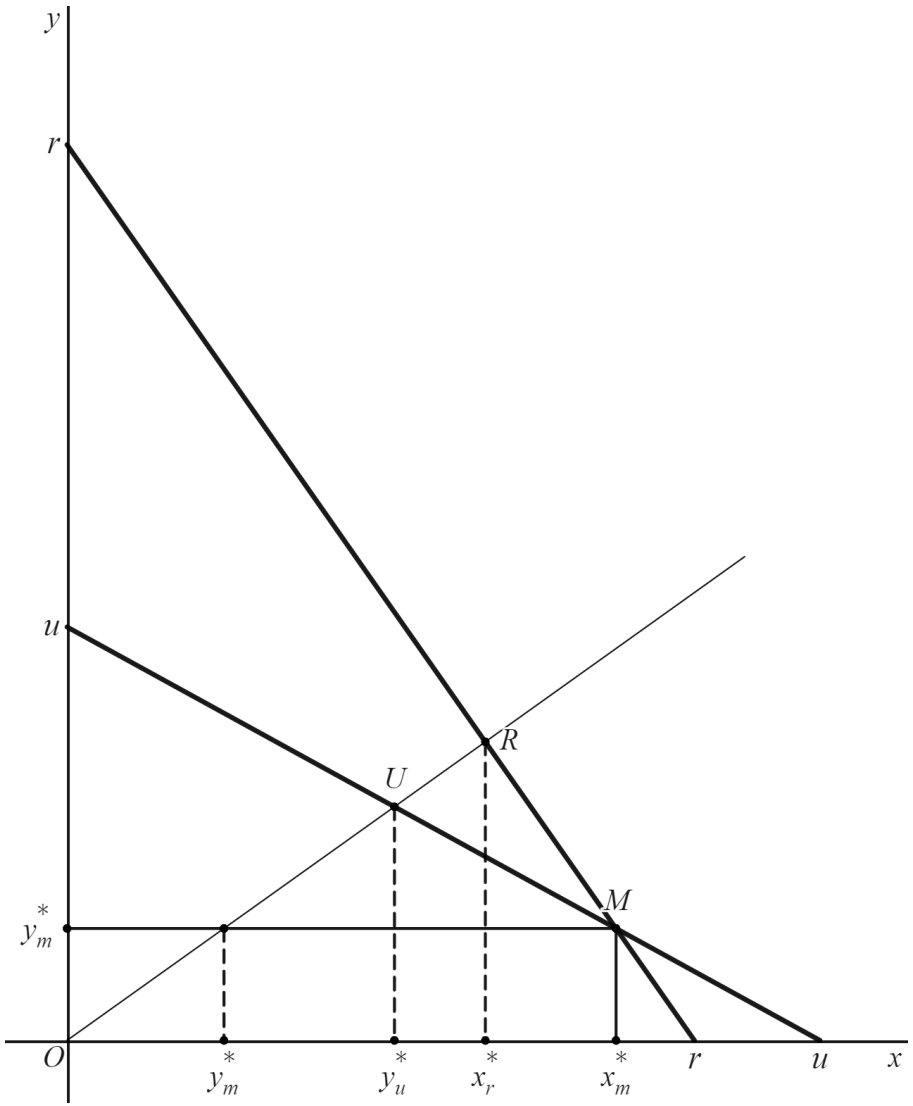

Figure 5: Ranking of Firm-level Output

Individual Firm Size and Profitability (Continued)

Proposition

Suppose consumer surplus is (uniquely) highest under private duopoly.Then, firm-level output as well as profit is highest when a firm is private and it operates in a mixed duopoly regime.

Outline

Competition

- **Efficiency gain** from privatization mainly stem from the **competition** among the efficient firms
- This gain fails to materialize in the absence of strong regulatory authority
- A **non-existent regulatory authority** can give incentive to the private firms to reap higher profit by forming a **collusion** among the firms

Competition (contd.)

- Private firm i places weight β on rival's profit and maximize weighted joint profit

$$\Pi_{r_i} \equiv \pi_{r_i} + \beta\pi_{r_j};\ i, j = 1, 2, i \neq j$$

- First order condition:

$$P(Q) + P'(Q)(x_{r_i} + \beta x_{r_j}) = C'_r(x_{r_i})$$

Competition (contd.)

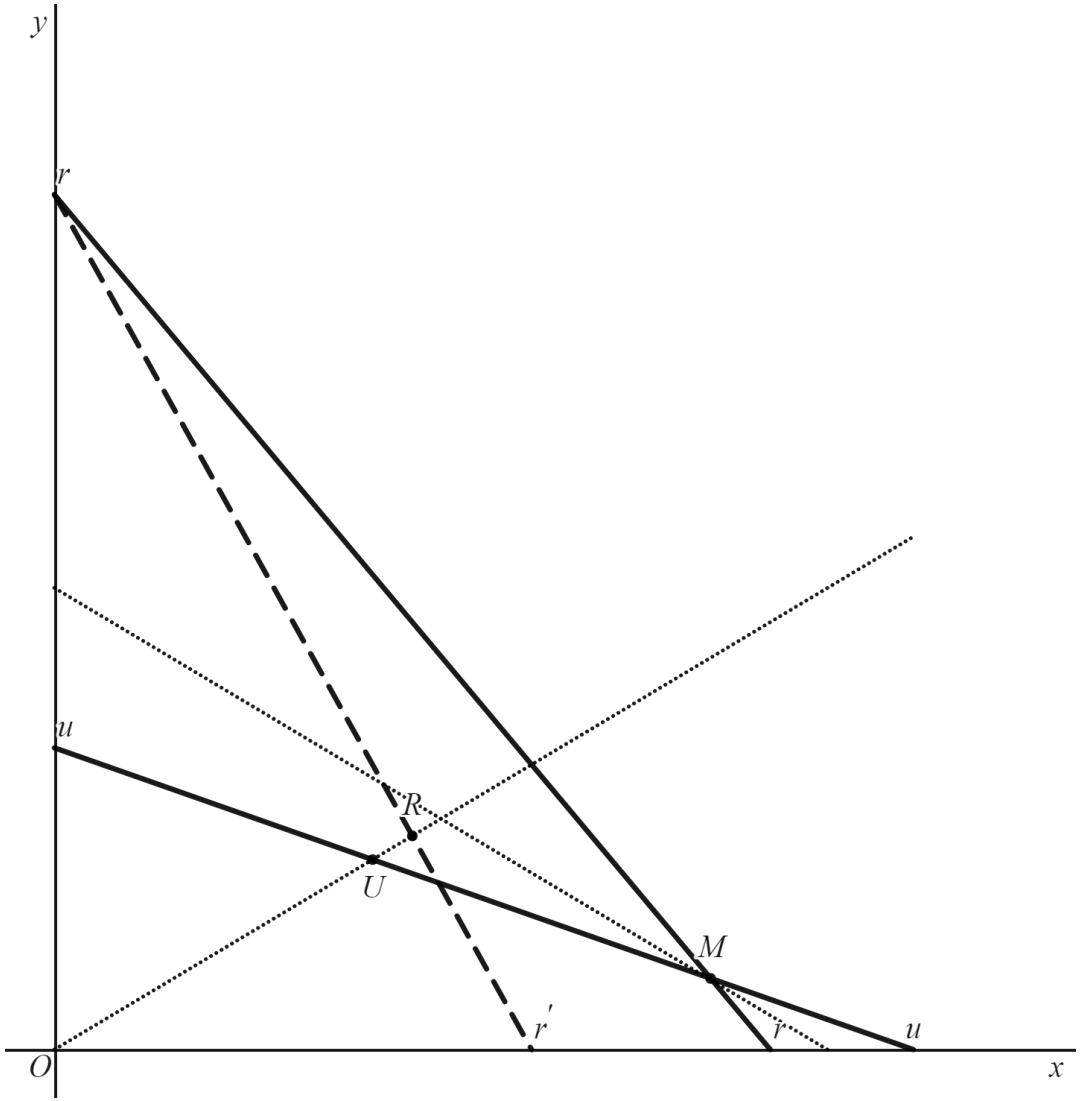

Figure 6: Lack of Competition

Competition (contd.)

Proposition

We assume linear demand and constant marginal costs - c_r and c_u for private and public firm respectively. Given cost inefficiency and profit orientation of the public firm, there exists a threshold level of $\beta - \hat{\beta}$, above which mixed duopoly gives higher consumer surplus compared to private duopoly. $\hat{\beta}$ is increasing in cost inefficiency and profit orientation of the public firm.

Outline

Alternative Objective Function

- Public firm maximizes consumer surplus subject to a break-even constraint (Bennett and La Manna, 2012)
- The first-order condition for the public firm gets replaced by the break-even condition:

$$P(Q)y - C_u(y) = 0$$

- Our main result holds in this specification as long as average cost $\frac{C_u(y)}{y}$ is increasing in y

Alternative Cost Function

- The new cost function for the public firm:

$$\tilde{C}_u(y) = (1-\lambda)C_u(y) + \lambda C_r(y)$$

- $\tilde{C}_u(.)$ suggests that as public firm becomes more profit-oriented (λ), its efficiency increases since $C_u(y) < C_r(y)$
- The new cost function inherits the same properties as $C_r(.)$ and $C_u(.)$, i.e. $\tilde{C}_u(0) = 0$, $\tilde{C}_u{}'(y) > 0$ and $\tilde{C}_u{}''(y) \geq 0$ for all $y \geq 0$.
- Our main result continues to hold with this new cost function, $\tilde{C}_u(y)$

Quality

- Higher prices lower consumer surplus but consumers are well compensated if higher prices are accompanied by higher quality products
- Adding quality dimension in cournot oligopoly
- Private firm offers high-quality, high-priced product whereas public firm offers low-quality, low-priced products
- Consumers pay a premium Δ for private firm's high quality products

Quality (contd.)

- Consumers' valuation, θ is distributed according to $F(\theta)$ in $\left[\underline{\theta}, \overline{\theta}\right]$
- If both firms are active in the market,

$$\theta + \Delta - p_r = \theta - p_u$$

- Common quality-adjusted price (ρ): ,

$$p_r - \Delta = p_u \equiv \rho$$

- Consumer with valuation θ buys the product if and only if $\theta \geq \rho$
- Aggregate demand function:

$$Q = 1 - F(\rho)$$

Quality (contd.)

- The objectives of the public and private firms remains the same.
- The first order conditions become:

$$\text{Private: } \rho(Q) + \Delta + \rho'(Q)x - C_r'(x) = 0$$
$$\text{Public: } \rho(Q) + \lambda\rho'(Q)y - C_u'(y) = 0$$

- Our result continues to hold when we add quality to the discussion

Outline

Conclusion

- Impact on consumers are important in policy decisions like privatization; more so for developing countries
- Which one is the best for consumer surplus – private, public, or a bit of both (mixed)?
- It might appear that mixed markets have the right balance: efficient private firms and welfare maximizing public firms
- We show that consumer surplus is typically not highest under mixed market
- Nevertheless, mixed markets exist presumably because privatization might be guided by other metrics. In terms of consumer surplus metric, mixed markets fare best when competition policies are lax