Eliciting Preferences in Random Satisficing Model*

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1 Introduction

Study of stochastic choice data is prevalent in analysis of economic decision-making. The existing literature on this is rich and powerful (see Block and Marschak, 1960; Luce and Suppes, 1965; Falmagne, 1978; Gul and Pesendorfer, 2006; Gul et al., 2014; Manzini and Mariotti, 2014; Aguiar et al., 2016). Several models of decision-making have been proposed and analysed to explain the variability in choice data. Additionally the literature of bounded rationality has introduced effects of cognitive constraints such as limited attention (see Manzini and Mariotti, 2014) or framing effects (see Rubinstein and Salant, 2012). Satisficing choice is one of the popular choice procedures (introduced by Simon (1955)) by which the decision-maker searches through available options until she finds a "good enough" one where she stops and chooses that alternative.

Due to its simple but intuitive appeal, satisficing choice rule has been extensively studied. However it has been observed that the model has less predictive power as the identification of the underlying primitives is not easy (for a discussion see Caplin et al. (2011)). In particular, it is impossible to identify if the underlying process is satisficing or utility maximization from a standard choice data. Further, as Aguiar et al. (2016) points out, without any further restrictions any stochastic choice data could be explained by a satisficing choice procedure. Thus it is desirable to find a testable model that *rationalizes* the observed stochastic choice by a satisficing process.

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* *Very preliminary.*

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In this paper we assume that the decision-maker has a given preference ordering and a satisficing level that is stochastic. It is possible to attribute such stochasticity to variable aspiration levels as in the example of "satisficing with perfect recall" illustrated in Rubinstein and Salant (2012). We assume that the decision-maker encounters the alternatives in an order and this order could be random. Examples of the situations where decision-makers face such ordered alternatives or *lists* are aplenty. For instance the products on shopping portals or in the supermarkets are organized in the form of lists. Further the menu cards in the restaurants offer the items available in the form of lists. It is also usual that such lists are variable or stochastic in nature. In any given list, the decision-maker searches sequentially until an alternative is found which has utility above the satisficing level. The decision-maker stops at that point and chooses that alternative. If the entire list is searched and no satisficing alternative is found then the best available option is chosen.

In line with the standard approach of revealed preference theory we assume that the underlying preference and random satisficing level are unobservable. We also assume that the *realized* list from which the choice is made is unobservable. However the distribution of lists is known to the analyst. In our model the choice is stochastic primarily because of two reasons: for a given list, the decision-maker has random threshold alternative and further the list is stochastic. We focus on two variants of random satisficing models which differ based on whether the preference ordering in general depends on the menu (i.e. the set from which the choice is made) or not. These models are called random satisficing model with menu-dependent preferences (RSM-MP) and random satisficing model (RSM).

Our main aim in this paper is to understand if this simple setting allows us to identify the underlying preference ordering from any choice data that is generated by a random satisficing model. Our notion of identification is the following: If a choice data has been generated by an RSM-MP or RSM model, if the underlying preference is unique. We say that the model is *generically* identified if the underlying preference is unique for the choice data generated by *almost*[1] any RSM-MP or RSM model. Our main result in this paper is that we are able to generically identify the model. We also provide comments on the general identification problem.

The closest to our paper is Aguiar et al. (2016), which identifies preference

[1] To be precise, we generically identify the model if we can show the uniqueness of the underlying preference for any choice data generated by any preference ordering and almost any random threshold function.

ordering only over the non-satisficing alternatives. Aguiar et al. (2016) however does not assume that the distribution of the orders over the set of alternatives is known to the analyst.

2 Definitions

2.1 Choice Data and the Satisficing Rule

Let X be a finite set of alternatives, with $|X| \geq 3$. A *menu* is a nonempty subset of X. Let $\mathcal{X} = 2^X \setminus \{\emptyset\}$ be the set of all menus. We consider a decision maker (henceforth, DM) who chooses an alternative out of those in a given menu. We assume that the DM has a strict, complete and transitive preference on X, denoted by $\succ$, with its weak part $\succsim$. Let $\mathcal{P}$ be the set of all such (strict) preferences on X. For each menu $A \in \mathcal{X}$, let $\succ|_A$ be the preference on A given by the restriction of $\succ \in \mathcal{P}$. A (stochastic) *choice function* or a *choice data* $p : X \times \mathcal{X} \to [0,1]$ specifies a probability distribution over each menu A such that $\sum_{a \in A} p(a, A) = 1$ and $p(b, A) = 0$ for every $b \in X \setminus A$.

For a given menu $A \in \mathcal{X}$, we assume that the DM encounters the alternatives in A as a sequence or list. Let $\mathcal{O}$ be the set of linear orders in X. When linear order $O \in \mathcal{O}$ represents the (deterministic) search order, "$a\ O\ b$" is interpreted as alternative a appearing prior to b in the list of alternatives. We assume that this order is fixed across all menus. When the DM follows the (deterministic) *satisficing rule* in a menu A, she has a *threshold* $t^* \in X \cup \{\varnothing\}$ along with her preference $\succ \in \mathcal{P}$ in mind. This threshold specifies the *satisficing set*, the set of alternatives weakly preferred to t^*, denoted by $S(\succ, t^*)$. When $t^* = \varnothing$, we assume that no alternative in X is satisficing under any preference. Thus, we have

$$S(\succ, t^*) := \begin{cases} \{a \in X \mid a \succsim t^*\} & \text{if } t^* \in X, \\ \emptyset & \text{if } t^* = \varnothing. \end{cases} \tag{1}$$

The satisficing rule chooses the first alternative among those in the satisficing set according to the search order O in A. If she finds no alternative in A is in the satisficing set, the satisficing rule chooses the most preferred alternative in A.[2] For simplicity, let $\bar{X} := X \cup \{\varnothing\}$. Then, for each $O \in \mathcal{O}$, each preference $\succ \in \mathcal{P}$ and each threshold $t^* \in \bar{X}$, the satisficing rule chooses the alternative from each

[2] Rubinstein and Salant (2012) call this rule the "perfect-recall satisficing."

menu $A \in \mathcal{X}$ given by

$$s(\succ, t^*; O, A) := \begin{cases} \max_O \big(S(\succ, t^*) \cap A\big) & \text{if } S(\succ, t^*) \cap A \neq \emptyset, \\ \max_\succ A & \text{if } S(\succ, t^*) \cap A = \emptyset. \end{cases} \tag{2}$$

2.2 Stochastic Search Order

In this paper, we consider a *stochastic search order* which remains the same across menus (Aguiar et al., 2016). Let γ be a probability distribution on $\mathcal{O}$. For each $A \in \mathcal{X}$, γ specifies the probability distribution over the search orders in each menu A. For each $a, b \in A$, with a slight abuse of notations, let $\gamma(a, b) := \sum_{O \in \{O' \in \mathcal{O} \,|\, aO'b\}} \gamma(O)$ be the probability that a appears prior to b in the menus containing both a and b. Note that this probability is independent of the menus. We assume that the analyst is able to observe the stochastic search order, but cannot observe the realized search order. Let Γ be the set of all stochastic search orders. We say that $\gamma \in \Gamma$ has a *full support* if for each $a \in X$, a is the first alternative with a positive probability, i.e., there exists $O \in \mathcal{O}$ such that $\gamma(O) > 0$ and $a\ O\ b$ for every $b \in X \setminus \{a\}$.[3] Let $\Gamma^{\text{fs}} \subset \Gamma$ be the set of all stochastic search orders with a full support.

2.3 Random Satisficing Rule

We consider the *random satisficing rule* in which the DM has a *random threshold* $t : \bar{X} \to [0, 1]$. For each $a \in \bar{X}$, $t(a)$ is the probability that a is the threshold in the satisficing behaviour. By definition, $\sum_{a \in \bar{X}} t(a) = 1$. Let $\mathcal{T}$ be the set of all random thresholds. We assume that the stochastic search order and the random threshold are distributed independently.

Given a stochastic search order $\gamma \in \Gamma$, for each preference $\succ \in \mathcal{P}$ and each random threshold $t \in \mathcal{T}$, the random satisficing rule induces a stochastic choice function given by a convex combination of the deterministic satisficing rules weighted by γ and t. For each menu $A \in \mathcal{X}$ and each $a \in A$, let $\sigma^\gamma(a, A; \succ, t) \in [0, 1]$ be the probability that a is chosen by the random satisficing rule with random threshold t in A under preference $\succ$, where $\sum_{a \in A} \sigma^\gamma(a, A; \succ, t) = 1$ holds. Formally, we have

$$\sigma^\gamma(a, A; \succ, t) := \sum_{O \in \mathcal{O}} \gamma(O) \sum_{t^* \in \bar{X}} t(t^*) \mathbf{1}\{a = s(\succ, t^*; O, A)\}.$$

[3] We follow the definition by Aguiar et al. (2016). Note that if γ has a full support, there may be $O \in \mathcal{O}$ such that $\gamma(O) = 0$.

We consider two models of random satisficing behaviour. In the second model, the DM's preference and the random threshold are fixed throughout all menus.

Definition 2.1 (RSM)**.** A choice data $p(a, A)$ has a *random satisficing model* (*RSM*) representation with stochastic search order $\gamma \in \Gamma$ if there exist a preference $\succ \in \mathcal{P}$ and a random threshold $t \in \mathcal{T}$ such that $p(a, A) = \sigma^{\gamma}(a, A; \succ, t)$ for every menu $A \in \mathcal{X}$ and every alternative $a \in A$.

The second model is much more permissive, and allows the DM's preference and the random threshold to change depending on the menus she faces.

Definition 2.2 (RSM-MP)**.** A choice data $p(a, A)$ has a *random satisficing model with menu-dependent preferences* (*RSM-MP*) representation with stochastic search order $\gamma \in \Gamma$ if for each menu $A \in \mathcal{X}$, there exist a preference $\succ \in \mathcal{P}$ and a random threshold $t \in \mathcal{T}$ such that $p(a, A) = \sigma^{\gamma}(a, A; \succ, t)$ for every alternative $a \in A$.

2.4 Rationalisation and Identification

Suppose that the stochastic search order $\gamma \in \Gamma$ is fixed. For each stochastic choice function $p(a, A)$ and each menu $A \in \mathcal{X}$, we say that the DM's preference $\succ \in \mathcal{P}$ *rationalises p in menu A* if there exists a random threshold $t \in \mathcal{T}$ such that the random satisficing rule induces p when the menu is A, i.e. $\sigma^{\gamma}(a, A; \succ, t) = p(a, A)$ for every $a \in A$. Let $T^A(p, \succ) := \{t \in \mathcal{T} \mid \sigma^{\gamma}(a, A; \succ, t) = p(a, A), \forall a \in A\}$ be the set of random thresholds which induce choice data p under preference $\succ$. By definition, p is rationalised by $\succ$ in menu A if and only if $T^A(p, \succ)$ is nonempty. The DM's preference $\succ \in \mathcal{P}$ *RSM-rationalises p* if there exists a random threshold $t \in \mathcal{T}$ fixed across all menus such that the random satisficing rule induces p, i.e. $\bigcap_{A \in \mathcal{X}} T^A(p, \succ) \neq \emptyset$. For each stochastic choice function $p(a, A)$ and each menu $A \in \mathcal{X}$, we say that the DM's preference $\succ \in \mathcal{P}$ *uniquely rationalises p in menu A* if another preference $\succ' \in \mathcal{P}$ rationalises $p(a, A)$ in A, then $\succ|_A = \succ'|_A$. The DM's preference $\succ \in \mathcal{P}$ *uniquely RSM-rationalises p* if no other preference RSM-rationalises p.

The idea of identification of the models is that any choice data induced by the random satisficing rule is uniquely rasionalised. If this property holds for choice data induced from almost all random thresholds, we say that the model is "generically" identified.

Definition 2.3 (Identification)**.** Let $\gamma \in \Gamma$ be a given stochastic search order.

1. The RSM-MP is *identified* if for each menu $A \in \mathcal{X}$, each preference $\succ \in \mathcal{P}$ and every random threshold $t \in \mathcal{T}$, the preference $\succ$ uniquely rationalises the choice data $\sigma^\gamma(a, A; \succ, t)$ induced in A by the random satisficing rule under $\succ$.
2. The RSM is *identified* if for each preference $\succ \in \mathcal{P}$ and every random threshold $t \in \mathcal{T}$, the preference $\succ$ uniquely RSM-rationalises the choice data $\sigma^\gamma(a, A; \succ, t)$.
3. The RSM-MP is *generically identified* if for each menu $A \in \mathcal{X}$, each preference $\succ \in \mathcal{P}$ and almost every random threshold $t \in \mathcal{T}$ (in the sense that the following property holds except on a subset with Lebesgue measure zero in $\mathcal{T}$), the preference $\succ$ uniquely rationlises the choice data $\sigma^\gamma(a, A; \succ, t)$ in A.
4. The RSM is *generically identified* if for each preference $\succ \in \mathcal{P}$ and almost every random threshold $t \in \mathcal{T}$, the preference $\succ$ uniquely RSM-rationalises the choice data $\sigma^\gamma(a, A; \succ, t)$.

Since RSM-MP is more permissive than RSM, if RSM-MP is (generically) identified, then RSM is also (generically, resp.) identified.

3 Preliminary

3.1 Equivalent Thresholds

The random satisficing rules with distinct preferences or distinct thresholds can induce the same choice data. In this subsection, we characterise such cases.

There are two cases in which the (non-random) satisficing rules induce the same choice data. First, by the definition (2), the outcome given by the satisficing rule depends on the satisficing set (1), rather than the threshold, as well as the preference, and if the satisficing set has a nonempty intersection with the menu, the outcome depends only on the satisficing set. For example, if the threshold t^* is the worst alternative according to a preference $\succ$, the satisficing set is X, and the satisficing rule picks up the first alternative in the list. This behaviour does not depend on the DM's preference itself, but on the fact the satisficing set is the entire set of alternatives. Second, the most-preferred alternative, say, a^* in menu A is chosen both when the threshold is a^* with the satisficing set being $\{a^*\}$, and when the satisficing set is empty.

For each menu $A \in \mathcal{X}$, each preference $\succ \in \mathcal{P}$ and each threshold $t^* \in \bar{X}$, let a revised threshold be

$$\tau(t^*;\succ,A) := \begin{cases} t^* & \text{if } t^* \in A, \\ \min_{\succ}\{a \in A \,|\, a \succ t^*\} & \text{if } t^* \in X \setminus A \text{ and } \{a \in A \,|\, a \succ t^*\} \neq \emptyset, \\ \max_{\succ} A & \text{if } t^* = \varnothing \text{ or } \{a \in A \,|\, a \succ t^*\} = \emptyset. \end{cases}$$

As an abuse of the notation, for a random threshold $t \in \mathcal{T}$, let $\tau(t;\succ,A)$ the random threshold defined accordingly. Note that, for each $t \in \mathcal{T}$, each $\succ \in \mathcal{P}$ and each $A \in \mathcal{X}$, the support of $\tau(t;\succ,A)$ is a subset of A. Then, the observations in the last paragraph shows that the random satisficing rule induces the same choice date with random thresholds t and $\tau(t;\succ,A)$.

Lemma 3.1. *For each stochastic search order $\gamma \in \Gamma$, each preference $\succ \in \mathcal{P}$, each menu $A \in \mathcal{X}$, and each random threshold $t, t' \in \mathcal{T}$, if $\tau(t;\succ,A) = \tau(t';\succ,A)$, then $\sigma^\gamma(a,A;\succ,t) = \sigma^\gamma(a,A;\succ,t')$ for each $a \in A$.*

When the stochastic search order γ has a full support, it is straightforward to see that the converse is also true.

Lemma 3.2. *For each stochastic search order $\gamma \in \Gamma^{\mathrm{fs}}$, each preference $\succ \in \mathcal{P}$, each menu $A \in \mathcal{X}$, and each random threshold $t, t' \in \mathcal{T}$, if $\sigma^\gamma(a,A;\succ,t) = \sigma^\gamma(a,A;\succ,t')$ for each $a \in A$, then $\tau(t;\succ,A) = \tau(t';\succ,A)$.*

Given these results, we say that two random thresholds $t, t' \in \mathcal{T}$ are *equivalent* under preference $\succ \in \mathcal{P}$ in menu A if $\tau(t;\succ,A) = \tau(t';\succ,A)$.

3.2 Virtual Satisficing Set

We observe that the outcome induced by the satisficing rule depends on the satisficing set with an exception that the empty satisficing set and the singleton satisficing set lead to the same alternative. Under any stochastic search order, the analyst can never distinguish the DM with no satisfactory alternative from the one with a single satisfactory alternative which is the most preferred in the menu.

For each menu $A \in \mathcal{X}$, each preference $\succ$ and each threshold $t^* \in \bar{X}$, we say that a nonempty subset $\tilde{S}^A(\succ,t^*)$ of A is the *virtual satisficing set* in A if

$$\tilde{S}^A(\succ,t^*) = \begin{cases} S(\succ,t^*) \cap A & \text{if } S(\succ,t^*) \cap A \neq \emptyset, \\ \{\max_{\succ} A\} & \text{if } S(\succ,t^*) \cap A = \emptyset. \end{cases}$$

When the satisficing set $S(\succ, t^*)$, defined in (1), has a nonempty intersection with A, the virtual satisficing set is the same as the satisficing set in A. When there is no intersection, we let the virtual satisficing set be the singleton for the sake of convenience. Since this replacement does not affect the observed satisficing behaviour, the virtual satisficing set simplifies and helps our analysis.

As an abuse of the notation, for each random threshold $t \in \mathcal{T}$, $\tilde{S}^A(\succ, t)$ be the probability distribution of the virtual satisficing sets. By Lemmas 3.1 and 3.2, we have the following observations.

Lemma 3.3. *For each stochastic search order $\gamma \in \Gamma$, each preference $\succ \in \mathcal{P}$, each menu $A \in \mathcal{X}$, and each random threshold $t, t' \in \mathcal{T}$, if $\tilde{S}^A(\succ, t) = \tilde{S}^A(\succ, t')$, then $\sigma^\gamma(a, A; \succ, t) = \sigma^\gamma(a, A; \succ, t')$ for each $a \in A$.*

Lemma 3.4. *For each stochastic search order $\gamma \in \Gamma^{\mathrm{fs}}$, each preference $\succ \in \mathcal{P}$, each menu $A \in \mathcal{X}$, and each random threshold $t, t' \in \mathcal{T}$, if $\sigma^\gamma(a, A; \succ, t) = \sigma^\gamma(a, A; \succ, t')$ for each $a \in A$, then $\tilde{S}^A(\succ, t) = \tilde{S}^A(\succ, t')$.*

4 Identification in Random Satisficing Rule

4.1 Stochastic Search Order with a Full Support

In this section, we assume that the given stochastic search order has a full support: $\gamma \in \Gamma^{\mathrm{fs}}$.

Proposition 4.1. *For each $\gamma \in \Gamma^{\mathrm{fs}}$, RSM-MP is generically identified.*

Proof of Proposition 4.1. Fix any menu $A \in \mathcal{X}$. Since $\gamma \in \Gamma^{\mathrm{fs}}$, it is clear that for each $t^* \in X$ and each $a \in X$, $\sigma^\gamma(a, A; \succ, t^*) > 0$ if and only if $a \succsim t^*$. This implies that the $|X|$ stochastic choice functions in $\{\sigma^\gamma(\cdot, A; \succ, t^*) \,|\, t^* \in X\}$ constitute vertices of a nondegenerate $(|X|-1)$-dimensional simplex. For each preference $\succ \in \mathcal{P}$ and each menu $A \in \mathcal{X}$, let $\bar{P}^\gamma(\succ; A) := \{\sigma^\gamma(\cdot, A; \succ, t) \,|\, t \in \mathcal{T}\}$ be this simplex, i.e., the convex hull of $|X|$ stochastic choice functions in $\{\sigma^\gamma(\cdot, A; \succ, t^*) \,|\, t^* \in X\}$. By Lemma 3.1, $\bar{P}^\gamma(\succ; A)$ is the set of choice data induced by the random satisficing model with some random thresholds under $\succ$. Consider two distinct preferences $\succ, \succ' \in \mathcal{P}$ such that $\succ \neq \succ'$. Then there exists $t^* \in X$ such that neither $\{a \in A \,|\, a \succsim t^*\} \subseteq \{a \in A \,|\, a \succsim' t^*\}$ nor $\{a \in A \,|\, a \succsim t^*\} \supseteq \{a \in A \,|\, a \succsim' t^*\}$. For such t^*, by Lemma 3.2, $\sigma^\gamma(\cdot, A; \succ, t^*) \notin \bar{P}^\gamma(\succ'; A)$ and $\sigma^\gamma(\cdot, A; \succ', t^*) \notin \bar{P}^\gamma(\succ; A)$. Therefore, two distinct simplices $\bar{P}^\gamma(\succ; A)$ and $\bar{P}^\gamma(\succ'; A)$ do not share at least a vertex, and $\bar{P}^\gamma(\succ'; A)$ has no intersection with the interior of $\bar{P}^\gamma(\succ; A)$. Since this holds for each pair of distinct preferences $\succ, \succ' \in \mathcal{P}$, if p is in the interior of $\bar{P}^\gamma(\succ; A)$, $\succ$ uniquely rationalises p in A. Since the interior of $\bar{P}^\gamma(\succ; A)$ is induced by all random thresholds $t \in \mathcal{T}$ such that $t(t^*) > 0$ for every $t^* \in \bar{X}$, RSM-MP is generically identified. $\square$

Proposition 4.1 states identification in a generic sense. It is, however, of interest what happens in "non-generic" cases. The observations in Section 3 show that the satisficing behaviour just depends on the virtual satisficing set, and thus if there are multiple preference-thresholds pairs that generate the same virtual satisficing set, the model cannot be identified. By Lemma 3.4, this natural limitation is the only reason for non-identification under the stochastic search order with a full support. This is formally stated as follows.

Proposition 4.2. *For each $\gamma \in \Gamma^{\mathrm{fs}}$, each menu $A \in \mathcal{X}$ and each preference $\succ, \succ' \in \mathcal{P}$, if both $\succ$ and $\succ'$ rationalise the same choice data p in A, then for each random threshold $t \in T^A(p, \succ)$, there exists $t' \in T^A(p, \succ')$ such that $\tau(t; \succ, A) = \tau(t; \succ', A)$ and $\tilde{S}^A(t; \succ, A) = \tilde{S}^A(t'; \succ', A)$.*

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