Patriarchy, Gender Norms and Household Labor Supply*

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Abstract

Generationally-linked households decide on consumption, time allocation and investment in child quality. The decision-problem is a two-stage game between the spouses with husbands first choosing whether or not to be patriarchal. Patriarchal husbands derive a payoff from asserting their right to be the primary decision-maker, and they make consumption and time allocation offers that wives can accept or reject. The former entails higher specialization in child quality production, restricted (possibly zero) labor supply and lower consumption for wives. The latter entails non-cooperative decision-making where child quality may suffer. When the husband opts not to be patriarchal, the spouses make choices jointly by pooling household income. The aggregate technology combines perfectly substitutable male and female labor, but firms may discriminate against the latter. The dynamic equilibrium typically features a secular transition from patriarchy to cooperative decision-making with a concomitant increase in female labor supply. For strong enough patriarchal identity, the transition path does not involve non-cooperative decision-making; for particularly strong identity payoff, a patriarchy trap may result. Transition out of patriarchy is delayed for lower female wages, higher labor market discrimination against women and higher return to education that ties women down to child quality production. We also examine the effect of women's identity formation from market employment.

KEYWORDS: Female Labour Force Participation, Patriarchy, Norms, Gender Wage Gap

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1 Introduction

This paper studies the effect of patriarchal norms on household decision-making and female labor supply. Our goal is to understand how low and stagnant female labor force participation in developing countries is the outcome of gender norms, and how economic forces shape those norms over time.

We construct a parsimonious model of household behavior where the choice to conform to and enforce gender norms is rationally made. Identical intergenerational households comprise of a husband (man/male), a wife (woman/female) and children. Households decide on the consumption of each spouse and the household public good, child quality, whose production depends on education expenditure and parental time towards child nurture. The latter comes at the cost of foregone labor earnings, and child-rearing efficiency may differ between the spouses. The household decision problem is a two-stage game between the spouses. A key innovation of the model is the choice husbands make regarding their identity, patriarchal versus egalitarian.

A patriarchal husband secures an identity payoff from asserting his right to be the primary decision-maker, and makes his wife a consumption-time allocation offer that she can either accept or reject. Acceptance entails residual claim to the household surplus, with the wife devoting disproportionate time towards child nurture and little, if any, time to market work. An offer that is rejected by the wife, on the other hand, leads to non-cooperative decision-making where each spouse maximizes their own utility taking as given the contribution by their spouse to child quality. Conversely, a household where the husband chooses to be egalitarian towards his wife functions akin to the unitary paradigm: spouses pool their incomes and jointly make consumption, education spending and child-nurture decisions.[1]

The household equilibrium, consequently, can be in of three regimes: unitary, patriarchal and non-cooperative. The exact outcome depends on the patriarchal identity payoff, which may or may not be conditioned by social norms, differential market returns to male and female labor, and relative efficiency in home production. When the female-to-male wage is sufficiently low, the prevalent outcome is patriarchy. At this low relative female wage, the reservation utility of women under the non-cooperative regime is so low that the husband is able to compensate her sufficiently while positioning himself as the patriarchal type. As the female wage rate improves relative to the male wage rate, the woman's participation constraint tightens. Yet, her income may

[1] Intra-household decision-making is often not unitary. In many such non-unitary models, the woman's bargaining power usually depends on her utility at the threat point, that is, her outside option. Majlesi (2016) studies this effect through exogenous labor demand shocks on women's jobs in the Mexican manufacturing sector. With better employment opportunities, women were found to exercise more decision-making power with regard to their private goods and services, major household expenditures and children's health. Increased demand for women's jobs was also associated with better health outcomes for children.

not be sufficiently high for the husband to cede his decision-making prerogative and benefit from income pooling as a unitary household. It is only when female wages rise sufficiently that pooling is the best action for the husband and the unitary household prevails.

This household model is incorporated into a dynamic framework that allows for human capital, wages and labor supply to change over time. We introduce an added feature, preference-based discrimination against female workers by firms. Following the seminal work of Becker (1971), we also assume that there is an additional cost to the firm of this unethical discrimination. While male and female labor are perfect substitutes in the production process, labor productivity that increases with the level of skill is assumed to be gender specific. This assumption is in line with the observed phenomenon that when required human capital levels are low, physical labor which differs among men and women can be a determining factor of productivities. Precisely, in those non-cognitive jobs, men experience a endogenously determined higher productivity. In this paper, our focus is on the discrimination within a generation ('horizontal' per se) instead of parental discrimination ('inter-generational'/'vertical') in terms of investment in education as well as child-rearing time between daughters and sons since the model guarantees the same human capital for girls and boys in every generation.

An increase in the level of human capital that reduces the gender wage gap induces a transition in household regimes over time. We observe that low education is associated with the prevalence of patriarchy, high education with unitary decision making. The intermediate range of human capital can lead to non-cooperative bargaining. Specifically, the dynamic equilibrium path features a secular transition from patriarchy to cooperative decision-making with a concomitant increase in women's labor supply. But when patriarchal identity is strong, the transition path does not involve non-cooperative decision-making; for particularly strong identity payoff, a patriarchy trap may result. In fact, an extremely high degree of market discrimination too may result in perpetual patriarchy and therefore very low level of female labor force participation even though there is no parental discrimination towards educating girls versus boys. Finally, our analysis clearly shows that transition out of patriarchy is delayed for higher returns to education as it ties women down to child quality production. It is also delayed for lower female wages and higher labor market discrimination as it lowers the pace of increase in the female wage rate relative to human capital increases.

Much has been written on female labor force participation, particularly in the context of developing countries where the gap relative to men is unusually large. In the labor market, demand-side discrimination by firms as well as supply-side constraints involving family decisions and societal norms have been identified as causes of differential participation rates (Klaesen, 2019). The relation between gender inequality in labor market outcomes and economic development operate through a multitude of factors. Kleven and Landais (2017) look at a composite database of 53 countries spanning 1967–2014, and find a large convergence in earnings of men and women over the path of development. This convergence primarily happens through wages and labor force participation, but the gap in hours worked changes little. Changes in educational attainment were found to have a modest direct effect compared to changes in fertility. However, an indirect effect of education is possible: educated parents typically also desire higher education for their children, underscoring a switch from child quantity to quality

Changing norms about women working outside the home also influence their work decisions. As Fernandez *et al.* (2004) show, a growing number of "modern men" with less traditional gender-role attitudes is a significant factor in the rise in female labor force participation. They also find evidence of intergenerational transmission of such values, wherein having a working mother affects a man's preferences about his wife's working behavior. A wide range of literature studies evidence of how norms dictate women's choices and household behavior. In a comparison between two ethnic groups in Burkina Faso, one of which is more constrained by patriarchal norms, Kevane and Wydick (2001) look at women's time allocation choices. They find that the time that women spend working in occupations ordained by the social norm is irresponsive to economic factors such as changes in the marginal productivity.

Gender norms often manifest in the form of societal behavioral prescriptions. One such dominant norm prevalent in developing societies is the notion that "a man should earn more than his wife" Bertrand *et al.* (2015) find evidence of the implications of such a norm in a sharp drop of the relative income distribution at half. Marriages where the woman is likely to earn more than her husband are less likely to happen. Even if they do, it is found that women often stay out of the labor force or "underwork" at a level below her potential earnings in order to comply with the norm. In couples where the wife does earn more, she also spends more time on household chores and child-rearing. Many such norms, which had likely been dormant till now, are starting to get threatened and more likely to bind as women's wages rise.

The effect on marriage rates is also studied by Bertrand *et al.* (2021) who lay down a theoretical model with random matching to explain the different trends for skilled and unskilled women. Social norms contribute to lower marriage rates for skilled women compared to unskilled women. The relationship between labor market outcomes and marriage rates for skilled women has been observed to be non-monotonic with skilled women in the US lately witnessing a revival of their marriage rates. A marriage penalty might also influence women's educational decisions as they balance labor and marriage market considerations.

Theoretical work assessing the connection between gender norms and female labor supply is relatively sparse. Close in spirit to our work is Ghosh and Thomas's (2020) framework of household decision making. They contrast choices made under patriarchy where the husband can prevent the wife from participating in the labor market, and conditional on that decisions are arrived at through household bargaining. The lack of independent income, in particular, eliminates the wife's bargaining power. While zero labor supply may be an equilibrium outcome of a patriarchal household in our model, the wife always retains some leverage. Moreover, different types of household behavior are nested in our framework and the dynamics focuses on its transition.

Also relevant is Doepke and Tertilt's (2009) analysis of how economic rights shifted towards married women in late nineteenth and early twentieth century advanced economies. In their model, even though men prefer to have more bargaining power for themselves, they may voluntarily relinquish some of that power in anticipation that their daughters, who they value just as much as their sons, would benefit in the future from greater say in household matters. The strength of this motive depends on returns to education and, therefore, it is the rise of skill-based economies that accounts for the transition in women's rights. A similar transition emerges in our model as men give up their patriarchal rights; it does not require mothers to value child quality more than fathers as in Doepke and Tertilt.

The rest of the paper is organized as follows. The household model is presented in Section 2 and equilibrium choices determined in Section 3. Section 4 introduces the production side of the economy that is then used for the dynamic analysis in Section 5. Section 6 concludes. An Appendix contains details of the results presented in Section 3.

2 The model

Time is discrete, $t = 1, 2, \dots \infty$, and the economy is populated by overlapping generations of households. Each household comprises of a couple, a man (husband) and a woman (wife). Individuals live for two periods, a passive childhood when they receive education and an active adulthood when they enter into a match with another adult to form a household, have $n \geq 1$ children, and make consumption, labor supply and child investment decisions. Parents value their own consumption and the quality or human capital of their children. While their own consumption yields a private payoff, child quality is a household public good. The production of child quality takes as inputs educational expenses and time spent by each parent on child-rearing.

The utility functions of person i, where $i = f, m$ for female and male respectively, who is an adult at time t is given as:

$$U_t^i = \ln c_t^i + \kappa_c \ln q_t \tag{1}$$

where child quality is produced according to:

$$q_t = \gamma(\bar{\tau} + \alpha_m \tau_t^m + \alpha_f \tau_t^f)^\theta (\bar{e} + e_t)^{1-\theta}, \quad \theta \in (0,1). \tag{2}$$

Here τ_t^m and τ_t^f denote time investment on each child by the man and the woman respectively, and e_t denotes educational expenditure. This expenditure is on a market-purchased input that is priced at p_e (taking the consumption good as numeraire). The positive intercepts in (2)m $\bar{\tau}$ and $\bar{e}$, ensure that a minimum level of child quality is produced even in the absence of discretionary parental contributions. This can be thought of as learning and experience acquired through interaction with one's social environment and peers as well as innate human capital. Even though the time spent by parents are perfect substitutes, the productivities of men and women can differ. Note that equation (1) shows that preferences are identical for both genders. Moreover, the elasticities of substitution both between consumption and child quality in the utility function as well as between the effective time input and education in child quality production are 1. It is assumed that parents treat all their children equally. There is no gender-based discrimination within the household. The number of children n is taken to be exogenous.

The decision problem that a household solves is determined by the outcome of a two-stage game between the spouses. Central to this approach is the assumption that husbands can choose whether or not be patriarchal, and behaving as a patriarch of the family means they exert their right to be the primary decision-maker.

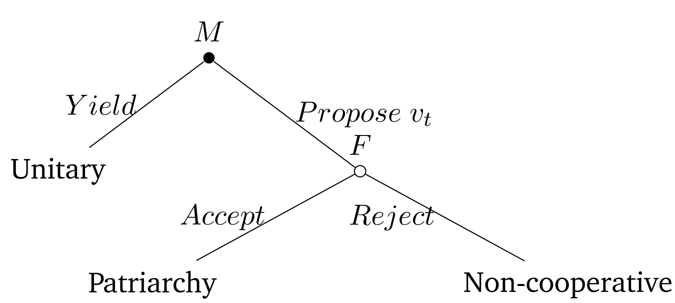

Figure 1: Game tree: Mode of decision-making

Accordingly, the husband moves first and chooses between two actions as outlined in Figure 1. If he relinquishes his patriarchal voice ("yield"), the spouses subsequently behave as a unitary household by pooling incomes and maximizing the sum of spousal utilities. If the husband exercises his patriarchal voice ("propose"), he proposes a vector $v_t \equiv \{c_t^m, c_t^f, e_t, \tau_t^m, \tau_t^f\}$, that is, an intra-household distribution of time and income. The wife can then either accept or reject the proposed v_t. If she accepts, then her payoff is realized according to v_t. Moreover, the man gets an additional patriarchal payoff $I_p \ln(\mu n \tau_t^f)$ from asserting his identity of the patriarchal head and adhering to the social norm that men are supposed to be the primary breadwinners of their families. The payoff is decreasing in the fraction of her time the wife spends working outside the home, thus reflecting the extent to which the husband is able to adhere to the norm. The payoff is positive only if the wife spends a minimum fraction $\frac{1}{\mu}$ of her time on the home good.

Were the wife to reject the husband's offer of v_t, household decision-making breaks down to a non-cooperative Cournot-Nash equilibrium, similar to the threat point described in Lundberg and Pollak (1993). In this equilibrium, both spouses may contribute towards their children in terms of educational expenditure and parental time, but each of them maximizes their own utility taking as given the actions of the other. It is to be noted that the wife's effective bargaining power is equilibrium dependent and can be viewed as endogenous in this structure. In particular, when the husband assumes a patriarchal identity, he relegates his wife to her reservation utility, in contrast to a more "egalitarian" decision-making process when he relinquishes his power to be the primary decision-maker.

The following subsections look at the three possible equilibrium outcomes in more detail.

2.1 Non-cooperative outcome

Each spouse maximizes his/her own personal utility taking as given the contributions of the other towards the household public good, child quality. This setup takes after the threat point depicted in Lundberg and Pollak (1993), but does not assume any 'separate spheres' with regard to scope of contribution. It allows for both spouses to contribute to their children both in terms of time spent in upbringing and educational expenditure. However, a separation of spheres wherein the educational expenditure and child-rearing time are borne entirely by separate parents may possibly happen at equilibrium.

Spouse i faces the problem of choosing $\{c_t^i, e_t^i, \tau_t^i\}$ maximizing (1) subject to their own budget constraint. Total educational spending on each child is $e_t = e_t^m + e_t^f$. The budget constraints for the husband and wife are

$$c_t^m + np_e e_t^m \leq (1 - n\tau_t^m) w_t^m, \tag{3}$$

$$c_t^f + np_e e_t^f \leq (1 - n\tau_t^f) w_t^f, \tag{4}$$

respectively. The optimization by each spouse yields response functions of spousal contributions to child quality, that is, $e_t^i = e_t^i(e_t^i, \tau_t^{-i})$ and $\tau_t^i = \tau_t^i(e_t^{-i}, \tau_t^{-i})$ for $i \in \{m, f\}$. A Cournot-Nash equilibrium would thus involve

$$e_t^m = e_t^m(e_t^f, \tau_t^f), \quad e_t^f = e_t^f(e_t^m, \tau_t^m) \tag{5}$$

$$\tau_t^m = \tau_t^m(e_t^f, \tau_t^f), \quad \tau_t^f = \tau_t^f(e_t^m, \tau_t^m) \tag{6}$$

The consumption levels are subsequently determined from the corresponding budget equations.

In the non-cooperative situation, the household does not engage in pooling of income. Hence it is necessary for both spouses to work ($\tau_t^m < 1/n, \tau_t^f < 1/n$) to be able to afford positive consumption. The specification for child quality production, however, allows for corner solutions in terms of education and time inputs by each parent. Notably, it is observed that both spouses extend positive contributions towards their children's education ($e_t^m > 0$, $e_t^f > 0$), and the household behaves as if it were pooling income. The consumption and, subsequently, utilities for both spouses are equal, irrespective of their own wages. The optimality conditions with respect to c_t^i and e_t^i turn out to be

$$\frac{1}{c_t^i} = \lambda_t^i, \tag{7}$$

$$\frac{\kappa_c(1-\theta)}{\bar{e} + e_t^m + e_t^f} = \lambda_t^m np_e = \lambda_t^m np_e, \tag{8}$$

respectively, where the λ's are the Lagrange multipliers associated with spousal budget constraints. Equations (7) and (8) together imply $\lambda_t^m = \lambda_t^f$ and $c_t^m = c_t^f$. Consequently, $U_t^m = U_t^f$. The pooling happens indirectly through the effect of the wage rate on the contribution to education. A rise in each spouse's own wage rate causes them to contribute more to education, and allows their spouse to cut back on their contribution to education and spend more on consumption. This is evident from the expressions for the contribution to education in such a case shown in Table 1, where they are decreasing in the spouse's wage rate. However, note from the conditions in Table 1 that in equilibrium both spouses contribute positive amounts when the relative wage $\frac{w_t^f}{w_t^m}$ takes intermediate values within an interval. As such, the non-cooperative outcome may not be the resultant equilibrium of the entire game in such cases.

Consider the case where both spouses make positive contributions towards both education and child-rearing. In this case, $\lambda_{1t}^m = \lambda_{1t}^f = 0$, and the optimality conditions for τ_t^m and τ_t^f are respectively

$$\frac{\kappa_c \theta \alpha_m}{\bar{\tau} + \alpha_m \tau_t^m + \alpha_f \tau_t^f} = \lambda_t^m n w_t^m, \tag{9}$$

$$\frac{\kappa_c \theta \alpha_f}{\bar{\tau} + \alpha_f \tau_t^m + \alpha_f \tau_t^f} = \lambda_t^f n w_t^f. \tag{10}$$

Since $\lambda_t^m = \lambda_t^f$ as shown previously, (9) and (10) together imply $\alpha_m/\alpha_f = w_t^m/w_t^f$, that is, neither spouse has a comparative advantage in either home production or wage labor outside the home. Thus, in the presence of a comparative advantage, a corner solution is inevitable, where at least one parent contributes to at most one of the two inputs to child quality, i.e. at least one among e_t^m, e_t^f, τ_t^m and τ_t^f will be zero. Table 1 shows the solutions restricted to the case where the woman has a comparative advantage in home production, i.e. $\alpha_m/\alpha_f < w_t^m/w_t^f$. Additionally, we imposes $\bar{e} = \bar{\tau} = 0$ in order to limit the number of possible outcomes; cases where parents do not invest in child quality do not occur under this assumption.

Equilibrium outcomes under non-cooperative decision-making depend on the two thresholds, B_1 and B_2:

$$B_1 = \frac{1 + \kappa_c \theta}{1 + \kappa_c (1 - \theta)}, \quad B_2 = 1 + \kappa_c.$$

Clearly, $B_2 > B_1$ and $B_1 > 1/B_2$. Both parents devote time to child-rearing when the relative productivity α_f/α_m lies in an intermediate range. Both spend on education when the wage ratio takes intermediate values. The cases and conditions when $\alpha_m/\alpha_f > w_t^m/w_t^f$ would mirror those shown here. Based on these decisions, the utilities of the spouses in the non-cooperative case can be specified as $U_{NC}^m(w_t^m, w_t^f)$ and $U_{NC}^f(w_t^m, w_t^f)$.

2.2 Patriarchy

The husband's offer of v_t, when he asserts his patriarchal prerogative to be the primary decision-maker, can be accepted or rejected by the wife. Rejection leads to a non-cooperative outcome. Thus, in order to sustain patriarchy, the husband needs to be able to match his wife's reservation utility $U_{NC}^m(w_t^m, w_t^f)$. And when the wife accepts v_t, the husband derives an additional pay-off $I_p \ln(\mu n \tau_t^f)$ that represents his gain from conforming to societal gender norms. The pay-off is increasing in the time spent by his wife on her children (at home). The husband's utility is given as:

$$U_t^{p,m} = \ln c_t^m + \kappa_c \ln q_t + I_p \ln(\mu n \tau_t^f) \tag{11}$$

$\frac{w_t^f}{w_t^m}$	$\frac{\alpha_f}{\alpha_m}$	$n\tau_t^m$	$n\tau_t^f$	E_t^m	E_t^f
$\in (B_1, B_2)$		0	$\in (0,1)$	> 0	> 0
$< B_1$	$> B_1$	0	$\in (0,1)$	> 0	0
$> B_2$	$> B_2$	0	$\in (0,1)$	0	> 0
$< \frac{1}{B_2}$	$< \frac{1}{B_2}$	$\in (0,1)$	0	> 0	0
	$\in (\frac{1}{B_2}, B_1)$	$\in (0,1)$	$\in (0,1)$	> 0	0

Table 1: Non-cooperative solution $\left(\alpha_m/\alpha_f < w_t^m/w_t^f\right)$

See Appendix A for the expressions for $\tau_t^m, \tau_t^f, E_t^m, E_t^f$. Interior solutions for all four variables is only possible when $\alpha_m/\alpha_f = w_t^m/w_t^f$. The remaining cases for $\alpha_m/\alpha_f > w_t^m/w_t^f$ can be worked out as mirror images as the above.

that he maximizes subject to the household budget constraint,

$$c_t^m + c_t^f + np_e e_t \leq (1 - n\tau_t^m)\, w_t^m + \left(1 - n\tau_t^f\right) w_t^f, \tag{12}$$

and the woman's participation constraint

$$\ln c_t^f + \kappa_c \ln q_t \geq U_{NC}^f(w_t^m, w_t^f). \tag{13}$$

Note that under patriarchy the man is better off on two counts. First, he gets the added social payoff from asserting his patriarchal identity. Secondly, he benefits from being the first mover or the proposer as he is able to extract the highest possible share of the surplus, relegating the woman to her reservation utility. The man faces a distinct trade-off in this context: he would like to restrict the woman from working outside to maximize his identity payoff, but doing so reduces overall household income which makes it more difficult to compensate the woman and sustain patriarchy.

Since, under patriarchy, the man values the social norm and also wields the decision-making power within the household, the woman's labor supply may be so restricted that a rise in the female wage rate does not necessarily translate into being able to work more outside the home. To illustrate this point, consider the case where at the optimum, $\tau_t^m = 0$ and $\tau_t^f = 1/n$, that is, the man spends his entire time endowment working outside the home, while the woman spends all her time on child-rearing. The conditions for such a case are respectively

$$\frac{\alpha_f}{\alpha_m} > \frac{\kappa_c \theta}{1 + \kappa_c(1-\theta)}, \tag{14}$$

$$\frac{\kappa_c \theta}{1 + \kappa_c(1-\theta)} w_t^m + \frac{I_p}{\lambda_t(w_t^m, w_t^f)} > w_t^f, \tag{15}$$

where $\lambda_t(w_t^m, w_t^f)$ is the solution to the Lagrange multiplier for the household budget constraint

$$\frac{1}{\lambda_t(w_t^m, w_t^f)} = \frac{w_t^m}{1 + \kappa_c(1-\theta)} \\ - \exp\left[U_{NC}^f(w_t^m, w_t^f) - \kappa_c \ln\left(\gamma\left(\frac{\alpha_f}{n}\right)^{\theta}\left(\frac{\kappa_c(1-\theta)w_t^m}{(1+\kappa_c(1-\theta))np_e}\right)^{1-\theta}\right)\right]$$

and (15) holds for a high enough I_p.

Since the woman does not work outside the home in this case, small changes in w_t^f do not affect her labor force participation. The time inputs to child-rearing are fixed and the educational input is rising in the man's wage rate. The binding participation constraint of the woman can be represented as

$$\ln c_t^f + \kappa_c \ln q_t(w_t^m) = U_{NC}^f(w_t^m, w_t^f), \tag{16}$$

where q_t is increasing in w_t^m. From (16), it is clear that *ceteris paribus*, an increase in w_t^f would lead to a corresponding increase in c_t^f, and the woman will not supply any labor as long as (14) and (15) continue to hold. There is thus a stagnation of the woman's labor supply. Even though, with a higher wage, the woman's reservation utility is higher, the man is able to compensate her enough to sustain patriarchy and maximize his payoff from the social norm.

2.3 Unitary household behavior

When the husband does not assert his patriarchal identity, we conceive of the household behaving in an egalitarian way. Accordingly, its behavior is guided by the "consensus principle" (Samuelson, 1956) where the weighted sum of spousal utilities

$$\max_{v_t} \quad \alpha U_t^m + (1-\alpha)U_t^f \tag{17}$$

is maximized subject to the household budget constraint (12). The relative importance of the spouses in household decision-making, α, determines solely the distribution of household consumption between the spouses. Since both spouses give equal importance to child quality, the time and education inputs are not affected by α.

Once again, note that a comparative advantage leads to a corner solution for labor supplied for at least one of the spouses. Both having an interior solution is only possible if $\alpha_f/\alpha_m = w_t^f/w_t^m$. Table 2 below summarizes the solution for the case $\alpha_m/\alpha_f < w_t^m/w_t^f$ with

$$B_3 = \frac{\kappa_c \theta}{1 + \kappa_c(1-\theta)}.$$

The remaining cases can be worked out as mirror images.

$\frac{w_t^f}{w_t^m}$	$\frac{\alpha_f}{\alpha_m}$	$n\tau_t^m$	$n\tau_t^f$	E_t
$> B_3$		0	$\in (0,1)$	$\frac{\kappa_c(1-\theta)}{1+\kappa_c}(w_t^m + w_t^f)$
$< B_3$	$> B_3$	0	1	$\frac{\kappa_c(1-\theta)}{1+\kappa_c(1-\theta)} w_t^m$
	$< B_3$	$\in (0,1)$	1	$\frac{\kappa_c(1-\theta)}{1+\kappa_c}\left(1 + \frac{\alpha_f}{\alpha_m}\right) w_t^m$

Table 2: Unitary household $\left(\alpha_m/\alpha_f < w_t^m/w_t^f\right)$

See Appendix C for the expressions for τ_t^m, τ_t^f. Both can have interior solutions only under $\alpha_m/\alpha_f < w_t^m/w_t^f$. Remaining cases for $\alpha_m/\alpha_f > w_t^m/w_t^f$ can be worked out as mirror images.

In the three outcomes discussed above, the patriarchal and the unitary household solution are Pareto-optimal by the definitions of their respective problems. In the non-cooperative case, each spouse does not fully internalize the value that the household as a whole places on the public good, that is, child quality, leading to inefficiencies.

3 Household Equilibrium

The equilibrium outcome signifies which of the three setups the household operates in. The equilibrium is determined from the indirect utilities of the two spouses at each terminal node. At node F (see Figure 1), the woman accepts the man's proposal if it exceeds (or matches) her reservation utility in the non-cooperative outcome, and rejects otherwise. At node M, the man faces the extensive margin choice of making an offer or settling for the unitary setup. Patriarchy would result only when the man is able to do better than in either the unitary or the non-cooperative outcome, while also ensuring the woman gets her reservation utility. To reiterate a point discussed earlier, in essence, the bargaining power of the woman in the household is being determined endogenously as an equilibrium of the game, even though the man is assumed to have the advantage of being the first mover. It is worthwhile examining how the equilibria depend on the earning potential (wages) of the spouses and other parameters of the model.

Figure 2 presents a numerical example. The equilibrium outcomes are mapped against the market wage rates of the man and the woman. The equilibrium appears to be strongly dependent on the relative wage w_t^f/w_t^m. With a rise in the relative wage, the household moves from patriarchy to unitary decision-making with an intermediate region of non-cooperative bargaining in between.

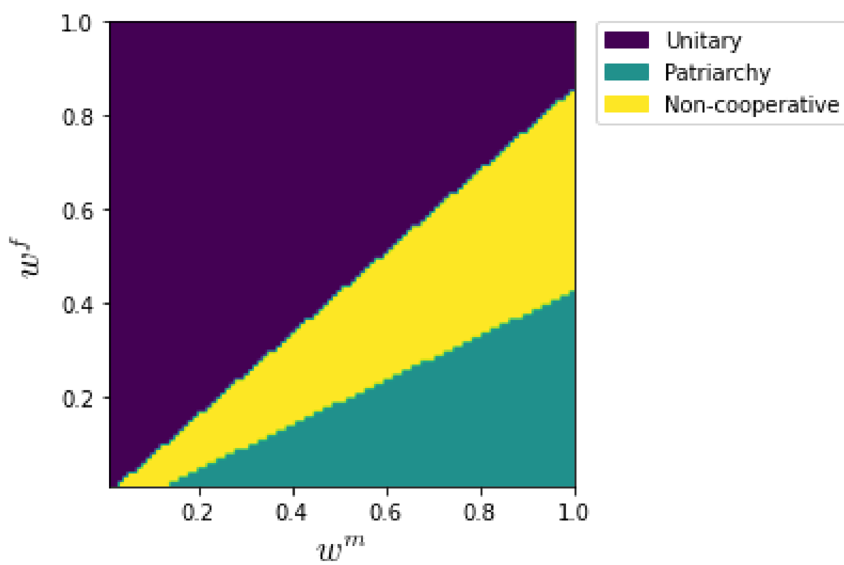

Figure 2: Household Equilibrium and Wages

Parameters: $\kappa_c = \gamma = 1, \theta = 0.3, \alpha_m = 0.4, \alpha_f = 0.6, \alpha = 0.5$
Note: The same color mapping applies hereafter.

When the woman's wage w_t^f is very low relative to the man's wage w_t^m, the prevalent outcome is patriarchy, given by the teal region towards the bottom-right. Due to her low earnings potential, the woman's reservation utility from the non-cooperative case is low. The husband is able to meet her participation constraint while himself being better off under patriarchy. As w_t^f rises relative to w_t^m, the woman's reservation utility becomes too high for the man to be compensate her sufficiently. At the same time, her wage may not be high enough for the man to benefit from pooling income and behave as a unitary household. Thus, the non-cooperative equilibrium results, given by the yellow region. Finally when the relative wage is high enough (blue region), income pooling is the best course of action for the man as well and the unitary household is the equilibrium outcome.

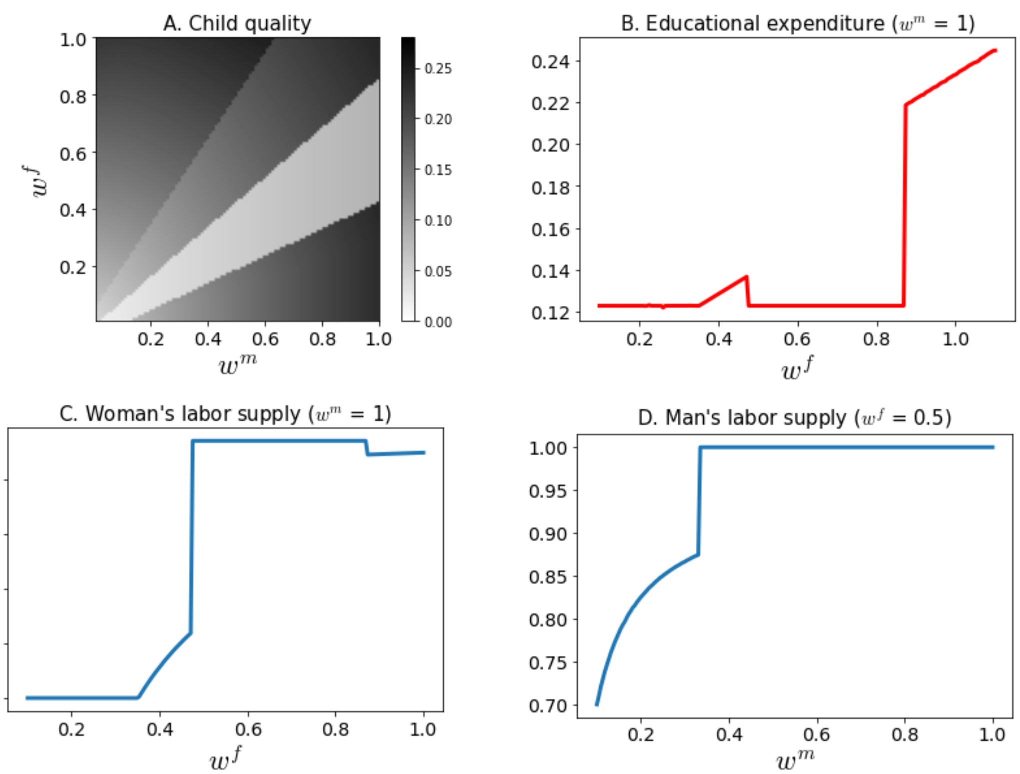

Figure 3: Household Equilibrium Outcomes

Notes: The plots correspond to the parametrization in Figure 2. Labor supplies are plotted at constant spousal wage. Even under patriarchy, the woman may supply a small amount of labor, as seen in the upward-sloping stretch around $w_t^f = 0.4$ (Panel C). This is reflected in the small spike in educational expenditure (Panel B). In A, resultant child quality is plotted (*Light:*Low, *Dark:*High). Child quality is distinctively lower under non-cooperative bargaining.

Figure 3 examines the other outcome variables at equilibrium corresponding to the same parametrization. The top left panel maps the resultant child quality over the same range of wages as in Figure 3. While in general it is increasing in wages of both spouses, the non-cooperative region in Figure 2 corresponds to markedly lower child quality, which as discussed results from disregarding externalities on the public good.

The bottom left panel plots the labor supply of the woman $(1 - n\tau_t^f)$ against her own wage keeping the man's wage constant. At very low w_t^f, patriarchy prevails and the man altogether restricts the woman from working outside home. This is where her labor supply stagnates. However, it is seen that at slightly higher levels, where patriarchy is still sustained, the man's proposal does involve the woman working for a certain small fraction of her time. Although this reduces his social payoff (which is decreasing in his wife's participation in the labor force), the additional income makes it easier for him to compensate her. The transition to the non-cooperative regime is marked by a sharp jump in the woman's labor supply. At high enough w_t^f, the household switches to a unitary regime, where her labor supply is marginally lower compared to the non-cooperative outcome, because of the higher value the household places on child quality, and hence time spent on child-rearing.

The top-right panel shows how the educational input per child varies with w_t^f at constant w_t^m. A sharp jump is seen at the transition from the non-cooperative to the unitary setup. The small spike within patriarchy corresponds to the initial gradual rise in the woman's labor supply as seen in the bottom-left panel. The bottom-right plots the man's labor supply at constant w_t^f. When relative wage $\frac{w_t^f}{w_t^m}$ is high, the unitary regime prevails, and the man is seen to devote some of his time to child-rearing. His labor supply is increasing and eventually spends his entire time endowment on wage labor.

It is to be noted that although the non-cooperative outcome serves a threat point in this model, it may not necessarily constitute an equilibrium. Unlike Figure 2 which shows a continuum of wage values for which the non-cooperative equilibrium results, in Figure 4, the household switches directly from patriarchy to the unitary setup. This is typically more likely when I_p is higher, and patriarchy is more attractive to the man.

It is apparent from Figures 2 and 4 that the nature of the equilibrium largely depends on the relative wage and seems to be devoid of any notable scale effects. However, the levels of the two wages do affect the composition of child quality inputs as seen in Figure 5. As the wages of both spouses rise (proportionally in the figure), child quality production becomes more education-intensive and less time-intensive. When wages are low enough, the household is too poor to make any positive discretionary contributions towards the children's education. As households get richer, parents are likely to spend less

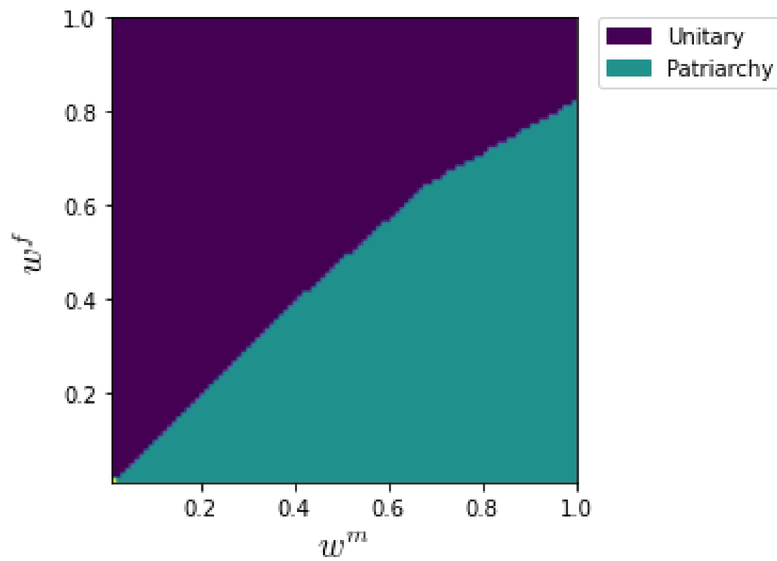

Figure 4: Equilibrium outcomes

Notes: Illustration of a case where the non-cooperative outcome does not appear as an equilibrium despite being the binding outside option of the woman in patriarchy. Such may typically happen with a high patriarchal pay-off to the man (I_p) or high price of education (p_e). See comparative statics (Section 3.1) for more details.

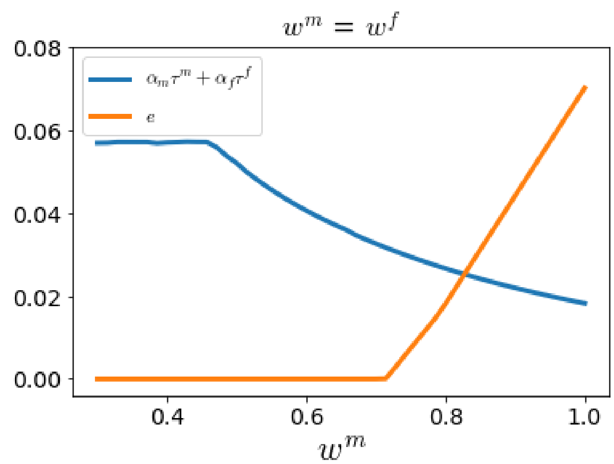

Figure 5: Inputs to child quality

Notes: The plot corresponds to the equilibria map in Figure 4. Child quality inputs along the line $w_t^m = w_t^f$ are plotted. The wage axis is truncated, but the initial horizontal stretches for both inputs extend from $w_t^m = 0$. The household starts making positive discretionary expenditures towards education around $w_t^f = w_t^m \approx 0.7$ (over $\bar{e}$). A constant wage ratio of 1 corresponds to the unitary regime throughout as seen in Figure 4. Lines through the patriarchal region also yielded similar results.

time rearing their children on their own and instead substitute it with market-purchased inputs such as education.

3.1 Comparative statics

In this section, we look at how the equilibrium results change with some of the parameters of interest in our problem, specifically the price of education p_e,[2] the man's patriarchal payoff I_p, and the woman's social payoff from being a careerist I_f.

While the time spent by parents in bringing up their children may be substitutable to some degree by certain market-purchased services such as daycare, the market for such services may be shallow and undeveloped, particularly in developing economies. The lack of a proper market for daycare facilities from which parents are willing to purchase hinders such substitution. This may partly explain why despite commanding higher wages in the labor market, parents (mothers in particular) still remain tied up at home with their child-rearing duties.

In this model, the market input to child quality is education. The lack of a proper market or the inability of parents to purchase the appropriate type/quality of the input will be reflected in its shadow price. Figure 6 shows that a rise in p_e leads to an expansion of the patriarchal region. The household purchases less education overall. In fact, the variation in education mirrors that of the woman's labor supply over w_t^f. Higher expenditure on education results from the increased purchasing power of the household when she devotes more of her time to wage labor. With a higher price of education, the patriarchal outcome becomes more restrictive for the woman in terms of her ability to work outside home. As education becomes less affordable, the household substitutes it with more time spent on child-rearing, in which the mother holds a comparative advantage. With patriarchy becoming more prevalent, the woman's labor supply stagnates for a longer range of values of w_t^f. Shallow markets for (or high underlying shadow costs to) substitutes to mothers' child-rearing efforts thus perpetuate patriarchal social norms.

In fact, the effect of a rise in p_e is found to be similar to that of a rise in I_p, the latter being shown in Figure 7. With patriarchy becoming more attractive

[2] Assuming efficient markets, a decrease in p_e can also be interpreted as an increase in the net returns to education.

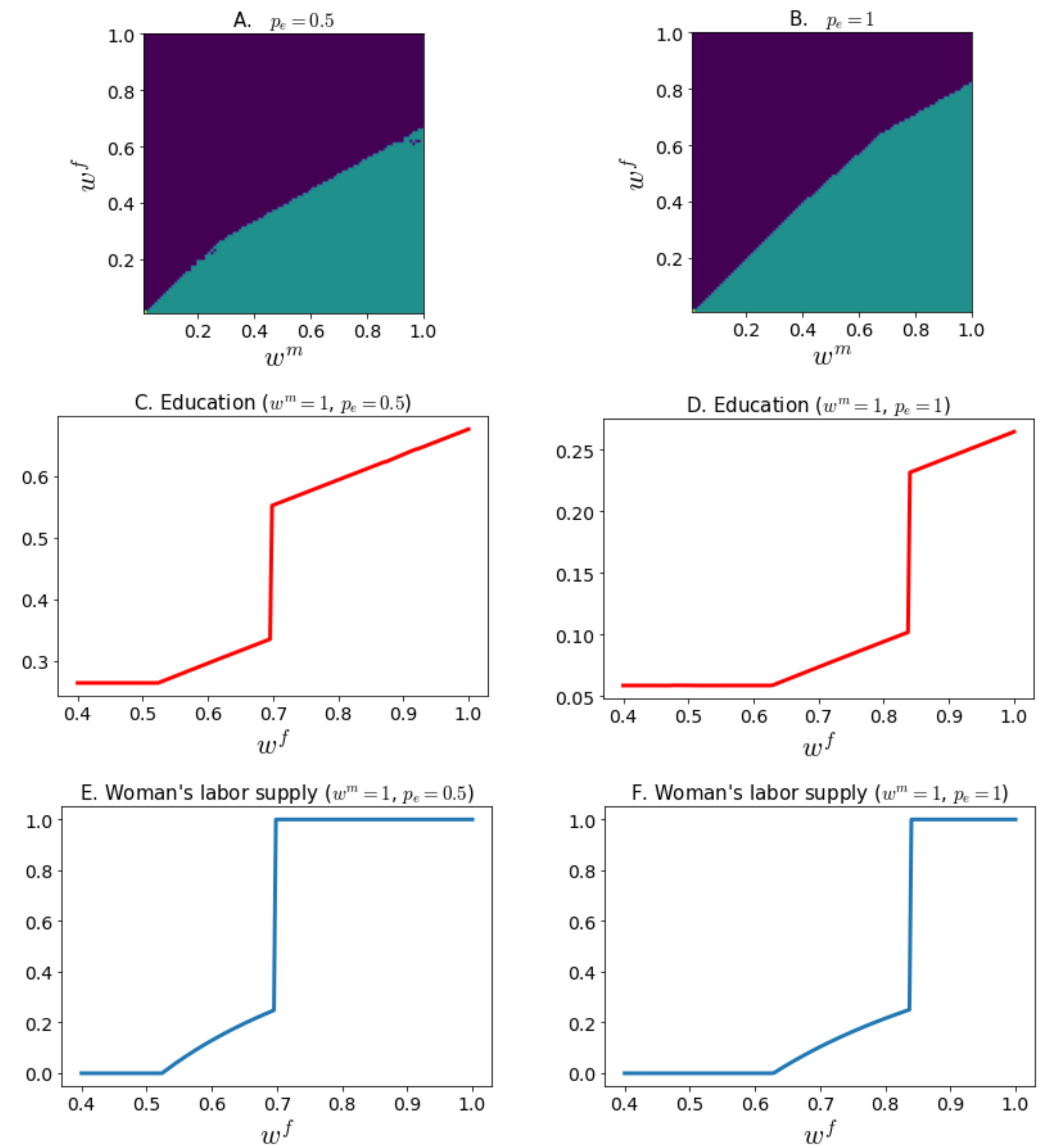

Figure 6: Comparative statics (p_e)

Notes: C and D plot the discretionary expenditure on children's education (over e). Higher price of education leads to expansion of the patriarchal (teal) region (Panels A&B). Education and woman's labor supply are plotted as constant male wage rate. With pricier education and increased prevalence of patriarchy, the household purchases less education overall.

to the man, he is better able to meet his wife's participation constraint. The patriarchal region expands and the woman's labor supply is found to reduce and stagnate longer.

Finally, we look at the effect of the woman's identity payoff from her career aspirations. The model allows for men to realize a social payoff from conform-

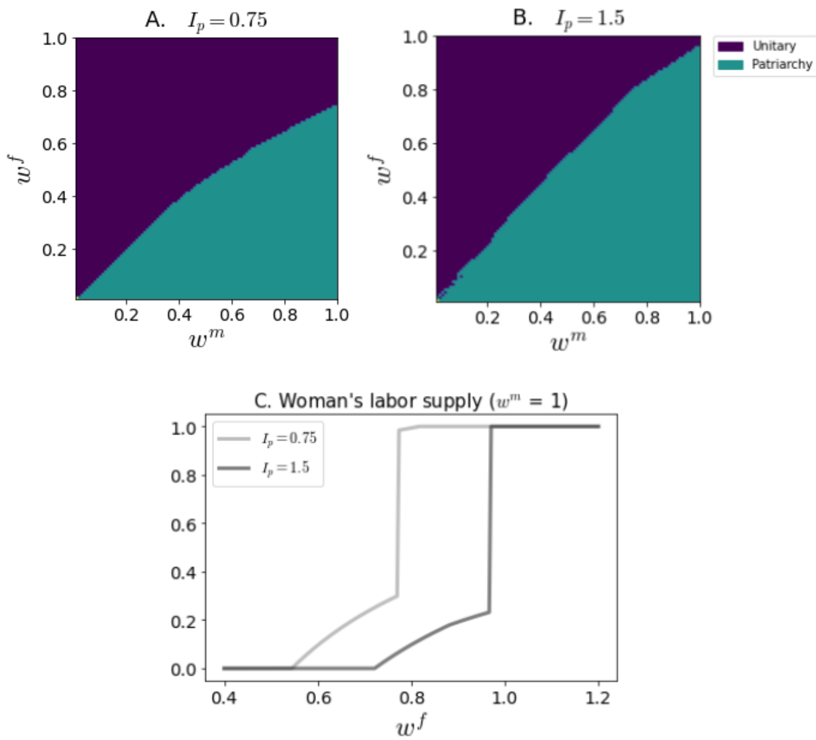

Figure 7: Comparative statics (I_p)

Notes: Higher identity pay-off from patriarchy makes it more attainable for the man. Woman's labor supply is plotted at a constant (spousal) male wage rate. The woman's labor supply stagnates for longer, and she needs a higher wage for the household to transition out of patriarchy.

ing to patriarchal norms. However, women may also be able to aspire to pursue a career, and derive a payoff from assuming the identity of a 'careerist'. A career however entails a loss of flexibility in how the woman may allocate her time. A career rewards continuity and experience in employment. For instance, she may have to commit a minimum number of hours to her job to be able to realize the gains from a career. Note that in some cases, e.g. in patriarchy, the woman may not be able to choose for herself, but valuing a career may affect her reservation utility. Thus, it is of interest to see its effect on household decision-making equilibria and outcomes. It is assumed that the woman realizes a career payoff of I_f of she is able to devote a fraction $\bar{s}$ to her time to paid work.

$$U_t^f = \begin{cases} \ln c_t^f + \kappa_c \ln q_t + I_f & \text{when } 1 - n\tau_t^f \geq \bar{s}, \\ \ln c_t^f + \kappa_c \ln q_t & \text{when } 1 - n\tau_t^f < \bar{s}. \end{cases}$$

Note that in all the preceding analyses, I_f has been set to zero to simplify the

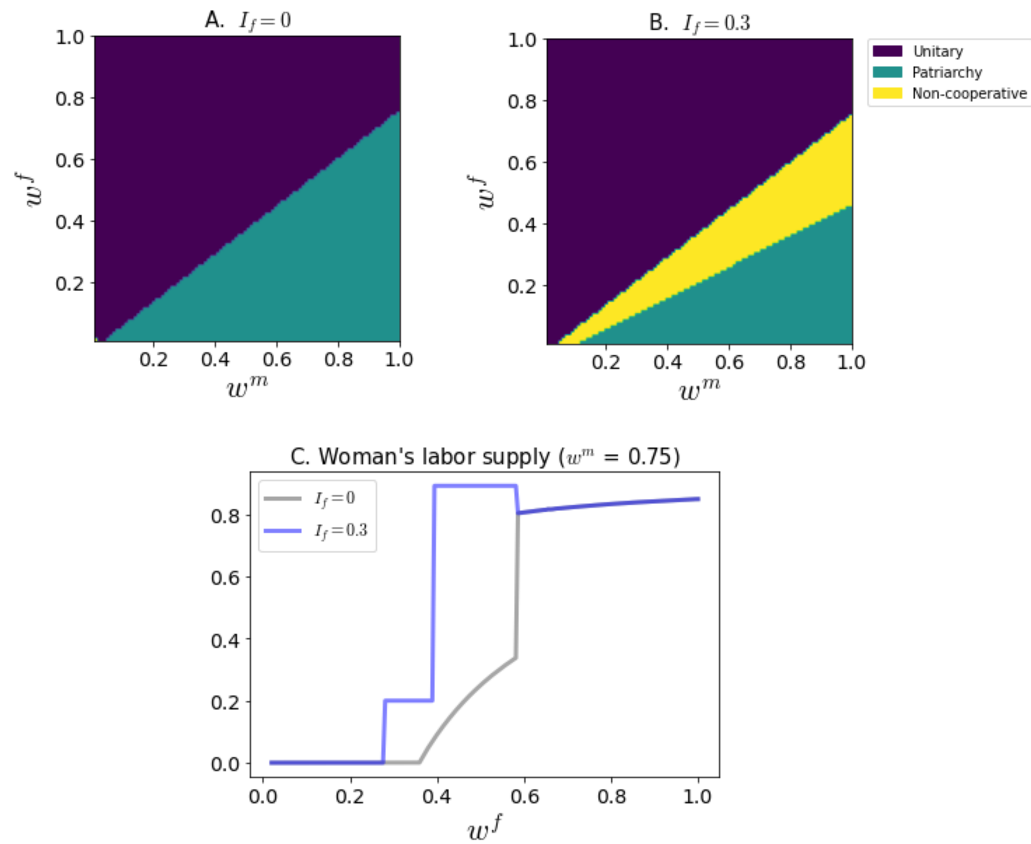

Figure 8: Comparative statics (Woman's identity payoff I_f) ($\bar{s} = 0.2$)

Notes: In both the non-cooperative and unitary regimes within this space, the woman exceeds the career threshold. Panel C plots the woman's labor supply at constant male wage. Along the horizontal stretch from around $w_t^f = 0.3$ to $w_t^f = 0.4$, the woman realizes a career payoff under patriarchy.

exposition. Figure 8 presents a possible implication of the woman valuing a career. Panel A shows that when $I_f = 0$ (i.e., the woman's career aspiration is 'switched off'), the equilibria comprise only patriarchy and unitary household behavior. However, in Panel B. where she assigns a positive value to the time commitment towards a career, a region of non-cooperative bargaining emerges, with a recession of the patriarchal region. This is essentially a manifestation of the conflict between the spouses' preferences. Since the woman now gets an additional social pay-off in the non-cooperative outcome (where her labor force participation exceeds the career threshold), the man finds it more difficult to adequately compensate under patriarchy. Thus the patriarchy region (teal) contracts. In Panel C, it is seen that even under patriarchy, the man may allow the woman to pursue her career aspirations. Around $w_t^f = 0.3$, the woman's labor supply jumps up to 0.2 (the assumed career threshold), where both the man and the woman realize their patriarchal and career payoffs respectively. With higher w_t^f, as patriarchy becomes unsustainable, female labor supply jumps up to its higher non-cooperative level.

4 Production

The final consumption good is produced using both male and female labor, which enter as perfect substitutes.

$$Y_t = Z_t^m L_t^m + Z_t^f L_t^f. \tag{18}$$

Z_t^m and Z_t^f denote the gender-specific labor productivity for men and women which increase linearly in their respective human capital. Additionally, the man's productivity is assumed to contain a positive intercept $\bar{A}_m$.

$$Z_t^m = \bar{A}_m + A_m h_t^m, \tag{19}$$

$$Z_t^f = A_f h_t^f. \tag{20}$$

Though the model does not involve multiple sectors of production, at low levels of human capital, 'non-cognitive' jobs constitute the primary occupations in the economy. $\bar{A}_m$ captures the higher productivity that men exhibit in such occupations with low intensity in human capital which primarily produce using physical labor. The education sector is not explicitly modelled and the educational inputs to child quality are assumed to be purchased from outside the economy in exchange for the consumer good.

The representative firm operates in perfectly competitive input and output markets and exercises a preference-based discrimination against women. This is represented by a tax τ on the female labor input in production, as in Becker (1971) and Cortes *et al.* (2018). The firm thus chooses the amount of labor inputs to maximize its quasi-profit.

$$\max_{\{L_t^m, L_t^f\}} \quad Y_t - w_t^m L_t^m - (1+\tau) w_t^f L_t^f. \tag{21}$$

The FOCs and (18) yield

$$w_t^m = Z_t^m, \tag{22}$$

$$w_t^f = \frac{Z_t^f}{1+\tau}. \tag{23}$$

The labor market equilibrium is given by

$$L_t^m = 1 - n\tau_t^m \left(w_t^m, w_t^f\right),\ L_t^f = 1 - n\tau_t^f \left(w_t^m, w_t^f\right).$$

where $\tau_t^m \left(w_t^m, w_t^f\right)$ and $\tau_t^f \left(w_t^m, w_t^f\right)$ derive from the household's optimization.

It is worth noting that this discriminatory wedge causes the household and the firm to substitute at different rates between male and female labor. This leads to an inefficiency whereby a part of the output produced by the firm is simply wasted.

5 Dynamics

This section looks at how the level of human capital evolves over generations. Note that human capital of a person which determines their labor productivity is essentially the child quality produced by their parents. Since the parents are assumed not to discriminate between sons and daughters, their contribute equally for their human capital formation, both in terms of education and child-rearing time. Hence $h_t^m = h_t^f$ for all generations. Note from (19) and (20) that when human capital levels are low, men have an advantage over women in terms of productivity. As human capital rises, the wage ratio converges to $\frac{1}{1+\tau}\frac{A_f}{A_m}$, which reflects both the differences in productivity and the discriminatory wedge.

The household's optimization expresses the education imparted to children and the parental time invested as functions of the parental wage rates, that is, $\tau_t^m \left(w_t^m, w_t^f\right)$, $\tau_t^f \left(w_t^m, w_t^f\right)$ and $e_t \left(w_t^m, w_t^f\right)$. The wage rates themselves are dependent on parental human capital as seen from (22) and (23). Thus children's human capital evolves as

$$h_{t+1} = g\left(h_t\right). \tag{24}$$

There may be one or multiple steady states and it is possible to have a steady state in any of the three regimes of household decision-making. Figure 9 illustrates some of these possibilities. Increases in the level of human capital

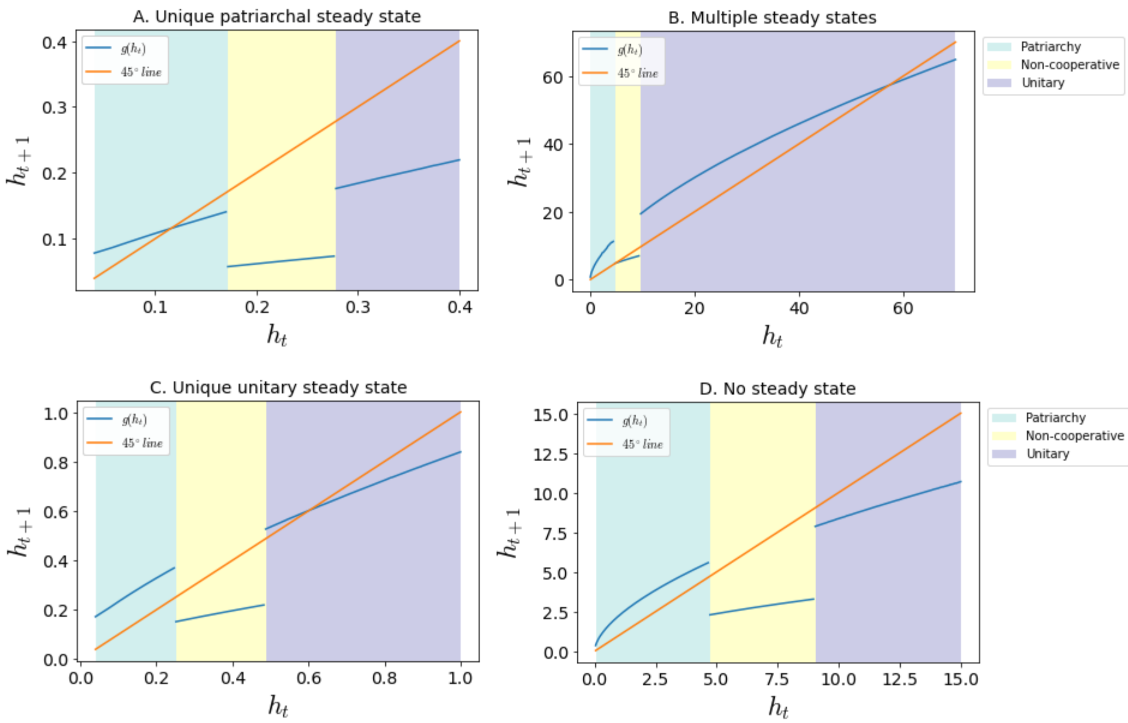

Figure 9: Evolution of human capital (possibilities)

Notes: Background shading represents the decision-making regime prevailing at equilibrium for every level of human capital. As the gender wage gap reduces with human capital development, households transition from patriarchy to non-cooperative to unitary decision-making.

narrows the gender wage gap and induces a switch in household regimes comparable to the equilibria maps seen in Section 3. Low human capital (low relative wage) corresponds to prevalence of patriarchy while high human capital leads to unitary decision making and an intermediate range of non-cooperative bargaining. On account of lower investment in the household public good (children's human capital) in the non-cooperative case, the function $g(h_t)$ exhibits discontinuities at the crossover points. In panel A, there is a unique steady state, one where patriarchy prevails. Panel C shows a unique steady state corresponding to the unitary regime. Panel B exhibits two steady states, one unitary and one non-cooperative. Both of them being stable, the non-cooperative region presents a low level equilibrium trap. A household starting from below it would be unable to ever transition to a unitary regime or the higher steady state. Panel D shows a case where no steady state exists.

The under-investment in human capital in the non-cooperative phase thus presents a hurdle in what would otherwise be a smooth convergence to the high steady state. In fact, even in the absence of a distinct low level equilibrium, a trap may result. This is illustrated using two cases in Figure 10. In both cases,

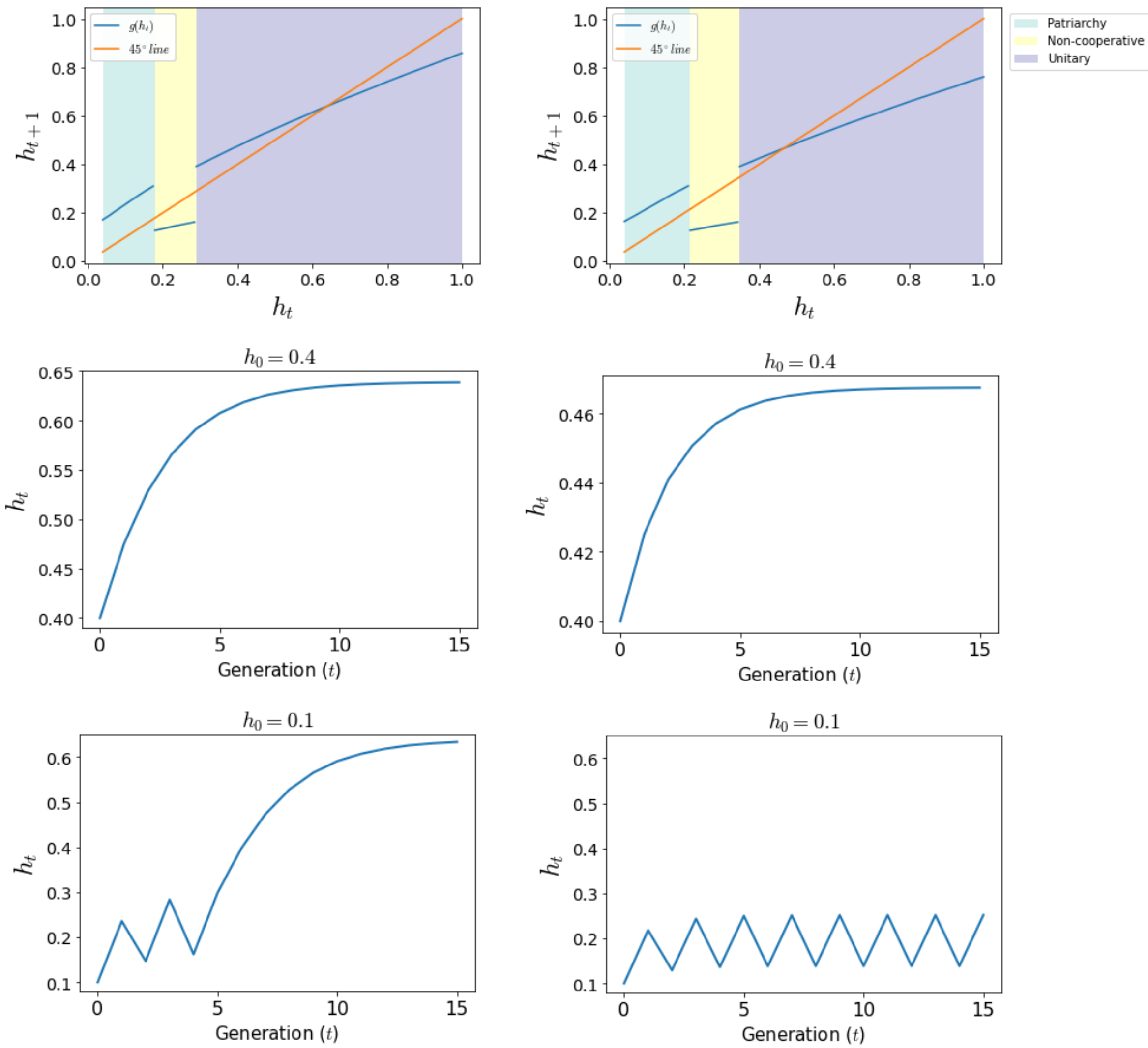

Figure 10: Low level traps (L: temporary, R: persistent)

Notes: Both cases present a unique unitary steady state. Panels C to F show the evolution of human capital over generations. For the one on the right hand side, the non-cooperative region is too wide a gap to be bridged. As long as the supremum of human capital in the patriarchal region does not directly lead to unitary decision-making in the next generation, the family dynasty is stuck in a persistent trap oscillating back and forth. On the left, the non-cooperative gap is much shorter, and the dynasty is eventually able to transition directly to the unitary regime

there is a unique steady state corresponding to the unitary regime. Starting out within the unitary regime itself, both cases witness a smooth convergence to the steady state, as seen in the middle panels. However, starting from patriarchy, the eventual outcomes in the two cases are vastly different (bottom panels). In the first case, the household is temporarily trapped as it oscillates between patriarchy and non-cooperative bargaining. But finally, following the fourth generation, it reaches a level of human capital high enough to promote it directly to the unitary regime. Thus, even though a trap may potentially exist, it does not persist for long. On the right hand side case, it is seen that the level of human capital formation under patriarchy is never high enough to allow it to transition directly to the unitary regime. The household is thus trapped in an endless loop switching between patriarchy and non-cooperative bargaining, never able to reach the higher steady state.

5.1 Effect of discrimination

The distortionary wedge τ affects the relative wage and has implications for the the regime which prevails and the ability to converge to the steady state. Figure 11 compares two cases, one with a positive discriminatory tax and one without any discrimination.

It is seen that discrimination causes the woman's wage to rise slower compared to the man's as human capital rises. The path that wages follow over a range of human capital is shown in Figure 12. With the lower trajectory, the household goes through longer phases of patriarchy and non-cooperative bargaining. Discrimination leads to a low level trap in this case, while in the absence of it, the household is eventually able to progress to the high steady state. Note that discrimination reduces the earnings potential for the woman given her level of human capital. However, this has zero to negligible effect on household income and consequently children's human capital formation under patriarchy, where the woman's labor supply is restricted. However, it has a marked effect on $g(h_t)$ in a unitary household, which leads to a decline in the steady state level of human capital in this case.

A very high degree of discrimination leads to a situation where the unique steady state corresponds to patriarchy. Such a case is presented in Figure 13. In the absence of discrimination, the household can converge over time to a steady state with unitary decision-making. However, introducing discrimination shifts the steady state to the patriarchal region, where the household re-

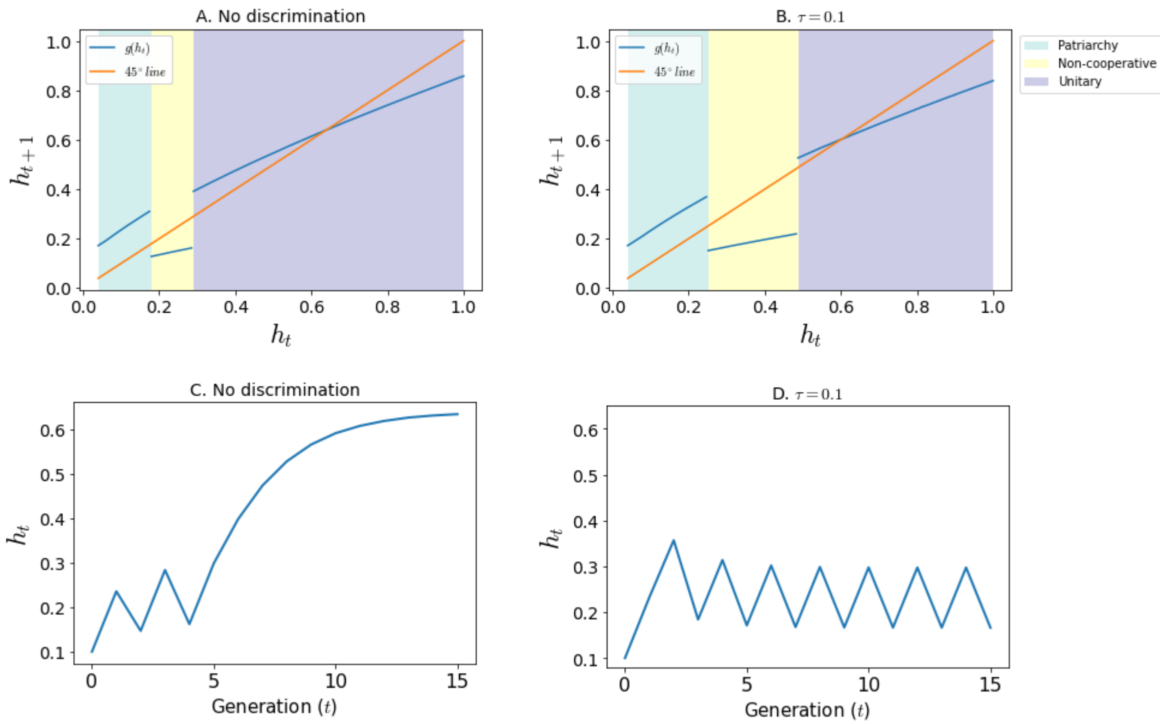

Figure 11: Discrimination in the labor market

Notes: As apparent from Figure 10, a persistent trap would be more likely with a flatter relative wage trajectory, i.e. where w_t^f/w_t^m rises slowly. Higher labor market discrimination flattens the trajectory, as seen in Figure 12.

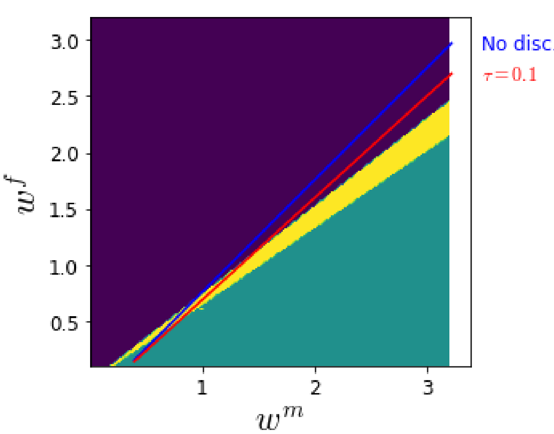

Figure 12: Trajectory of wages (discrimination vs. no discrimination)

Notes: Axes span the range of wages corresponding to the range of human capital in Figure 11. Higher discrimination in the labor market leads to a flatter wage trajectory (red line) and a wider range of human capital over which the non-cooperative outcome prevails.

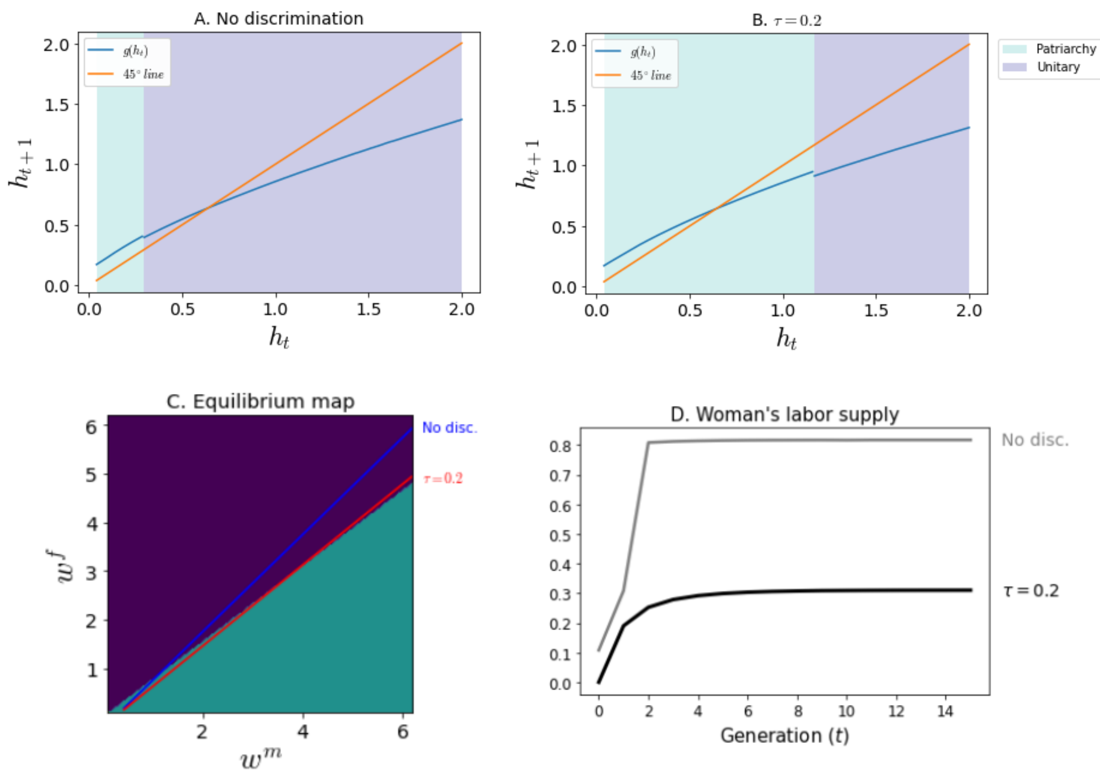

Figure 13: Discrimination in the labor market

Notes: High discrimination leads to unique patriarchal steady state. Panel C shows underlying wage trajectory. Difference in steady state human capital under patriarchal and unitary regimes is negligible (A and B). But woman's labor supply differs significantly. In the absence of discrimination, the household quickly enters the unitary regime and labor supply remains uniformly high at around 0.8. Under the patriarchy trap resulting from discrimination, it converges to a low but positive level.

mains trapped. The difference in resultant human capital (child quality) is negligible between the patriarchal and unitary regimes. However, there is a marked gap in the steady state levels of female labor supply. Under high discrimination, her labor supply converges to a low level, as entailed by her husband's patriarchal offer. In the absence of discrimination, the household quickly switches over to the unitary regime, where her labor supply is notably higher.

Thus, even though parents do not discriminate between their children for their human capital investments, the presence of discrimination in the labor market may potentially lead to perpetuation of patriarchy and simultaneously low labor force participation by women.

6 Conclusion

The paper develops a theory to account for low FLFP in many developing countries. Apart from disproportionate commitments by women towards household chores and raising children, patriarchal norms are often cited as a major reason behind low FLFP. We make patriarchal norms a centerpiece of our theory and show how its role in explaining female labor force participation changes as the relative wage of female to male labor changes over time. Our dynamic analysis of labor force participation also incorporates a demand side factor, discrimination by the firms towards female labor. Our analysis, through a parsimonious model of household behavior, generates a few important observations.

In the temporal equilibrium, our results show that the equilibrium mode of decision-making is heavily dependent on the woman's wage relative to the man's. Patriarchy prevails when the relative wage is low, unitary household behavior prevails when it is high, along with a possible intermediate region of non-cooperative decision-making. Although the non-cooperative outcome may not necessarily manifest as an equilibrium, it serves as a threat-point to the patriarchal arrangement. It becomes particularly pronounced when the woman values a career and its associated time commitment to the workforce, whereby the conflict of preferences between a 'traditional' husband and a 'careerist' wife may result in a breakdown of patriarchy. Our results also suggest that despite increases in women's educational attainment and consequently wages commanded in the market, women's labor force participation will continue to stagnate as long as patriarchy prevails in the household. An escape from patriarchy may be facilitated by changes in social norms, or greater affordability/availability of market-purchased substitutes to child-rearing, which otherwise keeps women occupied at home.

In the intertemporal equilibrium, generations are linked by parental human capital investments and the household equilibrium may change over time. We find that the dynamic equilibrium path features a secular transition from patriarchy to unitary household behavior, with a convergence of human capital. The otherwise smooth transition may be hindered by a few generations of non-cooperative bargaining in between. Further, a preference-based discrimination against women in the labor market impedes the rise in their relative wage. When the extent of this discrimination is high enough, a patriarchy trap results, with female labor force participation stagnating at extremely low levels.

Thus, our results indicate how labor market discrimination may lead to persistence of patriarchy even when parents do not discriminate between children in terms of human capital investments.

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Appendix

A Non-cooperative outcome

The Lagrangian for the optimization problems for the man and the woman respectively are

$$
\begin{aligned}
\mathcal{L}_t^m = & \ln(c_t^m) + \kappa_c \ln\left[\gamma(\alpha_m \tau_t^m + \alpha_f \tau_t^f)^\theta (e_t^m + e_t^f)^{1-\theta}\right] \\
& + \lambda_t^m \left[(1 - n\tau_t^m) w_t^m - c_t^m - np_e e_t^m\right] + \lambda_{1t}^m e_t^m + \lambda_{2t}^m \tau_t^m, \\
\mathcal{L}_t^f = & \ln(c_t^f) + \kappa_c \ln\left[\gamma(\alpha_m \tau_t^m + \alpha_f \tau_t^f)^\theta (e_t^m + e_t^f)^{1-\theta}\right] \\
& + \lambda_t^f \left[(1 - n\tau_t^f) w_t^f - c_t^f - np_e e_t^f\right] + \lambda_{1t}^f e_t^f + \lambda_{2t}^f \tau_t^f.
\end{aligned}
$$

The first-order condition for consumption for both spouses is thus

$$\frac{1}{c_t^i} = \lambda_t^i \quad \text{for} \quad i = m, f. \tag{25}$$

Note that, here, the intercepts for education and the time input on child-rearing have been suppressed i.e., $\bar{e}, \bar{\tau} = 0$. The more general specification with $\bar{e}, \bar{\tau} > 0$ can also be solved similarly. Therefore, all possibilities are covered by the following four cases.

Case 1: $\tau_t^m = 0$, $e_t^m, e_t^f, \tau_t^f > 0$.

Thus $\lambda_{1t}^m = \lambda_{1t}^f = \lambda_{2t}^f = 0$. From the man's and the woman's optimization problems respectively,

$$FOC\{e_t^m\} : \frac{\kappa_c(1-\theta)}{e_t^m + e_t^f} = \lambda_t^m np_e, \tag{26}$$

$$FOC\{e_t^f\} : \frac{\kappa_c(1-\theta)}{e_t^m + e_t^f} = \lambda_t^f np_e. \tag{27}$$

From (26) and (27), it is clear that $\lambda_t^m = \lambda_t^f$, and therefore,

$$E_t^m + E_t^f = \frac{\kappa_c(1-\theta)}{\lambda_t}, \tag{28}$$

where $E_t^i = np_e e_t^i$. Further, for the woman,

$$
\begin{aligned}
FOC\{\tau_t^f\} : \frac{\kappa_c \theta}{\tau_t^f} &= \lambda_t^f n w_t^f, \\
\Rightarrow n w_t^f \tau_t^f &= \frac{\kappa_c \theta}{\lambda_t^f}.
\end{aligned} \tag{29}
$$

Therefore, from the man's budget constraint,

$$w_t^m = \frac{1}{\lambda_t^m} + \frac{\kappa_c(1-\theta)}{\lambda_t^m} - E_t^f.$$

From the consolidated budget constraint of the household, we have

$$w_t^m + (1 - n\tau_t^f)w_t^f = c_t^m + c_t^f + E_t^m + E_t^f.$$

Using (25), (28) and (29), it follows that $\lambda_t = \frac{2+\kappa_c}{w_t^m + w_t^f}$. Plugging this value of λ_t in the respective FOCs and using the individual budget constraints, we get the educational expenditures and the time spent on children, as follows.

$$\begin{aligned}
E_t^m &= \frac{1+\kappa_c}{2+\kappa_c} w_t^m - \frac{1}{2+\kappa_c} w_t^f, \\
E_t^f &= \frac{1+\kappa_c(1-\theta)}{2+\kappa_c} w_t^f - \frac{1+\theta\kappa_c}{2+\kappa_c} w_t^m, \\
n\tau_t^f &= \frac{\kappa_c\theta}{(2+\kappa_c)w_t^f}(w_t^m + w_t^f).
\end{aligned}$$

It is clear that the educational expenditures are increasing in own wage rate and decreasing in spouse's wage rate. The time spent by the woman on child-rearing is decreasing in her own wage rate and increasing in her husband's. Moreover,

$$FOC\,\{\tau_t^m\} : \frac{\kappa_c\theta\alpha_m}{\alpha_f\tau_t^f} = \lambda_t n w_t^m - \lambda_{2t}^m. \tag{30}$$

Now, since $\tau_t^m = 0$ in this case, we have $\lambda_{2t}^m \geq 0$. This implies the following condition.

$$\lambda_t n w_t^m \geq \frac{\kappa_c\theta\alpha_m}{\alpha_f\tau_t^f}.$$

Using the values for λ_t and τ_t^f, this condition becomes

$$\frac{w_t^m}{w_t^f} \geq \frac{\alpha_m}{\alpha_f}.$$

Furthermore, $E_t^m, E_t^f > 0$ can respectively be rewritten as

$$\begin{aligned}
\frac{w_t^f}{w_t^m} &< 1 + \kappa_c, \\
\frac{w_t^f}{w_t^m} &> \frac{1+\theta\kappa_c}{1+\kappa_c(1-\theta)}.
\end{aligned}$$

Case 2: $\tau_t^m = e_t^f = 0$**,** $e_t^m, \tau_t^f > 0$**.**

Here, $\lambda_{2t}^m = \lambda_{1t}^f = 0$. From the man's optimization problem,

$$\begin{aligned} FOC\left\{e_t^m\right\}: \frac{\kappa_c(1-\theta)}{e_t^m} &= \lambda_t^m n p_e, \\ \Rightarrow E_t^m &= \frac{\kappa_c(1-\theta)}{\lambda_t^m}. \end{aligned} \tag{31}$$

In this case, the man's budget constraint is simply $w_t^m = c_t^m + E_t^m$. Using (25) and (31), we get

$$\lambda_t^m = \frac{1+\kappa_c(1-\theta)}{w_t^m}.$$

Similarly, from the woman's optimization,

$$\begin{aligned} FOC\left\{\tau_t^f\right\}: \frac{\kappa_c\theta}{\tau_t^f} &= \lambda_t^f n w_t^f, \\ \Longrightarrow \; n w_t^f \tau_t^f &= \frac{\kappa_c\theta}{\lambda_t^f}. \end{aligned} \tag{32}$$

The woman's budget constraint, in this case, is $w_t^f = n w_t^f \tau_t^f + c_t^f$. Using (25) and (32), we get

$$\lambda_t^f = \frac{1+\kappa_c\theta}{w_t^f}.$$

Plugging the values of λ_t^m and λ_t^f into (31) and (31) yields the educational expenditure by the man and the time spent on children by the woman, as follows

$$\begin{aligned} E_t^m &= \frac{\kappa_c(1-\theta)}{1+\kappa_c(1-\theta)} w_t^m, \\ n\tau_t^f &= \frac{\kappa_c\theta}{1+\kappa_c\theta}. \end{aligned}$$

Furthermore, the FOCs with respect to τ_t^m and e_t^f are

$$FOC\left\{\tau_t^m\right\}: \frac{\kappa_c\theta\alpha_m}{\alpha_f\tau_t^f} = \lambda_t^m n w_t^m - \lambda_{2t}^m, \tag{33}$$

$$FOC\left\{e_t^f\right\}: \frac{\kappa_c(1-\theta)}{e_t^m} = \lambda_t^f n p_e - \lambda_{1t}^f. \tag{34}$$

Using (33) and (34), $\lambda_{2t}^m, \lambda_{1t}^f \geq 0$ yield the conditions

$$\begin{aligned} \frac{\alpha_f}{\alpha_m} &\geq \frac{1+\kappa_c\theta}{1+\kappa_c(1-\theta)} \\ \frac{w_t^f}{w_t^m} &\leq \frac{1+\kappa_c\theta}{1+\kappa_c(1-\theta)} \end{aligned}$$

which together again imply

$$\frac{w_t^f}{w_t^m} \leq \frac{\alpha_f}{\alpha_m}.$$

Case 3: $e_t^m = \tau_t^m = 0, e_t^f, \tau_t^f > 0.$

Here, $\lambda_{1t}^f = \lambda_{2t}^f = 0$. From the woman's optimization problem,

$$FOC\left\{e_t^f\right\} : \frac{\kappa_c(1-\theta)}{e_t^f} = \lambda_t^f n p_e, \tag{35}$$

$$FOC\left\{\tau_t^f\right\} : \frac{\kappa_c\theta}{\tau_t^f} = \lambda_t^f n w_t. \tag{36}$$

Using (35) and (36), the woman's budget constraint can be re-written and λ_t^f derived as follows

$$\begin{aligned} w_t^f &= n w_t^f \tau_t^f + E_t^f + c_t^f, \\ \Rightarrow w_t^f &= \frac{1}{\lambda_t^f} + \frac{\kappa_c(1-\theta)}{\lambda_t^f} + \frac{\kappa_c\theta}{\lambda_t^f}, \\ \Rightarrow \lambda_t^f &= \frac{1+\kappa_c}{w_t^f}. \end{aligned}$$

Therefore educational expenditure and time input on children (both borne by the woman) become

$$\begin{aligned} E_t^f &= \frac{\kappa_c(1-\theta)}{1+\kappa_c} w_t^f, \\ n\tau_t^f &= \frac{\kappa_c\theta}{1+\kappa_c}. \end{aligned}$$

The man's budget constraint in this case is just $w_t^m = c_t^m$. Further, his optimality conditions are

$$FOC\left\{e_t^m\right\} : \frac{\kappa_c(1-\theta)}{e_t^f} = \lambda_t^m n p_e - \lambda_{1t}^m, \tag{37}$$

$$FOC\left\{\tau_t^m\right\} : \frac{\kappa_c\theta\alpha_m}{\alpha_f\tau_t^f} = \lambda_t^m n w_t^m - \lambda_{2t}^m. \tag{38}$$

Using (37) and (38), $\lambda_{1t}^m, \lambda_{2t}^m > 0$ yield the following conditions:

$$\frac{w_t^f}{w_t^m} > 1 + \kappa_c,$$

$$\frac{\alpha_f}{\alpha_m} > 1 + \kappa_c.$$

Note here that the case where $e_t^f = \tau_t^f = 0$, i.e., the one corresponding to row 4 of Table 1 is symmetric to the one discussed here, and can be solved similarly.

Case 4: $e_t^f = 0$, $e_t^m, \tau_t^m, \tau_t^f > 0$.

Here, $\lambda_{1t}^m, \lambda_{2t}^m, \lambda_{2t}^f > 0$. We get the following optimality conditions.

$$FOC\left\{e_t^m\right\}: \frac{\kappa_c(1-\theta)}{e_t^m} = \lambda_t^m n p_e, \tag{39}$$

$$FOC\left\{\tau_t^m\right\}: \frac{\kappa_c \theta \alpha_m}{\alpha_m \tau_t^m + \alpha_f \tau_t^f} = \lambda_t^m n w_t^m, \tag{40}$$

$$FOC\left\{\tau_t^f\right\}: \frac{\kappa_c \theta \alpha_f}{\alpha_m \tau_t^m + \alpha_f \tau_t^f} = \lambda_t^f n w_t^f. \tag{41}$$

From (40) and (41),

$$\frac{\alpha_m}{\lambda_t^m w_t^m} = \frac{\alpha_f}{\lambda_t^f w_t^f} = \bar{\lambda}_t \ (say). \tag{42}$$

Using (39) and (40), the man's budget constraint can be written as

$$\begin{aligned} w_t^m &= \frac{1}{\lambda_t^m} + \frac{\kappa_c(1-\theta)}{\lambda_t^m} + \frac{\kappa_c\theta}{\lambda_t^m} - \frac{\alpha_f}{\alpha_m} n w_t^m \tau_t^f, \\ \implies \lambda_t^m &= \frac{1+\kappa_c}{\frac{\alpha_f}{\alpha_m} n w_t^m \tau_t^f + w_t^m}. \end{aligned} \tag{43}$$

Using (41), the woman's budget constraint is written as

$$\begin{aligned} w_t^f &= \frac{1}{\lambda_t^f} + \frac{\kappa_c\theta}{\lambda_t^f} - \frac{\alpha_m}{\alpha_f} n w_t^f \tau_t^m, \\ \implies \lambda_t^f &= \frac{1+\kappa_c\theta}{\frac{\alpha_m}{\alpha_f} n w_t^f \tau_t^m + w_t^f}. \end{aligned} \tag{44}$$

Using the expressions for λ_t^m and λ_t^f in (42) yields

$$\frac{\alpha_f n \tau_t^f + \alpha_m}{1+\kappa_c} = \frac{\alpha_m n \tau_t^m + \alpha_f}{1+\kappa_c\theta} = \bar{\lambda}_t. \tag{45}$$

(45) subsequently yields the follwing two equations

$$n\alpha_f \tau_t^f = (1+\kappa_c)\bar{\lambda}_t - \alpha_m, \tag{46}$$

$$n\alpha_m \tau_t^m = (1+\kappa_c)\bar{\lambda}_t - \alpha_f. \tag{47}$$

Adding (46) and (47) gives

$$\kappa_c \theta \bar{\lambda}_t = [2 + \kappa_c(1+\theta)]\, \bar{\lambda}_t - (\alpha_m + \alpha_f),$$
$$\Longrightarrow \bar{\lambda}_t = \frac{\alpha_m + \alpha_f}{2 + \kappa_c}. \tag{48}$$

Therefore, using this value of λ_t in (42) yields

$$\lambda_t^m = \frac{\alpha_m(2+\kappa_c)}{(\alpha_m + \alpha_f) w_t^m}; \;\; \lambda_t^f = \frac{\alpha_f(2+\kappa_c)}{(\alpha_m + \alpha_f) w_t^f}.$$

Putting these values in (46) and (47) gives the solution for time and educational inputs to child quality.

$$n\tau_t^m = \frac{1+\kappa_c\theta}{2+\kappa_c} - \frac{\alpha_f}{\alpha_m} \cdot \frac{1+(1-\theta)\kappa_c}{2+\kappa_c},$$

$$n\tau_t^f = \frac{1+\kappa_c}{2+\kappa_c} - \frac{\alpha_m}{\alpha_f} \cdot \frac{1}{2+\kappa_c}.$$

Also,

$$E_t^m = \frac{\kappa_c(1-\theta)}{2+\kappa_c} \cdot \frac{\alpha_m + \alpha_f}{\alpha_m} w_t^m.$$

$\tau_t^m > 0$ and $\tau_t^f > 0$ correspond to the following conditions respectively:

$$\frac{\alpha_f}{\alpha_m} > \frac{1}{1+\kappa_c},$$

$$\frac{\alpha_f}{\alpha_m} < \frac{1+\kappa_c\theta}{1+(1-\theta)\kappa_c}.$$

Further,

$$\frac{\kappa_c(1-\theta)}{e_t^m} = \lambda_t^f n p_e - \lambda_{1t}^f.$$

Consequently, $\lambda_{1t}^f \geq 0$ implies

$$\frac{\alpha_f}{\alpha_m} \geq \frac{w_t^f}{w_t^m}.$$

B Patriarchy

The problem that the man, being the primary decision-maker, faces can be expressed as the following Lagrangian:

$$\begin{aligned}\mathcal{L}_t^{pat} &= \ln c_t^m + \kappa_c \ln \gamma(\alpha_m \tau_t^m + \alpha_f \tau_t^f)^\theta e_t^{1-\theta} + I_p \ln \mu n \tau_t^f \\ &+ \lambda_t[(1-n\tau_t^m)w_t^m + (1-n\tau_t^f)w_t^f - c_t^m - c_t^f - np_e e_t] \\ &+ \lambda_{1t}[\ln c_t^f + \kappa_c \ln \gamma(\alpha_m \tau_t^m + \alpha_f \tau_t^f)^\theta e_t^{1-\theta} - U_{NC}^f] \\ &+ \lambda_{1t}^m \tau_t^m + \lambda_{1t}^f \tau_t^f + \lambda_{2t}^m(1-n\tau_t^m) + \lambda_{2t}^f(1-n\tau_t^f).\end{aligned}$$

where λ_t and λ_{1t} are the Lagrangian multipliers with respect to the household budget constraint and the woman's participation constraint respectively, both of which will bind at optimum. In a case where $\tau_t^m = 0$ and $\tau_t^f = \frac{1}{n}$, we have $\lambda_{2t}^m = \lambda_{1t}^f = 0$. Therefore, the first-order optimality conditions are as follows.

$$FOC\left\{c_t^m\right\} : \frac{1}{c_t^m} = \lambda_t, \tag{49}$$

$$FOC\left\{c_t^f\right\} : \frac{\lambda_{1t}}{c_t^f} = \lambda_t, \tag{50}$$

$$FOC\left\{e_t\right\} : (1+\lambda_{1t})\frac{\kappa_c(1-\theta)}{e_t} = \lambda_t n p_e, \tag{51}$$

$$FOC\left\{\tau_t^m\right\} : (1+\lambda_{1t})\frac{\kappa_c\theta\alpha_m n}{\alpha_f} - \lambda_t n w_t^m + \lambda_{1t}^m = 0, \tag{52}$$

$$FOC\left\{\tau_t^f\right\} : (1+\lambda_{1t})\kappa_c\theta n + nI_p - \lambda_t n w_t^f - n\lambda_{2t}^f = 0. \tag{53}$$

When $\tau_t^m = 0$ and $\tau_t^f = 1/n$, the household budget constraint becomes:

$$\begin{aligned} w_t^m &= \frac{1}{\lambda_t} + \frac{\lambda_{1t}}{\lambda_t} + \frac{1+\lambda_{1t}}{\lambda_t}\kappa_c(1-\theta), \\ \implies \frac{1+\lambda_{1t}}{\lambda_t} &= \frac{w_t^m}{1+\kappa_c(1-\theta)}. \end{aligned} \tag{54}$$

Using this value in (51),

$$E_t = \frac{\kappa_c(1-\theta)}{1+\kappa_c(1-\theta)} w_t^m.$$

From (52), the condition $\lambda_{1t}^m \geq 0$ gives

$$\frac{\alpha_f}{\alpha_m} > \frac{\kappa_c\theta}{1+\kappa_c(1-\theta)}.$$

Using (53), the condition $\lambda^f_{2t} \geq 0$ can be rewritten as

$$\begin{aligned}
&\frac{1+\lambda_{1t}}{\lambda_t}\kappa_c\theta + \frac{I_p}{\lambda_t} \geq w^f_t, \\
\implies &\frac{\kappa_c\theta}{1+\kappa_c(1-\theta)}w^m_t + \frac{I_p}{\lambda_t} \geq w^f_t.
\end{aligned} \tag{55}$$

The value for λ_t is obtained using the binding participation constraint of the woman.

$$\ln(c^f_t) + \kappa_c \ln\left(\gamma(\alpha_m\tau^m_t + \alpha_f\tau^f_t)^\theta e_t^{1-\theta}\right) = U^f_{NC}(w^m_t, w^f_t), \tag{56}$$

$$\Rightarrow \ln\left(\frac{\lambda_{1t}}{\lambda_t}\right) + \kappa_c \ln\left(\gamma\left(\frac{\alpha_f}{n}\right)^\theta\left(\frac{\kappa_c(1-\theta)w^m_t}{(1+\kappa_c(1-\theta))np_e}\right)^{1-\theta}\right) = U^f_{NC}(w^m_t, w^f_t), \tag{57}$$

$$\Rightarrow \frac{\lambda_{1t}}{\lambda_t} = \exp\left(U^f_{NC}(w^m_t, w^f_t) - \kappa_c \ln\left(\gamma\left(\frac{\alpha_f}{n}\right)^\theta\left(\frac{\kappa_c(1-\theta)w^m_t}{(1+\kappa_c(1-\theta))np_e}\right)^{1-\theta}\right)\right), \tag{58}$$

$$\Rightarrow \frac{1}{\lambda_t(w^m_t, w^f_t)} = \frac{w^m_t}{1+\kappa_c(1-\theta)} - \exp\left(U^f_{NC}(w^m_t, w^f_t) - \kappa_c \ln\left(\gamma\left(\frac{\alpha_f}{n}\right)^\theta\left(\frac{\kappa_c(1-\theta)w^m_t}{(1+\kappa_c(1-\theta))np_e}\right)^{1-\theta}\right)\right), \tag{59}$$

where the last step uses the value for $(1+\lambda_{1t})/\lambda_t$ in (54).

C Unitary household behavior

The Lagrangian for the household's optimization problem is presented as follows

$$\begin{aligned}
\mathcal{L}^{unit}_t = {} & \alpha \ln c^m_t + (1-\alpha)\ln c^f_t + \kappa_c \ln\left(\gamma(\alpha_m\tau^m_t + \alpha_f\tau^f_t)^\theta e_t^{1-\theta}\right) \\
& + \lambda_t[(1-n\tau^m_t)w^m_t + (1-n\tau^f_t)w^f_t - c^m_t - c^f_t - np_e e_t] \\
& + \lambda^m_{1t}\tau^m_t + \lambda^f_{1t}\tau^f_t + \lambda^m_{2t}(1-n\tau^m_t) + \lambda^f_{2t}(1-n\tau^f_t).
\end{aligned}$$

Note that, similar to Appendix A, the constant non-discretionary inputs to education and child-rearing have been suppressed for convenience i.e., $\bar{e} = \bar{\tau}$. This leaves only 3 three cases of interest. The remaining cases are symmetric and can be solved similarly. In all possible cases, the FOCs with respect to their respective consumptions yield:

$$\frac{\alpha}{c^m_t} = \frac{1-\alpha}{c^f_t} = \lambda_t. \tag{60}$$

Case 1: $\tau^m_t = 0$, $\frac{1}{n} > \tau^f_t > 0$

Here, $\lambda_{2t}^m = \lambda_{1t}^f = \lambda_{2t}^f = 0$. The optimality conditions are as follows.

$$FOC\left\{\tau_t^m\right\} : \frac{\kappa_c \theta \alpha_m}{\alpha_f \tau_t^f} - \lambda_t n w_t^m + \lambda_{1t}^m = 0, \tag{61}$$

$$FOC\left\{\tau_t^f\right\} : \frac{\kappa_c \theta}{\tau_t^f} - \lambda_t n w_t^f = 0, \tag{62}$$

$$FOC\left\{e_t\right\} : \frac{\kappa_c(1-\theta)}{e_t} - \lambda_t n p_e = 0. \tag{63}$$

The household budget constraint, $(1-n\tau_t^m)w_t^m + (1-n\tau_t^f)w_t^f = c_t^m + c_t^f + np_e e_t$, can be re-written using equations (60)–(63) as follows

$$\begin{aligned} w_t^m + w_t^f &= \frac{\alpha}{\lambda_t} + \frac{1-\alpha}{\lambda_t} + \frac{\kappa_c \theta}{\lambda_t} + \frac{\kappa_c(1-\theta)}{\lambda_t}, \\ \implies \lambda_t &= \frac{1+\kappa_c}{w_t^m + w_t^f}. \end{aligned} \tag{64}$$

Using this value of λ_t in (63) and (62) yields the following solution.

$$\begin{aligned} n\tau_t^f &= \frac{\kappa_c \theta}{1+\kappa_c}\left(1 + \frac{w_t^m}{w_t^f}\right), \\ E_t &= \frac{\kappa_c(1-\theta)}{1+\kappa_c}(w_t^m + w_t^f). \end{aligned}$$

From (61), $\lambda_{1t}^m \geq 0$ gives

$$\frac{w_t^f}{w_t^m} \geq \frac{\alpha_f}{\alpha_m},$$

and $n\tau_t^f < 1$ gives:

$$\frac{w_t^f}{w_t^m} > \frac{\kappa_c \theta}{1+\kappa_c(1-\theta)}.$$

Case 2: $\tau_t^m = 0$, $\tau_t^f = \frac{1}{n}$.

Here, $\lambda_{2t}^m = \lambda_{1t}^f = 0$. The optimality conditions are as follows.

$$FOC\left\{\tau_t^m\right\} : \frac{\kappa_c \theta \alpha_m n}{\alpha_f} - \lambda_t n w_t^m + \lambda_{1t}^m = 0, \tag{65}$$

$$FOC\left\{\tau_t^f\right\} : \kappa_c \theta n - \lambda_t n w_t^f - \lambda_{2t}^f = 0, \tag{66}$$

$$FOC\left\{e_t\right\} : \frac{\kappa_c(1-\theta)}{e_t} - \lambda_t n p_e = 0. \tag{67}$$

In this case, the household budget constraint is rewritten as

$$w_t^m = \frac{\alpha}{\lambda_t} + \frac{1-\alpha}{\lambda_t} + \frac{\kappa_c(1-\theta)}{\lambda_t}, \tag{68}$$

$$\implies \lambda_t = \frac{1+\kappa_c(1-\theta)}{w_t^m}. \tag{69}$$

Therefore,

$$E_t = \frac{\kappa_c(1-\theta)}{1+\kappa_c(1-\theta)} w_t^m.$$

Using (65) and (66), $\lambda_{1t}^m, \lambda_{2t}^f \geq 0$ respectively yield the following conditions:

$$\frac{\alpha_f}{\alpha_m} \geq \frac{\kappa_c\theta}{1+\kappa_c(1-\theta)},$$

$$\frac{w_t^f}{w_t^m} \leq \frac{\kappa_c\theta}{1+\kappa_c(1-\theta)}.$$

Case 3: $0 < \tau_t^m < \frac{1}{n}$, $\tau_t^f = \frac{1}{n}$.

Here, $\lambda_{1t}^m = \lambda_{2t}^m = \lambda_{1t}^f = 0$. The optimality conditions are as follows.

$$FOC\left\{\tau_t^m\right\} : \frac{\kappa_c\theta\alpha_m n}{\alpha_m n\tau_t^m + \alpha_f} - \lambda_t n w_t^m = 0, \tag{70}$$

$$FOC\left\{\tau_t^f\right\} : \frac{\kappa_c\theta\alpha_f n}{\alpha_m n\tau_t^m + \alpha_f} - \lambda_t n w_t^f - \lambda_{2t}^f n = 0, \tag{71}$$

$$FOC\left\{e_t\right\} : \frac{\kappa_c(1-\theta)}{e_t} - \lambda_t n p_e = 0, \tag{72}$$

The household budget constraint is rewritten as,

$$w_t^m = \frac{\alpha}{\lambda_t} + \frac{1-\alpha}{\lambda_t} + \frac{\kappa_c\theta}{\lambda_t} - \frac{\alpha_f}{\alpha_m}w_t^m + \frac{\kappa_c(1-\theta)}{\lambda_t},$$

$$\implies \lambda_t = \frac{1+\kappa_c}{w_t^m\left(1+\frac{\alpha_f}{\alpha_m}\right)}. \tag{73}$$

Using this value of λ_t in (70) and (72) yields the following solution.

$$n\tau_t^m = \frac{\kappa_c\theta}{1+\kappa_c} - \frac{\alpha_f}{\alpha_m}\frac{1+\kappa_c(1-\theta)}{1+\kappa_c},$$

$$E_t = \frac{\kappa_c(1-\theta)}{1+\kappa_c}\left(1+\frac{\alpha_f}{\alpha_m}\right)w_t^m.$$

From (71), $\lambda_{2t}^f \geq 0$ implies

$$\frac{w_t^f}{w_t^m} \leq \frac{\alpha_f}{\alpha_m}, \tag{74}$$

and $n\tau_t^m > 0$ implies

$$\frac{\alpha_f}{\alpha_m} > \frac{\kappa_c \theta}{1 + \kappa_c(1-\theta)}. \tag{75}$$