Oligopolistic Competition and Corruption

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Abstract

This study explores the entry deterrence game between an incumbent firm and the potential entrant. In this model, the incumbent firm practices dubious means to deter the entry of other firms by bribing local officials. The bribe level can be considered as 'Height of the Entry Barrier.' We analyzed the three-stage incomplete information game based on Bertrand's competition in the differentiated goods market. We have shown that the Bertrand competition's optimal bribe level is lower than the Cournot competition when goods are complements or substitutes. This implies that the height of the entry barrier is more for the potential entrant in Cournot competition rather than Bertrand competition, irrespective of the nature of the goods. We can also infer that the magnitude of corruption is more under Cournot competition than Bertrand competition.

Keywords — Oligopoly, Bertrand Competition, Entry Deterrence, Industrial Organization, Corruption, Cournot Competition

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1 Introduction

Economies with poor governance and corruption are prone to the market outcome that lessens the market quality and shrinks the total surplus. These two elements lead to the biased distribution of the total surplus, as the incumbent use them to draw out the maximum surplus and deter entry. Entry deterrence can potentially bring inefficiency as it lowers the competition and reduces the overall welfare.

In many developed economies, firms practice lobbying to deter the entry of potential entrants, whereas in developing economies, firms pay a bribe to the government. However, lobbying is a legal activity in many countries, whereas bribery is illegal. In my study, I am analyzing that how difficult for a potential entrant to enter the market because an incumbent firm pays a bribe based on Bertrand. The amount of bribe is considered to be the "Height of entry barrier."

The term 'higher competition' signifies the ease of entry into the market. The ease of entry in the market is crucial because it defines the magnitude of the competition level present; otherwise, market welfare will be too low. So the ease of entry of a firm in the market is a vital element to decide the welfare of society. We compare the two setups of the oligopolistic model Bertrand and Cournot model. We want to analyze; What will happen to the competition level when there is Price Competition? What will happen to the competition level when there is Quantity Competition? So this study will answer the above two questions in the following sections.

Moreover, with the Cournot model, as we increase the number of firms, this will lead to an increase in the quantities and thereby decrease the price. Consider an example of forestry in which each district head can allow illegal logging in return for a bribe to increase the number of districts. The whole logging should increase, and thereby prices should fall. This study is empirically tested for Indonesia as well see (Cisneros, et. al., 2021). In Indonesia, between 2000 and 2008 number of districts almost doubled, with district splits occurring sporadically. (Cisneros, et. al., 2021) examine the impact of an increasing number of districts in a market over time. The results showed the effect on quantity using satellite data. They also demonstrated that the effects on prices from official production data. Suppose, in the given example, a local official asks for more bribes, and then this would be difficult for an incumbent to increase the bribe amount because it acts as a sunk cost for the incumbent. However, in our study, optimal bribe depends on the efficiency level of the potential entrant, which is unknown to the incumbent. I am interested in showing how just changing the way of competition can reduce the actual level of optimal bribe in the equilibrium. It is because this will help to explain the magnitude of corruption in different competitions.

Although entry deterrence can be done in different ways, other than bribing, it is concerned with the actions of the incumbent firms to restrict the entry of the new entrant into the market. These deterring strategies are practiced in several ways by the incumbent, ensuring that the incumbent acquires a significant market share, total surplus, and maximum market power. These strategies include excess capacity, limit pricing, predatory pricing, predatory acquisition, and switching costs.

The factor that may attract any firm to enter a particular market is anticipated profit. Profit depends upon several factors, two of which are the firm's payoff and customer number. Higher expected profits incentivize the new entrant to enter the market. So incumbent can 'tie up' with the customers to deter the entry of the potential buyer and reduce its payoff, which is referred to as preemptive deterrence. However, the branding and goodwill of the incumbent could also act as barriers to entry as consumers may be concerned with the brand's loyalty and taste towards the particular brand. This may create the scenario for the potential entrant to do a heavy price cut to fetch the customers, which again would be a resistance to entry. Moreover, the potential entrant may face a huge sunk cost to spread awareness about their product if the already existing firms have a large budget for advertising.

For instance, Monsanto's contract with its largest customers of Coke and Pepsi was the significant move to make the entry of the potential firm less attractive towards the soda markets as the potential market may faceless customers partially because of the Monsanto's contract and partly because of loyalty of the customers towards the Coke and Pepsi. This, in turn, would lessen the profit of the potential entrant. One more instance can be considered here when Xerox generated several patents that were left unused against the plain paper photocopying. It would be more challenging for the potential entrant to foresee the plain-paper photocopying. Patents would increase the entry cost and would act as entry deterrence for the potential firm.

Incumbent firms often strategically outdo production to limit the entry of potential firms. Excess capacity of the incumbent firm menace the potential firm to enter. This excess capacity reflects the incumbent firm's economies of scale and enables the incumbent firm to reduce the price level sufficiently. This strategy is most often seen in natural monopoly (Sharkey, William W., 1982); any industry has few key aspects that can be considered a monopolized industry, such as the necessity of a good or service, location for the production replaces alternatives, no storage possibility of the output, there should be economies of scale, production done by a single supplier.

Incumbent's production at positive profit attracts the new firms to enter the market. Limit pricing is a strategy to deter the entry of the new firm by strategically producing quantities at a lower price. Limiting the price will shrink the profit and disincentives the potential entrant from entering the market as the incumbent firm would already capture the significant market by lowering the price. This strategy would be most appropriate if the after-limit pricing profits are still higher than those risked if a rival entered the market; also, this strategy adheres to proper planning and sufficient machinery installation for credible results.

Both incumbent and the potential entrant do not have complete information about each other. Limit pricing serves as a signaling technique here for the potential when it's considering entering the market. The entrant would design the output matrix considering the values the incumbent firm reflects after limiting the price, leading to higher costs for the entrant.

Suppose a firm is operating at prices below the short-run marginal cost to keep the potential entrant reluctant from entering the market. In that case, predatory pricing is also known as Areeda-Turner Law. This strategical move alerts the potential entrant about the loss of demand that it can face if it enters. According to (Cabral et. al., 2008), this strategy would be fruitful for the incumbent even if the potential entrant does not carefully examine the situation.

If incumbent firms allow the entrant to enter the market, they may maximize the profit in the short run, but that may not persist in the long run. Then new firm would be under the impression that it is dominated by the incumbent firm and would try to bring more firms to the market. So following this strategy incumbent firm may secure itself from any such occurrence in the future. British Airways versus Virgin Atlantic during the 1980s can be considered here as an example.

Another way, such as Predatory acquisition, an incumbent firm deters the entry of an entrant. In this way, an incumbent firm tries to control the small firm by obtaining its sufficiently high share. Switching cost is another example of entry deterrence; in this case, a firm targets its customers and enforces an exit fee so that some financial hurdle deters the existing customers.

However, entry deterrence practices all over the world. For instance of Airbnb of Japan. In this case study, in 2016, lobbying by the incumbent industry created entry barriers for the new entrant, Airbnb. During that time, the government has released guidelines for home-sharing—called minpaku in Japanese—that could illegally make most Airbnb rentals in the country. After these guidelines, Airbnb hotels would only be allowed to rent to guests who stay at least a week. Another example is Walmart which has been lobbying with US officials since 2008 to enter the Indian market. According to the lobbying disclosure reports of Walmart, the company has spent on various lobbying activities approximately USD 25 million since 2008.

There are other scenarios as well, which motivates me to delve deeper and to analyze entry deterrence in Bertrand competition.

The following section reviews the literature devoted to this study.

2 Literature Review

The idea for the model presented in this study is borrowed from (Dastidar & Yano, 2021), where they examine the determinants of the height of entry barrier by using bribe as a proxy and characterizes the optimal bribe, and shows that this depends on the market size, the differentiation parameter (whether goods are substitutes or complement), and the extent of uncertainty. (Dastidar & Yano, 2021) explores the linkages between an optimal bribe and market welfare. This paper further shows that zero bribes need not maximize total surplus and market quality. However, in my study, I am incorporating the Bertrand model to compare the results with the Cournot model done by (Dastidar & Yano, 2021). Moreover, (Aliprанti et al.; 2014) compared the Cournot and Bertrand competition under vertically related market with two part-tariff under monopoly. They demonstrate that the standard conclusions are reversed and argue that welfare under Cournot is higher than the Bertrand model. Additionally, (Vives, 1999) bolsters the same argument by explaining that the nature of competition (Cournot or Bertrand) crucially affects the equilibrium outcomes.

The paper done by (Lee & Wang, 2007) also compared the entry deterrence game between Cournot and Bertrand competition. They argued that entry could be easily blockaded under the Cournot model than the Bertrand model in the differentiated goods market. They further investigated that entry can be easily blocked under the Bertrand model under close substitutes than the Cournot model. However, their model for this claim is of complete information. They have not considered the efficiency level factor of the entrant; otherwise, it will be an asymmetric information game. Furthermore, (Ishigaki & Hiroaki, 2000) shows that in a sequential game followed by simultaneous price game, in homogeneous goods market, it is impossible to have entry deterrence via informative advertising.

There is vast literature present on the entry deterrence model. (Gruca & Sudharshan, 1995) explain that entry deterrence strategy has significant aspects such as interconnection in the competitive environment, entry deterrence strategy's choice, corporate-level strategy, consequences for the incumbent. In regulated sectors, licensing procedures, safety standards, territorial restrictions, and other legal requirements may affect the entry of potential entrants. Some competition agencies have done entry barriers by issuing reports that study the regulations' effects on competition, identify less restrictive alternatives, and support appropriate changes (OECD, 2007). Futhermore, entry deterrence affects the market welfare too. (Goerke, L., 2017) points out the role of tax evasion in excessive entry prediction and welfare maximization. He analyzes that tax evasion lowers the overall welfare and can mitigate excessive entry. Consequently, he finds that fluctuation in excessive entry is controlled by the direct welfare consequences of tax evasion, and explains the relationship between tax evasion and the tax base. Instead of offering bribe, some incumbent firms do excess capacity installation to deter the entry of the potential entrant. (Maskin, E. S., 1999) analyses that how the incumbent uses the heavy capacity installation, which disincentives the potential firm from entering, as a tool to deter entry in uncertainty about demand or cost. (Harstad & Svensson, 2011) analyze the choices the firm can make when faced with regulatory constraints. This paper finds that bribery is more likely associated with the developing countries, which can keep them trapped in poverty forever, whereas lobbying is practiced in developed countries. (Athreya & Majumdar, 2005) attempt to analyze conditions under which the two government departments reorganize and serve both types of customers; initially, they were serving a particular type at a time. These conditions may improve efficiency. Moreover, (Lambert-Mogiliansky et. al., 2008) explore a game-theoretic model of petty corruption between entrepreneurs and bureaucrats with asymmetric information of the value of the project and explain the implications of welfare.

Moreover, (Ellison & Ellison, 2011) explain the model of entry deterrence related to the pharmaceutical industry before patent expiration. They argued that non-monotonic investment might result from strategic investments because these investments are in huge numbers unnecessarily. They also explained that in the medium-sized market incumbent has less motivation to deter the entry of potential entrants before the expiration of the patent.

Moreover, entry deterrence can be depicted through a linear city model with different cost functions. (Zhou, A., 2013) explained the spatial competition related to the linear city model. He explained that horizontal differentiation in the goods market in a linear city, higher transportation, and higher fixed cost are the critical elements for entry deterrence. He further argued that the optimal location for an incumbent, to deter the entry of potential entrants, will be the center point of the linear city.

The literature related to corruption is humongous but less related to our model. (Campos, et al., 2010) explains that about the 'bribe unavoidability' because corruption does not matter so much because of this feature. They also argued that social cost directly depends on the barrier created for potential entrants instead of the incumbent's marginal cost. On the other hand, (Sequeira & Djankov, 2013) explains the two types of corruption cost-reducing and cost-increasing. Cost reducing refers to 'collusion' whereas cost increasing refers to 'coercive.' They observed that adjustment in sourcing strategies of firms forces them to respond to each type of corruption. They found that the firms' behavior depends on the outcomes created by each type of corruption. However, (Emran & Shilpi, 2000*)* find the notion of optimal bribe that may result from the collaboration between the bureaucrat and the incumbents, which deters entry equilibrium conditions, explains the conditions of equilibrium in the case of cost heterogeneity and cost homogeneity. The paper by (Broadman & Recanatini, 2001) investigates illicit behavior and rent-seeking are affected by the market institutions. They corroborate the systematic link between the market institutions' development and incentives for corruption. They empirically validate that improvements in the democratic process and economic development mitigate corruption. (Kunieda & Shibata, 2014) explains the credit market imperfections considering it as a low-quality financial market. They further explained economic fluctuations depend on the quality of the financial market. They point out the stability of the economy's equilibrium which will be convergent when the degree of credit market imperfection is too low; otherwise, it will be divergent.

3 The Model

The motivation for the model for this research got from (Dastidar & Yano, 2021), in which they examine the determinants of the height of entry barrier by using bribe as a proxy and characterizes the optimal bribe, and shows that this depends on the market size, the differentiation parameter (whether goods are substitutes or complement), and the extent of uncertainty; based on Cournot competition.

In our study, we are doing the extension of (Dastidar & Yano, 2021) model by using Bertrand Competition and will compare the outcome from the Cournot competition see (Vives, 1999) and (Alipranti et al.; 2014).

In the model, we have two firms one is incumbent and another one is potential entrant competes in differentiated product market. Incumbent firm tries to deter entry of potential entrant by giving some bribe $'b'$ amount to local officials which will be the sunk cost for incumbent firm. The marginal cost of the entrant increases by $'hb'$. For the simplicity, we are considering $h = 1$. Here, b serves as a proxy for the 'height of entry barrier'.

Incumbent's Marginal Cost $= c$

Potential entrant's Marginal Cost $= c - \alpha + b$

Here, α is the efficiency level of the potential entrant.

We are assuming, α is distributed over $[0, \bar{\alpha}]$ with distribution function $F(.)$ and density function $f(.)$. When $(\alpha = 0)$ then it will be complete information game but when $(\alpha \neq 0)$ then it will be incomplete information game because incumbent firm doesn't know about the efficiency level. Asymmetric costs and incomplete information are may be because potential entrant may have superior technology which means low cost technology and access to the informal labor market due to the low bureaucracy.

Consider, $k^2 > 0$ which will be the entry cost for the potential entrant. Following are the marginal cost for the two firms:

$$c_1(q_1) = cq_1 + b$$

$$c_2(q_2) = (c - \alpha + b)q_2 + k^2$$

We are considering following three stages game:

1. In the first stage; the incumbent firm decides on a level of bribe, b (= 'height' of entry barrier).
2. In the second-stage; the entrant observes its marginal cost and then decides to enter or not to enter. Also, 2nd firm decides to enter iff it expects strictly positive profit in the third stage. (Dixit, 1980) & (Tirole, 1988).
3. If 2nd firm enters, then in the third-stage the firms play on Incomplete Information Bertrand Game in a differentiated goods market. If 2nd firm doesn't enter, then firm behaves like a monopolist.

Based on (Dixit, 1979), I consider the following utility function to get the direct demand functions:

$$U = a(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) + q_0$$

This utility function generates the following system of inverse demand function:

$$P_1 = a - q_1 - \gamma q_2 \dots\dots\dots\dots (i)$$

$$P_2 = a - \gamma q_1 - q_2 \dots\dots\dots\dots (ii)$$

Multiply equation (i) by γ to both the sides, we get—

$$\gamma P_1 = \gamma a - \gamma q_1 - \gamma^2 q_2 \dots\dots\dots\dots (iii)$$

Now subtract $(iii) - (ii)$, we get—

$$\gamma P_1 - P_2 = a(\gamma - 1) - \gamma^2 q_2 + q_2$$

$$\gamma^2 q_2 - q_2 = a(\gamma - 1) - \gamma P_1 + P_2$$

$$q_1(P_1, P_2) = \frac{a(\gamma - 1) - \gamma P_2 + P_1}{\gamma^2 - 1}$$

Similarly, we will get the direct demand function of q_2—

$$q_2(P_1, P_2) = \frac{a(\gamma - 1) - \gamma P_1 + P_2}{\gamma^2 - 1}$$

Here, $\gamma \in (-1, 1)$

Now we are solving the game by backward induction method. We first solve the third stage and then we proceed to second stage and first stage.

In the second stage, we will consider the following three cases:

i. When $\gamma = 0$, it means that goods are neutral.

ii. When $\gamma = 0.5$, it means that goods are substitutes.

iii. When $\gamma = -0.5$, it means that goods are complements.

3.1 Assumptions

1. $(a - c) > \frac{\mu(0)}{10}$. It implies that market size i.e. $(a - c)$ will be greater than one-tenth of expected price level of an entrant when the efficiency level is zero.
2. $(a - c) \geq 3k$. This implies that market size is greater than or equal to three times the entry fee of an entrant.
3. $A = a(\gamma - 1);\ B = 2 + \gamma;\ D = 4 - \gamma^2$

3.2 Third-Stage Equilibrium Analysis

In the third stage, the amount of bribe b has determined already and known to both firms. However, the efficiency level α is only known to the entrant only. In this scenario, firm 2 enters if and only if $\alpha \in [\alpha^*, \bar{\alpha}]$ in the second stage and expects a positive profit.

Consider, $\gamma \in (-1, 1)$

For any given b, Bayesian-Bertrand Nash equilibrium be:

$P_1(b)$ and $P_2(\alpha, b)$

In the second stage, 1st firm knows that 2nd firm enters if and only if $\alpha \geq \alpha^*$.

Hence, expected price for the 2nd firm be:

$$Exp.(P_2(\alpha, b) |\ \alpha \geq \alpha^*) = \int_{\alpha^*}^{\bar{\alpha}} \frac{P_2(\alpha, b) f(\alpha)}{1 - F(\alpha^*)} d\alpha$$

In the Bayesian-Nash equilibrium, where $\alpha \in [\alpha^*, \bar{\alpha}]$ we have the following:

$$P_2(\alpha, b) = \arg\max_{P_2 \geq 0} P_2 q_2(P_1, P_2) - [(c - \alpha + b) + k^2]$$

$$P_1(\alpha, b) = \arg\max_{P_1 \geq 0} P_1 q_1(P_1, P_2) - [c q_1(P_1, P_2) + b]$$

After solving, we will get the following results:

$$\boldsymbol{P_1(b) = \begin{cases} \frac{c-A}{2} + \frac{\gamma}{2}\left[\frac{(c-A)B + 2b - 2\mu(\alpha^*)}{D}\right] & \textit{if 2 enters} \\ \frac{a+c}{2} & \textit{if 2 doesn't enter} \end{cases}}$$

$$\boldsymbol{P_2(\alpha, b) = \begin{cases} \frac{c-\alpha+b-A}{2} + \frac{\gamma}{2}\left[\frac{c-A}{2} + \frac{\gamma}{2}\left[\frac{(c-A)B + 2b - 2\mu(\alpha^*)}{D}\right]\right] & \textit{if 2 enters} \\ 0 & \textit{if 2 doesn't enter} \end{cases}}$$

The equilibrium prices $P_1(b)$ and $P_2(\alpha, b)$ are positively correlated with the marginal cost $'c'$ and demand functions' intercept $'a'$.

In the following figures, P_1 is higher under substitute goods ($\gamma > 0$) than complementary goods ($\gamma < 0$). However, P_2 is lower under substitute goods ($\gamma > 0$) than complementary goods ($\gamma < 0$).

Figure 3.2.1 (a)

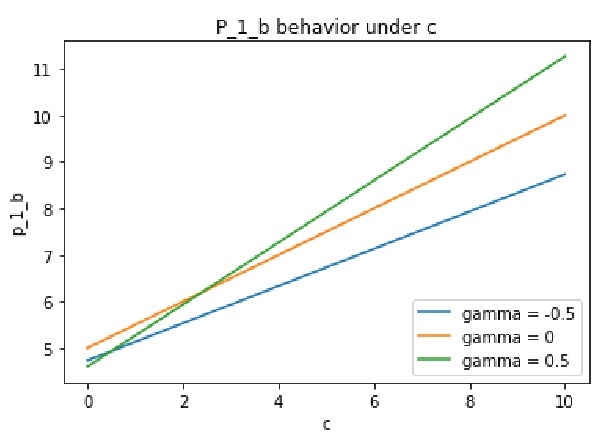


Figure 3.2.1 (b)

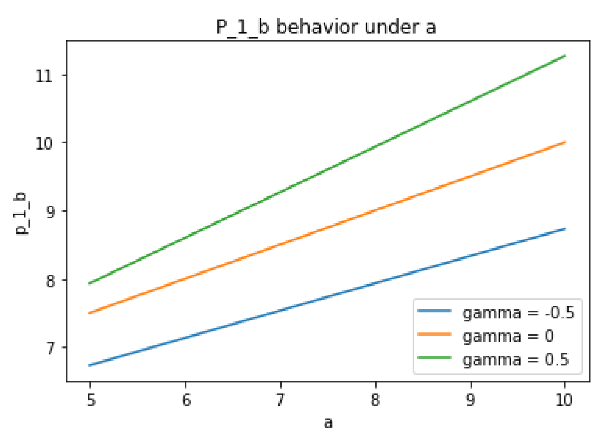


Figure 3.2.1 (c)

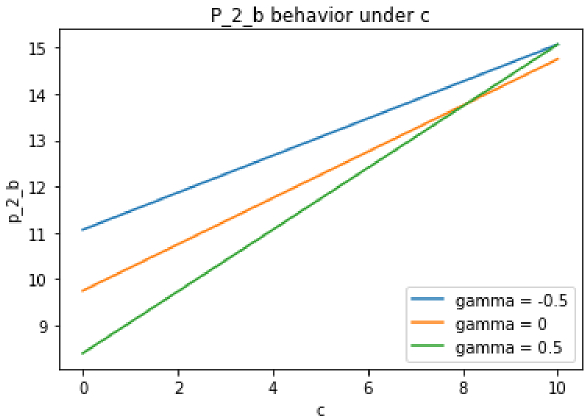
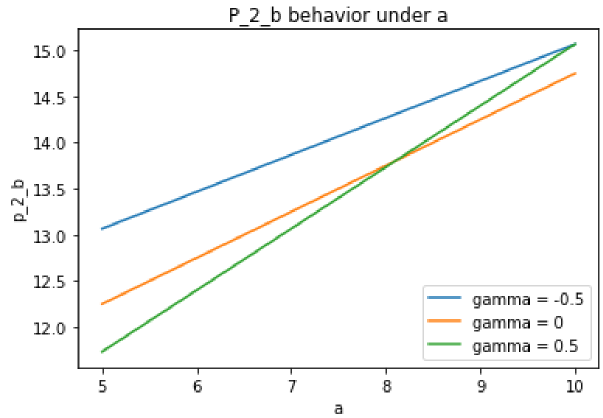


Figure 3.2.1 (d)



In the following sub-section, we are analyzing the second-stage equilibrium analysis for different values of $'\gamma'$.

3.3 When $\gamma = 0$ (Neutral Goods)

3.3.1 Second-Stage Equilibrium Analysis

Equilibrium prices will be:

$$P_1(b) = \frac{a+c}{2}$$

$$P_2(\alpha, b) = \begin{cases} \dfrac{a+c+b-\alpha}{2} & if\ 2\ enters \\ 0 \quad if\ \alpha \in [o, \alpha^*], & if\ 2\ doesn't\ enter\ for\ such\ \alpha \end{cases}$$

Then the equilibrium profits will be:

$$\pi_1(b) = \left(\frac{a-c}{2}\right)^2 - b$$

$$\pi_2(\alpha, b) = \left(\frac{-c+\alpha-b+a}{2}\right)^2 - k^2$$

Suppose Bribe is Zero i.e. $b = 0$ and $\alpha = 0$ then firm 2 decides to enter iff

$$\pi_2(0,0) = \left(\frac{a-c}{2}\right)^2 - k^2 > 0$$

$$= \frac{a-c}{2} > k$$

$$= a - c > 2k$$

Now, $b > 0$ and $\alpha = 0$

$$\pi_2(0, b) = \left(\frac{-c-b+a}{2}\right)^2 - k^2 > 0$$

$$= \frac{a-c-b}{2} > k$$

$$= a - c - b > 2k$$

Since, $\pi_2(0, b)$ is decreasing in b.

Therefore, there exists $\underline{b}$ such that $\pi_2(0, \underline{b}) = 0$

$$\Rightarrow \left(\frac{-c-\underline{b}+a}{2}\right)^2 - k^2 = 0$$

$$\Rightarrow \frac{a-c-\underline{b}}{2} = k$$

$$\boldsymbol{\underline{b} = a - c - 2k}$$

$\pi_2(0, b) > 0 \quad \forall \ b \in [0, \underline{b})$ then 2 will enter.

$\pi_2(\alpha, b)$ is strictly decreasing in b.

Also, $\pi_2(\alpha, b)$ is strictly increasing in α.

i.e. $\pi_2(\bar{\alpha}, b) > \pi_2(\alpha, b)$, for $b > \underline{b}$

Also, $\pi_2(\overline{\alpha}, b)$ is strictly decreasing in b, there exists $\bar{b}$ such that $\pi_2(\bar{\alpha}, \bar{b}) = 0$.

Now also note that using L'hospital's rule, we get—

$$\lim_{\alpha^* \to \overline{\alpha}} \mu(\alpha^*) = \bar{\alpha}$$

This means for $\alpha^* = \bar{\alpha}, b = \bar{b}$

$$\pi_2(\bar{\alpha}, \bar{b}) = \left(\frac{-c + \bar{\alpha} - \bar{b} + a}{2}\right)^2 - k^2 = 0$$

$$-c + \bar{\alpha} - \bar{b} + a = 2k$$

$$\boldsymbol{\bar{b} = a - c + \bar{\alpha} - 2k}$$

Profit of firm 2 will increase in terms of efficiency level (α) but it is decreasing in terms of b.

So, $\pi_2(\alpha, b) < 0$ when $b > \bar{b}$ so no one will enter.

If the optimal bribe $'b'$ is in between 0 and $\underline{b}$, and efficiency level is zero then all firms will enter. However, if the optimal bribe $'b'$ is in between $\underline{b}$ and $\bar{b}$, then those firms will enter whose efficiency level is in between α^* and $\bar{\alpha}$, it is because profit level be strictly positive in this case, and the probability of entry is $1 - F(\alpha^*)$. Moreover, when the optimal bribe level is more than $\bar{b}$, then no firm will enter because of negative profit, and the probability of entry is 0.

3.3.3 First-Stage Equilibrium Analysis

Now we are analyzing first stage equilibrium in which firm chooses bribe level $'b'$ such that its expected profit gets maximized. By using the above result, consider an optimal bribe level be b^*

When b lies in between 0 and $\underline{b}$ then potential entrant will enter because of positive profit then incumbent will get the duopoly profit i.e. $\pi_1(b)$.

When b lies in between $\underline{b}$ and $\bar{b}$, then $\alpha^* \in (0, \bar{\alpha})$. In this case, entrant will not enter if $\alpha \leq \alpha^*$ and probability for this case is $F(\alpha^*)$. This is the case of monopoly where incumbent firm sets price according to monopolist situation which gives the profit $\left(\frac{a-c}{2}\right)^2 - b$. Also, note that if $\alpha > \alpha^*$ then potential entrant will enter and probability is $1 - F(\alpha^*)$ and the payoff of incumbent will be duopolist's payoff i.e. $\pi_1(b)$.

When incumbent chooses b which is greater than $\bar{b}$ then no firm enters and the entry probability is zero. Incumbent behaves like a monopolist in this case and the payoff will be $\left(\frac{a-c}{2}\right)^2 - b$.

In the following equations, we summarized the result:

$$\boldsymbol{E_1(b)} = \begin{cases} \boldsymbol{\pi_1(b)} & \boldsymbol{if\ \ b \in [0, \underline{b}]} \\ \boldsymbol{F(\alpha^*)\left(\left(\frac{a-c}{2}\right)^2 - b\right) + [1 - F(\alpha^*)]\pi_1(b)} & \boldsymbol{if\ b \in [\underline{b}, \bar{b}]} \\ \boldsymbol{\left(\frac{a-c}{2}\right)^2} & \boldsymbol{if\ \ b \in [\bar{b}, \infty)} \end{cases}$$

3.3.4 Comparison between the Optimal Bribe under Cournot and Bertrand Model

(Dastidar & Yano, 2021) provides optimal bribe level under the Cournot competition is as follows:

$$\underline{b}^C = \frac{2(a-c)(2-\gamma)+\gamma^2\mu(0)-2(4-\gamma^2)k}{4}$$

$$\bar{b}^C = \frac{2(a-c)(2-\gamma)+4\bar{\alpha}-2(4-\gamma^2)k}{4}$$

When $\gamma = 0$; i.e. Goods are neutral.

$$\underline{b}^C = \frac{2(a-c)(2)+0-2(4)k}{4} = a-c-2k$$

$$\bar{b}^C = \frac{2(a-c)(2)+4\bar{\alpha}-2(4)k}{4} = a-c+\bar{\alpha}-2k$$

Now, Optimal bribe level under Bertrand competition is derived as follows:

$$\underline{b}^B = a-c-2k$$

$$\bar{b}^B = a-c+\bar{\alpha}-2k$$

We found that Optimal bribe level is same under Cournot and Bertrand Model when goods are neutral. This implies that $\underline{b}^C = \underline{b}^B$ and $\bar{b}^C = \bar{b}^B$ when $\gamma = 0$.

When goods are neutral, the optimal level of bribe is negatively related with marginal cost and positively related with intercept of the demand function. Moreover, the rate for both bribe levels i.e. lower level and upper level has same. In the given figures, upper bribe level is a parallel upward shift to lower bribe level and the differential intercept coefficient is efficiency level $'\alpha'$ of a potential entrant. This implies that when there is efficiency level exists for the potential entrant irrespective of the value of $'a'$ *and* $'c'$ then the incumbent has to increase the bribe level to deter the entry of potential entrant and the increase in bribe level that will depend on the efficiency level $'\alpha'$.

Figure 3.3.5 (a)

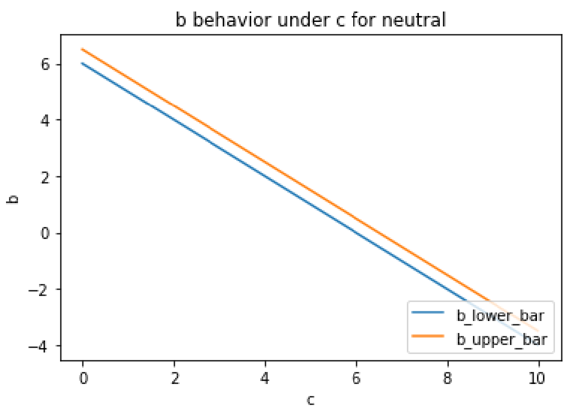
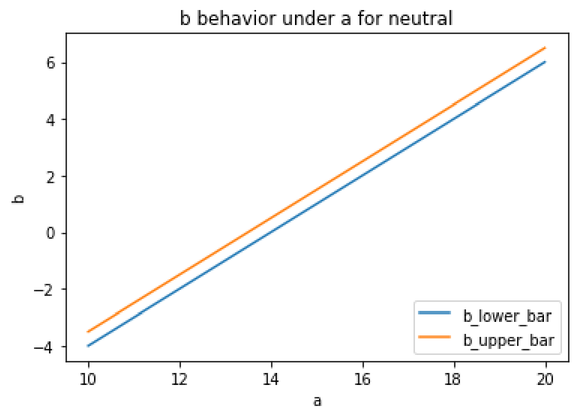


Figure 3.3.5 (b)



3.4 When $\gamma = 0.5$ (Substitute Goods)

3.4.1 Second-Stage Equilibrium Analysis

When $\gamma = 0.5$ then the value of:

$$A = -\frac{a}{2}$$

$$B = 2.5$$

$$D = 3.75$$

When 2 enters equilibrium price will be:

$$P_1(b) = \begin{cases} \frac{20a + 40c + 8b - 8\mu(\alpha^*)}{60} & if \ \ 2 \ enters \\ \frac{a + c}{2} & otherwise \end{cases}$$

$$P_2(\alpha, b) = \frac{160c + 80a + 128b - 120\alpha - 8\mu(\alpha^*)}{240}$$

Profit for the 2nd firm will be:

$$\pi_2(\alpha, b) = \left[\frac{80a - 80c + 120\alpha - 112b - 8\mu(\alpha^*)}{240}\right] q_2 - k^2$$

When $\gamma = 0.5$ then the value of q_2 will be:

$$q_2 = \frac{80a - 80c + 120\alpha - 112b - 8\mu(\alpha^*)}{180}$$

Therefore, $\pi_2(\alpha, b)$ will be:

$$\pi_2(\alpha, b) = \frac{\left(80a - 80c + 120\alpha - 112b - 8\mu(\alpha^*)\right)^2}{240 \times 180} - k^2$$

Suppose Bribe is Zero i.e. $b = 0$ and $\alpha = 0$ then firm 2 decides to enter iff

$$\pi_2(0,0) = \frac{\left(80a - 80c - 8\mu(0)\right)^2}{240 \times 180} - k^2 > 0$$

$$= 80a - 80c - 8\mu(0) > 208k$$

$$= 10(a - c) - \mu(0) > 26k$$

It holds from the assumptions: $a - c > \frac{\mu(0)}{10}$ and $a - c \geq 3k$.

Suppose $b > 0$ and $\alpha = 0$

$$\pi_2(0,b) = \frac{\left(80a - 80c - 112b - 8\mu(0)\right)^2}{240 \times 180} - k^2$$

$\pi_2(0,b)$ is decreasing in b.

Therefore, there exists $\underline{b}$ such that $\pi_2\left(0,\underline{b}\right) = 0$

$$\underline{\boldsymbol{b}} = \frac{\mathbf{10}(\boldsymbol{a} - \boldsymbol{c}) - \boldsymbol{\mu}(\mathbf{0}) - \mathbf{26}\boldsymbol{k}}{\mathbf{14}}$$

$\pi_2(0,b) > 0 \quad \forall\ b \in [0,\underline{b})$ then 2 will enter. Moreover, $\pi_2(0,b) < 0$ if $b > \underline{b}$.

$\pi_2(\alpha,b)$ is strictly decreasing in b. Also, $\pi_2(\alpha,b)$ is strictly increasing in α.

i.e. $\pi_2(\bar{\alpha},b) > \pi_2(\alpha,b)$, for $b > \underline{b}$.

Also, $\pi_2(\overline{\alpha},b)$ is strictly decreasing in b, there exists $\bar{b}$ such that $\pi_2\left(\bar{\alpha},\bar{b}\right) = 0$.

Now also note that using L'hospital's rule, we get—

$$\lim_{\alpha^* \to \bar{\alpha}} \mu(\alpha^*) = \bar{\alpha}$$

This means for $\alpha^* = \bar{\alpha}$, $b = \bar{b}$

$$\bar{\boldsymbol{b}} = \frac{\mathbf{10}(\boldsymbol{a} - \boldsymbol{c}) - \mathbf{26}\boldsymbol{k} + \mathbf{14}\,\bar{\boldsymbol{\alpha}}}{\mathbf{14}}$$

Profit of firm 2 will increase in terms of efficiency level (α) but it is decreasing in terms of b.

So, $\pi_2(\alpha,b) < 0$ when $b > \bar{b}$ so no one will enter.

If the optimal bribe $'b'$ is in between 0 and $\underline{b}$, and efficiency level is zero then all firms will enter.

However, if the optimal bribe $'b'$ is in between $\underline{b}$ and $\bar{b}$, then those firms will enter whose efficiency level is in between α^* and $\bar{\alpha}$, it is because profit level be strictly positive in this case, and the probability of entry is $1 - F(\alpha^*)$. Moreover, when the optimal bribe level is more than $\bar{b}$, then no firm will enter because of negative profit, and the probability of entry is 0.

3.4.3 First-Stage Equilibrium Analysis

Now we are analyzing first stage equilibrium in which firm chooses bribe level $'b'$ such that its expected profit gets maximized. By using the above result, consider an optimal bribe level be b^*

When b lies in between 0 and $\underline{b}$ then potential entrant will enter because of positive profit then incumbent will get the duopoly profit i.e. $\pi_1(b)$.

When b lies in between $\underline{b}$ and $\bar{b}$, then $\alpha^* \in (0, \bar{\alpha})$. In this case, entrant will not enter if $\alpha \leq \alpha^*$ and probability for this case is $F(\alpha^*)$. This is the case of monopoly where incumbent firm sets price according to monopolist situation which gives the profit $\left(\frac{a-c}{2}\right)^2 - b$. Also, note that if $\alpha > \alpha^*$ then potential entrant will enter and probability is $1 - F(\alpha^*)$ and the payoff of incumbent will be duopolist's payoff i.e. $\pi_1(b)$.

When incumbent chooses b which is greater than $\bar{b}$ then no firm enters and the entry probability is zero. Incumbent behaves like a monopolist in this case and the payoff will be $\left(\frac{a-c}{2}\right)^2 - b$.

In the following equations, we summarized the result:

$$\boldsymbol{E_1(b) = \begin{cases} \pi_1(b) & if \;\; b \in \left[0, \bar{b}\right] \\ F(\alpha^*)\left(\left(\frac{a-c}{2}\right)^2 - b\right) + [1 - F(\alpha^*)]\pi_1(b) & if \; b \in [\underline{b}, \bar{b}] \\ \left(\frac{a-c}{2}\right)^2 & if \;\; b \in [\bar{b}, \infty) \end{cases}}$$

3.4.4 Comparison between the Optimal Bribe under Cournot and Bertrand Model

(Dastidar & Yano, 2021) provides optimal bribe level under Cournot competition is as follows:

$$\underline{b}^C = \frac{2(a-c)(2-\gamma) + \gamma^2\mu(0) - 2(4-\gamma^2)k}{4}$$

$$\bar{b}^C = \frac{2(a-c)(2-\gamma) + 4\bar{\alpha} - 2(4-\gamma^2)k}{4}$$

When $\gamma = 0.5$; i.e. Goods are substitutes.

$$\underline{b}^C = \frac{2(a-c)(1.5) + (0.5)^2\mu(0) - 2(3.75)k}{4} = 0.75(a-c) + 0.0625\mu(0) - 1.8k$$

$$\bar{b}^C = \frac{2(a-c)(1.5) + 4\bar{\alpha} - 2(3.75)k}{4} = 0.75(a-c) + \bar{\alpha} - 1.75k$$

Now, Optimal bribe level under Bertrand competition is derived as follows:

$$\underline{b}^B = \frac{10(a-c) - \mu(0) - 26k}{14} = 0.71(a-c) - 0.07\mu(0) - 1.8k$$

$$\bar{b}^B = \frac{10(a-c) + 14\bar{\alpha} - 26k}{14} = 0.71(a-c) + \bar{\alpha} - 1.85k$$

By comparing the bribe level:

$$0.75(a-c) + 0.0625\mu(0) - 1.8k = \underline{b}^C > \underline{b}^B = 0.71(a-c) - 0.07\mu(0) - 1.8k$$

$$0.75(a-c) + \bar{\alpha} - 1.75k = \bar{b}^C > \bar{b}^B = 0.71(a-c) + \bar{\alpha} - 1.85k$$

We found that Optimal bribe level is greater under the Cournot than Bertrand Model when goods are substitutes. This implies that $\underline{b}^C > \underline{b}^B$ and $\bar{b}^C > \bar{b}^B$ when $\gamma = 0.5$.

When goods are substitute, then the optimal bribe level is negatively related with marginal cost and positively related with intercept of the demand function. Moreover, the rate for both bribe levels i.e. lower level and upper level has same in this case too. In the given figures, upper bribe level is a parallel upward shift to lower bribe level and the differential intercept coefficient is efficiency level $'\alpha'$ of a potential entrant. However, the shift is more than neutral goods case.

This implies that when there is efficiency level exists for the potential entrant irrespective of the value of $'a'$ *and* $'c'$ then the incumbent has to increase the bribe level to deter the entry of potential entrant and the increase in bribe level will depend on the efficiency level $'\alpha'$.

Figure 3.4.5 (a)

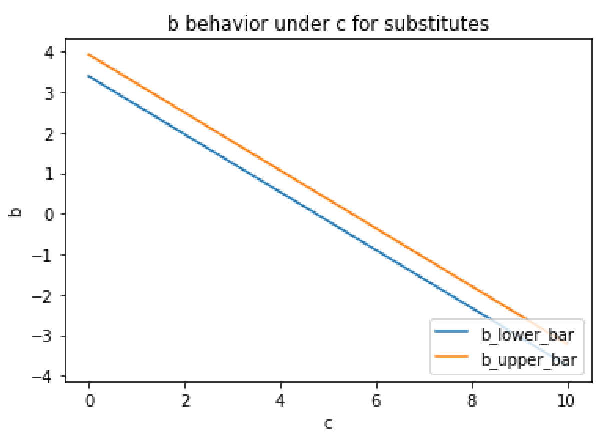
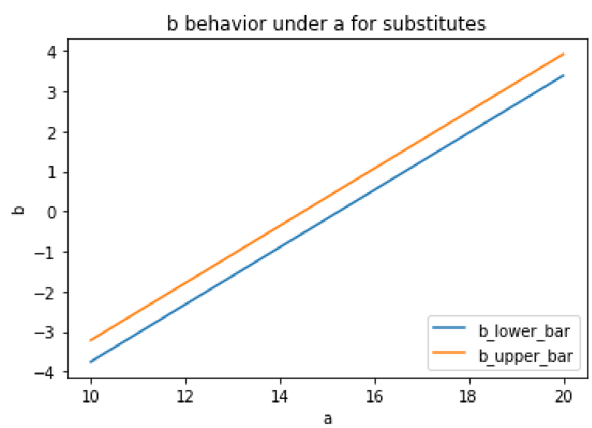


Figure 3.4.5 (b)



3.5 When $\gamma = -0.5$ (Complementary Goods)

3.5.1 Second-Stage Equilibrium Analysis

When $\gamma = -0.5$ then the value of:

$$A = -\frac{3a}{2}$$

$$B = \frac{3}{2}$$

$$D = 3.75$$

If 2 enters, then the equilibrium prices will be:

$$P_1(b) = \begin{cases} \frac{24c + 36a - 8b + 8\mu(\alpha^*)}{60} & if\ \ 2\ enters \\ \frac{a+c}{2} & otherwise \end{cases}$$

$$P_2(\alpha, b) = \frac{24c + 36a + 32b - 30\alpha - 2\mu(\alpha^*)}{60}$$

Profit for the 2nd firm will be:

$$\pi_2(\alpha, b) = \left[\frac{36a - 36c + 30\alpha - 28b - 2\mu(\alpha^*)}{60}\right] q_2 - k^2$$

When $\gamma = -0.5$ then the value of q_2 will be:

$$q_2 = \frac{36a - 36c + 30\alpha - 28b - 2\mu(\alpha^*)}{45}$$

Therefore, $\pi_2(\alpha, b)$ will be:

$$\pi_2(\alpha, b) = \frac{(36a - 36c + 30\alpha - 28b - 2\mu(\alpha^*))^2}{60 \times 45} - k^2$$

Suppose Bribe is Zero i.e. $b = 0$ and $\alpha = 0$ then firm 2 decides to enter iff

$$\pi_2(0,0) = \frac{(36a - 36c - 2\mu(0))^2}{60 \times 45} - k^2 > 0$$

$$= 36a - 36c - 2\mu(0) > 52k$$

$$= 18(a - c) - \mu(0) > 26k$$

It holds from the assumptions: $a - c > \frac{\mu(0)}{10}$ and $a - c \geq 3k$.

Suppose $b > 0$ and $\alpha = 0$

$$\pi_2(0, b) = \frac{(36a - 36c - 28b - 2\mu(0))^2}{60 \times 45} - k^2$$

$\pi_2(0, b)$ is decreasing in b.

Therefore, there exists $\underline{b}$ such that $\pi_2\left(0, \underline{b}\right) = 0$

$$\frac{(36a - 36c - 28b - 2\mu(0))^2}{60 \times 45} - k^2 = 0$$

$$36a - 36c - 28b - 2\mu(0) = 52k$$

$$\underline{b} = \frac{18(a - c) - \mu(0) - 26k}{14}$$

$\pi_2(0, b) > 0 \quad \forall \ b \in [0, \underline{b})$ then 2 will enter. Moreover, $\pi_2(0, b) < 0$ if $b > \underline{b}$.

$\pi_2(\alpha, b)$ is strictly decreasing in b. Also, $\pi_2(\alpha, b)$ is strictly increasing in α.

i.e. $\pi_2(\bar{\alpha}, b) > \pi_2(\alpha, b)$, for $b > \underline{b}$.

Also, $\pi_2(\overline{\alpha}, b)$ is strictly decreasing in b, there exists $\bar{b}$ such that $\pi_2(\bar{\alpha}, \bar{b}) = 0$.

Now also note that using L'hospital's rule, we get—

$$\lim_{\alpha^* \to \bar{\alpha}} \mu(\alpha^*) = \bar{\alpha}$$

This means for $\alpha^* = \bar{\alpha}, b = \bar{b}$

$$\frac{\left(36a - 36c + 30\bar{\alpha} - 28\bar{b} - 2\bar{\alpha}\right)^2}{60 \times 45} - k^2 = 0$$

$$36a - 36c + 30\alpha - 28\bar{b} - 2\bar{\alpha} = 52k$$

$$\bar{b} = \frac{18(a - c) + 14\bar{\alpha} - 26k}{14}$$

Profit of firm 2 will increase in terms of efficiency level (α) but it is decreasing in terms of b.

So, $\pi_2(\alpha, b) < 0$ when $b > \bar{b}$ so no one will enter.

If the optimal bribe $'b'$ is in between 0 and $\underline{b}$, and efficiency level is zero then all firms will enter. However, if the optimal bribe $'b'$ is in between $\underline{b}$ and $\bar{b}$, then those firms will enter whose efficiency level is in between α^* and $\bar{\alpha}$, it is because profit level be strictly positive in this case, and the probability of entry is $1 - F(\alpha^*)$. Moreover, when the optimal bribe level is more than $\bar{b}$, then no firm will enter because of negative profit, and the probability of entry is 0.

3.5.3 First-Stage Equilibrium Analysis

Now we are analyzing first stage equilibrium in which firm chooses bribe level $'b'$ such that its expected profit gets maximized. By using the above result, consider an optimal bribe level be b^*

When b lies in between 0 and $\underline{b}$ then potential entrant will enter because of positive profit then incumbent will get the duopoly profit i.e. $\pi_1(b)$.

When b lies in between $\underline{b}$ and $\bar{b}$, then $\alpha^* \in (0, \bar{\alpha})$. In this case, entrant will not enter if $\alpha \leq \alpha^*$ and probability for this case is $F(\alpha^*)$. This is the case of monopoly where incumbent firm sets price according to monopolist situation which gives the profit $\left(\frac{a-c}{2}\right)^2 - b$. Also, note that if $\alpha > \alpha^*$ then potential entrant will enter and probability is $1 - F(\alpha^*)$ and the payoff of incumbent will be duopolist's payoff i.e. $\pi_1(b)$.

When incumbent chooses b which is greater than $\bar{b}$ then no firm enters and the entry probability is zero. Incumbent behaves like a monopolist in this case and the payoff will be $\left(\frac{a-c}{2}\right)^2 - b$.

In the following equations, we summarized the result:

$$\boldsymbol{E_1(b)} = \begin{cases} \boldsymbol{\pi_1(b)} & \boldsymbol{if\ \ b \in [0, \bar{b}]} \\ \boldsymbol{F(\alpha^*)\left(\left(\frac{a-c}{2}\right)^2 - b\right) + [1 - F(\alpha^*)]\pi_1(b)} & \boldsymbol{if\ b \in [\underline{b}, \bar{b}]} \\ \boldsymbol{\left(\frac{a-c}{2}\right)^2} & \boldsymbol{if\ \ b \in [\bar{b}, \infty)} \end{cases}$$

3.5.4 Comparison between the Optimal Bribe under Cournot and Bertrand Model

(Dastidar & Yano, 2021) provides optimal bribe level under Cournot competition is as follows:

$$\underline{b}^C = \frac{2(a-c)(2-\gamma) + \gamma^2\mu(0) - 2(4-\gamma^2)k}{4}$$

$$\bar{b}^C = \frac{2(a-c)(2-\gamma) + 4\bar{\alpha} - 2(4-\gamma^2)k}{4}$$

When $\gamma = -0.5$; i.e. Goods are complements.

$$\underline{b}^C = \frac{2(a-c)(2.5) + (0.5)^2\mu(0) - 2(3.75)k}{4} = 1.25(a-c) + 0.0625\mu(0) - 1.75k$$

$$\bar{b}^C = \frac{2(a-c)(2.5) + 4\bar{\alpha} - 2(3.75)k}{4} = 1.25(a-c) + \bar{\alpha} - 1.75k$$

Now, Optimal bribe level under Bertrand competition is derived as follows:

$$\underline{b}^B = \frac{18(a-c) - \mu(0) - 26k}{14} = 1.2(a-c) - 0.071\mu(0) - 1.85k$$

$$\bar{b}^B = \frac{18(a-c) + 14\bar{\alpha} - 26k}{14} = 1.2(a-c) + \bar{\alpha} - 1.85k$$

By comparing the bribe level:

$$1.25(a-c) + 0.0625\mu(0) - 1.75k = \underline{b}^C > \underline{b}^B = 1.2(a-c) - 0.071\mu(0) - 1.85k$$

$$1.25(a-c) + \bar{\alpha} - 1.75k = \bar{b}^C > \bar{b}^B = 1.2(a-c) + \bar{\alpha} - 1.85k$$

We found that Optimal bribe level is greater under Cournot than Bertrand Model when goods are complements. This implies that $\underline{b}^C > \underline{b}^B$ and $\bar{b}^C > \bar{b}^B$ when $\gamma = -0.5$.

When goods are complements, then the optimal bribe level is negatively related with marginal cost and positively related with intercept of the demand function. Moreover, the rate for both bribe levels i.e. lower level and upper level has same in this case too. In the given figures, upper bribe level is a parallel upward shift to lower bribe level and the differential intercept coefficient is efficiency level $'\alpha'$ of a potential entrant. This implies that when there is efficiency level exists for the potential entrant irrespective of the value of $'a'\ and\ 'c'$ then the incumbent has to increase the bribe level to deter the entry of potential entrant and the increase in bribe level will depend on the efficiency level $'\alpha'$.

Figure 3.5.5 (a)

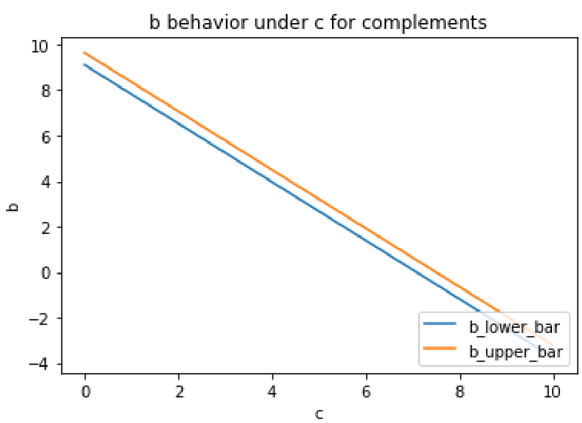
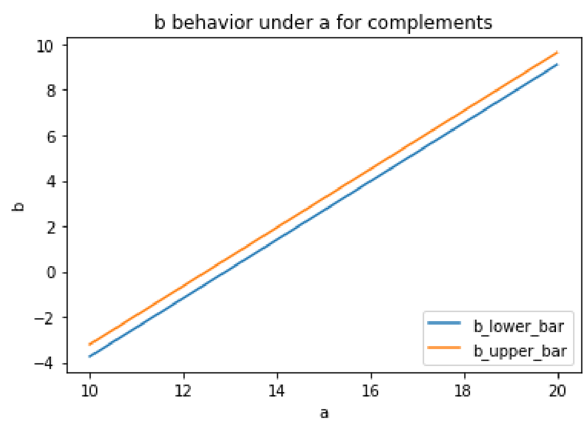


Figure 3.5.5 (b)



4 Conclusion

In this study, the analysis is based on three stage game. We firstly computed the Bayesian-Nash equilibrium of third stage. Then explained when 2nd firm will enter in the second stage, we showed that when the height of entry barrier is less than some threshold ($0 < b < \underline{b}$) then all firms will enter irrespective of efficiency level which implies that entry probability is one. However, when $\underline{b} < b < \bar{b}$ then some firms will enter and some firms will not. The firms who will enter has higher efficiency level ($\alpha > \alpha^*$) which implies that entry probability is $1 - F(\alpha^*)$. If the bribe is more than $\bar{b}$, then none of them will enter which implies that entry probability is zero in this case, it is referred as blockaded entry. After that we solved the expected payoff for first firm in the first stage equilibrium which depends on the bribe level.

We have considered all three possible cases: either good is neutral or complementary or substitute. Moreover, we have compared the optimal bribe level between the Cournot and Bertrand models and found that the optimal bribe level is the same in both models in the case of neutral goods.

However, we showed that the optimal bribe level under the Bertrand competition is lower than the Cournot competition when goods are complements or substitutes. This implies that the height of the entry barrier is more for the potential entrant in Cournot competition rather than Bertrand competition, irrespective of the nature of the goods. This means that firms can relatively easily enter into the market in Bertrand's case than Cournot's case. From this, we can understand that the Bertrand model provides more competition in the market than the Cournot model. The term 'more competition' explains the ease of entry into the market. . The ease of entry into the market is essential for market welfare because it defines what will be the level of competition that prevails in the market; otherwise, market welfare will be too low. The social welfare here can be depicted through the sum of consumer surplus, producer surplus, and government revenue. So the ease of entry or less magnitude of the level of entry barrier in the market for a firm is a vital element to decide the welfare of society. It is because higher will be competition, higher will be efficiency. In a nutshell, we have compared the two oligopolistic models, the Bertrand model, and the Cournot model. We have analyzed competition level will be more when there is Price Competition than Quantity Competition. We have shown how just changing the way of competition can reduce the actual level of optimal bribe in the equilibrium. This helps us to explain the magnitude of corruption in different competitions see (Cisneros, et. al., 2021). We can infer that the extent of corruption is more under the Cournot competition than the Bertrand competition. This will provide the policy implications for the regulators in the developing economy that they can ensure that if price competition is played in, then the welfare is relatively more than the Quantity competition.

However, one can take this study as prior research and make extensions for the different possibilities such as: What will happen if the cost of paying bribe for an incumbent firm is some function $\psi(b)$, where $\psi(.)$ is strictly increasing function?

From the future research perspective, one can check these results will hold or not by doing rigorous empirical studies. Also, what will happen to the market outcome when Government can impose a legal entry fee instead of a bribe paid by the incumbent? How will this legal fee be determined? What will happen if some regulator agency exists in a developing economy that creates a barrier for the incumbent firm the same as the incumbent is building for the potential entrant? For example, Competition Commission in India ensures an appropriate competition level in the market.

Moreover, one can compare what will happen to the market welfare, which is affected by optimal bribe in the case of the Bertrand and the Cournot model.

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APPENDIX

3.2 Third-Stage Equilibrium Analysis

In the third stage, the amount of bribe b has determined already and known to both firms. However, the efficiency level α is only known to the entrant only. In this scenario, firm 2 enters if and only if $\alpha \in [\alpha^*, \bar{\alpha}]$ in the second stage and expects a positive profit.

Consider, $\gamma \in (-1, 1)$

For any given b, Bayesian-Bertrand Nash equilibrium be:

$P_1(b)$ and $P_2(\alpha, b)$

In the second stage, 1st firm knows that 2nd firm enters if and only if $\alpha \geq \alpha^*$.

Hence, expected price for the 2nd firm be:

$$Exp.(P_2(\alpha, b)|\ \alpha \geq \alpha^*) = \int_{\alpha^*}^{\bar{\alpha}} \frac{P_2(\alpha, b)f(\alpha)}{1 - F(\alpha^*)} d\alpha$$

In the Bayesian-Nash equilibrium, where $\alpha \in [\alpha^*, \bar{\alpha}]$ we have the following:

$$P_2(\alpha, b) = \arg\max_{P_2 \geq 0} P_2 q_2(P_1, P_2) - [(c - \alpha + b) + k^2]$$

$$P_1(\alpha, b) = \arg\max_{P_1 \geq 0} P_1 q_1(P_1, P_2) - [c q_1(P_1, P_2) + b]$$

Now,

$$\arg\max_{P_2 \geq 0}(P_2 - c + \alpha - b) q_2(P_1, P_2) - k^2$$

From the direct demand functions:

$$\arg\max_{P_2\geq 0}(P_2 - c + \alpha - b)\left(\frac{a(\gamma-1)-\gamma P_1 + P_2}{\gamma^2-1}\right)$$

By differentiating with respect to P_2.

$$\frac{\partial}{\partial P_2}\left[(P_2 - c + \alpha - b)\left(\frac{a(\gamma-1)-\gamma P_1 + P_2}{\gamma^2-1}\right)\right] = 0$$

$$\Rightarrow \frac{a(\gamma-1)-\gamma P_1 + 2P_2 - c + \alpha - b}{(\gamma^2-1)} = 0$$

$$P_2(\alpha, b) = \frac{c - \alpha + b + P_1\gamma - a(\gamma-1)}{2} \dots\dots\dots (iii)$$

Now,

$$\arg\max_{P_1\geq 0}(P_1 - c)q_1(P_1, P_2) - b$$

$$\Rightarrow (P_1 - c)\left[\frac{a(\gamma-1)-\gamma P_2 + P_1}{(\gamma^2-1)}\right] - b$$

$$\Rightarrow (P_1 - c)\left[\frac{a(\gamma-1)-\gamma \int_{\alpha^*}^{\bar{\alpha}} \frac{P_2(\alpha,b)f(\alpha)}{1-F(\alpha^*)}d\alpha + P_1}{(\gamma^2-1)}\right] - b$$

$$\Rightarrow (P_1 - c)\left[\frac{a(\gamma-1)-\gamma P_2^{Exp.}(\alpha^*) + P_1}{(\gamma^2-1)}\right] - b$$

Differentiating with respect to P_1.

$$\frac{\partial}{\partial P_1}\left[(P_1 - c)\left[\frac{a(\gamma-1)-\gamma P_2^{Exp.}(\alpha^*) + P_1}{(\gamma^2-1)}\right] - b\right] = 0$$

$$\frac{a(\gamma-1)-\gamma P_2^{Exp.}(\alpha^*) + 2P_1 - c}{(\gamma^2-1)} = 0$$

$$P_1(b) = \frac{c - a(\gamma-1) + \gamma P_2^{Exp.}(\alpha^*)}{2} \dots\dots\dots (iv)$$

From (iii) & (iv)

$$P_2(\alpha, b) = \frac{c - \alpha + b - a(\gamma - 1)}{2} + \frac{\gamma}{2}\left(\frac{c - a(\gamma - 1) + \gamma P_2^{Exp.}(\alpha^*)}{2}\right)$$

As per our definition:

$$\int_{\alpha^*}^{\bar{\alpha}} P_2(\alpha, b)\frac{f(\alpha)}{1 - F(\alpha^*)}d\alpha = P_2^{Exp.}(\alpha^*)$$

∴ Above implies that-----

$$\int_{\alpha^*}^{\bar{\alpha}}\left[\frac{c - \alpha + b - a(\gamma - 1)}{2} + \frac{\gamma}{2}\left(\frac{c - a(\gamma - 1) + \gamma P_2^{Exp.}(\alpha^*)}{2}\right)\right]\frac{f(\alpha)}{1 - F(\alpha^*)}d\alpha$$

$$\Rightarrow\ \frac{1}{4}\int_{\alpha^*}^{\bar{\alpha}}\left[2c - 2\alpha + 2b - 2a(\gamma - 1) + \gamma\left(c - a(\gamma - 1) + \gamma P_2^{Exp.}(\alpha^*)\right)\right]\frac{f(\alpha)}{1 - F(\alpha^*)}d\alpha$$

$$\Rightarrow\ \frac{1}{4}\int_{\alpha^*}^{\bar{\alpha}}\left[2c + 2b + \gamma c + \gamma^2 P_2^{Exp.}(\alpha^*) - 2a(\gamma - 1) - \gamma a(\gamma - 1) - 2\alpha\right]\frac{f(\alpha)}{1 - F(\alpha^*)}d\alpha$$

$$\Rightarrow\ \frac{1}{4}\left[2c + 2b + \gamma c + \gamma^2 P_2^{Exp.}(\alpha^*) - a(\gamma - 1)(2 + \gamma)\right] - \frac{1}{2}\int_{\alpha^*}^{\bar{\alpha}}\frac{\alpha . f(\alpha)}{1 - F(\alpha^*)}d\alpha$$

$$\Rightarrow\ \frac{1}{4}\left[2c + 2b + \gamma c + \gamma^2 P_2^{Exp.}(\alpha^*) - a(\gamma - 1)(2 + \gamma)\right] - \frac{\mu(\alpha^*)}{2} = P_2^{Exp.}(\alpha^*)$$

$$\Rightarrow\ 2c + 2b + \gamma c + \gamma^2 P_2^{Exp.}(\alpha^*) - a(\gamma - 1)(2 + \gamma) - 2\mu(\alpha^*) = 4P_2^{Exp.}(\alpha^*)$$

$$P_2^{Exp.}(\alpha^*) = \frac{2c + 2b + \gamma c - a(\gamma - 1)(2 + \gamma) - 2\mu(\alpha^*)}{4 - \gamma^2} \ldots\ldots\ldots (v)$$

By using (iv) and (v), we get...

$$P_1(b) = \frac{c - a(\gamma - 1)}{2} + \frac{\gamma}{2}P_2^{Exp.}(\alpha^*)$$

$$P_1(b) = \frac{c - a(\gamma - 1)}{2} + \frac{\gamma}{2}\left(\frac{2c + 2b + \gamma c - a(\gamma - 1)(2 + \gamma) - 2\mu(\alpha^*)}{4 - \gamma^2}\right)$$

Let,

$$A = a(\gamma - 1)$$

$$B = 2 + \gamma$$

$$D = 4 - \gamma^2$$

$$P_1(b) = \frac{c - A}{2} + \frac{\gamma}{2}\left[\frac{cB + 2b - AB - 2\mu(\alpha^*)}{D}\right]$$

$$P_1(b) = \frac{c - A}{2} + \frac{\gamma}{2}\left[\frac{(c - A)B + 2b - 2\mu(\alpha^*)}{D}\right] \ldots\ldots\ldots (vi)$$

By using (vi) and (iii), we get---

$$P_2(\alpha, b) = \frac{c - \alpha + b - A}{2} + \frac{\gamma}{2}\left[\frac{c - A}{2} + \frac{\gamma}{2}\left[\frac{(c - A)B + 2b - 2\mu(\alpha^*)}{D}\right]\right]$$

Hence,

$$\boldsymbol{P_1(b) = \begin{cases} \dfrac{c - A}{2} + \dfrac{\gamma}{2}\left[\dfrac{(c - A)B + 2b - 2\mu(\alpha^*)}{D}\right] & \textit{if 2 enters} \\ \dfrac{a + c}{2} & \textit{if 2 doesn't enter} \end{cases}}$$

$$\boldsymbol{P_2(\alpha, b) = \begin{cases} \dfrac{c - \alpha + b - A}{2} + \dfrac{\gamma}{2}\left[\dfrac{c - A}{2} + \dfrac{\gamma}{2}\left[\dfrac{(c - A)B + 2b - 2\mu(\alpha^*)}{D}\right]\right] & \textit{if 2 enters} \\ 0 & \textit{if 2 doesn't enter} \end{cases}}$$

3.3 When $\gamma = 0$ (Neutral Goods)

3.3.1 Second-Stage Equilibrium Analysis

Equilibrium prices will be:

$$P_1(b) = \frac{a+c}{2}$$

$$P_2(\alpha, b) = \begin{cases} \dfrac{a+c+b-\alpha}{2} & if\ 2\ enters \\ 0 \qquad if\ \alpha \in [o, \alpha^*], & if\ 2\ doesn't\ enter\ for\ such\ \alpha \end{cases}$$

Then the equilibrium profits will be:

$$\pi_1(b) = P_1 q_1 - TC_1$$

$$= \left(\frac{c+a}{2}\right)(a - P_1) - cq_1 - b$$

$$= \left(\frac{c+a}{2}\right)\left(a - \frac{c+a}{2}\right) - c(a - P_1) - b$$

$$= \left(\frac{c+a}{2}\right)\left(a - \frac{c+a}{2}\right) - c\left(a - \left(\frac{c+a}{2}\right)\right) - b$$

$$= \left(a - \frac{c+a}{2}\right)\left(\frac{c+a}{2} - c\right) - b$$

$$= \left(\frac{2a-c-a}{2}\right)\left(\frac{c+a-2c}{2}\right) - b$$

$$\pi_1(b) = \left(\frac{a-c}{2}\right)^2 - b$$

$$\pi_2(\alpha, b) = \left(\frac{c-\alpha+b+a}{2}\right)(a - P_2) - (c - \alpha + b)q_2 - k^2$$

$$= (a - P_2)\left(\frac{c - \alpha + b + a - 2c + 2\alpha - 2b}{2}\right) - k^2$$

$$= \left(\frac{2a - c + \alpha - b - a}{2}\right)\left(\frac{-c + \alpha - b + a}{2}\right) - k^2$$

$$\pi_2(\alpha, b) = \left(\frac{-c + \alpha - b + a}{2}\right)^2 - k^2$$

Suppose Bribe is Zero i.e. $b = 0$ and $\alpha = 0$ then firm 2 decides to enter iff

$$\pi_2(0,0) = \left(\frac{a - c}{2}\right)^2 - k^2 > 0$$

$$= \frac{a - c}{2} > k$$

$$= a - c > 2k$$

Now, $b > 0$ and $\alpha = 0$

$$\pi_2(0, b) = \left(\frac{-c - b + a}{2}\right)^2 - k^2 > 0$$

$$= \frac{a - c - b}{2} > k$$

$$= a - c - b > 2k$$

Since, $\pi_2(0, b)$ is decreasing in b.

Therefore, there exists $\underline{b}$ such that $\pi_2\left(0, \underline{b}\right) = 0$

$$\Rightarrow \left(\frac{-c - \underline{b} + a}{2}\right)^2 - k^2 = 0$$

$$\Rightarrow \frac{a - c - \underline{b}}{2} = k$$

$$\boldsymbol{\underline{b} = a - c - 2k}$$

$\pi_2(0, b) > 0 \quad \forall\ b \in [0, \underline{b})$ then 2 will enter.

$\pi_2(\alpha, b)$ is strictly decreasing in b.

Also, $\pi_2(\alpha, b)$ is strictly increasing in α.

i.e. $\pi_2(\bar{\alpha}, b) > \pi_2(\alpha, b)$, for $b > \underline{b}$

Also, $\pi_2(\overline{\alpha}, b)$ is strictly decreasing in b, there exists $\bar{b}$ such that $\pi_2(\bar{\alpha}, \bar{b}) = 0$.

Now also note that using L'hospital's rule, we get—

$$\lim_{\alpha^* \to \bar{\alpha}} \mu(\alpha^*) = \bar{\alpha}$$

This means for $\alpha^* = \bar{\alpha}$, $b = \bar{b}$

$$\pi_2(\bar{\alpha}, \bar{b}) = \left(\frac{-c + \bar{\alpha} - \bar{b} + a}{2}\right)^2 - k^2 = 0$$

$$-c + \bar{\alpha} - \bar{b} + a = 2k$$

$$\boldsymbol{\bar{b} = a - c + \bar{\alpha} - 2k}$$

Profit of firm 2 will increase in terms of efficiency level (α) but it is decreasing in terms of b.

So, $\pi_2(\alpha, b) < 0$ when $b > \bar{b}$ so no one will enter.

3.4 When $\gamma = 0.5$ (Substitute Goods)

3.4.1 Second-Stage Equilibrium Analysis

When $\gamma = 0.5$ then the value of:

$$A = a(0.5 - 1) = -\frac{a}{2}$$

$$B = 2.5 = \frac{5}{2}$$

$$D = 4 - (0.5)^2 = 4 - 0.25 = 3.75$$

Then, $P_1(b)$ will be when 2 enters:

$$P_1(b) = \frac{1}{2}\left(c + \frac{a}{2}\right) + \frac{1}{4 \times 3.75}\left[\left(c + \frac{a}{2}\right)(2.5) + 2b - 2\mu(\alpha^*)\right]$$

$$P_1(b) = \frac{1}{2}\left(c + \frac{a}{2}\right) + \frac{1}{15}\left[2.5c + \frac{2.5a}{2} + 2b - 2\mu(\alpha^*)\right]$$

$$P_1(b) = \frac{2c + a}{4} + \frac{1}{30}[5c + 2.5a + 4b - 4\mu(\alpha^*)]$$

$$P_1(b) = \frac{30c + 15a + 10c + 5a + 8b - 8\mu(\alpha^*)}{60}$$

$$P_1(b) = \begin{cases} \dfrac{20a + 40c + 8b - 8\mu(\alpha^*)}{60} & if \;\; 2 \; enters \\ \dfrac{a + c}{2} & otherwise \end{cases}$$

Now solving for $P_2(b)$

$$P_2(\alpha, b) = \frac{c - \alpha + b + \frac{a}{2}}{2} + \frac{1}{4}\left[\frac{c + \frac{a}{2}}{2} + \frac{1}{4}\left[\frac{\left(c + \frac{a}{2}\right)2.5 + 2b - 2\mu(\alpha^*)}{3.75}\right]\right]$$

$$P_2(\alpha, b) = \frac{2c - 2\alpha + 2b + a}{4} + \frac{1}{4}\left[\frac{c}{2} + \frac{a}{4} + \frac{1}{15}\left(\left(\frac{2c + a}{2}\right)2.5 + 2b - 2\mu(\alpha^*)\right)\right]$$

$$P_2(\alpha, b) = \frac{2c - 2\alpha + 2b + a}{4} + \frac{1}{4}\left[\frac{c}{2} + \frac{a}{4} + \frac{1}{15}\left(\frac{10c + 5a}{4} + 2b - 2\mu(\alpha^*)\right)\right]$$

$$P_2(\alpha,b) = \frac{2c - 2\alpha + 2b + a}{4} + \frac{1}{4}\left[\frac{2c + a}{4} + \frac{1}{15}\left(\frac{10c + 5a + 8b - 8\mu(\alpha^*)}{4}\right)\right]$$

$$P_2(\alpha,b) = \frac{2c - 2\alpha + 2b + a}{4} + \frac{1}{16}\left[\frac{2c + a}{4} + \left(\frac{10c + 5a + 8b - 8\mu(\alpha^*)}{15}\right)\right]$$

$$P_2(\alpha,b) = \frac{120c - 120\alpha + 120b + 60a + 40c + 20a + 8b - 8\mu(\alpha^*)}{240}$$

$$P_2(\alpha,b) = \frac{160c + 80a + 128b - 120\alpha - 8\mu(\alpha^*)}{240}$$

Profit for the 2nd firm will be:

$$\pi_2(\alpha,b) = P_2 q_2 - TC_2$$

$$\pi_2(\alpha,b) = P_2 q_2 - (c - \alpha + b)q_2 - k^2$$

$$= [P_2 - (c - \alpha + b)]q_2 - k^2$$

$$= \left[\frac{160c + 80a + 128b - 120\alpha - 8\mu(\alpha^*)}{240} - (c - \alpha + b)\right] q_2 - k^2$$

$$= \left[\frac{160c + 80a + 128b - 120\alpha - 8\mu(\alpha^*) - 240c + 240\alpha - 240b}{240}\right] q_2 - k^2$$

$$\pi_2(\alpha,b) = \left[\frac{80a - 80c + 120\alpha - 112b - 8\mu(\alpha^*)}{240}\right] q_2 - k^2$$

Now solving for q_2:

When $\gamma = 0.5$ then the value of q_2 will be:

$$q_2 = \frac{a(\gamma - 1) - \gamma P_1 + P_2}{\gamma^2 - 1}$$

$$q_2 = \frac{-0.5a - 0.5P_1 + P_2}{-0.75}$$

$$q_2 = \frac{2}{3}[a + P_1 - 2P_2]$$

$$q_2 = \frac{2}{3}\left[a + \frac{20a + 40c + 8b - 8\mu(\alpha^*)}{60} - 2\left(\frac{160c + 80a + 128b - 120\alpha - 8\mu(\alpha^*)}{240}\right)\right]$$

$$q_2 = \frac{2}{3}\left[\frac{120a + 40a + 80c + 16b - 16\mu(\alpha^*) - 160c - 80a - 128b + 120\alpha + 8\mu(\alpha^*)}{120}\right]$$

$$q_2 = \frac{80a - 80c + 120\alpha - 112b - 8\mu(\alpha^*)}{180}$$

Therefore, $\pi_2(\alpha, b)$ will be:

$$\pi_2(\alpha, b) = \frac{\left(80a - 80c + 120\alpha - 112b - 8\mu(\alpha^*)\right)^2}{240 \times 180} - k^2$$

Suppose Bribe is Zero i.e. $b = 0$ and $\alpha = 0$ then firm 2 decides to enter iff

$$\pi_2(0,0) = \frac{\left(80a - 80c - 8\mu(0)\right)^2}{240 \times 180} - k^2 > 0$$

$$= 80a - 80c - 8\mu(0) > 208k$$

$$= 10(a - c) - \mu(0) > 26k$$

It holds from the assumptions: $a - c > \frac{\mu(0)}{10}$ and $a - c \geq 3k$.

Suppose $b > 0$ and $\alpha = 0$

$$\pi_2(0, b) = \frac{\left(80a - 80c - 112b - 8\mu(0)\right)^2}{240 \times 180} - k^2$$

$\pi_2(0, b)$ is decreasing in b.

Therefore, there exists $\underline{b}$ such that $\pi_2\left(0, \underline{b}\right) = 0$

$$\frac{\left(80a - 80c - 112\,\underline{b} - 8\mu(0)\right)^2}{240 \times 180} - k^2 = 0$$

$$\frac{80a - 80c - 112\,\underline{b} - 8\mu(0)}{208} = k$$

$$\underline{\boldsymbol{b}} = \frac{\mathbf{10}(\boldsymbol{a} - \boldsymbol{c}) - \boldsymbol{\mu}(\mathbf{0}) - \mathbf{26}\boldsymbol{k}}{\mathbf{14}}$$

$\pi_2(0, b) > 0 \quad \forall \; b \in [0, \underline{b})$ then 2 will enter. Moreover, $\pi_2(0, b) < 0$ if $b > \underline{b}$.

$\pi_2(\alpha, b)$ is strictly decreasing in b. Also, $\pi_2(\alpha, b)$ is strictly increasing in α.

i.e. $\pi_2(\bar{\alpha}, b) > \pi_2(\alpha, b)$, for $b > \underline{b}$.

Also, $\pi_2(\overline{\alpha}, b)$ is strictly decreasing in b, there exists $\bar{b}$ such that $\pi_2\left(\bar{\alpha}, \bar{b}\right) = 0$.

Now also note that using L'hospital's rule, we get—

$$\lim_{\alpha^* \to \bar{\alpha}} \mu(\alpha^*) = \bar{\alpha}$$

This means for $\alpha^* = \bar{\alpha}$, $b = \bar{b}$

$$\frac{\left(80a - 80c + 120\alpha - 112b - 8\mu(\alpha^*)\right)^2}{240 \times 180} - k^2 = 0$$

$$\frac{80a - 80c + 120\alpha - 112b - 8\,\bar{\alpha}}{208} = k$$

$$\bar{\boldsymbol{b}} = \frac{\mathbf{10}(\boldsymbol{a} - \boldsymbol{c}) - \mathbf{26}\boldsymbol{k} + \mathbf{14}\,\bar{\boldsymbol{\alpha}}}{\mathbf{14}}$$

Profit of firm 2 will increase in terms of efficiency level (α) but it is decreasing in terms of b.

So, $\pi_2(\alpha, b) < 0$ when $b > \bar{b}$ so no one will enter.

3.5 When $\gamma = -0.5$ (Complementary Goods)

3.5.1 Second-Stage Equilibrium Analysis

When $\gamma = -0.5$ then the value of:

$$A = a(-0.5 - 1) = -\frac{3a}{2}$$

$$B = \frac{3}{2}$$

$$D = 3.75$$

If 2 enters:

$$P_1(b) = \frac{1}{2}\left(c + \frac{3a}{2}\right) + \frac{(-1)}{4 \times 3.75}\left[\left(c + \frac{3a}{2}\right)\left(\frac{3}{2}\right) + 2b - 2\mu(\alpha^*)\right]$$

$$P_1(b) = \frac{2c + 3a}{4} + \frac{(-1)}{15}\left[\frac{3(2c + 3a)}{4} + 2b - 2\mu(\alpha^*)\right]$$

$$P_1(b) = \frac{2c + 3a}{4} + \frac{(-1)}{15}\left[\frac{6c + 9a + 8b - 8\mu(\alpha^*)}{4}\right]$$

$$P_1(b) = \frac{1}{4}\left(\frac{30c + 45a - 6c - 9a - 8b + 8\mu(\alpha^*)}{15}\right)$$

$$P_1(b) = \begin{cases} \dfrac{24c + 36a - 8b + 8\mu(\alpha^*)}{60} & if\ \ 2\ enters \\ \dfrac{a + c}{2} & otherwise \end{cases}$$

Now solving for $P_2(b)$

$$P_2(\alpha, b) = \frac{c - \alpha + b + \frac{3a}{2}}{2} + \frac{(-1)}{4}\left[\frac{c + \frac{3a}{2}}{2} + \frac{(-1)}{4 \times 3.75}\left[\left(c + \frac{3a}{2}\right)\left(\frac{3}{2}\right) + 2b - 2\mu(\alpha^*)\right]\right]$$

$$P_2(\alpha,b) = \frac{2c-2\alpha+2b+3a}{4} - \frac{1}{4}\left[\frac{2c+3a}{4} - \frac{1}{15}\left(\left(\frac{2c+3a}{4}\right)3 + 2b - 2\mu(\alpha^*)\right)\right]$$

$$P_2(\alpha,b) = \frac{2c-2\alpha+2b+3a}{4} - \frac{1}{4}\left[\frac{2c+3a}{4} - \frac{1}{15}\left(\frac{6c+9a+8b-8\mu(\alpha^*)}{4}\right)\right]$$

$$P_2(\alpha,b) = \frac{2c-2\alpha+2b+3a}{4} - \frac{1}{4}\left(\frac{1}{4}\right)\left(\frac{1}{15}\right)\left(30c+45a-6c-9a-8b+8\mu(\alpha^*)\right)$$

$$P_2(\alpha,b) = \frac{1}{4}\left(\frac{120c-120\alpha+120b+180a-30c-45a+6c+9a+8b-8\mu(\alpha^*)}{60}\right)$$

$$P_2(\alpha,b) = \frac{1}{4}\left(\frac{96c+144a+128b-120\alpha-8\mu(\alpha^*)}{60}\right)$$

$$P_2(\alpha,b) = \frac{24c+36a+32b-30\alpha-2\mu(\alpha^*)}{60}$$

Profit for the 2nd firm will be:

$$\pi_2(\alpha,b) = P_2q_2 - TC_2$$

$$\pi_2(\alpha,b) = P_2q_2 - (c-\alpha+b)q_2 - k^2$$

$$= [P_2 - (c-\alpha+b)]q_2 - k^2$$

$$= \left[\frac{24c+36a+32b-30\alpha-2\mu(\alpha^*)}{60} - (c-\alpha+b)\right]q_2 - k^2$$

$$= \left[\frac{24c+36a+32b-30\alpha-2\mu(\alpha^*)-60c+60\alpha-60b}{60}\right]q_2 - k^2$$

$$\pi_2(\alpha,b) = \left[\frac{36a-36c+30\alpha-28b-2\mu(\alpha^*)}{60}\right]q_2 - k^2$$

Now solving for q_2:

When $\gamma = -0.5$ then the value of q_2 will be:

$$q_2 = \frac{a(\gamma - 1) - \gamma P_1 + P_2}{\gamma^2 - 1}$$

$$q_2 = \frac{-1.5a + 0.5P_1 + P_2}{-0.75}$$

$$q_2 = \frac{2}{3}[3a - P_1 - 2P_2]$$

$$q_2 = \frac{2}{3}\left[3a - \left(\frac{24c + 36a - 8b + 8\mu(\alpha^*)}{60}\right) - 2\left(\frac{24c + 36a + 32b - 30\alpha - 2\mu(\alpha^*)}{60}\right)\right]$$

$$q_2 = \frac{2}{3}\left[\frac{180a - 24c - 36a + 8b - 8\mu(\alpha^*) - 48c - 72a - 64b + 60\alpha + 4\mu(\alpha^*)}{60}\right]$$

$$q_2 = \frac{2}{3}\left(\frac{72a - 72c - 56b - 4\mu(\alpha^*) + 60\alpha}{60}\right)$$

$$q_2 = \frac{36a - 36c + 30\alpha - 28b - 2\mu(\alpha^*)}{45}$$

Therefore, $\pi_2(\alpha, b)$ will be:

$$\pi_2(\alpha, b) = \frac{(36a - 36c + 30\alpha - 28b - 2\mu(\alpha^*))^2}{60 \times 45} - k^2$$

Suppose Bribe is Zero i.e. $b = 0$ and $\alpha = 0$ then firm 2 decides to enter iff

$$\pi_2(0,0) = \frac{(36a - 36c - 2\mu(0))^2}{60 \times 45} - k^2 > 0$$

$$= 36a - 36c - 2\mu(0) > 52k$$

$$= 18(a - c) - \mu(0) > 26k$$

It holds from the assumptions: $a - c > \frac{\mu(0)}{10}$ and $a - c \geq 3k$.

Suppose $b > 0$ and $\alpha = 0$

$$\pi_2(0, b) = \frac{(36a - 36c - 28b - 2\mu(0))^2}{60 \times 45} - k^2$$

$\pi_2(0, b)$ is decreasing in b.

Therefore, there exists $\underline{b}$ such that $\pi_2(0, \underline{b}) = 0$

$$\frac{(36a - 36c - 28b - 2\mu(0))^2}{60 \times 45} - k^2 = 0$$

$$36a - 36c - 28b - 2\mu(0) = 52k$$

$$\underline{b} = \frac{18(a - c) - \mu(0) - 26k}{14}$$

$\pi_2(0, b) > 0 \quad \forall \ b \in [0, \underline{b})$ then 2 will enter. Moreover, $\pi_2(0, b) < 0$ if $b > \underline{b}$.

$\pi_2(\alpha, b)$ is strictly decreasing in b. Also, $\pi_2(\alpha, b)$ is strictly increasing in α.

i.e. $\pi_2(\bar{\alpha}, b) > \pi_2(\alpha, b)$, for $b > \underline{b}$.

Also, $\pi_2(\overline{\alpha}, b)$ is strictly decreasing in b, there exists $\bar{b}$ such that $\pi_2(\bar{\alpha}, \bar{b}) = 0$.

Now also note that using L'hospital's rule, we get—

$$\lim_{\alpha^* \to \overline{\alpha}} \mu(\alpha^*) = \bar{\alpha}$$

This means for $\alpha^* = \bar{\alpha}, b = \overline{b}$

$$\frac{\left(36a - 36c + 30\bar{\alpha} - 28\overline{b} - 2\bar{\alpha}\right)^2}{60 \times 45} - k^2 = 0$$

$$36a - 36c + 30\alpha - 28\overline{b} - 2\bar{\alpha} = 52k$$

$$\bar{b} = \frac{18(a-c) + 14\bar{\alpha} - 26k}{14}$$

Profit of firm 2 will increase in terms of efficiency level (α) but it is decreasing in terms of b.

So, $\pi_2(\alpha, b) < 0$ when $b > \bar{b}$ so no one will enter.