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Affirmative Action in the Presence of a Creamy Layer

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Abstract

We examine affirmative action (AA) policies in a framework with statistical discrimination, where the target group, namely the "blacks", have a creamy layer (CLB) co-existing with a poorer section (PB). Further, the PB workers have to access an imperfect credit market if they want to invest in skill acquisition. We derive conditions for the existence of non-stereotyping, as well as patronizing equilibria (where employers hold the target group to lower standards via-a-vis the non-target group, the "whites"). We find that demographic shifts, whereby poorer blacks graduate to the "creamy layer", adversely affect all three groups under a patronising equilibrium, in the sense that a lower proportion is assigned to skilled jobs. Further, we demonstrate that a transition from identity-based to a class-based AA would be politically divisive, in that such a move would be opposed by at least two of the three groups, regardless of whether the equilibrium involves patronization, or not. In fact, CLBs would always be made worse off by such a transition. We also examine the implications of introducing a role model effect, as well as targeted education subsidies.

JEL No.: J71, J78, D78, D82.
Key Words: Affirmative action, patronizing equilibrium, creamy layer, class-based affirmative action, role model effects, demographic shifts, educational loans, subsidies.

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1 Introduction: The Central Issues

Affirmative action (AA) in favour of traditionally under-privileged groups has been pursued in many countries.[1] In this paper we focus on an aspect of affirmative action that has been relatively neglected in the literature, namely the fact that the intended beneficiaries typically contain people with very different income levels. While, on average, target groups are poorer compared to other groups,[2] they typically contain a substantial "creamy layer". Moreover, over time more and more members of the target groups have been entering the creamy layer.

Poverty levels among African-Americans in the US, for example, have fallen sharply over the last 50 years, from 41.8% in 1966, to 27.2% in 2012 (Pew Research Center, January 2014). In India, poverty rates among the scheduled castes, beneficiaries of AA, fell from 62.4% in 1993-94, to 31.5% in 2011-12. Similarly, poverty rates among the other beneficiaries of Indian AA, namely scheduled tribes and other backward castes (OBCs), declined by over 17 percentage points between 2003-04 and 2011-12, as opposed to an 11.6% decline for non-target groups (Business Standard, March 14, 2014, and also Times of India, May 5, 2015).[3]

The presence of the creamy layer makes affirmative action a politically divisive issue. This not only encourages the non-beneficiaries to question the very basis of affirmative action, it might prompt even relatively well off sections of society to demand affirmative action for themselves. In the Indian state of Gujarat, for example, the wealthy and upwardly mobile business community of "Patels" have started an agitation for OBC status, which would entitle them to benefit from affirmative action (Hindu, 26th August, 2015).[4]

In this paper we therefore examine affirmative action in a framework with a heterogeneous target group, leading naturally to several interesting political economy questions. For example, would the non-target group (henceforth called "whites" for convenience) align with the poorer members of the target group (henceforth "poor blacks", or PBs), or the richer members (henceforth "creamy layer blacks", or CLBs)? In the context of the US, Feagin and Porter (1995) discuss instances where whites opposed to affirmative action have formed coalitions with like-minded members of the black community. In a similar vein, one can ask if the interests of different members of the target group are going to be aligned, or polarized.[5] Moreover, how does

[1]The list of countries practicing some form of affirmative action includes, among others, the US, Canada, India, South Africa, New Zealand, Malaysia, China, Israel, and Sri Lanka. Affirmative action has been used since the 1960s in the US, considering race and sex in hiring criteria. Similarly the Canadian Employment Equity Act gives preferences to women, aboriginals, and minorities. In India, caste is a basis for reservation in government jobs and publicly funded educational institutions. In South Africa, the Employment Equity Act of 1998 mandated that companies with more than 50 employees must meet proportional quotas in their hiring of disadvantaged groups. In New Zealand affirmative action has given those of Maori descent better access to university and financial aid since 1993. Brazil has quotas for racial minorities and the poor in higher education.

[2]The current poverty rate among blacks is double that among whites in the US.

[3]Hnatkovska et al. (2012) and Deshpande and Ramachandran (2014) examine divergence along other dimensions as well, and reach conclusions with a similar flavour. In India, the cutoff income used for excluding beneficiaries has been increasing over time, from an yearly income of Rs 1 lakh in 1993, to Rs 6 lakhs in 2013, with the National Commission for Backward Classes proposing an even higher ceiling of Rs 10.5 lakh in 2015, indicating that the creamy layer beneficiaries are becoming richer.

[4]This prompted some politicians to argue that if an affluent community like the Patels were to benefit from AA, so should the non-target group, the "upper" castes (Manish Tewari, New Indian Express, 26th August 2015).

[5]Within the sociology and political science literature, Jackson (1987) finds considerable heterogeneity within blacks with regard to political preferences. For instance, he finds a black mayoral candidate received overwhelming support from the poorer segments of the black population, but not from affluent blacks. Like Nelson and Meranto

a demographic shift, whereby some PBs graduate to CLBs, affect these trade-offs? These issues are of interest since one may expect that affirmative action is more likely to be implemented whenever all the groups, in particular the whites and the CLBs, who comprise the elite, support such policy measures.

Another related issue is whether to base affirmative action policies on identity at all, rather than on income levels. While there is scope for debate on the sociological aspects of these two alternative policies, we are interested in a specifically economic aspect of this issue: the effect of income based policies on the intended beneficiaries - the creamy layer, as well as the poor. In the context of the recent Patel agitations in Gujarat, some commentators have argued that some sections of the agitators are really interested in pushing for income based affirmative action.[6]

One argument for continued affirmative action for the creamy layer is that poor members of the target group might get motivated by the success of the wealthier members of their group. Crocker et al. (1994), in their model of collective self-esteem, provide a sociological basis for such motivation effects by demonstrating that psychological well-being is linked to whether one can hold one's own "group" in "high regard". Thus, achievements by some members of the group would generate a feeling of well-being in other members (Allen (2000) and Schmermund et al., 2001). The empirical literature on role model effects, e.g. Solnick (1995), Constantine (1995), Patton (2009), Matsa and Miller (2011) and Kurtulus and Tomaskovic-Devey (2012), finds evidence of such motivation effects.[7]

Another issue of interest is the targeted educational credit policies, subsidizing poor minority group members' costs of education, followed by some countries. In the US, for example, the United Negro College Fund directly gives scholarships to disadvantaged African-Americans; it also funds Historically Black Colleges and Universities (http://www.collegescholarships.org/grants/african-american.htm). In China, ethnic minority students enrolled in ethnic minority-oriented specialties receive a monthly stipend.

Following Arrow (1972) and Coate and Loury (1993), we examine a model where the employers receive an imperfect signal regarding the skill level of the workers. Thus their belief regarding any worker is conditioned both by this signal, as well as their perception regarding the average skill level of the concerned group. Based on this belief they assign workers to either a skilled job (with higher wages), or to an unskilled one. The workers, in turn, may or may

(1977) and Wilson (1978), he finds that class differences within the black community may lead to low political mobilization and disunity among blacks as a group.

[6]In the US, when colleges were forbidden from explicit use of race-based quotas for admissions, some colleges began using "class-based" affirmative action (Sander 1997). However, following an adverse US Supreme Court ruling in 2003, class-based affirmative action has not been widely implemented. In Israel, on the other hand, four elite universities implemented a programme of class-based affirmative action, that ignored racial and ethnic criteria (Alon 2011). At the same time, from 2008 onwards, a proportion of seats in the Israeli civil service was reserved for Arabs (Haaretz, 2nd April, 2010). In Brazil, quotas for the poor coexist with those for racial minorities in federal universities and in some civil service jobs (the Economist, April 26, 2013). Highly-ranked French schools are required to maintain quotas for students from poorer families (Le Monde, December 17, 2008).

[7]For example, Kurtulus and Tomaskovic-Devey (2012) examine data on 20,000 firms during the period 1990-2003, finding that an increase in the share of women among the top management has a positive effect on the advancement of female middle management in later cohorts. Similarly, using data from 1997-2009, Matsa and Miller (2011) find that a rise in the share of women on boards of directors increases the share of women among top executives in later years. Chung (2000) develop a framework where labor market outcomes of black females who entered law school earlier, the role models, has informational value for later entrants.

not acquire the requisite skill depending on their own costs of doing so, as well as on the belief that the employers have about their group. Consequently, as is well known in the literature, the equilibrium may display statistical discrimination and stereo-typing.

Motivated by the actual heterogeneity in income within target groups ("blacks"), we extend Coate and Loury's(1993) framework to allow for this diversity. Thus while members of the richer sections of the target group, the so called creamy layer blacks (CLBs), as well as the "whites", can fund any training/educational costs out of their own pocket, the poorer blacks (PBs) have to access a credit market. Credit market imperfections ensure that the poor black workers have to pay a higher amount relative to the richer sections in case they want to get skilled. We demonstrate that there can be multiple equilibria, and examine conditions under which an equilibrium with stereo-typing, and even patronization (whereby the target groups are held to a lower standard) may, or may not exist.

We then use this framework to examine the issues introduced earlier. We first study the consequences of a demographic shift of the kind discussed before, whereby poorer members of the target group graduate to the creamy layer. We find that irrespective of whether the equilibrium involves stereo-typing or not, the creamy layer blacks are worse off, in the sense that a smaller proportion of such workers is assigned to the skilled job. In fact, under a patronizing equilibrium, *all* groups are 'worse off' as a result of the demographic shift. In the absence of any stereo-typing, however, the whites gain from such a demographic shift, though no other group does.

Turning to the role played by motivation effects, we find that, consistent with the popular perception, the presence of such a motivation effect does make the poor blacks better off provided the motivation effect is sufficiently strong. Interestingly, however, the presence of the motivation effect is not an unmixed blessing for the other groups. Irrespective of whether the equilibrium involves stereo-typing or not, the creamy layer blacks are worse off! The whites may also be worse off if the equilibrium is a patronising one.

We also examine several possible policy prescriptions, in particular the implications of switching to an income based affirmative action policy. Interestingly, such a switch would always make creamy layer blacks worse off. Moreover, at least one of the other two groups always oppose such a shift to class based affirmative action. In the absence of stereo-typing, the poor black workers are however better off under such a policy switch. This suggests one possible reason why income based affirmative action is such a hotly debated issue. Next, we examine the effect of the government providing targeted educational subsidies to PBs demonstrating that, under a patronising equilibrium, both the whites and the CLBs may be opposed to such a policy.

1.1 Literature Review

The literature closest to ours is the one on statistical discrimination, pioneered by Phelps (1972) and Arrow (1973).[8] The central idea behind statistical discrimination is that individual

[8] The literature on bias/discrimination has also explored some related ideas. Becker (1957) develops a theory of employment discrimination grounded in preference-based discrimination. In related work, Welch (1976) studies

attributes are not perfectly observable, so that employers use group attributes in making their decisions. The idea of statistical discrimination was further developed by Coate and Loury (1993), who use it to analyze the effect of affirmative action on stereo-types. Most strikingly, they find that affirmative action can lead to patronization whereby the target group can be held to a lower standard vis-a-vis the non-target group.

Turning to the theoretical literature,[9] Moro and Norman (2003) examine affirmative action in a framework where wages are endogenously determined, finding that affirmative action may improve the investment level by the discriminated workers in the worst equilibrium. Fryer (2007) examines firms with a hierarchical labour structure, showing that if an employer discriminates against a group of workers in her initial hiring, she may actually favor the successful members of that group when she promotes from within the firm. Fang and Norman (2006) show that in the presence of racial discrimination in public sector jobs, members of the discriminated groups may be better off in that they acquire a greater level of investment. Among other works, Lundberg (1991) examines the impact of affirmative action when regulators do not observe the firms' personnel policies, while Fryer and Loury (2013) examines the effect of group idenity being observable on the efficacy of diversity-enhancing policies. The present paper differs from the literature in that income heterogeneity among the target group, and the resulting credit market imperfections, play a central role in the analysis. Further, we then use this framework to examine several issues of interest, e.g. the effect of demographic shifts, and a switch to income based affirmative action, among others, that have been relatively unexplored in the literature.

The rest of the paper is organized as follows. In the next section we describe the framework analyzed in this paper. Next, while Section 3 examines the outcome without any affirmative action, Section 4 examines the outcome when affirmative action is present. In section 5 we examine the effect of a demographic shift whereby more blacks graduate to the creamy layer, while Section 6 examines the implication of a role model effect. In Section 7 we examine class-based affirmative action, while Section 8 examines targeted education subsidies. Finally, Section 9 concludes.

2 The Framework

The economy comprises three kinds of workers, λ_W white workers, λ_1 creamy layer blacks (henceforth CLBs) and λ_2 poor blacks (henceforth PBs), and a large number of firms. The number of workers is normalised to 1, so that $\lambda_W + \lambda_1 + \lambda_2 = 1$. Let λ_B denote the proportion of blacks in the population, and λ_R the proportion of the non-poor, so that $\lambda_B \equiv \lambda_1 + \lambda_2$, and $\lambda_R \equiv \lambda_W + \lambda_1$. All agents are risk neutral.

The workers are randomly matched to firms, with all workers finding a match. Following the matching process, the firms assign workers to either of two tasks, 1 or 2, where task 1 requires

sector specific employment quotas in a taste-based framework. Theories of bias can also be based on perception, e.g. Bertrand and Mullainathan (2004), and Banerjee et al. (2009).

[9] There is a large empirical literature on affirmative action which has examined, among other issues, the effect of affirmative action on improving black-white earning disparity. One can mention, among many others, Leonard (1984), Smith and Welch (1984), Welch (1989), etc.

skill, whereas task 2 does not. In task 1, the payoff of the principal is x_q (> 0) if the worker is skilled, and $-x_u$ (< 0) otherwise. Further, task 1 carries a positive wage of w, where w is exogenously given. In task 2 on the other hand, both the wages and returns are normalised to zero.

For all groups, acquiring the requisite skill is costly. This cost, denoted by c, is idiosyncratic and distributed over $[0, \infty)$ according to the distribution function $G(c)$, where $G(c)$ is continuously differentiable and identical for all three groups. For every worker assigned to firms, the firms can observe group identity, but they cannot observe whether the worker has acquired the necessary skill. The firms however do observe a signal regarding their skill level. The signal s, where $s \in [0, 1]$, has distribution $F_q(s)$ if the agent is $q$ualified (i.e. acquired the requisite skill), and $F_u(s)$ if the worker is un-qualified. Both $F_q(s)$ and $F_u(s)$ are twice continuously differentiable so that the associated density functions, $f_q(s)$ and $f_u(s)$ respectively, are well defined and continuous. Finally, define

$$\phi(s) = \frac{f_u(s)}{f_q(s)}.$$

The signal is informative in that a higher s signals that the agent is more likely to be qualified. This is formalised as

Assumption A1. $\phi(s)$ satisfies the monotone likelihood ratio property (henceforth MLRP), i.e. $\phi(s)$ is decreasing in s. Further, it satisfies the Inada conditions $\lim_{s \to 0} \phi(s) = \infty$ and $\lim_{s \to 1} \phi(s) = 0$.

Note that MLRP implies that $F_q(s)$ first order stochastically dominates $F_u(s)$, i.e. $F_u(s) \geq F_q(s)$, $\forall s$.

We next turn to the financing of skill acquisition. White and creamy layer black (CLB) workers can finance the cost of skill acquisition out of their own pockets. The poor black (PB) workers however must approach a competitive credit market for the requisite loan. A PB borrower can repay her loan out of her wages if she is assigned to task 1, but cannot do so is she is assigned to task 2. While the lenders can observe the group identity of the borrowers, they cannot observe whether the borrowers have been assigned to task 1, or task 2. In the presence of such informational asymmetries there can be strategic default, i.e. a worker can falsely claim to have been assigned to task 2, whereas in reality she had been assigned to task 1. The lenders can however prevent such default and ensure repayment by spending some additional amount. Let the total costs of lending an amount c *and* ensuring repayment (in case the borrower is assigned to task 1) be cm, where $m > 1$ is a measure of the imperfection in the credit market.

We then specify the utility function of the various agents. The payoff to a firm from a worker assigned to task 1 equals

$$\begin{cases} x_q - w, \text{ if the worker is skilled,} \\ -x_u - w, \text{ otherwise,} \end{cases}$$

and equals zero if the worker is assigned to task 2.

Next consider a worker who is either white, or a CLB. Her utility equals

$$\begin{cases} w-c, \text{ if she is skilled and assigned to task 1,} \\ -c, \text{ if she is skilled and assigned to task 2,} \\ w, \text{ if she is unskilled and assigned to task 1,} \\ 0, \text{ otherwise.} \end{cases}$$

Next let i_c denote the gross interest, i.e. the principal plus the interest, for a loan of c. We then consider the utility of a PB worker taking a loan of c at a gross interest of i_c. Denoting the competitive interest rate by i_c, the utility of such a worker is:

$$\begin{cases} w-i_c, \text{ if she is skilled and assigned to task 1,} \\ 0, \text{ if she is skilled and assigned to task 2,} \\ w, \text{ if she is unskilled and assigned to task 1,} \\ 0, \text{ otherwise.} \end{cases}$$

The timeline is as follows. Nature moves first, choosing the level of c for every worker. The workers themselves get to observe their own level of c, but the firms do not. Then the workers decide whether to acquire the skill required for task 1, or not. In the next stage the workers are matched to firms, when the firms get to observe a signal regarding the skill level of an worker assigned to it. Finally, the firms decide on task allocation.

3 No Affirmative Action

We first analyze the baseline model when there is no affirmative action (henceforth AA). Note that in the absence of AA, the outcomes for the three groups can be examined separately.

The firms' decisions: Consider a firm facing a worker of group i who emits a signal s. If the firm believes that a proportion π_i of the workers in this group are skilled, then the firm's belief that this particular worker is skilled, conditional on the signal s, is given by

$$A(\pi_i, s) \equiv \frac{\pi_i f_q(s)}{\pi_i f_q(s) + (1-\pi_i) f_u(s)}. \tag{1}$$

Hence the firm assigns this worker to task 1 if and only if the expected profits from doing so exceed the profits from assigning her to task 2 (which is normalised to zero), i.e. $A(\pi_i, s)x_q - (1 - A(\pi_i, s))x_u \geq 0$, i.e.

$$r \equiv \frac{x_q}{x_u} \geq \frac{1-\pi_i}{\pi_i}\phi(s). \tag{2}$$

Given assumption A1, the firms' decision is characterised by a cutoff $s_i(\pi_i)$ such that all workers with a signal greater than s_i are assigned to task 1, where s_i solves:

$$r \equiv \frac{x_q}{x_u} = \frac{1-\pi_i}{\pi_i}\phi(s_i). \tag{3}$$

From MLRP it follows that s_i is decreasing in π. Consequently the graph of $s^*(\pi)$, call it EE, is negatively sloped in $s - \pi$ space.

Let $\rho_i(s_i, \pi_i)$ denote the probability that a randomly drawn worker from group i is assigned to task 1, given that the cutoff signal for this group is s_i, i.e.

$$\rho_i(s_i, \pi_i) = \pi_i[1 - F_q(s_i)] + (1 - \pi_i)[1 - F_u(s_i)]. \tag{4}$$

The workers' decision: Next consider the decision problem facing a worker of type i, who believes that firms will assign her to task 1 if and only if she emits a signal of s_i, or higher. Denote $\beta(s) \equiv w(F_u(s) - F_q(s))$. It is straightforward to check that $\beta(0) = \beta(1) = 0$, so that $G(\beta(0)) = G(\beta(1)) = 0$. Further, given MLRP $G(\beta(s)) \geq 0$ and single peaked, and increasing if and only if $\phi(s) > 1$.

First consider workers who are not subject to credit constraints, i.e. white and CLB workers. For these workers let the opportunity cost of 1 unit of capital be 1. Hence such an worker with cost c acquires the skill if and only if $w(1 - F_q(s_i)) - c \geq w(1 - F_u(s_i))$, i.e.

$$c \leq \beta(s) \equiv w(F_u(s) - F_q(s)). \tag{5}$$

Recalling that c has a distribution $G(c)$, the proportion of type i workers getting educated

$$\pi_i = G(\beta(s_i)),\ i = 1, W. \tag{6}$$

Given the properties of $\beta(s)$, note that the graph of $\pi_i(s_i)$ in the $s - \pi$ space, call it WW_i, $i = 1, W$, is inversely U-shaped.

We next turn to the decision problem facing the poor black (PB) workers. The zero profit condition in the competitive loan market states that the expected cost of making a loan of c, i.e. cm, equals the expected returns $i_c(1 - F_q(s_2))$. This ensures that

$$i_c(1 - F_q(s_2)) = cm. \tag{7}$$

Consequently a worker with cost c takes a loan and acquires the skill if and only if $w(1 - F_q(s_2)) - i_c(1 - F_q(s_2)) \geq w(1 - F_u(s_2))$, i.e. $\beta(s_2) \geq i_c(1 - F_q(s_2)) = cm$. Thus, denoting $\hat{\beta}(s) \equiv \frac{\beta(s)}{m}$, the worker acquires the skill if and only if

$$\hat{\beta}(s_2) \equiv \frac{\beta(s_2)}{m} \geq c. \tag{8}$$

Thus the proportion of PB workers getting educated is given by:

$$\pi_2 = G(\hat{\beta}(s_2)). \tag{9}$$

Note that WW_W coincides with WW_1, and that WW_2 lies below $WW_i,\ i = W, 1$. Further, note that both curves peak at the same value of s, which we denote by $\tilde{s}$.

We are now in a position to define the notion of an equilibrium.

A configuration $(\bar{s}_W, \bar{\pi}_W, \bar{\rho}_W; \bar{s}_1, \bar{\pi}_1, \bar{\rho}_1; \bar{s}_2, \bar{\pi}_2, \bar{\rho}_2; \bar{i}_c)$ constitutes an *equilibrium* if and only if (a) the loan market clears, and, for all groups $i = 1, 2, W$, (b) given the cutoff $\bar{s}_i$, the proportion of group i workers acquiring the skill level is $\bar{\pi}_i$, and (c) given the level of skill acquisition $\bar{\pi}_i$, a cut-off of $\bar{s}_i$ maximises firm profits, i.e.

$$\pi_i = G(\beta(s(\pi_i))), \ i = 1, W, \tag{10}$$

$$\pi_2 = G((\hat{\beta}(s(\pi_2)))), \tag{11}$$

$$cm = i_c(1 - F_q(s(\pi_2))). \tag{12}$$

Henceforth we shall focus on equilibria that are *locally stable* where, for all groups, the absolute value of the slope of EE_i exceeds that of WW_i. For ease of exposition we shall refer to locally stable equilibria simply as equilibria. We shall be interested in two classes of equilibria, those with and without stereo-typing.

An equilibrium $(\bar{s}_W, \bar{\pi}_W, \bar{\rho}_W; \bar{s}_1, \bar{\pi}_1, \bar{\rho}_1; \bar{s}_2, \bar{\pi}_2, \bar{\rho}_2; \bar{i}_c)$ involves *no stereo-typing* if the CLB workers are held to the same standard as the white workers, and whereas the PB workers may be held to a higher standard, this arises solely from the fact that acquiring education is relatively more costly for the PB workers. Formally, we require that (a) $\bar{s}_W = \bar{s}_1$, and (b) there exists no other equilibrium that has a strictly lower level of s_2.

An equilibrium $(\bar{s}_W, \bar{\pi}_W, \bar{\rho}_W; \bar{s}_1, \bar{\pi}_1, \bar{\rho}_1; \bar{s}_2, \bar{\pi}_2, \bar{\rho}_2; \bar{i}_c)$ is said to involve *stereo-typing* if the poor black workers are held to a standard that is 'too' high vis-a-vis the white workers. Formally, we require that (a) $\bar{s}_W = \bar{s}_1 < \bar{s}_2$ and (b) there exists some other equilibrium which has a lower cut-off for the poor black workers.[10]

Moreover, so as to abstract from issues of coordination, we shall examine stable equilibria that dominate all other equilibria 'within its class', in a sense made formal below.

An equilibrium $(\bar{s}_W, \bar{\pi}_W, \bar{\rho}_W; \bar{s}_1, \bar{\pi}_1, \bar{\rho}_1; \bar{s}_2, \bar{\pi}_2, \bar{\rho}_2; \bar{i}_c)$ without stereo-typing is said to be *natural* (henceforth *NSTN*) if there exists no other equilibrium without stereo-typing that involves a strictly lower cutoff level for at least one of the groups.

An equilibrium $(\bar{s}_W, \bar{\pi}_W, \bar{\rho}_W; \bar{s}_1, \bar{\pi}_1, \bar{\rho}_1; \bar{s}_2, \bar{\pi}_2, \bar{\rho}_2; \bar{i}_c)$ with stereo-typing is said to be *natural* (henceforth *STN*) if there exists no other equilibrium with stereo-typing that involves a strictly lower cutoff level for at least one of the groups.

In Figure 1, $(\bar{s}_W = \bar{s}_1 = s', \bar{\pi}_W = \bar{\pi}_1 = \pi', \bar{s}_2 = s'', \bar{\pi}_2 = \pi'')$ constitutes an NSTN equilibrium, and $(\bar{s}_W = s', \bar{\pi}_W = \pi', \bar{s}_1 = s''', \bar{\pi}_1 = \pi''', \bar{s}_2 = s'''', \bar{\pi}_2 = \pi'''')$ constitutes an STN equilibrium.

Given the preceding discussion Proposition 1 below is immediate. It demonstrates, as is intuitive, that an increase in credit market inefficiency leads to a higher interest rate for the poor

[10] Of course, stereo-typing can involve the CLB workers as well. Given our focus on the poor black workers, the definition adopted by us helps crystallise our ideas.

blacks. Further, one finds that the presence or absence of stereo-typing affects the equilibrium interest rates, with the interest rates being higher in the presence of stereo-typing.

PROPOSITION 1. *Let assumption A1 hold.*

(a) Under either an STN, or an NSTN equilibrium, an increase in credit market inefficiency, i.e. in m, causes an increase in the interest rate $\bar{i}_c$. Further, the poor black workers are held to a higher cutoff.

(b) The equilibrium interest rate $\bar{i}_c$ is higher under an STN equilibrium compared to that under an NSTN equilibrium.

Proof. (a) With an increase in m, note that WW_2 shifts downwards so that $\bar{s}_2$ increases (since the equilibrium is locally stable). The result now follows since in any equilibrium the equilibrium gross interest is given by

$$i_c(1 - F_q(\bar{s}_2)) = cm.$$

(b) This follows since, by definition, the cutoff signal for the poor black workers under the STN equilibrium is greater than that under the NSTN equilibrium. □

4 Affirmative Action

We first introduce a formal definition of affirmative action (henceforth AA). To that end, let $P(s_i', \pi_i)$ denote an employer's expected payoff from a worker belonging to group i who emits a signal s_i':

$$P(s_i', \pi_i) = \pi_i[1 - F_q(s_i')]x_q - (1 - \pi_i)[1 - F_u(s_i')]x_u. \tag{13}$$

Let affirmative action involve mandating that the proportion of workers assigned to the skilled task, i.e. task 1, be the same for the black and the white workers, i.e.

$$\rho_W = \mu_1\rho_1 + \mu_2\rho_2,$$

where $\mu_i = \frac{\lambda_i}{\lambda_1 + \lambda 2}$, $i = 1, 2$.

We then define

$$\hat{\rho}(s) \equiv \rho(s, G(\beta(s)).$$

Thus $\hat{\rho}(s_i)$ denotes the fraction of group i workers being assigned to the skilled task hen the employers adopt a cutoff of s_i for this group, and $G(\beta(s_i)$ workers in this group invest in skill acquisition. Note that $\hat{\rho}(0) = 1$ and $\hat{\rho}(1) = 0$.

Assumption A2. $\hat{\rho}(s)$ is negatively sloped.

From Coate and Loury (1993) recall that in the absence of any credit market imperfections, given A2, affirmative action completely resolves the problem of stereo-typing (in the sense that the resulting equilibrium involves no stereo-typing, and can be sustained even after AA is withdrawn). In the present paper also this assumption plays a somewhat similar role, allowing us to consider a benchmark case where one can abstract from patronising equilibria (to be defined shortly). We shall of course also examine the outcome when this assumption is relaxed so that the equilibrium may involve patronisation.

We next turn to solving for equilibria in the presence of AA. Because of AA considerations, the employers' decisions in the various markets now become inter-linked, so that the constrained optimisation problem of an employer is:

$$\max_{s_1', s_2', s_3'} \sum_{i=W,1,2} \lambda_i P(s_i', \pi_i) + \gamma[\mu_1 \rho_1(s_1', \pi_1) + \mu_2 \rho_2(s_2', \pi_2) - \rho_W(s_W', \pi_W)], \tag{14}$$

where γ is the Lagrange multiplier on AA. The first order condition with respect to s_i' generates the cutoff s_i as a function of π_i. Denoting this function by $EE_i(\gamma)$, $i = 1, 2, W$, we have

$$\frac{x_q - \gamma/\lambda_W}{x_u + \gamma/\lambda_W} = \frac{1-\pi_W}{\pi_W}\phi(s_W) \quad : EE_W(\gamma), \tag{15}$$

$$\frac{x_q + \gamma\mu_1/\lambda_1}{x_u - \gamma\mu_1/\lambda_1} = \frac{x_q + \gamma/\lambda_B}{x_u - \gamma/\lambda_B} = \frac{1-\pi_1}{\pi_1}\phi(s_1) \quad : EE_1(\gamma), \tag{16}$$

$$\frac{x_q + \gamma\mu_2/\lambda_2}{x_u - \gamma\mu_2/\lambda_2} = \frac{x_q + \gamma/\lambda_B}{x_u - \gamma/\lambda_B} = \frac{1-\pi_2}{\pi_2}\phi(s_2) \quad : EE_2(\gamma). \tag{17}$$

Clearly, as a result of AA, employers act as if they have to pay a tax of $\frac{\gamma}{\lambda_W}$ on each white worker assigned to task one, and that they are receiving subsidies of $\frac{\gamma}{\lambda_B}$ on each CLB and PB worker they assign to task one.

Comparing with (3), where recall that (3) involves $r = \frac{1-\pi_i}{\pi_i}\phi(s_i)$, we have

Observation 1. *Fix $\gamma > 0$.*

(a) For any s_i, the π_i solving (15), is greater than the π_i solving (3), so that graphically $EE_W(\gamma)$ lies to the right of EE.

(b) From (16) and (17), $EE_1(\gamma)$ and $EE_2(\gamma)$ coincide.

(c) For any s_i, the π_i solving (16) (as well as (17)) is less than the π_i solving (3), so that $EE_i(\gamma)$, $i = 1, 2$, both lie to the left of EE.

An outcome $(s_W^*, \pi_W^*, \rho_W^*; s_1^*, \pi_1^*, \rho_1^*; s_2^*, \pi_2^*, \rho_2^*; i_c^*)$ constitutes an *equilibrium* with AA if and only if it satisfies

$$\pi_i = G(\beta(s(\pi_i))), i = 1, W, \tag{18}$$

$$\pi_2 = G(\hat{\beta}(s(\pi_2))), \tag{19}$$

$$cm = i_c(1 - F_q(s_2)), \tag{20}$$

$$\rho_W = \mu_1\rho_1 + \mu_2\rho_2. \tag{21}$$

Let γ^* denote the Lagrange multiplier associated with this equilibrium. Further, we shall restrict attention to equilibria that are locally stable, so that for all i, $EE_i(\gamma^*)$ intersects WW_i from above.

Under AA, we shall focus on two classes of equilibria.

First, we define a natural equilibrium without stereo-typing (henceforth NSTNAA equilibrium) as one where although the PB workers are held to a higher standard, i.e. a higher cutoff for the signal than the white workers, the higher standard follows solely from the fact that the PB workers have a higher cost of skill acquisition. Formally, an equilibrium $(s_W^*, \pi_W^*, \rho_W^*; s_1^*, \pi_1^*, \rho_1^*; s_2^*, \pi_2^*, \rho_2^*; i_c^*)$ is said to be NSTNAA if and only if (a) $s_W^* < s_2^*$, and (b) there exists no other equilibrium that satisfies (a) and moreover also involves a strictly lower standard for at least one of the three groups.

Next, following Coate and Loury (1993) we define the notion of a patronising equilibrium where the idea is that because of the AA constraint the employers adopt a lower standard for the poor black workers.[11]

An equilibrium $(s_W^*, \pi_W^*, \rho_W^*; s_1^*, \pi_1^*, \rho_1^*; s_2^*, \pi_2^*, \rho_2^*; i_c^*)$ is said to be a *natural patronizing equilibrium* (henceforth PNAA equilibrium), if and only if (a) $s_2^* < s_W^*$, and (b) there exists no other equilibrium that satisfies (a) and also involves a strictly lower standard for at least one of the three groups.

Proposition 2 below establishes some formal properties of an equilibrium under AA. To that end we define $\tilde{r}$ as solving

$$G(\beta(\tilde{s})) = \frac{\phi(\tilde{s})}{r + \phi(\tilde{s})}.$$

The proof of the following proposition can be found in the appendix. Given Proposition 2 below, we shall henceforth focus on equilibria with a positive Lagrange multiplier.

Proposition 2. *Let A1 and A2 hold.*

(a) Consider any $r > \tilde{r}$. Then there exists an open set of parameter values m such that an AA equilibrium with a positive Lagrange multiplier exists.

(b) Consider $r < \tilde{r}$. Then there exists an open set of parameter values of m and λ_W such that an AA equilibrium with a positive Lagrange multiplier exists.

(c) There can be at most one NSTNAA equilibrium. Further, no PNAA equilibrium exists.

We next turn to examining the properties of such equilibria. To begin with, we define the notion of two equilibria being close together. Consider an affirmative action equilibrium denoted by $(s_W^*, \pi_W^*, \rho_W^*; s_1^*, \pi_1^*, \rho_1^*; s_2^*, \pi_2^*, \rho_2^*; i_c^*)$, with a Lagrange multiplier γ^*. Thus this equilibrium solves (10), (11), (12), (15), (16) and (17) for $\gamma = \gamma^*$.

[11] Interestingly, the elite Indian enginneering schools, the IITs had begun to respond to AA requirements by admitting target group students who had scored only 6% in the entrance examinations, rather than allow target group vacancies to go unfilled (Times of India, July 14, 2015).

Let the cutoff signals solving (10), (11), (15), (16) and (17) be denoted by $\tilde{s}_i(\gamma)$, $i = 1, 2, W$. Define $\tilde{\pi}_i(\gamma)$ in a similar fashion.

We shall say that an equilibrium under AA, denoted $(s_W^*, \pi_W^*, \rho_W^*; s_1^*, \pi_1^*, \rho_1^*; s_2^*, \pi_2^*, \rho_2^*; i_c^*)$, is *close* to an equilibrium without AA, denoted $(\bar{s}_W, \bar{\pi}_W, \bar{\rho}_W; \bar{s}_1, \bar{\pi}_1, \bar{\rho}_1; \bar{s}_2, \bar{\pi}_2, \bar{\rho}_2; \bar{i}_c)$, if, $\tilde{s}_i(\gamma)$ converges to $\bar{s}_i$ as γ goes to zero.

The next proposition suggests two political problems that may possibly afflict affirmative action policies under assumption A2. First, we argue that whenever the equilibrium under AA is 'close' to the equilibrium without AA, the interests of the the whites and the CLBs (who are also likely to be quite vocal on the issue of AA), are necessarily opposed as regards whether AA should be imposed or not. In fact, the whites are worse off as far as being assigned to task 1 is concerned, whereas the creamy layer blacks are better off.

Suppose, for the sake of exposition, that for every group the over-riding criterion while deciding whether to support AA or not, is whether such a policy adversely affects the proportion of group members being assigned to the skilled task. In case this decision is controlled solely by the whites, our analysis suggests that it is unlikely to go through. Thus the outcome would depend on the kind of voice that the creamy layer blacks have.

Second, it suggests that, unlike under Coate and Loury (1993), removal of AA can lead to a worsening of the condition of the PB workers in terms of the proportion of such workers assigned to task 1.

PROPOSITION 3. *Let A1 and A2 hold.*

(a) Suppose that the equilibrium under AA, denote it by $(s_W^*, \pi_W^*, \rho_W^*; s_1^*, \pi_1^*, \rho_1^*; s_2^*, \pi_2^*, \rho_2^*; i_c^*)$, *is 'close' to, but different from, the NSTN equilibrium without AA, which is denoted by* $(\bar{s}_W, \bar{\pi}_W, \bar{\rho}_W; \bar{s}_1, \bar{\pi}_1, \bar{\rho}_1; \bar{s}_2, \bar{\pi}_2, \bar{\rho}_2; \bar{i}_c)$. *Then under AA, the interests of the white and the CLB workers are opposed in the sense that the CLBs prefer that AA happens, while the whites do not, i.e.* $\bar{\rho}_W > \rho_W^*$ *and* $\bar{\rho}_1 < \rho_1^*$.

(b) Assume that in the absence of affirmative action, the economy attains the NSTN equilibrium. Consider an NSTNAA equilibrium under AA. Once affirmative action is removed, the proportion of poor black workers assigned to task 1 will decline.

Proof. (a) Note that the AA constraint necessarily binds. Otherwise, the equilibrium will coincide with the NSTN outcome, where $\rho_2 < \rho_1 = \rho_W$, so that the AA constraint cannot be satisfied. Given that the AA binds, we have that the Lagrange multiplier under affirmative action, i.e. γ^*, must be positive. The argument follows from A2 and the fact that $\gamma^* > 0$.

(b) Note that under an NSTNAA, the AA constraint necessarily binds. Otherwise, the equilibrium will coincide with the NSTN outcome, where $\rho_2 < \rho_1 = \rho_W$, so that the AA constraint cannot be satisfied. Given that the AA binds, we have that the Lagrange multiplier under affirmative action must be positive. From Observation 1, it now follows that ρ_2 will decrease once AA is removed (so that the AA constraint does not bind). $\square$

We then establish some further properties of the NSTNAA equilibrium.

Proposition 4. *Let A1 and A2 hold. Denote the NSTN equilibrium by NSTN equilibrium by* $(\bar{s}_W, \bar{\pi}_W, \bar{\rho}_W; \bar{s}_1, \bar{\pi}_1, \bar{\rho}_1; \bar{s}_2, \bar{\pi}_2, \bar{\rho}_2; \bar{i}_c)$. *Under affirmative action, there exists an NSTNAA equilibrium* $(s_W^*, \pi_W^*, \rho_W^*; s_1^*, \pi_1^*, \rho_1^*; s_2^*, \pi_2^*, \rho_2^*; i_c^*)$ *where:*

(a) $\bar{s}_W < s_W^*,\ \bar{s}_1 > s_1^*,\ \bar{s}_2 > s_2^*,$

(b) $\bar{\rho}_W > \rho_W^*,\ \bar{\rho}_1 < \rho_1^*,\ \bar{\rho}_2 < \rho_2^*,$

(c) $\rho_1^* > \rho_W^* > \rho_2^*,$

(d) $s_2^* > s_W^* > s_1^*,$

(e) When $r > \tilde{r}$, $\bar{\pi}_W < \pi_W^*,\ \bar{\pi}_1 > \pi_1^*,\ \bar{\pi}_2 > \pi_2^*$, *whereas for* $r < \tilde{r}$, $\bar{\pi}_W < \pi_W^*,\ \bar{\pi}_1 > \pi_1^*,\ \bar{\pi}_2 > \pi_2^*$.

Proof. (a) Note that Proposition 4(a) follows from Observation 1 (see Figure 2).

(b) Given Proposition 4(a), Proposition 4(b) follows from the fact that $\hat{\rho}(s)$ is negatively sloped (from A2).

(c) and (d) Observe that from Proposition 4(a), $s_W^* > \bar{s}_W = \bar{s}_1 > s_1^*$. Hence given assumption A2, $\hat{\rho}(s_1^*) > \hat{\rho}(s_W^*)$. Further given that (a) $\hat{\rho}(s_1^*) > \hat{\rho}(s_W^*)$, and (b) from the AA constraint, $\hat{\rho}(s_W^*)$ equals the average of $\hat{\rho}(s_1^*)$ and $\hat{\rho}(s_2^*)$, we have that $\hat{\rho}(s_W^*) > \hat{\rho}(s_2^*)$. Next, given $\hat{\rho}(s)$ is decreasing, $s_2^* > s_W^* > s_1^*$.

(e) Finally, 4(e) follows from 4(a), and the fact that WW_i is increasing (respectively decreasing) over the relevant region for $r > \tilde{r}$ (respectively $r < \tilde{r}$). □

4.1 Patronising Equilibria

We next examine an economy where $\hat{\rho}(s)$ is non-monotonic. We shall find that, under some additional conditions, a PNAA equilibrium exists. Moreover, in the following sections we shall find that many of the comparative statics results, as well as the policy implications differ depending on whether one is considering an NSTNAA equilibrium, or a PNAA one.

Assumption A3. Let $\hat{\rho}(s)$ be increasing over some interval $(\underline{s}, \bar{s})$, where $0 < \underline{s} < \bar{s} < 1$, and decreasing otherwise.

Note that for $m = 1$, our framework coincides with Coate and Loury (1993). Consequently, from continuity, Proposition 4 in Coate and Loury (1993) guarantees the existence of a patronising equilibrium for an open set of parameter values (that requires λ_W to be sufficiently large) for m sufficiently close to 1.

Proposition 5. *Let A1 and A3 hold. Further, let m be sufficiently close to 1 so that a patronising equilibrium exists. Letting* $(\bar{s}_W, \bar{\pi}_W, \bar{\rho}_W; \bar{s}_1, \bar{\pi}_1, \bar{\rho}_1; \bar{s}_2, \bar{\pi}_2, \bar{\rho}_2; \bar{i}_c)$ *denote the NSTN equilibrium, the patronising equilibrium satisfies:*

(a) $\bar{s}_W < s_W^*,\ \bar{s}_1 > s_1^*,\ \bar{s}_2 > s_2^*,$

(b) $s_1^* < s_2^* < s_W^*,$

(c) $\rho_1^* > \rho_W^* > \rho_2^*$.

Proof.

(a) Proposition 5(a) follows from the fact that (a) one considers stable intersection points, and (b) with an increase in γ, $EE_W(\gamma)$ shifts to the right, and $EE_i(\gamma)$, $i = 1, 2$, shifts to the left (Observation 1).

(b) That $s_1^* < s_2^*$ follows from 5(a). Whereas $s_2^*(m) < s_W^*(m)$ follows from the fact that m is close to 1, and $s_2^* < s_W^*$ for $m = 1$ under the PNAA equilibrium.

(c) From 5(b), and the fact that $\hat{\rho}(s)$ is decreasing over the relevant range, we have that $\hat{\rho}(s_1^*) > \hat{\rho}(s_2^*)$. Finally, from the AA constraint we have that $\rho_1^* > \rho_W^* > \rho_2^*$. □

We next derive a technical lemma, which shall be used repeatedly in the subsequent analysis. Recall that $\tilde{s}_i(\gamma)$, $i = 1, 2, W$, solve (10), (11), (15), (16) and (17). Moreover, let $(\tilde{\pi}_i(s), s)$ belong to the graph of WW for $i = 1, W$, and to the graph of WW_2 for $i = 2$. We then define:

$Pr(s_i) \equiv \tilde{\pi}_i(s_i) f_q(s_i) + (1 - \tilde{\pi}_i(s_i)) f_u(s_i)$, $i = W, 1, 2$, and

$\theta_i(s_i) \equiv \frac{f_q'(s_i)}{f_q(s_i)} - \frac{f_u'(s_i)}{f_u(s_i)} + \frac{g(.)\beta'(s_i)}{\tilde{\pi}_i(s_i)(1-\tilde{\pi}_i(s_i))}$, $i = W, 1, 2$.

Further, let $s' \equiv \bar{s}_2$, where $\bar{s}_2$ denotes the cutoff signal for the PB workers under the NSTN equilibrium. Finally, define $\hat{s}$ as solving $\hat{\rho}(\hat{s}) = \hat{\rho}(s')$, where $\underline{s} \leq \hat{s} \leq \bar{s}$. Next we introduce

Assumption A4.

(a) $\lambda_W > \frac{(1-\tilde{\pi}_i(x))f_u(x)Pr(y)\theta_i(x)}{(1-\tilde{\pi}_i(x))f_u(x)Pr(y)\theta_i(x)+(1-\tilde{\pi}_W(y))f_u(y)Pr(x)\theta_W(y)}$, $i = 1, 2$, $\forall(x \leq s', y \geq \hat{s})$.

(b) $|\hat{\rho}'(x)| \geq |\hat{\rho}'(y)|$, $\forall(x \leq s', y \geq \hat{s})$.

Note that A4(a) is consistent with our existence result for a patronising equilibrium, since, from Coate and Loury (1993) we know that for $m = 1$, existence requires λ_W to be large.

LEMMA 1. *Assume that a patronizing equilibrium,* $(s_W^*, \pi_W^*, \rho_W^*; s_1^*, \pi_1^*, \rho_1^*; s_2^*, \pi_2^*, \rho_2^*; i_c^*)$, *where* $(s_1^* < s_2^* < \underline{s} < s_W^* < \bar{s})$, *exists. Then under assumption A4,* $|\frac{d\hat{\rho}(s_W)}{ds_W}\frac{\partial \tilde{s}_W(\gamma)}{\partial \gamma}| < |\frac{d\hat{\rho}(s_i)}{ds_i}\frac{\partial \tilde{s}_i(\gamma)}{\partial \gamma}|$, $i = 1, 2$.

We then use this framework to shed some light on several issues that have come up in recent discussions on affirmative action.

5 Demographic Shift: An Increase in the Relative Proportion of CLB Workers

As argued in the introduction, many countries are witnessing a demographic shift whereby poorer members of the target group are becoming relatively more affluent, so that the relative proportion of CLB workers is increasing vis-a-vis the PB workers. Interestingly, we find that with such a demographic shift, dissatisfaction with AA policies may grow among the various groups as the proportion of workers assigned to the skilled job declines. In fact, under a patronising equilibrium, all groups suffer a such decline.

Formally let λ_1 increase to λ_1', and λ_2 decrease to λ_2', so that $\lambda_1' + \lambda_2' = \lambda_1 + \lambda_2$. Define $\lambda' \equiv (\lambda_1', \lambda_2')$, and let the associated Lagrange multiplier be $\gamma^*(\lambda')$. Further, let $\rho_i^*(\bar{\lambda}, \bar{\gamma})$ solve (10), (11), (12), (15), (16) and (17), for $(\bar{\lambda}, \bar{\gamma}) \in \{(\lambda, \gamma^*), (\lambda', \gamma^*(\lambda'))\}$.

Proposition 6. *Let there be an increase in the relative proportion of the CLB workers, so that λ_1 increases to λ_1', and λ_2 decreases to λ_2'.*

(a) Let A1 and A2 hold. Further, suppose an NSTNAA equilibrium exists. Such a demographic shift increases the proportion of white workers assigned to task 1, and reduces the proportion of black workers, both poor as well as the CLBs, assigned to task 1.

(b) Let A1, A3 and A4 hold. Further, suppose that there exists a patronising equilibrium PNAA, denoted by $(s_W^*, \pi_W^*, \rho_W^*; s_1^*, \pi_1^*, \rho_1^*; s_2^*, \pi_2^*, \rho_2^*; i_c^*)$, where $s_1^* < s_2^* < \underline{s} < s_W^* < \bar{s}$. Following such a demographic change, the proportion of all three groups of workers assigned to task 1 declines.

Proof. (a) We first argue that $\gamma^*(\lambda') < \gamma^*$. Suppose not, so that $\gamma^*(\lambda') \geq \gamma^*$. Now given that (i) $\rho_1^*(\lambda, \gamma^*) > \rho_2^*(\lambda, \gamma^*)$ (from Proposition 4), and (ii) $\lambda_1' > \lambda_1$, $\lambda_2' < \lambda_2$ and $\lambda_1 + \lambda_2 = \lambda_1' + \lambda_2'$, we have that

$$\lambda_1'\rho_1^*(\lambda, \gamma^*) + \lambda_2'\rho_2^*(\lambda, \gamma^*) > \lambda_1\rho_1^*(\lambda, \gamma^*) + \lambda_2\rho_2^*(\lambda, \gamma^*).$$

Next recalling that AA implies that $\lambda_B\rho_W^*(\lambda, \gamma^*) = \lambda_1\rho_1^*(\lambda, \gamma^*) + \lambda_2\rho_2^*(\lambda, \gamma^*)$, and that $\rho_i^*(\lambda, \gamma^*) = \rho_i^*(\lambda', \gamma^*),\ i = 1, 2, W$, we then have that

$$\begin{aligned} \lambda_1'\rho_1^*(\lambda', \gamma^*) + \lambda_2'\rho_2^*(\lambda', \gamma^*) &= \lambda_1'\rho_1^*(\lambda, \gamma^*) + \lambda_2'\rho_2^*(\lambda, \gamma^*) > \lambda_1\rho_1^*(\lambda, \gamma^*) + \lambda_2\rho_2^*(\lambda, \gamma^*) \\ &= \lambda_B\rho_W^*(\lambda, \gamma^*) = \lambda_B\rho_W^*(\lambda', \gamma^*). \end{aligned} \tag{22}$$

Now given that $\gamma^*(\lambda') \geq \gamma^*$, from Observation 1 and Assumption 3 it follows that $\rho_i^*(\lambda', \gamma^*(\lambda')) \geq \rho_i^*(\lambda', \gamma^*), i = 1, 2$ and $\rho_W^*(\lambda', \gamma^*(\lambda')) \leq \rho_W^*(\lambda', \gamma^*)$. Hence from (22) it follows that

$$\lambda_1'\rho_1^*(\lambda', \gamma^*(\lambda')) + \lambda_2'\rho_2^*(\lambda, \gamma^*(\lambda')) > \lambda_B\rho_W^*(\lambda', \gamma^*(\lambda')),$$

which violates the AA constraint under the new demographic conditions. Hence it must be that $\gamma^*(\lambda') < \gamma^*$. Consequently, from Observation 1 and Assumption A2, the proportion of workers assigned to task 1 will decline for both the PB and the CLB workers, and increase for the white workers.

(b) We can argue as before, that (22) holds. Given Lemma 1, we know that a change in γ has a 'relatively greater' effect on $|\hat{\rho}_i'(s)|,\ i = 1, 2$, as compared to its impact on $|\hat{\rho}_W'(s)|$. Hence respecting the affirmative action constraint for λ' entails that $\gamma^*(\lambda') < \gamma^*$. Otherwise, while both the RHS and the LHS of (22) will increase if γ^* is replaced by $\gamma^*(\lambda')$ (given that $s_1^* < s_2^* < \underline{s} < s_W^* < \bar{s}$ and Assumption A3), from Lemma 1 the increase in the LHS will dominate that in the RHS, so that the AA constraint for λ' will be violated. Hence it must be that $\gamma^*(\lambda') < \gamma^*$. Consequently, from Observation 1, $s_i^*,\ i = 1, 2$, increases, and s_W^* decreases.

Finally, given that $s_1^* < s_2^* < \underline{s} < s_W^* < \bar{s}$ and Assumption 3, ρ_i^* decreases for all three groups, who therefore all have a lower probability of getting assigned to task 1. $\square$

What is the intuition for these results? As the CLB workers face a lower effective cost of education relative to the PB workers, their incentives for skill acquisition are higher. Since employers know this, they tend to adopt a lower standard for the CLB workers relative to the PB workers. Therefore, an increase in the proportion of CLBs increases the assignment rate for blacks, reducing the "shadow price" of equality, i.e. the Lagrange multiplier on the AA constraint. Thus the tax the AA constraint imposes on every white worker goes down; employers therefore lower standards for whites and raise them for blacks. The higher standards for blacks reduce their assignment rate to task 1, making them worse off. Under a non-stereo-typing equilibrium the assignment rate is monotonically decreasing in standards, so that the relaxation in standards for whites increases the proportion of whites assigned to task 1. However, under a patronising equilibrium, the relaxation of standards prompts whites to reduce their investment in skills to such an extent that the rate at which they are assigned to task 1 actually drops despite the relaxation in standards. In this case they become worse off.

6 Motivating Effect of CLBs

It is often argued that the success of the CLB workers might motivate the poor blacks via the role model effect (as argued earlier in the introduction), and thus incentivise them to acquire a greater level of skills. In the debate on extending affirmative action to the CLB workers, the motivation effect therefore provides a justification for doing so even when the creamy layer is not only reasonably large, but also increasing. We find that, consistent with the popular perception, the presence of such a motivation effect does make the poor blacks better off provided the motivation effect is sufficiently strong. Interestingly however, the presence of the motivation effect is not an unmixed blessing for the other groups. It makes the CLB workers worse off! The whites may also be worse off if the equilibrium is a patronising one.

Following Allen (1995) we focus on "moral role models" who affect the preference of PBs.[12] In order to formalise this aspect, we posit that a poor black worker's *effective* wage from being assigned to task 1 is $wh(\frac{\rho_1}{\rho_W})$.

Assumption 5. $h(\frac{\rho_1}{\rho_2})$ is once differentiable and increasing in $\frac{\rho_1}{\rho_W}$. Further, $h(1) = 1$.

We note the fact that $h(\frac{\rho_1}{\rho_W})$ is increasing in $\frac{\rho_1}{\rho_W}$ captures the motivation effect, since an increase in the relative success of the CLB workers increases the PB workers effective wage from task 1, and thus their incentive to get skilled.

Next consider an AA equilibrium in the presence of a motivation effect. Define $\tilde{\beta}(s, h(\frac{\rho_1}{\rho_W})) \equiv$

[12] This is in contrast to Chung (2000) who focuses on informational aspects of role models.

$\frac{\beta(s)h(\frac{\rho_1}{\rho_W})}{m}$. Note that a PB worker with cost c acquires the skill if and only if

$$\begin{aligned} wh(\frac{\rho_1}{\rho_W})(1-F_q(s_2)) - i_c &\geq wh(\frac{\rho_1}{\rho_W})(1-F_u(s_2)), \\ \text{i.e. } \tilde{\beta}(s, h(\frac{\rho_1}{\rho_W}))(1-F_q(s_2)) &\geq c. \end{aligned}$$

Thus the proportion of PBs getting educated is given by

$$\pi_2 = G(\tilde{\beta}(s, h(\frac{\rho_1}{\rho_W})). \tag{23}$$

In the presence of the motivation effect, let $(s_W^*(M), \pi_W^*(M); s_1^*(M), \pi_1^*(M); s_2^*(M), \pi_2^*(M); i_c^*(M))$ denote the equilibrium, and $\gamma^*(M)$ denote the associated Lagrange multiplier.

Proposition 7. *Suppose the poor black workers get motivated by the relative success of the creamy layer blacks.*

(a) Let A1, A2, and A5 hold. Further, let there be an NSTNAA equilibrium. Compared to the case where the motivation effect is absent, in the presence of the motivation the proportion of workers assigned to task 1 increases for the poor black and white workers, whereas it declines for the CLB workers.

(b) Let A1, A3, A4 and A5 hold. Further, suppose a patronising equilibrium PNAA exists. Then the motivation effect makes whites and CLBs worse off than they would have been in the absence of this effect. In addition, if the motivation effect is sufficiently strong, then the poor black workers are necessarily better off.

Proof. (a) Consider an NSTNAA equilibrium in the presence of the motivation effect. In the absence of the motivation effect, we know that $\rho_1^* > \rho_W^*$, so that $h(\frac{\rho_1^*}{\rho_W^*}) > 1$. Hence $G(\tilde{\beta}(s, h(\frac{\rho_1^*}{\rho_W^*})) = G(h(\frac{\rho_1^*}{\rho_W^*})\hat{\beta}(s)) > G(\hat{\beta}(s))$, so that the WW_2 curve shifts upwards in the presence of the motivation effect. Consequently, one has that $\tilde{s}_2(h(\frac{\rho_1^*}{\rho_W^*}), \gamma^*) < s_2^*$. Hence from assumption A2, it follows that $\tilde{\rho}_2(h(\frac{\rho_1^*}{\rho_W^*}), \gamma^*) > \rho_2^*$.

We next argue that $\gamma^* > \gamma^*(M)$. Suppose to the contrary that $\gamma^* \leq \gamma^*(M)$. Using the affirmative action constraint in the absence of the motivation effect, and the fact that $\tilde{\rho}_2(h(\frac{\rho_1^*}{\rho_W^*}), \gamma^*) > \rho_2^*$, we have that

$$\rho_W^* = \mu_1\rho_1^* + \mu_2\rho_2^* < \mu_1\rho_1^* + \mu_2\tilde{\rho}_2(h(\frac{\rho_1^*}{\rho_W^*}), \gamma^*).$$

Given that $\gamma^* \leq \gamma^*(M)$, from Observation 1 we then have that $\rho_W^*(M) \leq \rho_W^*$, $\rho_1^*(M) \geq \rho_1^*$ and $\rho_2^*(M) \geq \tilde{\rho}_2(h(\frac{\rho_1^*}{\rho_W^*}), \gamma^*)$. Hence one has that

$$\rho_W^*(M) < \mu_1\rho_1^*(M) + \mu_2\rho_2^*(M),$$

which violates the affirmative action constraint in the presence of the motivation effect. Hence $\gamma^* > \gamma^*(M)$, which in turn implies that $s_W^*(M) < s_W^*$ and $s_1^*(M) > s_1^*$. Hence given A2, it follows that $\rho_W^*(M) > \rho_W^*$ and $\rho_1^*(M) < \rho_1^*$. Finally, from (a) the AA constraint and (b) the fact that $\rho_W^*(M) > \rho_W^*$ and $\rho_1^*(M) < \rho_1^*$, it then follows that $\rho_2^*(M) > \rho_2^*$.

(b) We can argue as in part (a) that $\rho_W^*(M) < \mu_1\rho_1^*(M) + \mu_2\rho_2^*(M)$. We therefore require γ to change in such a way that the equality is restored. Note that this will not happen if γ increases; given Lemma 1, an increase in γ would cause the LHS of the above inequality to rise by less than it would increase the RHS, widening the gap. Therefore, γ must decrease. This shifts EE_W leftwards and EE_i,$i = 1, 2$ rightwards, decreasing ρ for whites and CLBs.

There are two conflicting effects for PBs: the direct motivation effect pushes up WW_2, making them better off, while the indirect effect of the rightward shift in EE_i because of the drop in γ to $\gamma^*(M) < \gamma^*$ tends to make them worse off. Define $s_2(M)$ to be the minimum s_2', where (s_2', π_2') is at the intersection of $WW_2(M)$ and EE. Let the motivation effect be large in the sense that $s_2(M) < s_2^*$. Given that $s_2^*(M) \leq s_2(M)$, it follows that $s_2^*(M) < s_2^*$. Hence, $\rho_2^*(M) > \rho^*$. $\square$

Intuitively, the motivation effect increases the PB workers' incentives to acquire skills. This in turn, by raising the assignment rate for blacks overall, lowers the shadow price of equality, so that the subsidy the AA constraint provides for assignment of black workers to task 1 falls. In response, standards for blacks are raised. Consequently, the CLB workers are worse off.

Turning to the white workers, the reduction in the shadow price of equality leads to a lower tax on assignment of whites to task 1, and standards are lowered for them. Under a non-stereotyping equilibrium, this increases their assignment to task 1 since their incentive for skill acquisition is not substantially affected. Whereas under a patronising equilibrium, the relaxation in standards for whites leads to a substantial reduction in the incentive to invest, reducing the assignment rate for white workers to task 1.

For the poor black workers, the general equilibrium effect of a rise in standards is overpowered by the direct motivation effect whenever the equilibrium does not involve patronisation. In the presence of patronisation, the effect is generally ambiguous. This is because under a patronising equilibrium the reduction in standards for the whites will be mitigated by the fact that the entrepreneurs will internalise the negative effect on white incentives of such a reduction. This in turn will tend to push up the standards for the blacks even further. Hence it is not clear if the direct effect of an increase in motivation will be sufficient to overturn the effect of an increase in standards. However, the direct effect dominates whenever the motivational effects are strong enough.

7 Class-based Affirmative Action

Among policy issues, we first consider the implications of switching to an affirmative action policy based on class/income (henceforth CAA), rather than on identity. As discussed earlier, there has been some debate in this regard both in India, as well as in the USA. How does the outcome under such a policy compare with the outcome under an identity based AA policy? We find that under such a policy the CLBs are definitely worse off. Moreover, in the absence of stereo-typing, the PBs are better-off, and the whites are worse off. Whereas in case of patronization the interests of the whites and the PBs are opposed.

A class based AA action policy mandates that the weighted average of the proportion of white *and* CLB workers being assigned to task 1, equal that of the PB workers. Defining $\lambda_1'' = \frac{\lambda_1}{\lambda_R}$ and $\lambda_W'' = \frac{\lambda_W}{\lambda_R}$, where $\lambda_R \equiv \lambda_1 + \lambda_W$, the CAA condition is formally given by:

$$\rho_2 = \lambda_1''\rho_1 + \lambda_W''\rho_W.$$

Let $(s_W^{**}, \pi_W^{**}, \rho_W^{**}; s_1^{**}, \pi_1^{**}, \rho_1^{**}; s_2^{**}, \pi_2^{**}, \rho_2^{**}; i_c^{**})$ denote an CAA equilibrium which is locally stable. Mimicing the earlier analysis the FOCs are given by:

$$\frac{x_q - \gamma/\lambda_R}{x_u + \gamma/\lambda_R} = \frac{1-\pi_W^{**}}{\pi_W^{**}}\phi(s_W^{**}) \quad : EE_W(\gamma), \tag{24}$$

$$\frac{x_q - \gamma/\lambda_R}{x_u + \gamma/\lambda_R} = \frac{1-\pi_1^{**}}{\pi_1^{**}}\phi(s_1^{**}) \quad : EE_1(\gamma), \tag{25}$$

$$\frac{x_q + \gamma/\lambda_1}{x_u - \gamma/\lambda_1} = \frac{1-\pi_2^{**}}{\pi_2^{**}}\phi(s_2^{**}) \quad : EE_2(\gamma). \tag{26}$$

Note that for ease of exposition we continue to use the notations $EE_i(\gamma)$ for this case as well. Clearly under CAA, $EE_1(\gamma)$ and $EE_W(\gamma)$ coincide, and both lie above EE, whereas $EE_2(\gamma)$ lie below EE.

Note that under an CAA policy not only are the white, and CLB workers are at par as far as the cost of getting skilled is concerned, the CAA policy also treats these two groups identically. Hence, in order to bring out the essential issues more sharply, we shall focus on symmetric equilibria where $s_1^{**} = s_W^{**}$, $\pi_1^{**} = \pi_W^{**}$ and $\rho_W^{**} = \rho_1^{**}$. Thus the CAA condition can be re-written as:

$$\rho_2^{**} = \rho_1^{**} = \rho_W^{**}.$$

As earlier, we can define a natural no stereo-typing equilibrium under CAA (denoted NST-NCAA) as one where (a) $s_2^{**} > s_i^{**}$, $i = 1, W$, and moreover the equilibrium is (b) locally stable, and (c) natural. We similarly define a natural patronising equilibrium under CAA (denoted PN-CAA) as one where (a) $s_2^{**} < s_i^{**}$, $i = 1, W$, and moreover the equilibrium is (b) locally stable, and (c) natural.

Proposition 8 below compares how the three groups fare under CAA vis-a-vis that under AA regarding getting assigned to task 1. We demonstrate that such a class based AA necessarily makes the CLB workers worse off compared to an identity based AA, irrespective of the nature of the equilibrium. The PB workers are however better off (and the white workers are worse off) if the equilibrium does not involve any stereo-typing. This suggests that in this case income based AA policies are unlikely to go through. Under a patronising equilibrium we find that interests of the PB and the white workers are necessarily opposed, though the effects on these two groups, relative to the AA policy, are ambiguous. In this case the CAA is unlikely to go through whenever the PB workers actually gain out of it, as both the white and CLB are worse off.

Proposition 8. *Let A1 hold.*

(a) *Let A2 hold, and suppose that an NSTNAA equilibrium exists. Compared to the NSTNAA equilibrium, under the NSTNCAA equilibrium, the proportion of workers assigned to task 1 is greater for the poor black workers, and lower for both the white, as well as the CLB workers.*

(b) *Let A3 and A4 hold, and suppose that a PNCAA equilibrium exists. Compared to the PNAA equilibrium, under the PNCAA equilibrium, the proportion of workers assigned to task 1 is lower for the CLB workers. Further, the interest of the whites and the poor black workers are opposed in the sense that if $\rho_2^{**} \leq \rho_2^*$ (respectively $\rho_2^{**} \geq \rho_2^*$), then $\rho_W^{**} \geq \rho_W^*$ (respectively $\rho_W^{**} \leq \rho_W^*$).*

Proof. (a) To begin with note that $s_1^{**} > s_1^*$ (this follows as $EE_2(\gamma)$ shifts down under AA, and shifts up under CAA). This implies that $\rho_1^* > \rho_1^{**}$ (from assumption A2). We next argue that $\rho_2^{**} > \rho_2^*$. Suppose to the contrary that $\rho_2^{**} \leq \rho_2^*$. This implies that $s_2^{**} \geq s_2^*$ (from assumption A2), so that $\gamma^* \geq \gamma^{**}$. This then implies that $\rho_W^{**} \geq \rho_W^*$. Next, given that $\rho_2^{**} = \rho_1^{**} = \rho_W^{**}$, we have that

$$\rho_W^{**} = \mu_1 \rho_1^{**} + \mu_2 \rho_2^{**}.$$

Thus,

$$\rho_W^* \leq \rho_W^{**} = \mu_1 \rho_1^{**} + \mu_2 \rho_2^{**} < \mu_1 \rho_1^* + \mu_2 \rho_2^*,$$

which contradicts the affirmative action constraint under AA. Hence, $\rho_2^{**} > \rho_2^*$. This in turn implies that $\gamma^* < \gamma^{**}$, so that from Observation 1 and A2, $\rho_W^{**} < \rho_W^*$.

(b) Mimicing the argument in case (a), note that $s_1^{**} > s_1^*$. This implies that $\rho_1^* > \rho_1^{**}$ since $\hat{\rho}(s)$ is negatively sloped over the range $(0, \underline{s})$ for both a PNAA, and a PNCAA equilibrium.

Next suppose that $\rho_2^{**} \leq \rho_2^*$. This implies that $s_2^{**} \geq s_2^*$ (since $\hat{\rho}(s)$ is negatively sloped over the range $(0, \underline{s})$ for both a PNAA and a PNCAA equilibrium), so that $\gamma^* \geq \gamma^{**}$. This then implies that $\rho_W^{**} \leq \rho_W^*$. A similar argument establishes that if $\rho_2^{**} \geq \rho_2^*$, then $\rho_W^{**} \leq \rho_W^*$. □

Intuitively, under class based affirmative action, the CLB workers no longer belong to the target group, unlike under identity based AA. Consequently, these workers face relatively higher standards and are worse off under IAA. In the absence of stereo-typing, this implies that the poor blacks would be better off as well. Otherwise, the shadow price of equality must be lower when AA is based on income, rather than on identity. This in turn would imply that the standards for the whites would be higher under identity-based AA, so that they would be worse off. This, however, violates the identity-based AA constraint, as both CLBs and PBs cannot be better off under identity-based AA, while the whites are worse off. Thus, CAA increases the shadow price of equality, making PBs better off, and whites worse off. Under patronization, if the poor blacks are worse off (respectively better-off) under CAA, then the shadow price of equality must be lower (respectively higher), so that the whites must be better off (respectively worse off).

8 Targeted Education Subsidy

We then examine policies aimed at redressing credit market imperfections. On the one hand, government policy may seek to directly improve the efficiency of the credit market. This can be formalised as a decline in m to m', $m' < m$. On the other hand, the government can seek to provide targeted education subsidies to the poor blacks. Most Canadian universities, for example, have exclusive scholarships earmarked for students of Aboriginal background. Similarly, New Zealand offers scholarships targeted at poor Maoris (Commission for Racial Equality, September 2006), while in Israel, blacks receive state-sponsored university tuition. Such targeted subsidies can be formalised as a reduction of c to, say αc, where $\alpha < 1$. Formally speaking these two policies are qualitatively similar, in that they both lead to an upward shifting of the WW_2 curve. Thus, for conciseness, we shall formalise both alternative policies as a decline in m.

We consider a decline in m to m', and let the associated Lagrange multiplier be $\gamma^*(m')$. Further, let $\rho_i^*(\bar{m}, \bar{\gamma})$ solve (10), (11), (12), (15), (16) and (17), for $\bar{m} \in \{m, m'\}$ and $\bar{\gamma} \in \{\gamma^*, \gamma^*(m')\}$.

Interestingly we find that such a subsidy necessarily makes the CLB workers worse off. The PB workers are better off in the absence of stereo-typing, and even under patronisation if the subsidy is large enough. The white workers are better off in the absence of stereo-typing, but worse off under a patronising equilibrium.

Proposition 9. *Suppose the government provides an educational subsidy to the poor black workers, so that m declines to m'.*

(a) Let A1-A2 hold. Further, suppose that an NSTNAA equilibrium exists. Then, an education subsidy increases the proportion of white and poor black workers assigned to task 1, and decreases the proportion of CLB workers assigned to task 1.

(b) Let A1, A3, A4 and A5 hold. Further, suppose that a patronizing equilibrium PNAA $(s_W^*, \pi_W^*, \rho_W^*; s_1^*, \pi_1^*, \rho_1^*; s_2^*, \pi_2^*, \rho_2^*; i_c^*)$ exists, where $s_1^* < s_2^* < \underline{s} < s_W^* < \bar{s}$. Then an education subsidy decreases the proportion of white and CLB workers assigned to task 1. If the education subsidy is sufficiently large, then the PBs are also better off.

Proof. (a) We first argue that $\gamma^*(m') < \gamma^*(m)$. Suppose not, so that $\gamma^*(m') \geq \gamma^*(m)$. Next note that $WW_2(m)$ lies below $WW_2(m')$, so that given Observation 1 and Assumption 3, we have that $\rho_2^*(m', \gamma^*(m)) > \rho_2^*(m, \gamma^*(m))$. Further, $\rho_W^*(m', \gamma^*(m)) = \rho_W^*(m, \gamma^*(m))$, and $\rho_1^*(m', \gamma^*(m)) = \rho_1^*(m, \gamma^*(m))$. Hence given the affirmative action constraint under m, we have that

$$\begin{aligned}\rho_W^*(m', \gamma^*(m)) &= \rho_W^*(m, \gamma^*(m)) = \mu_1 \rho_1^*(m, \gamma^*(m)) + \mu_2 \rho_2^*(m, \gamma^*(m)) \\ &< \mu_1 \rho_1^*(m', \gamma^*(m)) + \mu_2 \rho_2^*(m', \gamma^*(m)). \qquad (27)\end{aligned}$$

Next from Observation 1 and the fact that $\gamma^*(m') \geq \gamma^*(m)$, it follows that $s_W^*(m', \gamma^*(m')) \geq s_W^*(m, \gamma^*(m))$ and $s_i^*(m', \gamma^*(m')) \leq s_i^*(m, \gamma^*(m))$, $i = 1, 2$. Hence from Assumption 3, one has that $\rho_W^*(m', \gamma^*(m')) \leq s_W^*(m, \gamma^*(m))$ and $\rho_i^*(m', \gamma^*(m')) \geq \rho_i^*(m, \gamma^*(m))$, $i = 1, 2$. This, in conjunction with (27), yields

$$\begin{aligned}\rho_W^*(m', \gamma^*(m')) \leq \rho_W^*(m', \gamma^*(m)) &< \mu_1 \rho_1^*(m', \gamma^*(m)) + \mu_2 \rho_2^*(m', \gamma^*(m)) \\ &\leq \mu_1 \rho_1^*(m', \gamma^*(m')) + \mu_2 \rho_2^*(m', \gamma^*(m')),\end{aligned} \tag{28}$$

which violates the affirmative action constraint under m'. Therefore, it must be that $\gamma^*(m') < \gamma^*(m)$. Hence given Observation 1, $s_W^*(m', \gamma^*(m')) < s_W^*(m, \gamma^*(m))$ and $s_1^*(m', \gamma^*(m')) > s_1^*(m, \gamma^*(m))$. Hence from Assumption 3, one has that $\rho_W^*(m', \gamma^*(m')) > \rho_W^*(m, \gamma^*(m))$ and $\rho_1^*(m', \gamma^*(m')) < \rho_1^*(m, \gamma^*(m))$. Finally, the fact that $\rho_W^*(m', \gamma^*(m')) > \rho_W^*(m, \gamma^*(m))$ and $\rho_1^*(m', \gamma^*(m')) < \rho_1^*(m, \gamma^*(m))$ implies, in conjunction with the AA constraint, that $\rho_2^*(m', \gamma^*(m')) > \rho_2^*(m, \gamma^*(m))$.

(b) Arguing as before, it follows that $\rho_2^*(m', \gamma^*(m)) > \rho_2^*(m, \gamma^*(m))$, so that (27) holds, i.e. $\lambda_B \rho_W^*(m', \gamma^*(m)) < \lambda_1 \rho_1^*(m', \gamma^*(m)) + \lambda_2 \rho_2^*(m', \gamma^*(m))$. Given Lemma 1, we know that a change in γ has a relatively greater effect on $|\hat{\rho}_i'(s)|$, $i = 1, 2$ as compared to on $|\hat{\rho}_W'(s)|$. Hence respecting the affirmative action constraint for m', entails that $\gamma^*(m') < \gamma^*(m)$. Otherwise, while both the RHS and the LHS of (27) will increase (given that $s_1^* < s_2^* < \underline{s} < s_W^* < \bar{s}$ and Assumption 4), from Lemma 1 the increase in the RHS will dominate that in the LHS, so that the AA constraint for m' will be violated. Hence it must be that $\gamma^*(m') < \gamma^*(m)$. Consequently from Observation 1, s_W^* decreases, and s_1^* increases. Finally, given that $s_1^* < \underline{s} < s_W^* < \bar{s}$ and Assumption 4, ρ_1^* and ρ_W^* both decrease, so that the white and CLB workers have a lower probability of getting assigned to task 1.

For PB workers, the direct effect of a fall in m tends to make them better off, while the general equilibrium effect arising out of the AA constraint and the resultant fall in γ has the opposite effect. Define $s_2(m')$ to be the minimum s_2', where (s_2', π_2') is at the intersection of $WW_2(m')$ and EE. Let the educational subsidy be large in the sense that $s_2(m') < s_2^*$. Given that $s_2^*(m') \leq s_2(m')$, it follows that $s_2^*(m') < s_2^*$. Hence, $\rho_2^*(m') > \rho^*$. □

The intuitive explanation is similar to that for the motivation case. An educational subsidy (or equivalently an improvement in credit market conditions) reduces the effective cost of education for PBs, pushing up their acquisition of skills, and prompting employers to lower the standards required of them. This, in turn, makes them better off and reduces the shadow price of equality, with the same consequences that incorporating the motivation effect had.

9 Conclusion

We build on Coate and Loury (1993) to analyze a model with statistical discrimination where the target group exhibits heterogeneity in income. We find that multiple equilibria may exist, and derive conditions such that the equilibrium exhibits stereotyping, and even patronization (in the sense that employers patronize the target group by holding them to lower standards compared to others).

We then use this framework to examine a set of questions that are of importance to the countries practicing AA. We begin by examining the effect of a demographic shift whereby some poorer members of the target group graduate to the creamy layer. We find that such demographic shifts are bad for CLBs, in the sense that the proportion of CLBs assigned to the skilled job decreases. More surprisingly, even members of the target group, i.e. the PBs, are worse off as a result. Whites benefit, however, unless the equilibrium is a patronizing one, in which case they also become worse off.

We then examine a motivation/role model effect, so that the PBs become more motivated to acquire the necessary skills if the CLBs become more 'successful' vis-a-vis the whites. Interestingly, while the presence of such a motivation effect does make the poor blacks better off (provided the motivation effect is sufficiently strong), it is not an unmixed blessing for the other groups. It not only makes the CLB workers worse off, the whites may also be worse off if the equilibrium is a patronising one.

Turning to the impact of moving to class-based AA, we find that such a transition would be politically divisive, as it would be opposed by at least two of the three groups, with the CLBs always being worse off as a result. If there is no patronization in equilibrium, then such a transition would make the PBs better off, but hurt both the whites and the CLBs. Finally, we show that educational subsidies that target the PBs would hurt both the whites and the CLBs in a patronizing equilibrium, lowering the rate at which they are assigned to skilled jobs.

10 Appendix

Proof of Proposition 2. (a) Define $\hat{s}_i$ as solving

$$\hat{s}_i(\gamma) = \{\min s_i | s_i \text{ lies at the intersection of } \mathrm{EE_i}(\gamma) \text{ and } \mathrm{WW_i}\}.$$

Note that $\hat{s}_1(0) = \hat{s}_W(0)(= \bar{s}_1 = \bar{s}_W)$. Further, for all $r > \tilde{r}$, it is the case that $\hat{s}_i(0) < \tilde{s}$, $i = 1, W$. Further, from continuity, for all $r > \tilde{r}$ there exists $\tilde{m}(r)$ (> 1) such that $\hat{s}_2(0) = \bar{s}_2 < \tilde{s}$ for all m satisfying $1 < m < \tilde{m}(r)$. Note that $\bar{s}_1 = \bar{s}_W < \bar{s}_2$, so that from A2 one has that $\hat{\rho}(\bar{s}_1) > \hat{\rho}(\bar{s}_2)$. Consider any $r > \tilde{r}$, and m such that $1 < m < \tilde{m}(r)$. To show that there exists $\gamma^*(r, m) > 0$, such that $\hat{s}_i(\gamma^*)$ satisfies AA.

Let γ increase from zero. Note that $\hat{s}_i(\gamma)$ is continuous, with $\hat{s}_1(\gamma), \hat{s}_2(\gamma)$ being decreasing, and $\hat{s}_W(\gamma)$ being increasing in γ. Next define

$$D(\gamma) = \mu_1 \hat{\rho}(\hat{s}_1(\gamma)) + \mu_2 \hat{\rho}(\hat{s}_2(\gamma)) - \hat{\rho}(\hat{s}_W(\gamma)).$$

Note that

$$\begin{aligned} D(0) &= \mu_1 \hat{\rho}(\hat{s}_1(0)) + \mu_2 \hat{\rho}(\hat{s}_2(0)) - \hat{\rho}(\hat{s}_W(0)) \\ &= \mu_1 \hat{\rho}(\bar{s}_1) + \mu_2 \hat{\rho}(\bar{s}_2) - \hat{\rho}(\bar{s}_W) \\ &= \mu_2 \hat{\rho}(\bar{s}_2) - (1 - \mu_1)\hat{\rho}(\bar{s}_1) \text{ (since } \bar{s}_1 = \bar{s}_W) \\ &= \mu_2 (\hat{\rho}(\bar{s}_2) - \hat{\rho}(\bar{s}_1)) < 0, \end{aligned}$$

where the last inequality follows from assumption A2 and the fact that $\bar{s}_1 = \bar{s}_W < \bar{s}_2$. Further, as $\gamma \to \lambda_B x_u$, for $i = 1, 2$, $\hat{s}_i(\gamma)$ goes to zero (from A1), so that $\hat{\rho}(s_i)$ goes to 1, and consequently $[\mu_1 \hat{\rho}(\hat{s}_1(\gamma)) + \mu_2 \hat{\rho}(\hat{s}_2(\gamma))]|_{\gamma \to \lambda_B x_u} = 1$. Given that $\hat{\rho}(\hat{s}_W(\lambda_B x_u) < 1$, we have that $D(\gamma)|_{\gamma \to \lambda_B x_u} > 0$. Consequently, given that $D(0) < 0$, from the continuity of $D(\gamma)$ there exists $\gamma^* > 0$ such that $D(\gamma^*) = 0$.

(b). For all $r < \tilde{r}$, it is the case that $\hat{s}_1(0) = \hat{s}_W(0) > \tilde{s}$. Further, from continuity, for all $r < \tilde{r}$, there exists $\bar{m}(r)$ such that $\hat{s}_2(0) > \tilde{s}$ for all m satisfying $m < \bar{m}(r)$. Next note that there exists $\bar{\lambda}_B < 1$, such that for all $\lambda_B > \bar{\lambda}_B$, $EE_i(\gamma)$ and WW_i, $i = 1, 2$ intersect for all $\gamma \leq \lambda_W x_q$.[13] Consider any $r < \tilde{r}$, and m such that $1 < m < \tilde{m}(r)$. To show that for $\lambda_B > \bar{\lambda}_B(m, r)$, there exists $\gamma^*(r, m, \lambda_W) > 0$, such that $\hat{s}_i(\gamma^*)$ satisfies AA.

Define $D(\gamma)$ as before. As before $D(0) < 0$. Next let γ increase from zero. Next note that as γ goes to $\lambda_W x_q$, $\hat{s}_W(\gamma)$ goes to 1, so that $\hat{\rho}(\hat{s}_W(\gamma))$ goes to zero. Hence $D(\gamma)|_{\gamma \to \lambda_W x_q} > 0$ (since $\hat{\rho}(\hat{s}_W(\gamma))$ goes to 0, whereas $\mu_1 \hat{\rho}(\hat{s}_1(\gamma)) + \mu_2 \hat{\rho}(\tilde{s}_2(\gamma))$ is bounded away from 0). Thus there exists γ^* such that $D(\gamma^*) = 0$.

(c) We first establish that there can be at most one NSTNAA equilibrium. Consider an NSTNAA equilibrium $(s_W^*, \pi_W^*, \rho_W^*; s_1^*, \pi_1^*, \rho_1^*; s_2^*, \pi_2^*, \rho_2^*; i_c^*)$, with an associated Lagrange multiplier of γ^*. Note that the AA constraint implies that in the NSTNAA equilibrium $\rho_W^* = \mu_1 \rho_1^* + \mu_2 \rho_2^*$. Any other NSTNAA equilibrium must involve a different γ', where without loss of generality let $\gamma' > \gamma^*$. Then from Observation 1, the AA constraint cannot be satisfied.

Consider a potential PNAA equilibrium $(s_W^*, \pi_W^*, \rho_W^*; s_1^*, \pi_1^*, \rho_1^*; s_2^*, \pi_2^*, \rho_2^*; i_c^*)$. This involves $s_1^*, s_2^* < s_W^*$. From A2, this implies that $\rho_i^* > \rho_W^*, i = 1, 2$. Hence the AA constraint cannot be satisfied. $\square$

Proof of Lemma 1. Totally differentiating the WW and EE_W curves, with respect to s_W, π_W, and γ, and using Cramer's rule, we obtain

$$\frac{d\tilde{s}_W(\gamma)}{d\gamma} = \frac{Pr(\tilde{s}_W)}{(1 - \tilde{\pi}_W) f_u(\tilde{s}_W)(\lambda_W x_u + \gamma)\theta_W(\tilde{s}_W)}. \tag{29}$$

Note that by A1, $\frac{f_q'}{f_q} > \frac{f_u'}{f_u}$. Moreover, it can be shown that $\hat{\rho}'(s_W) > 0$ implies that $\beta'(s_W) > 0$.[14] Therefore, $\theta_W > 0$. Total differentiation of the EE_1 and WW curves yields, following a similar method,

$$\frac{d\tilde{s}_1}{d\gamma} = -\frac{Pr(\tilde{s}_1)}{(1 - \tilde{\pi}_1) f_u(\tilde{s}_1)(\lambda_B x_u - \gamma)\theta_1(\tilde{s}_1)} < 0. \tag{30}$$

Moreover, since $\tilde{s}_1 < \tilde{s}_W$, the inverted U shape of the WW curve and the fact that $\beta'(\tilde{s}_W) >$

[13] Note that as λ_B goes to 1, $\frac{x_q + \lambda_W x_q / \lambda_B}{x_u - \lambda_W x_q / \lambda_B}$ goes to x_q / x_u, so that $EE_i(\gamma)$ and WW_i intersect for $\gamma = \lambda_W x_q$ (since $EE_i(0)$ and WW_i intersect). Next given that EE_i, $i = 1, 2$ shifts outwards for an increase in γ, $EE_i(\gamma)$ and WW_i also intersect for all $\gamma < \lambda_W x_q$.

[14] We have $\beta(s) \equiv w(F_u(s) - F_q(s))$. Normalize $w = 1$ for simplicity. Recall that $\hat{\rho}(s) \equiv \rho(s, G(\beta(s))) \equiv G(\beta(s))[1 - F_q(s)] + [1 - G(\beta(s))][1 - F_u(s)]$. Hence differentiating with respect to s, one has $\hat{\rho}'(s) = g(\beta(s))\beta'(s)[1 - F_q(s)] - f_q(s)G(\beta(s)) - g(\beta(s))\beta'(s)[1 - F_u(s)] - f_u(s)[1 - G(\beta(s)] = g(\beta(s))\beta'(s)\beta(s) - f_q(s)G(\beta(s)) - f_u(s)[1 - G(\beta(s))]$. Note that in the last expression, the second and third terms are unambiguously negative as $G(\beta(s))$ lies between 0 and 1 and $f(s)$ is non-negative. Therefore, a necessary condition for $\hat{\rho}'(s)$ to be positive is that the first term in this expression be positive, that is, we must have $g(\beta(s))\beta'(s)\beta(s) > 0$. As $g(\beta(s))$ and $\beta(s)$ are necessarily non-negative, this implies that we must have $\beta'(s) > 0$ for $\hat{\rho}'(s) > 0$.

0, together imply that $\beta'(\tilde{s}_1) > 0$. Hence $\theta_1 > 0$.

Now (29) and (30) imply, by cross multiplication, that $\frac{d\tilde{s}_W}{d\gamma} < |\frac{d\tilde{s}_1}{d\gamma}|$ if and only if

$$\lambda_W > \frac{(1-\tilde{\pi}_1(\tilde{s}_1))f_u(\tilde{s}_1)Pr(\tilde{s}_W)\theta_1(\tilde{s}_1)}{(1-\tilde{\pi}_1(\tilde{s}_1))f_u(\tilde{s}_1)Pr(\tilde{s}_W)\theta_1(\tilde{s}_1)+(1-\tilde{\pi}_W(\tilde{s}_W))f_u(\tilde{s}_W)Pr(\tilde{s}_1)\theta_W(\tilde{s}_W)} - \frac{\gamma}{x_u}. \tag{31}$$

Since $\gamma > 0$, (31) follows from A4(a).

Similarly, total differentiation of the EE_2 and WW_2 curves yields

$$\frac{d\tilde{s}_2}{d\gamma} = -\frac{Pr(\tilde{s}_2)}{(1-\tilde{\pi}_2(\tilde{s}_2))f_u(\tilde{s}_2)(\lambda_B x_u - \gamma)\kappa(\tilde{s}_2)}, \tag{32}$$

where $\kappa(\tilde{s}_2) = \frac{f'_q(\tilde{s}_2)}{f_q(\tilde{s}_2)} - \frac{f'_u(\tilde{s}_2)}{f_u(\tilde{s}_2)} + \frac{g(.)\beta'(\tilde{s}_2)}{m\tilde{\pi}_2(1-\tilde{\pi}_2)}$. Moreover, (29) and (32) imply, by cross multiplication, that $\frac{d\tilde{s}_W}{d\gamma} < |\frac{d\tilde{s}_2}{d\gamma}|$ if and only if

$$\lambda_W > \frac{(1-\tilde{\pi}_2)f_u(\tilde{s}_2)Pr(\tilde{s}_W)\kappa(\tilde{s}_2)}{(1-\tilde{\pi}_2)f_u(\tilde{s}_2)Pr(\tilde{s}_W)\kappa + (1-\tilde{\pi}_W)f_u(\tilde{s}_W)Pr(\tilde{s}_2)\theta_W} - \frac{\gamma}{x_u}. \tag{33}$$

Moreover, given $\gamma > 0$, and that $\theta_2(\tilde{s}_2) > \kappa(\tilde{s}_2)$ (since $m > 1$), assumption A4(a) is sufficient for (33) to hold. Summarising the preceding arguments, we have

$$|\frac{d\tilde{s}_W(\gamma)}{d\gamma}| < |\frac{d\tilde{s}_i(\gamma)}{d\gamma}|, \quad i = 1, 2. \tag{34}$$

Next note that in any PNAA, we have $\hat{\rho}(s^*_W(\gamma)) > \hat{\rho}(s^*_2(\gamma)) = \hat{\rho}(s') = \hat{\rho}(\hat{s})$ where the inequality follows from Proposition 5(c), and the equalities follow from the definition of $\hat{s}$, where $\underline{s} < \hat{s} < \bar{s}$. Since $\hat{\rho}(s_W)$ is increasing in s_W, this implies that $s^*_W(\gamma) > \hat{s}$. Consequently, $\tilde{s}_W(\gamma) > \hat{s}$, for all $\gamma > \gamma^*$. Moreover, $\tilde{s}_2(\gamma)$ is decreasing in γ. Hence for any $\gamma > 0$, $\tilde{s}_2(\gamma)$ does not exceed s' (since $s' \equiv \tilde{s}_2(0)$). Since $\tilde{s}_1(\gamma)$ never exceeds $\tilde{s}_2(\gamma)$, for any γ, $\tilde{s}_1(\gamma)$ does not exceed s' either. Thus A4(b) immediately yields $|\frac{d\hat{\rho}(s_W)}{ds_W}| < |\frac{d\hat{\rho}(s_i)}{ds_i}|$, $i = 1, 2$. Combining this with (34), we obtain the result. □

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Figure 1: NSTN and STN equilibria

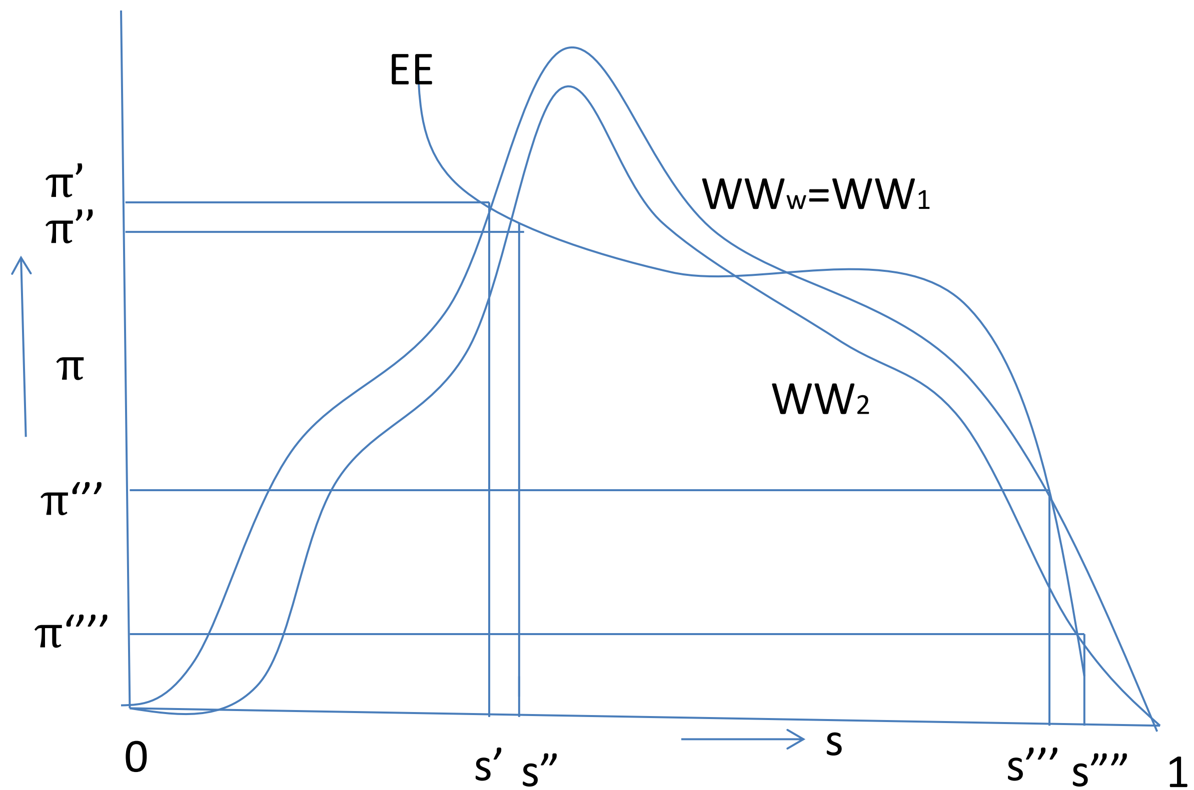

Figure 2:NSTN with AA