Asymptotics and simulation for heavy-tailed processes

SPEAKER: **Henrik Hult**, *KTH Royal Institute of Technology*
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Abstract

In this short course we will cover the following topics.

We shall start with an introduction to regular varying distributions for random variables and vectors. A key ingredient in the analysis of extreme events for heavy-tailed models is the *one big jump heuristic* which basically says that the sum of independent and identically distributed random variables is large most likely because precisely one of the variables is large. Fundamental results on the *one big jump heuristic* and Breiman's lemma will be covered. The content is partly based on [5, 4, 9, 10].

Next, we proceed to formulate regular variation on more general spaces and derive large deviations for heavy-tailed stochastic processes. I will present a survey of large deviations results for regularly varying processes and also make a connection to the weak convergence approach to large deviations. The content is partly based on [5, 6, 3, 8, 7].

The theory of large deviations is useful to obtain asymptotic approximations of tail probabilities, but approximations are sometimes crude. Simulation may lead to more accurate approximations but may be computationally costly. Variance reduction techniques, such as importance sampling, may significantly reduce the computational cost over standard Monte Carlo. Large deviations theory is crucial to design efficient simulation algorithms and this connection will be illustrated in the setting of a random walk with regularly varying steps. The content is partly based on [1].

Finally, we will present some recent work on designing efficient simulation algorithms for heavy-tailed random walks through Markov chain Monte Carlo. The basic idea is to sample from the conditional distribution of the random walk, given that the rare event occurs. This conditional distribution has the probability of the rare event as its normalizing constant and the goal is to estimate the normalizing constant from the sample. It seems that very efficient algorithms can be constructed in this way. The content is based on [2].

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Extremes of stationary sequences: clusters and spectral processes

SPEAKER: **Johan Segers**, *Université catholique de Louvain, Belgium*
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Abstract

Extreme-value asymptotics of a stationary sequence of random variables are governed by both the global and the local dependence structure of the sequence. In case of weak dependence, the sequence may be divided into approximately independent and identically distributed blocks. As in the case of independent random variables, this division into blocks induces a Poisson character for the occurrence of extremes. However, if the dependence within blocks is left unrestricted, extremes may arrive in batches or clusters, yielding compound Poisson limits. The distinctive feature of extremes of weakly dependent stationary sequences is therefore the appearance of clusters of extremes. The distribution of such clusters can be described via cluster functionals or cluster processes.

The asymptotic distribution of such clusters is fully determined by the conditional distribution of the process given that at a specific time point, an extreme value is produced. The limiting conditional distribution of the original sequence given such an event is called the tail process. The auto-normalized version of the tail process is called the spectral process. The spectral process plays for stationary processes the same role as the spectral measure in the context of regularly varying random vectors. Originating from a stationary process, the spectral process satisfies a remarkable system of equations that express the effect of a time shift.

For particular time series models, the spectral process readily reveals the underlying mechanisms of extremal dependence. For univariate regularly varying Markov chains, the spectral process is a multiplicative random walk, the increment being determined by the asymptotics of the Markov kernel at a large initial value. By stationarity, the forward and backward increments are linked through an adjoint relation between probability measures. For multivariate regularly varying Markov chains, the spectral process is still Markovian with a similar adjoint relation relating the forward and backward kernels. For infinite-order moving averages, the single-shock heuristic suggests that extremes are dominated by a single large innovation. This phenomenon is cristallized in the spectral process, being a discrete mixture over all the possible extreme scenarios.

On Adam Jakubowski's approach to proving asymptotic results for regularly varying sequences

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Abstract

In recent work [1, 4], an idea of Adam Jakubowski [2, 3] was used to prove infinite stable limit theory and precise large deviation results for sums of strictly stationary regularly varying sequences. The idea of Jakubowski consists of approximating tail probabilities of distributions for such sums with increasing index by the corresponding quantities for sums with fixed index. This idea can also be made to work for Laplace functionals of point processes, the distribution function of maxima and the characteristic functions of partial sums of stationary sequences. In each of these situations, extremal dependence manifests in the appearance of suitable cluster indices (extremal index for maxima, cluster index for sums,...). The proposed method can be easily understood and has the potential to function as heuristics for proving limit results for weakly dependent heavy-tailed sequences.

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Heavy tailed branching process with immigration

SPEAKER: **Zbigniew Palmowski** , *University of Wrocław, Poland*
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Abstract

In this talk we will analyze a branching process with immigration defined recursively by $X_t = \theta_t \circ X_{t-1} + B_t$ for a sequence (B_t) of i.i.d. random variables and random mappings $\theta_t \circ x := \theta_t(x) = \sum_{i=1}^{x} A_i^{(t)}$, with $(A_i^{(t)})_{i\in\mathbb{N}_0}$ being a sequence of $\mathbb{N}_0$-valued i.i.d. random variables independent of B_t. We assume that one of generic variables A and B has a regularly varying tail distribution. We identify the tail behaviour of the distribution of the stationary solution X_t. We also prove CLT for the partial sums that could be further generalized to FCLT. Finally, we also show that partial maxima have a Fréchet limiting distribution.

Regular variation for measures on general spaces

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CO-AUTHOR: H. Hult, *KTH Royal Institute of Technology, Sweden.*

Abstract

The analysis of rare event probabilities for random vectors and stochastic processes whose marginal distributions have Pareto-like tails naturally leads to regular variation for probability distributions. In this setting, regular variation bears a similarity to weak convergence of probability distributions and many useful results in the weak convergence theory have useful analogs for regular variation. The use of established convergence concepts, similar to weak convergence, for infinite measures requires unnatural modifications of the space in various ways in order to get the right topology. I will consider very concrete examples that illustrate the problems that appear and propose a more natural definition of regular variation that is valid on rather general metric spaces.

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Maxima of bivariate triangular arrays: Between asymptotic complete dependence and asymptotic independence lies the conditional extreme value model

SPEAKER: **Joyjit Roy**, *Cornell University, USA*
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CO-AUTHOR: Sidney I. Resnick, *Cornell University, USA.*

Abstract

In this talk, we propose a generalization of Hüsler and Reiss' intriguing result from 1989 and look at possible limit distributions of scaled and centered componentwise maxima of bivariate triangular arrays. As an aside, we also give new proofs and a corrected version of previous extensions of the result to the case of spherically symmetric random variables. We will also briefly talk about statistical applications of our results.

The Art of Seeking Hidden Risks

SPEAKER: **Sidney Resnick**, *Cornell University, Ithaca, NY, USA*
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Abstract

Assessing tail risks using the asymptotic models provided by multivariate extreme value theory has the danger that when asymptotic independence is present (as with the Gaussian copula model), the asymptotic model provides estimates of probabilities of joint tail regions that are zero. In diverse applications such as finance, telecommunications, insurance and environmental science, it may be difficult to believe in the absence of risk contagion. This problem can be partly ameliorated by using hidden regular variation which assumes a lower order asymptotic behavior on a subcone of the state space and this theory can be made more flexible by extensions in the following directions: (i) higher dimensions than two; (ii) where the lower order variation on a subcone is of extreme value type different from regular variation; and (iii) where the concept is extended to searching for lower order behavior on the complement of the support of the limit measure of regular variation. We discuss some challenges and potential applications and a flexible framework for regular variation on cones in a metric space initiated by [2] and utilized by [1].

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Extreme eigenvalues of random matrices

SPEAKER: **Arup Bose**, *Indian Statistical Institute Kolkata*
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Abstract

There is a substantial body of literature on the properties of extreme eigenvalues of large dimensional random matrices such as the Wigner matrix, sample covariance matrix, circulant matrix and the Toeplitz matrix. Compared to the behaviour of extreme values of iid observations, the almost sure behaviour and limit laws for extreme eigenvalues are much more complicated to derive and often the limits are not known in any explicit form.

We review some of the existing literature in this area and also mention a few interesting open issues.

Limit theory for the largest eigenvalues of sample covariance matrices with heavy-tails

SPEAKER: **Richard A. Davis**, *Columbia University*
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CO-AUTHOR(S): Oliver Pfaffel, Robert Stelzer

Abstract

We consider the joint limit distribution of the k largest eigenvalues of a sample covariance matrix based on a large random matrix X. The rows of X are given by independent copies of a linear process whose noise have regularly varying tails with index α in $(0, 4)$. It is shown that a point process based on the eigenvalues of the sample covariance matrix converges in distribution to a Poisson point process with intensity measure depending on the tail index α and the sum of squares of the linear coefficients in the filter. This result is extended to random coefficient models where the coefficients of the linear processes are a functions of an ergodic sequence. This setup allows for limited dependence between the rows. We also discuss the extension of these limiting results to nonlinear time series models including stochastic volatility and GARCH processes.

Extreme eigenvalues of random matrices with dependent entries

SPEAKER: **Rajat Subhra Hazra**, *University of Zürich, Switzerland*
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CO-AUTHOR(S): Arijit Chakrabarty, Bikramjit Das and Souvik Ghosh.

Abstract

In this talk we discuss extreme eigenvalues of two important symmetric matrices, Wigner and Sample Autocovariance. The behavior of extreme eigenvalues of Wigner matrices with light tailed and heavy tailed entries is well known. We present some results on extreme eigenvalues of Wigner matrices, when the input sequence is dependent and heavy tailed. The light tailed counterpart turns out to be much difficult to answer.

Sample Autocovariance matrix of a linear process can be thought of as a random Toeplitz matrix with dependent entries. Although there are some interesting results known for Toeplitz matrix with i.i.d. entries but the fluctuations of extreme eigenvalues of is still an open problem. We take an alternative route and derive a new representation of autocovariance matrix. This helps us to derive some rates of convergence of extreme eigenvalues using standard extreme value theory.

Model Uncertainty and Risk Aggregation

Speaker: **Paul Embrechts**, *ETH Zurich*
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Abstract

In this talk I will present a numerical algorithm for the calculation of inf-, and sup-bounds for risk measures of financial positions when only marginal information is available. As key example I will discuss the Value-at-Risk for the sum of one-period loss positions. The latter is motivated by the quantitative modelling of Operational Risk under Basel 2 and 3.

There is a VaR beyond usual approximations

Speaker: **Marie Kratz**, *ESSEC Business School Paris*
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Abstract

A normal approximation is often chosen in practice for the unknown distribution of the yearly log returns of financial assets, justified by the use of the CLT, when assuming aggregation of iid observations in the portfolio model. Such a choice of modeling, in particular using light tail distributions, has proven during the crisis of 2008/2009 to be an inadequate approximation when dealing with risk measures; as a consequence, it leads to a gross underestimation of the risks. The main objective of our study is to obtain the most accurate evaluations of risk measures when working on financial data under the presence of heavy tail and to provide practical solutions for accurately estimating high quantiles of aggregated risks. We explore new approaches to handle this problem, numerically as well as theoretically, based on properties of upper order statistics. We compare them with existing methods, for instance with one based on the Generalized Central Limit Theorem.

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Some asymptotic results for ruin probability in the presence of heavy-tailed claims

SPEAKER: **Corina Constantinescu**, *Institute for Financial and Actuarial Mathematics, University of Liverpool, UK*
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Abstract

In this talk we analyse the asymptotic behaviour of the ruin probability in a few insurance risk models for which the incoming cash flow (premiums, investment returns) is not of constant rate and the outgoing cash flow (claims) is modelled by heavy-tailed distributions.

Weighted Sums of Regularly Varying Random Variables with Dependent Weights

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Abstract

We study the tail behaviour of weighted sums of the form $\sum_{t=1}^{\infty} X_t \prod_{j=1}^{t} Y_j$, where $\{(X_t, Y_t)\}$ is an i.i.d. sequence following the bivariate Sarmanov distribution. We further assume that X_t has regularly varying tail. Under conditions similar to those used by Denisov and Zwart (2007) and modified by Hazra and Maulik (2012), we obtain the tail distribution of the random variables $\sup_{1 \leq k \leq n} \sum_{t=1}^{k} X_t \prod_{j=1}^{t} Y_j$ and $\sup_{n \geq 1} \sum_{t=1}^{n} X_t \prod_{j=1}^{t} Y_j$.

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Extremal serial dependence of time series

SPEAKER: **Holger Drees**, *University of Hamburg, Germany*
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Abstract

Modeling the dependence between consecutive observations in a time series plays a crucial role in risk management. For example, the risk of large losses from a financial investment is increased if extreme negative returns tend to occur in clusters, and heavy rainfall on several consecutive days could trigger a catastrophic flooding.

We will recall the so-called *coefficient of tail dependence* (introduced in [1]) as an important measure of the strength of serial dependence between extremes which allows for a refined characterization of dependence structures, especially in the case of asymptotic independence. A general class of empirical processes discussed in [2] enables us to analyze the asymptotic behavior of estimators of the coefficient of tail dependence in a unified framework. Bootstrap versions of these empirical processes yield asymptotic confidence intervals.

In an application it is shown how to use these results to discriminate between time series of GARCH-type and time series from common stochastic volatility models. An analysis of a time series of returns of the German blue stocks index however reveals that probably none of these time series models describe the extremal dependence structure accurately.

In an accompanying talk by Anja Janßen a new type of stochastic volatility models is introduced that allows for a greater flexibility in the modeling of tail dependence.

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Asymptotic Independence of Stochastic Volatility Models

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Abstract

Discrete-time stochastic volatility (SV) models have become a standard tool for the modeling of economic time series as they are able to reflect many of the well-known stylized facts of financial markets. With regard to their extremal behavior, the standard SV model specifications have in common that consecutive observations are asymptotically independent and thus their extremal index is equal to 1.

However, on a pre-asymptotic level SV models may still show a clustering of large values and we are therefore interested in the second order behavior of extremal dependence. Different concepts may be applied which allow for a finer analysis of asymptotic independence: See, in particular, the coefficient of tail dependence proposed by [1] and the notion of hidden regular variation, cf. [2]. However, the standard model specifications for SV models do not reflect the broad spectrum of possible second order behavior since their asymptotic properties are mainly determined by the heavy-tailed i.i.d. innovations. With a view towards empirical results for real life processes we suggest an alternative model which allows for more freedom in the asymptotic dependence structure. We analyze this model in the framework of hidden regular variation under the use of suitable general results.

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High-frequency sampled stable CARMA processes

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Abstract

We consider a continuous-time ARMA (CARMA) process driven by a stable Lévy process sampled at a high-frequency time grid $\{\Delta_n, 2\Delta_n, \ldots, n\Delta_n\}$ where the grid distance $\Delta_n \downarrow 0$ and $n\Delta_n \to \infty$. In this context of high frequency data within a long time interval we present different limit theorems. On the one hand, we study the asymptotic behavior of the properly normalized partial sum to a stable distribution. On the other hand, we derive the asymptotic behavior of the sample autocovariance function. In the case $\alpha = 2$ the sample autocovariance function is a consistent estimator. The last part of this talk is devoted to the limit behavior of different periodogram versions. The normalized and self-normalized versions of the periodogram converge to functions of stable random vectors. In contrast, the smoothed periodogram is a consistent estimator for the power transfer function and can be used to estimate the model parameters. Our results are similar to those for stable ARMA models in discrete time.

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Moments of Maxima of Stable Processes and Fields

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Abstract

A stochastic process $\{X_t\}_{t\in\mathbb{Z}^d}$ is called a (discrete parameter) stationary *symmetric α-stable* (SαS) random field if for each $m \geq 1$, for each $c_1, c_2, \dots, c_m \in \mathbb{R}$, and for each $u, t_1, t_2, \dots, t_m \in \mathbb{Z}^d$, $\sum_{k=1}^{m} c_k X_{t_k+u}$ follows an SαS distribution whose parameters do not depend on $u \in \mathbb{Z}^d$. For such a random field, we define the partial maxima sequence $M_n := \max_{\mathbf{0}\leq t\leq (n-1)\mathbf{1}} |X_t|$, $n \geq 1$, where $\mathbf{0} := (0, 0, \dots, 0), \mathbf{1} := (1, 1, \dots, 1) \in \mathbb{Z}^d$, and "$\leq$" denotes the componentwise inequality. It is easy to check that $E|M_n|^\beta < \infty$ if and only if $\beta < \alpha$. The following open problem was mentioned (for $d = 1$ case) in [3] in the context of uniform modulus of continuity of random fields: *can we give "sharp upper bound" on $E|M_n|^\beta$ for $\beta \in (0, \alpha)$?* Using a connection between the structure of a stationary SαS ($0 < \alpha < 2$) processes and random fields, and ergodic theory of nonsingular group actions (see [1, 2]), we have solved the above mentioned open problem for a big class of stable random fields $\{X_t\}_{t\in\mathbb{Z}^d}$. In fact, we have obtained the exact rates of growth of the β^{th} moment ($\beta \in (0, \alpha)$) of the partial maxima sequence under certain ergodic theoretic and group theoretic conditions on the underlying nonsingular action. These rates will be used to obtain finer results on the path properties (more precisely, the modulus of continuity) of stationary symmetric continuous parameter stable random fields with long memory.

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Minimal spectral representations of infinitely divisible and max–infinitely divisible processes

SPEAKER: **Stilian Stoev**, *University of Michigan, Ann Arbor, USA*
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Abstract

We introduce the notion of minimality for spectral representations of sum– and max–infinitely divisible processes and prove that the minimal spectral representation on a Borel space exists and is unique. This fact is used to show that a stationary, stochastically continuous, sum– or max–i.d. random process on $\mathbb{R}^d$ can be generated by a measure–preserving flow on a σ–finite Borel measure space and that this flow is unique. As a particular case, we characterize stationary, stochastically continuous, union–infinitely divisible random subsets of $\mathbb{R}^d$. We introduce several new classes of max–i.d. random fields including fields of Penrose type and fields associated to Poisson line processes.

Functional Central Limit Theorem for Heavy Tailed Stationary Infinitely Divisible Processes Generated by Conservative Flows

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Abstract

We establish functional central limit theorems for partial sum of certain sym- metric stationary infinitely divisible processes with regularly varying Lévy measures. The limit process is a new class of symmetric stable self-similar processes with stationary increments, that coincides, on a part of its parameter space, with a process previously described by Dombry and Guillotin-Plantard. The normalizing sequence and the limiting process are determined by the ergodic theoretical properties of the flow underlying the integral representation of the process, most importantly by its pointwise dual ergodicity. These properties can be interpreted as determining how long is the memory of the stationary infinitely divisible process. We also establish functional convergence, in a strong distributional sense, for conservative pointwise dual ergodic maps preserving an infinite measure.