

*International Conference on Computational
Operations Research
and Algorithmic Game Theory
January 21-23, 2025*

ICORAGT25



Organized by

Indian Statistical Institute, Delhi Centre

Contents

Welcome.....	3
Committees.....	5
Overview of Program.....	8
Abstract of the papers	17

Welcome to ICORAGT 2025

On behalf of the organizers of ICORAGT 25, I welcome you in the three days International conference on Computational Operations Research and Algorithmic Game Theory to be organized during January 21-23, 2025 at Indian Statistical Institute, Delhi Centre.

This International conference on Computational Operations Research and Algorithmic Game Theory (ICORAGT) aims at discussing new developments in the methods of decision making and promises to build an interaction between the users and the academic model developers by bringing them together. There have been important new developments in the computational techniques of Operations Research and game problems. This conference intends to review the current issues in the theory and applications of Operations Research and Game Theory to problems in business and industries. The objective of this conference is to provide a forum for new developments and applications of Operations Research and game theory. Leading scientists, experienced researchers and practitioners, as well as younger researchers will come together to exchange knowledge and to build scientific contacts. This conference is expected to have a broad international appeal, dealing with topics of fundamental importance in Operations Research and other related sciences (Economics, Physics, Management Science and Engineering). ICORAGT intends to present the state-of-the-art results and recent advances in these areas with a view to highlight possible future course of research in these areas.

This conference also intends to bring out a publication of selected and refereed papers in the Special Issue on Computational Operations Research and Algorithmic Game Theory in honor of Prof. T. Parthasarathy focused on "Computational Operations Research and Algorithmic Game Theory". We invite researchers and scholars from various disciplines to participate/present the paper in this conference and contribute their original research papers to this special issue. For details about this special issue visit <https://www.worldscientific.com/page/igtr/callforpapers02>

Also, book chapters/expository articles are invited for the book-volume 'Applied Operations Research, Game theory, and Decision Science' which is intended to be published in some reputed series of SPRINGER.

Authors are encouraged to submit papers as a book chapter/ expository articles that contribute to the advancement of knowledge in the field and offer practical insights for policymakers, industry

professionals, and researchers. All submissions will undergo a rigorous peer-review process to ensure the highest quality of published articles. Paper presented in the conference may be submitted as a book chapter before June 30, 2025 to the organizing committee chair Prof. S. K Neogy (e.mail: skn@isid.ac.in).

The symposium topics include (but not limited to):

- Unconstrained and constrained optimization.
- Game Theoretical applications of Operations Research
- Multi-objective and robust optimization
- Exact/heuristic hybrid methods, involving natural computing techniques in computational operations research
- Optimization in dynamic and/or noisy environments
- Optimization on graphs
- Large-scale optimization, in parallel and distributed computational environments
- Static and Dynamic games
- Algorithmic Game Theory
- Complementarity and Variational inequalities model in Game Theory
- Operations Research problems in Statistics and Game Theory
- Portfolio Optimization
- Stochastic Optimization
- Financial Optimization

Information about social events will be available to you at the time of registration.

S. K. Neogy
Organizing Committee Chair

Organizing Committee

S. K. Neogy (Chair), Indian Statistical Institute, Delhi Centre



R.B. Bapat, Indian Statistical Institute, Delhi Centre



Arunava Sen, Indian Statistical Institute, Delhi Centre



Prabal Roy Chowdhury, Indian Statistical Institute, Delhi Centre



Monisankar Bishnu, Indian Statistical Institute, Delhi Centre



K. Manjunatha Prasad, Manipal Academy of Higher Education, Manipal



Deepayan Sarkar, Indian Statistical Institute, Delhi Centre



Programme Co-ordinating Committee

Prabal Roy Chowdhury, R. B. Bapat, K. Manjunatha Prasad, Deepayan Sarkar

Facilities Committee

Samapan Padhi (Chair), Simmi Marwah, S. A. Srinivas, Pankaj Kumar Meena, Parama Gogoi, P. Sreejith, Dinesh Kumar, Rajes, Khusboo Singh, Praveen Pandey -- Convener



International Conference on Computational Operations Research and Algorithmic Game Theory

January 21-23, 2025

Program Overview

Inaugural Session Details

January 21, 2025 Time: 10:00 -10:30 Venue: Auditorium

Welcome address, Opening Remarks, About the Conference, Vote of Thanks

Group Photograph 10:30-10:45

Tea Break: 10:45 -11:00

Sessions Details

January 21, 2025 Time: 11:00 -13:00 Venue: Auditorium

Invited Session I

Chairman : Reinoud AMG Joosten, University of Twente, The Netherlands.

1.	Juan Enrique Martinez-Legaz (Universitat Autnoma de Barcelona, Spain), Four Types of Voronoi Diagrams
2.	Yasunori Kimura (Toho University, Funabashi, Japan) Convex minimization problem and approximating technique inspired by the cutting-plane method
3.	Lina Mallozzi (Department of Mathematics and Applications “R. Caccioppoli” University of Naples “Federico II”) Core solutions for normal form non-cooperative games
4.	Andreay Garnaev (Rutgers University, North Brunswick, NJ 08901, USA) Fairness as a criterion in game-theoretical models of wireless communication

Lunch Break: 13:00 -14:00

Venue: Guest House Lawn

January 21, 2025 Time: 14:00 -15:30 Venue: Auditorium

Invited Session II

Chairman: Yasunori Kimura, Toho University, Funabashi, Japan

1.	J. Dutta (Institute of Technology, Kanpur, India) Tea time notes on error bounds.
2.	Sanjeet Singh , (Indian Institute of Management Lucknow), An Innovative Stochastic Cross-Evaluation Framework for Building Composite Indicators
3.	Michal Feldman , (Tel Aviv University, Israel), Ambiguous Contracts

January 21, 2025 Time: 14.00 -15:30 Venue: Conference Room

Technical Session-I

Chairman : Michel De Lara (Cermics, École nationale des ponts et chaussées, IP Paris, France)

1.	Michel De Lara (Cermics, École nationale des ponts et chaussées, IP Paris, France) Games in Product Form
2.	Thirumulanathan D (Indian Institute of Technology Kanpur) KKT Reformulations for Single Leader and Multi-Follower Games
3.	Amit Kumar Bardhan (University of Delhi, Delhi) Frequency, Market Size, and Airline Competition
4.	Kishan Suthar (Indian Institute of Management Indore) Stochastic Generalized Nash Equilibrium under Shared Constraints: A CVaR-Based Approach

Tea: 15:30-15:45

January 21, 2025 Time: 15:45 -18:45 Venue: Auditorium

Best Paper award Session (Methodology/Application Category)

Chairman : Lina Mallozzi, Department of Mathematics and Applications “R. Caccioppoli”
University of Naples “Federico II”

1.	Anveksha Moar (University of Delhi) An algorithm to solve polytopic set optimization problem based on a partial set order relation. (Mehodology category)
2.	Rupesh K. Pandey (IIT Patna) A Generalization of Generalized Hukuhara Newton's Method for Interval-Valued Multiobjective Optimization Problems (Mehodology category)
3	Subham Poddar (IIT Patna) Inexact proximal point method with a Bregman regularization for quasiconvex multiobjective optimization problems via limiting subdifferential (Mehodology category)
4.	Pallabi Samal, IIT (ISM), Dhanbad, LP well-posedness for multidimensional bilevel controlled variational inequalities (Mehodology category)
5	Shivani Sain (IIT Patna) Characterizations of the solution set of nonsmooth semi-infinite programming problems on Hadamard manifold (Mehodology category)
6.	Sunil Kumar (ISI Chennai Centre) On Completely Mixed Games (Mehodology category)
7.	Prachi Sachan (Banaras Hindu University) On multiobjective optimization problems involving higher order strong convexity using directional convexificators (Mehodology category)
8	Vandana Singh (Institute of Science, Banaras Hindu University) On multiobjective semi-infinite programs with vanishing constraints and tangential subdifferentials
9.	Pooja Balhara (Birla Institute of Technology and Science, Pilani) Dependability-Based Analysis for Spectrum Sensing and Spectrum Access in Cognitive Radio Networks with Heterogeneous Traffic (Application Category)
10.	Sajal Ghosh (ISI Delhi Centre) On Solving a Larger Subclass of Linear Complementarity Problems by Lemke’s Method (Application Category)
11	Raina Ahuja (Thapar Institute of Engineering and Technology, Patiala, Punjab) Multi-criteria decision-making using a complete ranking of generalized trapezoidal fuzzy numbers: modified results (Application Category)

Tea: 15:30-16:00
January 21, 2025 Time: 16:00 -18:45 Venue: Conference Hall
Technical Session-II
Chairman: Prabal Roy Chowdhury, Indian Statistical Institute, Delhi Centre

1.	Abarar Ahamad Khan (Manipal University Jaipur, Jaipur) Combinatorial optimization of Balanced Transportation Problem Using Parametric Analysis
2.	Nandhini K (Division of Mathematics, Vellore Institute of Technology Chennai) A Matrix Analytic Method of Fluid queues with Adaptive Sleep Mode Strategies: To empower energy efficiency in 5G
3	Rishita Mishra (Manav Rachna University) COMPARATIVE ANALYSIS OF: NEW VOGEL'S APPROXIMATION METHOD WITH LOGICAL DEVELOPMENT OF VOGEL'S APPROXIMATION METHOD
4.	Awdhesh Kumar Bind (DR. B.R. Ambedkar National Institute of Technology Jalandhar, Punjab) A Modified Particle Swarm Optimization Algorithm to Solve the Solid Transportation Problem
5	Vaishaly Verma (Motilal Nehru National Institute of Technology Allahabad, Prayagraj , Uttar Pradesh) A novel approach for solving linear fractional programming problem in a Neutrosophic environment: Application in Production Planning Problem
6.	Saurabh Kumar Mishra (Madan Mohan Malaviya University of Technology, Gorakhpur, U.P) Optimizing Substitution Policies in EOQ-Based Inventory Model Considering Weighted Demand Factors: Promotion, Pricing, and Seasonality
7.	Neeraj Singh (Thapar Institute of Engineering and Technology Patiala) Interval-Valued Bilevel Optimization Problem with Equilibrium Constraints: Application to Risk Management
8.	Fida Parveen (Vellore Institute of Technology, Chennai) Enhancing Retrial Queue with Neutrosophic Sets: An Innovative Parametric Programming Approach to Uncertainty in Arrival, Service, and Retrial Parameters
9.	
10	

Conference Dinner: 7:30 pm onwards

Venue: Guest House Lawn

January 22, 2025 Time: 9:30 -11:30 Venue: Auditorium

Invited Session III

Chairman: Juan Enrique Martinez-Legaz, Universitat Autnoma de Barcelona, Spain

1.	Edith Elkind , University of Oxford, Towards fair and efficient public transportation: the bus stop model
2.	Bart de Keijzer (King's College London), Not Obviously Manipulable and Budget-Feasible Mechanism Design
3.	Reinoud Joosten (University of Twente, The Netherlands), Rogier Harmelink, Three person data sharing dilemmas.
4.	S. Dharmaraja , Indian Institute of Technology Delhi, India, Reliability Analysis of Communication Systems

Tea Break: 11:30 -12:00

January 22, 2025 Time: 12:00 -13:00 Venue: Auditorium

Invited Session IV

Chairman: Arunava Sen, Indian Statistical Institute Delhi Centre

1.	N. Hemachandra , (Indian Institute of Technology Bombay, India), Game theoretic analysis of cross channel retailing
2.	Vijay V. Vazirani , University of California, Irvine, The Assignment Game: Equitable Core Imputations

Tea Break: 11:30 -12:00

January 22, 2025 Time: 12:00 -13:00 Venue: Conference Room

Technical Session III

Chairman: Bart de Keijzer (King's College London)

1.	Vivek Laha (Institute of Science, Banaras Hindu University, Varanasi) SOME APPLICATIONS OF DIRECTIONAL CONVEXIFIERS IN NONSMOOTH OPTIMIZATION PROBLEMS
2.	Gaurav Uniyal (Indian Institute of Technology (Indian School of Mines), Dhanbad, India) Semi-infinite variational programming problem with Caputo-Fabrizio fractional derivative
3.	Vinay Singh (National Institute of Technology Mizoram, Chaltlang, Aizawl, Mizoram, India) Sufficient and duality results for nonsmooth multiobjective semi-infinite programming with switching constraints using tangential subdifferentials

Lunch Break: 13:00 -14:00
January 22, 2025 Time: 14:00 -16:00 Venue: Auditorium

Invited Session V

Chairman: S. K Neogy, Indian Statistical Institute, Delhi Centre

1.	Anurag Jayswal (Indian Institute of Technology (Indian School of Mines), Dhanbad-826004, Jharkhand, India), Exact penalty results for semi-infinite interval-valued mathematical programs with vanishing constraints.
2.	David Bartl , Silesian University in Opava, School of Business Administration in Karvina, Czech Republic, A New Nucleolus-Like Method to Compute the Priority Vector of a Pairwise Comparison Matrix
3.	Vikas Vikram Singh , (Indian Institute of Technology Delhi, India) Convex Approximations of Random Constrained Markov Decision Process
3.	Dipti Dubey , (Shiv Nadar University), On Almost Semimonotone Matrices

January 22, 2025 Time: 14:00 -16:00 Venue: Conference Room

Technical Session IV

Chairman: Vivek Laha (Institute of Science, Banaras Hindu University, Varanasi)

1.	Soham Das (Indian Institute of Technology Delhi, Hauz Khas, New Delhi) The Modified Solution Space Truncation Algorithm
2.	Rekha (Shiv Nadar Institution of Eminence) Compressive Sensing: A Paradigm Shift in Computer Tomography Image Reconstruction
3.	Saad Ashraf (Bennett University, Greater Noida, UP) Remuneration Strategies to Gig-based Delivery Workers for Competing Online Delivery Platforms
4.	Rimpi (Maitreyi College, University of Delhi) On Weak Sharpness and Optimality Conditions in Multiobjective Optimization through Convexifiers ⁴ .
5	Vimal Dixit (Jawaharlal Nehru University, New Mehrauli Road, New Delhi) Optimizing Hyperparameters in Quantum Neural Networks using the ANOVA Framework
6.	Vaibhav Kannojiya (Jawaharlal Nehru University, New Mehrauli Road, New Delhi) Application of Structured Sparsity Techniques in Natural Language Processing
7.	Manikandan Rangaswamy (Central University of Kerala, Kasaragod) Customer Decision-Making in M/M/1 Queueing-Inventory Systems with Dual Replenishment Policies
8..	Saleem Yousuf (Department of Mathematics and Computing, IIT (ISM), Dhanbad) Earth Moon Optimal Transfer Trajectory using Particle Swarm Optimization

Tea: 16:00-16:30

January 22, 2025 Time: 16:30 -18:15 Venue: Auditoriam

Technical Session V

Chairman: Reinoud AMG Joosten, University of Twente, The Netherlands.

1.	Yogendra Pandey (Satish Chandra College, Ballia, India.) Optimality conditions for robust nonsmooth multiobjective optimization problem using tangential subdifferentials
2.	Ajeet Kumar (Indian Institute of Technology (Indian School of Mines), Dhanbad, India) A parametric approach for robust semi-infinite interval-valued optimization problems
3.	Anjali Rawat (National Insitute Of Technology Mizoram, Chaltlang, Aizawl, Mizoram, India) A neurodynamic model for solving MPEC and its application to Stackelberg–Cournot–Nash equilibrium problem
4.	Poushali Das (Indian Institute of Technology Patna) Gap Functions and Error Bounds for a Class of Stochastic Vector Quasi-Variational Inequality Problems
5	Rishabh Pandey (National Insitute Of Technology Mizoram, Chaltlang, Aizawl, Mizoram, India) Sufficient and Duality For Nonsmooth Interval-Valued Bilevel Optimization Problem

January 22, 2025 Time: 16:30 -18:15

Venue: Conference Room

Technical Session VI

Chairman: Nagarajan Krishnamurthy (Indian Institute of Management (IIM), Indore)

1.	Manish Sarkhel (Indian Institute of Management, Sirmaur) Strategic Network Formation in Manufacturing & Distribution
2.	Debasish Ghorui (Ghani Khan Choudhury Institute of Engineering and Technology, Malda) On existence and computation of ϵ -Nash equilibrium strategies for generalized bimatrix game
3.	Shubham Singh (VIT-AP University, Amaravati, Andhra Pradesh, India.) A Relationship Between Multitime Fractional Variational-Like Inequalities and Optimization Problems
4.	Sudeep Singh Sanga (SV National Institute of Technology, Surat, Gujarat, India) Analysis of a Finite Multi-Server Queue with Admission Control and Customer Balking
5	Varun Kumar S G (Manipal Academy of Higher Education) Predicting and Explaining Electric Vehicle Adoption with Shapley Additive Explanation

January 23, 2025 Time: 9:30 -11:30 Venue: Auditorium

Invited Session VI

Chairman: Agnieszka Wiszniewska-Matyszek, University of Warsaw, Poland

1.	Nagarajan Krishnamurthy (Indian Institute of Management (IIM), Indore) Pure Nash Equilibria in Bimatrix Games
2.	Agnieszka Wiszniewska-Matyszek , University of Warsaw, Poland and Rajani Singh , Copenhagen Business School, Denmark, How to avoid the tragedy of the commons in the imperfect world
3.	Gajendra Pratap Singh (School of Computational and Integrative Sciences, Jawaharlal Nehru University, New Delhi-110067, India) Optimizing Graph Algorithms through Petri Net Modeling and Simulation
4.	Best paper award Ceremony

Tea Break: 11:30 -12:00

January 23, 2025 Time: 12:00 -13:00 Venue: Auditorium

Invited Session VII

Chairman: T E S Raghavan, University of Illinois at Chicago, USA

1.	T. E. S. Raghavan , (University of Illinois at Chicago, USA) An algorithm for computing the nucleolus for cyclic permutation games
2.	Pankaj Gupta (University of Delhi, India) Large-scale group decision-making under uncertainty

January 23, 2025 Time: 12:00 -13:00 Venue: Conference Hall

Technical Session VII

Chairman: Shanta Laishram, Indian Statistical Institute, Delhi Centre

1.	Shubham Kumar Singh (Indian Institute of Technology Patna) Duality for Nonsmooth Semidefinite Multiobjective Programming Problems with Equilibrium Constraints Using Convexifiers
2.	Bhawna Kohli (SGND Khalsa College, University of Delhi, Delhi) On Schaible dual for bilevel multiobjective fractional programming problem
3.	Akriti Dwivedi (Banaras Hindu University, Varanasi, India) ON APPROXIMATE SOLUTIONS FOR NONSMOOTH INTERVAL-VALUED MULTIOBJECTIVE OPTIMIZATION PROBLEMS WITH VANISHING CONSTRAINTS

Lunch Break: 13:00 -14:00

January 23, 2025 Time: 14:00 -16:00 Venue: Auditorium

Invited Session VIII

Chairman: C. S. Lalitha, (University of Delhi South Campus),

1.	S.K. Mishra (Banaras Hindu University, India) Duality for Mathematical Programs in terms of Tangential Subdifferentials
2.	C. S. Lalitha , (University of Delhi South Campus), Painlevé-Kuratowski Convergence of Solution Sets in Set Optimization
3.	Tadashi SAKUMA , Yamagata University, Japan, The pebble motion problem

January 23, 2025 Time: 14:00 -16:00 Venue: Conference Hall

Technical Session VIII

Chairman: Monisankar Bishnu, Indian Statistical Institute, Delhi Centre

1.	Prashant Jaiswal , Banaras Hindu University, Varanasi, India OPTIMALITY CONDITIONS FOR NONSMOOTH QUASIDIFFERENTIABLE MATHEMATICAL PROGRAMS WITH EQUILIBRIUM CONSTRAINTS IN BANACH SPACES
2.	Murari Kumar Roy (Graphic Era (Deemed to be University), Dehradun) Optimality and duality in multi-objective fractional programming problem using generalized convex functions
3.	Priyanka Bharati (Banaras Hindu University, Varanasi, India) OPTIMALITY CRITERIA FOR ROBUST MULTIOBJECTIVE OPTIMIZATION WITH VANISHING CONSTRAINTS UNDER UNCERTAIN DATA
4.	Gautam Beniwal (Manipal University Jaipur) Networks System with Unreliable Nodes
5.	Vidhi Tiwari (Graphic Era Deemed to be University) Food waste system reliability evaluation with Pythagorean fuzzy approach
6.	

Tea Break: 16:00 -16:15

January 23, 2025 Time: 16:15 -18:30 Venue: Auditorium

Technical Session IX

Chairman: Deepayan Sarkar, Indian Statistical Institute, Delhi Centre

1.	Mukil M (Vellore Institute of Technology, Chennai campus) Parametric Analysis of Neutrosophic Vacation Queues
2.	Merina Dhara (Indian Institute of Technology, Madras) Integrating ARMAX Forecasting with Game Theoretic Applications for Analyzing HRV Data in Strategic Decision-Making
3.	N Vimalraj (Vellore Institute of Technology, Chennai) Analysis of Single and Multiple Server Finite Capacity Neutrosophic Queueing Model
4.	Satyendra Kumar (SSSVS Government PG College Chunar, Mirzapur, Uttar Pradesh, Approximation of Common Fixed Points and Solution of Variational Inequality using a new Viscosity-extragradient method of S-iteration type
5.	Priya Sharma (University of Delhi, Delhi 110007, India) Integrating Machine Learning and Fuzzy Logic for Large-Scale Decision-Making Frameworks
6.	

January 23, 2025 Time: 16:15 -18:30 Venue: Conference Hall

Technical Session X

Chairman: Anurag Jayswal* (Indian Institute of Technology (Indian School of Mines), Dhanbad

1.	Prabhjot Kaur (UIET, Panjab University, Chandigarh, 160014, India) A bi-criteria bottleneck assignment problem with categorized tasks
2.	Dr. Anupam (Netaji Subhas University of Technology) Reliability analysis of energy management subsystem in a high altitude platform station
3.	Bhawna Kohli (SGND Khalsa College, University of Delhi, Delhi) On Schaible dual for bilevel multiobjective fractional programming problem
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ABSTRACT OF THE PAPERS

An algorithm for computing the nucleolus for cyclic permutation games

T. E. S. Raghavan,

University of Illinois at Chicago, USA

The question of allocating costs or benefits among the participants of a joint enterprise is frequently answered by determining the nucleolus of a related cooperative game. We discuss this issue in connection with multi-person decision situations in which finding the best course of joint action for a group of participants can be modeled by the well-known assignment optimization problem. The related transferable utility cooperative games are called permutation games. We consider the large subclass of so-called cyclic permutation games, and show how the nucleolus of an n -player cyclic permutation game can be computed by a known polynomially bounded algorithm directly from the underlying data. We also demonstrate that this approach might not work if for the group of all participants the optimal course of action is not given by a permutation consisting of a single cycle.

(Joint work with T. Solymosi and Stef Tijs)

Four Types of Voronoi Diagrams

Juan Enrique Martínez-Legaz

Universitat Autònoma de Barcelona, Spain

In addition to the classical Voronoi diagrams in \mathbb{R}^n , we delve into other variants: higher-order cells, farthest cells and power cells. Despite their diversity, all these Voronoi diagrams share a common feature: their cells are closed convex sets. For the classical case, we study the Voronoi inverse problem. Given a closed convex set $F \subseteq \mathbb{R}^n$ and a point $s \in F$, we study the family of sets $T \subseteq \mathbb{R}^n$ that contain s and satisfy the condition that the corresponding Voronoi cell $VT(s)$ of s with respect to T is precisely F . We explore the relationship between the elements of this family and the linear representations of F . Additionally, we provide explicit formulas for maximal and minimal elements within this family. Our investigation also involves studying the closure operator that assigns to each closed set T containing s the largest set $L(T) \subseteq \mathbb{R}^n$ containing s such that $VL(T)(s) = VT(s)$. Higher order Voronoi cells associate to each k -element subset of the set of sites the set of points for which the given subset consists of the k closest sites. We delve into the structure of k -order Voronoi cells and illustrate our theoretical findings through a case study involving two-dimensional higher-order Voronoi cells for four points. The farthest Voronoi cell of a point s , denoted as $FT(s)$, consists of all the points farther from s than from any other site. We explore farthest Voronoi cells and diagrams corresponding to arbitrary (possibly infinite) sets. Specifically, for a given arbitrary set T , we characterize those $s \in T$ such that $FT(s)$ is nonempty and

analyze the geometrical properties of $FT(s)$. Additionally, we characterize those sets T whose farthest Voronoi diagrams form tessellations of the Euclidean space, as well as those that can be expressed as $FT(s)$ for some $T \subseteq \mathbb{R}^n$ and some $s \in T$. Finally, we study power cells. Given a set $T \subseteq \mathbb{R}^n$ and a nonnegative function r defined on T , the power of $x \in \mathbb{R}^n$ with respect to the sphere with center $t \in T$ and radius $r(t)$ is $pr(x, t) := \|x - t\|^2 - r^2(t)$, with $\|\cdot\|$ denoting the Euclidean distance. The corresponding power cell of $s \in T$ is the set $C_{r,T}(s) := \{x \in \mathbb{R}^n : pr(x, s) \leq pr(x, t), \text{ for all } t \in T\}$. We investigate the structure of these power cells and explore assumptions on r that allow for generalizing known results on classical Voronoi cells.

Convex minimization problem and approximating technique inspired by the cutting-plane method

Yasunori Kimura

Toho University, Japan

The convex minimization problem is one of the central topics in nonlinear analysis. This topic has recently been studied in the setting of complete geodesic spaces.

We often consider the set of all fixed points of specific mapping as the constraint set for this minimization problem. We adopt nonexpansive mappings for this setting because the fixed point set of such a mapping is always closed and convex. It is a great advantage to study nonlinear and convex analysis. Because of these properties, we may apply various techniques in convex analysis and the approximation theory of fixed points. In this work, we propose a new technique inspired by the cutting-plane method to approximate a solution to a convex minimization problem over

Three person data sharing dilemmas

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Data sharing dilemmas arise if agents benefit from sharing data provided all other agent share as well, yet the outcome arising is Pareto dominated by another one. So, collective data sharing constitutes a Nash equilibrium yielding lower collective profits than another outcome. Archetypical games to model one-shot interaction in this framework are the Prisoner's Dilemma and the Stag Hunt.

We investigate intertemporal three person data sharing dilemmas in which agents share their data using a learning algorithm to obtain information or knowledge crucially instrumental in maximizing long run average profits. An agent is of either of two types which we fix throughout the analysis. One type benefits to a higher degree from having the own data go through the learning algorithm and less so from the spill-overs, i.e., other firms' data being used. The other type benefits more from the spill-overs and less from the own data being used.

Since learning over the algorithm has cumulative effects over time on the profits achievable, a one-shot game is not ideal for our purposes, nor is an infinitely repeated game using one and the same one-shot game as its basis. We develop therefore, a relevant stochastic game belonging to the class of games with endogenous stage payoffs and endogenous transition probabilities (ESP-ETP-games). We determine Nash equilibria under the limiting average reward criterion using the Folk Theorem approach developed originally for repeated games.

The underlying stage (or one-shot) games are inspired by the Prisoner's Dilemma and Stag Hunt. We can use standard generalizations of the former to the three person version, but for the Stag Hunt generalizations seem not readily available, so we design them ourselves.

We model four situations determined by the composition of this population regarding the firms' types i.e., either all are of one type, or one firm differs from the other two. The composition of the population influences the set of feasible rewards under the limiting average reward criterion but also the location of the so-called threat point. Clearly, this has consequences for the set of equilibrium rewards and the associated Nash equilibria.

Keywords: Data sharing dilemmas, stochastic games, endogenous stage payoffs, limiting average rewards, Folk Theorem, sustainable threats

Fairness as a criterion in game-theoretical models of wireless communication

Andrey Garnaev

Rutgers University, North Brunswick, NJ 08901, USA,

Traditionally, fairness metrics are used in wireless network engineering to determine whether users or applications get a fair share of the resources. In this presentation, we show how fairness metrics might be applied also to sort out some flaws that arise in the design of optimal scanning protocols in network security problems and communication problems based on traditional approaches maximizing the detection probability of the adversary and maximizing throughput at the receiver, respectively. Specifically, such a flaw in traditional protocol maximizing detection probability is that it might lead to focusing on scanning the most plausible nodes or bands for intrusion and neglecting to scan less plausible nodes or bands for intrusion due to restricted scanning resources. Meanwhile, such a flaw in traditional communication protocol maximizing throughput might lead to multiple equilibria and, so, instability in communication. We show that using alpha-fairness criteria allows the designing of optimal protocols,

which do not have such flaws. For each considered model we also show how to find an optimal protocol among a continuum of designed optimal protocols (a protocol per a fairness coefficient).

Ambiguous Contracts

Michal Feldman,

Tel Aviv University, Israel

In this work we explore the deliberate infusion of ambiguity into the design of contracts. We show that when the agent is ambiguity-averse and chooses an action that maximizes their max-min utility, then the principal can strictly gain from using an ambiguous contract. We provide insights into the structure of optimal contracts, and establish that optimal ambiguous contracts are composed of simple contracts. We also provide a characterization of ambiguity-proof classes of contracts. Finally, we show that when the agent considers mixed strategies, then there is no advantage in using an ambiguous contract.

(Joint work with Paul Duetting, Daniel Peretz and Larry Samuelson).

Towards fair and efficient public transportation: the bus stop model

Edith Elkind,

University of Oxford

We consider a stylized formal model of public transportation, where a set of agents need to travel along a given road, and there is a bus that runs the length of this road. Each agent has a left terminal and a right terminal between which they wish to travel; they can walk all the way, or walk to/from the nearest stop and use the bus for the rest of their journey. The bus can make a fixed number of stops, and the planner needs to select locations for these stops. We study notions of efficiency and fairness for this setting. First, we give a polynomial-time algorithm for computing a solution that minimizes the total travel time; our approach can capture further extensions of the base model, such as more general cost functions or existing infrastructure. Second, we develop a polynomial-time algorithm that outputs solutions with provable fairness guarantees (such as a variant of the justified representation axiom or α -approximate core). Our simulations indicate that our algorithm almost always outputs fair solutions, even for parameter regimes that do not admit theoretical guarantees.

(Based on joint work with Martin Bullinger and Mohamad Latifian, to appear in AAMAS'25.)

How to avoid the tragedy of the commons in the imperfect world

Agnieszka Wiszniewska-Matyskiel,

University of Warsaw, Poland and

Rajani Singh, Copenhagen Business School, Denmark

Our research question is whether it is possible and how to counteract "the tragedy of the commons" if facing various limitations of real world economies. To answer it, we derive regulatory tax-subsidy systems and self-enforcing environmental agreements in a problem of extraction of common renewable resources. The first considered limitation is that the feasible class of tax-subsidy systems may have a complicated form, e.g. there are transition periods for smooth reduction of fishing. Alternative limitation is that there is no institution that can impose taxes or subsidize, so sustainability can be achieved only by self-enforcing international agreements. Next limitation is in those agreements: we assume that it takes time to detect a defection. We study these enforcement tools in a continuous-time version of a Fish War type game with n countries, with fish indispensable for their economies. We calculate the social optimum, a Nash equilibrium and partial cooperation equilibria. The Nash equilibrium leads to depletion of fish, while the social optimum typically results in sustainability. For partial cooperation, only 2-country coalitions are stable. We calculate tax-subsidy systems that enforce maximization of joint payoff, also if there are additional constraints, and we propose an algorithm that looks for such a system in an arbitrary class of regulatory tax-subsidy systems. For the international agreement with imperfect monitoring, we are interested in the maximal detection delay for which the agreement remains self-enforcing. Counter-intuitively, the more players, the more stable the agreement

The talk is mainly based on results of [1].

[1] A. Wiszniewska-Matyskiel, R. Singh, 2024, How to avoid the tragedy of the commons in an imperfect world *Journal of Public Economic Theory* 2024;26:e12713, <https://doi.org/10.1111/jpet.12713>

Core solutions for normal form non-cooperative games

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We explore Core solutions for normal form non-cooperative games, focusing on the development of specific characteristic functions derived from coalition payoffs within these games. These characteristic functions are based on various assumptions regarding the responses of non-coalition members, resulting in different coalition values. As a result, multiple distinct characteristic functions can be constructed for

the class of normal form games. In this joint work with Robert P. Gilles (The Queen's University of Belfast) and Subhadip Chakrabarti (The Queen's University of Belfast), we first explore the different construction methods to build characteristic functions describing these related cooperative game-theoretic representations of normal form games, then we present a generalization of the λ -characteristic function that is founded on the idea that a coalition should have a first mover advantage. Under certain conditions it is proved that the generalised λ -Core is not empty and some economic applications are discussed.

A New Nucleolus-Like Method to Compute the Priority Vector of a Pairwise Comparison Matrix

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We first recall the classical notion of a cooperative game with transferable utility, and also the classical solution concept of nucleolus (Schmeidler, 1969) of the TU-game.

We then turn our attention to pairwise comparison methods, which are often used in multiple-criteria decision-making, e.g., in the Analytic Hierarchy Process (AHP) in particular: Given several objects, such as alternatives, the decision-maker (DM) evaluates them pairwise with respect to some criterion. The relative importance of the two elements in a pair is usually rated by a positive real number with multiplicative interpretation; that is, the number indicates how many times one element is better (or more important) than the other in the pair. By considering all pairs of the objects in this way, a pairwise comparison matrix (PCM) is obtained. The purpose is to find the priority vector of the given PCM; that is, numerical weights of the objects so that the ratio of two weights is close to the pairwise comparison value given by the DM when assessing the relative importance of the respective two elements. Saaty's Eigenvector Method (EVM) and the Geometric Mean Method (GMM) are prominently used to find the priority vector of the PCM.

In this paper, we allow the PCM entries to be elements of a divisible alo-group (Abelian linearly ordered group), cf. the "general unified framework for pairwise comparison matrices in multicriteria methods" by Cavallo and D'Apuzzo (2009), which includes the aforementioned positive real numbers as a special case. Then, while Saaty's EVM cannot be used in this setting due to its intrinsic properties, the GMM can easily be adapted to find the priority vector of the PCM with entries from a divisible alo-group (Cavallo & D'Apuzzo, 2012) and, to our best knowledge, is the only currently known method that can be used in this setting. Inspired by the aforementioned concept of nucleolus from cooperative game theory

(Schmeidler, 1969), we propose a new nucleolus-like method to compute the priority vector of a pairwise comparison matrix with entries from any divisible alo-group. The method utilizes the theory of linear programming in abstract spaces (Bartl, 2007).

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Not Obviously Manipulable and Budget-Feasible Mechanism Design

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When designing auctions, one generally has to carefully engineer mechanisms that incentivise truthful participation of involved agents. In this context, the usual assumptions of perfect rationality of the agents and unlimited monetary resources of the designer are typically overly generous. On the one hand, a class of mechanisms known as "Willy Wonka mechanisms" are known to properly incentivize agents that only deviate when that is "cognitively easy" for them, i.e., they are Not Obviously Manipulable (NOM). On the other hand, budget feasible mechanisms are developed to tackle settings where a budget limit applies, i.e., the payments made must be within a given budget.

In this talk, I discuss our recent research (jointly with Guido Schäfer, Artem Tsikiridis, and Carmine Ventrè) that addresses both limitations simultaneously. We focus on the paradigmatic setting of procurement auctions and derive tight bounds on the approximation guarantee of NOM budget-feasible mechanisms, for a general setting where an auctioneer can select the product or service offered from a set of agents, and where the auctioneer's valuation for the selected set belongs to a general class of monotone subadditive functions. We fully characterise the NOM budget feasible mechanisms in this domain. Our results separate the achievable approximation guarantees from those that can be obtained under classical strategy-proofness.

The Pebble Motion Problem

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Let G be a finite undirected graph with no multiple edges or loops. The vertex set of G and the edge set of G are denoted by $V(G)$ and $E(G)$, respectively. Let $P = \{1, \dots, n\}$ be a set of pebbles with $n < |V(G)|$. A *configuration* of P on G is defined as a function f from $V(G)$ to $\{0, 1, \dots, n\}$ with $\sum_i f(i) = n$, where $f(i)$ is a vertex occupied with the i th pebble for $1 \leq i \leq n$ and $V \setminus \{i \mid f(i) > 0\}$ is a set of unoccupied vertices. A *move* is defined as shifting a pebble from a vertex to some unoccupied neighbor. The *pebble motion problem* on the pair (G, P) is to decide whether a given configuration of pebbles reachable from another by executing a sequence of moves. The well-known puzzle named "15-puzzle" due to Loyd is a typical example of this problem where the graph G is a 4×4 -grid. The pebble motion problem is studied intensively, because of its considerable theoretical interest as well as its wide range of applications for computer science and robotics, such as the management of indivisible packets of data moving on wide-area communication network and motion planning of independent robots. In this talk, the speaker will introduce recent advancements in research on this problem, incorporating the results obtained thus far.

The Assignment Game: Equitable Core Imputations

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The classic 1971 paper of Shapley and Shubik characterized the core of the assignment game. We observe that a sub-coalition consisting of one player (or a set of players from the same side of the bipartition) can make zero profit, and therefore its profit under a core imputation can be an arbitrary amount. Hence an arbitrary core imputation makes no fairness guarantee at the level of individual agents. Can this deficiency be addressed by picking a "good" core imputation?

To arrive at an appropriate solution concept, we give specific criteria for picking a special core imputation, and we undertake a detailed comparison of four solution concepts. Leximin and leximax core imputations come out as clear winners; we define these to be equitable core imputations. These

imputations achieve ``fairness" in different ways: whereas leximin tries to make poor agents more rich, leximax tries to make rich agents less rich.

We give combinatorial strongly polynomial algorithms for computing these imputations via a novel adaptation of the classical primal-dual paradigm. The ``engine" driving them involves insights into core imputations obtained via complementarity. It will not be surprising if our work leads to new uses of this powerful technique. Furthermore, we expect more work on computing the leximin and leximax core imputations of other natural games.

Game theoretic analysis of cross channel retailing

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Cross-channel retailing refers to the situation where a customer takes advantage of one channel (for example, brick-and-mortar) to gather information but then purchases through a different channel (e-commerce) without compensating the channel that provided the information. This behavior can result in lost sales for the brick-and-mortar retailer (showrooming) or e-commerce retailer (webrooming) who provided the information. We first model this strategic interaction as a Stackelberg game, considering the power dynamics of the retailers (brick-and-mortar, e-commerce). This Stackelberg game model captures both the phenomena (showrooming and webrooming) and recovers some existing literature that commonly deals with showrooming only. We further consider the simultaneous move in the above strategic interaction and determine the Nash equilibrium. Equilibrium strategies are determined in terms of both retailers' price margins and sales efforts. We obtain several managerial insights for symmetric markets, i.e., equal market potentials, same cost of sales efforts, etc. We define social welfare as the sum of revenues of both channels. It turns out that the social welfare under the Stackelberg equilibrium dominates the Nash equilibrium. We also compare the social welfare of the individual retailers under different ranges of the retentive components. Furthermore, we observe that the best strategy of a retailer is to react to the strategies of the Stackelberg leader for individual revenue under equilibrium. We also discuss several interesting future research avenues.

(Based on ongoing work with Anirban Mitra, Mayank Jangid and Manu Gupta.)

Reliability Analysis of Communication Systems

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Reliability is defined to be the ability of a system or component to perform its required functions under stated conditions for a specified period of time. In the world of broadband network, different types of communication networks are facing a large demand due to its high-speed wireless data transmission, extensive coverage from base stations, and a luxury of upgrading protocol software. In this talk, an analytical model is discussed to determine reliability attributes of third generation and beyond Universal Mobile Telecommunication Systems (UMTS), Voice over Internet Protocol (VoIP), Long-Term Evolution Advanced (LTE-A), Vehicular Ad hoc Network (VANET), High-Altitude Platform (HAP) networks. These networks are modeled using stochastic models such as Markov chains, semi-Markov process, reliability block diagrams and Markov reward models to obtain these attributes. Numerical results illustrate the applicability of the proposed analytical model. In addition, it can help the guarantee of network connectivity after any failure, without over dimensioning the networks.

Aumann–Shapley pricing for baggage ancillary services

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This study explores the application of non-atomic game theory in developing a pricing strategy for airline baggage services within a joint cost setting. We propose a novel approach, based on the Aumann–Shapley pricing method, to determine baggage prices for the airline industry. Our pricing scheme aims to bring transparency and fairness to baggage pricing by considering capacity consumption as a fundamental factor. One of the key features of our approach is its adherence to the existing free baggage limit while offering an axiomatic pricing model. For a joint cost example, we demonstrate that our model preserves the axioms proposed in the pricing approach. Through a simulation study, we observe that the average revenue gain for the proposed pricing mechanism is 7–8%. In the subsequent section, we extend the pricing methodology to accommodate price differentiation for pre-booked and on-site baggage with a weighted value approach. Furthermore, our analysis demonstrates that the weights for the weighted pricing mechanism can be evaluated by a simpler Ramsey–Boiteux rule and weighted proportionality axiom approach. The proposed method can be generalized and applied to other freight pricing scenarios where both volume and weight play significant roles in cost determination.

Tea time notes on error bounds.

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The very title in fact tells the reader about the very nature of these notes. These notes came about through some tea time discussions with my former Phd. student. The question was simple. What are the simplest conditions in through which we can write down an error bound for a convex set defined by convex inequalities and affine inequalities and affine equalities. If we separate the affine part from the convex parts the answers are known and a lot of research is done on them. But to the best of our efforts we did not find a study considering all the form of inequalities at the same time. We found some interesting ways to approach it and we have provided two different approaches. The first approach depends on an estimate for the distance to the intersection of two convex sets. The second approach uses the very recently introduced notion of the Cone constraint qualification and using it to develop both local and global error bounds even in the absence of Slater constraint qualification.

Pure Nash Equilibria in Bimatrix Games

Nagarajan Krishnamurthy and Lina Mallozzi

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In this talk, we shall look at the existence of pure Nash equilibria in bimatrix games. Shapley (1964) showed that a matrix game has a pure saddle point if every 2×2 subgame has one. For bimatrix games, however, a similar condition on 2×2 subgames is not sufficient for the existence of pure Nash equilibria. With the addition of some interesting conditions on the structure of the bimatrix game, we show that pure Nash equilibria are guaranteed to exist. We shall discuss examples to illustrate our results as well as to show the necessity, sufficiency or otherwise of the conditions.

This is joint work with Lina Mallozzi, University of Naples - Federico II, Naples.

Convex Approximations of Random Constrained Markov Decision Process

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Constrained Markov decision processes (CMDPs) are used as a decision-making framework to study the long-run performance of a stochastic system. It is well-known that a stationary optimal policy of a CMDP problem under discounted cost criterion can be obtained by solving a linear programming

problem when running costs and transition probabilities are exactly known. In this paper, we consider a discounted cost CMDP problem where the running costs and transition probabilities are defined using random variables. Consequently, both the objective function and constraints become random. We use chance constraints to model these uncertainties and formulate the uncertain CMDP problem as a joint chance-constrained Markov decision process (JCCMDP). Under random running costs, we assume that the dependency among random constraint vectors is driven by a Gumbel-Hougaard copula. Using standard probability inequalities, we construct convex upper bound approximations of the JCCMDP problem under certain conditions on random running costs. In addition, we propose a linear programming problem whose optimal value gives a lower bound to the optimal value of the JCCMDP problem. When both running costs and transition probabilities are random, we define the latter variables as a sum of their means and random perturbations. Under mild conditions on the random perturbations and random running costs, we construct convex upper and lower bound approximations of the JCCMDP problem. We analyse the quality of the derived bounds through numerical experiments on a queueing control problem for random running costs. For the case when both running costs and transition probabilities are random, we choose randomly generated Markov decision problems called Garnets for numerical experiments.

Duality for Mathematical Programs in terms of Tangential Subdifferentials

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The talk presents duality results for nonsmooth mathematical programs with equilibrium constraints in terms of tangential subdifferentials. We focus on the Wolfe type and Mond-Weir type dual problems and provide usual duality results under convexity and generalized convexity assumptions for MPEC by using tangential subdifferentials.

Keywords: Mathematical programs, stationary points, nonsmooth analysis, tangential subdifferentials,

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Large-scale group decision-making under uncertainty

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The interest in multi-criteria group decision-making (MCGDM) has increased significantly across various fields, including management science, operations research, and industrial engineering. Recently, MCGDM processes have evolved into large-scale group decision-making, which can involve a large number of DMs, criteria, alternatives, or a combination of these elements. The increase in decision parameters complicates coordination and information integration, highlighting the need for advanced methods to deal with these complexities.

Optimizing Graph Algorithms through Petri Net Modeling and Simulation

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Petri Nets, introduced by Carl Adam Petri, a German computer scientist and mathematician, in 1962, offer a powerful framework for modelling of series, concurrent processes, synchronization and resource flows, which are often present in graph-based systems due to their special structure. By representing graph nodes and edges as places and transitions in a Petri Net, the dynamic flow of resources and information can be simulated and analysed effectively for the optimization of key graph problems like pathfinding, flow maximization, and graph matching, accommodating real-time changes and the evolving nature of complex systems. Graph algorithms are essential in various fields, such as network analysis, transportation systems, telecommunications, bioinformatics, computational biology and computer science, where they solve optimization problems like shortest path determination, biological network optimization, maximum flow, routing, and graph matching. While traditional graph optimization techniques are effective in many cases, they often fall short when it comes to handling real-time changes, concurrency, and the dynamic behaviors of complex systems. So, this is where Petri Net modelling comes into play, offering a novel approach to enhance and optimize these graph algorithms by addressing these limitations. In this talk, we will demonstrate how Petri Nets can be applied to several important graph algorithms, providing valuable insights into both the theoretical and practical aspects of these algorithms.

Painlevé-Kuratowski Convergence of Solution Sets in Set Optimization

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(Jointly with Anveksha Moar, Department of Mathematics, University of Delhi)

The hyperspace $K_{cc}(Y)$ consisting of all compact and convex subsets of a nontrivial normed space Y , does not form a vector space with respect to set addition and scalar multiplication, as there is no additive inverse in $K_{cc}(Y)$. To deal with such situations in hyperspaces, Wu [1] introduced a concept of null set, which is a form of zero element in the hyperspace. Wu [1] used this concept of null set to define partial orderings in $K_{cc}(Y)$ and introduced solution concepts for a set optimization problem where the image of set-valued objective map is a collection of subsets of $K_{cc}(Y)$. Moar et al. [2] introduced a notion of pseudo algebraic interior in the hyperspace to define a notion of weak set order relation and introduced a Gerstewitz type nonlinear scalarization function.

In this talk we discuss the convergence of solution sets of a set optimization problem where the set-order relation is based on the concept of null set. The perturbed problems involve perturbation both in the objective map and the feasible set. Upper and lower Painlevé-Kuratowski convergence results are established for minimal and weak minimal solution sets in the domain space.

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Exact penalty results for semi-infinite interval-valued mathematical programs with vanishing constraints

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In this talk, we focus on a semi-infinite interval-valued mathematical programs with vanishing constraints (SIMPVC) and construct a penalized unconstrained problem $(SIMPVC)_p$ associated with the considered (SIMPVC). In order to obtain exactness of the penalty function, we introduce

some new constraint qualifications and their inter-relationships. Further, we have shown that the penalty function is exact under the introduced SIMPVC-generalized pseudo-normality constraint qualification. An example is provided to verify the main result in the manuscript.

An Innovative Stochastic Cross-Evaluation Framework for Building Composite Indicators

Sanjeet Singh

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This talk presents an innovative framework for Composite Indicator (CI) construction, addressing key challenges in traditional Data Envelopment Analysis (DEA). While CIs are vital in multi-attribute decision making, conventional DEA models often allow excessive flexibility in weight allocation, enabling certain Decision-Making Units (DMUs) to manipulate weights and achieve biased efficiency scores, undermining evaluation reliability. To overcome this, we propose a novel cross-assessment method that incorporates peer-reviewed evaluations among DMUs, reducing bias and ensuring more robust assessments in deterministic settings. Extending this, we develop a stochastic model using chance-constrained programming to address real-world scenarios characterized by data uncertainty, enabling evaluations under probabilistic conditions, and facilitating future-oriented decision-making.

On Almost Semimonotone Matrices

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We examine almost (strictly) semimonotone matrices which are also either **Z**-matrices or matrices having Property (++) and show that both parts of Wendler's conjecture [Spec. Matrices 7 (2019) 291–303] hold for these matrices. We also discuss some interesting properties related to the structure of almost (strictly) semimonotone matrices and also present results pertaining to the existence and multiplicity of solutions to the linear complementarity problem.

Generalized Inverses associated with Networks

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The generalized inverses of matrices associated with networks play a prominent role in the study of network flow, electrical networks, defining new distances on graphs, and the Markov process. For example, if Q is the incidence matrix of a given network (multi-digraph) then $Qx = b$ represents net inflow at each vertices when the flow through the edges of the network is provided by x . As the matrix Q is not invertible and the linear system $Qx = b$ need not be consistent or with infinitely many solutions, it is of natural interest to find minimum inflow

through s_n arcs of the network to achieve net inflow at s_m vertices are as close as possible to the desired. Noting that such a solution is given with the help of the Moore-Penrose inverse, in this talk, we present several expressions for Q^{\dagger} and L^{\dagger} , which are computationally simpler by using network properties and different matrix methods.

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Abstract of the Contributed Talks

Games in Product Form

Benjamin Heymann (CRITEO, France)

Michel De Lara, Jean-Philippe Chancelier

(Cermics, École nationale des ponts et chaussées, IP Paris, France)

Game theory offers a mathematical formalism to study strategic interactions. In such models of competition, information (who knows what and before whom) plays a crucial role. In the fifties, H. W. Kuhn used trees to define a game in extensive form (GEF); in a GEF, the information of a player is represented by a partition of the player node moves [3]. In the seventies, H. S. Witsenhausen used agents, a product set and a product sigma-field to define the so-called Witsenhausen intrinsic model (WIM) in multi-agent stochastic control problems; in a WIM, the information of an agent is represented by a subfield of the product sigma-field [4, 5, 1].

In this talk, we introduce games in product form (GPF) as an alternative (based on WIM) to GEF [2]. we present Witsenhausen intrinsic model, with several illustrations. We discuss also classification of information structures and the potential for decomposition, be it hierarchical or parallel, with respect to subgroups of agents. We advocate the relevance of GPF for game theory by providing a case of game form that is playable, but that cannot be written on a tree, and by illustrating — with examples in principal-agent models in energy management — the easiness and flexibility of GPF to model games. Depending on the talk duration, we can also expose how to represent stochastic and Bayesian games inside the GPF formalism, provide a definition of perfect recall, of behavioral strategies “à la Aumann” and prove a Kuhn’s Equivalence Theorem for GPF. To sum up, games in product form are an alternative to games in extensive form defined on trees. They make it possible to define information constraints, among and within players, in an elegant concise way. By their product form, they are amenable to decomposition, hence relevant for algorithmic purposes.

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KKT Reformulations for Single Leader and Multi-Follower Games

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Abstract

We consider a bilevel optimization problem having a single leader and multiple followers. The followers choose their strategies simultaneously, and are assumed to converge to a Nash equilibrium strategy profile. We begin by providing a practical example of such a problem in an oligopoly setting. We then show the existence of a Nash equilibrium when the objective function of each follower is convex in its optimizing variable, and the feasible set is compact, convex, and nonempty. We then consider the KKT reformulation of the single leader multi-follower game (henceforth, SLMFG), and show using examples that the solutions of both the problems need not be the same, even when each of the followers' problem is convex. In particular, we show that the global minima of both the problems may differ if the follower's problem does not satisfy the Slater's condition. We then show that the local minima of the SLMFG and its KKT reformulation are the same if, in addition to convexity and Slater's constraints, the local minimum point remains a local minimum for every Lagrange multiplier in each of the followers' problem. Given that this condition is hard to verify in practice, we provide another condition for the local minima of the two problems to be the same using constant rank constraint qualification (CRCQ). We again show using examples that the local optima of the two problems may differ if the conditions are not satisfied.

Frequency, Market Size, and Airline Competition

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The selection of flight routes is a strategic decision for commercial airlines, who may seek to modify their existing networks to enhance efficiency and profitability. This consideration is particularly important during network expansion. There is a substantial body of literature on airline route choice and network design challenges (Wang et al., 2022); however, most studies on point-to-point network configuration tend to treat frequency decisions in one market as independent from those in other markets. This article aims to address this gap by concurrently examining two routes where passenger traffic varies based on the frequency of flights offered, albeit at different levels.

Predicting and Explaining Electric Vehicle Adoption with Shapley Additive Explanation

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This research investigates the application of SHAP (SHapley Additive exPlanations) values to interpret machine learning models for analyzing factors influencing electric vehicle (EV) adoption. Models are evaluated using a simulated dataset encompassing price, range, incentives, charging infrastructure, and brand reputation. SHAP values provide detailed insights into feature importance and their contributions to model predictions, enabling a transparent understanding of key drivers. Visualizations, including feature importance and dependence plots, are used to explain the factors influencing EV adoption, demonstrating the power of SHAP for interpreting complex machine learning models in predictive tasks.

A Matrix Analytic Method of *MMAP/PHF/1* Fluid queues with Adaptive Sleep Mode Strategies: To empower energy efficiency in 5G

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Energy is the lifeblood of human progress; without it, survival nature and driving innovation come to a halt. In recent years, energy consumption levels have risen due to technological development in wireless networks, in particular, 5G networks consume four times greater energy than 4G due to their ultra-dense network connectivity. Therefore, enhancing energy efficiency is required to overcome the energy crisis, by switching the components to low-power states (i.e., sleep mode) during low-activity periods to

conserve energy in our proposed model. The work is considered a *MMAP/PHF/1/FQ* model with two different types of sleeping strategies, namely SS1 and SS2. The analytical solution has been formulated using the matrix analytic method (MAM) and its performance indicators of the proposed queueing indices have been found to validate that our considered approach gives better performance.

COMPARATIVE ANALYSIS OF: NEW VOGEL'S APPROXIMATION METHOD WITH LOGICAL DEVELOPMENT OF VOGEL'S APPROXIMATION METHOD

Rishita Mishra¹

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Nowadays, Calculating efficient solution for Transportation problem plays a vital role in operation research. In the linear programming problem (LPP) transportation problem has many aims such as minimizing transportation cost, time etc. Transportation problem (TP) has key role in the industries, logistics etc. to get the best possible profit. There are some existing basic methods to find the initial basic feasible solution for the transportation problem such as North West Corner Rule (NWCR), Least Cost Method (LCM), and Vogel's Approximation Method (VAM). Transportation problem involves determining the most cost-effective way to transport goods from multiple sources (e.g., factories) to multiple destinations (e.g., warehouses) while meeting supply and demand constraints.

This research paper explores a comparative analysis of various modified Vogel's approximation method for obtaining the basic feasible solution. The study evaluates New Vogel's approximation method and Logical Development of Vogel's Approximation method and each of which introduce specific innovations to address limitations of traditional approaches. The results demonstrate that the modified methods offer significant improvements under certain conditions, with trade-offs in complexity and implementation ease. This research provides valuable insights into the practical applicability of these advanced methods and contributes to the ongoing development of efficient tools for solving transportation problems

A Modified Particle Swarm Optimization Algorithm to Solve the Solid Transportation Problem

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Particle swarm optimization is a nature-inspired optimization technique based on the social behavior of swarms. It optimizes by adjusting the positions and velocities of particles in the search space, relying on each particles individual experience and that of the neighbouring particles. Traditional methods, including optimization solvers such as LINGO, struggle to solve the high-dimensional nonlinear solid transportation problems (STPs), which are complex due to their multiple sources, destinations, and conveyances. Notably, the existing literature does not address the method for solving the problem using such metaheuristic algorithms. To fill this gap, this study

introduces a modified nonlinear solid particle swarm optimization designed to minimize the cost of nonlinear STPs. Also, this study proposes a novel initialization method for generating the initial solution of the nonlinear STP, enhancing the algorithms robustness and adaptability. The proposed algorithm tested two examples, small-scale and large scale nonlinear STPs, to illustrate its efficacy across various dimensions. A comparative analysis over different population sizes provide insights into its performance and convergence. The proposed algorithm demonstrates its strong convergence properties under varying population and iteration settings.

Sufficient and duality results for nonsmooth multiobjective semi-infinite programming with switching constraints using tangential subdifferentials

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Abstract

We investigate sufficient optimality condition for a nonsmooth multiobjective semi-infinite programming problem subject to switching constraints. Further, we formulate Mond-Weir duality model and establish the duality results using tangential subdifferentials as a tool.

We take up the following nonsmooth multiobjective semi-infinite programming problem subject to switching constraints:

$$\begin{aligned} \min \quad & f(x) = (f_1(x), \dots, f_m(x)) \\ \text{s.t.} \quad & g_s(x) \leq 0, & \forall s \in S, \\ & h_k(x) = 0, & \forall k \in K = \{1, \dots, q\}, \\ & G_i(x)H_i(x) = 0, & \forall i \in I = \{1, \dots, l\} \end{aligned} \tag{1}$$

where the index set S is an arbitrary nonempty set, not necessarily finite. The real-valued functions $f_j, j \in J = \{1, \dots, m\}, g_s, s \in S, h_k, k \in K, G_i$ and $H_i, i \in I$ are defined on \mathbb{R}^n and not necessary convex nor differentiable.

A Relationship Between Multitime Fractional Variational-Like Inequalities and Optimization Problems

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In this paper, we introduce a new class of variational inequalities, namely the weak multitime fractional variational-like and the multitime fractional variational-like. By using the so-called parametric approach, we establish the equivalence between the solutions to the introduced variational inequalities and (properly) efficient solutions of multitime multi-objective fractional variational problems. To validate our results, we also provide some examples.

On Completely Mixed Games

Parthasarathy Thiruvankatachari · Ravindran
Gomatam · Sunil Kumar

ISI Chennai Centre

Abstract A matrix game is considered completely mixed if all the optimal pairs of strategies in the game are completely mixed. In this paper, we establish that a matrix game A , with a value of zero, is completely mixed if and only if the value of the game associated with $A + D_i$ is positive for all i , where D_i represents a diagonal matrix where i^{th} diagonal entry is 1 and else 0. Additionally, we address Kaplan-sky's question from 1945 regarding whether an odd-ordered symmetric game can be completely mixed, and provide characterizations for odd-ordered skew-symmetric matrices to be completely mixed. Moreover, we demonstrate that if A is an almost skew-symmetric matrix and the game associated with A has value positive, then $A + D_i \in Q$ for all i , where D_i is a diagonal matrix whose i^{th} diagonal entry is 1 and else 0. Skew-symmetric matrices and almost skew-symmetric matrices with value positive fall under the class of P_0 and Q_0 , making them amenable to processing through Lemke's algorithm.

On multiobjective semi-infinite programs with vanishing constraints and tangential subdifferentials

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The aim of this article is to study a nonsmooth multiobjective semi-infinite programming problem with vanishing constraints (MSIPVC). We introduce the stationary point concept in terms of tangential subdifferentials and establish necessary and sufficient Karush-Kuhn-Tucker optimality conditions under suitable generalized constraint qualifications for efficient and other generalized efficient solutions for the MSIPVC in the sense of tangential subdifferentials. Further, we formulate Wolfe and Mond-Weir type dual problems and establish duality relations under convexity and generalized convexity assumptions in presence of tangentially convex functions. We also provide some examples that illustrate our results.

Neutrosophic Linear Fractional Optimization Problem using Fuzzy Goal Programming Approach

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This research article develops an algorithm to solve a neutrosophic linear fractional optimization problem (NLFOP), where the cost of the objective functions, the technology coefficients, and the resources are neutrosophic numbers (triangular neutrosophic numbers (TrNN) and trapezoidal neutrosophic numbers (TrpNN)). Two defuzzification methods are used here to convert the neutrosophic linear fractional optimization problem (NLFOP) into a multi-objective linear fractional optimization problem (MOLFOP). The fuzzy fractional objective function is converted into a crisp fractional objective function using the component-wise optimization method, so the single objective fuzzy fractional objective function is converted into multi-objective crisp fractional objective functions then these multi-objective fractional objective functions are further linearized by Charnes and Cooper technique and the linear fuzzy constraints are converted into crisp constraints using the aggregation ranking function strategy. Hence, the overall crisp multi-objective optimization problem is then solved using the fuzzy goal programming approach. In order to further explain our suggested algorithm, two numerical examples are solved one example is related to the triangular neutrosophic fuzzy numbers (TrNN) uncertain problem, and the second example is related to the trapezoidal neutrosophic fuzzy numbers (TrpNN) uncertain problem. In the end, outcomes obtained by our method are compared with the outcomes obtained using the other algorithm.

On Schaible dual for bilevel multiobjective fractional programming problem

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Abstract: In this paper, we associate Schaible type dual in terms of convexifactors to bilevel multiobjective fractional programming problem and connect the two with the help of various duality results via convexifactors.

Optimizing Substitution Policies in EOQ-Based Inventory Model Considering Weighted Demand Factors: Promotion, Pricing, and Seasonality

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In this article, a simplified extended version inventory model based on Economic Order Quantity (EOQ) is developed. The model is specifically designed for two substitute products identical cost structures with shortages and also considering weighted demand factors like promotion, pricing and seasonality. This

model considers independent replenishment policy. From available literature, most of the authors focused on deterministic demand models to achieve closed-form solutions and insightful results and products must be order at same time. They concluded that full substitution can never be optimal, and derived conditions for the optimality of either no or partial substitution. This article provides new results into inventory systems with two similar products by allowing shortage backordered with weighted demand factors.

This study proposes a methodology based on classical optimization techniques for an inventory management model utilizing the Economic Order Quantity (EOQ) framework. The aim is to optimize inventory control decisions and minimize the total average cost which includes ordering costs, holding costs, lost sale cost and substitution cost.

On existence and computation of ϵ -Nash equilibrium strategies for generalized bimatrix game

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In this paper, we investigate generalized bimatrix games and compute approximate Nash equilibrium using a vertical linear complementarity formulation. We define the concept of an approximate equilibrium point for polymatrix games, as well as its generalized version. Numerical examples are provided, where equilibrium points are computed within a complementarity framework using the Cottle-Dantzig algorithm.

Strategic Network Formation in Manufacturing & Distribution

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This paper discusses the impact of positive externalities due to cooperation on cost structures through networks among marginal firms in distribution channels. We particularly focus on the phenomenon of successive markups, that often leads to lower profits. To tackle the same marginal firms often use social ties to cooperate and consequently reduce their costs. For instance, Coca Cola's distribution channel involves employing local community members to manage and operate distribution kiosks, demonstrating a successful implementation of a decentralized distribution network. In our study, we consider two

distribution channels: the single firm model (SF) and the networked firm model (NF). The SF model refers to a scenario where a singular entity undertakes both manufacturing, distribution and retailing tasks. Conversely, the NF model encompasses a collection of firms, each assuming responsibility for their individual manufacturing and retailing ventures. The results obtained in the paper explain why decentralized distribution channels perform better than centralized distribution channels in specific industries in developing economies.

Stochastic Generalized Nash Equilibrium under Shared Constraints: A CVaR-Based Approach

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The equilibrium solution to an N-player game with non-disjoint strategy sets is referred to as a Generalized Nash Equilibrium (GNE). Due to the non-disjoint nature of the strategy sets, the uniqueness of such a solution is generally not guaranteed. However, under certain conditions, such as the quasi-concavity of the utility function and the convexity of non-disjoint strategy sets, uniqueness can be established. This study examines the shared constraints Generalized Nash game, where both the utility functions and strategy sets are subject to uncertainty. Previous studies have considered solving the Generalized Nash Equilibrium Problem (GNEP) using Quasi-Variational Inequality (QVI) methods. When uncertainty is introduced, it can be managed using various approaches. These include the almost-sure formulation, the expected value formulation, and risk measure considerations. In this study, we adopt the latter approach. We highlight the limitations of the expected value approach and present the benefits of risk-based measures such as Conditional Value-at-Risk (CVaR). The equilibrium solution is characterized in terms of players' risk aversion, and conditions for the existence of a unique equilibrium are identified.

ON MULTIOBJECTIVE OPTIMIZATION PROBLEM INVOLVING HIGHER ORDER STRONG CONVEXITY USING DIRECTIONAL CONVEXIFIATORS

Prachi Sachan* and Vivek Laha

Banaras Hindu University, Varanasi, India

This talk focuses to study the notion of strong convexity of higher order along continuity directions based on strong convexity of higher order and convexity along continuity directions. We characterize strong convexity of higher order along continuity directions of a function using the monotonicity of the associated directional convexificators. We have also studied multiobjective optimization problems involving strongly convex functions of higher order along continuity directions. We use vector variational inequalities of higher order in terms of directional convexificators to identify strict minimizers and semi-strict minimizers for the multiobjective optimization problems.

ON APPROXIMATE SOLUTIONS FOR NONSMOOTH INTERVAL-VALUED MULTIOBJECTIVE OPTIMIZATION PROBLEMS WITH VANISHING CONSTRAINTS.

Akriti Dwivedi and Vivek Laha

Banaras Hindu University, Varanasi, India

This talk aims to study the approximate weak and strong strongly stationary conditions for intervalvalued multiobjective mathematical programming with vanishing constraints (IVMMPVC). We begin by introducing approximate versions of several constraint and data qualifications for IVMMPVC and compare them. Next, we establish approximate necessary conditions termed approximate weak strongly stationary conditions and approximate strong strongly stationary conditions at type-2 E-quasi weakly Pareto and type-1 E-quasi Pareto solutions of IVMMPVC, respectively. Finally, we demonstrate that these approximate weak and strong strongly stationary conditions are also sufficient for optimality.

OPTIMALITY CONDITIONS FOR NONSMOOTH QUASIDIFFERENTIABLE MATHEMATICAL PROGRAMS WITH EQUILIBRIUM CONSTRAINTS IN BANACH SPACES

Prashant Jaiswal and Vivek Laha

Banaras Hindu University, Varanasi, India

This talk is devoted to the study of mathematical programs with equilibrium constraints involving quasidifferentiable functions in Banach spaces (abbreviated as, QMPEC). We present the Mangasarian Fromovitz constraints qualification in the framework of Banach spaces for QMPEC (abbreviated as, q.d.-ECMFCQ). The Karush-Kuhn-Tucker (KKT) type necessary optimality conditions for a local optimal solution are derived for such a nonsmooth mathematical programs with equilibrium constraints by employing q.d.-EC-MFCQ. Moreover, we establish sufficient optimality conditions under the assumptions of generalized convexity of the functions in terms of quasidifferential sum. Several nontrivial examples are furnished settings to demonstrate the validity of derived results.

OPTIMALITY CRITERIA FOR ROBUST MULTIOBJECTIVE OPTIMIZATION WITH VANISHING CONSTRAINTS UNDER UNCERTAIN DATA

Priyanka Bharati*1, Vivek Laha1

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In this talk, we study the multiobjective mathematical programs with vanishing constraints having uncertainty in constraint functions within the framework of robust optimization, denoted by UMMPVC. Even under relatively mild conditions, we demonstrate that UMMPVC fails to satisfy standard constraint qualifications such as Cottle constraint qualifications, Slater constraint qualifications, Mangasarian-Fromovitz constraint qualifications, linear independence constraint qualifications, linear objective constraint qualifications, and linear constraint qualifications. However, the standard generalized Guignard constraint qualification is sometimes satisfied. Further, We propose appropriate modifications

to the above mentioned constraint qualifications, establish connections among them, and derive Karush-Kuhn-Tucker type necessary optimality conditions for efficiency.

Gap Functions and Error Bounds for a Class of Stochastic Vector Quasi-Variational Inequality Problems

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In this article, we consider a class of stochastic vector quasi-variational in-equality problems (in short, SVQVIP) in the framework of Euclidean space. We transform the considered problem SVQVIP into a deterministic vector quasi-variational inequality problem (in short, DVQVIP) by employing the expected value (in short, EV) formulation for SVQVIP. We formulate various gap functions for DVQVIP, in particular, the residual gap function, the regularized gap function, and the Moreau-Yosida regularized gap function. These gap functions establish the necessary and sufficient conditions for the existence of a solution to DVQVIP. Moreover, we employ the residual gap function to derive the global error bound for the solution of DVQVIP under the assumption of strong monotonicity. In addition, under the suitable monotonicity hypotheses, the notions of regularized and Moreau-Yosida regularized gap functions are employed to derive the local error bounds for the solution of DVQVIP. Several non-trivial illustrative examples are provided to demonstrate the significance of the established results. To the best of our knowledge, this is the first time that the gap functions and error bounds for stochastic vector quasi-variational inequality problems have been explored in the Euclidean space setting.

Combinatorial optimization of Balanced Transportation Problem Using Parametric Analysis

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Combinatorial optimization plays a crucial role in identifying the best solution from a finite set of possibilities, selecting the most cost-effective routes and quantities using techniques like the Hungarian method, Simplex method, and Network flow algorithms. Combinatorial optimization of a balanced transportation problem under parametric analysis focuses on solving transportation challenges in a structured manner, particularly when certain parameters vary. The objective is to efficiently allocate resources to minimize such as cost or time while meeting supply and demand constraints. A balanced transportation problem ensures that total supply equals total demand, involving the transportation of goods from multiple sources to various destinations at the minimum possible cost. This problem is typically represented as a matrix, where each cell indicates the shipping cost. IBFS was carried out by Vogel's approximation method. Parametric analysis complements this by examining how changes in

parameters, such as costs, supply, or demand, impact the optimal solution, allowing for the assessment of solution sensitivity and robustness to parameter shifts.

On Solving a Larger Subclass of Linear Complementarity Problems by Lemke's Method

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It is well known that the success of Lemke's algorithm for solving a linear complementarity problem $LCP(q, M)$ depends on the matrix class M . Many researchers investigated a large class of matrices for which Lemke's algorithm computes a solution of the $LCP(q, M)$. In this paper, we follow a different approach for the class of LCP, which is not solvable by Lemke's algorithm. First, we construct an artificial $LCP(q_1, M_1)$ from $LCP(q, M)$ by adding some artificial variables and extra constraints, and show that the matrix M_1 belongs to the class of semimonotone matrices. However, $LCP(q_1, M_1)$ is not always solvable by Lemke's algorithm. Then, we construct another artificial $LCP(q_2, M_2)$ from $LCP(q_1, M_1)$ by adding some more artificial variables and extra constraints that satisfy Eaves condition. We show that the resulting artificial $LCP(q_2, M_2)$ is solvable by Lemke's algorithm. Given an $LCP(q, M)$, its solution can be obtained from the solution of the constructed artificial $LCP(q_2, M_2)$ with Eaves conditions. This approach leads to an innovative scheme for solving a large class of LCPs which are not solvable by Lemke's algorithm. Further, we also provide convergence results. The results obtained here can be used for broader applications of Lemke's algorithm.

An algorithm to solve polytopic set optimization problem based on a partial set order relation

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The primary aim of this paper is to present an algorithm for computing a weak m -minimizer and an m -minimizer for a class of set optimization problems. The solution concepts are based on m -order relation proposed by Karaman et al. (Positivity 22:783-802, 2018). The m -order relation is initially characterized in terms of support functions. Two scalar optimization problems are formulated to derive optimality conditions for weak m -minimizers and m -minimizers. Subsequently, the algorithm presented by Löhne and Schrage (Optimization 62(1):131-141, 2013) is modified to develop the algorithm. The paper includes numerical examples to validate the algorithm.

Earth-Moon Optimal Transfer Trajectory using Particle Swarm Optimization

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In the recent times, space trajectory optimization plays a pivotal role in designing transfer trajectories for various space missions. This process involves defining the objective function based on mission requirements and employing optimization methods to determine optimal solutions within the constraints of the mission. This study addresses the problem of identifying minimum-fuel optimal Earth- Moon transfers from a low Earth orbit (LEO) to a lunar orbit (LMO) within the framework of the bicircular restricted four-body problem (BCR4BP), utilizing a low-thrust propulsion system. The primary objective is to maximize the spacecraft's mass upon arrival at the Moon. The optimal control problem is first formulated, and the necessary conditions for optimality are derived using Pontryagin's Minimum Principle. To address the significant sensitivity of the state and costate equations in this problem, two distinct solution methods are proposed. The first method employs particle swarm optimization (PSO) for initializing the costate variables, followed by a shooting method to refine the solution and satisfy the problem's optimality constraints. Finally, numerical results for the transfer trajectories are analysed with respect to their geometry, transfer duration, and fuel efficiency.

Dependability-Based Analysis for Spectrum Sensing and Spectrum Access in Cognitive Radio Networks with Heterogeneous Traffic

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The Internet of Things (IoT) has experienced rapid growth in various applications, resulting in significant advancements that exhibit considerable variations in characteristics and requirements. Cognitive radio networks (CRNs) present a promising solution for ultra-reliable communication and dynamic spectrum sharing among IoT devices in 6G environment. The most critical task in CRNs is to identify unused spectrum opportunities, known as holes, across different times and locations. Addressing this challenge requires an effective spectrum sensing strategy at the medium access control (MAC) layer to optimize spectrum use while minimizing interference with licensed user signals. In this paper, we have proposed a

novel dynamic spectrum access scheme, which aims to address both spectrum availability and network reliability for various secondary user (SU) flows in IoT-centric CRNs. Our study examines the effect of random channel failure and their recovery on the performance of CRN. Moreover, we develop a continuous-time Markov chain (CTMC) model to examine the network performance across various key performance indicators (KPIs) in the presence of multiple channel failures and sensing errors. This analysis helps identify valuable trade-offs among the KPIs.

Sufficient and Duality For Nonsmooth Interval-Valued Bilevel Optimization Problem

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In this paper, we study a nonsmooth interval-valued bilevel optimization problem (P) , which is defined as follows:

$$(P) : \begin{cases} \min_{u,v} & F(u, v) = [F_L(u, v), F_U(u, v)] \\ \text{s. t.} & G_i(u, v) \leq 0, \quad \forall i \in I = \{1, 2, \dots, l\}, \\ & y \in \Upsilon(x), \end{cases}$$

where, for every $x \in \mathbb{R}^{n_1}$, $\Upsilon(x)$ represents the LU -solution set of the parametric interval-valued optimization problem,

$$(P_u) : \begin{cases} \min_v & f(u, v) = [f_L(u, v), f_U(u, v)] \\ \text{s. t.} & g_j(u, v) \leq 0, \quad \forall j \in J = \{1, 2, \dots, m\}, \end{cases}$$

where $F_L, F_U, f_L, f_U, G_i, g_j : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}$, $i \in I$, $j \in J$ are, given functions such that $F_L(u, v) \leq F_U(u, v)$, $\forall (u, v) \in E$, where $E := \{(u, v) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} : G_i(u, v) \leq 0, \quad \forall i \in I \text{ and } y \in \Upsilon(x)\}$ is the set of all feasible solutions of (P) .

Let us denote by $\Theta(u)$ the set of all feasible solutions of (P_x) , that is, the set defined by $\Theta(u) := \{v \in \mathbb{R}^{n_2} : g_j(u, v) \leq 0, \forall j \in J\}$. Then, for every $u \in \mathbb{R}^{n_1}$ and every $v \in \Theta(u)$, the lower level objective interval-valued function f fulfills the condition $f_L(u, v) \leq f_U(u, v)$ (as it follows from the definition of an interval-valued function).

Building on the necessary optimality conditions established by Dempe et al. [1], we derive sufficient optimality conditions for (P) . Furthermore, we establish various duality results by associating (P) with its Mond-Weir dual problem (MD) . The key tools utilized in our analysis are convexifiers and generalized invexity. To illustrate the applicability of our results, we include several examples.

Remuneration Strategies to Gig-based Delivery Workers for Competing Online Delivery Platforms

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Online delivery platforms (ODPs) often utilize a mixed workforce consisting of full-time employees and gig workers. Gig workers have no restrictions for exclusivity and can operate for multiple platforms. We consider an assignment problem where a set of gig workers is shared between two ODPs. The ODPs offer different delivery rates to the gig workers based on the urgency of their orders. Gig workers choose the most lucrative order. For delay in orders, the ODPs have to pay a lateness incentive to customers. This problem has been modelled as a Nash Equilibrium problem (NEP) using mixed integer linear programming (MILP). An iteration based solution algorithm is used to conduct numerical experiments for finding equilibrium strategies. Based on the results, the remuneration strategies an ODP should implement in different market conditions is proposed.

On a variant of Newton's method for interval-valued multiobjective optimization problems

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In this paper, we explore a class of interval-valued multiobjective optimization problems (IVMOP) and develop a corresponding multiobjective optimization problem (MOP). We use a parameter $q \in (0, 1)$ to define q -gH Hessian of interval-valued function. The objective function components in IVMOP are assumed to be continuously generalized Hukuhara (gH) differentiable, with the q -gH Hessians of these components assumed to be positive definite. We introduce a new variant of Newton's method to find

a weakly effective solution for IVMOP. The convergence of the sequence generated by this algorithm is analyzed under the assumption of continuity in the generalized Hukuhara derivatives of the objective function components. To illustrate the effectiveness of the proposed algorithm, we provide nontrivial numerical examples using MATLAB.

A parametric approach for robust semi-infinite interval-valued optimization problems

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In this paper, we consider a robust semi-infinite interval-valued optimization problem with inequality constraints having uncertain parameter. The parametric representation of the aforesaid problem is also consider in order to derive the necessary and sufficient optimality conditions. Furthermore, we formulate a mixed-type dual problem and derive duality results which associate the robust weak efficient solution of the primal and its dual problems. Several examples are given to illustrate the results in the manuscript. Joint work with Prof. Anurag Jayswal.

A neurodynamic model for solving MPEC and its application to Stackelberg–Cournot–Nash equilibrium problem

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A mathematical program with equilibrium constraints (MPEC) is a constrained optimization problem with Parametric Variational Inequalities (PVI) or complementarity systems as the major constraints. Typically, these constraints represent equilibrium phenomena that arises from economic and engineering applications. MPEC is generally defined in the following form:

$$\begin{aligned} \min \quad & f(x, y) \\ \text{s.t.} \quad & c(x) \leq 0, \\ & y \in S(x), \end{aligned}$$

where $f : \mathbb{R}^{l+m} \rightarrow \mathbb{R}$ and $c : \mathbb{R}^l \rightarrow \mathbb{R}$ are continuously differentiable functions. For each $x \in U_{ad} = \{x \in \mathbb{R}^l : c(x) \leq 0\}$, the set $S(x)$ represents the solutions of a variational inequality (VI) defined by the pair $(F(x, \cdot), K(x))$, where $F : \mathbb{R}^{l+m} \rightarrow \mathbb{R}^m$ is a continuously differentiable function and $K(x)$ is given by

$$K(x) = \{y \in \mathbb{R}^m : d_i(x, y) \geq 0, i = 1, \dots, l\}.$$

The function $d : \mathbb{R}^{l+m} \rightarrow \mathbb{R}^l$ is twice continuously differentiable and concave with respect to y . Specifically, $y \in S(x)$ iff $y \in K(x)$ and satisfies

$$(w - y)^T F(x, y) \geq 0 \quad \text{for all } w \in K(x)$$

A major challenge in handling MPEC problems arises from the fact that their feasible region is typically non-convex and non-smooth, primarily due to the presence of PVI or complementarity constraints. This non-convexity complicates the verification of optimality conditions, making the development of effective algorithms more difficult. Neural computing has emerged as a powerful tool for addressing optimization problems, particularly large-scale ones, in real-time. Various neural network models have been developed to solve linear and nonlinear problems. Unlike traditional methods, neural computing offers the advantage of rapid convergence to equilibrium points, which inspires the development of an efficient neural network-based algorithm for solving the MPEC problem.

This paper employs a neurodynamic model to solve MPEC problems. Using a smoothing technique, we first transform the original equilibrium constraint problem into an equivalent smooth nonlinear programming problem. Following this, we use the duality theory and the Karush-Kuhn-Tucker conditions to develop a neural network model that solves both the transformed nonlinear problem and its dual at the same time. Concerning the convergence behaviour and stability of the suggested neural network, the corresponding dynamical system is investigated. In this theoretical investigation, we demonstrate that the proposed neural network model is stable and converges to the local optimal solutions of the MPEC problem under mild conditions with the help of the Lyapunov stability theory and the LaSalle invariance principle. To assess the effectiveness of the model, several numerical examples are provided. The suggested model is also applied to the Stackelberg-Cournot-Nash equilibrium problem, a well-studied economic equilibrium problem, to demonstrate its practical significance. The findings highlight its potential as a significant tool for addressing MPEC problems in various real-world applications.

Semi-infinite variational programming problem with Caputo-Fabrizio fractional derivative

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In this work, we deal with a semi-infinite variational programming problem involving the Caputo-Fabrizio fractional derivative operator (SIVP). By using Slater's constraint qualification and some generalized convexity assumption, we first establish KKT necessary and sufficient optimality conditions for (SIVP). Later on, we study the Mond-Weir type dual model and discuss the several duality theorems. Additionally, some numerical examples have been given to support theoretical results.

Joint work with Prof. Anurag Jayswal.

On Approximate Variational Inequalities and Bilevel Programming Problems

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In this paper, we investigate a class of bilevel programming problems (BLPP) in the framework of Euclidean space. We derive relationships among the solutions of approximate Minty-type variational inequalities (AMTVI), approximate Stampacchia-type variational inequalities (ASTVI), and local ϵ -quasi solutions of the BLPP, under generalized approximate convexity assumptions, via limiting subdifferentials. Moreover, by employing the generalized Knaster-Kuratowski-Mazurkiewicz (KKM)-Fan's lemma, we derive some existence results for the solutions of AMTVI and ASTVI. We have

furnished suitable, non-trivial, illustrative examples to demonstrate the importance of the established results. To the best of our knowledge, there is no research paper available in the literature that explores relationships between the approximate variational inequalities and BLPP under the assumption of generalized approximate convexity by employing the powerful tool of Mordukhovich limiting subdifferentials.

Duality for Nonsmooth Semidefinite Multiobjective Programming Problems with Equilibrium Constraints Using Convexifiers

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In this article, we investigate the duality theorems for a class of nonsmooth semidefinite multiobjective programming problems with equilibrium constraints (in short, NSMPEC) via convexifiers. Utilizing the properties of convexifiers, we present Wolfe-type (in short, WMPEC) and Mond-Weir-type (in short, MWMPEC) dual models for the problem NSMPEC. Furthermore, we establish various duality theorems, such as weak, strong and strict converse duality theorems relating to the primal problem NSMPEC and the corresponding dual models, in terms of convexifiers. Numerous illustrative examples are furnished to demonstrate the importance of the established results. Furthermore, we discuss an application of semidefinite multiobjective programming problems in approximating K-means-type clustering problems. To the best of our knowledge, duality results presented in this paper for NSMPEC using convexifiers have not been explored before.

On LP well-posedness of multidimensional bilevel controlled variational inequalities

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In this paper, we introduce a new class of multidimensional bilevel controlled variational inequality problems (BVIP) and analyze their LP well-posedness and LP well-posedness in generalized sense using the monotonicity and hemicontinuity of the involved functionals. We also derive a sufficient condition for the generalized LP well-posedness by considering the non-emptiness and boundedness of the approximating solution set. Moreover, we have shown the generalized LP well-posedness with the help of the upper semicontinuity of the approximating solution set of the problem (BVIP). Additionally, some examples are constructed to demonstrate the theoretical results.

(Joint work with Prof. Anurag Jayswal and Prof. Jen-Chih Yao.)

Optimality and duality in multi-objective fractional programming problem using generalized convex functions

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In this article, we study a multi-objective fractional programming problem with generalized equality and inequality constraints in terms of contingent derivative over Banach space. Under appropriate assumptions, we obtain sufficient optimality conditions for the local weak minimizers, using generalized convexity assumptions under the framework of epiderivatives. Supportive examples are also presented corresponding to each duality model.

Characterizations of the Solution Set of Nonsmooth Semi-Infinite Programming Problems on Hadamard Manifolds

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This article is concerned with a class of nonsmooth semi-infinite programming problems on Hadamard manifolds (abbreviated as, (NSIP)). We introduce the Guignard constraint qualification (abbreviated as, (GCQ)) for (NSIP). Subsequently, by employing (GCQ), we establish the Karush-Kuhn-Tucker (abbreviated as, KKT) type necessary optimality conditions for (NSIP). Further, we derive that the Lagrangian function associated with a fixed Lagrange multiplier, corresponding to a known solution, remains constant on the solution set of (NSIP) under geodesic pseudoconvexity assumptions. In addition, we derive certain characterizations of the solution set of the considered problem (NSIP) within the framework of Hadamard manifolds. We provide illustrative examples that highlight the importance of our established results.

Customer Decision-Making in M/M/1 Queueing-Inventory Systems with Dual Replenishment Policies

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This study investigates a queueing-inventory system featuring two replenishment policies—normal and emergency replenishment—designed to manage inventory in response to varying customer demand. Customer arrivals follow a Poisson process with exponential service times. Customers engage in two

strategic behaviours: deciding, upon arrival, whether to join the queue and, after joining, whether to purchase. Normal replenishment is initiated when the inventory level reaches a threshold r , restoring the stock to level Q . In contrast, if the inventory level drops to a critical threshold $r_n < r$, an emergency replenishment is activated to quickly restock the inventory to $Q_n < Q$, minimizing the potential loss of customers. We derive a product-form solution for the system, identify equilibrium strategies, and optimize the service provider's revenue. In addition, we explore socially optimal strategies and quantify the inefficiency caused by the selfish behaviour of customers using the price of anarchy and the price of conservatism, assessing the system's inefficiency from the server's perspective. Numerical examples illustrate optimal reorder levels and quantities, providing valuable insights into inventory cycle lengths and offering practical and theoretical perspectives on managing queueing-inventory systems with strategic customer behaviour.

Parametric Analysis of Neutrosophic Vacation Queues

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This research investigates a novel approach to model and analyze vacation queueing systems under uncertainty using parametric programming within the framework of neutrosophic sets. Traditional queueing models often assume precise knowledge of system parameters, which may not accurately reflect real-world scenarios characterized by imprecision and uncertainty. Neutrosophy, with its inherent ability to handle indeterminacy, provides a robust framework for modeling such systems.

This study introduces a parametric programming approach to the neutrosophic vacation queueing model, where system parameters are represented as neutrosophic values, encompassing truth, indeterminacy, and falsity memberships. The proposed approach leverages the flexibility of parametric programming to efficiently explore the solution space and identify optimal system configurations under various uncertainty levels.

Integrating ARMAX Forecasting with Game Theoretic Applications for Analyzing HRV Data in Strategic Decision-Making

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In this study, heart rate variability has been analyzed with ARMA and ARMAX model. We show that our results outperforms existing ARIMA model. Model performance have been evaluated. We propose that HR data as exogenous variable can help in decision making on the physiological condition of an individual using game theoretic principle.

Interval-Valued Bilevel Optimization Problem with Equilibrium Constraints: Application to Risk Management

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This study investigates an interval-valued bilevel optimization problem with equilibrium constraints. Initially, we convert this complex problem into a scalar problem with a single level. Then, we use convexifiers to establish a suitable Abadie-type constraint qualification. Subsequently, we present the necessary optimality conditions that are always satisfied for a local LU-optimal solution. Additionally, under generalized convexity assumptions, we derive sufficient optimality conditions that guarantee a feasible point is LU-optimal, LU-weakly optimal, or both. To demonstrate the validity of our findings, we examine an everyday risk management problem.

SOME APPLICATIONS OF DIRECTIONAL CONVEXIFIERS IN NONSMOOTH OPTIMIZATION PROBLEMS

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Recently, Dempe and Pilecka (2015) introduced the notion of directional convexifiers in the context of bilevel programming problems for nonsmooth discontinuous functions. This notion generalizes the notion of convexifiers introduced by Jeyakumar and Luc (1999) for nonsmooth continuous functions. This talk deals with some recent applications of directional convexifiers in optimization problems, e.g. Lafhim and Kalmoun (2023), Sachan and Laha (2024), Mohapatra et al. (2024) etc.

Optimizing Hyperparameters in Quantum Neural Networks using the ANOVA Framework

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The advent of quantum computing has revolutionized the way of solving complex problems, especially with the help of Quantum Neural Networks (QNN). Quantum neural networks rely on the principles of quantum mechanics such as the principle of superposition which states that a quantum bit (qubit) can hold multiple states simultaneously and the principle of entanglement that states there is a mutual link between the qubits due to which information can be shared instantaneously. Quantum neural networks are applied in various domains such as classification, regression, and reinforcement learning. However, the QNNs can perform optimally only when the specified parameters are adjusted well to perform the learning process. These parameters are described as hyperparameters as they control the model's

performance. The hyperparameters in QNNs include the learning rate, the optimization algorithm, and how quantum circuits are designed.

In Classical machine learning, hyperparameters can be directly tuned with the help of grid search, random search, and Bayesian optimization. However, using these techniques has some difficulties when we apply them to quantum systems. Quantum devices are always constrained by the capabilities of the number of qubits accessed and are also affected by effects such as noise and decoherence. In addition, the specified hyperparameters for the quantum model include the ansatz type (the structure of the quantum circuit), entanglement strategy, and the quantum kernel, making it even more complex to optimize. These difficulties are significant now in the so-called Noisy Intermediate-Scale Quantum (NISQ) era, where quantum devices are still very limited. This is where the Analysis of Variance (ANOVA) framework comes into play. This statistical tool is used in quantum machine learning to solve these challenges. It helps in identifying how different hyperparameters and their combinations can impact the performance of QNN. This doesn't merely extend to minimizing which settings are important or which aren't; it also defines interactions between them. When combined with random forest models, this method provides one of the best ways to fine-tune these settings due to the granularity of the results.

This work deals with several significant hyperparameters such as ansatz type, entanglement scheme, initial learning rate, optimizer settings, and features map. These settings play different roles: the ansatz type and entanglement type determine the accuracy with which quantum circuit can derive the solution for the complex problems and the learning rate and optimizer used, will determine how fast the model will learn. Another important component is the feature maps that map the classical data into the quantum states. The study thoroughly examined different Quantum Neural Network (QNN) designs across multiple datasets using the ANOVA framework. The outcomes showed that this approach greatly improves QNNs' overall performance in addition to speeding up hyperparameter adjustment. In this way, researchers will enjoy flexibility in time and can focus only on the most significant contexts, while taking into account some of the specific features of the quantum systems, for instance, the limited coherence time of qubits or their error rate.

Using the ANOVA framework, it is possible to systematically investigate hyperparameters and determine which of them contributes most toward QNN performance. This targeted approach also helps to minimize the computational cost that is tied to many cross-tuning methodologies and therefore makes the implementation of QNNs on existing quantum platforms tractable. In Conclusion, the ANOVA framework bridges the gap between classical and quantum hyperparameter tuning and is a revolutionary tool for improving QNNs. It shows a practical approach to the issues inherent in quantum systems and promotes the development of quantum machine learning. This method not only optimizes the quantum

models' performances but also explains them to open up the possibility of more practical solutions for challenging problems in the real world.

On Weak Sharpness and Optimality Conditions in Multiobjective Optimization through Convexifiers

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(joint work with C.S. Lalitha)

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This work deals with the study of local weak sharp efficient solutions [1] for a multiobjective optimization problem. This notion is an extension of the notion of local weak sharp minima for scalar programming problems introduced by Ferris [2]. This notion is crucial while establishing convergence analysis of algorithms [3] and error bound property [4]. We derive necessary and sufficient optimality conditions for a local weak sharp efficient solution of a nonsmooth multiobjective optimization problem with both equality and inequality constraints through convexifiers [5, 6]. The Karush–Kuhn–Tucker (KKT) type necessary optimality conditions for local weak sharp efficient solutions are obtained by employing the weakest known constraint qualification, Guignard constraint qualification [7]. Further the sufficient optimality conditions are established by considering the objective and inequality constraint functions to be convex and quasiconvex [8], respectively while the equality constraints are additionally assumed to be both quasiconvex and quasiconcave. We also provide a scalar characterization for local weak sharp efficient solutions in terms of local weak sharp minima of an associated scalar problem obtained by using nonlinear Gerstewitz's function [9].

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Application of Structured Sparsity Techniques in Natural Language Processing

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The rapid growth in transformer-based architectures and other large-scale neural networks has revolutionized natural language processing (NLP). However, the computational and memory requirements of these models impose significant challenges, especially for deployment in resource-constrained environments. To address these challenges, this paper applies a family of structured sparsity inducing adaptive optimizers designed to reduce the size and complexity of deep learning models for text data, while maintaining or improving performance.

The paper (Deleu et al., 2023) extends popular adaptive optimization algorithms, such as Adam and RMSProp, by incorporating structured regularization techniques (L1 and L2 Norm). This approach dynamically prunes redundant groups of parameters (e.g., neurons, channels, or layers) during training, resulting in models with structured sparsity that are both computationally efficient and hardware-friendly. The main innovation is the use of group-based penalties, including Group Lasso and variants tailored for specific neural network architectures, to promote sparsity at various levels of granularity.

The proposed framework is particularly effective for transformer-based models, which are known for their overparameterization. By inducing sparsity in a structured manner, this approach not only reduces the parameter count but also eliminates redundant computations, thereby accelerating inference times.

In this work, we performed text classification using structured sparsity techniques with transformer-based model, we found reduced active parameters of the model i.e. active layers, neurons with mixed L1 and L2 regularization. In conclusion, Evaluation of this model demonstrated that its accuracy was comparable to that of standard transformer-based models. Additionally, the sparse model significantly reduced computational and memory costs, making it more efficient and better suited for classifying large text datasets.

Enhancing Retrial Queue with Neutrosophic Sets: An Innovative Parametric Programming Approach to Uncertainty in Arrival, Service, and Retrial Parameters

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This research paper explores the integration of neutrosophic sets into queuing theory, particularly on retrial queues. Here the arrival rate, service rate and retrial rate as single-valued trapezoidal neutrosophic numbers (SVTNN). The paper applies the concepts of α -cuts and Zadeh's extension principle to transform the neutrosophic queueing model into a crisp model using the (α, β, γ) -cut technique. This transformation allows for a deterministic analysis of the system, enhancing the descriptive power of retrial queue models. The study includes theoretical formulations, numerical examples, and comparative analysis, demonstrating the effectiveness of neutrosophic retrial queue models in capturing the complexity of uncertain environments.

Analysis of Single and Multiple Server Finite Capacity Neutrosophic Queueing Model

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This research work explores the use of neutrosophic queueing models to evaluate system performance, offering a more nuanced approach compared to traditional deterministic or probabilistic models. By incorporating truth, indeterminacy, and falsity, neutrosophic models provide a more comprehensive analysis of uncertainty in queueing systems. The work focuses on Single-Valued Trapezoidal Neutrosophic Numbers (SVTNNs). The model discussed is a Markovian queue with a single server and finite capacity, using neutrosophic methods to calculate performance metrics. The neutrosophic model is transformed into a deterministic model using Zadeh's extension principle and the (α, β, γ) -cut technique. An example is provided to illustrate the process.

The Modified Solution Space Truncation Algorithm

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We consider a stochastic discrete optimization problem where the objective is $\min_{x \in X} E[h(Y(x))]$, where Y is a random function of x , $x \in X$. The set X is a discrete and finite set, and thus x is a candidate solution for the stochastic discrete optimization problem. Given that Y is random, hence h is also random. Estimates of Y at any $x \in X$ are denoted by $\hat{y}(x)$. These estimates are used to obtain estimates of h , denoted by \hat{h} , as inputs in the quest for the optimal solution x^* , where $x^* \in X$. If this quest for x^* is carried out by obtaining the \hat{y} values through simulation, then this approach is known as simulation optimization. In practice, absence of relevant structural properties (like discrete convexity and Lipschitz continuity) of h can result in the performance of simulation optimization to be poor, especially if the simulations are computationally expensive. In this study, we devise an algorithm to use the property of discrete convexity to eliminate sub-optimal solutions, even without screening them, to reduce the solution space for simulation optimization. This work is an extension of the study of Das et al. 2021 [1], where the Solution Space Truncation (SST) algorithm was developed. The SST algorithm works by considering that h is a discrete convex function on X . The convexity of h is established through a strictly monotonic relationship between Y and x . The SST algorithm assumes very small variance of $\hat{y}(x)$, $\forall x \in X$. We relax this assumption and modify the SST algorithm, thus naming it the Modified Solution Space Truncation Algorithm (Mo-SST). We consider x and Y to be vectors. The algorithm consists of two passes, with the second pass starting after the end of the first one. The first pass starts with the smallest possible $x \in X$, and the second pass starts with the largest possible x in X' , where $X' \subseteq X$ and is the remaining solution space after the first pass. The second pass returns a set X'' , where $X'' \subseteq X'$, as the output. A suitable simulation optimization algorithm can then be applied on X'' for finding the optimal solution. The key ideas of the Mo-SST algorithm are:

- (1) The algorithm involves carrying out a t-test between $\hat{y}(x_u)$ and $\hat{y}(x_v)$, for any $x_u, x_v \in X$ such that $x_u \neq x_v$. Say, Y has a strictly increasing relationship with x , then \hat{y} should increase with the increase in x . Thus, if $x_u \prec x_v$, then $\hat{y}(x_u) \prec \hat{y}(x_v)$ ($\hat{y}(x_v)$ is element-wise greater than $\hat{y}(x_u)$ and each element-wise difference is statistically significant); otherwise we keep increasing the number of replications used to estimate $\hat{y}(x_u)$ and $\hat{y}(x_v)$ till $\hat{y}(x_u) \prec \hat{y}(x_v)$. The requirement of the t-test and incrementing the number of replications for calculating \hat{y} differentiate the Mo-SST algorithm from the SST algorithm.
- (2) In the first pass, for $x_u, x_v \in X$, if $x_u \prec x_v$ and the t-test finds that $\hat{y}(x_u) \prec \hat{y}(x_v)$, then (i) if $\hat{h}(x_v) < \hat{h}(x_u)$, we eliminate all solutions x such that $x \preceq x_u$ (ii) if $\hat{h}(x_v) > \hat{h}(x_u)$, we stop the first pass and move to the second pass with a reduced solution space X' .
- (3) Similarly, in the second pass, for $x'_u, x'_v \in X'$, if $x'_u \succ x'_v$ and the t-test finds that $\hat{y}(x'_u) \succ \hat{y}(x'_v)$, then (i) if $\hat{h}(x'_v) < \hat{h}(x'_u)$, we eliminate all solutions x such that $x \succeq x'_u$ (ii) if $\hat{h}(x'_v) > \hat{h}(x'_u)$, we stop the Mo-SST algorithm and obtain X'' as the reduced solution space.

We demonstrate the Mo-SST algorithm using a toy queueing problem. We consider two service stations in series, wherein a customer has to receive service at Station 1 before proceeding to receive service at Station 2. We consider Poisson arrivals to Station 1 and uniformly distributed service times at either station (setting the expected service times at Station 1 and Station 2 at t_1 and t_2 respectively). Station 1 has s_1 number of parallel servers and Station 2 has s_2 number of parallel servers. The number of parallel servers at either station can vary from 1 to 20. Thus, the number of possible combinations of (s_1, s_2) is 400. As the number of parallel servers at either station increases, the fraction of waiting time (denoted by W) decreases and the utilization (U) also decreases. Now, as the utilization decreases, the fraction of idle time (denoted by I) increases. Thus, with $x = (s_1, s_2)$, $X = \{(s_1, s_2) \mid 1 \leq s_1, s_2 \leq 20\}$, $Y = (\frac{1}{W}, I)$ and setting the objective as $\min_X (c_w E[W] + c_i E[I])$, where c_w and c_i are the costs per unit of expected fractions of waiting time and idle time respectively, we apply the Mo-SST algorithm with suitable parameterization. We obtain $X'' = \{(5, 6), (6, 5), (5, 7), (7, 5), (6, 6), (6, 7), (7, 6)\}$ as the reduced solution space.

Analysis of a Finite Multi-Server Queue with Admission Control and Customer Balking

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The multi-server queueing model finds applications across various domains, including railway stations, shopping malls, fuel stations, etc. Customers waiting for service often exhibit discouragement behaviors such as balking (refusing to join the queue), reneging (leaving the queue before service) during their wait. In finite-capacity queueing systems, admission control plays an important role in managing customer flow effectively. In this study, we investigate a finite-capacity multi-server queueing model with additional servers deployed during rush hours. The model incorporates an admission control F-policy, customer balking behavior, and server breakdowns. A mathematical analysis is conducted by formulating the steady-state Chapman–Kolmogorov (C–K) equations, which are solved using a recursive technique to establish the probability distributions for various system states.

These distributions are further utilized to derive key system performance measures, including the average number of customers in the system, the number of busy servers, and the probabilities of server idle and busy states. Moreover, a non-linear cost function is developed to determine optimal decision variables such as the number of permanent servers, admission control thresholds, and service rates. The particle swarm optimization (PSO) algorithm is employed to solve the cost optimization problem, enabling decision-makers to efficiently allocate resources and streamline system operations. The findings of this study demonstrate the practical applicability of the proposed model in real-world scenarios, such as electric vehicle charging stations (EVCSs).

An innovative method for evaluating Pythagorean fuzzy reliability

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Reducing waste, enhancing safety, and increasing the capacity of food systems to adapt to climate change and population growth are essential for a secure, healthy, and sustainable food supply. Consequently, reliability is essential in daily life and system design, since it involves assessing the probability of a system's components functioning well. Evaluating reliability in engineering systems is very essential. However, conventional testing sometimes neglects to account for the system's unpredictability, including mistakes, ambiguities, and data inadequacies. This work presents an innovative method for evaluating Pythagorean fuzzy reliability, using lifespan parameters to create triangular Pythagorean fuzzy numbers and to describe fuzzy variables within the context of the Weibull distribution and universal generating function. This approach is then used in the food waste system to evaluate Pythagorean fuzzy reliability, sensitivity, mean time to failure, and cost analysis. The arithmetic mean operator is also used, assigning equal weight to different time periods. A tabular and graphical depiction of the findings is also provided for clarification.

Compressive Sensing: A Paradigm Shift in Computer Tomography Image Reconstruction

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Computer tomography (CT) is one of the most important and popular diagnostic medical imaging tools in different health scenarios. The biggest disadvantage of CT based image reconstruction is of hazardous due to X-rays. For many decades the efforts have been made to reduce the X-ray dosage in acquiring the projection data which is the main input for the CT image reconstruction algorithm. Over the years, there have been many efforts to develop different CT image reconstruction algorithms. Apart from filtered backprojection (FBP) (which is the most commercially used algorithm), there have been iterative reconstruction algorithm using different approaches of optimization techniques. Recently, deep learning-based algorithms are gaining a lot of attention. In all these approaches the focus is to reduce the X-ray dosage and attain superior image quality.

In this paper, our focus is to target the medical scenario where a patient has to go through several times for CT image reconstruction over a long period of time. In such case, our target is to reduce the X-ray

dosage by acquiring less number of projection data after every subsequent CT image reconstruction. We would like to extract the information from previous reconstructed images in such a way that it helps in reducing the number of projection data for the next CT image reconstruction. This will directly benefit in reducing the X-ray dosage. To construct such a framework, compressive sensing technique is the best suited technique. Traditionally, compressive sensing is based on the concept of compression by acquiring minimalistic data for reconstruction. Compressive sensing is an inverse problem, solving linear system of equations $y = Ax$, whereas $y \in R^N$, $x \in R^M$, $A \in R^{N \times M}$ and $N \ll M$. In compressive sensing framework, only known information is y and we need to estimate A to obtain x . The matrix A is called as system matrix and plays the most crucial role in perfect reconstruction, i.e. obtaining the x . Essentially for perfect reconstruction in compressive sensing system matrix must satisfy certain known properties, such as restricted isometry property or mutual coherence. For real applications, to find such a matrix is like finding a needle in haystack. In our research work, we are trying to establish a mathematical framework for estimation of system matrix which satisfies the property of mutual coherence for CT image reconstruction. Once we have such a system matrix then as a next step our target is to use optimization technique for solving CT image reconstruction as a linear system of equations problem for perfect / almost perfect reconstruction.

A Genetic Algorithm for Multicommodity Stochastic-flow

Networks System with Unreliable Nodes

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Many real-life systems can be modelled as stochastic-flow networks, where resource flows traverse through unreliable nodes and arcs. This paper addresses the problem of optimizing resource flow allocation and control strategies in multicommodity stochastic-flow networks under budget constraints. To evaluate the reliability of such networks, we develop a genetic algorithm (GA) that identifies every vector with limited capacity meeting the specified demand and budget constraints. The system reliability is then resolute based on these vectors, ensuring the maximization of the probability that sink nodes' demands are satisfied as resource flows transmit from source nodes. The proposed GA is designed to efficiently seek the optimal allocation strategy, balancing computational complexity with effectiveness. To demonstrate the efficacy of the algorithm, a numerical example is presented. Results show that the GA not only achieves high reliability but also exhibits better temporal efficiency, making it applicable to larger and more complex networks

Optimality conditions for robust nonsmooth multiobjective optimization problem using tangential subdifferentials

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In this article, we use the worst-case approach for finding approximate efficient solutions of the robust multiobjective programming problem. The considered robust multiobjective programming problem with uncertainty in both the objective and constraint functions. We establish both necessary and sufficient optimality conditions for a feasible solution to be an approximate efficient solution using tangential subdifferentials.

Integrating Machine Learning and Fuzzy Logic for Large-Scale Decision- Making Frameworks

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This study presents a novel Fuzzy Large-Scale Decision Making (FLSDM) framework designed to address the complexities of handling a large number of criteria in decision-making scenarios. In an environment of Triangular Spherical Fuzzy Numbers (STFN), an integrated feature selection technique is suggested to find the most relevant criteria from a large set. The Integrated Determination of Objective Criteria Weights (IDOCRIW) and Additive Ratio Assessment (ARAS) methods are extended for the STFN environment to calculate criteria weight and alternative ranking, respectively. The application of the proposed framework is demonstrated by ranking sustainable energy sources based on a comprehensive set of sustainability criteria. A thorough analysis is carried out to assess the developed model's efficacy and robustness, emphasizing its potential for handling high-dimensional decision-making challenges.

**Approximation of Common Fixed Points and Solution of Variational Inequality using a new
Viscosity-extragradient method of S-iteration type**

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In this article, we have introduced a new viscosity-extragradient method of S-iteration type iterative technique by combining S-iteration process, viscosity method and extragradient method for approximating common elements of the set of fixed points of nonexpansive mapping, set of fixed points of a nearly nonexpansive mapping and set of solutions of variational inequality problem formed by monotone Lipschitz continuous mappings in Hilbert spaces. A strong convergence result for the sequence generated by the proposed iteration process is established.

A bi-criteria bottleneck assignment problem with categorized tasks

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Many real world problems give rise to the projects that involve sequential execution of tasks as some of them may not be executed without completing the others in hierarchy. Such problems give rise to the multi-stage assignment problems. As the name suggests in a multi-stage assignment problem, different sets of tasks may need to be executed in different stages and a collective pre-defined objective is to be optimized. If the decision maker wishes to assign each of the tasks to an agent which is most suitable to execute it, then the problem becomes more practical in nature and helps him to take efficient decisions. But the best suitability may not be achievable for each task involved in the project, and therefore, the decision maker seeks for the compromised solutions that optimize a collective (comprising of all the stages) pre-defined objective and maximize the minimum achievable level of satisfaction corresponding to various agent-task assignments, simultaneously. In this paper, such a bi-criteria bottleneck two-stage assignment problem is discussed which has two different sets of tasks, one consisting of the primary tasks which are to be executed in Stage-I of the problem and the other one consisting of secondary tasks, to be executed in Stage-II of the problem (after the completion of Stage-I). However, in both of these stages, the tasks can be executed in parallel. Further, it is assumed that the tentative execution time

for each agent-task link is known to the decision maker. The objective of the problem is to minimize the sum of execution times of both the stages and to maximize the minimum suitability of various agents to execute various tasks, simultaneously. An efficient iterative algorithm is developed to find all the non-dominated/Pareto optimal solutions assignment schedules and their corresponding points in the criteria space generated by both the objectives. Numerical illustrations are provided to validate the theory of the algorithm. Also, the proposed algorithm is coded in MATLAB and the Pareto frontiers of various instances (small to large sized) of the problem are obtained.

Reliability analysis of energy management subsystem in a high altitude platform station

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High Altitude Platform Station (HAPS) is a network node that functions at an altitude of around 17-20 km in the stratosphere. It plays a crucial role in the provision of communication services. The increasing need for faster data rates and better performance has prompted wireless network providers to consider energy management in High Altitude Platform Station (HAPS) over Internet of Things (IoT). Energy management is a basic requirement for solar-powered HAPS, which has not been paid attention in recent times. This HAPS architecture suffers from limited and unreliable energy management resources. Although there are many competent technologies working for the purpose of capacity enhancement and reliability of the Energy Management Subsystem (EMS), such as optimization of propulsion energy and energy optimization technologies, there is still room and need for further improvement. In this work, we introduce a three-level hierarchical model to evaluate the reliability characteristics of EMS in HAPS. Two different methods, Continuous-Time Markov Chains (CTMCs) and Reliability Block Diagrams (RBDs), are used to model this hierarchical model of EMS in HAPS. CTMC-based modeling is used to evaluate performance metrics, such as the reliability function and Mean Time To Failure (MTTF), at two different levels by using k-out-of-n stochastic modeling technique. Further, the proposed k-out-of-n models for repairable and non-repairable batteries and solar cells are subject to an analytical investigation, and their validity is confirmed using discrete event simulation. The precision in estimating reliability and temporal attributes for diverse input data, employing important parameters, is attained through the widely recognized ML models such as K-Nearest Neighbors (KNN) regression, polynomial regression, etc.