Conference on Analysis

(*December 23-25, 2011*)
Venue : Indian Statistical Institute, New Delhi

In honour of Professor K. R. Parthasarathy

Conference Schedule

December 23rd

09:00-09:15	Getting together
09:15-09:30	Welcome and opening remarks by Director, Indian Statistical Institute
09:30-10:20	S. R. S. Varadhan *Large deviations for random graphs and random matrices*
10:30-11:20	B. V. Rajarama Bhat *Roots of states*
11:20-12:00	Tea
12:00-12:50	T. R. Ramadas *The chiral boson and function theory on the unit disc*
13:00-14:00	Lunch
14:10-15:00	K. B. Sinha *Master Equation, stochastic Schrodinger equation–a review and history*
15:15-16:00	Felicitation
16:00	Tea

December 24th

09:30-10:20	J. M. Lindsay *Quantum stochastic analysis*
10:30-11:20	V. S. Sunder *From graphs to free probability*
11:20-12:00	Tea
12:00-12:50	D. Goswami *Quantum isometry groups: an overview*
13:00-14:00	Lunch
14:10-15:00	M. S. Raghunathan *On horocycle flows*
15:10-16:00	Manjunath Krishnapur *Determinantal point processes*
16:10-17:00	S. G. Dani *Asymptotics of probability measures on vector spaces, and applications*
17:00-17:30	Tea
19:00	Dinner at India International Centre (by invitation)

December 25th

09:30-10:20	Jaikumar R.	
	Communication complexity	
10:30-11:20	G. Misra	
	The Bergman kernel for the symmetrized poly-disc	
11:20-12:00	Tea	
12:00-12:50	M. Madiman	
	Some recent themes in the intertwining of information theory and probability	
13:00-14:00	Lunch	
14:10-15:00	Claus Koestler	
	The wonderful world of quantum coin tosses	
15:10-16:00	V. P. Belavkin	
	An introduction into modular theory of stochastic integration	
16:00-16:30	Tea	

Abstracts of Talks

Large deviations for random graphs and random matrices

We consider dense random graphs, with a large number of vertices in which each edge is turned on independently of others with probability p. The large deviation probabilities for the number of triangles, or other finite subgraphs, in the entire graph to be much different from their expected number is calculated. There is a phase transition in the types of graphs that contribute to this small probability. In the case of triangles, if the difference is smaller than a certain proportion, the graphs are similar random graphs as the original one but with a different value of p. If the the number of triangles is larger than the expected number by more than a certain critical proportion the contributions comes from cliques where a higher density of edges show up among a smaller group of vertices.

The techniques developed here allows us to estimate the probability of having a certain number of eigenvalues of order n for a random symmetric matrix of i.i.d entries for which a typically all eigenvalue are of order $\sqrt{n}$. The proofs for both results depend on the notion of graph limits and Szemeredi's regularity lemma.

Roots of states

Given a state ϕ on a unital C^*-algebra $\mathcal{A}$ we look at unital quantum dynamical semigroups $\{\tau_t\}_{t\geq 0}$ on $\mathcal{A}$ such that $\tau_{t_0}(\cdot) = \phi(\cdot)I$ for some $t_0 > 0$. We see that for the von Neumann algebra $\mathcal{B}(\mathcal{H})$, such quantum dynamical semi-groups (with pure states) dilate to semigroups of unital $*$-endomorphisms (E_0-semigroups) in standard form and conversely all E_0-semigroups in standard form arise this way. I am talking about a specific problem in the context of a general theory.

The chiral boson and function theory on the unit disc

Conformal field theories (CFTs) are related (in Mathematics) to algebraic geometry, infinite-dimensional Lie algebras and probability and (in Physics) to critical phenomena and string theory. From a Mathematical point of view much of the formalisation has been from the point of view of algebra - in fact using formal power series. I will give a definition of the simplest "chiral" or "holomorphic" CFT using elementary function theory. If time permits, I will also explain operator product expansions.

Master Equation, Stochastic Schrodinger Equation – a review and history

Beginning with Physicists' efforts(particularly Pauli's) to describe non-equilibrium phenomena, an attempt will be made to describe a trajectory of the evolutions of mathematical ideas, culminating in the Hudson- Parthasarathy Equation. Along the way, one possible precise mathematical meaning to Pauli's "random wave approximation" appears.

Quantum stochastic analysis

Quantum stochastic analysis was pioneered by Hudson and Parthasarathy in the nineteen-eighties. Their theory was founded on the following insight. There is a deep relationship between two iconic equations: the formal Itô relation for Brownian increments

$$(dB_t)^2 = dt,$$

and the canonical commutation relations of quantum mechanics

$$pq - qp = -i\,\hbar.$$

The purpose of this talk is to describe quantum stochastic analysis from a modern perspective. This involves a nonadapted departure from Itô calculus on the one hand, and its fuller embrace on the other. New commutation relations play a significant role.

Apart from the Indian and British Schools of Quantum Probability (which include many academic descendants of Hudson and Parthasarathy), significant contributions to the development of the theory have come from Italy, Russia, Germany and the Strasbourg School in France.

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From graphs to free probability

We investigate a construction which associates a finite von Neumann algebra $M(\Gamma, \mu)$ to a finite weighted graph (Γ, μ). This also yields a 'natural' example of a Fock-type model of an operator with a free Poisson distribution.

Pleasantly, but not surprisingly, the von Neumann algebra associated to to a 'flower with n petals' is the group von Neumann algebra of the free group on n generators.

In general, the algebra $M(\Gamma, \mu)$ is a free product, with amalgamation over a finite-dimensional abelian subalgebra corresponding to the vertex set, over algebras associated to subgraphs 'with one edge' (actually a pair of dual edges).

Quantum isometry groups: an overview

I shall briefly explain with examples the notion of quantum isometry groups for classical and noncommutative manifolds and metric spaces formulated by myself and my collaborators over the last few years.

On horocycle flows

In this talk I will give representation theoretic proof of the unique ergodicity of horocycle flows on compact quotients of semisimple groups. This is a special case of a theorem first proved by Dani (Dani's theorem itself is a consequence of a theorem due to Ratner).

Determinantal point processes

Determinantal point processes are a general class of random point sets inspired by Quantum physics. They occur in surprisingly many different situations in combinatorics, probability and statistical physics. We give a survey of the subject, emphasizing some general properties as well as the plethora of examples.

Asymptotics of probability measures on vector spaces and applications

Given a sequence of probability measures $\{\mu_n\}$ on the euclidean space $V = \mathbb{R}^d$, and a sequence $\{T_n\}$ of linear transformations of V, we study the limit points of the sequence $\{T_N(\mu_n)\}$. Results on these measures, and especially their supports, are involved in many problems concerning probability measures on Lie groups, including convergence of types, study of factors and roots, and concentration functions. In this talk we describe the results and discuss the applications.

Communication complexity

We will provide an introduction to the area of Communication Complexity, give examples of how it is connected to various other areas of Theoretical Computer Science, and show how tools from Analysis, Linear Algebra and Information Theory are applied in determining the communication complexity of problems.

The Bergman kernel for the symmetrized poly-disc

This is a joint work with S. Shyam Roy and G. Zhang.

Some recent themes in the intertwining of information theory and probability

Two areas in which Professor KRP has made major contributions have been in the foundations of probability theory, and in information theory (both in the classical and quantum setting, though we will not venture into the quantum world). The recognition that these two fields are intertwined at a fundamental level goes back to Kolmogorov, Pinsker, Shannon, and their contemporaries, but the breadth and depth of this intertwining has been uncovered further over the subsequent decades. Rather than discussing well known aspects of connections between probability and information theory such as the theory of large deviations, we will focus on outlining some relatively new directions. First, following work of Barron and others, we will outline how classical limit theorems (particularly the central limit theorem) can be viewed as phenomena of entropy increase (like the "second law of thermodynamics"). Second, we will discuss an information-theoretic approach to the study of log-concave measures, particularly in high but finite dimension, which is also closely related to geometric functional analysis and the theory of concentration of measure. In particular, we will describe a quantitative Shannon-McMillan-Breiman type phenomenon for log-concave (not necessarily stationary) measures, as well as a reverse entropy power inequality for the more general class of convex measures.

The wonderful world of quantum coin tosses

The outcome of one coin toss is a subject of classical probability, but more than one coin toss may occur in a quantum world, depending on whether these coin tosses commute with each other or not. My talk will introduce into such quantum coin tosses and address recent progress on the study their joint distributions. Most parts of my talk should be accessible to a wider mathematical audience.

An introduction into modular theory of stochastic integration

We review the basic concepts of stochastic integration and reformulate them in terms of a Banach four-normed *-algebra with the associative product given by the stochastic covariation. We show that this nonunital algebra has two nilpotent first and second order *-ideals with the C*-normed quotient algebra, being a generalization of the C*-algebra corresponding to the only one nontrivial operator norm. The noncommutative generalization of this algebra called B*-algebra leads to the *-algebraic theory of quantum stochastic integration developed in [1-4]. The main notions and results of classical and quantum stochastic analysis are reformulated in this unifying approach. The general Lévy process is defined in terms of the modular B*-Itô algebra and the corresponding quantum stochastic Master equation on the predual space of a

W*-Algebra is derived as a noncommutative version of the Zakai equation driven by the process. This is done by a noncommutative analogue of the Girsanov transformation, which we introduce in full generality here.

References:

1. *A new form and *-algebraic structure of quantum stochastic integrals in Fock space*, Rendiconti del Seminario Matematico e Fisico di Milano, **LVIII** (1988), 177–193.

2. *A quantum nonadapted Ito formula and stochastic analysis in Fock scale*, J of Funct Analysis, **102 no. 2** (1991), 414–447.

3. *Chaotic states and stochastic integrations in quantum systems*, Usp. Mat. Nauk, **47** (1992), 47–106; translation in: Russian Math. Surveys, **1** (1992), 53–116.

4. *Quantum Lévy-Itô Algebras and Noncommutative Stochastic Analysis*, Stochastics, to be published.