The Political Intergenerational Welfare State*

Monisankar Bishnu
Indian Statistical Institute, Delhi &
Australian National University†

Min Wang
Peking University‡

February 7, 2017

Abstract

Using a three-period overlapping generations economy framework, we characterize an intergenerational welfare state with endogenous education and pension under voting. We show that although politically establishing Pay-As-You-Go (PAYG) social security in isolation in a dynamically efficient economy will always reduce the capital investment and therefore the social welfare as expected, in contrast politically implementing education-pension policy package instead can improve both human and physical capital accumulation and social welfare over laissez faire. However for this the political influence of the old has to be small thus limiting the size of the PAYG social security program.

Keywords : Education, Pension, Probabilistic Voting

JEL Classification: D91, E6, H3, H52, H55, I21

*We would like to thank the editor (B Ravikumar), and the anonymous associate editor and referees for their extremely valuable suggestions. We also thank Torben Andersen, Costas Azariadis, Joydeep Bhattacharya, Michelle Boldrin, Sylvain Chassang, Raj Chetty, Juan Carlos Cordoba, David De la Croix, Ayşe İmrohoroğlu, Murat Iyigun, Frederico Gil Sander, Todd Sandler, Tridib Sharma, Aloysius Siow, Zheng Michael Song, Carlos Urrutia, Cheng Wang and Itzhak Zilcha for their numerous helpful comments and suggestions. Thanks are also due to Marina Azzimonti, Pierre-Daniel Sarte and Jorge Soares for sharing their codes which helped us formulating our codes used in the earlier version of our work.

†Economics and Planning Unit, Indian Statistical Institute 7, S.J.S. Sansanwal Marg, New Delhi 110 016, India. Email: mbishnu@isid.ac.in.

‡China Center for Economic Research, National School of Development, Peking University, Beijing 100871, China, Email: wangmin@nsd.pku.edu.cn.

1 Introduction

For more than a century, old age pension benefit has served as one of the most important transfers that one receives in the time of need when she is no longer working. However, the demographical changes such as aging[1] and population growth coupled with the emergence of different political views resulted in a serious concern being raised about the solvency of social security programs in many countries. Further, in theory, the standard Aaron-Samuelson result guarantees that implementing pay-as-you-go (PAYG) social security can not be welfare improving if the economy is dynamically efficient and therefore on the grounds of efficiency, it has no support.[2] Naturally, the continuation of PAYG social security is therefore at the heart of the challenges and have ignited policy debates in many countries throughout the world.[3]

However, in reality, pension is not implemented in isolation and as a backward intergenerational transfer, it is deeply interwined with the forward intergenerational transfer, namely, public education. Becker and Murphy (1988) actually connects education investment made by the parents with social security by considering this as a trade among generations: children receive education from their parents and in exchange pay for their old age benefits. Moreover, since the old-age social security benefits under PAYG very much depends on the labor productivity of the future generations, reexamining PAYG social security program along with education investment has naturally gained momentum in recent years.

Rangel (2003) focuses on the issue of sustainability when both forward and backward intergenerational goods are present and shows that backward intergenerational goods, such as social security, play a crucial role in sustaining investment in forward intergenerational goods like education: without them investment in forward intergenerational goods is inefficiently low, but with them optimal investment is possible. Boldrin and Montes (2005) confirms that when credit market for education loan does not exist, the establishment of PAYG social security, coupled with public education, can be justified as a means to implement an intergenerational transfer scheme that replicates complete market allocations and therefore can act as a substitute for the missing education-loans market.[4] In a relatively recent work,

[1]For example, in developed countries, older population will on average reach one fourth of their total population in three decades (See World Population Prospects, United Nations, 2013).

[2]There can be other rationale behind continuing PAYG social security even in a dynamically effieicnt economy, for e.g., when people suffer from myopia (see for e.g., Feldstein (1985), Andersen and Bhattacharya (2011)).

[3]See Cooley and Soares (1999), Boldrin and Rustichini (2000), Orszag and Stiglitz (2001), Feldstein (2005) for example.

[4]Wang (2013) extends their study by endogenizing the imperfection of the credit market and shows that the result could hold for a wide range of parameters even when borrowing constraints for education loan endogenously arise as the result of limited commitment. Docquier et al. (2007) shows that because of

Andersen and Bhattacharya (2015) points out that if the economy is dynamically efficient, the golden rule level of human capital is higher than what is achievable with complete education-loan markets alone. Further, they show that a carefully designed education-pension welfare package can improve the welfare compared to the laissez faire outcome, and owing to the intergenerational human capital externality, the pension component of such a package will be eventually phased out entirely. In their study, PAYG social security is an important means to get the welfare improving education-pension package off the ground.[5]

However, as noted by Boldrin et al. (1999), most of the factors that affect the PAYG social security are political. Existing work, such as Gonzalez-Eiras and Niepelt (2008) and Lancia and Russo (2015), has shown that PAYG social security can be sustained in a political equilibrium. In these studies, the political decision is made under an aggregate preference of various groups of voters in the economy and as long as the old voter has sufficiently large political power, PAYG social security program, either independently or together with public education program, can survive the political process. A natural question that arises is, can the political outcome be further rationalized on efficiency grounds? What impact does political establishment of a policy package consisting of these two public programs have on the level of human and physical capital investment as well as on social welfare? To answer these questions, in this paper we construct a tractable general equilibrium political economy model that is dynamically efficient.

The framework that we consider is a standard three-period overlapping generations economy. Young agents receive education investment either from private funding or from the government. Agents reap the benefits of education, pay education tax and pension tax when they are middle aged, and receive social security benefits when old. We model electoral competition under the assumption of probabilistic voting, which as will be shown later can capture intergenerational cooperation and conflict through competition between education subsidy and social security. In the political economy, voting power is distributed intergen-

the externalities in human capital accumulation, allocations of human and physical capital in competitive equilibrium differ from the planner's and a possibility naturally arises where the laissez-faire equilibrium experiences higher physical capital accumulation but lower human capital accumulation compared to the planner's allocations. However, Bishnu (2013) shows that if the origin of non-optimality of human capital accumulation is consumption externality, the possibility that the accumulation of human and physical capital in a laissez-faire will differ from the planner's in an opposite direction is not at all feasible. That is, overaccumulation (underaccumulation) of human capital will always be accompanied by overaccumulation (underaccumulation) in physical capital.

[5]The literature indeed started much earlier. According to Pogue and Sgontz (1977), pay-as-you-go (PAYG) social security creates incentives for public investment in education. Becker and Murphy (1988) also suggest that PAYG social security strengthens the political support among the current working agents for public investment in education. Some of the papers that also study education and social security are Richman and Stagner (1986), Cremer et al. (1992), Kaganovich and Zilcha (1999), Pecchenino and Utendorf (1999), Poutvaara (2006) among others.

erationally where voters hold rational expectations and are allowed to sequentially choose the policies. We focus on the Markov Perfect Equilibrium (henceforth MPE), under which the voting outcomes depend on a set of fundamental state variables.[6] Here, the dynamic politico–economic equilibria are founded on competitive equilibrium with subgame perfect tax rates and transfers since the voters are not bound by their past political decisions.

Under this political economy framework, we concentrate on the welfare implications of the political decisions including how the pension-education policy package would affect human and physical capital accumulation as well as social welfare, and also how these two programs interact with each other. The analysis is very relevant for developing countries which are taking steps to establish welfare state in line with the developed countries. For example, China announced an ambitious plan to provide public education and social security program for the rural population which is around 700 million and remains outside the welfare state system. In 2001 China shifted the responsibility of funding rural compulsory education from rural farmers to the county-level government, and announced in 2009 that it would establish a rural social security program in 10% of the rural area and reach complete coverage by 2020. Similarly, in India only a small fraction of working force was covered by pension, however, recently more employees are being brought under a new public pension scheme implemented in 2010 which has been modified again in 2015.

As argued by Mulligan and Sala-i-Martin (1999), the political clout of the old people has grown beyond what was predicted by the evolution of the demographics. They interpret the data in favor of the old as an increase in the political power enjoyed by those citizens (also see Preston, 1984; Lindert, 1994). In Song et al. (2012), it is shown that the voter's turnout in the US has been falling however, the participation of the old is increasing. Further, they report that the share of votes cast by voters of more than 61 years of age is expected to reach 50% by the year 2050 in the OECD countries. This rise of the old generation's political power is undoubtedly changing the pattern of the intergenerational distribution of social expenditure. Hence in this paper we particularly focus on the impact of the intergenerational distribution of political power on the expenditure on two substantially heavy intergenerational programs and its consequences on the evolution of the economy.

We consider two stylized human capital investment models to investigate the political choices on the two public programs - (i) a model with private investment in education made by the altruistic parents who have preference toward children's human capital, and (ii) a model with private investment in education made by the young agents who borrow funds for education from a perfectly competitive credit market and pay back the loan when

[6]For a discussion on MPE in overlapping generations framework, see Krusell et al. (1997) and Bhaskar (1998) among others.

middle aged. Our models clearly shows that, although politically establishing PAYG social security alone in a dynamically efficient economy not only reduces human and physical capital investment but also hurts the social welfare, the welfare implications completely differ when we consider the more realistic setup where both PAYG social security and public education programs are set in the political agenda. We show that when private education investment is funded by altruistic parents, politically implementing the two-policy package could generate opposing effects on capital investment in the short run when the political influence of the old is small enough so that the size of social security program is limited. Since rational voters take into account the positive general equilibrium effects of education investment, such as the increase of future interest rate, the public funding for education is more generous than private funding, thus generating a higher level of human capital. In the short run, this higher level of public investment in education, together with PAYG social security, depresses the physical capital investment. However, the long run may see a reversal in this result. We show that the higher level of human capital eventually may expand the family income which in turn will improve both the human and physical capital in the long run and therefore ultimately guarantee a higher level of steady state welfare for a representative agent.

Moreover, when we extend our analysis to a situation where instead of relying on altruistic parents, in the laissez-faire economy, agents can borrow funds for education from a perfect capital market as in Boldrin and Montes (2005), we can show that if the political influence of the old is sufficiently small, politically implementing the two-policy package could simultaneously increase the human and physical capital investment both in the short and long run, and also the social welfare. A point to note here is that funding in education through generation has an edge over borrowing through the market since the market rate is higher than the population growth rate under dynamic efficiency. While that logic is valid in ours as well as in Andersen and Bhattacharya (2015), besides the normal periodic channel of improvement in the levels of capital under some reasonable influence of the old, higher level of human capital stock in our analysis is partly guaranteed through the general equilibrium effect that is internalized by the agents. On the other hand in Andersen and Bhattacharya (2015), the higher level of human capital is partly due to the existence of human capital externality that is not internalized by the agents. Nevertheless if the political power of the old in our model is sufficiently large so that the PAYG social security benefit is too generous, the political decisions reduce the investment in both the two capitals and therefore the social welfare. Thus, a welfare improvement is possible even from the best equilirium outcome that a competitive equilibrium can generate.[7]

[7]Boldrin et al. (1999) mentions that "....a PAYG system, even if smaller than at present, is necessary for achieving intergenerational efficiency and fairness and to sustain long-run growth, at least while we do not

Our paper is also an addition to the stream of literature that deals with the public expenditure programs in a political economy framework. The literature on pension as an isolated instrument in a voting setup in the generational context is quite rich (see for e.g., Cooley and Soares (1999), Boldrin and Rustichini (2000), Forni (2004), Bassetto (2008), Gonzalez-Eiras and Niepelt (2008), Mateos-Planas (2008), Song (2011), Chen and Song (2014) among others).[8] However studies incorporating both pension and education, the two oppositely directed intergenerational goods into a voting setup is relatively more recent. Kaganovich and Zilcha (2012) studies a political economy encompassing education and social security where only public education is determined through voting but the social security tax rate is exogenously given. They show that compared to PAYG, the fully funded social security system produces political support for a relatively higher education funding, and hence generates higher rates of physical and human capital accumulation and economic growth. Two recent papers allow both public education and social security to be endogenously determined by voting; Gonzalez-Eiras and Niepelt (2012) studies how the demographic ageing – increased longevity and reduced fertility – would affect the economic growth and Lancia and Russo (2015) mainly focuses on the conditions under which simultaneous implementation of public education and social security can survive though voting in a small open economy.[9] On the basis of the political economy setup, though our model is close to these studies, our focus is on completely different aspects - we study how politically implementing public programs would impact the evolution of the economy both in the short and in the long run.

Further, unlike Lancia and Russo (2015) and other prior studies, we consider the general equilibrium effects of political decisions. General equilibrium effects are costly to ignore especially when we are considering the whole life-span of the agents. In some of the voting papers that have different frameworks than us, general equilibrium is shown as an important channel.[10] While fixing the factor prices, a prominent feature of a small open economy,

find means better than public expenditure for financing the accumulation of human capital." Our findings in a political equilibrium framework somewhat suggests their claim. In our model, a limited PAYG program along with public education may turn out to be beneficial for the society.

[8]For a survey on political economy model with social security which also discusses the importance of general equilibrium effects, see Galasso and Profeta (2002).

[9]In Gonzalez-Eiras and Niepelt (2012), demographic ageing increases the GDP share of social security transfer with the negative implication for growth. On the other hand, when retirement age is also determined by the political process, demographic ageing could delay the retirement of the old which increases the return of education investment. As a consequence public education investment rises, benefiting the growth. The main mechanism in Lancia and Russo (2015) is that because the middle-aged expect that investment in the human capital of future generations can expand future pension possibilities, the public education program can be sustained together with social security in the political equilibrium. In an open economy framework where the proportion of expenditure on one public program to the total expenditure is artificially fixed at some arbitrary level (and therefore voting is essentially on a single instrument), Naito (2012) also focuses on the sustainability of public education and social security under majority voting on labor income tax.

[10]For example, in Tabellini (1991) which models public debt, in Cooley and Soares (1999), Bassetto

is common in the literature, allowing the general equilibrium effects in a voting essentially generates a completely different political mechanism. For example, our model shows that once general equilibrium effects are incorporated, the public education program may reduce the present value of future social security benefits because it drives up the future interest rate which in turn offsets the increase in future pension possibilities, and the middle-aged would support the public education program because the increased future interest rate at the same time expands the return on their savings.

The rest of the paper is organized as follows. Section 2 presents the baseline laissez-faire economy, where the private education investment is funded by altruistic parents along with the political economy model. We there examine the policy implications of the intergenerational transfers. We in section 3 re-examine the model results by considering a setup that the private education investment is funded by education-loans market. While section 4 concludes, proofs are presented in the Appendix A and B.

2 Education Funding via Altruism

We consider an economy that consists of an infinite sequence of three-period lived overlapping generations, an initial old generation and an initial middle-aged generation. An agent who is working at period t, that is a middle-aged agent in t, is called a generation-t agent. In each generation, there is a continuum of identical agents of measure one. Physical capital, K_t, and human capital, H_t, are owned respectively by the old and the middle-aged agents. (K_0, H_0) are exogenously given and define the initial state of the economy. There is a single final good produced by a constant returns to scale production function $F(K_t, H_t)$ with standard Inada conditions satisfied. The human capital is produced by a concave function $h_{t+1} = h(e_t)$ with $h(0) = 0$, $h'(e_t) > 0$ and $\lim_{e_t \to 0} h'(e_t) = \infty$, where e_t is the physical resource invested in education. Defining $k_t \equiv K_t/N_t$ and $h_t \equiv H_t/N_t$ in which N_t is the population of generation-t agents, output per middle-aged agent at time t can be expressed as an intensive form $f(k_t, h_t) = F(k_t, h_t)$. Since the measure of the members of each generation is one, we have $k_t = K_t$ and $h_t = H_t$. The final good can either be consumed in the period it is produced, or it can be saved to provide capital in the following period. Capital is conveniently assumed to depreciate fully between periods. Young agents supply labor inelastically in competitive labor markets, earning a wage of w_t at time t; similarly, capital is traded in competitive capital markets, and earns a gross real return of R_{t+1} between t and $t+1$.

(2008), Gonzalez-Eiras and Niepelt (2008) and Song (2011) which focus on pension, Gonzalez-Eiras and Niepelt (2012) which analyzes the impact of ageing on economic growth, general equilibrium effects are crucial.

We assume an agent born in period $t-1$ draws utility from (c_t^m, c_{t+1}^o), denoting consumption at middle and old age,

$$\mathcal{U} \equiv u(c_t^m) + \delta \left[u\left(c_{t+1}^o\right) + \phi u\left(h_{t+1}\right) \right] \tag{1}$$

where $\delta \in (0,1)$ is the standard discount factor and $\phi \geq 0$ represents the relative weight assigned to the utility that an old agent enjoys from her children's human capital, expressing parents' altruism towards their offspring.[11] Instantaneous utility function u is continuously differentiable, strictly increasing and strictly concave.

2.1 Competitive Equilibrium

An agent when middle-aged allocates her labor income among consumption, saving and investment in education for her children. Saving s_t while middle-aged gives a return of $s_t R_{t+1}$ in the next period when the agent is old. These imply the following life-cycle budget constraints for a representative generation-t agent

$$c_t^m = w_t h_t - s_t - e_t \tag{2}$$

$$c_{t+1}^o = s_t R_{t+1} \tag{3}$$

where e_t is the education investment. Thus given the factor prices, human capital production technology $h_t = h\left(e_{t-1}\right)$ and the budget constraints stated above, when a generation$-t$ agent maximizes her utility (1) with respect to $\{s_t, e_t\}$, we arrive at the following first order conditions[12]:

$$u'\left(w_t h_t - s_t^L - e_t^L\right) = \delta R_{t+1} u'\left(R_{t+1} s_t^L\right) \tag{4}$$

$$u'(w_t h_t - s_t^L - e_t^L) = \delta\phi u'\left(h_{t+1}\right) h'\left(e_t^L\right). \tag{5}$$

[11] Utility specification that represents altruism through the level of human capital of the next generation is very common and vastly used in the literature. Our specification is simple and standard, for example, as in Kaganovich and Zilcha (1999), Pecchenino and Utendorf (1999), Glomm and Kaganovich (2003, 2008) and in line with Glomm and Ravikumar (1992), De la Croix and Doepke (2003) among others. Mookherjee and Ray (2002, 2010) also assume a particular form of altruism where parents have preference over the income of the kids which depends on their occupation/education level funded by the parents.

[12] In section 2, superscript L represents the optimal and equilibrium values of the concerned variables in the laissez-faire economy. Similarly, the superscripts P and X are used in our analysis to represent the optimal and equilibrium values of the concerned variables in the political economy of pension and the political economy of education - pension as a package respectively. In section 3 where education is funded through a credit market under the laissez-faire setup, L, P and X have been replaced by l, p and x.

While equation (4) represents the optimal intertemporal consumption allocation, that is the standard Euler's equation, (5) indicates that the marginal sacrifice in utility from investing in education of the descendents is equal to the marginal benefit from utility gain adjusted to the gain in the level of human capital of their descendents.

The competitive equilibrium is defined by $w_t = \partial f(k_t, h_t)/\partial h_t$, $R_t = \partial f(k_t, h_t)/\partial k_t$, (4), (5) and general equilibrium condition $s_t = k_{t+1}$. To derive closed-form solutions of political equilibrium, we must impose functional form restrictions. As is standard, throughout the paper, we assume that the human capital is produced by $h_{t+1} = h(e_t) = Be_t^{\beta}$, $B > 0$, $\beta \in (0,1)$, and the final good production function f takes the Cobb-Douglas form, i.e., $f(k_t, h_t) = Ak_t^{\alpha} h_t^{1-\alpha}$, $A > 0$, $\alpha \in (0,1)$. In a competitive setup, this guarantees that the wage rate $w_t = \partial f(k_t, h_t)/\partial h_t = (1-\alpha) Ak_t^{\alpha} h_t^{-\alpha}$ and the gross real rate of return on capital $R_t = \partial f(k_t, h_t)/\partial k_t = \alpha A k_t^{\alpha-1} h_t^{1-\alpha}$. To derive the analytical results, we assume that the felicity utility function is logarithmic, that is, $u(\cdot) = \log(\cdot)$.[13]

By solving the above two first order conditions under logarithmic utility, we have $e_t^L = \beta\delta\phi w_t h_t/(1+\delta+\beta\delta\phi)$ and $s_t^L = \delta w_t h_t/(1+\delta+\beta\delta\phi)$. Substituting equilibrium factor prices and the general-equilibrium condition $s_t = k_{t+1}$ into the optimal solutions of e_t^L and s_t^L, we can obtain a two-dimensional first-order dynamical system of the economy:

$$k_{t+1}^L = \frac{\delta(1-\alpha)A}{1+\delta+\beta\delta\phi}\left(k_t^L\right)^{\alpha}\left(h_t^L\right)^{1-\alpha}, \tag{6}$$

$$h_{t+1}^L = B\left[\frac{\beta\delta\phi(1-\alpha)A}{1+\delta+\beta\delta\phi}\right]^{\beta}\left(k_t^L\right)^{\alpha\beta}\left(h_t^L\right)^{\beta-\alpha\beta} \tag{7}$$

Denote $\overline{k}^L$ and $\overline{h}^L$ the steady state values of physical and human capital.[14]

Lemma 1 *In the laissez-faire economy, there exists a unique and stable non-trivial steady state equilibrium of* $\left(\overline{k}^L, \overline{h}^L\right)$ *with*

$$\frac{\overline{k}^L}{\overline{h}^L} = \left[\frac{\delta(1-\alpha)A}{1+\delta+\beta\delta\phi}\right]^{\frac{1}{1-\alpha}}$$

[13]For analytical tractability, this assumption of logarithmic utility is standard in the studies of probabilistic voting. While the assumption restricts the political equilibrium results to be unvarying over time, more general utility function, for e.g., CRRA (constant relative risk aversion), can also be used to show our claims but in that case we have to rely solely on simulation.

[14]It can be shown that

$$\overline{k}^L = B^{\frac{1}{1-\beta}}(\beta\phi)^{\frac{\beta}{1-\beta}}\left[\frac{\delta(1-\alpha)A}{1+\delta+\beta\delta\phi}\right]^{\frac{1}{(1-\alpha)(1-\beta)}} \text{ and } \overline{h}^L = B^{\frac{1}{1-\beta}}(\beta\phi)^{\frac{\beta}{1-\beta}}\left[\frac{\delta(1-\alpha)A}{1+\delta+\beta\delta\phi}\right]^{\frac{\beta}{(1-\alpha)(1-\beta)}}.$$

Assumption: The economy is dynamically efficient.

As standard in the literature, the sufficient condition for the steady state equilibrium to be dynamic efficient is that the gross rate of return on physical capital be larger than or equal to one, i.e. $\overline{R}^L \equiv \alpha A \left(\overline{k}^L/\overline{h}^L\right)^{\alpha-1} \geq 1$. Hence the laissez-faire economy is dynamic efficient if and only if

$$\frac{\alpha\left(1+\delta+\beta\delta\phi\right)}{\delta\left(1-\alpha\right)} \geq 1. \tag{8}$$

For reasonable values of α, β, δ and ϕ, the condition is easily satisfied and we assume it always holds so that the Laissez-faire economy is dynamically efficient.

2.2 Political Economy

In this section we consider the same framework as before but introduce two oppositely directed intergenerational policies, namely public education and PAYG social security. We bring these two instruments together through a political process where the economy politically determines the size of these programs to be implemented. In particular, in the political economy, both PAYG social security and public education are simultaneously determined through the political process, and the public education spending is a perfect substitute for the private investment in education.

If we shut down the voting on public education by setting public spending in education equal to zero, we have an economy where agents only vote for the PAYG social security and make full private investment in education as in the laissez-faire economy. That PAYG social security is not desirable in a dynamically efficient economy is the well-known Aaron-Samuelson result from public economics (Aaron, 1966). Not surprisingly, the political decision in such an economy will always hurt the long run social welfare. However in the real world, PAYG social security generally does not appear in isolation on the political agenda. In the early stages of development, the government of developed countries invested a reasonable amount mainly in the form of subsidy in education; however there was no old age care for elderly people until recently, for example, 1940s in the USA. In fact, provision of education subsidy without old age pension support is still a common form of government intervention in less developed countries. Hence examining the welfare impacts of politically establishing PAYG social security has to take into account the fact that on the political agenda, both the forward and backward intergenerational goods are present, that is, the economy is two-armed. As shown below, the welfare implications become completely different when we allow the pension-education policy package to be simultaneously determined through the political process.

Suppose the government imposes a proportional education tax rate θ_t and a proportional social security tax rate τ_t at each t on the wage income of middle-aged workers to finance the public education program for the present young, e_t^g, and the PAYG social security benefit for the present old, b_t. The fiscal programs of subsidy need to satisfy the period-wise balanced budget condition, i.e., $e_t^g = \theta_t w_t h_t$ and $b_t = \tau_t w_t h_t$. The tax rates θ_t and τ_t are determined by a repeated political process (to be discussed below in detail) at the beginning of each period. After the political process of voting is complete and both the two tax rates are set, agents make their decisions on consumption, education investment and savings. We assume the public education investment is a perfect substitute of private education investment.

Given the factor prices, government policies and the human capital production technology, a generation-t agent's optimization problem now is to maximize (1) subject to

$$\begin{aligned} c_t^m &= (1-\theta_t-\tau_t)w_t h_t - s_t - (e_t - e_t^g) && (9)\\ c_{t+1}^o &= s_t R_{t+1} + b_{t+1} && (10)\\ e_t &\geq e_t^g. && (11) \end{aligned}$$

where e_t is the total education investment and $e_t - e_t^g$ thus is the private education investment funded by parents. However as shown in Appendix B, the political support for public education spending is more generous than private education spending in the absence of public education program, and therefore the private investment in education is optimally driven to the zero corner and education investment eventually is fully funded by the government, i.e., $e_t = e_t^g$ in equilibrium. For the sake of simplicity, we drop private education investment decision $(e_t - e_t^g)$ from agents' choice set. Then given θ_t and τ_t, the first order condition with respect to s_t is as follows:

$$u'\left[(1-\theta_t-\tau_t)\, w_t h_t - s_t^X\right] = \delta R_{t+1} u'\left(R_{t+1} s_t^X + b_{t+1}\right). \tag{12}$$

Under logarithmic utility, we can solve $s_t^X = \left\{\delta\left[(1-\tau_t-\theta_t)w_t h_t\right] - b_{t+1}/R_{t+1}\right\}/(1+\delta)$.

2.2.1 Choice of tax rates

We move to solve the political equilibrium, that is, we find out the optimal θ_t and τ_t under a repeated voting process where only middle-aged and old participate. Abiding by the standard practice, we disallow the young's participation in the voting process due to age-restriction. At the beginning of each period, the contemporaneous tax rates are determined by a candidate who is democratically elected by all the current voters. We assume that the size of the program is determined in a probabilistic-voting (see Lindbeck and Weibull, 1987)

framework.

Under this political setting, there are two political candidates who are competing in an election. These two office-seeking candidates first announce their policy platform which in our model is announcing the tax rates and the transfers that obey the balanced budget condition. The candidates cannot commit to future policies. Voters, in our model only middle aged and the old, then choose their preferred candidate that depends not only on the announcement of the policies by the candidates but also on their ideology. This idiosyncratic ideological bias reflects the voters view about the candidates position (say for example, candidates social stance about some issues or private characteristics) that is not related to the platform that they proposed. Thus though the candidates know about the choice of one particular generation's preferences towards their proposed fiscal policies, they are still uncertain about the voters' ultimate decision since ideologically agents can very much differ and vote differently within the cohort. In all these probabilistic voting models, the uncertainty of the office-seeking candidates needs a probabilistic description of the agents' choice behavior. Since winning the election is the only aim of the candidates, in the probabilistic-voting Nash equilibrium, the two candidates will seek to maximize their vote shares propose the same policy platform. Under some reasonable assumptions, this policy platform actually maximizes a weighted average of the welfare of all voters, in which the weights assigned to different groups of voters reflect the size or the political power of different generations.[15]

When deciding on which candidate to support, voters anticipate the effects of the candidate's policy platform on equilibrium prices, future political decisions and their own welfare. As we mentioned earlier we focus on the MPE where agents (middle aged and old) can form perfect foresight on the policies which depend on the set of state variables of the economy. For rest of the analysis, we use the notation S^t to denote the set of state variables in period t, i.e., $S^t \equiv \{k_t, h_t\}$.

From the foregoing discussion, the political decision on the equilibrium education policy can be derived by maximizing the weighted sum of the indirect utilities of two generations, i.e.,

$$\max_{\theta_t, \tau_t \in [0,1)} \mathcal{W}\left(\mathcal{S}^t; \theta_t, \tau_t\right) = \omega V_t^o\left(\mathcal{S}^t; \theta_t, \tau_t\right) + V_t^m\left(\mathcal{S}^t; \theta_t, \tau_t\right), \tag{13}$$

where V_t^m and V_t^o denote the welfare of the middle aged and the old respectively at period t given the state $\mathcal{S}^t$. The parameter ω is the effective political weight assigned to the old relative to a middle-aged agent by the political candidates. In line with the explanation by Song et al. (2012), this weight captures the relative political clout of each generation,

[15] The whole voting mechanism and the derivation of (13) have been shown in some of the previous papers (see for e.g., the Appendix of Song et al. (2000) or the Appendix of Lancia and Russo (2015) which uses a very similar framework to our paper).

reflecting, on one hand, its relative size and on the other hand, exogenous group-specific characteristics, such as the voting turnout or the salience of the fiscal policy for that group relative to other issues. In our model, the young receive education, but are not allowed to vote for public education expenditure. It should be noted that although the young have no role in the voting process, the middle-aged have an incentive to invest in education for the future generation since they directly derive utility out of the level of education accumulated by their descendents. Not only that, agents also could acquire higher returns on their saving in the next period through general equilibrium effects of public education investment.

To characterize the political decision, we first consider the welfare implications of the choice of education tax and social security tax respectively on various groups of voters, i.e., the middle-aged workers and the old retirees. These welfare implications induce group-specific preferences over policies. In the second step, we solve the equilibrium taxes by aggregating these preferences in the political process.

Evidently, given τ_t, the education tax θ_t imposed in period t has no welfare effect on the current old, i.e., $\partial V_t^o/\partial\theta_t = 0$. This follows directly from the fact that all the variables, i.e., h_t, R_t and s_{t-1}, in the utility function of the old in period t, that is in $V_t^o = u\left(R_t s_{t-1}\right) + \phi u\left(h_t\right)$, are pre-determined in t. For the middle aged, the welfare effect of education tax is more complex. Differentiating $V_t^m\left(\cdot\right)$ with respect to θ_t yields

$$\begin{aligned}\frac{\partial V_t^m}{\partial\theta_t} &= \underbrace{-u'\left(c_t^m\right)w_t h_t}_{\mathcal{A}} + \underbrace{\delta\phi u'\left(h_{t+1}\right)\frac{\partial h_{t+1}}{\partial\theta_t}}_{\mathcal{B}} \\ &\quad +\delta s_t u'\left(c_{t+1}^o\right)\left(\underbrace{\frac{\partial R_{t+1}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial\theta_t}}_{\mathcal{C}} + \underbrace{\frac{\partial R_{t+1}}{\partial h_{t+1}}\frac{\partial h_{t+1}}{\partial\theta_t}}_{\mathcal{D}}\right) + \underbrace{\delta u'\left(c_{t+1}^o\right)\frac{\partial\left(\tau_{t+1}w_{t+1}h_{t+1}\right)}{\partial\theta_t}}_{\mathcal{E}}.\end{aligned} \tag{14}$$

Note that the effect of θ_t on the savings of the middle aged cancels out by the envelope theorem. The first negative term $\mathcal{A}$ reflects the cost of investment in public education. The second term $\mathcal{B}$ captures the positive effect of public education through tax enjoyed by the parental generation because of altruism.

The two terms $\mathcal{C}$ and $\mathcal{D}$ reflect the general equilibrium effect of public education tax on the rate of interest through the channel of physical and human capital respectively. On one hand, by directing funds as a forward intergenerational good to the next generation, the education tax θ_t reduces private savings in period t and consequently also reduces the physical capital investment, which eventually leads to an increase in the rate of interest in the next period. That is, $\partial R_{t+1}/\partial k_{t+1} < 0$ along with $\partial k_{t+1}/\partial\theta_t < 0$. On the other hand, channelizing more funds towards the education of the next generation necessarily increases

the level of human capital of the descendents, which in turn, also increases the rate of interest, i.e., $\partial R_{t+1}/\partial h_{t+1} > 0$ along with $\partial h_{t+1}/\partial \theta_t > 0$.[16] The aggregate general equilibrium effect of public education investment on interest rate, as well as the total return of middle-age saving, is thus positive for the middle-aged workers with $\mathcal{C} > 0$ and $\mathcal{D} > 0$.

The last term $\mathcal{E}$ represents an extra general equilibrium effect of education tax where

$$\begin{aligned} \frac{\partial\left(\tau_{t+1}w_{t+1}h_{t+1}\right)}{\partial\theta_t} &= \tau_{t+1}w_{t+1}\frac{\partial h_{t+1}}{\partial\theta_t} + w_{t+1}h_{t+1}\frac{\partial\tau_{t+1}}{\partial\theta_t} \\ &\quad +\tau_{t+1}h_{t+1}\left(\frac{\partial w_{t+1}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial\theta_t} + \frac{\partial w_{t+1}}{\partial h_{t+1}}\frac{\partial h_{t+1}}{\partial\theta_t}\right). \end{aligned} \tag{15}$$

As we can see $\mathcal{E}$ captures the welfare effect of education tax that the middle aged agents pay on the social security benefit that they will receive in the future when old. As discussed previously, the general equilibrium effects of public education are $\partial k_{t+1}/\partial\theta_t < 0$ and $\partial h_{t+1}/\partial\theta_t > 0$, along with $\partial w_{t+1}/\partial k_{t+1} > 0$ and $\partial w_{t+1}/\partial h_{t+1} < 0$. Given the sign of these effects along with the fact that $\partial\tau_{t+1}/\partial\theta_t = 0$ (as will be shown below), the sign of $\mathcal{E}$ becomes indeterminate.

Note that since establishing a public education program has no economic gain to the current old retirees, it is the middle aged agents who decide the education investment in the political process. As a voter, when deciding on investment in education, besides the altruism effect, the rational middle-aged agent also takes into account the general equilibrium effects of education investment, $\mathcal{C} + \mathcal{D} + \mathcal{E}$. Intuitively if the negative term in $\mathcal{E}$, $\tau_{t+1}h_{t+1}\left(\partial w_{t+1}/\partial k_{t+1}\right)\left(\partial k_{t+1}/\partial\theta_t\right)$, is small so that the aggregate general equilibrium effects $\mathcal{C} + \mathcal{D} + \mathcal{E}$ is positive, the education investment optimally chosen in the political process is higher than the private education investment. It is true in equilibrium as afore-discussed and also shown in Appendix B. If we disregard the general equilibrium effect of education investment in the voting process, i.e., $\mathcal{C} + \mathcal{D} + \mathcal{E}$ in (14), the investment in public education would be exactly equal to the investment in private education, adjusted to per unit tax.

We now proceed to consider the welfare implication of choice of social security tax. Differentiating the utility of the old with respect to τ_t yields $\partial V_t^o/\partial\tau_t = u'\left(c_t^o\right)w_t h_t > 0$. Since the old benefit from the social security program without bearing any cost, it is obvious that they always prefer a tax rate that is as high as possible. By using the government's balanced budget condition $b_t = \tau_t w_t h_t$, market clearing condition $k_{t+1} = s_t$, and the factor

[16] Note that under our specific functional form of the human capital production, $\partial h_{t+1}/\partial\theta_t = B\beta e_t^{\beta-1}w_t h_t$.

prices, the welfare effect of social security tax τ_t on a generation$-t$ agent is[17]

$$\frac{\partial V_t^m}{\partial \tau_t} = \underbrace{-u'\left(c_t^m\right) w_t h_t}_{\mathcal{F}} + \delta u'\left(c_{t+1}^o\right) \left(\underbrace{\tau_{t+1} w_{t+1} \frac{\partial h_{t+1}}{\partial \tau_t}}_{\mathcal{G}} + \underbrace{w_{t+1} h_{t+1} \frac{\partial \tau_{t+1}}{\partial \tau_t}}_{\mathcal{H}} + \mathcal{I} \right) + \underbrace{\delta \phi u'\left(h_{t+1}\right) \frac{\partial h_{t+1}}{\partial \tau_t}}_{\mathcal{J}} \tag{16}$$

where

$$\mathcal{I} = s_t \left(\underbrace{\frac{\partial R_{t+1}}{\partial k_{t+1}} \frac{\partial k_{t+1}}{\partial \tau_t}}_{\mathcal{I}^1} + \underbrace{\frac{\partial R_{t+1}}{\partial h_{t+1}} \frac{\partial h_{t+1}}{\partial \tau_t}}_{\mathcal{I}^2} \right) + \tau_{t+1} h_{t+1} \left(\underbrace{\frac{\partial w_{t+1}}{\partial k_{t+1}} \frac{\partial k_{t+1}}{\partial \tau_t}}_{\mathcal{I}^3} + \underbrace{\frac{\partial w_{t+1}}{\partial h_{t+1}} \frac{\partial h_{t+1}}{\partial \tau_t}}_{\mathcal{I}^4} \right).$$

The first negative term $\mathcal{F}$ reflects the direct cost of social security contributions. The second term $\mathcal{G}$ captures the effect of social security tax which reduces the level of education of the next generation and consequently the transfer benefits that the present middle aged generation receives when old. This effect can be explicitly derived from the expression of equilibrium education tax rate which is as shown below in equation (19). Evidently PAYG social security program competes with public education for government revenue and therefore crowds out at least a part of the public funding for the public education program, thus depressing the human capital accumulation. While the third term $\mathcal{H}$ as will be shown below equals to zero under logarithmic utility, $\mathcal{J}$ is negative since the reduction in human capital due to increase in the pension tax adversely affects the agents via altruism. The last term $\mathcal{I}$ reflects the general equilibrium effect of social security tax through the channel of physical and human capital on the factor prices. By shifting income from the middle-aged to the old, the PAYG social security reduces savings as well as the physical capital investment. The effect of social security tax on the physical capital investment is thus negative i.e., $\partial k_{t+1}/\partial \tau_t < 0$. Here the movement of capital in terms of backward intergenerational goods not only decreases the level of physical capital in the future and thus increasing the rate of interest, but also decreases the level of investment in human capital for the next generation, consequently reducing human capital in the future. This in turn leads to an opposite effect (compared to physical capital) on the rate of interest. We see a similar thing happening with the other factor price, namely wages. While wages fall owing to the fact that there is a loss in physical capital due to backward transfer, we will also expect the price of human capital to be augmented since human capital also goes down in the future. Thus, unlike the previous case of education tax rate, there is always a tension between the two opposite

[17]By the envelope theorem, the effect of τ_t on the savings and education investment of the middle aged cancels out.

effects of two types of capital on the factor prices. Therefore $\mathcal{I}^1$ and $\mathcal{I}^4$ are positive and $\mathcal{I}^2$ and $\mathcal{I}^3$ are negative, with the result that the sign of the aggregate general equilibrium effect of the social security tax, $\mathcal{I}$ becomes ambiguous. Indeed this particular result has support from the existing literature. As emphasized by, for e.g., Boldrin and Rustichini (2000) and Gonzalez-Eiras and Niepelt (2008), the general equilibrium effect is very crucial in sustaining the social security program in equilibrium. Our analysis in a different framework, in fact, shows that it is possible for the general equilibrium effect to be positive and therefore help make the PAYG social security program sustainable.

2.2.2 Political equilibrium

Under MPE, the equilibrium tax rates are the fixed points in

$$\left\{\theta_t\left(\mathcal{S}^t\right),\tau_t\left(\mathcal{S}^t\right)\right\}=\arg\max_{\theta_t,\tau_t\in[0,1)}\mathcal{W}\left(\mathcal{S}^t;\theta_t,\tau_t\right).$$

We first substitute the factor prices, government's balanced budgets for the two programs, the equilibrium condition for market clearing and the private optimal savings into $\mathcal{W}\left(\mathcal{S}^t;\theta_t,\tau_t\right)$. By omitting the terms that are independent of the policy parameters θ_t and τ_t, the value functions of the middle aged and the old can be re-written as[18]

$$\begin{aligned}V_t^m\left(\mathcal{S}^t;\theta_t,\tau_t\right) &\simeq (1+\alpha\delta)\ln\left[\frac{1-\tau_t-\theta_t}{\alpha(1+\delta)+(1-\alpha)\tau_{t+1}}\right]+(1+\delta)\ln\left[\frac{\alpha+(1-\alpha)\tau_{t+1}}{\alpha(1+\delta)+(1-\alpha)\tau_{t+1}}\right]\\ &\quad +\beta\delta(1-\alpha+\phi)\ln(\theta_t),\\ V_t^o\left(\mathcal{S}^t;\theta_t,\tau_t\right) &\simeq \ln\left[\alpha+\tau_t(1-\alpha)\right].\end{aligned}$$

It can be noted that the value function of the middle aged depends on both the current policy choices and the future social security tax rate. As standard in the literature (see for e.g., Gonzalez-Eiras and Niepelt (2008)), future equilibrium policy τ_{t+1} will be independent of current political choice of θ_t and τ_t under the logarithmic utility.[19] To prove that we first make a conjecture that this is true and then we verify that it is indeed the case. Then by omitting the term τ_{t+1}, the political objective function $W\left(\mathcal{S}^t;\theta_t,\tau_t\right)$ reduces to

$$\begin{aligned}\mathcal{W}\left(\mathcal{S}^t;\theta_t,\tau_t\right) &\simeq \omega\ln\left[\alpha+\tau_t(1-\alpha)\right]+(1+\alpha\delta)\ln(1-\theta_t-\tau_t)\\ &\quad +\beta\delta(1-\alpha+\phi)\ln(\theta_t).\end{aligned}\tag{17}$$

[18] In all that follows, we will use the notation $\simeq$ to denote the effective value function that contains the relevant fiscal parameter but not the other irrelevant terms.

[19] It is however not true if the utility function is not logarithmic because the equilibrium τ_{t+1} then depends on k_{t+1} and h_{t+1} which are affected by the current political choice of θ_t and τ_t.

The optimal tax rates are simultaneously determined under a repeated probabilistic voting. We solve for the two tax rates by simultaneously solving the two first order conditions $\partial \mathcal{W}\left(\mathcal{S}^t;\theta_t,\tau_t\right)/\partial\theta_t = 0$ and $\partial \mathcal{W}\left(\mathcal{S}^t;\theta_t,\tau_t\right)/\partial\tau_t = 0$, i.e.,

$$\tau_t = \frac{\omega\left(1-\theta_t\right)-\alpha\left(1+\alpha\delta\right)/\left(1-\alpha\right)}{1+\alpha\delta+\omega} \tag{18}$$

$$\theta_t = \frac{\beta\delta\left(1-\alpha+\phi\right)\left(1-\tau_t\right)}{\Omega}, \tag{19}$$

where $\Omega = 1+\alpha\delta+\beta\delta-\alpha\beta\delta+\beta\delta\phi$.

Solving the MPE for this probabilistic-voting problem gives us the following lemma.

Lemma 2 *In a political economy with public education and PAYG social security being determined in a probabilistic-voting setting, there exists a unique set of instruments* $\left(\theta^X,\tau^X\right)$ *under MPE where*

$$\theta^X = \frac{\beta\delta\left(1-\alpha+\phi\right)/\left(1-\alpha\right)}{\omega+\Omega}\ \forall t$$

$$\tau^X = \frac{\omega-\alpha\Omega/\left(1-\alpha\right)}{\omega+\Omega}\ \forall t.$$

Under logarithmic utility, the equilibrium social security tax is invariant of the states of the economy and therefore τ_{t+1} is independent of θ_t and τ_t, verifying the conjecture. Note that the equilibrium social security tax τ^X is monotonically increasing in the relative political power of the old retirees ω, and is non-negative if and only if $\omega \geq \alpha\Omega/\left(1-\alpha\right)$. In all that follows we assume $\omega \geq \alpha\Omega/\left(1-\alpha\right)$. Under this repeated election game in every period, every subgame thus has a unique and the same Nash equilibrium. Thus the short run taxes that have been obtained simultaneously at each period t, are valid not only for each t but also valid for the long run. Given the two equilibrium tax rates, we can then obtain a two-dimensional first-order dynamical system of the economy, which is

$$k_{t+1}^X = \frac{\alpha\delta\left(1+\alpha\delta\right)A}{\omega+\alpha\delta\left(\omega+\Omega\right)}\left(k_t^X\right)^{\alpha}\left(h_t^X\right)^{1-\alpha} \tag{20}$$

$$h_{t+1}^X = B\left[\frac{A\beta\delta\left(1-\alpha+\phi\right)}{\omega+\Omega}\right]^{\beta}\left(k_t^X\right)^{\alpha\beta}\left(h_t^X\right)^{\beta-\alpha\beta}. \tag{21}$$

Finally, as a special case, we consider a scenario where only PAYG social security program is implemented. In this economy the government does not provide any public education support to young people, that is, we set $\theta_t = 0\ \forall t$ and education is completely privately funded where $e_t^P = \beta\delta\phi\left[\left(1-\tau_t\right)w_t h_t + b_{t+1}/R_{t+1}\right]/\left(1+\delta+\beta\delta\phi\right)$. Following the same exercise as above, we can solve the political equilibrium of the social security tax rate τ^P, which turns

out to be equal to τ^X. Moreover, the evolution of the physical and human capital in the economy can be shown to be

$$k_{t+1}^P = \frac{\alpha\delta}{\alpha\delta + \omega\left(1 + \alpha\delta + \beta\delta\phi\right)/\Omega} A\left(k_t^P\right)^{\alpha}\left(h_t^P\right)^{1-\alpha} \tag{22}$$

$$h_{t+1}^P = B\left\{\frac{A\omega\beta\delta\phi}{\left[\alpha\delta + \omega\left(1 + \alpha\delta + \beta\delta\phi\right)/\Omega\right](\omega + \Omega)}\right\}^{\beta}\left(k_t^P\right)^{\alpha\beta}\left(h_t^P\right)^{\beta-\alpha\beta}. \tag{23}$$

Following the same proof for Lemma 1, it can be easily verified that there exists a unique and stable non-trivial steady state equilibrium in both of the two political economies presented above with

$$\frac{\overline{k}^X}{\overline{h}^X} = \left[\frac{\alpha\delta\left(1 + \alpha\delta\right)A}{\omega + \alpha\delta\left(\omega + \Omega\right)}\right]^{\frac{1}{1-\alpha}} \text{ and } \frac{\overline{k}^P}{\overline{h}^P} = \left[\frac{A\alpha\delta}{\alpha\delta + \omega\left(1 + \alpha\delta + \beta\delta\phi\right)/\Omega}\right]^{\frac{1}{1-\alpha}}.$$

2.2.3 Welfare implications

When the government is short-lived and commitment issue is subtle, the outcome in the period-wise voting represents the true wishes of an economy in terms of the allocation of resources. In this subsection we proceed to explore how the political decisions affect the accumulation of physical and human capital, as well as the social welfare of the economy. We first examine the special case where PAYG social security is the only instrument implemented in a political equilibrium. In what follows, we concentrate on the effects of the intergenerational transfer policy on the accumulation of both human and physical capital and the welfare of the representative agent in the short as well as in the long run.

By short run we mean the immediate effect in the very next period and for the long run, we mean the steady state. More specifically, when examining the short-run effects of any policy on the level of capital investment and social welfare, we assume that the state of the economy at any period t same for both the laissez-faire and the political economy, that means at t the level of k_t and h_t are the same in these two economics. Then we compare both the physical and human capital investment in the next period, $t+1$, of the two economies. Here we actually compare two parallel economies with the same state at period t and we claim that a political decision can increase investment in capital (both physical and human) or social welfare in the short run if and only if the investments in the next period is higher in the political economy than in the laissez-faire economy. As for the long run policy effects, we directly compare the steady state equilibrium between the laissez-faire and the political economy. We present our first proposition below.

Proposition 1 *In a dynamically efficient laissez-faire economy, politically establishing a PAYG social security reduces both the physical capital investment and the level of human capital in the short run, and also reduces the steady-state ratio of physical capital to human capital. That is, given any pair of* (k_t, h_t), $k_{t+1}^P < k_{t+1}^L$, $h_{t+1}^P < h_{t+1}^L$, *and* $\overline{k}^P/\overline{h}^P \leq \overline{k}^L/\overline{h}^L$.

As presented in the Corollary below, the above result is consistent with the well-known Aaron (1966) - Samuelson (1958) result even though we have a model with both physical and human capital and they are endogenously determined.[20] Our analysis shows that along with the physical capital, human capital too declines under this political equilibrium. When parents completely rely on their own income to make the investment decision, they will increase investment in the children's education if and only if the present value of the life-cycle income increases. When education is privately funded, for a middle aged agent, social security program reduces the current income by $\tau_t w_t h_t$ but increases her future income by $\tau_{t+1} w_{t+1} h_{t+1}$. Thus whether politically establishing a PAYG social security program will increase the education investment completely depends on whether the benefits from future social security can dominate the tax loss at middle age, i.e., $\tau_{t+1} w_{t+1} h_{t+1}/R_{t+1} > \tau_t w_t h_t$, or equivalently, in this setup, $k_{t+1}^P/\alpha > A\left(k_t^P\right)^{\alpha}\left(h_t^P\right)^{1-\alpha}$. As shown in the proof, this condition is exactly opposite to the condition needed for dynamic efficiency under the laissez-faire economy. Since the political establishment of PAYG social security reduces short-run capital accumulations for any pair of (k_t, h_t), it will guarantee lower level of physical and human capital for all periods afterwards once it is implemented. The physical capital however declines more than human capital, leading to a fall in the steady state ratio of physical to human capital. Hence the political economy is also dynamically efficient. However recall that maximizing lifetime welfare of a representative agent is equivalent to maximizing the present lifecycle income $(1-\tau_t)w_t h_t + b_{t+1}/R_{t+1}$. As discussed above, compared to the laissez-faire economy with same (k_t, h_t), in the political economy, the life-cycle income of an agent decreases. Therefore, the lifecycle income at the steady state under the political economy will certainty be lower than that in the laissez-faire at the steady state. Hence, from the above discussion we can conclude the following.

Corollary 1 *Politically establishing PAYG social security in a dynamically efficient laissez-faire economy cannot improve the long run (steady-state) social welfare.*

[20] Since in many countries, public education proceeded social security by decades or even centuries, one may be interested in the question of whether adding social security to a political economy that already has an established public education program can improve welfare or not. To answer this question, we can firstly solve the model results of the political economy where only public education program is implemented, that is, we set $\tau_t = 0\ \forall t$, and then compare the results to that of the political economy where both of the two public programs are present. It can be shown that the results of Proposition 1 exactly hold for this scenario.

Next we consider the main focus of our paper - the case of establishing PAYG social security program along with a public education program, that is, implementing an education - pension package through political equilibrium. The immediate short run effects in this case differ from the long run. We start with presenting the results for the short run.

Proposition 2 *In a dynamically efficient laissez-faire economy, politically establishing a PAYG social security, together with a public education program,*

1) always reduces the short-run physical capital investment, i.e., $k_{t+1}^X < k_{t+1}^L$ *for any given pair of* (k_t, h_t)*, but*

2) can increase the short-run human capital investment, i.e., $h_{t+1}^X > h_{t+1}^L$ *for any given pair of* (k_t, h_t)*, if and only if* $\omega \in [\alpha\Omega/(1-\alpha), \widetilde{\omega}]$*, where*

$$\widetilde{\omega} = \frac{(1+\delta+\alpha\phi\beta\delta)(1-\alpha+\phi)}{\phi(1-\alpha)} - (1+\alpha\delta).$$

In contrast to the case of politically establishing PAYG social security alone, Proposition 2 shows when both of the forward and backward intergenerational transfers are brought into the political agenda, the political outcome in the short run would generate two opposing effects on the accumulation of the two different capitals. The result is intuitive. As discussed previously, when agents vote for public education program, they take the general equilibrium effects of education investment into account and therefore will vote for a more generous funding support than what a private decision can provide. On the other hand, politically establishing PAYG social security program, as shown in equation (19), would discourage public education investment. The human capital investment thus can be enhanced (compared to the laissez-faire private human capital investment) if and only if the PAYG social security is limited and therefore its effect on public education investment is also limited.

Since in the short run we observe an increase in human capital but a fall in the physical capital, naturally the question that arises is what about the long-run consequences of the two-policy package? Once the policy package has been approved by the political process, will the physical capital investment be always lower than that in the laissez-faire economy? More importantly, what is the long-run impact of the two-policy package on the social welfare? To answer these questions, we compare the steady state political equilibrium of the two economies.

Proposition 3 *Simultaneously establishing PAYG social security and public education program through political voting,*

1) can increase steady-state human capital, i.e., $\overline{h}^X \geq \overline{h}^L$*, if and only if* $\omega \leq \overline{\omega}_h$ *and*

2) can increase steady-state physical capital, i.e., $\overline{k}^X \geq \overline{k}^L$*, if and only if* $\omega \leq \overline{\omega}_k$*, where* $\overline{\omega}_h > \overline{\omega}_k$*, but*

3) always reduces the steady-state ratio of physical capital to human capital, i.e., $\overline{k}^X/\overline{h}^X \leq \overline{k}^L/\overline{h}^L$*.*

It can be easily verified that both the capitals at the steady state monotonically decreases in ω. However Proposition 3 shows that when we compare it to the steady state under the laissez-faire, the existence of three regimes that the two-policy package exerts different long-run impacts on the economy: 1) $\omega < \overline{\omega}_k$, the two armed intergenerational policy increase both of the human capital and physical capital at the steady state; 2) $\omega \in [\overline{\omega}_k, \overline{\omega}_h]$, the policy package increases steady-state physical capital investment, but reduces steady-state human capital investment; 3) $\omega \geq \overline{\omega}_h$, the policy package reduces both of the two steady-state capitals. However in all of the three regimes, the policy package reduces the steady state ratio of physical capital to human capital, leading the economy remains in the regime of dynamic efficiency.

Note that the golden rule resource allocation is solved by maximizing the steady state utility of a representative agent subject to the resource constraint of the economy $Ak^{\alpha}h^{1-\alpha} = c^m + c^o + e + k$. It can be shown that golden rule resource allocation (denoted by the superscript $*$) needs to satisfy

$$\partial f\left(k^*, h^*\right)/\partial k = 1 \tag{24}$$

$$\delta\phi u'\left(h^*\right) h'\left(e_t^*\right) + u'\left(c^{m,*}\right) h'\left(e_t^*\right) \partial f\left(k^*, h^*\right)/\partial h = u'\left(c^{m,*}\right). \tag{25}$$

Equation (24) is the standard textbook result which states that the social planner can fund the physical capital investment at the cost of 1 unit of good. Comparing (25) to the private decision in the laissez-faire economy (5), it is clear that the social planner takes into account an extra marginal benefit of education investment $u'\left(c^{m,*}\right) h'\left(e_t^*\right) \partial f\left(k^*, h^*\right)/\partial h$, which is the marginal benefit of education investment on the output of the economy and is not realized in the agent's decision process under the competitive equilibrium. Thus, because of this extra marginal benefit, ceteris paribus, the planner will allocate more resources in education investment than the agents in the laissez-faire economy. Since the laissez-faire economy is dynamically efficient with $\partial f\left(k^L, h^L\right)/\partial k > 1$, there will be underinvestment in both physical and human capital in the steady-state laissez-faire equilibrium compared to the golden rule allocation. Hence if the politically chosen policy reduces both physical and human capital further, it will hurt the welfare of the representative agent at the steady state. This is also the reason why, as discussed previously, implementing PAYG social security policy alone cannot be welfare improving in the long run. Evidently, a necessary condition for the

political decision to be welfare improving in the long run is that at least one of the two capitals increases at the steady state compared to the steady state under the laissez-faire.

It can be noted that in the short run, physical and human capital investment cannot be simultaneously increased, while in the long run, as long as ω falls under the regime 1, i.e., $\omega < \overline{\omega}_k$, both capital investment can be increased. This is precisely the difference between the results of Proposition 3 and Proposition 2. The existence of regime 1 implies that the physical capital investment falls down initially at the beginning of the implementation of the two public programs as they depress savings initially in the short run. However, as long as $\omega < \overline{\omega}_k$ so that the political power of the old is sufficiently small, it can catch up eventually because the increased human capital drives up future income directly as well as indirectly and therefore the middle-aged agents are capable of allocating more income in savings which guarantees more investment in physical capital in the future. Moreover, since both the human and physical capital increase at the steady state, the total output of the economy will increase and therefore as afore-discussed agents may enjoy a higher level of welfare at the steady state. In contrast, both of the human and physical capital decrease at the steady state when $\omega \geq \overline{\omega}_h$ so that the political power of the old is sufficiently large. The welfare implication of regime 3 is same with that of the case of politically establishing PAYG social security alone. Hence in this regime, the political establishment of PAYG social security coupled with public education cannot be welfare improving in the long run. The message in regime 2 is mixed, but since both of the steady state capitals monotonically decreases in ω, there may exist some $\omega \in [\overline{\omega}_k, \overline{\omega}_h]$, in which the welfare ranking reverses.

Since analytical comparison of steady-state welfare becomes intractable, we, in the following, use a numerical example to demonstrate the steady-state welfare of a representative agent in different model economies. In this example, each period lasts 25 years, i.e., the remaining life expectancy of an agent entering the workforce is about 50 years. A and B are scale parameters and both are set at 5. As is standard, $\delta = 0.99^{25} = 0.7778$ and $\alpha = 0.3$. Finally we let $\beta = 0.5$ and $\phi = 0.5$. For this parameterization, we can compute $\overline{\omega}_k = 1.05$ and $\overline{\omega}_h = 2.40$.

Figure 1 compares the steady-state welfare of the three economies. As afore-discussed, if we consider PAYG social security as the only one voting choice in a dynamic efficient economy, politically establishing PAYG social security always reduce the long-run welfare. Moreover as shown in the figure, the welfare loss monotonically increases from $\omega = \alpha\Omega/(1-\alpha)$. However if PAYG social security is politically determined together with public education, the policy package can generate a higher steady state welfare for all $\omega's$ that are smaller than 1.64, which locates between $\overline{\omega}_k$ and $\overline{\omega}_h$. The overall message of Proposition 3 and Figure 1 is that when the long run social welfare is concerned, there exists a situation where a mild

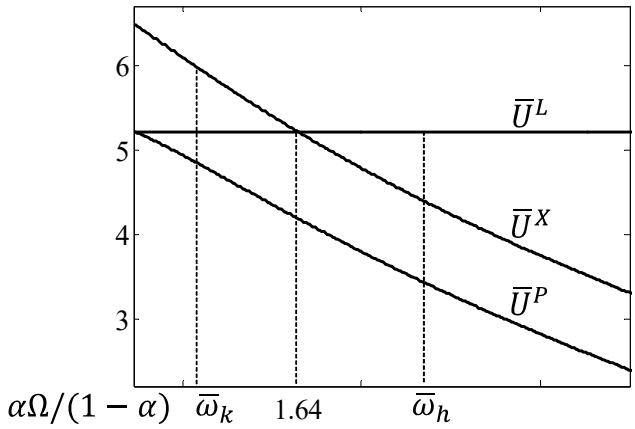

Figure 1: Comparison of steady-state welfare

social security scheme is better compared to a situation with no social security. This result indicates a positive side of the existence of public social security where a mutual benefit of higher level of capital investment (compared to laissez-faire) and a limited social security can very well coexist. However, when the influence of the old steadily increases and becomes substantial, it demands thick social security benefits which may hurt the investment of human and physical capital and result in an capital level that is lower than the level that the laissez-faire guarantees.

3 Education Funding via Loan Market

While private investment (made by parents) and public funding (made by the government) are the two main sources of funds for education, providing a perfect capital market to borrow funds for education is becoming popular. Thus the literature is also incorporating the source of this funding (see for e.g., Boldrin and Montes (2005) and Andersen and Bhattacharya (2015)) to get the best possible equilibrium that a competitive equilibrium is capable of generating. In those papers, agents are not altruistic and have no preference over children's human capital, but they themselves borrow to invest in education when they are young and repay the debts accrued when they are middle aged. While our main interest in this section is to examine whether the results derived in the previous section are robust to this alternative setup of education investment, an added advantage of this study lies in the fact that if we can show that an improvement in welfare over the laissez-faire regime is at all possible, we will be certain that the political economy is capable of generating higher welfare than the best possible laissez-faire solution.

Thus in the absence of altruism, the utility function is given by

$$\mathcal{U} \equiv u(c_t^m) + \delta u\left(c_{t+1}^o\right) \tag{26}$$

and the life-cycle budget constraints for an agent born in period $t-1$ changes to

$$\begin{aligned} 0 &\leq e_{t-1} \leq \frac{w_t h_t}{R_t}, & (27)\\ c_t^m + \frac{c_{t+1}^o}{R_{t+1}} &= w_t h_t - e_{t-1} R_t. & (28) \end{aligned}$$

We assume that the competitive credit market is perfect so that the agents are free to borrow or save, and commit to repaying their loans. A representative agent maximizes (26) subject to (27) - (28). The first order conditions can be simplified to

$$\begin{aligned} u'\left(w_t h_t - s_t^l - e_{t-1}^l R_t\right) &= \delta R_{t+1} u'\left(R_{t+1} s_t^l\right) & (29)\\ w_t h'\left(e_{t-1}^l\right) &= R_t. & (30) \end{aligned}$$

As before, equation (29) represents the optimal intertemporal consumption allocation. Equation (30) indicates that the marginal return from human capital investment is equal to the marginal cost of financing education via the credit market.

The competitive equilibrium is defined by $w_t = \partial f\left(k_t, h_t\right)/\partial h_t$, $R_t = \partial f\left(k_t, h_t\right)/\partial k_t$, (29), (30) and the credit market clearing condition $s_t = e_t + k_{t+1}$, that is, the aggregate saving in equilibrium finances investment in physical and human capital.

By solving the above two first order conditions under logarithmic utility, we have $s_t^l = \delta\left(w_t h_t - e_{t-1}^l R_t\right)/\left(1+\delta\right)$ and $e_{t-1}^l = \beta\left(1-\alpha\right)k_t/\alpha$. Moreover, using equilibrium factor prices and the fact that the general-equilibrium condition $s_t = e_t + k_{t+1}$ holds at every t, we finally can obtain a two-dimensional first-order dynamical system of this economy,

$$\begin{aligned} k_{t+1}^l &= \frac{\alpha\delta}{\left(1+\delta\right)\Phi} A\left(k_t^l\right)^{\alpha}\left(h_t^l\right)^{1-\alpha} & (31)\\ h_{t+1}^l &= B\left[\frac{\beta\delta\left(1-\alpha\right)}{\left(1+\delta\right)\Phi} A\right]^{\beta}\left(k_t^l\right)^{\alpha\beta}\left(h_t^l\right)^{\beta-\alpha\beta}, & (32) \end{aligned}$$

where $\Phi = \left(\alpha+\beta-\alpha\beta\right)/\left[\left(1-\alpha\right)\left(1-\beta\right)\right]$. Let $\overline{k}^l$ and $\overline{h}^l$ denote the steady state values of physical and human capital. It is easy to show that there exists a unique non-trivial steady state.

As in section 2.2.3, maximizing the social planner's problem at the steady state can yield the golden rule resource allocation for the economy which requires the steady-state

gross rate of return on both physical and human capital investment to be equal to one, i.e., $\alpha A\left(k^*/h^*\right)^{\alpha-1} = w^* h'\left(e^*\right) = 1$. Hence the sufficient condition for the steady state equilibrium to be dynamically efficient is same as before, i.e. $\overline{R}^l \equiv \alpha A\left(\overline{k}^l/\overline{h}^l\right)^{\alpha-1} \geq 1$. This means that the parametric restriction needed for the laissez-faire economy to be dynamically efficient is

$$\Phi \equiv \frac{\alpha+\beta-\alpha\beta}{(1-\alpha)(1-\beta)} \geq \frac{\delta}{1+\delta}.$$

Again, for reasonable values of α, β and δ, the condition is easily satisfied and we assume it holds in all that follows. Just as in the altruism model, it is also clear that there will be underaccumulation of both physical and human capital at the steady state under the education-loans market when the laissez-faire economy is dynamically efficient, and a necessary condition for the politically chosen policy to be welfare improving is that it can at least increase one of the two capitals at the steady state.

Let us now focus on the political outcome of the intergenerational transfer policies. As before, to characterize the political decision, we first consider the welfare implications of the choice of education tax and social security tax on the middle-aged workers and the old retirees, and then solve the equilibrium taxes by aggregating these preferences in the political process. Actually the first step is same as in 2.2.2 by imposing $\phi = 0$. Similarly, by excluding the private education investment from the agent's choice set, the equilibrium tax rates of (τ_t^x, θ_t^x) and equilibrium capital investment $\left(k_{t+1}^x, h_{t+1}^x\right)$ can also be directly derived by imposing $\phi = 0$ on the equilibrium solutions under the altruism model.[21] As for the political solution of the case when only PAYG social security is voted, the equilibrium tax rate has to be resolved by maximizing the probabilistic-voting problem, which yields

$$\tau_t^p = \frac{\omega - (\alpha+\beta-\alpha\beta)\,\Omega'/(1-\alpha-\beta+\alpha\beta)}{\omega+\Omega'} \;\forall t.$$

where $\Omega' = 1+\alpha\delta+\beta\delta-\alpha\beta\delta$.[22]

[21] For simplicity, here we also exclude the private education investment from the agent's choice set. In contrast to the altruism model, the private education investment does not have to take the corner solution under this setup. It can be shown that we get a corner solution for private education investment if and only the political power of the old ω is smaller than a threshold value, which in turn is strictly larger than the minimum value of ω, $\alpha\Omega'/(1-\alpha)$. Hence, the main insights of the paper, that simultaneously establishing public education and PAYG social security increases both human and physical capital and as a consequence also increases steady-state welfare when ω is sufficiently small, hold true in the presence of private education.

[22] By omitting the terms independent of the policy parameter τ_t, the political objective function $W\left(\mathcal{S}^t;\tau_t\right)$ under logarithmic utility reduces to

$$\mathcal{W}\left(\mathcal{S}^t;\tau_t\right) \simeq \omega \ln\left[\alpha+\beta(1-\alpha)(1-\tau_t)+\tau_t(1-\alpha)\right] + \Omega' \ln(1-\tau_t).$$

We now examine the welfare implications of the political outcomes. As in the altruism model, when the laissez-faire economy is dynamically efficient, given any pair of (k_t, h_t), politically establishing the PAYG social security alone reduces both physical and human capital investment, as well as the steady-state capital ratio, i.e., $k_{t+1}^p < k_{t+1}^l$, $h_{t+1}^p < h_{t+1}^l$, and $\overline{k}^p/\overline{h}^p \leq \overline{k}^l/\overline{h}^l$. The result is exactly the same as before and the consequences of it is that the political voting of PAYG social security in isolation reduces the long-run welfare of agents. However when PAYG social security and public education are simultaneously determined in the political process, the welfare implication of the policy package has some differences, although the main insights remain the same as before. It can be easily shown that [23]

Proposition 4 *In a dynamically efficient economy, politically establishing a PAYG social security together with a public education program*

1) can increase short-run human capital accumulation, i.e., $h_{t+1}^x \geq h_{t+1}^x$ *for any given pair of* (k_t, h_t)*, if and only if* $\omega \leq \widetilde{\omega}_h$*, and*

2) can increase short-run physical capital accumulation, i.e., $k_{t+1}^x \geq k_{t+1}^x$ *for any given pair of* (k_t, h_t)*, if and only if* $\omega \leq \widetilde{\omega}_k$*, and*

3) can increase the steady state ratio of physical capital to human capital, i.e., $\overline{k}^x/\overline{h}^x \geq \overline{k}^l/\overline{h}^l$*, if and only if* $\omega \leq \widetilde{\omega}_k$*; where* $\widetilde{\omega}_h = (1+\delta)\,\Phi - \Omega'$*, and* $\widetilde{\omega}_k = (1+\delta)\,\Phi - \alpha\delta\Omega'/(1+\alpha\delta)$*.*

In the same model environment, Lancia and Russo (2015) in fact show that simultaneous implementation of public education and social security can survive through voting in a small open economy where both interest rate and wage rate are exogenously given. This part can be seen as an extension of their result to the general equilibrium setup which also incorporates the welfare angle of the analysis. It can be shown that $\widetilde{\omega}_h \geq \alpha\Omega'/(1-\alpha)$ if and only if $\beta + \beta(1+\alpha\delta)/(1-\alpha) + \beta^2\delta > 1$, the condition which can easily be satisfied and we assume it holds true. Moreover it is obvious from the above expression that $\widetilde{\omega}_k > \widetilde{\omega}_h$. Then given the voting equilibrium, we also have three regimes under the setup of education-loans market: 1) $\omega \in [\alpha\Omega'/(1-\alpha),\ \widetilde{\omega}_h]$, the two armed intergenerational policy increases both human and physical capital in the short run; 2) $\omega \in [\widetilde{\omega}_h,\ \widetilde{\omega}_k]$, the policy package increases physical capital investment but reduces human capital investment in the short run; 3) $\omega \geq \widetilde{\omega}_k$, the policy package reduces both human and physical capital in the short run. Different from the previous case, the politically chosen policy package can change the laissez-faire economy which is initially dynamically efficient to a political economy that is dynamically inefficient when the political power of the old is limited, i.e., $\omega \leq \widetilde{\omega}_k$. That means in the long run

[23] The proof for the following proposition has been omitted since it is straightfoward.

although both capital investment increases, the physical capital can increase more than the human capital after implementation of an education-pension policy.

Regime 1 has an interesting implication that when the private education investment is funded by credit market and the political power of the old is sufficiently small, the political outcome can increase the short-run investment of both capitals instead of only increasing the human capital investment as in the altruism model. While perhaps surprising at first, the result is intuitive. Recall that under the public education program, the government imposes an education tax on the middle-aged and uses the proceeds to pay for the education expenses of the young. From the perspective of the young born in period t, \$1 received from the middle-aged today implies a payment (tax) obligation of exactly \$1 to the next young generation born in period $t+1$. However, if the young borrow from the credit market to fund the education investment, she has to pay $\$R_{t+1}$ next period for every \$1 borrowed on the education-loans market. Since $R_{t+1} > 1$ in the dynamically efficient economy, it is socially cheaper for education to be funded via intergenerational transfers (at gross return 1) than for it to be funded via the education-loans market. That explains why the policy package can increase the human capital investment in regimes 1 and 2. Not only that, if we substitute the credit market clearing condition, $s_t = e_t + k_{t+1}$, into (28), it is clear that the middle-aged agent under education-loans regime not only needs to pay back the education loan taken when young, but also needs to fund the education investment of next generation via the credit market. In contrast, as shown in (10), the middle-aged agent under the policy regime only needs to pay for the education funding of next generation in the form of tax $\theta_t w_t h_t$, which in the meanwhile can also be considered as the repayment of the education funding she received when young. This comparison shows that the agent under policy regime may have less burden on education-related expenditure and therefore has a higher ability to fund physical capital investment. It is especially true when ω is small so that the size of PAYG social security is limited and the negative effect of social security on physical capital investment is dominated by the positive effect generated by the cost saving program of public education.

On the other hand, when agents simultaneously vote for both social security and public education, the two programs compete with each other for the budget revenue and the old will affect the public education program indirectly via their preference over PAYG social security program. Since the present old benefit from the social security program without bearing any cost at present, they always prefer a PAYG social security program to be as large as possible. Therefore as the political power of the old increases, the size of PAYG social security program increases as well, which eventually guarantees a sacrifice of the generosity of public education program. Hence, as in the altruism model, when ω increases, both human

and physical capital investment decreases, but human capital investment falls before physical capital investment starts falling under the model of education-loans. Eventually when ω is sufficiently large as in regime 3, the political intergenerational transfers reduce both human and physical capital investment.

Finally we examine the long run welfare implication of the two-policy package. Since the short-run impacts on human and physical capital investment head towards the same direction in regimes 1 and 3, both human and physical capital investment increase in regime 1 but fall in regime 3 at the steady state. Consequently comparing to the laissez-faire economy, the steady-state welfare under the policy package may rise and fall in regimes 1 and 3 respectively, and there exists a cutoff ω, after which the welfare ranking reverses. If we proceed to run the simulation example under the setup of education-loans market, we will get exactly the same picture of welfare ranking as in Figure 1. Hence, although some quantitative differences arise in the welfare effects on capital investment under the setup of education-loan market, the main message of the paper remains valid: politically establishing a package consisting of public education and social security may have the ability to generate higher social welfare than the complete market allocations in the long run if and only if the influence of the old is limited. Therefore we conclude that our claim is robust to the setup of private education investment.

4 Conclusion

Using a political economy setup, this paper studies the effects of the intergenerational transfer policies where both education and social security policies are determined endogenously. In our analysis, the government is short-lived and cannot commit to the future policies. Agents on the other hand are subject to intergenerational cooperation and conflict on the issue of intergenerational transfers.

Our results under the political economy setup is in line with the well known results that if the economy is dynamically efficient, on the grounds of efficiency, PAYG social security should be rejected. If PAYG social security program is implemented, we show that along with the physical capital, the human capital production also declines, despite some positive general equilibrium effects. However, if an education - pension package is implemented, in the short run, an economy can experience a boost in investment in education but it will be accompanied by a fall in physical capital. But due to the positive general equilibrium effects that the agents internalize while voting, eventually the economy can produce more of both types of capital. Not only that, an education - pension package is capable of producing higher welfare too. This however happens only when the influence of the old in the political process

is limited. Thus a mutual benefit of higher level of education (compared to laissez-faire) and a limited social security can well coexist and it is welfare improving. A substantial social security program however is not cost-less - it chokes this interplay of mutual benefits.

The above result is derived under the setup of laissez-faire where altruism motivates investment in the education of the next generation. However if we replace the altruism source of financing by providing a perfect capital market that allows the agents to borrow funds for their own education, the qualitative prescription that a package consists of education - pension can be welfare improving over the laissez-faire if the political influence of the old is limited will still remain valid. Along with the general equilibrium effects, the additional channel that helps securing higher level of capital as well as welfare is the funding through the generational arrangement (here tax) that is cheaper than the market interest rate under the assumption of dynamic efficiency.

Thus our paper hints at the basic fact that unless the influence of the old is restricted, with the present demographic change, there might be a natural tendency to see public education under pressure while pension is improved. In our model public education produces better allocation and improved welfare than the situation when only private education is present. As we mentioned, historically as well as in developing countries, the public education program proceeded the social security by decades or even century. The paper also confirms that adding social security up to the public education program cannot be welfare improving.

Couple of points need worth mentioning here. The key mechanism of the paper is that the political support for education investment could guarantee more of both types of capital as well as welfare than private education investment can generate under the condition that the political power of the old is limited. As long as it is possible to boost education investment under the political process, the future income can be increased so that the agents can invest more in human and physical capital in the future and the welfare can be improved. This mechanism is not expected to be different even if we consider a more general utility function, for e.g., constant relative risk aversion (CRRA) model. Of course, under the assumption of CRRA utility function, the tax rates may not be constant anymore, but the key mechanism remains the same and thus we do not think CRRA setup would change the main results of the paper.

Further, this paper contributes to the debate on how social security interacts with education, and what effects these two intergenerational goods have on the economy, both in the short and long run. As the intergenerational distribution of political power is tied to the demographic change, which in turn is determined by the changing pattern of fertility and longevity, a natural extension of this study is to accommodate individual choice of fertility and longevity. We leave this for future study.

Appendix A

Proof of Lemma 1. By linearizing the dynamic systems around the steady state, we have

$$\begin{pmatrix} k_{t+1}^L - \overline{k}^L \\ h_{t+1}^L - \overline{h}^L \end{pmatrix} = \begin{pmatrix} \alpha & (1-\alpha)\,\overline{k}^L/\overline{h}^L \\ \alpha\beta\overline{h}^L/\overline{k}^L & \beta\,(1-\alpha) \end{pmatrix} \begin{pmatrix} k_t^L - \overline{k}^L \\ h_t^L - \overline{h}^L \end{pmatrix}.$$

It can be easily shown that the eigenvalues of the Jacobian matrix are 0 and $\alpha + \beta - \alpha\beta$, both of which are non-negative and less than one. Hence the nontrivial steady state is a sink and thus stable. ■

Proof of Proposition 1. To prove the proposition, it is enough to show that given (8) and $\omega \geq \alpha\Omega/(1-\alpha)$, $k_{t+1}^P \leq k_{t+1}^L$ and $e_t^P \leq e_t^L$ for any pair of (k_t, h_t). Note that

$$\frac{k_{t+1}^L}{k_{t+1}^P} = \frac{1-\alpha}{\alpha\,(1+\delta+\beta\delta\phi)}\left[\alpha\delta + \frac{\omega\,(1+\alpha\delta+\beta\delta\phi)}{\Omega}\right]$$

which monotonically increases in ω and is equal to 1 when $\omega = \alpha\Omega/(1-\alpha)$. Hence for any pair of (k_t, h_t), $k_{t+1}^P > k_{t+1}^L$ if and only if $\alpha\Omega/(1-\alpha)$. On the other hand, when we compare the two optimal solutions of education investment e_t^P and e_t^L, it is clear that $e_t^P \leq e_t^L$ if and only if $b_{t+1}/R_{t+1} - \tau_t w_t h_t \leq 0$, which by substituting the equilibrium values becomes $k_{t+1}^P/\alpha - Ak_t^\alpha h_t^{1-\alpha} \leq 0$. Using (22), the condition further can be reduced to

$$G(\omega) \equiv \frac{\delta}{\alpha\delta + \omega\,(1+\alpha\delta+\beta\delta\phi)/\Omega} \leq 1.$$

Evidently $G(\omega)$ monotonically decreases in ω and takes the maximal value when $\omega = \alpha\Omega/(1-\alpha)$, with

$$\max G(\omega) = \frac{(1-\alpha)\,\delta}{\alpha\,(1-\alpha)\,\delta + \alpha\,(1+\alpha\delta+\beta\delta\phi)}.$$

Given the dynamic efficiency condition (8) requires $\alpha\,(1+\alpha\delta+\beta\delta\phi) \geq (1-\alpha)\,\delta$, it is obvious that $\max G(\omega) < 1$ and therefore $e_t^P \leq e_t^L$ holds for any pair of (k_t, h_t).

To prove $\overline{k}^P/\overline{h}^P \leq \overline{k}^L/\overline{h}^L$, it is enough to show

$$J(\omega) \equiv \frac{\alpha\,(1+\delta+\beta\delta\phi)/(1-\alpha)}{\alpha\delta + \omega\,(1+\alpha\delta+\beta\delta\phi)/\Omega} \leq 1.$$

Same as above, $J(\omega)$ monotonically decreases in ω and takes the maximal value when $\omega = \alpha\Omega/(1-\alpha)$. It can be easily verified that $\max J(\omega) = 1$. Hence the proof. ■

Proof of Proposition 2. It can be verified that

$$\frac{e_t^L}{e_t^X} = \frac{\phi\left(1-\alpha\right)}{1+\delta+\beta\delta\phi}\left(\beta\delta + \frac{1+\alpha\delta+\omega}{1-\alpha+\phi}\right)$$

monotonically increases in ω and equal to 1 when $\omega = \widetilde{\omega}$. Hence for any pair of (k_t, h_t), $e_t^L \leq e_t^X$ if and only if $\omega \leq \widetilde{\omega}$. Moreover with some algebra, we can show that

$$\widetilde{\omega} - \frac{\alpha\Omega}{1-\alpha} = \frac{1+\delta+\delta\phi}{\phi} > 0.$$

Similarly it can be verified that k_{t+1}^L/k_{t+1}^X monotonically increases in ω and strictly larger than 1 when $\omega = \alpha\Omega/\left(1-\alpha\right)$, which completes the proof. ■

Proof of Proposition 3. First, it can be shown that

$$\frac{\overline{h}^X}{\overline{h}^L} = \Psi_1^{\frac{\beta}{1-\beta}}\Psi_2^{\frac{\alpha\beta}{(1-\alpha)(1-\beta)}} \text{ and } \frac{\overline{k}^X}{\overline{k}^L} = \Psi_1^{\frac{\beta}{1-\beta}}\Psi_2^{\frac{1-\beta+\alpha\beta}{(1-\alpha)(1-\beta)}}$$

where

$$\begin{aligned}\Psi_1 &= \frac{\left(1-\alpha+\phi\right)\left(1+\delta+\beta\delta\phi\right)}{\phi\left(1-\alpha\right)\left(\omega+\Omega\right)} \\ \Psi_2 &= \frac{\alpha\left(1+\alpha\delta\right)\left(1+\delta+\beta\delta\phi\right)}{\left(1-\alpha\right)\left(\omega+\alpha\delta\omega+\alpha\delta\Omega\right)}.\end{aligned}$$

Clearly both $\overline{h}^X/\overline{h}^L$ and $\overline{k}^X/\overline{k}^L$ monotonically decrease in ω and we define $(\overline{\omega}_k, \overline{\omega}_h)$ by $\overline{k}^X/\overline{k}^L\Big|_{\omega=\overline{\omega}_k} = 1$ and $\overline{k}^X/\overline{k}^L\Big|_{\omega=\overline{\omega}_h} = 1$. Note that $\overline{k}^X/\overline{k}^L = (\Psi_2)^{\frac{1}{1-\alpha}}\,\overline{h}^X/\overline{h}^L$. Hence to prove $\overline{\omega}_h > \overline{\omega}_k$, it is enough to show $\Psi_2 < 1$ so that $\overline{k}^X/\overline{k}^L < \overline{h}^X/\overline{h}^L$ for any $\omega \geq \alpha\Omega/\left(1-\alpha\right)$. Some algebra can show that $\Psi_2 < 1$ when $\omega = \alpha\Omega/\left(1-\alpha\right)$. Since Ψ_2 monotonically decreases in ω, we have $\Psi_2 < 1$ for any $\omega \geq \alpha\Omega/\left(1-\alpha\right)$, which completes the proof for the first two parts of the proposition.

$\overline{k}^X/\overline{h}^X \leq \overline{k}^L/\overline{h}^L$ is equivalent with

$$M\left(\omega\right) \equiv \frac{\alpha\left(1+\alpha\delta\right)\left(1+\delta+\beta\delta\phi\right)/\left(1-\alpha\right)}{\omega+\alpha\delta\left(\omega+\Omega\right)} \leq 1,$$

and $M\left(\omega\right)$ takes the maximal value when $\omega = \alpha\Omega/\left(1-\alpha\right)$. Some algebra can yield

$$\max M(\omega) = \frac{\left(1+\alpha\delta\right)\left(1+\delta\right)+\left(1+\alpha\delta\right)\beta\delta\phi}{\left(1+\delta\right)\left(1+\alpha\delta\right)+\left(1+\delta\right)\left[\beta\delta\left(1-\alpha\right)+\beta\delta\phi\right]}$$

which is obviously smaller than 1. Hence the proof. ■

5 Appendix B

Optimal solution of private education investment under the two-policy package:

When both private and public education investment are present, agents maximize (1) subject to (9) - (11), which yields first order conditions as follows

$$\begin{aligned} u'\left[(1-\theta_t-\tau_t)\,w_t h_t - s_t^X - \left(e_t^X - e_t^g\right)\right] &= \delta R_{t+1} u'\left(R_{t+1}s_t^X + b_{t+1}\right) \\ u'\left[(1-\theta_t-\tau_t)\,w_t h_t - s_t^X - \left(e_t^X - e_t^g\right)\right] &\geq \delta\phi u'\left(h_{t+1}\right) h'\left(e_t^X\right), \; = \text{ if } e_t^X \geq e_t^g. \end{aligned}$$

To show the optimal private investment in education is zero corner, i.e., $e_t^X = e_t^g$, it is enough to show

$$u'\left[(1-\theta_t-\tau_t)\,w_t h_t - s_t^X\right] \geq \delta\phi u'\left(h_{t+1}\right) h'\left(e_t^X\right),$$

which under the assumption of logarithmic utility and government's balanced budget condition turns to be

$$\frac{1}{(1-\theta_t-\tau_t)\,w_t h_t - s_t^X} \geq \frac{\beta\delta\phi}{\theta_t w_t h_t}$$

By substituting equilibrium tax rates θ_t^X, τ_t^X, and general equilibrium condition $k_{t+1}^X = s_t^X$, the above inequality changes to

$$\frac{\omega + \alpha\delta\left(\omega + \Omega\right)}{\omega\left(1+\alpha\delta\right)} \geq \frac{\phi}{1-\alpha+\phi},$$

which with some algebra is equivalent to

$$(1-\alpha)\left(\omega + \alpha\delta\omega + \alpha\delta\Omega\right) + \alpha\delta\phi\Omega \geq 0.$$

Hence the proof.

References

[1] Aaron, H., 1966. The social insurance paradox. Canadian Journal of Economics and Political Science 32(3), 371—374.

[2] Andersen, T., Bhattacharya, J., 2011. On myopia as rationale for social security. Economic Theory. 47, 135–158.

[3] Andersen, T., Bhattacharya, J., 2015. The Intergenerational Welfare State and the Rise and Fall of Pay-as-you-go Pensions**.** Economic Journal. Forthcoming.

[4] Bassetto, M., 2008. Political Economy of Taxation in an Overlapping-Generations Economy. Review of Economic Dynamics. 11, 18-43.

[5] Bhaskar, V. 1998. Informational constraints and the overlapping generations model: folk and anti-folk theorems. Review of Economic Studies. 65, 135–149.

[6] Becker, G. S., Murphy, K. M., 1988. The Family and the State. *Journal of Law and Economics*. 31, 1–18.

[7] Bishnu, M., 2013. Linking Consumption Externalities with Optimal Accumulation of Human and Physical Capital and Intergenerational Transfers. Journal of Economic Theory. 148, 720-742.

[8] Boldrin, M., Donaldo, J., Jimeno, J., Perachhi, F., 1999. The Future of Pensions in Europe. Economic Policy. 14 (29), 287-320.

[9] Boldrin, M., Montes, A., 2005. The Intergenerational State Education and Pensions. Review of Economic Studies. 72, 651-664.

[10] Boldrin, M., Rustichini, A., 2000. Political Equilibria with Social Security. Review of Economic Dynamics. 3, 41–78.

[11] Chen, K., Song, Z., 2014. Markovian Social Security in Unequal Societies. The Scandinavian Journal of Economics. 116(4), 982-1011.

[12] Cooley, T. F., Soares, J., 1999. A Positive Theory of Social Security Based on Reputation. Journal of Political Economy. 107, 135-160.

[13] Cremer, H., Kessler, D. and Pestieau, P., 1992. Intergenerational Transfers within the Family. European Economic Review. 36, 1–16.

[14] De La Croix, D., Doepke, M., 2003. Inequality and Growth: Why Differential Fertility Matters. American Economic Review. 93, 1091-1113.

[15] Docquier, F., Paddison, O., Pestieau, P., 2007. Optimal Accumulation in an Endogenous Growth Setting with Human Capital. Journal of Economic Theory. 134, 361-378.

[16] Feldstein, M., 1985. The Optimal Level of Social Security Benefits. Quarterly Journal of Economics. 100(2), 303-320.

[17] Feldstein, M., 2005. Structural Reform of Social Security. Journal of Economic Perspectives. 19, 33-55.

[18] Forni, L., 2005. Social Security as Markov Equilibrium in OLG Models. Review of Economic Dynamics. 8(1), 178-194.

[19] Galasso, V., Profeta, P., 2002. The Political Economy of Social Security. European Journal of Political Economy. 18, 1-29.

[20] Glomm, G. and Kaganovich, M., 2003. Distributional Effects of Public Education in an Economy with Public Pensions. International Economic Review. 44, 917-937.

[21] Glomm, G. and Kaganovich, M., 2008. Social Security, Public Education and the Growth–Inequality Relationship. European Economic Review. 52, 1009-1034.

[22] Glomm, G., Ravikumar, B., 1992. Public vs. Private Investment in Human Capital: Endogenous Growth and Income Inequality. Journal of Political Economy. 100, 818-834.

[23] Gonzalez - Eiras, M., Niepelt, D., 2008 The Future of Social Security. *Journal of Monetary Economics*. 55, 197-218.

[24] Gonzalez - Eiras, M., Niepelt, D., 2012. Ageing, Government Budgets, Retirement, and Growth. European Economic Review. 56, 2012, 97-115.

[25] Kaganovich, M., Zilcha, I., 1999. Education, Social Security and Growth. Journal of Public Economics. 71, 289-309.

[26] Kaganovich, M., Zilcha, I., 2012. Pay-as-you-go or Funded Social Security? A General Equilibrium Comparison. Journal of Economic Dynamics and Control. 36, 455-467.

[27] Krusell, P., Quadrini, V., Rios-Rull, J.-V., 1997. Politico-economic Qquilibrium and Economic Growth. Journal of Economic Dynamics and Control. 21 (1), 243–272.

[28] Lancia, F., Russo, A., 2015. Public Education and Pensions in Democracy: A Political Economy Theory. Journal of the European Economic Association. Forthcoming.

[29] Lindbeck, A., Weibull, J. W., 1987. Balanced-budget Redistribution as the Outcome of Political Competition", Public Choice, 52, 273-297.

[30] Lindert, P., 1994. The Rise of Social Spending, 1880-1930. *Explorations in Economic History*. 31, 1-37.

[31] Mateos-Planas, X., 2008. A Quantitative Theory of Social Security without Commitment. Journal of Public Economics. 92(3-4), 652-671.

[32] Mookherjee, D., Ray, D., 2002. Is Equality Stable?. American Economic Review Papers and Proceedings. 92, 253-259.

[33] Mookherjee, D., Ray, D., 2010. Inequality and Markets: Some Implications of Occupational Diversity. American Economic Journal: Microeconomics. 2, 38-76.

[34] Mulligan, C., Sala-I-Martin, X., 1999. Gerontocracy, Retirement, and Social Security. Working Paper 7117, NBER.

[35] Naito, K., 2012. Two-sided Intergenerational Transfer Policy and Economic Development: A Politico-economic Approach. Journal of Economic Dynamics and Control. 36, 1340-1348.

[36] Orszag, P. R., Stiglitz, J. E., 2001. Rethinking pension reform: ten myths about social security systems, in: Holzmann, R., Stiglitz, J. (Eds.), New Ideas about Old Age Security: Toward Sustainable Pension Systems in the 21st Century. The World Bank, Washington, D.C.

[37] Pecchenino, R. A., Utendorf, K. R., 1999. Social Security, Social welfare and the Aging Population. Journal of Population Economics. 12, 607–623.

[38] Pogue, T. F., Sgontz, L. G., 1977. Social Security and Investment in Human Capital. National Tax Journal. 30,157–169.

[39] Poutvaara, P., 2006. On the Political Economy of Social Security and Public Education. Journal of Population Economics. 19, 345-365.

[40] Preston, S. H., 1984. Children and the Elderly: Divergent Paths for America's Dependents. Demography. 21, 435-457.

[41] Rangel, A., 2003. Forward and Backward Intergenerational Goods: Why is Social Security Good for the Environment. American Economic Review. 93, 813-834.

[42] Richman, H. A., Stagner, M. W., 1986. Children: Treasured Resource or Forgotten Minority, in: A. Pifer and L. Bronte (eds.) Our Aging Society: Paradox and Promise (New York: Norton), 161–179.

[43] Soares, J., 2003. Self-interest and public funding of education. Journal of Public Economics. 87, 703–727.

[44] Soares, J., 2006. A Dynamic General Equilibrium Analysis of the Political Economy of Public Education. Journal of Population Economics. 19,367–389.

[45] Song, Z., 2011. The Dynamics of Inequality and Social Security in General Equilibrium. Review of Economic Dynamics, 14, 613-635.

[46] Song, Z., Storesletten, K. and Zilibotti, F., 2012. Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt. Econometrica. 80, 2785–2803.

[47] Tabellini, G., 1991. The Politics of Intergenerational Redistribution. Journal of Political Economy. 99, 335-357.

[48] UNITED NATIONS (2013), Department of Economic and Social Affairs, Population Division (2013). World Population Prospects: The 2012 Revision, DVD Edition.

[49] Wang, M., 2014. Optimal Education Policy under Endogenous Borrowing Constraints. Economic Theory. 55, 135-159.