Redistributive Taxation and Administrative Costs

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Abstract

I analyze the impact of administrative costs on the determination of the optimal rate of redistributive direct and indirect taxes. In the case of direct taxes it is shown that for any tax rate, an increase in the administrative cost coefficient amounts to a reduction in the effective rate of taxation leading to higher output and profits at the expense of worker welfare. Social welfare declines with an increase in the cost coefficient. The optimal rate of tax may decrease with an increase in the cost coefficient.

Indirect taxes are neutral as the increased costs borne by the entrepreneurs and the increased purchasing power of workers who are the recipients of the tax transfer nullify one another leaving output constant. A leakage in tax revenue that administrative cost represents makes indirect taxes non-neutral. An increase in the administrative cost coefficient amounts to an increase in the effective tax rate in an equivalent economy where the leakage rate is 100 percent, i.e. all the tax is lost in administration. This increase in the cost coefficient results in a decline in output and profits if leisure and output are strong substitutes. However in case leisure and output are complements, the increase in the cost coefficient can result in an increase in output and profits, but at the expense of worker welfare. In the event of an increased cost coefficient a suitable decrease in the tax rate restores the economy to the equilibrium at the original cost coefficient. Unlike in the case of direct taxes, social welfare remains constant or increases with an increase in the administrative cost coefficient. In case it remains constant, the optimal indirect tax rate decreases with an increase in tax leakage.

1 Introduction

Optimal tax theory started with the question: how can the government raise a fixed amount of tax revenue with a limited tax base, while minimizing efficiency costs(Ramsey 1927)? Till fairly recently, this body of work did not address several important aspects of real world tax regimes like costs of tax administration, compliance etc.(Slemrod 1990).

In the spirit of Heller(1974), and Yitzhaki(1979) this paper analyzes the impact of one such real world feature, namely a leakage in tax revenue, on social welfare and the optimal tax rate. In many countries a large proportion of the public fund does not reach the intended recipients. In India, for example, studies have shown that for every rupee spent on public works, only fifteen paise reaches the target beneficiary. The impact of such a leakage, interpreted as the administrative cost coefficient, on optimal rates of direct and indirect tax is therefore a problem worthy of attention. As in Yitzhaki, once incurred the administrative costs are deemed to leave the economy without further impact in terms of expenditure by the beneficiaries of the leakage.

The analysis is carried out in the context of an Arrow-Debreu(1954) economy with a continuum of entrepreneurs and workers, and a government which levies a direct tax on the profit of entrepreneurs or an indirect tax on sales. [1] The proceeds of the tax are redistributed to workers with an exogenously determined cost coefficient. As mentioned earlier the leakage does not re-enter the economy in any fashion.

In the case of direct taxes it is shown that for any tax rate, an increased administrative cost coefficient amounts to a reduction in the effective rate of taxation leading to higher output and profits at the expense of worker welfare. Social welfare declines with an increase in the cost coefficient. The optimal rate of tax may decrease with an increase in the cost coefficient.

Indirect taxes are neutral in our economy as the increased costs borne by the entrepreneurs and the increased purchasing power of workers who are the recipients of the tax transfer nullify one another leaving output constant. An administrative cost makes indirect taxes non-neutral. An increase in the cost coefficient amounts to an increase in the tax rate in an equivalent economy where the coefficient is 100 percent, i.e. all the tax is lost. This increase in leakage results in a decline in output and profits, and maybe even worker welfare, if leisure and output are strong substitutes. However in case leisure and output are complements, the increase in the cost coefficient can result in an increase in output and profits. In the event of increased leakage, a suitable decrease in the tax rate restores the economy to the original equilibrium. Unlike in the case of direct taxes, social welfare remains constant or increases with an increase in the administrative cost coefficient. In case it remains constant, the optimal indirect tax rate decreases with an increase in tax leakage.

[1] This way of modeling the economic agents of the economy owes its origin to the classical school of political economy starting with Ricardo(1817).

The results of the paper imply that in a country where business is largely controlled by a class that does not directly engage in production, an anticipated increase in the administrative cost coefficient might call for a downward revision of tax rates, especially if indirect taxes constitute the lion's share of tax revenue.

The rest of the paper is organized as follows: the first half is devoted to the direct tax model and the second half to the indirect tax model. In section 2 the baseline model with direct taxes is delineated, existence and uniqueness of equilibrium established and impact of a redistributive tax characterized for a robust sub-class of economies. Next an exogenous leakage is introduced and an equivalence established with the model without leakage. Finally the implications of leakage for output, profits and worker welfare and consequently for the optimal tax rate are drawn out. In Section 3, a similar analysis is carried out for an indirect tax regime. Section 4 presents concluding remarks.

2 Model with Direct Taxes

We consider an economy lasting two time periods $t \in T = \{0, 1\}$. The economy consists of a continuum of entrepreneurs (E), and workers(W). There are two goods in the economy - labor l(leisure $\bar{l}$) and corn c. Workers are endowed with L units of labor in period 1 and entrepreneurs possess technology that converts labor supplied by the worker in Period 1 into corn in Period 2. This technology is given by $f : R_+ \to R_+$ where f satisfies the usual assumptions of differentiability, strict concavity, impossibility of free production, and possibility of no production. We assume that $f^{'}(0) = \infty$, i.e. the marginal product at zero input level is infinite and that $f^{'}(L) > 0$.

Workers like corn and leisure, while entrepreneurs maximize profits. The worker has a standard CES utility function given by $u^w(\bar{l}, c) = (\bar{l}^{\rho} + c^{\rho})^{-\frac{1}{\rho}}$.

2.1 Government

The government levies a tax $\tau \in [0, 1]$ on the surplus of entrepreneurs and transfers it to the worker in order to maximize a differentiable, perfectly concave, non-decreasing social welfare function defined on the worker welfare and entrepreneur profits.

2.2 Sequence of Activities

2.2.1 Period 1

The following activities take place in the order listed:
(1) The labor market meets. The worker exchanges labor for future corn.
(2) The labor sold by the worker is employed in the production of corn.

2.2.2 Period 2

The following activities take place in the order listed:
(1) Production of corn is realized.
(2) The entrepreneur redeems his corn obligation to the worker and pays (corn) tax to the government.
(3) The tax is transferred to the worker and consumption takes place.

Let $q_l^W \equiv$ quantity of labor sold by a worker in Period 1, and $q_c^W \equiv$ quantity of corn demanded by a worker in Period 2. The set of actions available to the worker (q_l^W, q_c^W) is denoted by q^W. Let $q_l^E \equiv$ quantity of labor demanded by an entrepreneur in Period 1, $q_c^E \equiv$ quantity of corn sold by the entrepreneur in period 2. The set of actions available to an entrepreneur (q_l^E, q_c^E) is denoted by q^E. Given the tax rate τ, the actions of the entrepreneur induce a tax obligation of $(f(q_l^E) - q_c^E)\tau$ units of corn in Period 2. Let $p \equiv$ the relative price of price of labor to the price of corn, i.e. the real wage.

2.3 The Budget Set of a Worker

The set of feasible allocations A^W of leisure and corn are given by the following equations:

Labor sold to entrepreneur $\leq$ Labor endowment

$$q_l^W \leq L \qquad (1)$$

Leisure consumed = Labor endowment - Labor sold

$$A_{\bar{l}}^W = L - q_l^W \qquad (2)$$

Corn consumed = Corn bought + corn obtained through transfers

$$A_c^W \equiv q_k^W = pq_l^W + (f(q_l^W) - pq_l^W)\tau \qquad (3)$$

The set of allocations A^W corresponding to actions q^W that satisfy these constraints is denoted by $\Sigma^W(p)$ and is called the budget set of the worker.

2.4 The Budget Set of an Entrepreneur

We define A^E, the final allocation of an entrepreneur as follows: $A_c^E = (f(q_l^E) - q_c^E)(1-\tau)$ where $S \equiv f(q_l^E) - q_c^E$ is the gross surplus of the firm and $\pi \equiv A_c^E$ the profit. The constraints on the set of actions q^E available to an entrepreneur given p are as follows:

In Period 1: Cost of labor demanded = Future corn sold

$$pq_l^E = q_c^E \qquad (1)$$

In Period 2: Corn sold + Corn paid as tax $\leq$ corn produced

$$q_c^E + (f(q_l^E) - q_c^E)\tau \leq f(q_l^E) \qquad (2)$$

i.e.

$$q_c^E(1-\tau) \leq f(q_l^E)(1-\tau) \qquad (2')$$

The set of A_c^E corresponding to actions q^E that satisfy these constraints is denoted by $\Sigma^E(p)$ and is called the budget set of the entrepreneur.

2.5 Equilibrium

A vector of allocations, and prices $(A^W, A^E; p)$ is an equilibrium for a given τ if:

(1) All workers are optimal on their budget sets, i.e. for workers

$$A^W \in \Sigma^W(p)$$

and

$$\hat{A^W} \in \Sigma^W(p) \Rightarrow u^W(\hat{A}^W) \leq u^W(A^W)$$

For entrepreneurs

$$A^E \in \Sigma^E(p)$$

$$\hat{A^E} \in \Sigma^E(p) \Rightarrow \hat{A}^E \leq A^E$$

(2)All markets clear, i.e. in the labor market

$$q_l^E = q_l^W$$

In the corn market

$$q_c^E = q_c^W$$

2.6 Equations of equilibrium

The first order condition for utility maximization for workers is

$$\frac{MU_{\bar{l}}}{MU_c}(A^W) \equiv g = p(1-\tau) + \tau f^{'}$$

[2]

The first order condition for profit maximization for entrepreneurs is

$$f^{'}(q_l^E) = p$$

These two equations along with the budget constraints of the workers and entrepreneurs and the market clearing condition constitute the equations of equilibrium.

[2] The boundary equilibrium at which $q_l^W = 0$ is ruled out by the assumption $f^{'}(0) = \infty$. The boundary equilibrium at which $q_l^W = L$ is ruled out by the assumption $g(0, c) = \infty$. Therefore the assumption that at equilibrium $g = p(1-\tau)+\tau f^{'}$ is without loss of generality.

2.7 Existence and Uniqueness of Equilibrium

Theorem 1: *For all $\tau \in [0,1]$ there exists an equilibrium.*

Proof: (i) The proof for $\tau \in [0,1)$ proceeds in two steps. We first show that the excess demand for labor, $ED_l = q_l^E - q_l^W$, is strictly positive for sufficiently small p and strictly negative for sufficiently large p. Given the continuity of the excess demand function and Walras' Law, this establishes the existence of a price at which all equilibrium conditions are satisfied.[3]

Note that the labor demand function is given by

$$f^{'}(q_l^E) = p$$

Given the assumption that $f^{'}(L) > 0$, for sufficiently small p, the quantity demanded will be greater than L the highest possible level of the supply of labor. This shows that for sufficiently small p the excess demand for labor is strictly positive.

To show that for sufficiently large p, the excess demand for labor is strictly negative we show that for large enough p, the demand for corn is greater than $f(L)$, the physical limit of production in the economy, i.e. there is an excess demand for corn. From this it follows that there is an excess supply of labor at that price.

Note that given the assumptions that $g(0,c) = \infty, \forall c > 0$, and that u^W is strictly concave, it follows that $\exists B$ s.t. $g(L, f(L)) < B$.

For large enough p the worker will be able to to buy $f(L)$, the physical limit of production. For such a p, we define the threshold labor supply $l(p)$ as the quantity of labor that would enable the worker to buy exactly $f(L)$ units of corn. $l(p)$ is given by the equation

$$pl(p)(1-\tau) + \tau f(l(p)) = f(L)$$

Note that that $l(p)$ is bounded above by $\frac{f(L)}{p(1-\tau)}$ and that for any $\tau \in [0,1)$, there exists a $p^\star$ large enough that the worker is able to buy $f(L)$ and

$$p^\star(1-\tau) > B \qquad [1]$$

$$g(L - \frac{f(L)}{p^\star(1-\tau)}; f(L)) < B \qquad [2]$$

Condition 2 states that the upper limit on $l(p)$ the threshold labor supply is small enough that the upper bound on $g(L, f(L))$ continues to apply at the consumption point $(L - \frac{f(L)}{p^\star(1-\tau)}; f(L))$. At this $p^\star$ the slope of the worker's budget line $m(p^\star, l) = p^\star(1-\tau) + \tau f^{'}(l)$ is greater than the slope of the indifference curve at the consumption point $(L - l(p^\star); f(L))$. To see this note

$$m(p^\star, L - l(p^\star)) > p^\star(1-\tau) > B$$

[3] Recall there are only two goods in our model.

$$B > g(L - \frac{f(L)}{p^\star(1-\tau)}; f(L)) > g(L - l(p^\star); f(L))$$

The last inequality follows from the observation that g declines as the amount of leisure consumed increases while corn consumed stays the same. Therefore the worker will optimize at a choice of leisure greater than $l(p^\star)$, and demand an amount of corn greater than $f(L)$, the physical limit. Thus at $p^\star$, there is excess demand for corn and excess supply of labor. By the continuity of the excess demand function, there exists a price at which equilibrium exists.

(ii) For $\tau = 1$, the worker will get all the corn produced in return for the labor supplied. So the objective function is $u^W(L - q_l^W; f(q_l^W))$. By the maximum principle, an optimum exists and the choice of q_l^W is independent of p.

The entrepreneur is indifferent about employing any quantity of labor at any prevailing wage rate, including q_l^{W*} the quantity of labor the worker wants to supply. Therefore any positive real wage along with q_l^{W*} constitutes an equilibrium when $\tau = 1$.

QED

Theorem 2: *Consider an economy with* $f^{'} + lf^{''} > 0, \forall l \in [0, L]$. *For all* $\tau \in [0,1]$ *the equilibrium is unique.*

Proof: From the equations of equilibrium it follows that $q_l^\star, q_k^\star$ constitute an equilibrium at price $p^\star$ iff

$$p^\star q_l^\star(1-\tau) + \tau f(q_l^\star) = q_k^\star$$

$$f^{'}(q_l^\star) = p^\star$$

$$g(L - q_l^\star, q_k^\star) = p^\star(1-\tau) + \tau f^{'}(q_l^\star)$$

$$q_l^\star \leq L; q_k^\star \leq f(q_l^\star)$$

From this it follows that at equilibrium

$$g(L - q_l^\star; f^{'}(q_l^\star)q_l^\star(1-\tau) + \tau f(q_l^\star)) = f^{'}(q_l^\star) \qquad [*]$$

From the first two equations of equilibrium it follows that

$$f^{'} q_l^\star(1-\tau) + \tau f(q_l^\star) = q_k^\star$$

From this it follows that

$$\frac{\partial q_k^\star}{\partial q_l^*} = f^{'} + (1-\tau)lf^{''} > 0$$

Given the assumption that $f^{'} + lf^{''} > 0 \ \ \forall l \in [0, L]$, $\frac{\partial q_k^\star}{\partial q_l^*}$, i.e. for a given tax rate if there are two equilibria, the one with the higher output will also have a higher corn consumption on the part of the worker.

Now we prove the result by contradiction. Suppose there are two equilibria for a given τ. At the higher output equilibrium $\overline{E}$, the worker has less leisure, and more corn, therefore the LHS of [*] is higher in $\overline{E}$ compared to the lower output equilibrium. But $f^{'}$ declines with an increase in labor, so the RHS of [*] is lower in $\overline{E}$ compared to the lower output equilibrium. This implies that [*] cannot be satisfied for both the equilibria.

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2.8 Impact of change in tax rate

Theorem 3: *Consider an economy with $f^{'} + lf^{''} > 0 \;\; \forall l \in [0, L]$. In this economy an increase in tax rate will lead to a decrease in output.*

Proof: From theorem 1 and 2, we know that there is a unique equilibrium therefore it is possible to examine the outcome of a tax increase. As shown earlier

$$g(L - q_l^\star; f^{'}(q_l^\star)q_l^\star(1-\tau) + \tau f(q_l^\star)) = f^{'}(q_l^\star) \qquad [*]$$

We first note that given the concavity of f,

$$\frac{\partial q_k^\star}{\partial \tau} = (f - f^{'} q_l^\star) > 0 \qquad [1]$$

i.e. $q_k^\star$ is a positive function of τ. As shown in the proof of Theorem 2

$$\frac{\partial q_k^\star}{\partial q_l^*} = f^{'} + (1-\tau)(f^{'} + lf^{''}) > 0 \qquad [2]$$

i.e. $q_k^\star$ is a positive function of $q_l^\star$.

Suppose an increase in tax rate results does not lead to a decrease in the quantity of labor supplied. Then from [1] and [2] the worker must be consuming more corn and no less leisure. This implies that g the marginal rate of substitution of corn for leisure will go up. On the other hand the marginal product $f^{'}$ will stay the same or decrease as the quantity of labor has not gone up. Thus the equilibrium equation will not be satisfied.

QED

Corollary 1: *The real wage and worker's welfare increase with the tax rate.*

Proof: From Theorem 3 it follows that as the tax rate increases, output falls. Therefore marginal product must increase, and by the equation of equilibrium, so must the real wage p. Notice

$$\frac{\partial q_k^W}{\partial p} = q_l^W(1-\tau) \geq 0 \;\; \forall q_l^W \in [0, L]$$

This means that for any quantity of leisure consumed the worker has at least as much corn at a higher real wage, and strictly more for all $q_l^W \in (0, L]$. Therefore the budget set at a higher real wage is a superset of the budget set at a lower real wage, and the worker must be better off.

QED

Corollary 2: *Profits fall with an increase in the tax rate.*

Proof: From Theorem 3, output falls with an increase in the tax rate. The surplus of the entrepreneur is given by

$$\pi = (f(q_l^\star) - p^\star q_l^\star)(1-\tau)$$

Substituting the entrepreneur's profit maximizing condition we get

$$\pi^* = (f(q_l^\star) - f'(q_l^\star)q_l^\star)(1-\tau)$$

From the concavity of $f, (f(l) - f'(l)l)$, decreases with a decrease in output. $(1-\tau)$ also decreases with an increase in τ. The result follows

QED

2.8.1 Example 1

Consider an economy characterized by the following parameters: $L = 10; f(l) = 30l - \frac{1}{2}l^2; u^W = \bar{l}^{\frac{1}{3}} c^{\frac{2}{3}}$. In this economy $l^\star = \frac{110-\sqrt{4900+1200\tau}}{6-\tau}$. It can be checked that as tax increases, output falls, real wage increases, the worker is better off, and profits falls.

2.9 The Direct Tax Model with Administrative Cost

We now consider a variation of the model in which the tax collected from the entrepreneurs is not entirely transferred to the intended recipients, the workers. A proportion $\kappa \in [0,1]$ of tax revenue is lost on account of administrative costs. This variation implies two changes in the equations of equilibrium. Firstly, the amount of corn consumed by the worker is now given by the equation

$$q_k^W = pq_l^W + (f(q_l^W) - pq_l^W)\tau(1-\kappa)$$

For this reason the workers' utility maximization equation must factor the leakage, i.e. now

$$\frac{MU_l}{MU_c}(A^W) \equiv g = p(1-\tau(1-\kappa)) + \tau(1-\kappa)f^{'}$$

The profit maximization condition of the entrepreneur remains the same.

As in Theorem 1, it can be shown that there exists an equilibrium for all $\tau \in [0,1]$ and $\kappa \in [0,1]$. Further, as in Theorem 2, if $f^{'} + lf^{''} > 0 \ \ \forall l \in [0, L]$, then the equilibrium is unique.

2.10 Comparing the Direct Tax Models With and Without Administrative Cost

Theorem 4: *An equilibrium at tax rate $\tau \in [0,1]$ with cost coefficient $\kappa \in [0,1]$ is identical in terms of output, real wage, worker welfare, and gross surplus S of entrepreneurs to an equilibrium at tax rate $\tau(1-\kappa)$ without leakage.*

Proof: Follows easily from the observation that the equations of equilibrium in the the model with tax rate τ and cost coefficient κ are identical to the equations of equilibrium in the the model with tax rate $\tau(1-\kappa)$ and no leakage.

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The equivalence classes of τ and κ that yield the same equilibrium output, real wage, worker welfare, and gross surplus of entrepreneurs are given by the equation $\tau(1-\kappa) = C$ and indicated by Figure 1. Note $\frac{d\tau}{d\kappa} = \frac{\tau}{1-\kappa} \geq 0$. From Theorem 3, output declines and real wage and worker welfare increase with an increase in the tax rate. Therefore output and gross surplus decline and real wage and worker welfare increase in the NW direction of the equivalence map.

The net profit of entrepreneurs does not share the same equivalence class as the worker's welfare, or the surplus. This is because $\pi = (1-\tau)S$. Therefore the net profit of entrepreneurs is lower at the equilibrium with tax rate $\tau \in (0,1]$ with cost coefficient $\kappa \in (0,1]$ compared to the equilibrium at tax rate $\tau(1-\kappa)$ with $\kappa = 0$. In both equilibria, the gross surplus is the same, as the output and real wage are identical, but in the equilibrium with tax rate $\tau \in (0,1]$ with cost coefficient $\kappa \in (0,1]$ entrepreneurs are worse off as they incur tax at the rate τ as opposed to $\tau(1-\kappa)$. In fact, as we move along the workers' indifference curve given by $\tau(1-\kappa) = C$, the gross surplus remains the same but profit decreases.

Corollary 1: *The slope of the indifference curve of profits is positive and lower than the slope of the indifference curve of the worker at every point in the $\tau - \kappa$ space.*

Proof: The indifference curve of S, the gross surplus of entrepreneurs is given by the equation

$$\frac{\partial S}{\partial \tau}\triangle\tau + \frac{\partial S}{\partial \kappa}\triangle\kappa = 0$$

Therefore

$$\frac{d\kappa}{d\tau} = -\frac{\frac{\partial S}{\partial \tau}}{\frac{\partial S}{\partial \kappa}}$$

We know $\pi = (1-\tau)S$. Therefore

$$\frac{\partial \pi}{\partial \tau} = (1-\tau)\frac{\partial S}{\partial \tau} - S \quad [1]$$

$$\frac{\partial \pi}{\partial \kappa} = (1-\tau)\frac{\partial S}{\partial \kappa} \quad [2]$$

The indifference cuve of profits is given by the equation

$$\frac{\partial \pi}{\partial \tau}\triangle\tau + \frac{\partial \pi}{\partial \kappa}\triangle\kappa = 0$$

Substituting from [1] and [2] we get

$$[(1-\tau)\frac{\partial S}{\partial \tau} - S]\triangle\tau + (1-\tau)\frac{\partial S}{\partial \kappa}\triangle\kappa = 0$$

This implies

$$\frac{d\kappa}{d\tau} = -\frac{\frac{\partial S}{\partial \tau}}{\frac{\partial S}{\partial \kappa}} + \frac{S}{(1-\tau)\frac{\partial S}{\partial \kappa}}$$

We know $\frac{\partial S}{\partial \kappa} > 0$ and $\frac{\partial S}{\partial \tau} < 0$. Therefore

$$\Rightarrow \frac{d\kappa}{d\tau} > -\frac{\frac{\partial S}{\partial \tau}}{\frac{\partial S}{\partial \kappa}} > 0$$

$$\Rightarrow \frac{d\tau}{d\kappa} < -\frac{\frac{\partial S}{\partial \kappa}}{\frac{\partial S}{\partial \tau}}$$

The result follows.

QED

From Corollary 1, the equivalence classes of τ and κ that yield the same equilibrium profit will have a lower slope than the equivalence classes of τ and κ that yield the same welfare and surplus. They are also shown in the same figure.[4].

[4]I have conducted the analysis with a constant κ. However the result on existence would go through with κ being any continuous function of the tax revenue. The result on uniqueness and decline in output with an increase in tax rate would go through with κ being any non-increasing function of tax revenue. The more complicated model has been omitted for expositional simplicity.

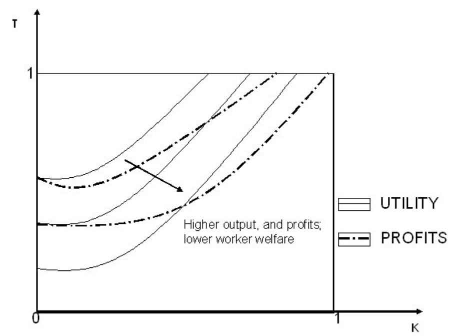

Figure 1: $\tau - \kappa$ Equivalence curves with direct taxes

Corollary 2: *Assume the government is maximizing a differentiable, perfectly concave, non-decreasing social welfare function defined on the worker welfare and entrepreneur profits by choosing $\tau \in [0,1]$ for an exogenously fixed κ. The social welfare cannot increase as κ increases. If social welfare at the lower rate of leakage is maximized at a strictly positive tax rate, then it must strictly decrease with an increase in leakage.*

Proof: Consider any two cost coefficients κ_1 and κ_2 with $\kappa_2 > \kappa_1$. Consider any $\tau > 0$. For any economy defined by (τ, κ_2), there exists an economy $(\tau^{'}, \kappa_1)$ such that $\tau^{'}(1-\kappa_1) = \tau(1-\kappa_2)$. At $(\tau^{'}, \kappa_1)$, the worker welfare is the same and profits higher than at (τ, κ_2). Therefore the equilibrium at $(\tau^{'}, \kappa_1)$ pareto dominates the equilibrium at (τ, κ_2). If $\tau = 0$ then the social welfare is the same in both cases. The result follows.

QED

As the following example demonstrates it is possible that the optimal tax rate falls with the level of leakage in the economy. An open problem is to identify general conditions under which the phenomenon of this example holds.

2.10.1 Example 2

Consider the economy of Example 1 characterized by the following parameters: $L = 10; f(l) = 30l - \frac{1}{2}l^2; u^W = \bar{l}^{\frac{1}{3}} c^{\frac{2}{3}}$. Suppose the government is choosing τ to maximize a social welfare function defined over worker utility and profits given by $\omega = 100000 u^{.95} \pi^{.05}$. We numerically compute the optimal choice of τ at two levels of leakage $\kappa_1 = 0$ and $\kappa_2 = 0.3$. As can be seen from Table 1 and Table 2 in the Appendix, the optimal tax rate at $\kappa_1 = 0$ is 0.4 with a corresponding social welfare of 422135 utils. The optimal tax rate at $\kappa_2 = 0.3$ is 0.21 with a corresponding social welfare of 418678 utils. The optimal tax rate and the level of social welfare have fallen with an increase in the administrative cost coefficient.

3 Model with Indirect Taxes

Consider a variation of the previous model in which the government levies an indirect tax on sales instead of a direct tax on corn surplus. For every unit of corn sold, $\tau \in R_+$ units are collected by the government and transferred to the worker.

3.1 The Budget Set of a Worker

We define A^W, the final allocation of a worker as follows: $A_l^W = L - q_l^W$, and $A_c^W \equiv q_k^W = pq_l^W(1+\tau)$. The constraints on the set of actions q^W available to a worker given p are as follows:

In Period 1:

Labor sold to entrepreneur $\leq$ Labor endowment

$$q_l^W \leq L \qquad (1)$$

Corn purchased = Wage Income

$$q_c^W = p(1+\tau)q_l^W \qquad (2)$$

The set of allocations A^W corresponding to actions q^W that satisfy these constraints is denoted by $\Sigma^W(p)$ and is called the budget set of the worker.

3.2 The Budget Set of an Entrepreneur

We define A^E, the final allocation of an entrepreneur as follows: $A_c^E = f(q_l^E) - q_c^E$. The constraints on the set of actions q^E available to an entrepreneur given p are as follows:

In Period 1: Cost of labor demanded = Future corn sold

$$p(1+\tau)q_l^E = q_c^E \qquad (1)$$

In Period 2: Corn sold + Corn paid as tax $\leq$ corn produced

$$q_k^E = q_c^E(1+\tau) \leq f(q_l^E) \qquad (2)$$

The set of A_c^E corresponding to actions q^E that satisfy these constraints is denoted by $\Sigma^E(p)$ and is called the budget set of the entrepreneur.

3.3 Equilibrium

A vector of allocations, prices and policy $(A^W, A^E; p)$ for $\tau \in R_+$ is an equilibrium if:

(1) All workers are optimal on their budget sets, i.e. for workers

$$A^W \in \Sigma^W(p)$$

and

$$\hat{A^W} \in \Sigma^W(p) \Rightarrow u^W(\hat{A}^W) \leq u^W(A^W)$$

For entrepreneurs

$$A^E \in \Sigma^E(p)$$

$$\hat{A^E} \in \Sigma^E(p) \Rightarrow \hat{A}^E \leq A^E$$

(2)All markets clear, i.e. in the labor market

$$q_l^E = q_l^W$$

In the corn market

$$q_c^E = q_c^W$$

3.4 Equations of equilibrium

The first order condition for utility maximization for workers is

$$\frac{MU_l}{MU_c}(A^W) \equiv g = p(1+\tau)$$

The first order condition for profit maximization for entrepreneurs is

$$f^{'}(q_l^E) = p(1+\tau)$$

These two equations along with the budget constraints of the workers and entrepreneurs and the market clearing condition constitute the equations of equilibrium. The tax rate is the exogenous variable chosen by the government.

The proof for existence of equilibrium $\forall \tau \geq 0$ proceeds along similar lines as the proof of Theorem 1. As in the proof of Theorem 2, for economies with $f^{'} + lf^{''} > 0$, the equilibrium is unique.

3.5 Impact of change in tax rate

Theorem 5: *A change in the tax rate has no impact on the level of output, worker welfare, or profit.*

Proof: Consider two economies, one with tax rate τ and the other with tax rate $\tau^{'}$. We prove the result by showing that there exists an equilibrium in the economy with tax rate τ if and only if there exists a corresponding equilibrium in the economy with tax rate $\tau^{'}$ with the same level of output, worker welfare, and profit.

Notice that the equilibrium equations at the tax rate τ with price p are identical to the equilibrium equations at the tax rate $\tau^{'}$ at a price equal to $\frac{p(1+\tau)}{1+\tau}$. Therefore the two equilibria are identical except for the scaling of prices.

QED

3.6 The Indirect Tax Model with Complete Transfer Leakage

We now consider a variation of the model in which the tax collected from the entrepreneurs is not transferred to the intended recipients, the workers. A proportion $\kappa \in [0, 1]$ of tax revenue is lost due to administrative costs and constitutes a leakage from the economy.

This variation implies two changes in the equations of equilibrium. Firstly, the amount of corn consumed by the worker is now given by the equation

$$q_k^W = pq_l^W(1 + (1 - \kappa)\tau)$$

For this reason the workers' utility maximization equation must factor the leakage, i.e. now

$$\frac{MU_l}{MU_c}(A^W) \equiv g = p(1 + (1 - \kappa)\tau)$$

The profit maximization condition of the entrepreneur remains the same, i.e.

$$f^{'}(q_l^E) = p(1 + \tau)$$

The proof for existence and uniqueness of equilibrium proceeds along similar lines as the proof of Theorem 1 and Theorem 2.

In this model the invariance of the economy with respect to taxes is no longer obtained. We first present the results for the case where $\kappa = 1$, i.e. all the corn collected as tax is spent on administration.

Theorem 6: *Consider an economy where workers' utility is characterized by the form $u^W = (l^\rho + c^\rho)^{-\frac{1}{\rho}}$ with $0 < \rho < 1$, and where $\kappa = 1$. In this economy, output falls with an increase in tax rates.*

Proof: From the equations of equilibrium

$$(1 + \tau)g(L - q_l^\star; \frac{f^{'}(q_l^\star)q_l^\star}{1 + \tau}) = f^{'}(q_l^\star)$$

Computing the MRS for the CES utility function we get

$$(1+\tau)(\frac{(L-q_l^\star)(1+\tau)}{f^{'}(q_l^\star)q_l^\star})^{\rho-1} = f^{'}(q_l^\star)$$

$$\Leftrightarrow (1+\tau)^\rho(\frac{L-q_l^\star}{q_l^\star})^{\rho-1} = (f^{'}(q_l^\star))^\rho$$

Denoting $\frac{L-q_l^\star}{q_l^\star}$ by ω and differentiating both sides with respect to τ we get

$$\frac{dq_l^\star}{d\tau}[\frac{(1+\tau)^\rho(\rho-1)L\omega^{\rho-2}}{q_l^{\star 2}} + \rho(f^{'}(q_l^\star))^{\rho-1}f^{''}(q_l^\star)] = \omega^{\rho-1}\rho(1+\tau)^{\rho-1}$$

The term in brackets on the LHS is negative as $0<\rho<1$ and f is concave. The term on the RHS is positive. Therefore $\frac{dq_l^\star}{d\tau}$ must be negative.

QED

Theorem 7: *Consider an economy where workers' utility is characterized by the form* $u^W = (l^\rho + c^\rho)^{-\frac{1}{\rho}}$ *with* $0<\rho<1$, *and where* $\kappa = 1$. *In this economy, profits decrease with an increase in the tax rate.*

Proof: The profit of the entrepreneur is given by

$$\pi = f(q_l^E) - q_c^E$$

Substituting the entrepreneur's budget set we get

$$\pi = f(q_l^E) - pq_l^E(1+\tau)$$

Substituting the entrepreneur's profit maximizing condition we get

$$\pi^* = f(q_l^E) - f'(q_l^E)q_l^E$$

From the concavity of f, the profit increases with output. From Theorem 5, output decreases with an increase in the tax rate. The conclusion follows.

QED

No conclusion can be arrived at about the welfare of the worker from an increase in taxes. We present an example where an increase in taxes results in a decrease in output and a Pareto decline in the economy.

3.6.1 Example 3

Consider an economy characterized by the following parameters: $L = 10; f(l) = 30l - \frac{1}{2}l^2; u^W = (\bar{l}^{0.5} + c^{0.5})^{-2}$. In this economy $l^\star = \frac{\tau+41-\sqrt{481+\tau^2+82\tau}}{2}$. It can be checked that as tax increases, output falls, real wage decreases, the entrepreneur and worker are worse off, and tax revenue decreases.

However in the indirect tax model output need not necessarily go down with an increase in taxes.

3.6.2 Increase in Output with the Tax Rate

In the following economy output increases with an increase in the tax rate.

3.6.3 Example 4

Consider an economy described by the following parameters. $L = 1; u^W = min\{\bar{l}, c\}; f(l) = 10l - \frac{l^2}{2}$. [5]In this economy the equilibrium labor supply is given by

$$q_l^\star = \frac{11 + \tau - \sqrt{\tau^2 + 18\tau + 117}}{2}$$

The derivative with respect to τ

$$\frac{dq_l^\star}{d\tau} = \frac{1}{2} - \frac{\sqrt{\tau^2 + 18\tau + 81}}{\sqrt{\tau^2 + 18\tau + 117}}$$

is greater than zero. The equilibrium labor supplied is less than the endowment at tax rates up to one million! In this economy output and profits increase with the tax rate. From the worker's utility function the optimization implies that at equilibrium

$$L - q_l^\star = q_k^\star$$

Therefore the indirect utility $v^W = min\{L - q_l^\star, q_k^\star\}$ is a declining function of τ.

3.7 The Indirect Tax Model with Partial Transfer Leakage

We now map any economy characterized by $[\tau, \kappa]$ with $\tau \in R_+$ and $\kappa \in [0, 1]$ to an economy with tax rate $\tau^{'}$ and $\kappa^{'} = 1$.

Theorem 8: *The indirect tax economy with tax rate $\tau \in R_+$ and $\kappa \in [0, 1]$ is equivalent in terms of output, worker welfare, and entrepreneur profit to the indirect tax economy with tax rate $\tau^{'} = \frac{k\tau}{1+(1-\kappa)\tau}$ and $\kappa^{'} = 1$.*

Proof: Solving the equations of equilibrium for the economy with tax τ and $\kappa \in [0, 1]$ we get

$$g(L - q_l^W; f^{'}(q_l^W)q_l^W \frac{(1 + (1 - \kappa)\tau)}{1 + \tau}) \frac{1 + \tau}{1 + (1 - \kappa)\tau} = f^{'}(q_l^W)$$

Recall the equilibrium condition for the indirect tax economy with tax $\tau^{'}$ and $\kappa^{'} = 1$ is

[5]Notice that Leontieff utilities do not fall within the space of utility functions chosen in the specification of the model. The analysis of the economy with Leontieff utilities can be used to understand the behaviour of 'near-Leontieff economies', which do fall within the ambit of the specification, and whose outcomes would be close to the outcomes of the Leontieff economy, by continuity of all our functional forms.

$$g(L - q_l^W; \frac{f^{'}(q_l^W)q_l^W}{1+\tau^{'}})(1+\tau^{'}) = f^{'}(q_l^W)$$

Setting $\tau^{'} = \frac{k\tau}{1+(1-\kappa)\tau}$, it is easy to see that the two economies will have the same output and consumption by the worker, and therefore worker welfare.

The price in the economy with tax τ and $\kappa \in [0,1]$ is given by the equation

$$f^{'}(q_l^W) = p(1+\tau) \quad [1]$$

In the economy with tax rate $\tau^{'} = \frac{k\tau}{1+(1-\kappa)\tau}$ and $\kappa^{'} = 1$ the corresponding equation is

$$f^{'}(q_l^W) = p^{'}\frac{1+\tau}{1+(1-\kappa)\tau} \quad [2]$$

The LHS of [1] and [2] are equal as shown through the equilibrium condition of the two economies. It follows that

$$p = \frac{p^{'}}{1+(1-\kappa)\tau}$$

The entrepreneur profit in the economy with tax τ and $\kappa \in [0,1]$ in terms of $p^{'}$ is given by

$$f(q_l^\star) - pq_l^\star(1+\tau) = f(q_l^\star) - p^{'}q_l^\star\frac{1+\tau}{1+(1-\kappa)\tau}$$

This is also the entrepreneur profit in the economy with tax rate $\tau^{'} = \frac{k\tau}{1+(1-\kappa)\tau}$ and $\kappa^{'} = 1$.

QED

Corollary 1: *Consider an economy where workers' utility is characterized by the form* $u^W = (l^\rho + c^\rho)^{-\frac{1}{\rho}}$ *with* $0 < \rho < 1$. *At a given* $\tau > 0$, *as we increase the cost coefficient* κ, *the output and profits decrease.*

Proof: Notice that for any τ, κ the corresponding effective rate $\tau^{'}$ in the economy with $\kappa^{'} = 1$ is an increasing function of κ.

$$\frac{\partial \tau^{'}}{\partial \kappa} = \frac{\tau + \tau^2}{1+(1-\kappa)\tau} > 0$$

Therefore as we increase κ the resulting equilibrium is the same as the equilibrium in the economy with $\kappa^{'} = 1$ and a tax rate $\tau^{'} > \tau$. The result follows from Theorem 5 and Theorem 6.

QED

Corollary 2: *Consider an economy where workers' utility is characterized by the form* $u^W = (l^\rho + c^\rho)^{-\frac{1}{\rho}}$ *with* $0 < \rho < 1$. *At a given* $\kappa > 0$, *as we decrease the tax rate* τ, *the output and profits increase.*

Proof: Notice that for any τ, κ the corresponding effective rate $\tau^{'}$ in the economy with $\kappa^{'} = 1$ is an increasing function of τ.

$$\frac{\partial \tau^{'}}{\partial \tau} = \frac{\kappa \tau}{1 + (1-\kappa)\tau} > 0$$

As we decrease τ the resulting equilibrium is the same as the equilibrium in the economy with $\kappa^{'} = 1$ and a tax rate $\tau^{'} < \tau$. The result follows from Theorem 5 and Theorem 6.

QED

Corollary 3: *Consider the economy of Example 4.* $L = 1; u^W = min\{\bar{l}, c\}; f(l) = 10l - \frac{l^2}{2}$. *At a given* $\tau \in [0, 1000000]$[6]*, as we increase the cost coefficient* κ*, the output and profits increase.*

Proof: As shown in Corollary 1, when we increase κ the resulting equilibrium is the same as the equilibrium in the economy with $\kappa = 1$ and a tax rate $\tau^{'} > \tau$. The result follows from the conclusion of Example 4.

QED

Corollary 4: *Consider the economy of Example 4.* $L = 1; u^W = min\{\bar{l}, c\}; f(l) = 10l - \frac{l^2}{2}$. *At a given* $\kappa \in (0, 1)$*, as we increase the tax rate in the range* $[0, 1000000]$*, the output and profits increase.*

Proof: As shown in Corollary 1, as we increase κ the resulting equilibrium is the same as the equilibrium in the economy with $\kappa = 1$ and a tax rate $\tau^{'} > \tau$. The result follows from the conclusion of Example 4.

QED

The equivalence classes of τ and κ that yield the same equilibrium outcomes are indicated by Figure 2a and 2b.

[6] The bound on τ is set to ensure nonnegative values of the variables at equilibrium.

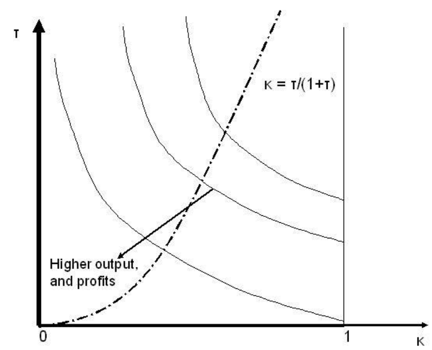

Figure 2: $\tau - \kappa$ Equivalence curves with indirect taxes for strong substitutes

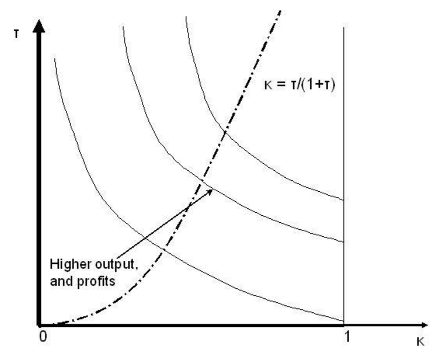

Figure 3: $\tau - \kappa$ Equivalence curves with indirect taxes for strong complements

From the equivalence map it is clear that any economy characterized by a rate of tax τ and cost coefficient κ can be mapped on to an equivalent economy with

$$\tau^{'} = \frac{k\tau}{1 + (1-\kappa)\tau}; \kappa^{'} = 1$$

Notice that the indifference curve for a given $\tau^{'}$ with $\kappa^{'} = 1$, is a hyperbola with an asymptote at $\frac{\tau^{'}}{1+\tau^{'}}$.

3.8 Increase of Social Welfare With an Increase in the Administrative Cost Coefficient

Let $W : R_+^2 \longrightarrow R_+$ be a continuous social welfare function defined on worker utility and entrepreneur profit. Let us assume there is an upper bound on the tax rate given by $\overline{\tau}$.

Given the differentiability of all functions, W is a continuous function of $\tau \in [0, \overline{\tau}]$ at any given leakage $\kappa \in [0, 1]$. Therefore by the maximum principle, there exists a maximum social welfare at any κ.

Suppose we increase the leakage coefficient to $\kappa' > \kappa$. From Theorem 8 and the equivalence map it is clear that all combinations of utility and profit possible at κ are also possible at κ'. However there are some combinations of utility and profit possible at κ' that are not possible at κ as the tax rate that would establish equivalence lies outside the domain. Therefore social welfare cannot decrease with an increase in the leakage rate.

From example 4, it is clear that an increase in the tax rate at any given leakage could result in an increase in profit and an increase in welfare for a suitably defined social welfare function. Therefore it is possible that social welfare increases with an increase in leakage.

If social welfare remains constant with an increase in κ, then there exists an optimal tax lower than the optimal tax chosen at the lower cost coefficient which will achieve the maximum.

4 Concluding Remarks

The paper highlights some counterintuitive results of the administrative costs of taxation. In a direct tax regime, for any given level of tax an increase in the cost coefficient always leads to an increase of output and profits as workers, anticipating the loss of transfers are willing to work more. However this results in a reduction of real wages and worker welfare. The social welfare defined over workers' utility and profits declines as the cost coefficient increases. A government which chooses tax rates to make an optimal trade-off between worker welfare and profits may choose to reduce taxes in the event of a greater cost coefficient. Finding conditions under which this response is always appropriate is an open question.

In an indirect tax regime on the other hand the outcome of an increase in the administrative cost coefficient depends on the degree of substitutability of leisure and output in the utility function of the worker. If leisure and output are strong substitutes, then an increase of the cost coefficient reduces output and profits. If they are strong complements then an increase of the cost coefficient can increase output and profits. In either event, the maximal social welfare defined over workers' utility and profits cannot decline as the cost coefficient increases, and may be strictly greater in some cases. If an increase in the cost coefficient does not result in a decline of the maximal social welfare then the optimal tax rate declines unless there are multiple optima and the government shifts to a different optimal tax trajectory.

5 References

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6 Appendix

Table 1: Outcomes with No Administrative Cost

Tax Rate	Labor	Corn Consumption	Utility	Profit	Welfare
0.00	6.67	155.56	43.21	22.22	417938
0.10	6.64	157.23	43.65	19.81	419622
0.20	6.6	158.88	44.09	17.45	420933
0.30	6.57	160.49	44.52	15.13	421804
0.36	6.56	161.45	44.77	13.76	422076
0.37	6.55	161.61	44.81	13.53	422100
0.38	6.55	161.76	44.86	13.30	422118
0.39	6.55	161.92	44.90	13.08	422130
0.40	6.54	162.08	44.94	12.85	422135
0.41	6.54	162.24	44.98	12.63	422133
0.42	6.54	162.39	45.02	12.40	422124
0.43	6.54	162.55	45.07	12.18	422108
0.44	6.53	162.71	45.11	11.95	422084
0.45	6.53	162.86	45.15	11.73	422053
0.50	6.52	163.64	45.35	10.61	421771
0.60	6.49	165.17	45.76	8.42	420453
0.70	6.46	166.67	46.16	6.26	417698
0.80	6.43	168.15	46.55	4.14	412439
0.90	6.41	169.60	46.93	2.05	401351
1.00	6.38	171.04	47.31	0.00	0

Table 2: Outcomes with Thirty Percent Administrative Cost Coefficient

Tax Rate	Labor	Corn Consumption	Utility	Profit	Welfare
0.00	6.67	155.56	43.21	22.22	417938
0.10	6.64	156.73	43.52	19.87	418466
0.16	6.63	157.43	43.71	18.47	418635
0.17	6.63	157.55	43.74	18.24	418651
0.18	6.63	157.66	43.77	18.01	418663
0.19	6.63	157.78	43.80	17.78	418672
0.20	6.62	157.89	43.83	17.55	418677
0.21	6.62	158.01	43.86	17.31	418678
0.22	6.62	158.12	43.89	17.08	418676
0.23	6.62	158.24	43.92	16.85	418669
0.24	6.61	158.35	43.95	16.62	418658
0.30	6.60	159.04	44.13	15.25	418502
0.40	6.58	160.17	44.43	12.99	417843
0.50	6.56	161.29	44.73	10.76	416545
0.60	6.54	162.39	45.02	8.55	414354
0.70	6.52	163.48	45.31	6.37	410800
0.80	6.50	164.56	45.60	4.22	404840
0.90	6.48	165.62	45.88	2.10	393225
1.00	6.46	166.67	46.16	0.00	0